Occulter-Based High-Contrast Exoplanet Imaging: Design, Scaling, and Performance Verification

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Abstract

Over the last two decades, a large number of exoplanets have been confirmed with the rate of discovery increasing in recent years primarily as new instruments with improved sensitivities have become available. Direct imaging of an Earth-like planet is now an important goal of the science community. This is a challenging problem for two primary reasons. First, the intensity ratio between the bright star and its dim Earth-like companion is expected to be approximately ten orders of magnitude and, second, the angular separation to the star is very small.

An external occulter is a specially-shaped spacecraft that is flown in formation with a telescope in order to block most of the starlight before it reaches the entrance pupil thereby allowing planetary light outside of the occulter’s inner working angle to become visible. Designing a shape for the occulter spacecraft to enable suppression over a wavelength band of interest requires modeling through scalar diffraction theory. Typical designs feature occulters that are tens of meters across at a separation of tens of thousands of kilometers from the space telescope.

In this dissertation, we focus on occulter design and scaling to enable experimental optical verification of occulters in the laboratory. We provide experimental results that establish a $10^{-5}$ suppression level in the pupil and $10^{-10}$ contrast in the focal plane, which are both approximately two orders of magnitude below the ideal performance of the testbed. We use numerical simulation to study the sensitivity of the occulter design in the laboratory and determine that performance is feature-size limited. We provide the design of a longer and flight-like occulter experiment, and study its sensitivity to determine the expected performance.
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“We may mount from this dull Earth; and viewing it from on high, consider whether Nature has laid out all her cost and finery upon this small speck of Dirt. So, like Travellers into other distant countries, we shall be better able to judge of what’s done at home, know how to make a true estimate of, and set its own value upon every thing. We shall be less apt to admire what this World calls great, shall nobly despise those Trifles the generality of Men set their Affections on, when we know that there are a multitude of such Earths inhabited and adorn’d as well as our own.”

— Christiaan Huygens, *Celestial Worlds Discover’d*
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Chapter 1

Introduction

1.1 Exoplanet science

Cosmic pluralism is the philosophical belief that there are many other worlds similar to Earth in the universe and has been debated from the very beginning of recorded history with the ancient Greeks and Romans. In 6th century BC, Thales and his student Anaximander posited a metaphysical argument on the infinite nature of the universe. As an evolution of this argument, Epicureans believed on the basis of their atomist philosophy in the existence of innumerable worlds, some of which were certain to harbour life, and questioned the existence of the gods [44]. Interestingly, some Stoics were pluralists but believed in a succession of worlds in time. Opposing pluralism were Aristotelians who found it incompatible with their physical interpretation of the natural place, and believed in the uniqueness of Earth’s creation. Early Christian scholars adopted Aristotle’s ideas and rejected pluralism, though by the 13th century this stance began to be challenged on a theological basis [39]. In his seminal work, On the Revolutions of the Heavenly Spheres, Copernicus developed a heliocentric model of the universe that could reconcile the observed planetary motions more simply than the prevailing Ptolemaic system at the time but not once mentioned the possibility
of pluralism. It was Giordano Bruno who in his *De l’infinito universo et mondi* in 1584, formulated the modern idea of *exoplanets*, that is planets like Earth that are in orbit around different stars scattered throughout an infinite universe.

With the invention of the telescope, Galileo announced in his *Sidereus Nuncius* observations of the mountainous surface of the Moon, many more observable stars, and four moons in orbit around Jupiter. His and subsequent observations with the aid of ever-improving telescopes [158] had profound philosophical implications in constraining and evolving the nature of the debate. Many later preeminent scientists and thinkers including Huygens, Herschel, and Franklin believed that our solar system and the universe are teeming with life. In his *Cosmotheoros*¹, Huygens speculated among other things that the existence of life on other worlds depended on the availability of water in liquid form and was careful to label his conjectures as such. Over the past half century, robotic exploration missions in our solar system have so far not discovered traces of life outside Earth even though evidence of liquid water has been found. The search has now expanded outside of our own solar system, primarily focusing on possible rocky exoplanets located in the *habitable zone* – the orbital region in space around the host star where the range of temperatures would enable liquid water to exist on the surface of an exoplanet [89].

The first discovery of an exoplanet² around a Sun-like star was 51 Pegasi b, announced in 1995 by Mayor and Queloz [118], using a radial velocity (RV) method. This early discovery of a Jupiter-sized planet with a very short period was radically different from the example set by our own solar system. This discovery had an immediate impact on planetary formation theories and led to development of planetary

¹The first edition in English, 1698, bears the title *The Celestial Worlds Discover’d: or, conjectures concerning the inhabitants, plants, and productions of the worlds in the planets.*

²An earlier confirmed exoplanet discovery was actually in 1992 [184], although this was orbiting PSR 1257 – a pulsar star. Predating even this was HD114662b in 1989 [97], a suspected Brown dwarf whose mass was recently constrained to categorize it as an exoplanet. Gamma Cephei Ab, a planet in a binary system, was first discovered in 1988 [26], but was only confirmed later in 2002 [73].
known exoplanets in relation to the planets

Figure 1.1: Summary of the mass and semi-major axis parameters of confirmed exoplanets by September 2014 with the detection methods outlined. Also shown for comparison are the parameters of the planets in our solar system. Over 1800 exoplanets have been confirmed out of which nearly 600 were found using radial velocity methods, 1100 from transiting photometry, and 40 from direct imaging. Data retrieved from exoplanet.eu [142].

migration theory (see e.g., [115]). Since then more than 1800 exoplanets have been confirmed with the rate of discovery accelerating in recent years as new instruments with better sensitivities have become available. Out of these about 1100 are distinct exoplanetary systems, with multiple exoplanets discovered in nearly 500 systems [142]. Figure 1.1 summarizes the exoplanets confirmed to date by method of detection and compares their parameters with the planets in our solar system.

