ESSAYS IN THE POSITIVE THEORY OF POLICY CHOICE

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Abstract

This manuscript consists of three essays in the positive theory of policy choice. The first essay, titled *Equilibrium False Consciousness*, focuses on policy choice through majoritarian voting in a model of class conflict and social mobility. It studies a new model of social mobility with two types of voters: high income voters and low income voters. All voters are fully rational and care only about their economic payoff. However, the main result of the chapter is the existence of an equilibrium (for large electorates) where some low income voters cast their ballot for the right wing policy despite knowing that the left wing policy gives them a higher expected payoff. The chapter provides a new explanation for why rational low income voters may oppose redistribution on the basis of their preferences and expectations regarding the prospect of upward mobility.

The second essay, titled *Incomplete Policymaking: Making Healthcare Policy 2009-2010*, explains the emergence of incomplete policies as the outcome of a dynamic logrolling problem between policymakers. The idea that incomplete policies may emerge as a partial solution to the dynamic logrolling problem is new to the formal literature on policy-making. The theory is developed through a narrative of the healthcare policy negotiations that took place in the U.S. Congress between 2009 and 2010.

The third and final essay, titled *Coordination and Development in Dictatorships*, develops a theory of inefficient policy choice by authoritarian regimes, and highlights the tradeoff between the benefits of economic coordination for rulers vis-a-vis the costs of political coordination. One important implication of the theory is the result that part of the impetus
for democratization may emerge as a consequence of the efficiency gains associated with eliminating some of the intensity of political conflict. This is in sharp contrast to most of the previous formal literature, which emphasizes social conflict at the expense of modernization in explaining the emergence of democracy.
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Chapter 1

Introduction

This manuscript consists of three essays in the positive theory of policy choice. The first essay, titled *Equilibrium False Consciousness*, focuses on policy choice through majoritarian voting in a model of class conflict and social mobility. The second essay, titled *Incomplete Policymaking: Making Healthcare Policy 2009-2010*, explains the emergence of incomplete policies as the outcome of a dynamic logrolling problem between policymakers. The theory is developed through a narrative of the healthcare policy negotiations that took place in the United States Congress between 2009 and 2010. Finally, the third essay, titled *Coordination and Development in Dictatorships*, develops a theory of inefficient policy choice by authoritarian regimes, and highlights the tradeoff between the benefits of economic coordination for rulers vis-a-vis the costs of political coordination. The essays are described in greater detail as follows.

Chapter 2: Equilibrium False Consciousness

This chapter studies a simple model of class conflict in the context of social mobility. The model features two types of voters, high income voters and low income voters. All voters are fully rational and care only about their economic payoff. However, the main result of the model is the existence of an equilibrium (for large electorates) where some low income voters cast their vote for the right wing policy despite knowing that the left wing policy gives them a higher expected payoff.
The model has two periods. In the first period, the right wing policy is implemented. Under this policy, each low income voter has the chance of receiving an opportunity to permanently become a high income earner. If he is talented, then he is able to seize the opportunity and climb the economic ladder. But there is also a chance that he will discover that he is untalented, and destined to remain a low income earner. At the end of the first period, an election is held to decide whether to continue with the incumbent right wing policy, or to switch to the left wing policy in the next period. Low income voters who successfully climbed the economic ladder all vote to keep the right wing policy, while those who discovered that they are untalented vote for the left wing policy. Low income voters who did not get an opportunity to be rich in the first period may end up voting for the right wing policy despite knowing that their expected payoff is higher under the left wing policy. This is because they condition their vote on the event that it matters: if the election is close, then it is likely that the right wing policy provides the economic opportunity with high probability. Thus, these voters behave as if they have a false consciousness about their “prospect of upward mobility” (POUM). A central result of the model is the existence of sequences of equilibria (indexed by the size of the electorate) along which it becomes certain that the right wing policy wins re-election. Thus, the model exhibits aggregation failure, i.e. the failure of the election to aggregate the preferences of the majority of voters.

Chapter 3: Incomplete Policymaking

This chapter applies a model that I developed jointly with Juan Ortner (see Acharya and Ortner 2010) to study the recent healthcare reform negotiations that took place in the United States Congress between 2009 and 2010. The chapter first argues that the healthcare reform laws that were ultimately implemented constitute an incomplete policy when evaluated on two dimensions that liberal and moderate Democrats alike considered important. One dimension, considered relatively more important by liberal Democrats, is the health insurance coverage rate. The other dimension, considered relatively more important by more moderate
Democrats (such as members of the Blue Dog coalition in the House, or the more fiscally conservative members of the Senate), is the rapidly rising cost of medical care. The chapter asks: Why were these two factions of Democratic policymakers unable to arrive at a policy outcome that lies on the *coverage-savings frontier*? The explanation that it advances highlights the importance of commitment problems in dynamic logrolling. The model is closely related to the bilateral bargaining model of Acharya and Ortner (2010), with some important differences. While Acharya and Ortner (2010) study a model with an infinite horizon, this chapter studies a bargaining game with a finite horizon. In addition, bargaining takes place over a policy-space with an endogenous status quo, whereas in Acharya and Ortner (2010), the agents bargain over a surplus.

**Chapter 4: Coordination and Development in Dictatorships**

Why do so many authoritarian regimes choose inefficient policies? This chapter argues that in some circumstances, implementing the efficient policy also reduces the political power of the ruling elite by improving the ability of the disenfranchised citizens to coordinate a revolutionary uprising. Unlike existing work (e.g. Acemoglu and Robinson 2006), it explicitly models the coordination problem, generating several new results and hypotheses that have not yet been empirically tested.

The basic idea advanced in this chapter is the following. Facing the threat of overthrow, the regime may have to make large concessions to the citizens, and in expectation may be better off from preserving the inefficient status quo. This implies that a *political hold-up problem* is in effect: the regime does not implement a policy that increases productivity because the policy also empowers its political adversaries, namely the disenfranchised citizens. This is analogous to the *hold-up problem* in bargaining theory, where one party does not take an action that would increase total surplus because that action would also improve the bargaining power of the counter-party.

One important implication of the model is the result that part of the impetus for democratization can be generated by the gains in efficiency that arise due to eliminating some of
the intensity of political conflict. This is in sharp contrast to models such as Acemoglu and Robinson (2000b) that emphasize social conflict at the expense of modernization in explaining the rise of democracy. (See Boix, 2011, and references therein, on the debate over the validity of the modernization hypothesis.)
Chapter 2

Equilibrium False Consciousness

2.1 Introduction

Political scientists have long recognized the importance of voters’ beliefs about the attainability of future prosperity—e.g. the “American Dream”—in shaping their political opinions and voting patterns (e.g., Hochschild 1981; Bartels 2008a). Nevertheless, the bulk of research on social mobility in the United States has been conducted by sociologists and economists.¹ For example, the sociologist Thomas DiPrete (2007) argues that many young Americans overestimate their chances of becoming rich, and claims that the degree of overestimation is considerably large whether one uses subjective or objective definitions of the word “rich.” Thus, in Marxist parlance, DiPrete argues that many American voters appear to have a false consciousness about their prospect of upward mobility (POUM).²

¹It should be noted, however, that most of these literatures are largely concerned with measuring social mobility or providing theories to explain upward and downward mobility, rather than understanding individuals’ views on mobility and how these views inform political behavior. See, for example, Morgan, Grusky and Fields (2011), Kerbo (2011), Bowles, Gintis and Osborne Groves (2005), Gottschalk and Spolaore (2002), Piketty (2000), Fields and Ok (1996), Jencks (1979) Lipset and Bendix (1991) and Becker and Tomes (1979) as well as findings of the Panel Study on Income Dynamics (PSID).

²The term false consciousness, goes back to Engels (1893) in his “Letter to Franz Mehring,” and is used by Marxist scholars to describe how capitalist processes (i) mislead the proletariat into having false beliefs, and (ii) instill them with values that conflict with their material interest. However, the most widespread modern use of the term refers to the unjustifiably optimistic beliefs that low and middle income citizens have about their POUM (see, e.g., Benabou and Ok 2001).
There is no doubt that such overly optimistic beliefs can have important consequences for public policy via their effect on public opinion and voting behavior. Perhaps the most important of these consequences concerns voters’ preferences for redistribution and their tolerance for inequality. In fact, almost four decades ago Albert Hirschman (1973) drew the connection between beliefs about social mobility and tolerance for income inequality by explaining what he called the *tunnel effect*. Hirschman described how a low income individual could make inferences about his own mobility prospects by observing the experiences of his neighbors, relatives, and friends. If he sees some of these people climbing the economic ladder, he forms optimistic beliefs about his own chances, which in turn leads him to tolerate current inequality. By extending Hirschman’s theory to its logical end, one can argue that such inferences may cause low income voters to oppose redistribution if such policies have some *temporal persistence* (in the sense that once instituted, these policies are difficult to reverse). Indeed, these voters may prefer the policy that in reality is more likely to make them worse off.

In light of this possibility, the question that I address in this chapter is the following. Suppose there are two policies, a more redistributive left wing policy, and a less redistributive right wing policy; and, suppose that the right wing policy is currently in effect. Consider a low income individual that overestimates his chances of upward mobility under the right wing policy, and on this basis votes to re-elect it. If instead this individual—and all other individuals—that had correct beliefs and voted “rationally,” would his voting behavior be necessarily different? Given the centrality of the debate in the American political behavior literature over the extent to which voters are rational (and how voter rationality affects public policy), the answer to this question has clear substantive importance.\(^3\) However, before

explaining my answer to the above question, a brief discussion about the meaning of the word “rational,” and the distinction between *strategic* and *naive* voting, will be useful.

In the case where the individual described above has overly optimistic beliefs about his chances of upward mobility, and on that basis votes for the right wing policy, the individual performs an expected utility calculation (using his mistakenly optimistic probabilities) and votes for the policy that gives him the higher utility. Comparing the expected utilities associated with the two policies, given one’s beliefs, and then voting for the policy that yields the higher utility is called *naive* voting (or *sincere* voting). Under the standard definition of “rationality,” having mistakenly optimistic beliefs can in fact be rational.4 Nevertheless, most political commentators and rational choice theorists construe rationality as requiring that the individual’s original (prior) beliefs are not mistaken, which in this case implies that voters correctly assess their chances of upward mobility.5 Moreover, it appears that most political commentators implicitly equate rationality in voting with having correct assessments of the likelihood of each outcome associated with the various policies and voting naively (Shenkman 2008; Bartels 2008b). However, this is not the definition of voter rationality that rational choice theorists have come to use. On the contrary, rational choice theorists refer to voting behavior as rational only if it is part of some *equilibrium* to the voting game in which all voters have correct assessments of the probability of each outcome associated with the various policies. Such voting behavior is called *strategic* voting, and it is a well-known fact that strategic voting is generally not equivalent to naive voting (Austen-Smith and Banks 1996). Thus, there is a conflict in the way political commentators use the word “rational,”

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4In fact, the term *rationality* in economic theory refers to preferences satisfying the expected utility axioms of Savage (1954), which may be the case even for the mistakenly optimistic voter. In Savage’s framework, a decision maker might still be rational even if he believes that probability of getting heads as the outcome of a fair coin toss is more (or less) than 50%. The reason is because in Savage’s world, probabilities are *subjective*, i.e. *Bayesian*, as opposed to *frequentist*. For an abridged discussion of this distinction, see Morris (1995, p.231-234).

5Often, the justification for this much stronger notion of rationality is to add the assumptions of de Finetti’s (1937) exchangeability theorem to the Savage axioms, though it is difficult to interpret de Finetti’s assumptions in the context of one-shot individual decision making.
and the way rational choice theorists use it. In this chapter, I opt to use the customary
definition of the word “rational,” i.e. the one used in rational choice theory.

My main finding is that rational voting behavior for a large number of low income voters
may not be necessarily different from naive voting behavior with mistakenly optimistic beliefs
about the prospect of upward mobility. In fact, the first main result of this chapter is a
behavioral equivalence result: there exists an equilibrium in which several low income voters,
who are rational and correctly assess their chances of upward mobility, cast their ballots
exactly as if they were naive voters with mistakenly optimistic beliefs. In other words, if
one had to draw inferences only from data on the equilibrium behavior of the strategic low
income voters in my model, one could not rule out the possibility that these are actually
individuals who overestimate their chance of climbing the economic ladder and vote naively.
Thus, my model exhibits what one might call “equilibrium false consciousness” about the
prospect of upward mobility.

The following is an informal description of the model. There are two periods in time, and
two types of voters: high income voters and low income voters. In the first period, a right
wing policy is in effect, and an election is held to decide whether to continue with it in the
following period, or to switch to a left wing policy. Under the right wing policy, each low
income voter has probability $\delta$ of receiving an opportunity to permanently become a high
income earner. Each voter that receives such an opportunity is able to take advantage of it,
and successfully become a high income earner, with probability $p$. Those who cannot take
advantage of the opportunity once are destined to fail again in the future, should another
such opportunity come their way. Thus, $p$ can be interpreted as representing the uncertainty
that each voter has about his talent: voters do not know whether or not they are talented
enough to climb the economic ladder unless the receive an opportunity to do so. But all they
need is one opportunity to discover their real abilities. While all voters know the success
rate $p$, they are uncertain about $\delta$ is. This uncertainty can be taken to reflect uncertainty
about whether the right wing policy “works well.” If $\delta$ is high, then the right wing policy works well, whereas if $\delta$ is low then it does not. Each voter experiences the consequences of the first period policy only for himself; thus, each voter is uninformed about who (and how many others) received the opportunity to climb the economic ladder, and whether or not they succeeded.

The left wing policy, on the other hand, might have different implications for social mobility and/or economic growth, but is more redistributive than the right wing policy.

I then study the equilibria of the model using two refinements that are natural for voting models such as the present one. First, I restrict attention to symmetric equilibria. This means that voters who are identical in terms of their income, life experience, and the information that they possess, all vote identically. Second, I only study equilibria in weakly undominated strategies. This enables me to rule out equilibria in which the behavior of the electorate as a whole is such that no voter’s individual vote ever matters; therefore, disillusioned by how powerless he is, every voter is (in equilibrium) indifferent between voting for either policy.

Despite these refinements, the model has multiple equilibria. However, not surprisingly, in all of these equilibria, high income voters all vote to keep the right wing policy while low income voters who learned that they are not talented all vote for the left wing policy. This implies that the equilibria differ in how low income voters who did not receive the opportunity to climb the economic ladder in the first period vote. These voters have a more complicated voting decision. I refer to them as $L^0$ voters. If these voters believe that $\delta$ is high, then they believe that the right wing policy works well, and they would like to continue with it in the next period. However, in reality, the right wing policy is very likely to not work well—in other words, the expected value of $\delta$ is low—and the $L^0$ voters are aware of this. Thus, they do not hold mistakenly optimistic beliefs. Rather, they hold the correct beliefs, and they actually prefer to switch to the left wing policy. Yet, I show that at the time of voting, there is an equilibrium in which the $L^0$ voters cast their vote to re-elect the
right wing policy. If they do so, and the size of the electorate is large, then as a result of their behavior, the right wing policy is almost certain to win re-election. This is true even when the vast majority of voters prefer the left wing policy. Thus, the model exhibits election failure.6

In light of this result (and the simplicity of the model used to derive it), one might question what such an abstract model can tell us about the real-life behavior of voters. After all, the real world is much more complicated than the model. (For example, one might point to the fact that in the real world, there is a wider range of income types rather than just two; there more policies—and, in particular, more complicated policies—to choose between; there is great heterogeneity in the rate at which voters receive economic opportunities, and this may depend on their race, gender or social status; and, the real world, voters receive information from various sources while policy-makers receive influence from various lobby groups.) Although the model that I present abstracts from many important factors, it highlights the important role that social mobility—and beliefs about the connection between policy and mobility—plays in shaping political behavior. If one agrees that the world described above is a reasonable and interesting abstraction of the real world, one might be interested in the question: “How does a rational voter behave in such a world?” Then, whether or not one accepts that the model has descriptive value, there is one important implication of it that cannot be dismissed. At the very least, it suggests that for democracy to succeed, we might have to place some peculiar demands on voters: that they not be overoptimistic and naive, but that they also not be fully rational.

6This is in contrast to Feddersen and Pesendorfer (1997), who find that in the most natural specification of their model, elections succeed in aggregating information even when nearly all voters behave strategically and vote against their interest.
2.1.1 Related Literature

My model is related to other studies of how social mobility plays a role in shaping political ideology and voting behavior. One of the most influential contributions on this topic has been Piketty’s (1995) model of dynastic learning. Piketty describes the quest of a rational individual, in a world of uncertainty, as he gropes for information on which to establish his ideology. Piketty’s individuals use their own (and their ancestor’s) social mobility experiences to make inferences about the relative importance of luck and effort in determining economic success. Piketty finds that some dynasties converge to the right wing belief that luck is relatively unimportant, and come to oppose redistribution, while others converge to the left wing belief that luck is also important, and come to support redistribution. Alesina and La Ferrara (2005) provide some empirical evidence to support Piketty’s conclusion that experiences of inter-generational upward mobility reduce the support for redistribution.

Piketty’s (1995) rational agent is not purely self-interested; instead, his preferences also contain an ideological component. Perhaps more in line with the agenda of this chapter, Benabou and Ok (2001) address the question of whether purely self-interested low income voters with rational expectations about their POUM can vote against redistribution. They show that if the income transition function between periods is strictly concave, then such behavior is possible. However, Benabou and Ok’s story is considerably different from the one in this chapter. While in my model there is a tension between a voter’s true (naive) preferences and his equilibrium behavior, no such tension exists in the Benabou-Ok model. Thus, the Benabou-Ok model does not exhibit election failure. Another key difference is that in the Benabou-Ok model, agents have the potential for downward mobility, whereas this is not the case in my model. Benabou and Ok take their model to the data, and find that as a result of the potential for downward mobility, the “POUM effect is probably dominated by the demand for social insurance.” Thus, their findings on the POUM are consistent with those of Putterman (1997), who tests various theories for why low income voters might
oppose redistributive policies.\footnote{Among other rational choice theories of anti-redistribution are Roemer (1998) and Roemer and Lee (2004), who argue in favor of the multi-dimensionality of political competition, in particular the ideological role of voter racism in diminishing the equilibrium size of government. Roemer and Lee (2004) show that their model fits the data extremely well.}

My model is also related to the literature on strategic voting in large elections, particularly models of election failure. For example, Feddersen and Pesendorfer (1997) show that in elections with asymmetric information, the fraction of voters whose vote depends on their private information converges to zero as the size of the electorate converges to infinity. However, in their baseline model, the outcome of the election is the same as it would be if all private information were common knowledge. Thus, Feddersen and Pesendorfer provide an argument in favor of democracy that does not require the assumption that voters vote naively, as in Condorcet’s Jury Theorem (Miller 1986). However, Feddersen and Pesendorfer also show that if the distribution of private information is uncertain, then the model has election failure. Gul and Pesendorfer (2009) also provide a model of election failure with uninformed strategic voters and strategic policy choice for one of the two candidates. In their model, voters have small personality preferences for one of two candidates. Nevertheless, in equilibrium, voters cast their votes entirely on the basis of their personality preferences. Moreover, if the strategic candidate enjoys greater bias in his favor, then he is able to win on a platform that is favored only by a minority of voters. Thus, the model also has policy divergence.

\section{2.2 The Political Environment}

There are two periods and two policies, a right-wing policy and a left wing policy. In the first period, the right wing policy is implemented and an election is held to decide whether to continue with it in the next period, or to switch to the left wing policy. There are $n + 1$ low income voters and $\lambda(n + 1)$ high income voters, and all voters must vote for one of the two
policies. Assumption 1 below states that low income voters form the majority. Assumption 2 states that $\lambda$ and $n$ are chosen so that total population is odd.

**Assumption 1.** $0 < \lambda < 1$.

**Assumption 2.** $\mathcal{N} \equiv \{ \tilde{n} \in \mathbb{N} \mid (1 + \lambda)(\tilde{n} + 1) \text{ is an odd number} \} \neq \emptyset$ and $n \in \mathcal{N}$.

Assumption 2 can always be satisfied when $\lambda$ is the ratio of even and odd integers, and it implies that the election cannot end in a tie. Also, note that the assumption does not restrict us to studying only small electorates since $\mathcal{N} \neq \emptyset$ implies that $\mathcal{N}$ is unbounded.

Under the left wing policy, all high income voters receive a payoff $y^h$ while all low income voters receive a payoff $y^l$. Under the right wing policy, each high income has payoff $y^h$ while each low income voter has probability $\delta$ of receiving an opportunity to become a high income voter. If a voter receives an opportunity to climb the economic ladder, and he is talented, then he receives a payoff $y^h$. If he is not talented, or if he does not receive such an opportunity, then his payoff is $y^l$. Each low income voter has prior probability $p$ of being talented. If a voter becomes a high income earner in the first period, he remains a high income earner in the second period. Voters who receive an opportunity to become high income earners in the first period and are unsuccessful learn that they would also be unsuccessful in the following period. Voters who do not receive this opportunity in the first period learn nothing new about their talents. The next assumption states that each voter is more likely than not to be talented, but he is not certain to climb the economic ladder if given the opportunity.

**Assumption 3.** $1/2 < p < 1$.

Voters do not directly observe $\delta$, nor does any voter observe the consequences of the first period policy for any other voter. Instead, voters believe that $\delta$ is a random variable with distribution $F$. If the right wing policy is re-elected, then the value of $\delta$ in the second period is the same as in the first. Let $\bar{\delta}$ denote the expected value of $\delta$ according to $F$. Conditional
on $\delta$, the expected payoff from the right wing policy for a low income voter is given by

$$y(\delta) = (1 - \delta p)y^l + \delta py^h.$$ 

Therefore, the unconditional expected payoff to re-electing the right wing policy for a low income voter who did not receive the economic opportunity is $y(\delta)$. His expected payoff to electing the left wing policy is simply $y^l$. I now make an assumption about voter preferences.

**Assumption 4.** (i) $y^l < y^l < y^h$ and (ii) $y(\delta) < y^l < y(1)$

Assumption 4(i) states that voters who remain low income types in the first period receive a higher payoff under the left wing policy than under the right wing policy. On the other hand, high income voters receive a lower payoff under the left wing policy, but their payoff is always at least as large as the payoff of low income voters. Thus, low income voters who learn that they are not talented prefer the left wing policy while high income voters prefer the right wing policy. Assumption 4(ii) states that a low income voter who did not receive the economic opportunity in the first period prefers the left wing policy because the right wing policy does not provide the economic opportunity with high enough probability. However, if he expected to receive the opportunity with high enough probability, then he would actually prefer the right wing policy. Since $y(\delta)$ is strictly increasing in $\delta$, there is a cutoff $\delta^*$ such that a low income voter who has not yet discovered his talents would prefer the right wing policy if the expected value of $\delta$ was above $\delta^*$, and would prefer the left wing policy if it was below $\delta^*$.

Finally, I make a technical assumption about the distribution of $\delta$. The assumption guarantees that $F$ has full support on $[0, 1]$, and the relative likelihood of any two states $\delta$ and $\delta'$ is bounded above and below.

**Assumption 5.** $F$ has density $f$ and

$$\exists \nu > 0 \text{ s.t. } 1/\nu > f(\delta) > \nu \forall \delta \in [0, 1].$$
I now provide some concreteness to the above assumptions by way of a numerical example. The example also provides some additional clarification as to what is and is not being implicitly postulated in Assumptions 1 - 5. It shows that the assumptions are perfectly consistent with the idea that the left wing policy yields a higher expected average payoff than the right wing policy. It also shows that the assumptions are consistent with the opposite claim that the right wing policy yields a higher expected average payoff. It then illustrates that the assumptions are consistent with the idea that both the left and right wing policies provide low income voters with the opportunity for social mobility, but that these opportunities are not independent of policy. Readers who are already persuaded may skip the example and proceed to the next section.

**Example.** Suppose that the payoffs represent income, and that the left wing policy is one that taxes at 100% and redistributes all revenue as untargeted transfers. More concretely, assume

\[ \lambda = \frac{1}{20}, \ y^l = \$45,000, \ y^h = \$200,000 \] \text{ and } \[\bar{y}^l = \bar{y}^h = \frac{\lambda}{1+\lambda} y^h + (1 - \frac{\lambda}{1+\lambda}) y^l \approx \$52,400.\]

Thus, high income voters earn $200,000 and form slightly less than 5% of the population, while low income voters earn $45,000. These numbers roughly reflect features of the U.S. income distribution in 2003 according to the U.S. Census Bureau’s income data reported in 2005. Median household income was approximately $45,000 while the 97.5th percentile income was within the $200,000 ball-park.

Next, assume that in expectation, the right wing policy has the effect of giving each low income earner the opportunity to become a high income earner with a 1% chance, and each low income earner is able to take advantage of this opportunity with a \(2/3\) probability; i.e.,

\[ \bar{\delta} = 1/100 \text{ and } p = 2/3. \]
These numbers are consistent with Assumptions 1 – 5, and in particular with Assumption 4, as shown by the following:

\[ y^l = \$45,000 < y(\bar{\delta}) \approx \$46,000 < \bar{y}^l = y^h \approx \$52,400 \]
\[ < y(1) = \$148,000 < y^h = \$200,000 \]

As mentioned before, Assumption 4 implies that there is a cutoff \( \delta^* \) such that low income voters who have never received the opportunity to raise their income prefer the right wing policy if \( \bar{\delta} > \delta^* \) and prefer the left wing policy if \( \bar{\delta} < \delta^* \). In this example, the critical threshold is \( \delta^* \approx 0.07 \), i.e. low income voters would prefer the right wing policy only if they expected to have at least about a 7% chance of receiving the opportunity to be rich under that policy.