The RV method is an indirect observation technique that measures the gravitational influence of the orbiting planet on its host star through Doppler spectroscopy. Therefore, the host star itself is in an orbit about the barycenter, the common center
of gravity of that solar system which is typically very close to (often inside of) the host star. The Doppler signal consists of the edge-on component of motion with respect to Earth – thus, as the star’s relative motion changes over the course of its orbit towards and away from the observer, the wavelength changes and this shift can be measured in the stellar spectrum. Most of the early exoplanetary discoveries were found through surveys using this method [133, 179], and to date nearly 600 confirmed exoplanets have been found using RV [142]. The Doppler signal is more pronounced for large planets at short orbital periods to the host star and this introduced an inherent bias in the exoplanets discovered, particularly the early datasets [111]. The sensitivity has improved in recent surveys such as HARPS [169], which at present theoretically allows for detection of planets in the habitable zone with only about five times Earth mass. For face-on systems, the stellar wobble around the barycenter can be directly measured although this requires extraordinarily accurate astrometric measurements. Several candidates were identified using astrometry from ground telescopes but were not confirmed through follow-up observations. The GAIA space mission, launched in 2013, will provide an astrometric all-sky survey and has the sensitivity required to identify long-period giant planets within 200 parsec [41].

A technique called transit photometry [34] has to date found the majority of the known exoplanets. When a planet goes across the host star, a transit signal can be detected in the form of a dip in the stellar flux. The magnitude of the dip corresponds to the diameter of the exoplanet and the length and frequency of the signal dip corresponds to orbital parameters. In particular, planets that have a smaller separation from their target star will exhibit the transiting signal more frequently and will be easier to detect. Two early space missions using transit photometry were MOST [117] and CoRoT [11]. Since then the Kepler space mission has had tremendous success in increasing the number of known exoplanets since it was launched in 2009 [14, 15]. Among the discovered exoplanets with Kepler, two Earth-size planets have already
been identified [57]. This technique can be used to obtain limited spectra under certain conditions [35]. Aside from all the confirmed exoplanets, Kepler has identified nearly 3000 additional candidates, the vast majority of which are expected to become confirmed in the near-term [6], including more Earth-size planets. TESS is a follow-up space mission to Kepler to launch in 2017 and will feature a wider field of search and higher sensitivity for Earth-like planets [134].

Another indirect detection method is microlensing, an observational event due to gravity whose brightening effect was computed by Einstein in 1936 [49]. A typical lensing event requires the precise alignment of a source star and a lensing star in the foreground. A planetary companion of the lensing star introduces a shorter time-scale modification on the lensing light transmission profile, and the presence of the companion can be inferred. This method was first applied in 2013 towards the detection of an exoplanet around OGLE-2003-BLG-235L/MOA-2003-BLG-53L [10]. Due to the precise nature of the alignment required, a large number of stars, typically in the direction of the galactic core where they are more numerous, must be monitored for lensing events. The method is sensitive to planets in the 1-10 AU range, and has been adopted as an instrument on the upcoming WFIRST-AFTA mission to provide a census of exoplanet parameters [161].

As seen from the clustering in Figure 1.1, each discovery method has clear biases in terms of the parameters of the discovered exoplanets and requires completeness measures to analyze the actual distribution of exoplanet populations [18]. A large number of exoplanets have already been discovered from the initial RV surveys and using Kepler. These, together with the upcoming surveys including in particular TESS and the microlensing instrument on WFIRST-AFTA, is expected to provide a rich statistical dataset of exoplanets that will help classify the frequency of planets with different parameters. Already based on Kepler candidates, preliminary analyses have shown that rocky Earth-like planets are expected to be very common [46, 56,
Additionally, the measured rate of occurrence of different types of exoplanets, particularly in known multiple planetary systems [52, 101], has been used to constrain planet-formation models [65].

Indirect detection methods cannot provide sufficient information to characterize exoplanetary atmospheric composition. In order to obtain photons from the planet itself, direct imaging techniques are needed. The first confirmed planetary system using a direct imaging technique was HR 8799 using observations in the infrared on the Keck and Gemini Telescopes [112]. Initially three planets (HR 8799b, c, and d) were discovered in 2008 with separations 24-68 AU and 5-13 Jupiter masses, with HR 8799d only announced later in 2010 [113]. Also in 2008, Fomalhaut b was directly imaged at a separation of 116 AU from its host star using a coronagraph on the Hubble Space Telescope [79]. This was confirmed by independent follow-up observations [58, 164]. Beta Pictoris b was discovered at a somewhat smaller 9 AU separation [96]. Since these initial discoveries, to date nearly 40 planets have been discovered using direct imaging. As the parameters of the initial discoveries suggest, direct imaging to date tends to be more sensitive to self-luminous planets at a high angular separation from their host star and is therefore complementary to the indirect detection methods. These direct imaging observations have informed exoplanet atmosphere models [143] and mechanisms for the formation of gas giants in very wide orbits [45].

Figure 1.2 shows the contrast ratio for planets in our solar system with the Sun as seen from 10 parsecs away obtained from theoretical modeling via simple Planck curves and uniform wavelength reflectivity at the planetary surface [43]. Measurement of Earthshine, reflected Earth light off the Moon, has been compared with this theoretical model [144]. Additionally, analysis of the empirical absorption lines due to water, carbon-dioxide, ozone, and others from the Earth serves as a useful model for detecting such biomarkers on an Earth-like planet. The absorption wavelengths are spread over a wide range, but a range spanning the optical and near-infrared
Figure 1.2: Model spectrum comparison of the Sun and the planets as seen from 10 parsec [167]. The model uses Planck emission and uniform albedo reflectance. An approximate $10^{10}$ contrast ratio is expected between the Sun and the Earth in the optical wavelengths.
wavelengths has been generally accepted to be the most indicative of biomarkers.