Under the parameter values above, the expected average income under the right wing policy is \$53,700, which is about 2.5% higher than it is under the left wing policy. However, one could change the example by assuming that the left wing policy produces more growth than I have assumed, e.g. one could set \( \bar{y}^l = y^h = \$54,500 \), leaving all other parameters the same, and Assumptions 1 – 5 would continue to be satisfied. This alternative assumption would be satisfied if, for instance, the left wing policy gave high income voters \$225,000 and low income voters \$46,000, and then redistributed all income to the mean. Alternatively, it would also be the case if the left wing policy gave each low income voter a 1.5% chance, in expectation, of becoming a high income voter; gave high income voters \$204,300 and low income voters \$45,500; and then redistributed all income to the mean. In either case, the expected average income under the right wing policy is about 2% lower than under the left wing policy.

The example shows that the assumptions do not implicitly take a position on whether the right wing policy produces more or less economic growth than the left wing policy, nor
are they inconsistent with the idea that both the right wing policy and the left wing policy provide low income voters with the opportunity for social mobility. However, they do rule out the case where the rate at which low income voters receive economic opportunities is independent of policy. For example, if social mobility has nothing to do with policy, and the left and right wing policies differ only in their distributive consequences, then Assumption 4 would always be violated if low income voters are weakly risk averse. This is because their expected income under the two policies would be the same, but the right wing policy is riskier than the left wing policy.

2.3 Voting Behavior

Let \( \theta = (y^l, y^h, y'^l, y'^h, \lambda, F, p) \) denote the tuple of all parameters of the model other than \( n \), and denote the voting game by \( G(n, \theta) \). All voters are rational, so each voter conditions his vote on the event that he is pivotal (i.e. the event that his vote will decide the election). A strategy for a voter is the probability with which he votes to re-elect the incumbent policy. The equilibrium concept throughout this chapter is symmetric equilibrium in weakly undominated strategies. In models such as the present one, it is customary to refer to such an equilibrium as a voting equilibrium.

In the remainder of this section, I establish the existence of voting equilibrium of the game \( G(n, \theta) \) when \( (n, \theta) \) satisfies Assumptions 1 – 5. I also report some basic properties of all voting equilibria. I then establish the existence of pure strategy voting equilibrium in large electorates. Finally, I study the aggregation properties of voting equilibrium for the case of large electorates.

2.3.1 Existence of Equilibrium

Call the set of voters that started off as high income earners \( H \). These voters have a weakly dominant strategy to vote for the right wing policy. This implies that the policy is re-elected
if and only if more than

\[ n \times q_n \equiv \frac{(1 - \lambda)(n + 1) - 1}{2} \]

low income voters vote for it. Equation (2.1) implicitly defines the quantity \( q_n \). Observe that \( \{q_n\}_{n \in \mathbb{N}} \) is a strictly increasing sequence that converges to

\[ q_\infty \equiv \frac{1 - \lambda}{2}. \]  

(2.2)

Next, partition the set of low income voters into three categories, \( L^+ \), \( L^- \) and \( L^0 \). Voters in category \( L^+ \) are those who became high income earners in the first period. These voters remain high income earners in the second period, so like voters in \( H \), they too have a weakly dominant strategy to vote for the right wing policy. On the other hand, voters in \( L^- \) are low income earners who learnt that they are not talented. These voters have a weakly dominant strategy to vote for the left wing policy. Finally, voters in \( L^0 \) are those who did not get the opportunity to climb the economic ladder in the first period. These voters would be interested in knowing what the value of \( \delta \) is, but they are unable to observe it. I use \( x^i \) to denote the probability with which voter \( i \in L^0 \) votes for the right wing policy. Since a voting equilibrium is a symmetric equilibrium, this means that in equilibrium \( x^i = x \) for all \( i \in L^0 \).

The following theorem establishes the existence of a voting equilibrium to the game \( G(n, \theta) \) and records the equilibrium behavior of voters in \( H \), \( L^+ \) and \( L^- \) described above.

**Proposition 1.** For every parameter profile \((n, \theta)\) satisfying Assumptions 1–5, a voting equilibrium to the game \( G(n, \theta) \) exists.

In every voting equilibrium, voters in \( H \) and \( L^+ \) vote for the right wing policy, while voters in \( L^- \) vote for the left wing policy.

**Proof.** The previous paragraph argued that voters in \( H \) and \( L^+ \) find it weakly dominant to vote for the right wing policy while voters in \( L^- \) find it weakly dominant to vote for the left wing policy. Therefore, given a symmetric strategy \( x \) for voters in \( L^0 \), a randomly drawn
low income voter casts his ballot for the right wing policy with probability

\[ \pi(\delta, x) = \delta p + (1 - \delta) x. \] (2.3)

Voters in \( L^0 \) are fully rational and vote strategically, so they vote as if they are pivotal. Given \( x \) and \( n \), the probability that a low income voter is pivotal is

\[ \varphi(\delta|x, n) = \binom{n}{n q_n} (\pi(\delta, x))^{n q_n} (1 - \pi(\delta, x))^{n (1 - q_n)}. \] (2.4)

Then, conditional on being pivotal, the distribution of \( \delta \) is given by

\[ f^\text{piv}(\delta|x, n) = \frac{\varphi(\delta|x, n) f(\delta)}{\int_0^1 \varphi(\omega|x, n) f(\omega) d\omega}, \] (2.5)

which is well-defined because \( 0 < \pi(\delta, x) < 1 \) for all \((\delta, x)\). Therefore, conditional on being pivotal, the expected value of \( \delta \) is

\[ \bar{\delta}^\text{piv}(x, n) = \int_0^1 \delta f^\text{piv}(\delta|x, n) d\delta \]

\[ = \frac{\int_0^1 \delta \cdot \varphi(\delta|x, n) f(\delta) d\delta}{\int_0^1 \varphi(\omega|x, n) f(\omega) d\omega} \]

\[ = \frac{\int_0^1 \delta \cdot (\pi(\delta, x))^{n q_n} (1 - \pi(\delta, x))^{n (1 - q_n)} f(\delta) d\delta}{\int_0^1 (\pi(\omega, x))^{n q_n} (1 - \pi(\omega, x))^{n (1 - q_n)} f(\omega) d\omega}. \] (2.6)

Note that \( \bar{\delta}^\text{piv}(x, n) \) is continuous in \( x \). By Assumption 4, there is a threshold \( \delta^* \in (0, 1) \) such that if \( \bar{\delta}^\text{piv}(0, n) \leq \delta^* \) then \( x = 0 \) is a voting equilibrium; if \( \bar{\delta}^\text{piv}(1, n) \geq \delta^* \) then \( x = 1 \) is a voting equilibrium; and if \( \bar{\delta}^\text{piv}(0, n) > \delta^* > \bar{\delta}^\text{piv}(1, n) \) then the intermediate value theorem implies that there is a number \( \bar{x} \in (0, 1) \) such that \( \bar{\delta}^\text{piv}(\bar{x}, n) = \delta^* \). Clearly \( \bar{x} \) is a voting equilibrium.

Since Proposition 1 establishes the behavior of voters in \( H, L^+ \) and \( L^- \) for all voting equilibria, a voting equilibrium is fully described by the behavior of voters in \( L^0 \). In other words, it is fully described by the equilibrium value of \( x \). Therefore, throughout the remainder of the chapter, I will identify a voting equilibrium with its value for \( x \).
2.3.2 Analysis for Large Electorates

I now turn to the analysis of large electorates. Let \( x \) be a symmetric strategy. If \( x \neq p \) then \( \pi(\delta, x) \) is strictly monotonic in \( \delta \), so there is a unique value of \( \delta \) that minimizes \(|\pi(\delta, x) - q_\infty|\).

I denote this value by \( \delta^\dagger(x) \). The following proposition is the key mathematical result that I use to study voting behavior in large electorates.

**Proposition 2.** Let \( \{(n, \theta)\}_{n \in \mathbb{N}} \) be any sequence of parameter profiles satisfying Assumptions 1–5. Fix a symmetric strategy \( x \neq p \). Then for all \( \epsilon > 0 \), there exists \( \rho > 0 \) and a number \( N \) such that \( n \geq N \) implies

\[
|\bar{\delta}^{\text{piv}}(\bar{x}, n) - \delta^\dagger(\bar{x})| \leq 2\epsilon \quad \forall \bar{x} \in B_\rho(x) \equiv \{ \bar{x} \in [0, 1] : |x - \bar{x}| \leq \rho \}.
\]

The proposition implies that \( \bar{\delta}^{\text{piv}}(x, n) \) converges pointwise to \( \delta^\dagger(x) \). It also establishes a property of this convergence that can be interpreted as *almost* locally uniform convergence. If the number \( \rho \) does not depend on \( \epsilon \), then the convergence is locally uniform. But in principle \( \rho \) may depend on \( \epsilon \), in which case it may not be locally uniform. Nevertheless, this almost-locally uniform convergence property will suffice for our purpose of studying the limit properties of voting equilibria.

Although the proposition rules out the case where \( x = p \), the value of \( \bar{\delta}^{\text{piv}}(x, n) \) for this case is trivial. If \( x = p \), then \( f^{\text{piv}}(\cdot|x, n) \) is equal to the unconditional distribution \( f \), since \( \varphi(\delta|x, n) \) is independent of \( \delta \). So \( \bar{\delta}^{\text{piv}}(p, n) = \bar{\delta} \) for all \( n \).

The proof of Proposition 2 below shows that if \( x \neq p \), then the conditional distribution \( f^{\text{piv}}(\cdot|x, n) \) converges to Dirac mass at \( \delta^\dagger(x) \) as \( n \) gets large. The proof proceeds by first proving a uniform convergence result that is crucial for showing the almost-locally uniform convergence result of the proposition. It then applies a technique inspired by the proof of Lemma 1 in Feddersen and Pesendorfer (1997). Throughout the proof, and the rest of the chapter, we will use the notation \( B_\rho(x) \) to denote the closed interval defined above.
Proof of Proposition 2. For $n \in \mathbb{N} \cup \{\infty\}$, define the functions $h^n : [0, 1] \to \mathbb{R}$ by

$$h^n(\tilde{\pi}) = \tilde{\pi}^{q_n}(1 - \tilde{\pi})^{1-q_n}.$$ 

We will make use of the following easy-to-verify facts.

**Fact 1.** The sequence $\{q_n\}$ is strictly increasing and converges to $q_\infty$.

**Fact 2.** For all $n \in \mathbb{N} \cup \{\infty\}, n > 1$, the composite function $h^n(\pi(\cdot, \cdot)) : [0, 1]^2 \to \mathbb{R}$ is uniformly continuous.

**Fact 3.** For all $x \in [0, 1]$ and all $n \in \mathbb{N} \cup \{\infty\}$, the function $h^n(\pi(\cdot, x)) : [0, 1] \to \mathbb{R}$ is single-peaked and maximized by the value of $\delta \in [0, 1]$ that minimizes $|\pi(\delta, x) - q_n|$. Thus, $\delta^*(x)$ maximizes $h^n(\pi(\delta, x))$.

**Fact 4.** $\varphi(\delta|x, n) = \left(\frac{n}{q_n}\right)(h(\pi(\delta, x)))^n$.

These facts are useful in establishing the following lemmata.

**Lemma 1.** $\{h^n(\pi(\cdot, \cdot))\}_{n \in \mathbb{N}}$ converges uniformly to $h^\infty(\pi(\cdot, \cdot))$.

*Proof.** This can be proven by piecewise applying Dini’s theorem (see, e.g., Aliprantis and Border, 2006, Theorem 2.66, p. 54). The need to apply the theorem piecewise arises because the pointwise convergence of $\{h^n(\pi(\cdot, \cdot))\}$ to $h^\infty(\pi(\cdot, \cdot))$ may only be piecewise monotonic, as we will show.

First, we show that $\{h^n(\pi(\cdot, x))\}$ converges pointwise to $h^\infty(\pi(\cdot, x))$. Note that if $(\delta, x) = (0, 0)$ or $(0, 1)$, then $h^n(\pi(\delta, x)) = h^\infty(\pi(\delta, x))$ for all $n \in \mathbb{N}$. So let $(\delta, x) \neq (0, 0)$ or $(0, 1)$. Then we have

$$|h^\infty(\pi(\delta, x)) - h^n(\pi(\delta, x))| = |(\pi(\delta, x))^{q_\infty}(1 - \pi(\delta, x))^{1-q_\infty} - (\pi(\delta, x))^{q_n}(1 - \pi(\delta, x))^{1-q_n}|$$

$$= (\pi(\delta, x))^{q_\infty}(1 - \pi(\delta, x))^{1-q_\infty} \left|1 - \left(\frac{\pi(\delta, x)}{1 - \pi(\delta, x)}\right)^{q_n - q_\infty}\right|$$

But since $q_n \to q_\infty$ by Fact 1, we have

$$\lim \left(\frac{\pi(\delta, x)}{1 - \pi(\delta, x)}\right)^{q_n - q_\infty} = 1.$$
Therefore \( \{h^n(\pi(\cdot, x))\} \) converges pointwise to \( h^\infty(\pi(\cdot, x)) \).

Next, we show that the convergence is (piecewise) monotonic. Now define the sets

\[
X_1 = \{(x, \delta) \in [0,1]^2 : \pi(\delta, x) \leq 1/2 \}
\]
\[
X_2 = \{(x, \delta) \in [0,1]^2 : \pi(\delta, x) \geq 1/2 \}
\]

and the functions \( h^n_1(\pi(\cdot, \cdot)) : X_1 \to \mathbb{R} \) and \( h^n_2(\pi(\cdot, \cdot)) : X_2 \to \mathbb{R} \) such that \( h^n_1(\pi(\delta, x)) \equiv h^n(\pi(\delta, x)) \), \( j = 1, 2 \), \( n \in \mathcal{N} \cup \{\infty\} \). All of these functions are continuous, and we have shown above that \( \{h^n_j(\pi(\cdot, x))\} \) converges pointwise to \( h^\infty_j(\pi(\cdot, x)) \), \( j = 1, 2 \). Moreover, these sequences are monotonic:

\[
\forall (x, \delta) \in X_1 \enspace h^n_1(\pi(\delta, x)) \geq h^{n'}_1(\pi(\delta, x)) \quad n < n'
\]
\[
\forall (x, \delta) \in X_2 \enspace h^n_2(\pi(\delta, x)) \leq h^{n'}_2(\pi(\delta, x)) \quad n < n'
\]

The first line is a consequence of Fact 1 and \( \pi(\delta, x) \leq 1/2 \) for all \((x, \delta) \in X_1\), by definition of \( X_1 \). The second line is a consequence of Fact 1 and \( \pi(\delta, x) \geq 1/2 \) for all \((x, \delta) \in X_2\). Therefore, Dini’s theorem implies that \( \{h^n_j(\pi(\cdot, \cdot))\} \) converges uniformly to \( h^\infty_j(\pi(\cdot, \cdot)) \), \( j = 1, 2 \). But then \( \{h^n(\pi(\cdot, \cdot))\} \) must converge uniformly to \( h^\infty(\pi(\cdot, \cdot)) \).

\[\square\]

**Lemma 2.** Fix \( x \neq p \) as in the hypothesis of the proposition. Fix \( \epsilon > 0 \) small enough so that \( \Delta_\epsilon(x) \neq [0,1] \). Then there exists \( \eta_\epsilon \in (0,1) \), \( \rho' > 0 \) and a number \( N' \) such that for all \( n \geq N' \)

\[
h^\infty(\pi(\delta^\dagger(x), x)) - \eta_\epsilon > \sup_{\delta \notin \Delta_\epsilon(x)} h^n(\pi(\delta, \tilde{x})) \quad \forall \tilde{x} \in B_{\rho'}(x).
\]

(2.7)

Furthermore, for all \( n \in \mathcal{N} \cup \{\infty\} \), define the set

\[
\Omega^n_\epsilon(\tilde{x}) = \{\omega \in \Delta_\epsilon(x) : |h^\infty(\pi(\delta^\dagger(x), x)) - h^n(\pi(\omega, \tilde{x}))| \leq \eta_\epsilon/2\}.
\]

(2.8)

Then, there exists \( \mu > 0 \), \( \rho'' > 0 \), and a number \( N'' \) such that for all \( n \geq N'' \) and all \( \tilde{x} \in B_{\rho''}(x) \), the set \( \Omega^n_\epsilon(\tilde{x}) \) contains an interval of length at least \( \mu \).
Proof. When $\epsilon$ is small, the single-peakedness and continuity of $h^\infty(\pi(\cdot, x))$, together with Fact 3, implies that there is a number $\eta_\epsilon > 0$ such that

$$h^\infty(\pi(\delta^t(x), x)) - 2\eta_\epsilon > \sup_{\delta \in \Delta_e(x)} h^\infty(\pi(\delta, x)). \quad (2.9)$$

Since $1 \geq h^\infty(\pi(\delta, x)) \geq 0$ for all $(\delta, x) \in [0, 1]^2$, we know that $\eta_\epsilon < 1$. Uniform convergence from Lemma 1 implies the existence of $N'$ such that for all $n \geq N'$

$$|h^n(\pi(\delta, \tilde{x})) - h^\infty(\pi(\delta, x))| < \eta_\epsilon / 3 \quad \forall (\delta, \tilde{x}) \in [0, 1]^2. \quad (2.10)$$

By the uniform continuity of $h^\infty(\pi(\cdot, \cdot))$ there exists $\rho' > 0$ such that

$$|h^n(\pi(\delta, \tilde{x})) - h^n(\pi(\delta, \tilde{x})))| < \eta_\epsilon / 3 \quad \forall (\delta, \tilde{x}) \in [0, 1] \times B_{\rho'}(x). \quad (2.11)$$

Combining (2.10) and (2.11) using the triangle inequality implies that for $n \geq N'$

$$|h^n(\pi(\delta, \tilde{x})) - h^\infty(\pi(\delta, x))| < 2\eta_\epsilon / 3 \quad \forall (\delta, \tilde{x}) \in [0, 1] \times B_{\rho'}(x). \quad (2.12)$$

Thus, for all $n \geq N'$

$$\eta_\epsilon > \sup_{\delta \in \Delta_e(x)} |h^n(\pi(\delta, \tilde{x})) - h^\infty(\pi(\delta, x))|$$

$$\geq \sup_{\delta \in \Delta_e(x)} h^n(\pi(\delta, \tilde{x})) - \sup_{\delta \in \Delta_e(x)} h^\infty(\pi(\delta, x)) \quad \forall \tilde{x} \in B_{\rho'}(x) \quad (2.13)$$

Combining (2.9) and (2.13) shows that for all $n \geq N'$

$$h^\infty(\pi(\delta^t(x), x)) - \eta_\epsilon = (h^\infty(\pi(\delta^t(x), x)) - 2\eta_\epsilon) + \eta_\epsilon$$

$$> \sup_{\delta \in \Delta_e(x)} h^\infty(\pi(\delta, x)) + \eta_\epsilon > \sup_{\delta \in \Delta_e(x)} h^n(\pi(\delta, \tilde{x})) \quad \forall \tilde{x} \in B_{\rho'}(x)$$

which establishes (2.7).

To prove the second statement, first define for each $\tilde{x} \in [0, 1]$ the following sets

$$\Omega^*_\epsilon(\tilde{x}) = \{ \omega \in \Delta_e(x) : |h^\infty(\pi(\delta^t(x), x)) - h^\infty(\pi(\omega, x))| \leq \eta_\epsilon / 8 \}$$

$$\Omega^*_\epsilon^*(\tilde{x}) = \{ \omega \in \Delta_e(x) : |h^\infty(\pi(\delta^t(x), x)) - h^\infty(\pi(\omega, \tilde{x}))| \leq \eta_\epsilon / 4 \}$$
Note how the constraints defining each of these sets differ. Since $h^\infty(\pi(\cdot,x))$ is continuous and single-peaked, and maximized by $\delta^\dagger(x)$, the set $\Omega^*_\epsilon(x)$ is a nonempty interval. Let $\mu > 0$ denote its length.

Since $h^\infty(\pi(\cdot,\cdot))$ is uniformly continuous, there exists $\rho'' > 0$ such that

$$|h^\infty(\pi(\omega,x)) - h^\infty(\pi(\omega,\bar{x}))| < \eta_k/8 \quad \forall \bar{x} \in B_{\rho'}(x)$$

(2.14)

We combine this with the constraint that defines $\Omega^*_\epsilon(x)$ to get

$$\forall \omega \in \Omega^*_\epsilon(x) \quad |h^\infty(\pi(\delta^\dagger(x),x)) - h^\infty(\pi(\omega,\bar{x}))|$$

$$\leq |h^\infty(\pi(\delta^\dagger(x),x)) - h^\infty(\pi(\omega,x))| + |h^\infty(\pi(\omega,x)) - h^\infty(\pi(\omega,\bar{x}))|$$

$$\leq \eta_k/8 + \eta_k/8 = \eta_k/4 \quad \forall \bar{x} \in B_{\rho'}(x).$$

This proves that $\Omega^*_\epsilon(x) \subseteq \Omega^{**}_\epsilon(\bar{x})$ for all $\bar{x} \in B_{\rho'}(x)$.

Since $\{h^n(\pi(\cdot,\cdot))\}$ converges uniformly to $h^\infty(\pi(\cdot,\cdot))$, there exists a number $N''$ such that $n \geq N''$ implies

$$|h^\infty(\pi(\omega,\bar{x})) - h^n(\pi(\omega,\bar{x}))| \leq \eta_k/4 \quad \forall (\omega, \bar{x}) \in [0,1]^2.$$

Combining this with the constraint that defines $\Omega^{**}_\epsilon(\bar{x})$ implies that for all $n \geq N''$

$$\forall \omega \in \Omega^{**}_\epsilon(\bar{x}) \quad |h^\infty(\pi(\delta^\dagger(x),x)) - h^n(\pi(\omega,\bar{x}))|$$

$$\leq |h^\infty(\pi(\delta^\dagger(x),x)) - h^\infty(\pi(\omega,\bar{x}))| + |h^\infty(\pi(\omega,\bar{x})) - h^n(\pi(\omega,\bar{x}))|$$

$$\leq \eta_k/4 + \eta_k/4 = \eta_k/2 \quad \forall \bar{x} \in [0,1].$$

Thus we have proven that for all $n \geq N''$ and all $\bar{x} \in B_{\rho'}(x)$

$$\Omega^*_\epsilon(x) \subseteq \Omega^{**}_\epsilon(\bar{x}) \subseteq \Omega^n_{\epsilon}(\bar{x}).$$

Therefore, for all $n \geq N''$ and all $\bar{x} \in B_{\rho'}(x)$, the set $\Omega^n_{\epsilon}(\bar{x})$ must contain an interval of length at least $\mu$. \qed
Lemma 3. Fix $x \neq p$ as in the hypothesis of the proposition. Fix $\epsilon > 0$. Then there exists $\rho > 0$, and $N$ such that $n \geq N$ implies

$$\int_{\delta \in \Delta_\epsilon(x)} f^{\text{piv}}(\delta|\bar{x}, n) d\delta > 1 - \epsilon \quad \forall \bar{x} \in B_\rho(x) \quad (2.15)$$

Proof. If $\Delta_\epsilon(x) = [0, 1]$, then the result holds trivially since $\int_{\delta \in \Delta_\epsilon(x)} f^{\text{piv}}(\delta|\bar{x}, n) d\delta = 1$ for all $\bar{x}$. Therefore, suppose that $\Delta_\epsilon(x) \neq [0, 1]$. Let $N = \max\{N', N''\}$ and $\rho = \min\{\rho', \rho''\}$, where $\rho', \rho'', N'$ and $N''$ are the numbers defined in the previous lemma. Then for all $n \geq N$ and all $\bar{x} \in B_\rho(x)$, we have

$$\int_{\delta \notin \Delta_\epsilon(x)} f^{\text{piv}}(\delta|\bar{x}, n) d\delta = \frac{\int_{\delta \notin \Delta_\epsilon(x)} \varphi(\delta|\bar{x}, n) f(\delta) d\delta}{\int_{\omega \in \Omega^n(\bar{x})} \varphi(\omega) d\omega} \leq \sup_{\delta \notin \Delta_\epsilon(x)} \varphi(\delta|\bar{x}, n) \int_{\delta \notin \Delta_\epsilon(x)} f(\delta) d\delta \leq \sup_{\delta \notin \Delta_\epsilon(x)} \varphi(\delta|\bar{x}, n) \int_{\omega \in \Omega^n(\bar{x})} f(\omega) d\omega \leq \left( \frac{h^\infty(\pi(\delta^\dagger(x), x)) - \eta_k}{h^\infty(\pi(\delta^\dagger(x), x)) - \eta_k/2} \right)^n \frac{1}{\mu \nu} \leq \left( \frac{1 - \eta_k}{1 - \eta_k/2} \right)^n \frac{1}{\mu \nu}. \quad (2.16)$$

The first inequality follows because $\Omega^n(\bar{x}) \subset [0, 1]$ for all $\bar{x}$. The second follows by bounding the integrals in the numerator and denominator. The third follows by bounding the integrals in the numerator and denominator using Assumption 5 and Lemma 2. The fourth follows by Fact 4 and Lemma 2. Finally, the fifth inequality follows by the fact that the left side is strictly increasing in $h^\infty$. Thus (2.16) implies the desired result by choosing $n \geq N$ for $N$ large enough.

The proof of Proposition 2 now follows from Lemma 3. Write

$$\delta^{\text{piv}}(\bar{x}, n) = \int_{\delta \notin \Delta_\epsilon(x)} \delta f^{\text{piv}}(\delta|\bar{x}, n) d\delta + \int_{\delta \in \Delta_\epsilon(x)} \delta f^{\text{piv}}(\delta|\bar{x}, n) d\delta. \quad (2.17)$$
and observe that for $n \geq N$ and $\tilde{x} \in B_\rho(x)$, this quantity is bounded above by $\epsilon + (\delta^\dagger(x) + \epsilon) = \delta^\dagger(x) + 2\epsilon$ and bounded below by $(\delta^\dagger(x) - \epsilon)(1 - \epsilon) > \delta^\dagger(x) - 2\epsilon$, both of which follow from Lemma 3. Thus for all $\tilde{x} \in B_\rho(x)$, the conditional expectation $\tilde{\delta}_piv(x, n)$ is (weakly) within $2\epsilon$ of $\delta^\dagger(x)$.

\[ \Box \]

2.3.3 Pure Strategy Equilibrium

Proposition 2 can be used to show that for large electorates there is always a pure strategy voting equilibrium to the game $G(n, \theta)$. Remarkably, $x = 1$ is always part of a voting equilibrium when the electorate is large. On the other hand, $x = 0$ is not part of a voting equilibrium in large electorates if $\delta^*$ is low enough—specifically, if it is below $\delta^\dagger(0) = q_\infty/p$. (Recall that $\delta^*$ is the value of $\delta$ that solves $y(\delta) = \overline{y}. \)$ This means that $x = 0$ is not part of a voting equilibrium if the unconditional expected payoff to the right wing policy is not much lower than the payoff to the left wing policy. These are, in fact, the main results of this chapter, and they are proven in the following proposition.
Proposition 3. Let \( \{(n, \theta)\}_{n \in \mathbb{N}} \) be any sequence of parameter profiles each satisfying Assumptions 1–5. Then there is a number \( N \) such that \( n \geq N \) implies

(i) \( x = 1 \) is part of a voting equilibrium to the game \( G(n, \theta) \)

(ii) \( x = 0 \) is part of a voting equilibrium to the game \( G(n, \theta) \) if \( \delta^* > \delta^\dagger(0) \), and is not part of any voting equilibrium to the game \( G(n, \theta) \) if \( \delta^* < \delta^\dagger(0) \).

Proof. Note that \( \pi(\delta, 1) \) is strictly decreasing in \( \delta \) and \( \pi(1, 1) = p > 1/2 > q_\infty \). Therefore, \( \delta^\dagger(1) = 1 \). By Proposition 2 it follows that if \( n \) is large, then \( x = 1 \) is part of a voting equilibrium to the game \( G(n, \theta) \). Next, note that \( \pi(\delta, 0) \) is strictly increasing in \( \delta \) over the entire interval \([0, 1]\), and ranges from 0 to \( p \). (See, e.g., Figure 1.) Therefore, if \( \delta^\dagger(0) < \delta^* \), then Proposition 2 implies that \( x = 0 \) is part of a voting equilibrium to the game \( G(n, \theta) \) for large \( n \). On the other hand, if \( \delta^\dagger(0) > \delta^* \), then Proposition 2 implies \( x = 0 \) cannot be part of a voting equilibrium to the game \( G(n, \theta) \) if \( n \) is large.

The proof of Proposition 3 shows that it is a trivial consequence of the analytical result established in Proposition 2. In explaining the intuition for the result, it is instructive to begin by studying the result for the \( x = 0 \) case. First, observe that in light of Proposition 2, the condition \( \delta^0 > \delta^* \) states that if the election is close, then a voter’s conditional expectation of \( \delta \) is likely to be higher than \( \delta^* \). Next, suppose that \( x = 0 \) is part of a voting equilibrium. In this case, none of the \( L^0 \) voters cast their ballots for the right wing policy. Therefore, the election is not likely to be close unless many low income voters received the economic opportunity, became rich, and voted for the right wing policy. But this means that the conditional expectation of \( \delta \) cannot be too small. When \( \delta^* \) is larger than this conditional expectation, the \( L^0 \) voters are happy to vote for the left wing policy. But when it is smaller, then they would actually like to switch their votes to the right wing policy.

The main result of this chapter is that \( x = 1 \) is always part of a voting equilibrium when the electorate is large. When all voters in \( L^0 \) vote to re-elect the right wing policy, the
election is unlikely to be close: the right wing policy should win by a large margin. But if the election does turn out to be close, then a large fraction of the electorate must have voted for the left wing policy. Since only the untalented voters in $L^-$ vote for the left wing policy, it must be that a large number of voters discovered that they are untalented. This can only be the case when a large number of voters received the economic opportunity, and some succeeded while others did not. In other words, $\delta$ must be large. But in this case, it makes sense—from the perspective of the $L^0$ voters—to vote for the right wing policy. Therefore, with a large electorate, $x = 1$ is in fact part of voting equilibrium.

### 2.3.4 Limit Equilibrium

We now study the properties of all voting equilibria (not just pure strategy equilibria) of the game $G(n, \theta)$ when $n$ is large. To do this, we define a “limit equilibrium.”

Proposition 1 implies that under Assumptions 1–5, the game $G(n, \theta)$ has a voting equilibrium. Let $\{(n, \theta)\}_{n \in \mathcal{N}}$ denote a sequence of parameter profiles each satisfying these assumptions. We say that a number $z \in [0, 1]$ is a limit equilibrium of this sequence if there
exists a sequence \( \{x_n\}_{n \in \mathbb{N}} \) of corresponding voting equilibria (meaning that \( x = x_n \) is part of a voting equilibrium to the game \( G(n, \theta) \) for each \( n \in \mathbb{N} \)) that has a subsequence that converges to \( z \).\(^8\) Immediately, we know that limit equilibria must exist since every sequence of voting equilibria \( \{x_n\}_{n \in \mathbb{N}} \) is contained in the unit interval. (Also, Proposition 3 implies that \( x = 1 \) is a limit equilibrium.) The main result of this section characterizes the entire set of limit equilibria.

Before stating this result, we provide some graphical intuition. Figure 2 depicts \( \delta^\dagger(x) \) as a function of \( x \). Here, we have defined \( \delta^\dagger(p) = \delta \) and included the point \((p, \delta)\). Also, we know that \( \delta^\text{piv}(x, n) \) is a continuous function of \( x \) for all \( n \). Therefore, the figure also depicts \( \delta^\text{piv}(x, n) \) as a function of \( x \) for a large fixed value of \( n \). This is represented by the dotted curve. Since \( n \) is large, this line is drawn close to \( \delta^\dagger(x) \). (Recall that Proposition 2 states that \( \delta^\text{piv}(x, n) \) converges pointwise to \( \delta^\dagger(x) \).) Furthermore, one possible value for \( \delta^* \) is depicted by the blue line. In this case, \( \delta^* > \delta^\dagger(0) \) and both \( x = 0 \) and \( x = 1 \) are voting equilibria. Notice, however, that there is one other equilibrium at the point where the blue line crosses the dotted line. In this equilibrium, the value of \( x \) is close to \( p \). Next, consider the situation where \( \delta^* \) is given by the red line, so that \( \delta^* < \delta^\dagger(0) \). In this case, \( x = 1 \) is the only pure strategy equilibrium. But, there are also two other mixed strategy equilibria, which correspond to the two points where the red line crosses the dotted line. Consequently, Figure 2 suggests that with large electorates, there are generically three equilibria of the kind described above. We will show that these equilibria in Figure 2 are (very close to) the limit equilibria of the model.

When \( \delta^* < \delta^\dagger(0) \), let \( x^* \) denote the value of \( x \) that solves \( \pi(x, \delta^*) = q_{\infty} \). (This is the value of \( x \) at which the red line in Figure 2 cross the graph of \( \delta^\dagger(x) \).)

\(^8\)Note that this definition is different from the one in Gul and Pesendorfer (2009), where a limit equilibrium is simply the limit of a sequence of voting equilibria, not the limit of subsequences of voting equilibria. Thus, the present definition asks less of a limit equilibrium than the Gul-Pesendorfer definition. This makes the set of limit equilibria potentially larger. Nevertheless, we will see that the set of limit equilibria is the same under both definitions.
Proposition 4. Let \( \{(n, \theta)\}_{n \in \mathbb{N}} \) be any sequence of parameter profiles each satisfying Assumptions 1–5. If \( \delta^* > \delta^\dagger(0) \) then the set of limit equilibria of this sequence is \( \{0, p, 1\} \). On the other hand, if \( \delta^* < \delta^\dagger(0) \) then the set of limit equilibria is \( \{x^*, p, 1\} \).

Proof. The claims that 1 is always a limit equilibrium, and 0 is a limit equilibrium when \( \delta^* > \delta^\dagger(0) \), both follow from Proposition 3. Therefore, we must prove the following three claims: (i) \( p \) is always a limit equilibrium, (ii) \( x^* \) is a limit equilibrium when \( \delta^* < \delta^\dagger(0) \), and (iii) there are no other limit equilibria.

(i) We first show that \( p \) is a limit equilibrium. Define \( \delta^\dagger(p) = \bar{\delta} \) as in Figure 2. Then fix a small \( \epsilon > 0 \). Proposition 2 implies that \( \{\delta^\text{piv}(x, n)\}_{n \in \mathbb{N}} \) converges pointwise to \( \delta^\dagger(x) \). That means \( \delta^\text{piv}(p - \epsilon, n) \) converges to 0 while \( \delta^\text{piv}(p + \epsilon, n) \) converges to 1. Therefore, since \( \delta^* \in (0, 1) \), there is a number \( n \) large enough such that \( \delta^\text{piv}(p - \epsilon, n) < \delta^*/2 \) and \( \delta^\text{piv}(p + \epsilon, n) > (1 + \delta^*)/2 \). But since \( \delta^\text{piv}(x, n) \) is continuous in \( x \) for all \( n \), the intermediate value theorem implies that for \( n \) large enough, there exists a number \( x \in [p - \epsilon, p + \epsilon] \) such that \( \delta^\text{piv}(x, n) = \delta^* \). Thus \( x \) is a voting equilibrium to the game \( G(n, \theta) \). (To be precise, we would say that \( x \) is part of a voting equilibrium to the game \( G(n, \theta) \); but recall that this abuse of terminology is justified by Proposition 1, which enables us to identify a voting equilibrium with its value for \( x \).)

Now start with \( \epsilon > 0 \) very small, and construct the sequence \( \{\epsilon/k\}_{k=1}^\infty \). By the procedure above, we can associate with each \( \epsilon/k \), a number \( n \) and a voting equilibrium \( x_n \) to the game \( G(n, \theta) \). Moreover, we can use the procedure to construct a sequence of voting equilibria \( \{x_n\} \) where the \( k \)th element of this sequence is associated with \( \epsilon/k \). Then observe that, by construction, this sequence of voting equilibria converges to \( p \) since the sequence \( \{\epsilon/k\} \) converges to 0. Therefore, \( p \) must be a limit equilibrium.

(ii) The proof that \( x^* \) is also a limit equilibrium when \( \delta^* < \delta^\dagger(0) \) is exactly analogous, and omitted.

(iii) Finally, we prove that there are no other limit equilibria besides the ones reported.
in the proposition. Indeed, suppose that there were another limit equilibrium, and call it \( z \).

Suppose \( \delta^* > \delta^\dagger(0) \), so that \( z \in (0,1) \), \( z \neq p \). Since \( \delta^* \in (\delta^\dagger(0), 1) \), for \( \epsilon > 0 \) small enough we know that \( \delta^\dagger(z) \) is bounded away from \( \delta^* \); in particular

\[
|\delta^\dagger(z) - \delta^*| > 2\epsilon. \tag{2.18}
\]

(See the graph of \( \delta^\dagger(x) \) in Figure 2.) Since \( z \) is a limit equilibrium, there is a sequence of voting equilibria \( \{x_n\} \) that has a subsequence that converges to \( z \). Denote that subsequence by \( \{x_k\}_{k \in \mathcal{K} \subseteq \mathcal{N}} \), where \( \mathcal{K} \) is an infinite set. Then, by Proposition 2, there exists \( \rho > 0 \) and \( N \) such that \( n \geq N \) implies

\[
|\bar{\delta}^\text{piv}(x, n) - \delta^\dagger(z)| \leq 2\epsilon \quad \forall x \in B_\rho(z). \tag{2.19}
\]

The inequalities (2.18) and (2.19) imply that for all \( n \geq N \)

\[
|\delta^* - \bar{\delta}^\text{piv}(x, n)| \geq \epsilon \quad \forall x \in B_\rho(z).
\]

This in turn implies that there is an index \( k \in \mathcal{K}, k \geq N \), such that \( x_k \in (0,1) \) and \( |\delta^* - \bar{\delta}^\text{piv}(x_k, k)| \geq \epsilon \). But then \( x_k \) cannot be part of a voting equilibrium to the game \( G(k, \theta) \). Contradiction.

We can use an exactly analogous argument in the case of \( \delta^* < \delta^\dagger(x) \) to show that there can be no other limit equilibria besides \( x^*, p \) and 1.

An important implication of Proposition 4 is that when \( n \) is large, every voting equilibrium of the game \( G(n, \theta) \) has to be close to a limit equilibrium. To understand why, ask the following question: If \( \{x_n\}_{n \in \mathcal{N}} \) is a sequence of voting equilibria, can it have a subsequence of voting equilibria that remains bounded away from any of the limit equilibria? The answer is “no,” because that subsequence would in turn contain a sub-subsequence that converges to a limit that is bounded away from any of the limit equilibria. But since this sub-subsequence is also a subsequence of \( \{x_n\}_{n \in \mathcal{N}} \), its limit would have to be a limit equilibrium. Contradiction.

\[\square\]
This implies that when \( n \) is large, every voting equilibrium of the game \( G(n, \theta) \) has to be close to one of the limit equilibria.

### 2.3.5 The Election Outcome

Consider all of the assumptions as before, except now suppose that the realization of \( \delta \) is public knowledge for the voters. Call this modified game \( G^0(n, \theta) \). Then, in any voting equilibrium to \( G^0(n, \theta) \), all voters not in \( L^0 \) vote exactly as before. In particular, members of \( H \) and \( L^+ \) vote for the right wing policy while members of \( L^- \) vote for the left wing policy. Members of \( L^0 \), however, vote for the left wing policy if \( \delta < \delta^* \) and vote for the right wing policy if \( \delta > \delta^* \).

Now, suppose that \( \{x_k\}_{k \in K \subseteq N} \) is a subsequence of voting equilibria associated with the sequence of games \( \{G(k, \theta)\}_{k \in K \subseteq N} \). Let this subsequence converge to the limit equilibrium \( z \). If, for almost all realizations of \( \delta \), the probability that the game \( G(k, \theta) \) has the same election outcome as the game \( G^0(k, \theta) \) converges to 1 as \( k \to \infty \), then \( z \) is said to exhibit election success. If, on the other hand, the set of \( \delta \) such that this probability converges to 0 as \( k \to \infty \) has positive measure (according to \( F \)), then \( z \) is said to exhibit election failure. We are interested in knowing what values of \( z \) aggregate information, and what values exhibit election failure.

**Proposition 5.** The limit equilibrium \( z = p, 1 \) exhibits election failure. If \( z = 0, x^* \) is a limit equilibrium, then it exhibits election success.

**Proof.** The proof relies on a simple application of the law of large numbers. Suppose \( \{x_k\} \) is a subsequence of voting equilibria that converges to \( z \). If \( z = p \) or \( z = 1 \), we have \( \lim \pi(\delta, x_k) > q_\infty \) for all \( \delta \). Therefore, the right wing policy wins the election with probability 1 in game \( G(k, \theta) \) as \( k \to \infty \). But for a range of small values of \( \delta \), the left wing policy wins with probability 1 in the game \( G^0(k, \theta) \) as \( k \to \infty \). Therefore, the limit equilibria \( p \) and \( 1 \) exhibit election failure.
Now, suppose $z = 0$, and 0 is a limit equilibrium. If $\delta p > q$, then the right wing policy wins with probability 1 in the game $G(k, \theta)$ as $k \to \infty$. But if this is the case, the fraction of voters who vote for the right wing policy in the game $G^0(k, \theta)$ is also strictly larger than $1/2$ as $k \to \infty$. On the other hand, if $\delta p < q^\infty$, then the left wing policy wins with probability 1 in the game $G(k, \theta)$ as $k \to \infty$. But in this case, the fraction of voters who vote for the left wing policy in the game $G(k, \theta)$ is strictly smaller than $1/2$ as $k \to \infty$. Therefore, the limit equilibrium $z = 0$ exhibits election success.

Finally, suppose $z = x^*$ and $x^*$ is a limit equilibrium. If $\delta p + (1 - \delta)x^* > q^\infty$, then the right wing policy wins with probability 1 in the game $G(k, \theta)$ as $k \to \infty$. But in this case, $\delta > \delta^*$, so that the right wing policy wins in the game $G^0(k, \theta)$ with probability 1 as $k \to \infty$. If $\delta p + (1 - \delta)x^* < q^\infty$, then the left wing policy wins with probability 1 in the game $G(k, \theta)$ as $k \to \infty$. But it also wins with probability 1 in the game $G^0(k, \theta)$ as $k \to \infty$. Therefore, the limit equilibrium $z = x^*$ also exhibits election success.

### 2.4 Discussion

In this section, I address two possible criticisms of the model. First, I address the fact that the results require the assumption that the right wing policy is implemented in the first period. Second, I discuss the fact that the results require full rationality from voters who are only imperfectly informed about the effectiveness of the right wing policy. I also explain how the model’s results might provide insights into the behavior of voters that are only boundedly rational.

#### 2.4.1 Left Wing Policy First

The results of the model hinge on the assumption that the right wing policy is implemented in the first period. It is easy to show that if the left wing policy is implemented first, or if there is only one period and voters must choose between the two policies without experiencing either
policy, then the left wing policy always wins the election. What justifies the assumption that voters experience one policy first? And, why is this assumed in the model to be the right wing policy?

First, note that the model in which voters choose between two policies without experiencing either policy is less realistic than any model in which voters experience one of the two policies first. In almost all elections, voters choose between an incumbent party or policy that they have experienced, and a challenger that they have not. In particular, Fiorina (1981) has made the case that voters will vote retroactively: they evaluate the incumbent policy on the basis of their personal experience under that policy. Thus, my model falls very much within the paradigm of retrospective voting, and the assumption that one of the two policies is experienced first is substantiated by Fiorina’s compelling argument for the retrospective voting model.

Second, in light of the fact that the first period policy is always (likely to be) re-elected, one could criticize the model for being a model of status quo bias rather than a model of right-wing bias. I have two different responses to this criticism. First, it may very well be the case that disapproval of left wing policies by American voters is significantly the result of a status quo bias. For example, Meltzer and Richard (1978) argue that in the past century, the United States progressively adopted left wing policies to replace status quo policies that were defended by right wing conservatives. Meltzer and Richard point to Roosevelt’s New Deal and Johnson’s Great Society—both examples of policy agendas that saw intense opposition when they were first introduced, but whose programs, such as Social Security and Medicare, enjoy intense popular support today. Thus, the assumption that the right wing policy comes first may be an assumption that is consistent with historical fact. If this is the case, then it may be quite challenging to distinguish status quo bias from right wing bias.

Alternatively, one might justify the assumption that the right wing policy is implemented first by making the model more realistic, for example, by introducing politicians and lobbies.
To see how this can be done, suppose that instead of directly electing policy, voters elect candidates that can be one of three types: a commitment type that always implements the left wing policy, a commitment type that always implements the right wing policy, or a strategic type that cares only about being in office. Suppose that all types occur with positive probability, but the probability of either commitment type is small. The strategic type receives a payoff of 1 from each period in office, and a payoff of 0 when not in office. The election after the first period is between the incumbent politician and a random challenger. Clearly, the only equilibrium is one where the strategic type implements the left wing policy in both periods. Under this plan of action, the incumbent’s re-election is certain, whereas under any other plan he is re-elected with probability smaller than 1.

Now, suppose that there is a lobby group that can commit to paying politicians up to a certain amount for implementing the right wing policy in the first period. One can show that there is an equilibrium in which the strategic type implements the right wing policy in both periods. As a corollary to Propositions 3 and 5, there is an equilibrium strategy profile for the voters in which politicians with such a plan of action are returned to office with some probability $\psi_n$, where $\psi_n \to 1$. Thus, absent any contribution from the lobby, the strategic politician has an expected future payoff of $\psi_n$ if voters play this strategy profile. The politician’s highest expected future payoff from deviating to any other plan is 1. Thus, for large electorates, there is an equilibrium in which the lobby offers the politician a contingent transfer $1 - \psi_n$ for implementing the right wing policy in the first period, the strategic politician accepts the offer, all $L^0$ voters cast their vote for the incumbent, and if the strategic politician is re-elected, he implements the right wing policy in the second period as well. Observe that in large electorates, the equilibrium contribution of the lobby to the politician is small.\(^9\)

\(^9\)This implication is consistent with an empirical observation known in the public choice literature as the “Tullock paradox.” Tullock (1980) points out that bribes to corrupt politicians tend to be quite small in comparison to the rents received in exchange.
2.4.2 Information, Rationality and Bounded Rationality

Another concern with the model might be that it requires the seemingly peculiar combination of voters being fully rational and relatively uninformed about the effectiveness of the right wing policy. But, in fact, this is not a peculiar combination at all. Even economists and sociologists—despite all of the methodological tools available to them—do not always find it easy to predict the mobility consequences of particular policies, let alone a bundle of right wing or left wing policies. Why should we expect rational voters to be perfectly informed?

Additionally, although the model studies only the benchmark case of full rationality, there are many ways in which its results provide insight into the behavior of voters that are only boundedly rational. Observe that a number of stylized facts are consistent with many of the intermediate conclusions of the model. Voters often believe that their vote “matters,” which implies that they expect elections to be close (pivotal voting). Indeed, many elections are close, even though they are not decided by one vote. Voters also often believe that there are many other voters who share their traits, and they expect these voters to vote like they do (equilibrium is symmetric). A boundedly rational low income voter who is inclined to vote for the right wing policy might infer from past close elections that the left wing policy should not have received so many votes. Such a voter might think that it must have received as many votes as it did because several people discovered that they are untalented or lazy, or found themselves prone to squandering the many opportunities that the right wing policy gave them while it was in effect, and thus decided to vote for the left wing policy ($L^-$ voters). Based on these expectations, the voter would infer that his chances for upward mobility are good, even though (he believes that) he has not yet been lucky enough to receive the opportunity to climb the economic ladder. If such a voter perceives himself to be hard-working and talented ($p > 1/2$), then he may find reason to vote for the right wing policy.
2.5 Conclusion

I offered a model in which low income voters who care only about their economic payoff may, in one particular equilibrium, vote for to re-elect a right wing policy that they believe is worse for them than the competing left wing policy. This occurs because voters are rational, so they must condition their vote on being pivotal: In the event that every vote for the winning policy matters, low income voters believe that the right wing policy gives them greater chance of upward mobility than it actually does. Thus, behavior under this equilibrium exhibits a false consciousness about the prospect of upward mobility (POUM). More importantly, this equilibrium exhibits a behavioral equivalence between rational strategic voting and optimistic naive voting.

One important question that the model in this chapter leaves unanswered is whether the equilibrium behavior of the voters in the model can be sustained over time. In particular, it would be interesting to study an infinite horizon extension of the model and ask whether there is an equilibrium path in which the behavioral equivalence between rational strategic voting and optimistic naive voting continues to hold in the long run. I conjecture that it may not, because the optimistic voter has the potential to eventually learn that his beliefs were too optimistic. On the other hand, the rational voter has nothing to learn. Thus, it may be the case that election failure will continue to hold in the long run if voters are fully rational, but will not not hold if they are optimistic and naive. However, analyzing strategic voting in dynamic elections has been challenging and this extension is clearly not straightforward.
Chapter 3

Incomplete Policymaking

Making Healthcare Policy 2009-2010

3.1 Introduction

In March 2010, President Obama signed into law the Patient Protection and Affordable Care Act (PPACA) and the Health Care and Education Reconciliation Act of 2010. These two pieces of legislation together constitute the most sweeping healthcare reform laws enacted since the introduction of Medicare and Medicaid by President Johnson in 1965. Yet, despite their enormous impact on the healthcare industry, critics across the ideological spectrum have attacked these policies as constituting only an incomplete solution to the nation’s healthcare problems. For example, liberal Democrats complain that the new laws do not guarantee health insurance coverage for all US residents, as would a universal single payer system. On the other hand, even moderate Democrats complain that the new laws do little to control the rapidly rising costs of medical care. Moreover, some of these critics have also suggested that more could have been done to control these costs without reducing the target number of insured individuals.
If the above allegations are true, then the 2010 acts present a puzzle: What explains why the Democratic majorities of the 111th Congress did not come to an agreement that lies on the *coverage-savings frontier*? In other words, why didn’t President Obama, along with the Democratic majorities of the House and Senate, choose a policy that generates the largest possible cost-savings given the number of individuals it seeks to insure? My basic argument is that the final (incomplete) agreement between liberal and moderate Democrats was the only possible outcome given the commitment problems inherent to dynamic logrolling.

To better understand the issues surrounding the 2010 healthcare acts, consider the following facts.

- The 2010 acts, in particular the PPACA, introduce an *individual mandate* requiring all individuals covered by the new laws to purchase health insurance, or else pay a fine. However, despite the mandate, the expected fraction of US residents with health insurance does not rise to 100%. This is in some part because not everyone is covered by the law, and in remaining part because some individuals will opt to pay the fine. According to a Congressional Budget Office (CBO) report produced at the time the healthcare bills were passed, the new laws are expected to reduce the number of uninsured residents from 50 million in 2010 to 23 million in 2019, leaving a substantial fraction of the population uninsured (see CBO 2010).\(^1\) More recent estimates by the CBO are also consistent with these numbers. For example, numbers reported in CBO (2011) imply that only about 72% of nonelderly individuals who are projected to not be covered by health insurance in the absence of the new laws will have health insurance in 2021.