Direct imaging from ground-based instruments can take advantage of large apertures allowing for imaging at smaller angular separations. An early ground-based integral field spectrograph (IFS) is Project 1640, which was outfitted on the 5-m telescope at the Palomar Observatory [75] and was used to obtain low-resolution spectra of HR 8799 [125]. The SPHERE instrument is now operating at the VLT-Paranal and features an IFS [8]. GPI is a new IFS recently installed on the 8-m telescope at Gemini South [107] and features better angular separation and contrast allowing for direct imaging of gas giants at separations similar to those in the solar system. The instrument has seen first light imaging Beta Pictoris b [106]. A team here at Princeton is currently building an IFS for usage with the 8-m Subaru telescope with deployment expected in 2016 [68]. This instrument features mitigation of spectral cross-talk, improved dispersion uniformity, and mechanical stability. Therefore, the next few years will likely see a substantial number of directly imaged self-luminous gas giants being discovered.

Atmospheric turbulence limits the performance of any ground-based instrument and makes optical observations of rocky planets practically impossible. For this purpose, observations must be made from a space-based telescope. The James Webb Space Telescope (JWST) will feature a 7-aperture non-redundant aperture mask that will provide some direct imaging capability in the infrared [156]. JWST’s successor, the afore-mentioned 2.4-m WFIRST-AFTA mission, has been announced to be outfitted with a coronagraphic instrument to enable high-contrast direct imaging of planets, possibly including dim rocky planets, around nearby stars in the optical wavelengths [161].
1.2 High contrast direct imaging

The ultimate goal of exoplanetary science is the direct imaging of an Earth-like planet in the habitable zone. The problem of direct imaging of an Earth-like exoplanet is, however, a formidable challenge for the following reasons:

- at the optical wavelengths, rocky planets are visible only through reflected light from the host star which represents ten orders of magnitude contrast difference to the host star;
- the desired habitable zone is at a small angular separation from the host star, requiring high-contrast imaging capability down to approximately 75 milliarc-seconds (mas);
- the diffraction wings of the host star are many orders of magnitude brighter than the target planet for any reasonably-sized telescope.

All of these difficulties can be seen clearly in Figure 1.3. The peak of the planetary point spread function is $10^{10}$ dimmer than the host star as predicted from the emission model in Figure 1.2 [43]. The planetary PSF is offset by 100 mas, the angular separation corresponding to a companion planet at 1 AU from a star located at 10 parsec (for a 4-m telescope at 600 nm this angular separation is about $3 \lambda/D$). As can be seen, this means that the diffraction wings of the star are orders of magnitude brighter than the planet’s peak which is therefore not visible. Increasing the size of the telescope can help as the diffraction ringing is compressed, but would require a telescope with a diameter on the order of kilometers to resolve the planet and star simultaneously. For stars that are located further than this example star, the angular separation becomes smaller. Lastly, whereas observation in the infrared provides a smaller contrast differential it also involves a wider spread of the diffraction wings.

To address the problem of direct imaging of the dim planetary companion therefore requires techniques for starlight suppression – reducing the contribution of the
Figure 1.3: Cross-section comparison of the diffraction-limited point spread function for a star and an Earth-like planet. The star is located at 10 parsec distance. The companion planet is 10 orders of magnitude dimmer than the star and orbits at 1 AU separation. For stars that are further away the angular separation decreases while a smaller telescope will exhibit wider diffraction rings. Contrast across the y-axis is the normalized intensity with respect to the stellar peak.
stellar point spread function while leaving the planetary peak signal unaffected. Over the years, several concepts have been proposed that will allow for direct high-contrast imaging of a dim planetary companion. Coronagraphs were invented by Lyot in 1931 to observe the solar corona without the need of an eclipse [105]. The original Lyot coronagraph featured an occulting spot at the image plane and a stop at a conjugate pupil plane to reduce the total amount of starlight by two orders of magnitude. Thus coronagraphs are devices that apply phase and amplitude changes at the image and pupil planes of the telescope to enable high-contrast imaging. In 1965, Slepian proposed prolate spheroidal apodization functions at the exit pupil to increase contrast at the image plane; the prolate spheroidal function maximally concentrates energy across the center of the frequency domain [157]. Since then, many different types of coronagraphs have been developed, and designs have been proposed to permit exoplanetary imaging. At Princeton, our group has used shaped-pupil coronagraphs which are one of the simplest coronagraph classes – they are a binary mask placed at the exit pupil to reshape the stellar point spread function to enable regions of high contrast. Spergel proposed the first such coronagraph in 2000, based on the Fourier transform property of a Gaussian profile [162]. Kasdin created an improved shaped pupil coronagraph by using a binary realization of the prolate spheroidal apodization to create a region of high-contrast along one axis [86]. The region of high-contrast is relatively small and would require multiple observations with a rotated pupil. Vanderbei re-wrote this as a linear optimization problem that resulted in shaped pupils with significantly increased regions of high-contrast [88, 173, 176]. We make particular note here of a family of structurally-connected shaped pupil coronagraph masks that approximate a circularly symmetric apodization profile via a starshape [175]. More recently, these optimization techniques were generalized to account for arbitrary pupil shapes [28] enabling potential usage of such shaped pupil coronagraphs on the upcoming WFIRST-AFTA [135]. Another class of coronagraphs promises to
increase the total throughput by using two mirrors in series whose shape is optimized to induce an equivalent amplitude attenuation profile at the expense of potentially increased sensitivity to misalignment errors [7, 69, 171].

Whereas coronagraph designs are theoretically capable of achieving contrast ratios necessary for imaging Earth-like planets, in practice wavefront errors introduced by surface aberrations of the optics degrade performance to orders of magnitude above the coronagraph’s design by leakage of the stellar point spread function. To recover the region of high-contrast (called the dark hole) in the presence of optical aberrations, techniques from adaptive optics have been used with deformable mirrors (DM) [109]. It was shown that dark holes can be created through speckle-nulling via sequential conjugation and energy minimization [12, 168], that broadband performance can be adequately maintained over small 10% bands [60, 131], that two DMs in series can create symmetrical dark holes [132], and filtering increases the efficiency of estimation [67, 90]. A space mission could thus use a coronagraph and DM combination to allow imaging of exoplanets.