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\(^1\)These numbers include unauthorized immigrants. Excluding these immigrants from the calculation, the number of uninsured authorized residents in 2010 was 44.7 million, and the expected number of uninsured authorized residents in 2019 is 14.4 million.
• While the 2010 acts are expected to reduce the Federal deficit,\textsuperscript{2} they are expected to do little to control the rising cost of medical care for businesses and individuals. The acts’ main provisions for attacking these costs are a number of pilot projects. Although the CBO’s estimates of the savings accruing from implementation of the new healthcare laws do not take into account expectations for the success or failure of these projects, a report commissioned by Business Roundtable states that the 2010 acts “could potentially reduce the rate of future healthcare cost increases by 15% to 20% when fully phased in by 2019,” but only under the most optimistic assumption that all of the cost-saving pilot programs succeed and are entirely replicated by the market (see Business Roundtable, 2009).\textsuperscript{3}

• Viewed alongside a universal single payer healthcare system (e.g. the United States National Health Care Act, H.R. 676, pending a vote in Congress), the 2010 acts do in fact appear to fall short of lying on the coverage-savings frontier. To see why, consider the following comparison. First, although CBO has not assessed the monetary impact of any universal single payer healthcare system in nearly twenty years, a 1993 study using Medicare’s payment rates showed that a universal single payer system with co-payments would actually reduce total national health expenditures by approximately 2% while providing healthcare coverage to 100% of US residents (CBO 1993). The

\textsuperscript{2}CBO (2010) estimates the fiscal impact of the healthcare bills to be a net total reduction in the budget deficit of $138 billion over the 2010-2019 period. Recent estimates for a slightly different time period are even more optimistic. For example, CBO (2011) estimates a reduction in the budget deficit of $210 billion over the 2012-2021 period. These savings are generated by new taxes included in the legislation and the elimination of some wasteful spending, such as overpayments to private insurers by the Medicare Advantage program.

\textsuperscript{3}Some of the pilot projects test incremental changes to the current medical care delivery system, while others test broader changes such as moving from the current “fee-for-service” payment system to a system of “bundled payments.” Such a change may help mitigate the incentives for doctors to prescribe more tests and treatments rather than better tests and treatments. See “Testing, Testing,” by Atul Gawade in The New Yorker, December 14, 2009, for an informal description of some of the projects.
bulk of savings in these estimates are associated with the implementation of a universal preventative care program, reductions in insurance company overheads, and a transition from the “fee for service” payment system (which, for the most part, is still currently in place) to a system of “bundled payments” for a variety of basic treatments. In contrast to these estimates, a recent study by the Department of Health and Human Services (HHS) finds that the 2010 laws are actually expected to raise national healthcare expenditures by approximately $234 billion in the 2010-2019 period, despite the fact that fewer individuals would have healthcare coverage under these laws than under a universal single payer system (see HHS 2009). This is not surprising given that the three main sources of savings identified by the CBO for a universal single payer system are not present in the 2010 laws. Consequently, it is difficult to argue that the 2010 acts do in fact lie on the coverage-savings frontier. Specifically, we cannot rule out the possibility that there were additional (or alternative) legislative measures that the 111th Congress could have included in the 2010 bills (or implemented in lieu of the 2010 bills) that would have increased both coverage and savings.

What then led the Democratic members of the 111th Congress to implement such an incomplete policy despite having partisan majorities in both chambers? One argument is that the final policy that President Obama signed into law should not be viewed as incomplete if one is willing to consider the fact that public opinion was fiercely against any of the serious healthcare measures that were being debated during this period. For instance, one could argue that the 111th Congress implemented the most complete policy possible subject to

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4 Like the CBO’s estimates, the HHS estimates do not account for the expected effects of the pilot projects. But, as the report commissioned by Business Roundtable suggests, even under optimistic assumptions the expected effect on costs is modest at best.

5 Not to mention, a filibuster-proof majority in the Senate for a brief period after Sen. Al Franken (D-MN) was sworn into office.
re-election constraints. This explanation, however, is hard to defend since in March 2010 when the PPACA passed the House, only 32% of the American public approved of the law, while 54% disapproved.\footnote{See, e.g., \url{http://www.cbsnews.com/8301-503544_162-20010453-503544.html}.} If the 111th Congress was decidedly willing to go as much against public opinion as it did in passing the PPACA, why did it not go all the way and complete the agreement?

To answer these questions, I use insights from the model of partial agreements developed by Acharya and Ortner (2010) in the context of a bilateral bargaining problem. My basic explanation for the emergence of incomplete policies such as the 2010 healthcare acts is illustrated as follows. President Obama entered office in 2009 amidst a severe recession and two expensive wars. He knew that in order to pass healthcare reform, he would need the support of the more moderate Blue Dog Democrats in Congress. In better economic times (and/or times of peace) he and the fiscally conservative Blue Dogs might have been able to strike an agreement whereby a universal single payer healthcare bill (or at least the public option) would be passed in exchange for a major reduction in government spending in other areas. But the severity of the recession—and, in particular, the (perceived) necessity of fiscal stimulus—meant that negotiating any changes in the size of government was essentially infeasible in the 2009-2010 period. Would the Blue Dogs have accepted a promise from the President to lower future spending in exchange for a healthcare bill that covered 100% of US residents today? No, because such a promise would not be credible. On the other hand, would it have made sense to wait for the recession to clear before doing anything on healthcare? Also no, for reasons that will become clear from the formal analysis that follows. For now, the explanation can be informally summarized by saying that according to the narrative of my model, the Democrats of the 111th Congress deliberately came to an
incomplete agreement in 2010 so that they could leave just enough issues on healthcare open to be able to logroll these with fiscal policy issues later (e.g., in the event that the recession cleared and the Democrats were able to hold on to their majority following the midterm election).

### 3.2 A Narrative Model

A Democratic president, whom I call $P$, enters office with partisan majorities in Congress. He is eager to implement his liberal policy agenda, but to do this he requires the support of the more moderate members of his Congressional coalition, in particular a Blue Dog senator whom I call $S$. The president and senator negotiate two issues over two periods, $t = 1, 2$. One can think of these periods as representing the first and second half of the president’s term, specifically the periods before and after the midterm election. I describe the issues, and the negotiations that take place in these periods, as follows.

#### 3.2.1 The Issues

The two issues that the president and senator negotiate are healthcare reform and the size of government. A policy on the size of government is represented by a number $z \in [0, 1]$ with higher numbers representing larger government. The status quo policy is $z^{\text{sq}} = 1$ so that government is large to begin with. A policy on the healthcare issue is represented by a pair of numbers $(x, y)$. The first component $x$ represents the fraction of currently uninsured citizens that are expected to gain health insurance coverage under the policy. The second component $y$ represents the percent of total possible savings in healthcare costs that is expected to be realized under the stated policy, for the government and those who already have insurance. One can think of these savings as arising from the removal of waste in the medical services...
Figure 1: The blue triangle represents the set $\Delta$. The point $(x', y')$ is a complete healthcare policy, since it lies on the coverage-savings frontier. The point $(x'', y'')$ is an incomplete healthcare policy since all of the points in the red triangle are policies where coverage and savings are both higher than under $(x'', y'')$.

delivery system. For example, if the maximum possible savings under any new policy is $\$Y$, then $y$ represents the fraction of those $Y$ dollars that would be saved under the policy $(x, y)$. The set of feasible policies over which the president and senator negotiate is

$$\Delta = \{(x, y) : x, y \geq 0 \text{ and } x + y \leq \psi\}.$$  

The constraint $x + y \leq \psi$ represents a tradeoff: if coverage is extended to more individuals—presumably through some combination of subsidies and mandates—then the maximum possible savings for the government and those who already have insurance is lower. The idea here is that large savings targets might be inconsistent with the legislative measures required to provide coverage to a large number of citizens. For simplicity, I take $\psi \equiv 1$. Thus, the blue triangle in Figure 1 represents the feasible policies $\Delta$.

\footnote{An alternative interpretation that is consistent with the description of the model that follows would treat $x$ as representing the spending floor mandate on insurance companies, and $y$ as the reduction in the budget deficit resulting from the elimination of only government waste. The spending floor mandate in the 2010 healthcare laws requires insurance companies to spend at least 80% of their premiums on medical care or “quality improvement” for their policyholders.}
Say that a healthcare policy \((x, y) \in \Delta\) is *complete* if \(x + y = \psi\). Under a complete policy, coverage is extended to the maximum number of currently uninsured citizens given the savings target \(y\). Thus, a complete policy is one that lies on the coverage-savings frontier. An example of a complete policy is the point \((x', y')\) in Figure 1. On the other hand, a policy \((x, y) \in \Delta\) such that \(x + y < \psi\) is said to be *incomplete* because there are legislative measures, or alternative policies, that could be enacted to increase both coverage and savings. Such a policy lies below the coverage-savings frontier. An example is the point \((x'', y'')\) in Figure 1. Observe that all of the points in the small red triangle to the northeast of the vertex \((x'', y'')\) represent policies under which both coverage and savings are higher. The status quo policy on healthcare is obviously the incomplete policy \((x^{sq}, y^{sq}) = (0, 0)\): coverage increases by 0% and healthcare costs are reduced by 0%.

### 3.2.2 First Period Negotiations

In the first period, the president sets the agenda and negotiates with the senator over healthcare policy, but not the size of government. I motivate this assumption by supposing that the president entered his term amidst a severe recession. He and the senator agreed on the need for bold fiscal action, and given his decision to focus his attention on passing healthcare legislation, nothing (outside of healthcare reform) can be done in the first period to substantially reduce the size of government. Thus, fiscal policy remains off the negotiating table, and the status quo policy \(z^{sq} = 1\) is implemented.

The assumption that the president sets the agenda means that he offers a healthcare policy \((x, y) \in \Delta\) to the senator, who must then either accept or reject the offer. If the senator accepts the offer, then the offered policy \((x, y)\) is implemented. If the senator rejects the offer, then the status quo policy \((x^{sq}, y^{sq})\) is implemented. I denote the policy that is implemented in period 1 by \((x_1, y_1, z_1)\). (Recall that \(z_1 = z^{sq}\) in this period, since the
president and senator negotiate only healthcare policy.

3.2.3 Second Period Negotiations

The second period begins with the midterm election. I assume that the Democratic majority survives the election with probability $\delta \in (0, 1)$. With complementary probability $1 - \delta$, the Republicans take control of Congress. If this happens, then there is legislative gridlock, no new policies are enacted for the remainder of the term, and the previous period policy $(x_1, y_1, z_1)$ is re-implemented. On the other hand, if the Democratic majority survives the midterm election, then the president and senator continue to negotiate policy. However, in this period, the senator sets the agenda; that is, he makes a policy proposal, and the president decides whether to accept or reject.

I assume that in this period, the economy recovers with probability $p \in (0, 1)$. If the economy does not recover, then as before, the president and senator negotiate only healthcare policy with the first period fiscal policy $z_1 = z^{eq} = 1$ being automatically re-implemented. However, I place one important constraint on the senator’s healthcare policy proposal: if the healthcare policy that was implemented in the first period is $(x_1, y_1)$, then the senator can only propose healthcare policies from the set

$$\Delta(x_1, y_1) \equiv \{(x, y) \in \Delta : x \geq x_1 \text{ and } y \geq y_1\}.$$

In other words, coverage and savings cannot decrease.\(^8\) If the senator’s offer is accepted, then the healthcare policy that is implemented is the offered policy. If the offer is rejected, then the policy that prevails is the first period policy $(x_1, y_1)$.

Now suppose that the economy recovers. In this case, the senator includes a proposal

\(^8\)One justification for this assumption is the idea that once a policy is introduced, it is difficult to reverse because it creates endogenous interests. Coate and Morris (1999) examine the theoretical basis for this argument, providing some conditions under which it is justified.
$z \in [0, 1]$ on the size of government along with his offer $(x, y) \in \Delta(x_1, y_1)$ on healthcare. The president must either accept the senator’s combined proposals, or reject them wholly. If the president accepts, then the proposed policies are implemented. If the president rejects, then the first period policy $(x_1, y_1, z_1)$ is re-implemented. I denote the policy that is implemented in the second period by $(x_2, y_2, z_2)$.

### 3.2.4 Payoffs

With $(x_t, y_t, z_t)$ denoting the policy that is implemented in period $t = 1, 2$, I assume that the period $t$ payoffs to the president and senator are

$$u^P_t = x_t + rz_t \quad \text{and} \quad u^S_t = ry_t + (1 - z_t) \quad (3.1)$$

where $0 < r < 1$. These payoffs have three important features. First, the president cares more about extending coverage to the uninsured than about the size of government, while the senator cares more about the size of government than improving efficiency in healthcare delivery system. Second, the president has a preference for larger government, whereas the senator has a preference for smaller government. Third, the president does not care about improving efficiency, and the senator does not care about increasing coverage. Though this third feature is an abstraction, the main substantive results of the model would not change if I assumed that the president had a small preference to increase efficiency while the senator had a small preference to extend coverage.

### 3.3 Equilibrium Analysis

The equilibrium concept for solving the model is Subgame Perfect Equilibrium. In this section, I establish the existence of equilibrium and characterize its properties using backward induction.
3.3.1 Second Period Analysis

I begin by characterizing the equilibrium outcomes and payoffs for period 2. I will show that the senator’s equilibrium offer is always the unique feasible offer that maximizes his payoff subject to the president receiving weakly greater than his rejection payoff. In other words, the senator’s equilibrium offer is the unique solution to a constrained maximization problem. In the case where the Democrats survive the midterm election, and the economy recovers, this maximization problem is

\[
\begin{align*}
\max_{(x,y,z) \in \Delta(x_1,y_1) \times [0,1]} & \quad ry + 1 - z \\
\text{s.t.} & \quad x + rz \geq x_1 + r \\
\end{align*}
\tag{P1}
\]

Since the objective function is continuous, and the constraints define a nonempty compact set, this problem has a solution. The following lemma characterizes the solution, showing that it is unique.

**Lemma 1.** The unique solution to (P1), written as a function of the period 1 policy \((x_1, y_1)\), is

\[
(x, y, z) = (\xi(x_1, y_1), 1 - \xi(x_1, y_1), \zeta(x_1, y_1)) \quad \text{where}
\]

\[
\begin{align*}
\xi(x_1, y_1) &= x_1 + r & \text{and} & \zeta(x_1, y_1) &= 0 & \text{if} & \quad x_1 + y_1 \leq 1 - r \\
\xi(x_1, y_1) &= 1 - y_1 & \text{and} & \zeta(x_1, y_1) &= 1 - \frac{1-x_1-y_1}{r} & \text{if} & \quad x_1 + y_1 \geq 1 - r \\
\end{align*}
\tag{3.2}
\]

**Proof.** Clearly, we must have \(x + y = 1\) at the solution to (P1). In other words, the senator’s healthcare proposal must be a complete policy; otherwise, if a policy \((x, y)\) with \(x + y < 1\) is feasible, then the objective function can be increased without violating any of the constraints. Therefore, we can substitute \(y = 1 - x\) into the objective function, and solve the maximization problem over \(x\) and \(z\). After this substitution, the objective function is strictly decreasing in both \(x\) and \(z\). This implies that at the solution, the constraint (IC-P)
must hold with equality, i.e. \( x = x_1 + r(1 - z) \). After substituting this expression for \( x \), the objective function is strictly decreasing in \( z \). Therefore, the solution to (P1) occurs at the smallest value of \( z \) for which the constraints can be satisfied. This immediately yields (3.2) as the unique solution to (P1).

With the solution to (P1) at our disposal, it is straightforward to compute the unique equilibrium payoffs in period 2. Standard arguments can be employed to establish the following lemma.

**Lemma 2.** Suppose the Democrats retain control of Congress. If the economy does not recover, then the unique period 2 equilibrium payoffs are \( x_1 + r \) for the president, and \( r(1 - x_1) \) for the senator. If the economy recovers, then the unique period 2 equilibrium payoffs are \( \xi(x_1, y_1) + r\zeta(x_1, y_1) \) for the president, and \( r(1 - \xi(x_1, y_1)) + (1 - \zeta(x_1, y_1)) \) for the senator, where \( \xi \) and \( \zeta \) are defined in Lemma 1.

*Proof.* See Appendix A.1. □

The proof of Lemma 2 is straightforward. In the case where the economy recovers, the result is proven by showing that the unique equilibrium offer by the senator is the solution to Lemma 1, and that this offer must be accepted.

Given the period 1 policy \((x_1, y_1)\), Lemma 2 implies that the period 2 expected equilibrium payoffs to the players are unique, and are given by

\[
V^P(x_1, y_1) = \delta \left[ p(\xi(x_1, y_1) + r\zeta(x_1, y_1)) + (1 - p)(x_1 + r) \right] \\
+ (1 - \delta) (x_1 + r) \\
V^S(x_1, y_1) = \delta \left[ p(r(1 - \xi(x_1, y_1)) + (1 - \zeta(x_1, y_1)) + (1 - p)r(1 - x_1)) \right] \\
+ (1 - \delta) ry_1
\]

(3.3)
These payoffs will be used to establish the uniqueness of period 1 equilibrium payoffs in the following section.

### 3.3.2 First Period Analysis

In this section, I will show that an equilibrium exists. Moreover, the president’s equilibrium offer is generically unique. In every equilibrium, the president’s offer is accepted by the senator. And, for a range of parameter values, the president’s equilibrium offer is an *incomplete* policy.

To prove these results, I will first show that the president’s equilibrium offer is a feasible offer that maximizes his payoff subject to the senator receiving a payoff weakly larger than his payoff to rejecting. In other words, the president’s equilibrium offer is one that solves a constrained maximization problem, similar to the senator’s period 2 problem (P1) above. Consequently, the main result of this section will show that the solution to this constrained maximization problem is generically unique, and is sometimes an incomplete policy. I first state and solve the constrained maximization problem. It is then straightforward to show that an offer is an equilibrium offer if and only if it solves this optimization problem.

To state the optimization problem, define the payoffs that the players receive from the senator rejecting the president’s offer. These are

\[
V^P \equiv r + V^P(0, 0) = 2r \\
V^S \equiv V^S(0, 0) = \delta \left[p(r(1 - r) + 1) + (1 - p)r \right] = \delta p(1 - r^2) + \delta r
\]

Thus, the president’s optimization problem described above is:

\[
\max_{(x, y) \in \Delta} x + r + V^P(x, y) \quad \text{(P2)} \\
\quad \text{s.t. } ry + V^S(x, y) \geq V^S \quad \text{(IC-S)}
\]
The following Lemma is the key technical result that drives the main qualitative results of this chapter. It characterizes the solutions to problem (P2).

**Lemma 3.** If

\[ r^2 < \frac{\delta p}{2 - \delta(1 - p)} \]  

then the unique solution to (P2) is

\[ x = \frac{1}{2} (2 - \delta)(1 - r) \quad \text{and} \quad y = \frac{1}{2} \delta(1 - r). \]  

If (3.5) holds with reverse inequality, then the unique solution to (P2) is

\[ x = \frac{1}{2} \left( 2 - \delta - \delta p \left( \frac{1}{r} - r \right) \right) \quad \text{and} \quad y = \frac{1}{2} \left( \delta + \delta p \left( \frac{1}{r} - r \right) \right) \]  

If the two sides of (3.5) are equal, then (3.6) and (3.7) are the only solutions to (P2).

**Proof.** Suppose we add the constraint \( x + y \leq 1 - r \) to the problem (P2). Call the new problem that includes this constraint (P2a). This problem has a solution, since it maximizes a continuous function on a nonempty compact set. Since all of the constraints must be satisfied, the value of the objective function at a solution \((x, y)\) is

\[ x + r + \delta p(x + r) + \delta(1 - p)(x + r) + (1 - \delta)(x + r) = 2(x + r) \]  

which we obtain by substituting \( V^P(x, y) \) into the objective function in (P2). After substituting \( V^S(x, y) \) and \( V^p \), the constraint (IC-S) also simplifies to

\[ ry + \delta \left( p(r(1 - x - r) + 1) + (1 - p)r(1 - x) \right) + (1 - \delta)ry \geq \delta p(1 - r^2) + \delta r \]

\[ \iff (2 - \delta)ry \geq \delta rx. \]  

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Since $x + y \leq 1 - r$ is also a constraint in (P2a), we have
\[
x \leq 1 - r - y \leq 1 - r - \frac{\delta x}{2 - \delta} \\
\Rightarrow \left(1 + \frac{\delta}{2 - \delta}\right)x \leq 1 - r
\] (3.10)

Since the objective function in (3.8) is maximized for the largest feasible value of $x$, inequalities (3.9) and (3.10) along with the constraint $x + y \leq 1 - r$ imply that the solution to (P2a) is given by (3.6). Note that the value of the objective function in (P1) at these values of $x$ and $y$ is
\[
2 - \delta + \delta r.
\] (3.11)

Now suppose we added the constraint $x + y \geq 1 - r$ to the problem (P2). Call this problem (P2b). Like problem (P2a), this problem also has a solution, and after substituting $V^P(x, y)$, the value of the objective function at a solution $(x, y)$ is given by (3.8). After substituting $V^S(x, y)$ and $V^S$, the constraint (IC-S) reduces in this case to
\[
\left[(1 + \delta p)r - \frac{\delta p}{r} + (1 - \delta)r\right]y + \left[\delta(1 - p)r + \frac{\delta p}{r}\right](1 - x) \geq V^S
\] (3.12)

I now consider two cases: the case where $\alpha$, the coefficient of $y$ in the inequality above, is nonnegative, and the case where it is nonpositive. If $\alpha$ is nonnegative and the inequality in (3.12) is strict, then the objective function in (3.8) can be increased by raising $x$ (and lowering $y$, if necessary) without violating any of the constraints. (Notice that if (3.12) is strict and $\alpha$ is nonnegative, then $x < 1$ since $\beta > 0$ and $V^S > 0$; thus, there is room to raise $x$.) Therefore, (3.12) must hold with equality. Furthermore, if $x + y < 1$ then the objective function in (P2) can be raised by increasing both $x$ and $y$, so that (3.12) continues to hold with equality. Therefore, we must have $x + y = 1$. This implies that we can substitute $y = 1 - x$ into (3.12) and solve it as an equality to find the values of $x$ and $y$ that solve
These values are given by (3.7). Furthermore, it is easy to verify that $\alpha$ is nonnegative if and only if the left hand side of (3.5) is weakly smaller than the right hand side. Finally, the value of the objective function when $x$ and $y$ are given by (3.7) is

$$2 - \delta + 2r - \delta p \left( \frac{1}{r} - r \right). \quad (3.13)$$

Now consider the case where $\alpha$ is nonpositive, i.e. (3.5) does not hold. Since we have $x + y \geq 1 - r$ as a constraint, we need $y \geq 1 - r - x$. Combining this with (3.12) and the assumption that $\alpha$ is nonpositive gives us

$$(\alpha + \beta)(1 - x) \geq V^S + \alpha r$$

$$\iff x \leq 1 - \frac{V^S + \alpha r}{\alpha + \beta} = \frac{1}{2} (2 - \delta) (1 - r). \quad (3.14)$$

(The equivalence in (3.14) follows because $\alpha + \beta > 0$.) Since the objective function is maximized for the largest feasible value of $x$, and the constraints $x + y \geq 1 - r$ and (3.12) must be satisfied, we obtain (3.6) as a solution to the problem (P2b) when $\alpha$ is nonpositive.

To conclude the proof, note that if (3.5) holds, then (3.11) is strictly larger than (3.13). If (3.5) holds with reverse inequality, then (3.11) is strictly smaller than (3.13). If the two sides of (3.5) are actually equal, then (3.11) is equal to (3.13).

Lemma 3 shows that if (3.5) does not hold, then a solution to (P2) is given by (3.7), which defines a complete policy, since at the stated values of $x$ and $y$ we have $x + y = 1$. However, the key feature to note in the solution to (P2) is that if (3.5) does hold, then the unique solution to (P2) is (3.6), which is an incomplete policy. This is because for these values of $x$ and $y$, we have $x + y = 1 - r$. Since the following proposition states that every first period equilibrium outcome is a solution to (P2), an immediate corollary of the analysis is that if (3.5) holds, then the unique equilibrium healthcare policy that is implemented in period 1 is an incomplete policy.

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Proposition 1. In every equilibrium, the president offers a solution to (P2), and the senator accepts this offer.

Proof. See Appendix A.2. □

The final proposition reports what by now is obvious.

Proposition 2. An equilibrium exists.

Proof. Consider the following strategy profile. In period 1, the president offers a solution to (P2) and the senator accepts an offer if and only if accepting gives him a payoff weakly larger than his payoff to rejecting. In period 2, if the economy does not recover then the senator proposes \((x_1, 1 - x_1)\) and the president accepts all feasible offers. If the economy does recover, then the senator proposes the solution to (P1) and the president accepts an offer if and only if accepting gives him a payoff weakly larger than his payoff to rejecting. This is clearly an equilibrium. □

I finish the analysis by explaining how we might expect the results in this chapter to be robust to a variety of natural alterations and extensions. For example, I stated (without proof) that even if the president received a small additional payoff from generating savings in healthcare expenditures, and the senator received a small additional payoff from increasing health insurance coverage, all of the qualitative results would continue to hold. This is because logrolling incentives would be slightly weaker, but they would still be present, and it is evident from the analysis here, and the analysis in Acharya and Ortner (2010), that such incentives are the driving force for partial or incomplete agreements in bargaining situations. In addition, the analysis in Acharya and Ortner (2010) shows that the payoffs in (1) can be generalized to \(x_t + r^P z_t\) for the president and \(r^S y_t + (1 - z_t)\) for the senator, where \(r^P, r^S \in (0, 1)\) but \(r^P\) need not equal \(r^S\).
Second, Acharya and Ortner (2010) showed how similar results can be obtained in an infinite horizon model. Although I have assumed here that the game ends after the president’s first term is over, this assumption per se is unimportant. In fact, I conjecture that my results would continue to hold provided the payoffs that the players receive from future events (such as future elections and terms in office) are “sufficiently independent” of the events that take place in the president’s first term. By this I mean the following. Suppose payoffs to future events are bounded and independent of what happens in the president’s first term, and let \( \tilde{V}^P \) and \( \tilde{V}^S \) denote the discounted value of these future payoffs. Then, adding \( \tilde{V}^P \) and \( \tilde{V}^S \) to all of the current payoffs would not change the results because \( \tilde{V}^P \) and \( \tilde{V}^S \) are unaffected by \((x_t, y_t, z_t)_{t=1,2}\). However, even if they were affected slightly by \((x_t, y_t, z_t)_{t=1,2}\), I conjecture that the main qualitative results of the model would not be different.

Finally, although I assumed that the senator is the agenda setter in period 2, I could obtain similar results if instead I assumed that the period 2 agenda setter was randomly chosen. The intuition for this assertion is also made clear in Acharya and Ortner (2010). What is important is for the senator to be chosen as the period 2 agenda setter with nonzero probability.

### 3.4 Conclusion

This chapter studied the dynamic logrolling problem using a formal narrative of the health-care reform negotiations that recently took place. Although the model is highly stylized (and specific to the particular case that I study), the main ideas that I present are actually very fundamental, and quite general, as shown in the closely related work of Acharya and Ortner (2010). These authors study the dynamic logrolling problem in greater generality and abstraction, and show that commitment problems can generate significant delays in bargaining.
situations where the players have strong logrolling incentives. They also show that if the players are allowed to reach partial agreements, then such agreements may sometimes obtain as the unique outcome of the players’ strategic interactions. The emergence of incomplete policies, as I have described here, is an example of these more general ideas.

Appendix

A.1

I first prove the following claim.

Claim. If \((x_1, y_1)\) is an incomplete healthcare policy, then there is no equilibrium outcome in which the president rejects the senator’s offer.

Proof of Claim. First consider the case where the economy does not recover, and the senator proposes \((x, y) \in \Delta(x_1, y_1)\). Assume, for the sake of contradiction that there is an equilibrium outcome in which the president rejects the offer. Rejecting gives the president a payoff \(x_1 + r\) and the senator a payoff \(ry_1\). Now consider a deviation by the senator to the offer \((x_1 + \varepsilon, 1 - x_1 - \varepsilon)\) where \(\varepsilon \in (0, 1 - x_1 - y_1)\). Note that this deviation is feasible by the assumption that \((x_1, y_1)\) is an incomplete policy. Since this offer gives the president a payoff \(x_1 + \varepsilon + r > x_1 + r\), he must accept it. This implies that the deviation gives the senator a payoff \(r(1 - x_1 - \varepsilon) > ry_1\) and is therefore profitable.

Next, consider the case where the economy recovers. In this case, the senator offers policies on both healthcare and the size of government \((x, y, z)\) with \((x, y) \in \Delta(x_1, y_1)\) and \(z \in [0, 1]\). Again, suppose for the sake of contradiction that there is an equilibrium outcome in which the president rejects the offer. In this equilibrium, the president’s payoff is \(x_1 + r\) and the senator’s payoff is \(ry_1\). If \(\varepsilon \in (0, 1 - x_1 - y_1)\), then the offer \((x_1 + \varepsilon, 1 - x_1 - \varepsilon, 1)\) is feasible, and a profitable deviation for the senator. This is because it gives the president
a payoff $x_1 + \varepsilon + r > x_1 + r$, and hence must be accepted; therefore, it gives the senator a payoff $r(1 - x_1 - \varepsilon) > ry_1$ and is hence profitable.

I now return to the proof of Lemma 2. First, consider the case where the economy does not recover. If the period 1 healthcare policy $(x_1, y_1)$ is a complete policy, then $\Delta(x_1, y_1)$ is a singleton, and the senator can only propose $(x, y) = (x_1, y_1) = (x_1, 1 - x_1)$. Whether the president accepts or rejects this offer, the payoffs are $x_1 + r$ for the president and $r(1 - x_1)$ for the senator. If, on the other hand, the period 1 healthcare policy $(x_1, y_1)$ is an incomplete policy, then suppose for the sake of contradiction that there is an equilibrium in which the senator receives a payoff smaller than his highest possible payoff $r(1 - x_1)$, and let $\varepsilon > 0$ be the total shortfall in the senator’s payoff under this equilibrium. Consider a deviation by the senator from this equilibrium to the offer $(x_1 + \varepsilon, 1 - x_1 - \varepsilon)$, where $\varepsilon \in (0, \max\{\varepsilon, 1 - x_1 - y_1\})$. Note that this deviation is feasible since the first period healthcare policy is incomplete by hypothesis. However, the deviation is also profitable since accepting the offer gives the president a payoff $x_1 + \varepsilon + r > x_1 + r$, and hence he must accept; thus, making the offer gives the senator a payoff $r(1 - x_1 - \varepsilon) > r(1 - x_1) - \varepsilon$ and is therefore profitable. This yields the necessary contradiction. Consequently, the senator’s unique equilibrium payoff is his largest feasible payoff $r(1 - x_1)$, and this in turn implies that the president’s unique equilibrium payoff is $x_1 + r$.

Now consider the case where the economy recovers, and let $(\tilde{x}, \tilde{y}, \tilde{z})$ be the solution to (P1) characterized in Lemma 1. Clearly $(\tilde{x}, \tilde{y}, \tilde{z})$ is an equilibrium offer. Given the findings of Lemmas 1 and the Claim above, the result is proven by showing that if an offer $(x, y, z)$ does not solve (P1), then it is not an equilibrium offer. To show this, first assume that the period 1 healthcare policy $(x_1, y_1)$ is a complete policy. Lemma 1 implies that the only equilibrium offer is $(x_1, y_1, 1)$, since all other feasible offers give the president a payoff smaller than his rejection payoff $x_1 + r$, and hence must be rejected. Next, assume that
the period 1 healthcare policy \((x_1, y_1)\) is an incomplete policy. If a feasible offer \((x, y, z)\) does not solve \((P1)\), then either \((i)\) it does not satisfy \((IC-P)\), or \((ii)\) it satisfies \((IC-P)\), but the value of the objective function under \((x, y, z)\) falls short of its value under \((\tilde{x}, \tilde{y}, \tilde{z})\) by some quantity \(\varepsilon > 0\). If case \((i)\) holds, then the Claim above implies that \((x, y, z)\) is not an equilibrium offer. If case \((ii)\) holds, then suppose for the sake of contradiction that \((x, y, z)\) is an equilibrium offer. The Claim implies that it must be accepted, giving the senator a payoff \((r\tilde{y} + 1 - \tilde{z}) - \varepsilon\). Now, consider a deviation by the senator to the offer \((\tilde{x}, \tilde{y}, \tilde{z} + \varepsilon)\), where \(\varepsilon \in (0, \min\{\tilde{z}, 1 - x_1 - y_1\})\). (Such a deviation is feasible by the hypothesis that \((x_1, y_1)\) is an incomplete policy, so that \(\tilde{z}\) can at most equal \(1 - \frac{1-x_1-y_1}{r} < 1\).) In equilibrium, the president must accept the offer \((\tilde{x}, \tilde{y}, \tilde{z} + \varepsilon)\) since it gives him a larger payoff than his payoff to rejecting it:

\[
\tilde{x} + r\tilde{z} + r\varepsilon > \tilde{x} + r\tilde{z} = x_1 + r,
\]

(A1)

where the equality follows because Lemma 1 shows that the offer \((\tilde{x}, \tilde{y}, \tilde{z})\) satisfies \((IC-P)\) with equality. Therefore, making the offer \((\tilde{x}, \tilde{y}, \tilde{z} + \varepsilon)\) gives the senator a larger payoff than making the offer \((x, y, z)\):

\[
r\tilde{y} + 1 - \tilde{z} - \varepsilon > r\tilde{y} + 1 - \tilde{z} - \varepsilon.
\]

(A2)

Consequently, \((x, y, z)\) is not an equilibrium offer.

A.2

Proof of Proposition 1. Let \((x^*, y^*)\) denote a solution to \((P2)\) and note that from Lemma 3 we have \(x^* > 0\) and \(y^* < 1\) for all solutions \((x^*, y^*)\). I first show that there is no equilibrium outcome in which the senator rejects the president’s offer. Suppose there was an equilibrium with such an outcome. In this equilibrium, the payoffs are \(V^P\) for the president and \(V^S\) for the senator. Now consider a deviation by the president to an offer \((x^* - \varepsilon, y^* + \varepsilon)\) where \(\varepsilon \in (0, \min\{x^*, 1 - y^*\})\) so that the deviation is feasible. In equilibrium, the senator must
accept this offer since accepting gives him a payoff

$$r(y^* + \varepsilon) + V^S(x^* - \varepsilon, y^* + \varepsilon) > ry^* + V^S(x^*, y^*) = V^S.$$  \((A3)\)

The inequality follows because \(V^S(x, y)\) is decreasing in \(x\) and increasing in \(y\). The equality follows because the proof of Lemma 3 shows that the constraint (IC-S) holds with equality at any solution \((x^*, y^*)\) to the problem (P2). However, since the senator must accept the offer \((x^* - \varepsilon, y^* + \varepsilon)\), this offer is a profitable deviation for the president because it gives him a payoff

$$\left(x^* - \varepsilon\right) + r + V^P(x^* - \varepsilon, y^* + \varepsilon) = 2\left(x^* - \varepsilon + r\right) > 2r = V^P,$$  \((A4)\)

where the inequality follows because \(\varepsilon < x^*\). Thus, there is no equilibrium outcome in which the senator rejects the president’s offer.

Now I show that \((x, y)\) is a solution to (P2) if and only if it is an equilibrium offer. Clearly, any solution to (P2) is an equilibrium offer. Conversely, assume that \((x, y)\) is an equilibrium offer; and, suppose for the sake of contradiction that it is not a solution to (P2). By the argument above, if \((x, y)\) is an equilibrium offer, then it must be accepted; thus, the president’s payoff to making the offer is \(x + r + V^P(x, y)\). However, because \((x, y)\) is not a solution to (P2), one of the following must be true: \((i)\) it violates (IC-S), or \((ii)\) the value of the objective function under \((x, y)\) falls short of its value under a solution \((x^*, y^*)\) by some quantity \(\varepsilon > 0\). If case \((i)\) holds, then \((x, y)\) cannot be part of an equilibrium because it gives the senator a payoff lower than his rejection payoff, and I argued above that in every equilibrium outcome, the senator must accept the president’s offer. If case \((ii)\) holds, then consider a deviation by the president to the offer \((x^* - \varepsilon, y^* + \varepsilon)\) where \(\varepsilon \in (0, \min\{\varepsilon/2, x^*, 1 - y^*\})\) so that the deviation is feasible. This offer must be accepted, since it gives the senator a payoff strictly greater than his rejection payoff as in (A3) above. But this means that it is a profitable deviation for the president because it gives him a larger
payoff than his payoff under the offer \((x, y)\):

\[
x^* - \varepsilon + r + V^P(x^* - \varepsilon, y^* + \varepsilon) = 2(x^* + r) - 2\varepsilon > 2(x^* + r) - \varepsilon
\]

\[
= (x^* + r + V^P(x^*, y^*)) - \varepsilon = x + r + V^P(x, y).
\]

(A5)

The last equality follows by the definition of \(\varepsilon\). Consequently, even in case \((ii)\), the offer \((x, y)\) cannot be an equilibrium offer.
Chapter 4
Coordination and Development in Dictatorships

4.1 Introduction

A classical puzzle in comparative political economy asks the following question: Why don’t authoritarian rulers implement efficient policies and later use their political power to appropriate a large share of the output? Put another way, if rulers represent the interests of a political elite, who have large stakes in the economy, then why don’t they govern their countries in the same way as CEOs manage profit-maximizing corporations? Are rulers not like CEOs, the political elite like shareholders, and citizens like workers? At what point does the analogy break down?

In some cases, authoritarian rulers do enact productive reforms. Take, for example, Lee Kuan Yew’s five decade long corporate rule of Singapore, Deng Xiaoping’s economic reforms in China in the 1980’s, and Suharto’s “New Order” for Indonesia from the 1960’s to the 1990’s.¹ These and many other 20th Century Asian rulers embraced the analogy between state and corporation, and their economic policies exhibited many aspects of the profit-seeking motive that we see in decentralized capitalist economies. However, even a cursory

glance at the cross section of authoritarian regimes in history, and in the present, would show that the Asian experience is an isolated one. More often than not, authoritarian rulers go out of their way to block economic reforms in favor of preserving the inefficient status quo. For such regimes, the analogy between state and corporation appears too simplistic and flawed.

With the exception of relatively recent papers by Acemoglu and Robinson (2006, 2000a), these questions have received little attention from the formal literature in political economy. The key idea that is advanced by the Acemoglu-Robinson papers is that authoritarian regimes sometimes choose to maintain the inefficient status quo because implementing the efficient policy would ‘change the balance of power’ in society in a way that favors the regime’s adversaries. The important hurdle for the regime in these models is that even though the efficient policy raises total income, it also empowers the opponents of the regime in demanding a more favorable distribution of that income, so that the ruling elite are made worse off by implementing the policy. Although this shift in political power is at the heart of their argument, Acemoglu and Robinson do not explicitly model the mechanism by which it occurs; instead, they exogenously assume that as a result of adopting the efficient policy, the citizens are more likely to (be able to) replace the existing regime.

In the standard models of redistributive politics under democratic institutions (e.g. Meltzer and Richard 1981) the idea of ‘changing political power’ is elegantly captured by changing the identity of the median voter. As poorer segments of the population gain the franchise, the median voter becomes a lower-income individual, who demands higher levels of redistribution. In the case of authoritarian polities, however, the absence of a standard institutional mechanism like voting makes it difficult to formalize a corresponding notion of ‘changing political power’. Moreover, as Boix (2009) reminds us, there are several different kinds of authoritarian regimes, each with different institutions for power sharing and policy choice. A second challenge is then to abstract from these many differences, a develop a model
of policy choice that relies only on the few important commonalities across the heterogeneity in types of authoritarian regimes.

Politics is about group conflict, and in nearly all authoritarian regimes the central political conflict is between those in power, namely the ruling elite, and the politically mobilized but disenfranchised citizens. The elite are interested in preserving their authority, while citizens seek concessions from the elite in the form of a more favorable income distribution. Citizens may be able to exact these concessions by staging an uprising that threatens to topple the regime. Without the ability to organize and protest, these disenfranchised citizens are relatively powerless and have few other alternatives. The easier it is for them to organize, the more powerful they are, and the more likely it is that they will get the concessions they seek. The central thesis of this chapter is the proposition that citizens are powerful when their information is precise and their beliefs are not too varied, for it is only then that they can coordinate their actions and succeed in drawing concessions.

If this argument is correct, then it can be used to account for some situations in which authoritarian regimes choose not to implement efficient policies. Many policies that improve efficiency also have the effect improving citizens' ability to coordinate. Examples include the freedom to share information, the building of roads, the expansion of literacy and the promotion of newspapers and other types of media. Cell phones and the internet, for example, can be used to reduce information asymmetries and market frictions that cause inefficiency. But these tools can also be used to coordinate other non-market activities. This implies that in deciding whether or not to implement an efficient policy, the regime must weigh the size of the surplus that the policy creates against the cost of empowering its political adversaries, namely the disenfranchised citizens. That the Chinese government allows its citizens to use email but forbids them from using Facebook or Twitter is not paradoxical: the regime is making a rational decision, weighing the benefits of these utilities against their costs. It is not surprising that the utilities that it blocks have relatively high coordination value, while
those that it allows have relatively high productive value.

The model in this chapter captures some of these strategic considerations. The following is an informal description of the setup: An authoritarian elite must decide whether to implement a policy that increases the income of all agents in society. The regime must make this decision before learning its strength. The strength of the regime is reflected by a parameter that determines the amount of concessions that the elite will have to make to the citizens in the face of a revolutionary uprising, should such an uprising take place. Citizens’ beliefs about the strength of the regime are always somewhat varied, but the efficient policy has the effect of reducing this variation. When the regime is very weak or very strong, then all citizens either attack the regime or stay at home, because although their beliefs are somewhat heterogeneous, they all agree that attacking is either worth the cost or not. But suppose the strength of the regime is in an intermediate range. If the citizens’ beliefs are very diffuse, then it will be difficult for them to coordinate their actions: some will want to attack the regime while others will not. If these beliefs are not so diffuse, then it is more likely that all of the citizens will choose the same action. Under a set of plausible assumptions, the elite prefer the citizens’ beliefs to be more heterogeneous. In other words, the efficient policy also comes with a cost. The main result of the chapter shows when the surplus created by the efficient policy is not very large, the elite do not implement the efficient policy.

I then present three extensions to the model. The first extension considers an economy where income has two sources: natural resource rents and industrial output. I assume that the role of the efficient policy is to reduce market frictions so that unemployed inputs can be united to produce output. I show that if income from natural resources is large in proportion to industrial income, then the regime will not implement the efficient policy. Notice that this result is consistent with the empirical finding of the Resource Curse (see, e.g., Ross 1999). Conversely, if the economy is already enjoying high levels of human capital, then the efficiency gains from reducing the market frictions are large enough to entice the elite
to implement the efficient policy. This result provides some insight into why a number of authoritarian regimes, particular those in East Asia, have adopted comparatively efficient policies.

The second extension endogenizes the level of inequality. I show that if the elite are able to establish rents over the benefits of economic reform, then they are more enticed to adopt the efficient policy. The primary observation of this extension suggests that there is an endogenous relationship between income, inequality and revolutionary activity.

The third extension studies the elites’ incentives to democratize. Unlike much of the past work on democratization (e.g. Acemoglu and Robinson 2000b; Lizzeri and Persico 2004; Llavador and Oxoby 2005), this extension shows that part of the impetus for democratization may arise from the potential for increased efficiency. For example, consider an authoritarian regime in which the elite are not adopting an efficient policy because they are concerned about its effect on the ability of the disenfranchised citizens to organize. If the elite were to democratize, they would lose some of their income due to redistribution; but neither they nor the new government would feel the need to block the efficient policy. Therefore, depending on how intense they expect these redistributive pressures to be, the elite may voluntarily democratize so that they can enjoy the surplus created by implementing the efficient policy. Therefore, this extension provides a theoretical argument that is consistent with the observed correlation between democracy and development.²

4.1.1 Related Literature

Acemoglu and Robinson (2006) regenerated interest in the topic of inefficient dictatorships by revisiting Gerschenkron’s (1970) study of economic backwardness in Eastern Europe. They argued that rulers may not implement efficient policies because these policies often increase the odds that they will be replaced. Using a very different model, they arrived

²See Przeworski and Limongi (1997), Boix and Stokes (2003), Acemoglu et. al. (2008) and Boix (2011) for some contributions to the debate over Modernization Theory.
at results qualitatively similar to the ones in this chapter. However, their model did not explain what it was about the efficient policy that enabled challengers to more easily replace the rulers; nor did their model address why the ruler and citizens could not bargain out of the inefficiency created by the ruler electing to not implement the efficient policy. Although this latter shortcoming was recognized by Acemoglu (2003), who used Nash’s cooperative bargaining model to argue that the inefficiency was unavoidable, he left open the question of whether the inefficiency could persist in a strategic, noncooperative framework. This chapter uses a new framework to study the same problem, and fills both of these important gaps in the Acemoglu-Robinson project. First, it argues that although the distribution of income in an authoritarian society represents an implicit bargain between rulers and citizens, the timing constraints on bargaining my create inefficiencies as in the economic hold-up problem: authoritarian rulers must make their policy decisions before they resolve their political-economic conflict with the disenfranchised citizens. Second, and more importantly, the present paper takes a concrete position on how power might shift as a result of introducing the efficient policy to the economy. The policy has the effect of creating a convergence in the beliefs of disenfranchised citizens, which in turn enables them to better coordinate their actions. Put simply, the paper highlights the importance of coordination.

Few other papers have stressed the tension between the need for coordination in economic development, and the effect improved coordination on political conflict in societies with low levels of political mobilization (and, to my knowledge, there are no formal models that directly capture this tension). Huber, Rueschemeyer and Stephens (1993) argued that economic progress, specifically through industrialization, lowers the power of the landed elite and improves the ability of the working class to organize. The industrial process itself requires a certain degree of coordination. To produce output, a number of different inputs must be united. This requires the owners of the inputs to be able to communicate prices. When information and communication channels are scarce, market frictions, such as search fric-
tions and information asymmetries, inhibit economic coordination and result in inefficiency. Indeed, the view that market activity is inherently about coordination goes back to the work of Leon Walras (1874). Under the Walrasian perspective, it is natural to assume that some economic reforms improve efficiency by eliminating the frictions that create informational asymmetries and inhibit market coordination. Jensen (2007), for example, shows that the introduction of cell phones to southern India eliminated waste in the fishing industry by creating a dramatic reduction in price dispersion. At the same time, there is no doubt that the introduction of technologies such as cell phones creates opportunities not only to coordinate market activity, but also to coordinate non-market activities such as revolutionary uprising.

There are also a number of recent papers that are related to the present paper. Boix (2009), for instance, argues that the primary concern for authoritarian rulers is coordination against their regime. Cosgel, Miceli and Rubin (2009) provide a historical example of an authoritarian regime blocking a productive technology because of the costs resulting from improved coordination. These authors observe that while the Ottoman empire enthusiastically adopted advancements in military technology, it also blocked the arrival of the printing press for nearly three hundred years. Cosgel, Miceli and Rubin then argue that while military technologies reduced the expected value of revolution for the disenfranchised citizens, the printing press would have increased its expected value.

Bueno de Mesquita and Smith (2009) and Campante and Do (2010) present evidence that is remarkably consistent with the predictions of this chapter. Bueno de Mesquita and Smith argue that political leaders often suppress public goods such as the freedom of assembly and information because of their concern with the threat of revolution. The authors present empirical evidence showing that when government revenue comes primarily from natural resources and foreign aid, rather than activities that require more labor inputs, then governments are less likely to grant these freedoms. Conversely, if society relies relatively more on labor-intensive sources of income, then authoritarian regimes are more likely to
grant the freedoms of assembly and information, and they are also more likely to democratize. Campante and Do show that there is a negative correlation between the geographic concentration of the population and post-tax inequality in authoritarian regimes. They propose that high geographic concentration makes revolutionary threats more likely, which in turn require the elite to provide citizens with a more favorable distribution of income. In this regard, their argument shares with this chapter the idea that the final distribution of income in an authoritarian society represents the outcome of an implicit bargain between rulers and citizens. Their finding that geographic concentration is an important predictor of post-tax income is also consistent with the hypothesis that economic progress, particularly through industrialization, is accompanied by urbanization; this in turn results in a reduction in informational frictions, the consequent convergence of beliefs, and improved coordination.

4.2 Model

There is a political elite, and a continuum of citizens of unit mass indexed by \( i \in I \). Under the status quo, denoted \( \pi = 0 \), the elite have income \( y^r_0 = \theta \bar{y}_0 \), while each citizen has income \( y^p_0 = (1 - \theta) \bar{y}_0 \). The quantity \( \bar{y}_0 \) is the total income while \( \theta \in (0, 1) \) is the fraction of income in the hands of the elite. An alternative to the status quo is an efficient policy, denote \( \pi = 1 \), under which the income of all agents is augmented by a factor \( \alpha > 0 \). Therefore, under the efficient policy the elite income rises to \( y^r_1 = \theta \bar{y}_1 = \theta (1 + \alpha) \bar{y}_0 \) while each citizen’s income becomes \( y^p_1 = (1 - \theta) \bar{y}_1 = (1 - \theta)(1 + \alpha) \bar{y}_0 \). The game begins with the elite, who are in power, deciding whether to implement the efficient policy, \( \pi = 1 \), or continue with the status quo, \( \pi = 0 \). Adopting the policy has no direct cost, but influences citizens’ beliefs as follows.

After the regime sets policy, each citizen imperfectly observes the state of the political economy \( \mu \in [0, 1] \), updates his beliefs about \( \mu \), and then decides whether or not to participate in a revolutionary uprising. The state \( \mu \) is the fraction of their income that the elite are able to keep if the citizens succeed in overthrowing the regime. The remaining fraction \( 1 - \mu \) of
elite income is appropriated by those who participated in the revolution and divided equally among the participants.\(^3\) I assume that \(\mu\) is distributed uniformly in the unit interval, and I denote its distribution \(\mathbb{F}\). I then model citizens as imperfectly observing the state by assuming that if \(\mu\) is the true state, then each citizen \(i\) observes only a private signal \(x_i\) drawn uniformly and independently across agents from the interval \(B_\eta(\mu) = [\mu - \eta, \mu + \eta]\).

If the ruler adopted the efficient policy, then \(\eta = \eta_1\) while if he did not, then \(\eta = \eta_0\). The next assumption states that adopting the reform always lowers the noise with which citizens observe the true state, but that there is always some variation in beliefs.

**Assumption 1.** \(0 < \eta_1 < \eta_0\).

Denote the set of citizens who participate in an uprising by \(I^a\) and the measure of this set by \(\xi\).\(^4\) If \(I^a = \emptyset\) then the game ends with each player receiving a payoff equal to his income. Otherwise, the game proceeds as follows.