Another mission architecture that has been proposed even before the coronagraph architecture is a space interferometer [16]. The basic idea is that light collected at two apertures can be interfered with phase shift leading to regions of destructive interference where a planetary signal could be detected. A space-based mid-infrared interferometer with several arms obtained via precise formation flight has been proposed in the form of NASA’s TPF-I [59] and ESA’s Darwin [36]. Laboratory experiments have demonstrated that it is possible to obtain nulling in the visible wavelengths as well, although alignment tolerances are very stringent – on the 1 nm level [180]. A related architecture is the visible nulling coronograph, which uses a single aperture and emulates baseline separation by beam shearing [103, 147].
In this dissertation, we are concerned with a third space mission architecture that would enable direct high-contrast imaging, namely external occulters, sometimes also referred to as starshades. The occulter concept is summarized in Figure 1.4 and involves placing a screen between the target star and the observing telescope to block starlight prior to reaching the telescope pupil. Diffraction at the edges of the occulter screen becomes a limiting factor that must be considered in the design. The occulter concept goes back several decades, having been first proposed for solar study [51] as an improvement on Lyot’s coronagraph, but it has only been over the last ten years that technological development has shown them to be realistic contenders for a space mission architecture dedicated to imaging exoplanets. An occulter mission addresses the tight alignment tolerances of interferometers and sidesteps the limiting optical aberrations for coronagraph architectures.

Lyman Spitzer, at Princeton, wrote a famous article in Scientific American in 1962 [163], in which he first proposed the broad concept of a space telescope. He advocated not being constrained by atmospheric effects as is the case with ground-based observations. These advantages include observations in the ultra-violet and shorter wavelengths, diffraction-limited observations, and minimization of gravita-
tional mechanical distortions. Spitzer’s efforts in particular led to the development of the Hubble Space Telescope in the 1980s. The second half of his article is devoted to discussing the possibility of direct imaging of exoplanets, including the proposal [40] for usage of an occulting disk flown along the line-of-sight of the telescope to the observed star to create a shadow at the telescope pupil and limit starlight. Additionally, he suggested designing the occulter’s edge to darken the shadow beyond the level of a simple circular edge.

A decade later, NASA commissioned a study to investigate the feasibility of an occulter mission [185] based on Spitzer’s idea. Although a separate analysis that studied the detectability of exoplanets is mentioned but not detailed, it appears that the occulter is still envisioned as a disk\(^3\) which alone would be insufficient to directly image rocky planets. The final baseline reference numbers are surprisingly similar to today’s concepts, with a 100,000 km separation, a 50-m occulter, and a 3-m telescope. A deployment method for such a structure in space is also sketched out and an orbit about one of the Lunar libration point is considered.

The idea of an occulting disk was discussed in different forms [9, 50, 53]. Marchal, in 1985, deserves special mention for designing the edge of the occulter to mitigate diffraction [110]. He used an analytical approach to first show the insufficient starlight suppression of a circular disk and then derived a star-shaped occulter screen that creates dark zones in the shadow. He proposed occulter screens at 100,000 to 1,000,000 km separation and ranging in corresponding diameter from 100 to 800 m. The geometrical constraints of his solutions, however, limit imaging only in the outer regions of neighbouring solar systems.

Copi and Starkman [38] proposed a partially transmitting occulter screen. They used a dark circular region, with a polynomial apodization function. On this basis,\(^3\)Presumably the solution was to combine the disk with Spitzer’s other proposed solution, i.e., use both the disk and apodize the telescope pupil. Recent studies have shown that, although promising [25, 104], this hybrid solution appears to be problematic in terms of tolerancing alignment [19].
they proposed the BOSS mission concept of a 70 m × 70 m occulter screen that would provide 5 orders of magnitude stellar suppression. This could be used with an 8-m telescope to image rocky planets in the very nearest solar systems and gas giants further away. A square apodized aperture was also considered [78]. One problem with this concept is that manufacturing such a large partially apodized screen with sufficient accuracy is not possible.

Cash first envisioned the New Worlds Observer as a large-scale pinhole camera in space [33]. The pinhole camera consists of a large 1000 meter star shade spacecraft with a designed small hole at the center that would limit diffraction. A telescope spacecraft at a distance of 200,000 km would collect the photons by moving through the space that forms a large-scale pinhole image. Simmons [149] showed that by applying Babinet’s Principle a complementary occulter to the pinhole star shade can be designed that is equivalent in spatial extent to the hole, thus substantially reducing the size of the required structure in space. Cash then proposed a hypergaussian apodization profile that could be turned, using a technique developed for star-shaped coronagraph masks [175], into a petalized occulter [30]. Vanderbei used optimization techniques to generalize the design of the occulter edge reducing the size and improving the performance of the occulter [172].

Several mission designs have been proposed and designed since then. The key
parameters of these missions are summarized in Table 1.1. The New Worlds Oberver (NWO) is based on a hypergaussian design [31] featuring an 50 m occulter at a 80,000 km separation. A Telescope for Habitable Exoplanets and Interstellar/intergalactic Astronomy (THEIA) is based on optimized designs [81] – to reduce its size further, this was proposed as a two-distance design with the occulter operating at the nominal distance for imaging at the blue-end wavelengths and moving to a close separation for characterization in the red-end. The occulter diameter for THEIA was 40 m operating at a nominal 55,000 km.

Both THEIA and NWO are flagship-class missions that require a major optical telescope. Smaller probe-class missions have also been proposed. The Occulting Ozone Observatory (O3) is an occulter designed for a smaller, cheaper 1.1 m telescope [84]. As such, O3 is designed to obtain spectra using photometric filters since the smaller aperture does not have the sensitivity to allow for a spectrometer. The New Worlds Probe (NWP) took a different approach and proposed to operate in the near and mid-infrared using an occulter with the JWST [32, 159].

Occulter feature a geometrical inner working angle outside of which a planetary signal would be undisturbed by the occulter screen. This geometric inner working angle for an occulter of radius $R$ flown a distance $z$ in front of the telescope is given by the simple relation

$$\text{IWA} = \tan^{-1}\left(\frac{R}{z}\right) \approx \frac{R}{z} \quad (1.3.1)$$

If the planet is not blocked by one of the occulter petals, it could also theoretically be visible in between the petals.

In Figure 1.5, we show simulations using the parameters of the THEIA mission to illustrate the effect of an occulter on starlight suppression. The simulation is at
600 nm using a 4-m telescope centered in the shadow as shown in Figure 1.5(a). The shadow is shown in log-scale at the telescope pupil and is normalized with respect to the intensity of the light in the absence of the occulter. The log-scale picture shows that at the center of the shadow there is a dark hole, a region slightly larger than the telescope where the intensity of the shadow is suppressed by ten orders of magnitude (this normalized intensity in the pupil plane is called suppression for occulters). The telescope must be maintained over the duration of observations in the center of the shadow with alignment tolerance being given by the amount of oversizing of the dark hole. This amount is typically on the order of 1 m (compared to an interferometer for example which requires 1 nm alignment precision). In Figure 1.5b, we see the image formed by the telescope when collecting light centered in the shadow. This image is normalized with respect to the peak of the image formed when no occulter is in the line-of-sight (this normalized intensity in the image plane is called contrast for occulters). We see that the peak of the stellar point spread function compared to that
in Figure 1.3 has been reduced by approximately ten orders of magnitude. Thus the offset planet’s point spread function would no longer be obscured by the diffraction wings of the star’s point spread function. The inner working angle described in Equation 1.3.1 outside which the planet signal could be imaged is shown by the red circle. Because all the light has been blocked prior to the telescope pupil, optical aberrations at the telescope would not cause starlight leakage into other regions of the image plane (as opposed to an internal coronagraph, for which this is the main challenge).

There are, however, significant challenges that must be overcome to demonstrate the feasibility of occulters related to both the optical performance of the occulter system and the formation flying. Technical demonstrations on all aspects of occulter development are currently underway. Work that has been done to date includes:

- optical modeling and tolerancing of the necessary manufacturing precision of the occulter edges and alignment [5, 21, 47, 63, 64, 146, 145];

- analysis of optical scattering from third-body sources including modeling of the edge curvatures [4, 29, 114];

- experimental optical verification of occulter designs both in the laboratory and in the field [22, 23, 99, 62, 137, 138, 140, 152, 153, 154];

- mechanical modeling and prototype development of occulter structure and deployment mechanism; optical metrology of individual and deployed petals [83, 85, 87, 166, 181];

- development of orbit trajectories, optimal scheduling, and multiple-occulter formations [77, 92, 93, 94, 139];

- simulations and analysis of formation flying during observations including position sensing [23, 48, 98, 102, 123, 124, 150, 155, 151].
1.4 Overview of occulter validation efforts

This dissertation’s contributions are in validation of occulter optical models through comparison to experimental results, and providing a better understanding of the factors limiting the performance of scaled occulters. Here we review some of the experimental results obtained in other occulter testbeds to date. Some of these results will be referred for comparison throughout the dissertation.

The first set of occulter experiments was conducted was at the University of Colorado. In this experiment a scaled occulter was illuminated by broadband, collimated solar light through the use of a heliostat. A suppression level of $5 \times 10^{-6}$ was measured initially [140]. It was believed that the limiting factor of the performance was aerosol scattering, so the experiment was subsequently conducted in a vacuum tank, for which a suppression of $10^{-7}$ was reported [99]. In this experiment, contrast-calibrated images were also shown to demonstrate the detection of a planetary signal at a contrast level of $5 \times 10^{-9}$. Over-resolved contrast-calibrated images feature bright spots at the tips and bases of the occulter petals as well as the supporting wires. Additionally, different occulter shapes were tested including a metal-fabricated occulter and a 42-petal silicon occulter with more petals. Both performed worse than the 16-petal silicon occulter, which suggests to us that the accuracy of the manufacturing process is a limiting factor.

A subsequent occulter experiment was setup at Northrop Grumman, the first to operate at realistic flight Fresnel numbers. This consisted of a supercontinuum broadband laser beam collimated on a silicon occulter in a vacuum tank providing approximately 40-m of total propagation distance [137]. The performance metric of this experiment was contrast in the image plane with a resolution factor similar to that expected for a directly scaled mission. Measured radially-averaged contrast at the 77 mas-equivalent inner working angle of the occulter was $2.6 \times 10^{-8}$ [138].

Recently, a series of desert field tests were conducted for large-scale occulters.
These were conducted in the desert, and used over-sized occulters allowing resolving of light scattering off the ground and the occulter support stand. A contrast level of $8 \times 10^{-8}$ was measured at a 100 mas-equivalent separation [62].

Technological demonstrations have also been performed to demonstrating the capability to manufacture individual petals [87], and their assembly towards a full-occulter shape [181]. Measurements of the manufactured petals and assembly have shown that the necessary $10^{-10}$ target contrast can be maintained with the manufacturing capabilities available.

1.5 Scalar diffraction theory

Diffraction theory is a consequence of the wave model of light. The first recorded mention that light has a wave-like motion is due to Hooke in 1665 who, in his Micrographia stated without supporting empirical evidence that light motion exhibits three main features, namely, that it is “quick”, “vibrative”, and “short”. It was Huygens who first used the wave-theory to explain observed phenomena. In a series of lectures at the French Royal Academy, later published in 1690 as a Treatise on Light, he postulated that light is a longitudinal wave similar to sound, with a finite speed, and travelling through an ether medium. He used geometrical constructs to demonstrate that secondary spherical sources on the wavefront can account for the phenomena of reflection and refraction. Newton, however, advocated the corpuscular theory according to which light is made up of small particles that possess kinetic energy. Thus, Huygens’ theory did not really catch on until 1804 when Thomas Young presented his double-slit experiment which demonstrated the formation of bright and dark fringing. Interference based on the wave theory of light could very accurately predict the locations of the fringes [188].