Once the citizens make their decisions, they select a leader \(l \in I^a\) to bargain with the regime. The leader and the regime both learn the true state \(\mu\) and they observe the fraction \(\xi\) of citizens that protested. The elite then offer the attackers a transfer \(t_\pi(\mu, \xi)\) per individual in exchange for withdrawing their protest. I assume that transfers cannot be targeted so that if the leader accepts the regime’s offer, each individual \(i \in I^a\) receives the same transfer \(t_\pi(\mu, \xi)\). I also allow negative transfers (\(t_\pi(\mu, \xi) < 0\)). In other words, the regime may offer the protestors a punishment, e.g. a fine, which the leader may find more appealing to accept than continuing to fight. If the leader rejects the regime’s offer, then a confrontation between the regime and the members of \(I^a\) takes place. This results in the insurgents succeeding to

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\(^3\)If the citizens come to power following a successful revolution, they are unable to adopt the reform in the event that the regime did not adopt it at the start of the game. The qualitative results of the model would not change if I relaxed this assumption.

\(^4\)I am implicitly assuming a \(\sigma\)-algebra on which the law of large numbers holds (Judd (1985)).
overthrow the regime with probability

\[ p(\xi) = \begin{cases} 
1 & \text{if } \xi = 1 \\
z\xi & \text{if } \xi < 1 
\end{cases} \quad (4.1) \]

where \( z \in (0, 1) \). In other words, if the citizens unanimously attack the regime, then their revolt will succeed with certainty; otherwise, it will succeed only with probability \( z\xi \). Notice that \( z < 1 \) implies that the probability of a successful revolution discontinuously drops when an infinitesimal fraction of citizens choose not to attack the regime.

A citizen who fights the regime incurs a cost \( c\bar{y}_\pi \) of fighting. This cost may include the opportunity cost to fighting as well as physical costs such as the destruction of personal assets. The following assumption is equivalent to stating that although fighting the regime is costly, citizens never end up with a negative payoff.

**Assumption 2.** \( 0 < c < 1 - \theta \).

If the citizens fight the regime, then the expected payoff to the elite is \( y^e_\pi = p(\xi)\mu y^e_\pi + (1 - p(\xi))y^e_\pi \) while the expected payoff to each citizen is

\[ y^a_\pi = y^p_\pi + \frac{p(\xi)(1 - \mu)\theta}{\xi} \bar{y}_\pi - c\bar{y}_\pi \quad (4.2) \]

so long as \( \xi > 0 \). (See the next section for comments about the case where \( \xi = 0 \).) If the regime offers the protestors a transfer \( t_\pi(\mu, \xi) \) each, and the leader accepts on their behalf, then the members of \( I^a \) each receive the payoff \( y^p_\pi + t_\pi(\mu, \xi) \) while the elite get the payoff \( y^r_\pi - \xi t_\pi(\mu, \xi) \). Of course, the transfer must be feasible:

\[ t_\pi(\mu, \xi) \in T_\pi(\xi) \equiv \{ t \in \mathbb{R} | -\xi(1 - \theta)\bar{y}_\pi \leq xt \leq \theta \bar{y}_\pi \}. \quad (4.3) \]

Notice that the assumptions guarantee that the leader \( l \) always has a payoff equal to that of the other members of \( I^a \) (by the assumption that the transfers \( t_\pi(\mu, \xi) \) cannot be targeted). Moreover, the citizens know that the leader is fully informed: he learns the true state \( \mu \) and perfectly observes \( \xi \). Consequently, the leader acts as a perfect agent for the
members of $I^a$. These observations imply that the process by which the leader is selected is inconsequential and can be left unspecified.\footnote{An interesting area for future work is to delve into the agency problems that arise between revolutionary leaders and their followers. See Bueno de Mesquita (2010) for work in this direction.}

The timing of the game is summarized as follows:

1. The regime decides whether to adopt the efficient policy or preserve the inefficient status quo, i.e. chooses $\pi \in \{0, 1\}$.

2. Each citizen privately and imperfectly observes the state $\mu$ and decides whether or not to participate in an uprising against the regime, i.e. chooses from the set \{attack, refrain\}.

3. If nobody revolts (i.e. $I^a = \emptyset$), then the game ends with each citizen getting $y^p_\mu$ and the elite getting $y^r_\mu$. Otherwise, the protestors select a leader $l \in I^a$ to bargain with the regime. The regime then offers the protestors a feasible transfer $t_\pi(\mu, \xi) \in T_\pi(\xi)$.

4. The leader $l$ either accepts the offer or fights, i.e. chooses from \{accept, fight\}. If the leader accepts an offer $t_\pi(\mu, \xi)$, then the game ends with each protestor $i \in I^a$ getting $y^p_\mu + t_\pi(\mu, \xi)$, each citizen $i \in I \setminus I^a$ getting $y^p_\pi$, and the elite getting $y^r_\pi + t_\pi(\mu, \xi)$. If the leader rejects the offer, then a confrontation takes place in which the protestors succeed in overthrowing the regime with probability $p(\xi)$. Each protestor $i \in I^a$ gets the expected payoff $y^a_\pi$, each citizen $i \in I \setminus I^a$ gets $y^p_\pi$ and the elite get $y^r_\pi$.

### 4.2.1 Equilibrium Concept

Before defining the equilibrium concept, we address two problems that arise when $\xi = 0$. First, the payoff in (4.2) is not defined when the probability of overthrow is given by (4.1). It is natural to assume that when a zero mass of individuals attack the regime ($\xi = 0$), the
payoff to each citizen \( i \in I^a \) from fighting is

\[
\lim_{\xi \to 0} y^a_{\pi} = \lim_{\xi \to 0} \left( y_p^\pi + \frac{p(\xi)(1 - \mu)\theta}{\xi} \bar{y}_\pi - c\bar{y}_\pi \right) = y_p^\pi + z(1 - \mu)\theta \bar{y}_\pi - c\bar{y}_\pi
\]

which is to say that the payoff from fighting is continuous at \( \xi = 0 \). A perfect Bayesian equilibrium of the game is then defined as follows.

**Definition 1.** A strategy profile of the game consists of a policy choice \( \pi \in \{0, 1\} \); a collection of mappings \( \{\sigma_i^\pi : B_\pi \to \{\text{attack, refrain}\}\}_{i \in I, \pi = 0, 1} \), where \( B_\pi = [-\eta_\pi, 1 + \eta_\pi] \) is the signal space; a pair of mappings \( \{t_\pi : [0, 1]^2 \to \mathbb{R}\}_{\pi = 0, 1} \) s.t. \( t_\pi(\mu, \xi) \in T_\pi(\xi) \) for all \( \pi, \mu, \xi \); and a collection of mappings \( \{\varphi_\pi(\cdot, \cdot | \xi) : [0, 1] \times T_\pi(\xi) \to \{\text{accept, fight}\}\}_{\xi \in [0, 1], \pi = 0, 1} \), where \( \varphi_\pi(\mu, t|\xi) \) is the leader’s choice of accepting or rejecting the offer \( t \) when the state is \( \mu \) and \( \xi \) fraction of citizens have protested. A perfect Bayesian equilibrium of the game is a strategy profile that is sequentially rational with respect to the players’ beliefs, which are updated by Bayes rule.

Next, notice that \( \xi = 0 \) is consistent with \( I^a \neq \emptyset \) so that the regime may offer a transfer to the zero mass of citizens in \( I^a \). In these circumstances, the regime is always indifferent between making any transfer \( t \) and fighting, since it survives with certainty and all feasible transfers are costless: \( \xi t = 0 \) for all \( t \). However, notice that if the regime offered \( t_\pi(\mu, 0) \) high enough, it would be generating perverse incentives for the citizens to attack when they expect \( \xi = 0 \). Thus, the regime’s indifference generates equilibria that seem implausible. I rule out such equilibria by focusing on strategy profiles where \( t_\pi(\mu, \cdot) \) are continuous in \( \xi \) at \( \xi = 0 \) for all \( \mu \) and \( \pi \).

**Definition 2.** A perfect Bayesian equilibrium is said to be a continuous if for all \( \mu \) and \( \pi \), the regime uses a transfer schedule \( t_\pi(\mu, \cdot) \) that is continuous in \( \xi \) at \( \xi = 0 \).

The next sections focuses on establishing existence and characterizing the properties of continuous perfect Bayesian equilibrium.
4.3 Results

The game can be solved by backward induction. The first lemma, stated below, shows that in every equilibrium of the game, the probability of a successful overthrow is zero. This is a consequence of the efficiency of bargaining. Because avoiding a confrontation generates a surplus $\xi c\bar{y}$, the regime is able to send all of the protestors home by offering them a transfer that their leader accepts. In other words, fighting is costly, and the citizens and elite can save this cost by bargaining. The model implies that once the citizens have decided whether or not to attack, all of the bargaining power is with the regime, since the elite make a single take-it-or-leave-it offer. Therefore, the elite are able to extract the whole surplus $\xi c\bar{y}$ whenever $\xi > 0$.

**Lemma 1.** In any equilibrium of the game, the final payoff to every citizen $i \in I^a$ is equal to $y^a_i$ defined in (4.2). Moreover, a zero fraction of citizens fight the regime.

*Proof.* See Appendix A.1. □

The lemma implies that a successful overthrow never occurs in the model: either the citizens do not protest, or if they do protest then the regime makes the necessary concessions for them to withdraw.\(^6\)

Because the payoffs to threatening the regime are pinned down by the uniqueness of equilibrium payoffs in the bargaining subgame, the incentives for participating in a revolution can be studied by comparing the payoff $y^a_i$ to the payoff $y^p_i$. The next result shows that (almost) all citizen types have a unique equilibrium action in deciding whether or not to attack the regime.\(^7\) The following assumption, which states that the variation in citizens’

\(^6\)Like Acemölugu and Robinson (2000b), the model does not provide an explanation for why popular movements in some countries may actually succeed in overthrowing the incumbent regime. However, it is easy to incorporate this outcome by introducing common perturbations to the citizens’ final payoffs, where these perturbations are private information to the leader $l$. Of course, we would need to maintain the assumption that the elite are risk neutral.

\(^7\)I say ‘almost’ because there is only one signal value $x$ at which a citizen is indifferent between attacking
beliefs is never too high, will be useful in proving this result.

**Assumption 3.** $\eta_0 < \min\{1 - c/z\theta, c/\theta\}$.

What is the significance of the bound in Assumption 3? Lemma A of Appendix A.2 shows that if $\mu$ were perfectly observable ($\eta_0 = \eta_1 = 0$), then for each citizen $i \in I$ it would be a strictly dominant stage action to revolt when $\mu < 1 - c/z\theta$ and a strictly dominant stage action to stay at home when $\mu > 1 - c/\theta$. In other words, there are two regions in which all citizens have strictly dominant stage actions. Assumption 3 states that if the observation of $\mu$ is noisy (e.g. $\eta_0 > \eta_1 > 0$ as in Assumption 1), then for each dominance region there is a value of $\mu \in [0, 1]$ such that *every* citizen receives a signal in that region. Now notice that for any signal $x \in [\eta_\pi, 1 - \eta_\pi]$, the expectation of $\mu$ conditional on $x$ is $\hat{\mu}_x = \mathbb{E}[\mu|x] = x$. Because the payoff $y'_a$ is linear in $\mu$, the bound in Assumption 3 implies that there are signal values low enough such that it is dominant to revolt, and there are signal values high enough such that it is dominant to not revolt. But what about intermediate signal values, say in the interval $[1 - c/z\theta, 1 - c/\theta]$? If $\mu$ were perfectly observable, then there would be multiple equilibria leading to different payoffs (see Lemma A). But $\mu$ is not perfectly observable, and standard arguments will be used to show that almost every citizen type has a unique equilibrium action.

**Lemma 2.** In any equilibrium of the game, citizens with signals

$$x < \tilde{x}(\theta) \equiv 1 - c/z\theta$$

attack the regime, while citizens with signals $x > \tilde{x}(\theta)$ do not. Citizens with signal $\tilde{x}(\theta)$ are indifferent between attacking and not attacking the regime.

**Proof.** See Appendix A.3. □

and staying at home, as will be shown in the next result.
Interestingly, the threshold $\tilde{x}(\theta)$ is equal to the threshold defining the dominance region for attacking in the game with common certainty about $\mu$ (see Lemma A). In other words, there is no equilibrium in which a citizen who receives a signal in the interval $[1-c/z\theta,1-c/\theta]$ attacks the regime. The reason for this peculiarity is made clear in the proof of Lemma 2. Recall that the jump in $p(\xi)$ occurs at $\xi = 1$: only when all agents attack the regime are the citizens able to enjoy the discontinuously higher success rate for revolution. In fact, the proof of Lemma 2 shows that if the probability in (4.1) were generalized to

$$p(\xi) = \begin{cases} 
\xi & \text{if } \xi \geq \xi^p \\
z\xi & \text{if } \xi < \xi^p 
\end{cases}$$

(4.4)

for arbitrary $\xi^p \in (0,1]$, then the result of Lemma 2 would hold but with a different attacking threshold $\tilde{x}$ that depended not just on $c$, $z$ and $\theta$ but also on $\xi^p$ and $\eta_\pi$. Moreover, this threshold would be strictly decreasing in $\xi^p$ and strictly increasing in $\eta_\pi$. (See Appendix A.3 for the details.)

Although it is possible to analyze the properties of equilibria under the more general probability distribution given by (4.4), the special case where $\xi^p = 1$ is appealing for two reasons. First, it has qualitative appeal because it captures the idea that even a small amount of (passive) support for the regime benefits it discontinuously. When the total mass of citizens are against the regime, the regime has no support and collapses with certainty. But any small fraction of citizens not fighting the regime significantly improves its chances of surviving in the event of a confrontation. Second, the assumption that $\xi^p = 1$ generates the lowest possible attacking threshold $\tilde{x}$ among all $\xi^p \in (0,1]$. For an activity as risky as revolution, one might expect citizens to use the most conservative (i.e. the lowest possible) attacking threshold.

Suppose that the regime adopts the efficient policy. Then, the probability of an uncertain revolution threat decreases by $z(\eta_0-\eta_1)$, which is less than the amount that the probability of a certain revolution threat increases, $\eta_0 - \eta_1$. Therefore, the probability of a regime collapse
is higher when the elite implement the efficient policy than when they do not. Since the elite know that they will make fewer concessions in the face of an uncertain threat than in the face of a certain threat, their expected payoff from adopting the policy is lower than their expected payoff from maintaining the status quo provided the efficiency gain from adopting the the policy is not too large. This is in fact the main result of the paper.

**Proposition 1.** There is $\alpha > 0$ such that in any equilibrium of the game, the regime chooses the efficient policy $\pi = 1$ if $\alpha > \alpha$ and keeps the status quo $\pi = 0$ if $\alpha < \alpha$.

**Proof.** Lemma 2 implies that the fraction of citizens that attack the regime when the state is $\mu$ is given by

$$
\xi_{\pi}(\mu) = \begin{cases} 
1 & \text{if } \mu \in [0, \bar{x}(\theta) - \eta]\n \frac{1}{\bar{x}(\theta) - \mu + \eta} & \text{if } \mu \in [\bar{x}(\theta) - \eta, \bar{x}(\theta) + \eta]\n 0 & \text{if } \mu \in [\bar{x}(\theta) + \eta, 1].
\end{cases}
$$

(4.5)

Lemma 1 states that a zero fraction of citizens fight the regime in any reasonable equilibrium, and each citizen $i \in I^a$ receives the final payoff equal to $y_{\pi}^a$. Therefore, when the state is $\mu \in [0, \bar{x}(\theta) + \eta]$, the leader $l$ must be accepting the offer

$$
t_{\pi}^*(\mu, \xi(\mu)) = \begin{cases} 
(1 - \mu)\theta y_{\pi} - c\bar{y}_{\pi} & \text{if } \mu \in [0, \bar{x}(\theta) - \eta]\n z(1 - \mu)\theta \bar{y}_{\pi} - c\bar{y}_{\pi} & \text{if } \mu \in [\bar{x}(\theta) - \eta, \bar{x}(\theta) + \eta].
\end{cases}
$$

(See the proof of Lemma 1 for the calculation of $t_{\pi}^*(\mu, \xi)$..) If $\mu \in [\bar{x}(\theta) + \eta, 1]$ then $\xi_{\pi}(\mu) = 0$ and the elite get payoff $\theta\bar{y}_{\pi}$. Since the distribution of $\mu$ is uniform on $[0, 1]$, the expected payoff to the elite from policy $\pi$ is

$$
v(\theta, \bar{x}(\theta), \bar{y}_{\pi}, \eta) = \left[ \int_{0}^{\bar{x}(\theta) - \eta} \left( \mu + \frac{c}{\theta} \right) d\mu + \int_{\bar{x}(\theta) + \eta}^{1} \left( 1 - \frac{\bar{x}(\theta) - \mu + \eta}{2\eta} \right) \left( 1 - \mu - \frac{c}{\theta} \right) d\mu + \int_{\bar{x}(\theta) + \eta}^{1} \frac{d\mu}{\eta} \theta\bar{y}_{\pi} \right]
$$

$$
= \left[ (1 - \bar{x}(\theta))^2 \left( \frac{1}{2} - z \right) + (1 - \bar{x}(\theta))(z + \eta(1 - z)) + \frac{1}{2} + (\eta)^2 \left( \frac{1}{2} - \frac{z}{3} \right) \right] \theta\bar{y}_{\pi}
$$

(4.6)
where \( \tilde{x}(\theta) = 1 - c/z\theta \) from Lemma 2. We can then show that \( v(\theta, \tilde{x}(\theta), \bar{y}_\pi, \eta_\pi) \) is strictly increasing in \( \eta_\pi \):

\[
\frac{\partial v(\theta, \tilde{x}(\theta), \bar{y}_\pi, \eta_\pi)}{\partial \eta_\pi} = \left[ c \left( \frac{1 - z}{z} \right) + \eta_\pi \theta \left( 1 - \frac{2z}{3} \right) \right] \bar{y}_\pi > 0.
\] (4.7)

Thus \( v(\theta, \tilde{x}(\theta), \bar{y}_0, \eta_0) > v(\theta, \tilde{x}(\theta), \bar{y}_1, \eta_1) \) by Assumption 1. Clearly \( v(\theta, \tilde{x}(\theta), (1 + \alpha)\bar{y}_0, \eta_1) \) is strictly increasing and unbounded in \( \alpha \). Therefore, the solution to

\[
v(\theta, \tilde{x}(\theta), \bar{y}_0, \eta_0) = v(\theta, \tilde{x}(\theta), (1 + \alpha)\bar{y}_0, \eta_1)
\] (4.8)

is some \( \alpha = \alpha > 0 \). The result then follows from the definition of \( \alpha \) and the fact that the right side of (4.8) is strictly increasing in \( \alpha \): if \( \alpha < \alpha \) then \( v(\theta, \tilde{x}(\theta), \bar{y}_0, \eta_0) > v(\theta, \tilde{x}(\theta), \bar{y}_1, \eta_1) \) whereas if \( \alpha > \alpha \) we have \( v(\theta, \tilde{x}(\theta), \bar{y}_0, \eta_0) < v(\theta, \tilde{x}(\theta), \bar{y}_1, \eta_1) \).

Proposition 1 states that when the gains from implementing the efficient policy are not very large, \( \alpha < \alpha \), then the political hold-up problem is in effect: the regime does not adopt the efficient policy because the increase in output that it would generate is not sufficient to offset the increased risk of a coordinated uprising. If \( \alpha > \alpha \) then these gains are large enough, and the regime adopts the efficient policy despite the greater likelihood of a strong attack.

To show that the equilibrium set is nonempty, it is easy to verify that the following strategy profile is a reasonable equilibrium of the game. The regime adopts the policy \( \pi = 1 \) if \( \alpha \geq \alpha \); otherwise, it chooses to maintain the status quo, \( \pi = 0 \). All citizens who receive signals \( x < \tilde{x}(\theta) \) attack the regime while citizens who receive signals \( x \geq \tilde{x}(\theta) \) refrain from attacking. If a fraction \( \xi \in [0, 1] \) citizens attack the regime, then the regime offers transfers \( t^*_\pi(\mu, \xi) \) defined in (A2) of Appendix A.1. The leader \( l \) accepts any transfer \( t \geq t^*_\pi(\mu, \xi) \) and rejects all other offers. This immediately implies existence of equilibrium.

**Corollary 1.** There exists an equilibrium to the game.
Finally, establishing the following comparative statics results for the threshold $\alpha$ is straightforward. As the fraction of income held by the elite $\theta$ rises, then $\alpha$ also increases and the regime blocks the efficient policy for a larger range of $\alpha$. Interestingly, $\alpha$ is also higher for a higher cost of attack $c$ and a lower effectiveness of attack $z$. This is because the regime is able to appropriate each protestor’s cost of attack $c$ through bargaining, while if $z$ is low, then it is already difficult for the citizens to launch an effective attack and the elite do not have much to gain from adopting the efficient policy. Finally, as $\eta_0$ increases or $\eta_1$ decreases, $\alpha$ increases. The regime is more likely to protect the status quo if the efficient policy reduces the heterogeneity in citizens’ beliefs. These results are shown in Appendix A.5.

4.4 Extensions

Throughout this section, and particularly for Section 4.4.3, it will be useful to assume that the elite are a homogenous minority with mass $n < 1$. This guarantees that each elite person has income $y_\pi^r/n$. Denote by $\lambda = n/(1 + n) < 1/2$ the share of the elite in the population. The following assumption guarantees that each elite person is richer than any citizen.

Assumption 4. $\lambda < \theta$.

4.4.1 Natural Resources, Human Capital and Market Frictions

Suppose that there are two sources of income: natural resource rents $R$, which may include agrarian rents; and industrial output $Y(K, L, A)$ written as a function of its inputs, capital $K$, labor $L$, and human capital $A$. The output function $Y$ is strictly increasing in each input, and homogenous of degree one in capital and labor.

The total capital and labor endowments of society are $\bar{K}$ and $\bar{L}$. Under the status quo, again denoted $\pi = 0$, there are (unmodeled) market frictions that result in only a fraction $\delta \in (0, 1)$ of the capital and labor stocks being matched to produce output. The remaining
fraction $1 - \delta$ of these inputs are unemployed. This implies that the total industrial output under the status quo is only

$$Y_0 = Y(\delta \bar{K}, \delta \bar{L}, A) = \delta Y(\bar{K}, \bar{L}, A) \equiv \delta \bar{Y} \tag{4.9}$$

where the second equality follows from homogeneity of degree one.

The regime has the opportunity to implement a policy, again denoted $\pi = 1$, that eliminates the market frictions. Under this policy, the unemployed capital and labor are united so that total industrial income rises to the efficient level $Y_1 = \bar{Y}$. Therefore, total income under policy $\pi \in \{0, 1\}$ is

$$\bar{y}_\pi = Y_\pi + R. \tag{4.10}$$

As before, assume that the political elite have income $y^e_\pi = \theta \bar{y}_\pi$ when the policy is $\pi \in \{0, 1\}$, while each citizen $i \in I$ has income $y^p_\pi = (1 - \theta)\bar{y}_\pi$. Also, assume that the political elite and citizens play the game described in Section 4.2. Then Lemma 1, A and 2 hold as stated, and a counterpart to Proposition 1 is the following.

**Proposition 2.** Let $\rho = R/\bar{Y}$ be the ratio of natural resource rents to potential industrial output. There exist $\bar{\rho} > 0$ and $\bar{\delta}(\rho) \in (0, 1)$ for all $\rho \in [0, \bar{\rho})$ such that in any equilibrium of the game, the regime chooses the efficient policy $\pi = 1$ if $\rho < \bar{\rho}$ and $\delta < \bar{\delta}(\rho)$, and keeps the status quo $\pi = 0$ if $\rho \geq \bar{\rho}$ or if $\rho < \bar{\rho}$ and $\delta > \bar{\delta}(\rho)$.

**Proof.** The proof is analogous to the proof of Proposition 1. Let $\psi(\theta, \eta_\pi)$ denote the term in square brackets in equation (4.6) above. Then the expected payoff to the elite from adopting the efficient policy is $v(\theta, \bar{x}(\theta), \bar{Y} + R, \eta_1) = \psi(\theta, \eta_1)\theta(\bar{Y} + R)$ while their expected payoff from preserving the status quo is $v(\theta, \bar{x}(\theta), \delta \bar{Y} + R, \eta_0) = \psi(\theta, \eta_0)\theta(\delta \bar{Y} + R)$. Define

$$\bar{\rho} = \frac{\psi(\theta, \eta_1)}{\psi(\theta, \eta_0) - \psi(\theta, \eta_1)}. \tag{4.11}$$

It is then easy to verify that $\rho \geq \bar{\rho}$ implies $v(\theta, \bar{x}(\theta), \bar{Y} + R, \eta_1) < v(\theta, \bar{x}(\theta), \delta \bar{Y} + R, \eta_0)$ for
all $\delta \in (0, 1)$. On the other hand, suppose $\rho < \bar{\rho}$. Then for $\delta$ small enough

$$\rho[\psi(\theta, \eta_0) - \psi(\theta, \eta_1)] < \psi(\theta, \eta_1) - \delta \psi(\theta, \eta_0)$$

$$\iff v(\theta, \bar{x}(\theta), \delta \bar{Y} + R, \eta_0) < v(\theta, \bar{x}(\theta), \bar{Y} + R, \eta_1)$$  \hspace{0.5cm} (4.12)$$

whereas for $\delta$ close enough to 1, the inequalities are reversed. (This follows from (4.7) in the proof of Proposition 1.) Since the left side of (4.12) is continuous and strictly increasing in $\delta$, there exists $\delta(\rho) \in (0, 1)$ such that the two sides of the inequality are made equal. Moreover, this threshold has the stated properties.

The qualitative implications of Proposition 2 can be understood by establishing some additional comparative statics. Suppose $\rho < \bar{\rho}$. It follows from the implicit function theorem that the threshold $\delta(\rho)$ is decreasing in $\rho$. To see this, recall that $\delta$ equates the two sides of the first inequality in (4.12). So as $\rho$ increases, $\delta(\rho)$ must decrease. Consequently, if natural resource rents $R$ constitute a large source of income in the economy, then Proposition 2 states that the set of parameter values for which the regime is willing to adopt the efficient policy is small. This is consistent with the Resource Curse. Conversely, suppose that the level of human capital $A$ increases. The only effect of human capital on $\delta(\rho)$ is through its effect on $\rho$. Since $\rho$ decreases as $A$ increases, authoritarian societies that enjoy higher levels of human capital are more likely to implement efficient policies than societies that experience relatively lower levels of human capital. This may account for why several authoritarian regimes in East Asia have implemented many economic reforms.

The intuitions for Propositions 1 and 2 are closely related. Under the assumptions of this section, the maximum gain in output as a result of implementing the efficient policy is $(1 - \delta)\bar{Y}$, which is bounded by $\bar{Y}$. So when natural resource rents $R$ are very large, the percentage increase in income as a result of implementing the efficient policy cannot be very large. In fact, for a fixed value of $\bar{Y}$, there is a minimum value for $R$ (consequently a minimum value for $\rho$) such that this percent increase must be lower than $\bar{\pi}$ for all $\delta \in (0, 1)$.
If the actual $R$ is lower than this value, then for $\delta$ low enough, the percentage gain in output can be greater than $\bar{\alpha}$. The threshold $\bar{\delta}(\rho)$ determines how low $\delta$ must be for this to be true. In light of this discussion, I will hereafter relate low values of $\alpha$ to situations where natural resource rents are a large source of income, and situations where the level of human capital is low.

4.4.2 Establishing Rents

I have so far assumed that if the regime adopts the efficient policy then the incomes of all agents rise symmetrically. Suppose instead that the elite can establish ownership over the surplus created by new policy. Specifically, the policy raises the elite income to $y_1^e = (\theta + \alpha \beta)\bar{y}_0$ and each citizen’s income to $y_1^p = (1 - \theta + \alpha(1 - \beta))\bar{y}_0$. If the elite adopt the efficient policy, then they also choose $\beta \in [\theta, \bar{\beta}]$ where $\theta < \bar{\beta} \leq 1$ before the citizens decide whether or not to revolt. The case where $\beta = \theta$ corresponds to the original model of Section 4.2 where the policy has a symmetric effect on all agents’ incomes. For any $\beta \in [\theta, \bar{\beta}]$, total income under the efficient policy is always $\bar{y}_1 = (1 + \alpha)\bar{y}_0$ so that the role of $\beta$ is only to change the degree of inequality. In particular, choosing higher values of $\beta$ can be interpreted as establishing greater rents over the surplus generated by the efficient policy. Given $\beta$, the fraction of total income controlled by the elite is

$$\theta_\beta = \frac{\theta + \alpha \beta}{1 + \alpha}.$$  \hfill (4.13)

As the game has now been modified, the equilibrium concept can also be modified in the obvious way to include the choice of $\beta$. Furthermore, the following assumptions will replace Assumptions 2 and 3 throughout the remainder of this paper.

Assumption 2’. $0 < c < 1 - \theta_\beta$

Assumption 3’. $\eta_0 < \{1 - c/z\theta, c/\theta_\beta\}$. 

One may think that giving the citizens ownership over the benefits of the efficient policy may deter them from attacking the regime, and therefore the regime will be more willing to adopt the policy. However, the next result shows that the opposite is true: the inefficiency is in fact mitigated by allowing the elite to claim greater rents for themselves.

**Proposition 3.** In the game where the regime can also choose $\beta \in [\theta, \bar{\beta}]$, there exists $\alpha \in (0, \bar{\alpha})$ such that in any equilibrium, the regime preserves the status quo $\pi = 0$ if $\alpha < \alpha$ and adopts the efficient policy $\pi = 1$ if $\alpha > \alpha$. If the regime implements the efficient policy $\pi = 1$ then they always choose $\beta = \bar{\beta}$. Moreover, $\alpha$ is strictly decreasing in $\bar{\beta}$.

**Proof.** Observe that for all $\beta \in [\theta, \bar{\beta}]$ we have $\theta \beta \leq \bar{\beta} \beta < 1$, and therefore $c/\theta \beta \leq c/\bar{\beta} \leq c/\theta$. This implies

$$0 < c < 1 - \theta \beta \text{ and } \eta_0 < \{1 - c/z \theta, c/\theta \beta\} \forall \beta \in [\theta, \bar{\beta}]. \quad (4.14)$$

Therefore, by the same argument used to prove Lemmas 1, A and 2, in any continuous Perfect Bayesian equilibrium to the game where the regime chooses $\beta \in [\theta, \bar{\beta}]$, the citizens use the attacking threshold $\tilde{x}(\theta \beta) = 1 - c/z \theta \beta < 1$ if the efficient policy is implemented and the attacking threshold $\tilde{x}(\theta \beta) = 1 - c/z \theta$ if it is not. Conditional on adopting the efficient policy, the payoff to the elite is strictly increasing in $\beta$:

$$\frac{\partial v(\theta \beta, \tilde{x}(\theta \beta), \bar{y}, \eta)}{\partial \beta} = \frac{\partial \theta \beta}{\partial \beta} \left[\frac{\partial v(\theta \beta, \tilde{x}(\theta \beta), \bar{y}_1, \eta_1)}{\partial \theta \beta}\right] > 0 \quad (4.15)$$

since $\partial \theta \beta / \partial \beta = \alpha / (1 + \alpha) > 0$ and the term in square brackets is positive, as shown in equation (A22) of Appendix A.5. This implies that the regime will choose $\beta = \bar{\beta}$. In addition, we have

$$v(\theta \beta, \tilde{x}(\theta \beta), (1 + \bar{\pi})\bar{y}_0, \eta_1) > v(\theta, \tilde{x}(\theta), (1 + \bar{\pi})\bar{y}_0, \eta_1) = v(\theta, \tilde{x}(\theta), \bar{y}_0, \eta_0) \quad (4.16)$$

where the inequality follows again from (A22), and the equality follows from the definition of $\bar{\pi}$ in the proof of Proposition 1. It is easy to see that $\bar{\beta} > \theta$ implies that $\theta \beta$ is strictly
increasing in $\alpha$. Therefore, $v(\theta, \bar{\beta}, \tilde{x}(\theta), (1 + \alpha)\bar{y}_0, \eta_1)$ is strictly increasing and unbounded (above or below) in $\alpha \in (-\infty, \infty)$. Consequently, the solution to

$$v(\theta, \bar{\beta}, \tilde{x}(\theta), (1 + \alpha)\bar{y}_0, \eta_1) = v(\theta, \bar{x}(\theta), \bar{y}_0, \eta_0)$$

must be some $\alpha = \underline{\alpha} < \bar{\alpha}$. The right hand side of (4.17) is constant in both $\alpha$ and $\bar{\beta}$, while the left hand side is strictly increasing in both. Therefore, by the implicit function theorem, $\alpha$ is strictly decreasing in $\bar{\beta}$.

It is left to verify that $\underline{\alpha} > 0$. To see this, note that if $\alpha = 0$ then

$$v(\theta, \bar{\beta}, \tilde{x}(\theta), (1 + \alpha)\bar{y}_0, \eta_1) = v(\theta, \bar{x}(\theta), \bar{y}_0, \eta_1) < v(\theta, \bar{x}(\theta), \bar{y}_0, \eta_0).$$

Therefore, it must be that $\underline{\alpha} > 0$. The result follows from the definition of $\underline{\alpha}$ and the fact that $v(\theta, \bar{\beta}, \tilde{x}(\theta), (1 + \alpha)\bar{y}_0, \eta_1)$ is strictly increasing in $\alpha$.

Proposition 3 states that the inefficiency is mitigated by allowing the elite to make the benefits of economic reform less equitable. However, notice that although the elite are more willing to adopt the efficient policy when they are able to establish greater ownership over the surplus that it generates, adopting the new policy nevertheless empowers the citizens, politically, by raising the probability with which they launch an effective uprising; that is, it increases their political power.

The proposition establishes the comparative statics of $\underline{\alpha}$ with respect to $\bar{\beta}$. Appendix A.5 shows that the comparative statics of $\underline{\alpha}$ with respect to $\theta, c, z, \eta_0$ and $\eta_1$ are the same as the comparative statics of $\bar{\alpha}$ with respect to these variables. Moreover, the fact that the equilibrium set is nonempty for this extension follows almost instantly from the strengthening of Assumptions 2 and 3, the characterization of the regime’s initial choices in Proposition 2 and the fact that the equilibrium set is nonempty for the baseline model in Section 4.2.
4.4.3 Democratization

The last result showed that if the elite are able to establish rents over the surplus created by the efficient policy, then they may be enticed to adopt it in spite of the increased risk of a coordinated attack. In this section, I enable the elite to altogether eliminate the risk of a coordinated attack by transitioning to democracy. The main result of this section will show that the elite may find it in their economic interest to step out of the political limelight if by doing so they are guaranteed to keep a large share of their income.

The elite first decide whether or not to democratize, \( \omega \in \{0, 1\} \), where \( \omega = 1 \) indicates democratization. If they choose to keep the regime authoritarian, \( \omega = 0 \), then the game proceeds as in the previous section, i.e. they decide whether or not to adopt the efficient policy, \( \pi \in \{0, 1\} \), and if \( \pi = 1 \) they also choose \( \beta \in [\theta, \bar{\beta}] \) and so forth. If the elite decide to democratize, \( \omega = 1 \), then a randomly selected citizen sets the tax rate \( \tau \in [0, \bar{\tau}] \). As in standard models of redistributive politics under democracy (e.g. Meltzer and Richard 1981), I assume that taxes are linear, purely redistributive, and take the form of transfers from the rich to the poor. I also assume that taxation causes no distortions and redistributive transfers cannot be targeted. Since the citizens are a homogenous majority, the tax rate that is implemented is a Condorcet-winner, and can be found by maximizing a citizen’s post-fisc income. Following redistribution, the game ends.

As in the previous section, the equilibrium concept can be extended in the obvious way to include the elite’s choice of \( \omega \in \{0, 1\} \), and any individual citizen’s choice of \( \tau \in [0, \bar{\tau}] \). Assumption 4 implies that \( 0 < 1 - \lambda/\theta < 1 \). The following assumption then ensures that redistributive taxation does not totally eliminate inequality, and taxes are never so high that an elite person becomes poorer than a regular citizen.

Assumption 5. \( \bar{\tau} < 1 - \lambda/\theta \).
The upper bound \( \bar{\tau} \) on the tax rate may be viewed as a capacity constraint or may reflect the fact that in reality democracies do not totally expropriate the rich.\(^8\) The assumptions made so far imply that the the post-fisc income of the elite is simply given by \( y_{D\pi} = (1 - \tau)y_{r\pi}^e \), with each individual elite person getting \( y_{D\pi}^e = y_{r\pi}^e / n \). On the other hand, the income of each citizen is given by \( y_{p\pi} = y_{p\pi}^\pi + \tau y_{r\pi}^\pi \).

**Proposition 4.** For every \( \alpha > 0 \) and \( \bar{\beta} \in (\theta, 1] \), there exists a tax threshold \( \tau^*(\alpha, \bar{\beta}) \in (0, 1] \) such that in any equilibrium of the game, the elite (i) democratize \( \omega = 1 \), adopt the efficient policy \( \pi = 1 \), and set \( \beta = \bar{\beta} \) when \( \bar{\tau} < \tau^*(\alpha, \bar{\beta}) \); (ii) remain authoritarian \( \omega = 0 \), and preserve the status quo \( \pi = 0 \), when \( \alpha < \bar{\alpha} \) and \( \bar{\tau} > \tau^*(\alpha, \bar{\beta}) \); and (iii) remain authoritarian \( \omega = 0 \), adopt the efficient policy \( \pi = 1 \), and set \( \beta = \bar{\beta} \) when \( \alpha > \bar{\alpha} \) and \( \bar{\tau} > \tau^*(\alpha, \bar{\beta}) \). Moreover, \( \tau^*(\alpha, \bar{\beta}) \) is continuous in \( \alpha \) and \( \bar{\beta} \) and if \( \tau^*(\alpha, \bar{\beta}) < 1 - \lambda / \theta \) then \( \tau^*(\alpha, \bar{\beta}) \) is strictly increasing in both \( \alpha \) and \( \bar{\beta} \), otherwise it is constant.

*Proof.* See Appendix A.4. \( \square \)

The following additional comparative statics results are shown in Appendix A.5. Whenever \( \tau^*(\alpha, \bar{\beta}) \) is less than \( 1 - \lambda / \theta \), it is strictly increasing in \( \theta \) and \( z \) and strictly decreasing in \( c \). Therefore, all other things equal, countries with relatively higher starting inequality are more likely to go democratic. Appendix A.5 also shows that if \( \tau^*(\alpha, \bar{\beta}) < 1 - \lambda / \theta \), then \( \alpha < \bar{\alpha} \) then \( \tau^*(\alpha, \bar{\beta}) \) is decreasing in \( \eta_1 \) and if \( \alpha > \bar{\alpha} \) then \( \tau^*(\alpha, \bar{\beta}) \) is decreasing in \( \eta_0 \).

Figure 1 plots the function \( \tau^*(\alpha, \bar{\beta}) \) against \( \alpha \), and the dotted line marks the productivity level \( \alpha = \bar{\alpha} \). In the region below \( \tau^* \) the elite democratize. In the region above \( \tau^* \) and to the left of the dotted line, the regime remains authoritarian and does not adopt the efficient policy, while in the region above \( \tau^* \) and to the right of the dotted line, the regime remains democratic but adopts the efficient policy.

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\(^8\)See Putterman (1997) for an evaluation of a number of explanations as to why redistribution may be limited. Explanations not covered by Putterman can be found in Roemer (1998), Acemoglu and Robinson (2008) and Besley and Persson (2010).
We can use Proposition 4 and Figure 1 to generate a number of empirical predictions that are summarized as follows.

**Political and Economic Backwardness.** Recall from Section 4.4.1 that one might expect $\alpha$ to be low in natural resource rich countries or peasant societies with low levels of human capital. The reliance on natural resources or land for income implies that $\bar{\tau}$ is high because these resources can more easily be expropriated (e.g. through land reforms) than other sources of income, especially those that are human capital-intensive. Therefore, one would expect countries with these characteristics to lie on the top left of Figure 1: the political regime remains authoritarian and blocks economic reform.

**Political Backwardness and Economic Development.** When the level of human capital is high and society relies relatively little on natural resources or land as the source of income, then it is likely to have a high growth potential $\alpha$. If the society is relatively ethnically or culturally homogenous, or if there is very little inter-group conflict in spite of heterogeneity, then $\bar{\tau}$ may be high. This could either be because group conflict tends to adversely affect the
level of redistribution in society (e.g., Roemer, Lee and van der Straeten 2007) or because group fractionalization is conducive to corruption (e.g., Mauro 1995) and therefore suppresses redistribution. The Asian cases mentioned in the introduction may lie on the top right of Figure 1: the regime adopts productive reforms, but does not democratize.

*The Relationship between Taxation and Development.* Besley and Persson (2009) observe that the wealthiest countries are also those with the highest levels of taxation. Figure 1 is consistent with this observation. Because $\tau^*(\alpha, \bar{\beta})$ is strictly increasing in $\alpha$, democracies with relatively higher levels of taxation will also have a high (realized) growth potential $\alpha$. Poorer democracies have relatively lower levels of redistribution, while in authoritarian countries there is essentially no redistribution.

### 4.5 Conclusion

This chapter has argued that some authoritarian regimes will not adopt a new technology or institute an economic reform because doing so would empower the disenfranchised citizens by improving their ability to coordinate a revolutionary uprising against the incumbent regime. Facing the threat of overthrow, the elite may have to make large concessions to the citizens, and in expectation they may be better off from not adopting the reform than from adopting it. If they do not adopt the reform, then the political hold-up problem is in effect: the elite do not take an action that increases productivity because the same action also increases the political power of the citizens. This is similar to the hold-up problem in bargaining theory, where one party does not take an action that would increase total surplus because that action also improves the bargaining power of her adversary.

The baseline model was enriched to show that a reliance on natural resources for income may inhibit future progress, as regimes become less likely to adopt the efficient policy. On the other hand, a society that is endowed with high levels of human capital is more likely
to enjoy the benefits of economic reform even when the regime is authoritarian. However, such societies are also very likely to transition to democracy. A second application of the model provided a new rationale for why rising levels of income might be associated with rising levels of inequality as well as increases in political power and mobilization among the disenfranchised. Finally, a third application showed that the potential for economic progress may provide the impetus for democratization.

Appendix

A.1

Proof of Lemma 1. There are two cases: (i) $\xi > 0$ and (ii) $\xi = 0$ but $I^a \neq \emptyset$.

Case (i): ($\xi > 0$). Denote by $t^*_\pi(\mu, \xi)$ the value of the transfer that makes any citizen $i \in I^a$ that learns the true state $\mu$ indifferent between accepting and rejecting the regime’s offer. Also define

$$z(\xi) = \begin{cases} 
1 & \text{when } \xi \geq \xi^p \\
z & \text{when } \xi < \xi^p. 
\end{cases} \quad (A1)$$

where $\xi^p = 1$. (The argument of this proof will actually carry through when the probability of success is given by (4.4) in the main text, rather than (4.1) and $\xi^p \in (0, 1]$ is arbitrary.)

Thus $t^*_\pi(\mu, \xi)$ solves

$$y^p_\pi + t^*_\pi(\mu, \xi) = y^p_\pi + z(\xi)(1 - \mu)\theta y_\pi - c y_\pi$$

$$\Rightarrow t^*_\pi(\mu, \xi) = z(\xi)(1 - \mu)\theta y_\pi - c y_\pi. \quad (A2)$$

Note that this transfer is always feasible, and in particular $t^*_\pi(\mu, \xi) \in \text{int } T_\pi(\xi) \forall \pi, \xi$.

In equilibrium, the leader $l$ rejects any offer strictly less than $t^*_\pi(\mu, \xi)$ and accepts any offer strictly greater. To show that the regime will not make an offer strictly less than $t^*_\pi(\mu, \xi)$, it is sufficient to show that their payoff from making an offer $t^*_\pi(\mu, \xi) + \epsilon y_\pi$ with $0 < \epsilon < c$ is strictly greater than the payoff they receive from fighting. Their payoff from making the
offer \( t^*_n(\mu, \xi) + \epsilon y_n \) is

\[
\theta y_n - \xi t^*_n(\mu, \xi) - \xi \epsilon y_n = \theta y_n - z(\xi) \theta y_n + z(\xi) \mu \theta y_n + \xi c y_n - \xi \epsilon y_n
\]

\[
> \theta y_n - z(\xi) \theta y_n + z(\xi) \mu \theta y_n = p(\xi) \mu \theta y_n + (1 - p(\xi)) \theta y_n
\]

which is the payoff from fighting. The first equality holds from substituting \( t^*_n(\mu, \xi) \) and rearranging. The inequality holds because \( c > \epsilon \) by assumption. The final equality holds from replacing \( p(\xi) = z(\xi) \xi \). Thus, the regime never makes an offer less than \( t^*_n(\mu, \xi) \). The regime also doesn’t make an offer strictly greater than \( t^*_n(\mu, \xi) \), since it could deviate to a slightly lower offer (but one that is still greater than \( t^*_n(\mu, \xi) \)) and the citizens would still accept. Such a deviation is clearly profitable.

Therefore, in equilibrium the elite can only be offering \( t^*_n(\mu, \xi) \). Note that although the citizens are indifferent between accepting this offer and rejecting it, in any equilibrium they must in fact be accepting it. For suppose they were rejecting; then the regime could deviate to the offer \( t^*_n(\mu, \xi) + \epsilon y_n \), which would be accepted, and which we showed above would be preferable to fighting. Thus each citizen \( i \in I^a \) receives a payoff equal to \( y^a_n \), by definition of \( t^*_n(\mu, \xi) \), and the fraction of citizens that fight the regime is zero.

Case (ii): \((\xi = 0, I^a \neq \emptyset)\). In this case, the regime is guaranteed to survive. Therefore, it is indifferent between fighting and making any feasible offer \( t \), since even if the citizens were to accept, it would cost the regime nothing: \( \xi t = 0 \). However, in equilibrium (see Definition 2), the regime makes the offer \( t_n(\mu, \xi) = \lim_{\xi \to 0} t^*_n(\mu, \xi) = z(1 - \mu) \theta y_n - c y_n \), and the leader either accepts or rejects: either way, all citizens \( i \in I^a \) receive a payoff equal to \( y^a_n \).

A.2

Lemma A. Replace Assumption 1 with the assumption that \( \eta_1 = \eta_0 = 0 \). Then there are thresholds \( 0 < \underline{\mu} < \overline{\mu} < 1 \), such that if \( \mu > \overline{\mu} \), then it is a stage dominant action for each citizen to not revolt; if \( \mu < \underline{\mu} \), then it is a stage dominant action for each citizen to revolt; and if \( \mu \in [\underline{\mu}, \overline{\mu}] \), then there are multiple reasonable equilibria that lead to different payoffs.
Proof. As in the proof of Lemma 1, let $\xi^p = 1$. (Again, the argument of this proof will hold when $p(\xi)$ is given by (4.4) rather than (4.1) and $\xi^p \in (0, 1]$ is arbitrary.) By Lemma 1, in any equilibrium of the game, all citizens who participate in an uprising are guaranteed to get a payoff equal to $y^a$. Observe that $y^p \preceq y^a$ if and only if

$$0 \preceq p(\xi)(1 - \mu)\theta/\xi - c.$$  \hspace{1cm} (A3)

Then the threshold $\mu = \bar{\mu}$ exists if and only if it satisfies

$$(1 - \mu)\theta = c.$$ \hspace{1cm} (A4)

Here we are evaluating the right side of (A3) assuming $\xi \geq \xi^p$ because for not revolting to be a dominant action it must be profitable at every $\xi$ and if it is profitable at any $\xi \geq \xi^p$ then it is profitable everywhere. To see why the dominance region exists, note that at $\mu = 1$, the left hand side of (A4) is strictly less than the right. Note that $0 < 1 - c/z\theta$ follows from Assumptions 1 and 3, implies that $c < z\theta$, so that at $\mu = 0$ the left side of (A4) is strictly greater than the right. The existence of $\bar{\mu}$ thus follows from the intermediate value theorem and the strict monotonicity of the left side of (A4) in $\mu$. Showing that there is $\underline{\mu}$ solving $z(1 - \mu)\theta = c$ for $\mu$, and having the properties described in the Lemma, follows a similar argument. The fact that $\underline{\mu} < \bar{\mu}$ follows from the assumption that $z < 1$. Finally, observe that if $\mu \leq \underline{\mu}$, then revolting is a best response if $\xi \geq \xi^p$ agents revolt and if $\mu \geq \underline{\mu}$, not revolting is a best response if $\xi < \xi^p$ agents revolt. Therefore, on $[\underline{\mu}, \bar{\mu}]$ there are multiple reasonable equilibria that lead to different payoffs. \hfill \QED

A.3

Proof of Lemma 2. I prove the lemma in three steps. The first step is to show that the game has strategic complementarity. The second step establishes a simple continuity result that is required in the proof strategy appearing in Morris and Shin (1998). The final step is to replicate the argument of Morris and Shin (1998) in the present setup.
In the course of the proof it will be insightful to assume that the probability of a successful uprising is more generally given by equation (4.4) in the main text, where \( \xi^p \in (0, 1] \) is arbitrary, rather than by (4.1). The result will then follow by substituting \( \xi^p = 1 \). Also recall that the proofs of Lemma 1 and A show that the results hold even under this generalization.

For citizen \( i \in I \), the decision to participate in a revolutionary attack is represented by the function \( \sigma^i : B_\pi \to \{\text{attack, refrain}\} \) where \( B_\pi = [-\eta_\pi, 1 + \eta_\pi] \) is the set of possible signals. An action profile \( \{\sigma^i\}_{i \in I} \) gives rise to an aggregate action \( \sigma_\pi : B_\pi \to \mathbb{R}_+ \), which is the density of citizens who revolt as a function of the signals they receive. An agent with signal \( x \) attacks the regime if \( \mathbb{E}[y^a_\pi|x] > y^p_\pi \) and does not attack if the reverse inequality holds.

For \( x \in [\eta_\pi, 1 - \eta_\pi] \) we have \( \hat{\mu}_x \equiv \mathbb{E}[\mu|x] = x \) and thus

\[
\mathbb{E}[y^a_\pi|x] > y^p_\pi \iff \frac{p(\xi)(1 - x)\theta}{\xi} - c > 0.
\]

Therefore, for a citizen with signal \( x \in [\eta_\pi, 1 - \eta_\pi] \), the payoff to attacking the regime can be renormalized to \( p(\xi)(1 - x)\theta/\xi - c \) and the payoff to not attacking will be 0. I do this for the remainder of this proof. In a reasonable equilibrium, each \( \sigma^i_\pi(x) \) is a best response to the associated aggregate action \( \sigma_\pi \) at every \( x \) given \( \hat{\mu}_x \).