In his memoir to the French Academy of Sciences in 1818, Fresnel combined Huy-
We consider Huygens' wavelets\footnote{Fresnel and Arago also corrected Huygens' longitudinal light wave model by showing that a transverse wave model is necessary to account for birefringence in calcite [3].} with the idea that they could constructively and destructively interfere. He made assumptions about the amplitude and phases of the illuminating Huygens wavelets, and in doing so he was able to predict the diffraction patterns very accurately. Poisson, a member of the committee evaluating his paper, was skeptical of the theory and showed that it predicted a bright spot at the center of the shadow cast by an opaque disk. Arago, who chaired the committee, performed the experiment and discovered the spot predicted by Poisson.

Kirchoff proposed an integral theorem derived using Green’s theorem and showed that the form of amplitude and phase that Fresnel assumed was a mathematical consequence of the wave nature of light. Nonetheless, the boundary conditions he used were shown by Sommerfeld to be inconsistent. Sommerfeld rederived Kirchoff’s integral theorem with a different form of Green’s function [66].

Occulter optical models are based on scalar diffraction theory, which assumes that each orthogonal component of the electric field is an independent solution of the wave equations. To see this, we follow the derivation in Goodman [66] starting with Maxwell’s equations which represent the most general propagation of electromagnetic waves, where \( E \) and \( H \) are the vector forms of the electric and magnetic fields respectively:

\[
\nabla \times E = -\mu \frac{\partial H}{\partial t}, \\
\nabla \times H = \epsilon \frac{\partial E}{\partial t}, \\
\nabla \cdot E = 0, \\
\nabla \cdot B = 0.
\]

The above equations assume a charge-free medium of propagation and \( \epsilon \) and \( \mu \) are the permittivity and permeability of the medium respectively. We apply the curl
operator to the left and right sides of the Maxwell-Faraday Equation 1.5.1 and the vector identity

\[ \nabla \times (\nabla \times \mathbf{E}) = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \]  

To derive scalar propagation theory the following assumptions about the propagation medium are made: linear, isotropic, homogeneous, and nondispersive. These are all good assumptions for propagation of light in space as is the case with an occulter. We then obtain identical wave equations for both \( \mathbf{E} \) and \( \mathbf{H} \):

\[ \nabla^2 \mathbf{E} - \frac{n^2}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \]  
\[ \nabla^2 \mathbf{H} - \frac{n^2}{c^2} \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0 \]

where \( n = \sqrt{\epsilon/\epsilon_0} \) and \( c = 1/\sqrt{\mu_0 \epsilon_0} \). Each of the rectangular components of the electric and magnetic fields satisfies the wave equation. Then a single scalar wave equation can be written with \( u(x, y, z, t) \) representing any of the scalar components:

\[ \nabla^2 u(x, y, z, t) - \frac{n^2}{c^2} \frac{\partial^2 u(x, y, z, t)}{\partial t^2} = 0 \]

If the medium is non-dispersive, or if boundary conditions (e.g., from an aperture) are present then a coupling term between the components of the field appears. A scalar propagation equation cannot be written and instead we would need to propagate the entire vector, for example using finite-difference methods [121] or Fourier modal analysis [186]. The boundary condition approximation in particular depends on the area of the aperture compared to the wavelength of light – if the aperture is large then errors introduced due to coupling at the boundaries are relatively small.

Next, eliminating the temporal component of the scalar field, we can write the
Helmholtz wave equation:

\[(\nabla^2 + k^2)U(x, y, z) = 0\]  \hspace{1cm} (1.5.9)

where the wave-number \( k = \frac{2\pi}{\lambda} \). We can use Green’s theorem to convert Helmholtz’s equation to integral form:

\[\int \int \int_V (G\nabla^2 U - U\nabla^2 G) \, dv = \int \int_S \left( G\frac{\partial U}{\partial n} - U\frac{\partial G}{\partial n} \right) \, ds \]  \hspace{1cm} (1.5.10)

where \( S \) is a closed surface with a volume \( V \) and \( n \) is the normal direction at surface points along \( S \). \( U \) and \( G \) and their partial derivatives must be continuous. The choice of a particular form of Green’s function \( G \) is somewhat arbitrary and leads to different integral formulations of diffraction theory. The following is the choice by Sommerfeld, is internally consistent, and recovers the Huygens-Fresnel Principle

\[G = \frac{e^{ikr_{01}}}{r_{01}} - \frac{e^{ik\bar{r}_{01}}}{\bar{r}_{01}}\]  \hspace{1cm} (1.5.11)

where \( P_1 \) is a point on the aperture, \( P_0 \) is the point of observation, and \( \bar{P}_0 \) is a mirror point to \( P_0 \) on the opposite side of the aperture. This is illustrated in Figure 1.6. Application of this choice of Green’s function gives the following Rayleigh-Sommerfeld diffraction integral of the first kind:

\[U(x, y, z) = \frac{1}{i\lambda} \int \int_{\Sigma} U(u, v, 0) \frac{z}{r_{01}} \frac{e^{ikr_{01}}}{r_{01}} \, dudv.\]  \hspace{1cm} (1.5.12)

We can interpret this diffraction integral as the superposition of diverging spherical waves across the aperture \( \Sigma \) to obtain the electric field at an observation point. Thus, Equation 1.5.12 is a mathematical formulation of the Huygens-Fresnel principle. This
Figure 1.6: Geometry of Green’s function choice leading to Rayleigh-Sommerfeld diffraction integral of the first kind. $P_1$ is a point on the diffracting aperture $P_0$ is the observation point, $P_0$ is a mirror point behind the aperture, and $r_{01}$ is the distance between $P_0$ and $P_1$.

can be expressed in Cartesian coordinates by substitution of:

$$r_{01} = \sqrt{z^2 + (x-u)^2 + (y-v)^2}.$$ (1.5.13)