**Step 1.** Take an action profile \( \{\sigma^i_\pi\}_{i \in I} \) and let the associated aggregate action be denoted \( \sigma_\pi \). Given \( \sigma_\pi \), the fraction of citizens that participate in the uprising is

\[
\xi(\mu, \sigma_\pi) = \frac{1}{2\eta_\pi} \int_{\mu - \eta_\pi}^{\mu + \eta_\pi} \sigma_\pi(x)dx.
\] \hspace{1cm} (A5)

Now define the set \( R_\pi(\sigma_\pi) = \{\mu|\xi(\mu, \sigma_\pi) \geq \xi^p\} \), which is the set of states at which the critical mass \( \xi^p \) is met. By Lemma 1, the expected reasonable equilibrium payoff to participating is

\[
u(x, \sigma_\pi) = \frac{1}{2\eta_\pi} \int_{x - \eta_\pi}^{x + \eta_\pi} \frac{p(\xi)(1 - \mu)\theta}{\xi} d\mu - c
= z(1 - x)\theta + \frac{1}{2\eta_\pi} \int_{R(\sigma_\pi) \cap [x - \eta_\pi, x + \eta_\pi]} (1 - z)(1 - \mu)\theta d\mu - c \] \hspace{1cm} (A6)

so long as \( x \in [\eta_\pi, 1 - \eta_\pi] \). This is because \( \mathbb{E}[\mu|x] = x \) when \( x \in [\eta_\pi, 1 - \eta_\pi] \), which implies
that participating guarantees at least $z(1 - x)\theta$ throughout the posterior support of $\mu$. (For $x \notin [-\eta_\pi, 1 - \eta_\pi]$ the bounds of integration can be adjusted accordingly.)

Expression (A6) can be used to show that the game has strategic complementarity. Indeed, suppose that for aggregate actions $\sigma_\pi$ and $\sigma'_\pi$, we have $\sigma_\pi(x) \geq \sigma'_\pi(x)$ for all $x$. Then $\xi(\mu, \sigma_\pi) \geq \xi(\mu, \sigma'_\pi)$ for all $\mu$. This implies $R_\pi(\sigma'_\pi) \subseteq R_\pi(\sigma_\pi)$, which in turn implies $u(x, \sigma_\pi) \geq u(x, \sigma'_\pi)$. Thus, the game has strategic complementarity.

**Step 2.** Define a *threshold action* to be an action rule $\sigma_{i, s}^{i, \pi} : B_\pi \to \{\text{attack, refrain}\}$ in which a citizen with signal $x \in B_\pi$ participates in the revolution if and only if $x < s$ for some $s \in \mathbb{R}$. In a symmetric action profile of threshold actions with threshold $s$, the aggregate action is given by the step function

$$
\sigma_{\pi, s}(x) = \begin{cases} 
1 & \text{if } x < s \\
0 & \text{if } x \geq s.
\end{cases}
$$

We now show that $u(s, \sigma_{\pi, s})$ is continuous and strictly decreasing in $s$ for all $s \in [0, 1]$. The assumption that $x$ takes a uniform distribution on $[\mu - \eta_\pi, \mu + \eta_\pi]$ implies that

$$
\xi(\mu, \sigma_{\pi, s}) = \begin{cases} 
1 & \text{if } \mu < s - \eta_\pi \\
\frac{1}{2} \left[ 1 - \frac{\mu - s}{\eta_\pi} \right] & \text{if } \mu \in [s - \eta_\pi, s + \eta_\pi] \\
0 & \text{if } \mu > s + \eta_\pi
\end{cases}
$$

(A8)

Therefore, the value of $\mu$ at which $\xi(\mu, \sigma_{\pi, s})$ crosses $\xi^p$ is

$$
\mu = s + \eta_\pi (1 - 2\xi^p)
$$

which we note is weakly greater than $s - \eta_\pi$. Substituting this in equation (A6),

$$
u(s, \sigma_{\pi, s}) = z(1 - s)\theta + \frac{1}{2\eta_\pi} \int_{s - \eta_\pi}^{s + \eta_\pi} (1 - z)(1 - \mu)\theta d\mu - c.
$$

(A10)

The first term is strictly decreasing in $s$ and the integrand is strictly decreasing in $\mu$ so that $u(s, \sigma_{\pi, s})$ is strictly decreasing in $s$. Continuity is immediate.

The remaining argument is essentially identical to the proof of Lemma 3 in Morris and Shin (1998). Instead of using the generalized result in Morris and Shin (2003), we provide the model-specific proof here for completeness.
Step 3. The previous step implies that there is a unique \( x^*_\pi \) such that \( u(x^*_\pi, \sigma_\pi, x^*_\pi) = 0 \). To see this note that by Lemma A, for \( s < \mu - \eta \pi \) we have \( u(s, \sigma_\pi, s) > 0 \) and for \( s > \bar{\mu}_\pi + \eta \pi \) we have \( u(s, \sigma_\pi, s) < 0 \). Strict monotonicity (and continuity) of \( u(s, \sigma_\pi, s) \) from Step 2 above, and the intermediate value theorem, imply the existence of the unique \( x^*_\pi \).

Now take any equilibrium of the game and let \( \{\sigma_\pi\}_{\pi=0,1} \), be the two aggregate actions associated with it. Let

\[
\underline{x}_\pi = \inf\{x|\sigma_\pi(x) < 1\} \quad \text{and} \quad \bar{x}_\pi = \sup\{x|\sigma_\pi(x) > 0\}.
\]

There must exist \( \nu_\pi \geq 0 \) such that for all \( x \in (\underline{x}_\pi, \bar{x}_\pi + \nu_\pi) \),\(^9\) citizens use an aggregate action that puts positive weight on not revolting. Take any such \( x \) in this interval. It must be that \( u(x, \sigma_\pi) \leq 0 \). Since the payoff in (A6) is continuous in \( x \), we also have \( u(\underline{x}_\pi, \sigma_\pi) \leq 0 \). Since \( \sigma_\pi(x) \geq \sigma_{\pi, \underline{x}_\pi}(x) \) for all \( x \), strategic complementarity from Step 1 implies that \( u(\underline{x}_\pi, \sigma_{\pi, \underline{x}_\pi}) \leq u(\underline{x}_\pi, \sigma_\pi) \leq 0 \). And, from Step 2 above, it follows that \( \underline{x}_\pi \geq x^*_\pi \).

By a similar argument we can show that \( \bar{x}_\pi \leq x^*_\pi \), which then implies \( \bar{x}_\pi \leq \underline{x}_\pi \). But notice that

\[
\bar{x}_\pi \geq \sup\{x|0 < \sigma_\pi(x) < 1\} \geq \inf\{x|0 < \sigma_\pi(x) < 1\} \geq \underline{x}_\pi
\]

which is then only consistent with \( \underline{x}_\pi = \bar{x}_\pi = x^*_\pi \). Thus the aggregate action associated with any reasonable equilibrium is \( \sigma_{\pi, x^*_\pi} \). Since the payoff to attacking \( u(x, \sigma_{\pi, x^*_\pi}) \) is decreasing in signal \( x \) and \( x = x^*_\pi \) solves \( u(x, \sigma_{\pi, x^*_\pi}) = 0 \), every best reply to the aggregate action \( \sigma_{\pi, x^*_\pi} \) must prescribe attacking the regime when \( x < x^*_\pi \) and not attacking when \( x > x^*_\pi \). Citizens of type \( x = x^*_\pi \) are indifferent between attacking the regime and staying at home.

---

\(^9\)We identify \( (\underline{x}_\pi, \bar{x}_\pi] \) with the singleton \( \{\underline{x}_\pi\} \).
We have left to calculate \( x^*_n \). By definition, \( x^*_n \) satisfies

\[
\begin{align*}
  u(x^*_n, \sigma_n, x^*_n) &= 0 \\
  \iff z(1 - x^*_n)\theta + \frac{1}{2\eta}\int_{x^*_n - \eta_n}^{x^*_n + \eta_n (1 - 2\xi)} (1 - z)(1 - \mu)\theta d\mu - c &= 0 \\
  \iff x^*_n &= 1 - \frac{c}{\theta(1 - \xi)(1 - \xi)} + \frac{\xi(1 - \xi)(1 - z)}{1 - \xi(1 - z)}\eta_n.
\end{align*}
\]

(A11)

It is easily verified that \( x^*_n \) is decreasing in \( \xi \). Therefore it is minimized at \( \xi = 1 \) at which point it equals \( 1 - c/z\theta > 0 \). It is bounded by its value at \( \xi = 0 \), which is \( 1 - c/\theta < 1 \).

\[ \square \]

A.4

Lemma B. If the regime transitions to democracy \( \omega = 1 \), then in any equilibrium of the subgame that follows, the ruler adopts the efficient policy \( \pi = 1 \), sets \( \beta = \bar{\beta} \) and the citizens always set \( \tau = \bar{\tau} \). Consequently, the equilibrium payoff to the elite from democracy is

\[
w(\bar{\tau}, \theta_{\bar{\beta}}, (1 + \alpha)\bar{y}_0) = (1 - \bar{\tau})\bar{\beta}(1 + \alpha)\bar{y}_0.
\]

(A12)

Proof. The payoff to each citizen from democracy is

\[
y_{D\pi}^p = (1 - \hat{\theta}(1 - \tau))\bar{y}_\pi
\]

(A13)

where \( \hat{\theta} = \theta \) if \( \pi = 0 \), and \( \hat{\theta} = \theta_{\bar{\beta}} \) if \( \pi = 1 \). This payoff is strictly increasing in \( \tau \). Therefore, no matter what the regime does, the citizens set \( \tau = \bar{\tau} \). Consequently, the elite’s payoff to the elite from transitioning to democracy is

\[
y_{D\pi}^e = (1 - \bar{\tau})\hat{\theta}\bar{y}_\pi
\]

(A14)

which is increasing in \( \hat{\theta} \). Thus the elite adopt the efficient policy and set \( \beta = \bar{\beta} \).

\[ \square \]

Lemma C. \( \eta_0 \) can take any value in the interval \( (0, z\theta/(\theta_{\bar{\beta}} + z\theta)) \) and \( c \) can take any value in the interval \( (\eta_0\theta_{\bar{\beta}}, z(1 - \eta_0)\theta) \). Further, \( \eta_0 \) and \( c \) can only take values in these respective intervals.

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Proof. Define \( \Delta = \{(\eta, c) \in \mathbb{R}_+^2 \mid \eta < \min \{1 - c/z\theta, c/\theta\} \} \), which is the set of pairs \((\eta, c)\) for which Assumption 3’ can be satisfied. This gives rise to the set

\[
\Delta_{\eta} = \{\eta \in \mathbb{R}_+ \mid \exists c \text{ such that } (\eta, c) \in \Delta\}
\]

\[
= \left\{ \eta \left| 0 < \eta < \frac{z\theta}{\theta\beta + z\theta} \right. \right\};
\]

and for each \( \eta \in \Delta_{\eta} \) we can define the set

\[
\Delta_c(\eta) = \{c \in \mathbb{R}_+ \mid (c, \eta) \in \Delta\}
\]

\[
= \{c \mid \eta\theta\beta < c < z(1 - \eta)\theta\} = (\eta\theta\beta, z(1 - \eta)\theta)
\]

which is the range of values \( c \) that satisfy Assumption 3’ for some value of \( \eta \) for which it can be satisfied. Note that this interval is smaller for \( \eta = \eta_0 \) than it is for \( \eta = \eta_1 \). Therefore, the assumptions of the model require \( \eta_0 \in \Delta_{\eta} \) and

\[
c \in \Delta_c(\eta_0) = (\eta_0\theta\beta, z(1 - \eta_0)\theta)
\]

From these, it is clear that \( \eta_0 \) can take any value in the set \( \Delta_{\eta} \) and \( c \) can take any value in the set \( \Delta_c(\eta_0) \).

Lemma D. The expected equilibrium payoff to the elite conditional the regime remaining authoritarian, \( \omega = 0 \), is strictly increasing in \( c \).

Proof. By Proposition 3, if the regime remains authoritarian, then in any equilibrium of the subgame that follows, citizens use the attacking threshold \( \tilde{x}(\theta) \) and the regime chooses \( \beta = \bar{\beta} \) if they adopt the efficient policy. The expected payoff to the elite from remaining authoritarian is thus strictly increasing in \( c \):

\[
\frac{\partial v(\hat{\theta}, \tilde{x}(\hat{\theta}), \bar{y}_\pi, \eta_\pi)}{\partial c} = 1 - \frac{c}{z\hat{\theta}} + \left( \frac{c}{z\hat{\theta} + \eta_\pi} \right) \left( \frac{1 - z}{z} \right) > 1 - \frac{c}{z\hat{\theta}} > 0.
\]

where \( \hat{\theta} = \theta \) if the regime does not adopt the efficient policy and \( \hat{\theta} = \theta\beta \) (\( = \theta\bar{\beta} \) in equilibrium) if it does.
Lemma E. For every $\alpha > 0$ and $\bar{\beta} \in (0, 1]$, there exists a tax threshold $\tau^*(\alpha, \bar{\beta}) \in (0, 1]$ such that in any equilibrium of the game, the elite (i) democratize, $\omega = 1$, adopt the efficient policy, $\pi = 1$, and set $\beta = \bar{\beta}$ when $\bar{\tau} < \tau^*(\alpha, \bar{\beta})$; (ii) remain authoritarian, $\omega = 0$, and preserve the status quo, $\pi = 0$, when $\alpha < \alpha$ and $\bar{\tau} > \tau^*(\alpha, \bar{\beta})$; and (iii) remain authoritarian, $\omega = 0$, adopt the efficient policy, $\pi = 1$, and set $\beta = \bar{\beta}$ when $\alpha > \alpha$ and $\bar{\tau} > \tau^*(\alpha, \bar{\beta})$.

Moreover, $\tau^*(\alpha, \bar{\beta})$ is continuous in $\alpha$ and $\bar{\beta}$.

Proof. Let $\tau^o : \mathbb{R}_+ \times (0, 1] \to \mathbb{R}$ be the function defined by $\tau^o(\alpha, \bar{\beta}) = \tau$ solving

\begin{align}
 v(\theta, \bar{x}(\theta), \bar{y}_0, \eta_0) &= w(\tau, \theta, (1 + \alpha)\bar{y}_0) \text{ if } \alpha < \alpha \\
 v(\theta, \bar{x}(\theta), (1 + \alpha)\bar{y}_0, \eta_0) &= w(\tau, \theta, (1 + \alpha)\bar{y}_0) \text{ if } \alpha \geq \alpha
\end{align}

(A16) (A17)

Unique solutions to these equations exist because $w(\cdot, \theta, (1 + \alpha)\bar{y}_0)$ is continuous, strictly decreasing in $\tau$ for $\tau \in (-\infty, +\infty)$, and the image of $(-\infty, +\infty)$ under $w(\cdot, \theta, (1 + \alpha)\bar{y}_0)$ is $(-\infty, +\infty)$. In addition, it is obvious that $\tau^o$ is continuous in $\alpha$ (including at $\alpha = \alpha$) and $\bar{\beta}$.

We now show that $\tau^o(\alpha, \bar{\beta}) > 0$ for all $\alpha \geq 0$. Note that for all $\alpha < \alpha$, we have

\begin{align}
 v(\theta, \bar{x}(\theta), \bar{y}_0, \eta_0) &< \lim_{c \to z(1-\eta_0)\theta} v(\theta, \bar{x}(\theta), \bar{y}_0, \eta_0) = \left[1 - \frac{z\eta_0^2}{3}\right] \theta \bar{y}_0 \\
 &< \theta(1 + \alpha)\bar{y}_0 = w(0, \theta, (1 + \alpha)\bar{y}_0),
\end{align}

(A18)

where the first inequality follows from Lemmas C and D. For all $\alpha \geq \alpha$ we have

\begin{align}
 v(\theta, \bar{x}(\theta), (1 + \alpha)\bar{y}_0, \eta_1) &< \lim_{c \to z(1-\eta_1)\theta} v(\theta, \bar{x}(\theta), (1 + \alpha)\bar{y}_0, \eta_1) \\
 &< \lim_{c \to z(1-\eta_1)\theta} v(\theta, \bar{x}(\theta), (1 + \alpha)\bar{y}_0, \eta_1) = \left[1 - \frac{z\eta_1^2}{3}\right] \theta(1 + \alpha)\bar{y}_0 < \theta(1 + \alpha)\bar{y}_0
\end{align}

(A19)

where again the first two inequalities follow from Lemmas C and D and the fact that $\eta_1 < \eta_0$ (Assumption 1). Since $w(\cdot, \theta, (1 + \alpha)\bar{y}_0)$ is unboundedly decreasing in $\tau$, this proves that $\tau^o(\alpha, \bar{\beta}) > 0$ for all $\alpha \geq 0$. 

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If we define $\tau^*(\alpha, \beta) = \min\{\tau^0(\alpha, \beta), 1 - \lambda/\theta\}$ then $\tau^*(\alpha, \beta)$ satisfies the properties stated in the proposition. This follows from Lemma B, Proposition 3 and the definition of $\alpha$, and the fact that $w(\cdot, \beta, (1 + \alpha)\bar{y}_0)$ is strictly decreasing in $\tau$.

Lemma E. If $\alpha < \alpha_0$ then $\tau^0(\alpha, \beta)$ is strictly increasing in $\alpha$ and $\beta$.

Proof. To see this, observe that the left side of (A16) is constant in $\alpha$, $\beta$ and $\tau$ while the right side is strictly increasing in $\alpha$ and $\beta$ but decreasing in $\tau$. (It is strictly increasing in $\beta$ because it depends on $\beta$ only through $\theta_{\beta}$ and it is strictly increasing in $\theta_{\beta}$.) So the result follows trivially from the implicit function theorem.

Lemma F. If $\alpha \geq \alpha_0$ then $\tau^0(\alpha, \beta)$ is strictly increasing in $\alpha$ and $\beta$.

Proof. In this case, we have

$$\tau^0(\alpha, \beta) = 1 - v(\theta_{\beta}, \bar{x}(\theta_{\beta}), 1, \eta_1)/\theta_{\beta}, \quad (A20)$$

so that $\tau^0$ depends on $\alpha$ and $\beta$ only through its dependence on $\theta_{\beta}$. Since $\theta_{\beta}$ is strictly increasing in both $\alpha$ and $\beta$, both $\partial \tau^0/\partial \alpha$ and $\partial \tau^0/\partial \beta$ have the same as the sign of the derivative of the right side of $(A20)$ with respect to $\theta_{\beta}$. That derivative is

$$\frac{1}{\theta_{\beta}} \left[ \frac{v(\theta_{\beta}, \bar{x}(\theta_{\beta}), 1, \eta_1) - \theta_{\beta} v_{\theta_{\beta}}(\theta_{\beta}, \bar{x}(\theta_{\beta}), 1, \eta_1)}{\theta_{\beta}} \right]$$

where $v_{\theta_{\beta}}$ denotes the derivative of $v$ with respect to $\theta_{\beta}$, and is given by $(A22)$ with $\theta_{\beta}$ substituted for $\theta$. The term in square brackets is

$$(1 - \bar{x}(\theta_{\beta}))^2 \left( \frac{1}{2} - z \right) + (1 - \bar{x}(\theta_{\beta}))(z + \eta_1(1 - z)) + (1 - \bar{x}(\theta_{\beta}))^2 \left( \frac{1}{2} - z \right)$$

$$= (1 - \bar{x}(\theta_{\beta})) \left[ 1 - \bar{x}(\theta_{\beta}) - z + 2\bar{x}(\theta_{\beta})z + \eta_1(1 - z) \right]$$

$$> (1 - \bar{x}(\theta_{\beta})) \left[ 1 - \bar{x}(\theta_{\beta}) - z + 2\bar{x}(\theta_{\beta})z \right]$$

and it is not hard to verify that the term in square brackets following the inequality is strictly convex in $z$ when $z > 0$. (Substitute back $\bar{x}(\theta_{\beta}) = 1 - c/z\theta_{\beta}$ and take the second partial
derivative with respect to \( z \).) In fact, we can set the first derivative equal to 0 and solve for \( z \) to find the global minimum on the interval \((0, 1)\). The solution is \( z^* = \left(\frac{c}{\theta \beta} \right)^{\frac{1}{2}} \). The term in square brackets (following the inequality) evaluated at \( z^* \) is \( \left( \frac{2}{\theta \beta} \right) \left( \left( \frac{c}{\theta \beta} \right)^{\frac{1}{2}} - \left( \frac{c}{\theta \beta} \right) \right) > 0 \) since \( 0 < c < \theta \beta \). Therefore, \( \tau^*(\alpha, \beta) \) is strictly increasing in both \( \alpha \) and \( \beta \).

It follows that \( \tau^*(\alpha, \beta) \) is strictly increasing in \( \alpha \) and \( \beta \) whenever \( \tau^*(\alpha, \beta) < 1 - \lambda/\theta \). Otherwise, it is constant in these variables.

**A.5**

This section contains comparative statics results for Sections 4.3, 4.4.2 and 4.4.3.

**Comparative Statics for \( \alpha \).** Observe that

\[
v(\theta, \bar{x}(\theta), \bar{y}_0, \eta_0) - v(\theta, \bar{x}(\theta), (1 + \alpha)\bar{y}_0, \eta_1) = \left[ v(\theta, \bar{x}(\theta), 1, \eta_0) - v(\theta, \bar{x}(\theta), 1, \eta_1)(1 + \alpha) \right] \bar{y}_0. \tag{A21}
\]

Since equation (4.7) of the main text showed that \( \partial v(\theta, \bar{x}(\theta), \bar{y}_\pi, \eta_\pi) / \partial \eta_\pi > 0 \), the comparative statics result for \( \eta_0 \) and \( \eta_1 \) mentioned in the main text follow immediately from the implicit function theorem, since (A21) is strictly decreasing in \( \alpha \).

Next, observe from equation (4.7) in the main text that \( \partial^2 v(\theta, \bar{x}(\theta), \bar{y}_\pi, \eta_\pi)/\partial \eta_\pi \partial \theta > 0 \).

Moreover, we have

\[
\frac{\partial v(\theta, \bar{x}(\theta), \bar{y}_\pi, \eta_\pi)}{\partial \theta} = v_1(\theta, \bar{x}(\theta), \bar{y}_\pi, \eta_\pi) + v_2(\theta, \bar{x}(\theta), \bar{y}_\pi, \eta_\pi)\bar{x}'(\theta)
\]

\[
= \left[ (1 - \bar{x}(\theta))^2 \left( z - \frac{1}{2} \right) + \left( \frac{1}{2} + (\eta_\pi)^2 \right) \frac{1}{2} \left( \frac{1}{2} - \frac{z}{3} \right) \right] \bar{y} > 0 \tag{A22}
\]

Therefore, (A21) is strictly increasing in \( \theta \). So, again by the implicit function theorem, \( \bar{x} \) is strictly increasing in \( \theta \). The remaining comparative statics results (discussed in the final paragraph of Section 4.4.3) follow in a similar way from verifying that

\[
\frac{\partial^2 v(\theta, \bar{x}(\theta), \bar{y}_\pi, \eta_\pi)}{\partial \eta_\pi \partial \theta} < 0, \quad \frac{\partial v(\theta, \bar{x}(\theta), \bar{y}_\pi, \eta_\pi)}{\partial \eta_\pi} < 0, \quad \frac{\partial^2 v(\theta, \bar{x}(\theta), \bar{y}_\pi, \eta_\pi)}{\partial \eta_\pi \partial \eta_\pi} > 0, \quad \frac{\partial v(\theta, \bar{x}(\theta), \bar{y}_\pi, \eta_\pi)}{\partial c} > 1 - c/\theta > 0.
\]
Comparative Statics for $\alpha$. Note that $d\theta_\beta/d\theta = 1/(1 + \alpha)$ so that

$$
\frac{\partial}{\partial \theta} \left( v(\theta, \tilde{x}(\theta), \bar{y}_0, \eta_0) - v(\theta_\beta, \tilde{x}(\theta_\beta), (1 + \alpha)\bar{y}_0, \eta_1) \right)
= \frac{\partial}{\partial \theta} \left[ v(\theta, \tilde{x}(\theta), 1, \eta_0) - v(\theta, \tilde{x}(\theta), 1, \eta_1) \right] > 0. \tag{A23}
$$

Therefore, by an argument similar to the case of $\bar{\alpha}$, we can conclude that $\alpha$ is also strictly increasing in $\theta$. Moreover, because $\theta_\beta$ does not depend on $c$, $z$, $\eta_0$ or $\eta_1$, the comparative statics results for $\alpha$ with respect to these variables are the same as the results for $\bar{\alpha}$.

Comparative Statics Results for Section 4.4.3. We know that $\theta_\beta$ is strictly increasing in $\theta$ and we showed that (A20) is strictly increasing in $\theta_\beta$. Since $\tau^o(\alpha, \bar{\beta})$ depends on $\theta$ only through $\theta_\beta$, it must be strictly increasing in $\theta$ when $\alpha \geq \alpha$. If $\alpha < \alpha$ then

$$
\frac{\partial}{\partial \theta} \left( v(\theta, \tilde{x}(\theta), \bar{y}_0, \eta_0) - w(\tau, \theta_\beta, (1 + \alpha)\bar{y}_0) \right)
= \frac{\partial}{\partial \theta} \left( v(\theta, \tilde{x}(\theta), \bar{y}_0, \eta_0) - w(\tau, \theta, \bar{y}_0) \right)
$$

so that $\tau^o(\alpha, \bar{\beta})$ must be increasing in $\theta$ when $\alpha < \alpha$ as well. Since (A15) shows that the left sides of (A16) and (A17) are strictly increasing in $c$ and it is easy to verify that they are strictly decreasing in $z$, then $\tau^o(\alpha, \bar{\beta})$ is strictly decreasing in $c$ and strictly increasing $z$. Since (A9) shows that the left sides of (A16) and (A17) are strictly increasing in $\eta_0$ and $\eta_1$ respectively, $\tau^o(\alpha, \bar{\beta})$ is strictly decreasing in $\eta_0$ when $\alpha < \alpha$ and constant when $\alpha \geq \alpha$; and $\tau^o(\alpha, \bar{\beta})$ is strictly decreasing in $\eta_1$ when $\alpha > \alpha$ and constant when $\alpha \leq \alpha$. 

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Bibliography