Numerical calculation of the Rayleigh-Sommerfeld diffraction integral in Equation 1.5.12 is possible and several algorithms have been proposed. These include separating the quickly oscillating portions of the kernel [177], using scaled convolution methods [122], or reformulating the integral as a Fourier transform of a generalized pupil function [71]. Nonetheless, these methods are slow especially for the sampling intervals necessary to accurately evaluate occulter diffraction. Additional approximations can be made by re-writing $r_{01}$ using the binomial expansion:

$$r_{01} = z \sqrt{1 + \left(\frac{x-u}{z}\right)^2 + \left(\frac{y-v}{z}\right)^2}.$$ (1.5.14)

$$\approx z \left(1 + \frac{1}{2} \left(\frac{x-u}{z}\right)^2 + \frac{1}{2} \left(\frac{y-v}{z}\right)^2\right).$$ (1.5.15)
The condition for the accuracy of the approximation is the following:

\[ z^3 \gg \frac{\pi}{4\lambda} \left( (x-u)^2 + (y-v)^2 \right)_{\text{max}}. \]  

(1.5.16)

Substituting the approximation for \( r_{01} \) in Equation 1.5.12 gives the Fresnel diffraction integral:

\[ U(x, y, z) = e^{ikz} i\lambda z \int \int_{\Sigma} U(u, v, 0) e^{\frac{ik}{\lambda z} ((x-u)^2 + (y-v)^2)} dudv. \]  

(1.5.17)

We should mention here that higher-order terms must be kept for some coronagraphs as a more accurate alternative to the Fresnel approximation [171]. For a far-away aperture satisfying the following condition,

\[ z \gg \frac{k}{2} \left( u^2 + v^2 \right)_{\text{max}} \]  

(1.5.18)

we can ignore the quadratic terms in Equation 1.5.17, and obtain the Fraunhofer diffraction integral which features only a linear phase factor in the kernel:

\[ U(x, y, z) = e^{ikz} i\lambda z \int \int_{\Sigma} U(u, v, 0) e^{\frac{ik}{\lambda z} (xu+yv)} dudv. \]  

(1.5.19)

For a typical occulter propagation with a radius of 15-30 m and a propagation distance of 30-80,000 km (see Table 1.1) the Fresnel approximation condition in Equation 1.5.16 is easily met by several orders of magnitude, but the Fraunhofer approximation condition in Equation is not satisfied at all. Therefore, for optical propagations with an occulter we must use Fresnel diffraction and perform the integration in Equation 1.5.17. An occulter experiment provides a verification of the assumptions behind both the scalar propagation and the Fresnel approximation.

One further form of the Fresnel diffraction integral in Equation 1.5.17 will be of
particular interest. This form arises for the case of circular symmetry. We use polar coordinates \((r, \theta)\) across the aperture plane and \((\rho, \phi)\) across the observation plane.

The consequence of circular symmetry is that there is no angular dependence in the electric field expression, that is \(U(u, v, 0) = U(r, 0)\) and \(U(x, y, z) = U(\rho, z)\). In polar coordinates, the Fresnel diffraction integral (assuming a finite radial extent \(R\) of the aperture) becomes

\[
U(\rho, z) = \frac{e^{ikz}}{i\lambda z} \int_0^R \int_0^{2\pi} U(r, 0) e^{i\frac{kr}{\lambda z}(r^2 + \rho^2)} e^{-i\frac{kn}{\lambda z}r\rho \cos \theta} r\,dr\,d\theta 
\]

\[
(1.5.20)
\]

\[
= \frac{2\pi e^{ikz}}{i\lambda z} \int_0^R U(r, 0) e^{i\frac{kr}{\lambda z}(r^2 + \rho^2)} J_0 \left( \frac{2\pi r\rho}{\lambda z} \right) r\,dr.
\]

\[
(1.5.21)
\]

where \(J_0\) is the zeroth order Bessel function of the first kind and as shown is due to the angular integral.

### 1.6 Polarization effects

The previous discussion of scalar diffraction theory assumes that the diffracting screen is infinitely thin and perfectly conducting material. It also assumes that the input beam is unpolarized.

Electromagnetic waves can become polarized through reflection and through scattering [13]. In fact, polarimetry instruments are being developed as a method for exoplanet imaging and characterization based on the fact that the host star is a broadband thermal radiator without a preferred polarization whereas light from an exoplanet becomes polarized as it is scattered in its atmosphere [183].

An important concern for high-contrast imaging is polarization via reflection from the primary telescope mirror reducing contrast for the starlight suppression systems designed for unpolarized light inside the telescope [17]. For an occulter mission, polarization due to reflections inside the telescope is not expected to reduce performance.
because the starlight has already been suppressed prior to the pupil. For a possible polarimetric instrument, the telescope-induced polarization can be calibrated.

Polarization can still affect performance for an occulter. Throughout this dissertation, we have used scalar diffraction theory to model the performance of occulters. However, rigorous diffraction theory [13] studies the diffraction of the orthogonal components of the electromagnetic field. Each of the polarized components must be propagated independently and added incoherently at the telescope pupil reducing the performance in the optimized shadow. Luckily, because of the small angles involved in forward diffraction scatter for an occulter geometry this effect is expected to be small. However, because of the suppression levels necessary for high-contrast imaging, quantification of this effect is important and will be the subject of a future study.

Starlight travelling through the interstellar medium can become polarized by scattering from dust particles between the target star and the occulter. Theoretical models show that the direction of polarization depends on the grain size and direction of orientation, and is more pronounced for targets farther away [100]. Empirical polarization maps can be consulted for all the desired target stars for an occulter to determine the expected degree of polarization of the input field [116]. Typical strong polarizations are of one part in $10^4$ [182] relative intensity difference at the input of the occulter. We expect this to be a secondary effect to the orthogonal component incoherence of the forward diffraction scatter even assuming unpolarized input light.

Polarization must also be considered for diffraction and reflection off the finite thickness occulter edges for the Sun, with the suppression degradation contribution due to each of the orthogonal components. Current studies are under-way to determine optimal edge shapes and Sun-occulter angles to minimize such stray light [114].
1.7 Matrix Fourier Transforms

One method of evaluation of the Fresnel diffraction integrals is as Fourier transforms
of the pupil function and the quadratic phase factor. Many algorithms exist for evalua-
tion of the Fourier Transform, including the Fast Fourier Transform (FFT) [37, 129].
We detail here the mathematical formulation of a Matrix Fourier Transform (MFT)
method described by Soummer at al. [160]. We use this method for optical propaga-
tions because it is very convenient for occulters: it permits setting different sampling
intervals of the planes of propagation, reduces memory usage to allow implementa-
tion of the grids necessary for accurate occulter propagations, and is simple to implement.

Suppose we wish to find the Fourier transform of a function $U(x, y)$. This can be
written as:

$$
\hat{U}(\hat{x}, \hat{y}) = \int_{-\infty}^{+\infty} U(x, y) e^{-2\pi i (x \hat{x} + y \hat{y})} dx dy.
$$

(1.7.1)

Where $(\hat{x}, \hat{y})$ are the spatial frequency components corresponding to $(x, y)$. We can
write the discrete Fourier transform as a matrix multiplication as follows:

$$
\hat{U}(\hat{x}, \hat{y}) = \frac{m}{N_AN_B} e^{-2\pi i \hat{x} \hat{x}^T} \cdot U(x, y) \cdot e^{-2\pi i \hat{y} \hat{y}^T}.
$$

(1.7.2)

In Equation 1.7.2 we have discretized the original Equation 1.7.1 using column vectors
as follows by choice of number of samples $N_A$ and $N_B$, with an output plane size $m$:

$$
x = [x_0, x_1, \ldots, x_k, \ldots, x_{N_A - 1}]^T, \quad x_k = \frac{k - N_A/2}{N_A}
$$

(1.7.3)

$$
y = [y_0, y_1, \ldots, y_k, \ldots, y_{N_A - 1}]^T, \quad y_l = \frac{k - N_A/2}{N_A}
$$

(1.7.4)

$$
\hat{x} = [\hat{x}_0, \hat{x}_1, \ldots, \hat{x}_l, \ldots, \hat{x}_{N_B - 1}]^T, \quad \hat{x}_l = (l - N_B/2) \frac{m}{N_B}
$$

(1.7.5)

$$
\hat{y} = [\hat{y}_0, \hat{y}_1, \ldots, \hat{y}_l, \ldots, \hat{y}_{N_A - 1}]^T, \quad \hat{y}_l = (l - N_B/2) \frac{m}{N_B}
$$

(1.7.6)
Evaluating Equation 1.7.1 thus consists of two matrix products. In the case that \( N_A = N_B = N \), the MFT number of operations is of order \( 8N \), whereas the FFT is of order \( 5 \log_2(N) \), so we see that we trade-off some computational speed for choice of arbitrary sampling intervals.

1.8 Dissertation organization and contributions

This dissertation comprises a body of work primarily oriented towards laboratory verification of occulter performance to demonstrate our ability to design and operate realistic occulter models. We use existing occulter design theory, and apply this theory towards experimental verification of scaled occulters. We provide empirical results for an existing occulter experiment of measured suppression at the level of \( 10^{-5} \) compared to a theoretical level of approximately \( 10^{-7} \); similarly we provide measured contrast levels of \( 10^{-10} \) compared to approximately \( 10^{-12} \) expected. These results are compared to experimental baselines to demonstrate improvement by design of the shape. We make use of the empirical image plane results to identify limiting factors of performance and provide a more rigorous analysis of their effect on the experiment than available for other occulter experiments. We then demonstrate how a realistic flight occulter can be scaled to laboratory size and we apply the optical models validated experimentally to predict performance for such a proposed testbed design. The dissertation is organized as follows, and novel contributions are outlined:

Chapter 2. One-dimensional diffraction propagation models used to design occulters are summarized. The method of optimal design of occulters is discussed, with different constraints and objective function variations. To illustrate this occulter theory, we use a novel design example created for a small-scale technology demonstrator occulter mission.
Chapter 3. The approach used to scale occulters to laboratory scale is demonstrated, and it is shown that the diffraction integral remains mathematically identical to that expected in space. We discuss the design parameters, scaled dimensions, and expected performance for an occulter experiment which was oversized in its design to enable scaling to a propagation distance-limited testbed. We also outline the theoretical performance loss between the circularly symmetric design and the 16-petal realization used in the experiment.

Chapter 4. Empirical results are presented that demonstrate the performance of the optimized occulter. These include measurements at both the pupil and image plane of a camera-telescope. As a baseline we use a circular occulter shape to compare the performance experimentally. We also compare the results with theoretical predictions. A negative experimental result using a mask on glass shows the sensitivity of occulters to phase errors.

Chapter 5. A diffractive sensitivity analysis is performed to identify the experimental limits of the occulter performance in the testbed. We introduce a two-dimensional diffraction model which allows us to break the circular symmetry inherent in the one-dimensional models used to design the occulter. This analysis demonstrates that accuracy of the edge features is the main limiting factor of performance in the testbed, and we provide a solution to increase performance.

Chapter 6. We present the design of a new experiment which features a longer propagation distance enabling testing of occulters with similar Fresnel numbers as expected for a flight design. We numerically verify each step of the scaling process. Based on the experimentally-validated model we perform a diffractive sensitivity analysis of the performance of the occulter design in the new testbed focusing in particular on the effect of manufacturing precision and propagation distance. To assist in the building of the testbed, we also perform diffractive analyses for sizing the enclosure.
This requires considering diffractive models for the input and output tunnels.

**Chapter 7.** Implications of this work are discussed and future steps including demonstration of formation flight alignment and position sensing in the new testbed are outlined.
Chapter 2

Occulter Theory and Design

In this chapter, we summarize occulter propagation theory using one-dimensional integrals based on circular symmetry. The chapter is organized as follows. In §2.1, we start with a complementary apodized hole and show how using Babinet’s Principle we can compute the diffraction of an apodized occulter screen. In §2.2, we demonstrate how flower-shaped occulters with petals can be used to approximate the diffraction pattern of the designed, circularly symmetric occulter apodization profile. We show propagation to the image plane to obtain point-spread functions in §2.3. We briefly summarize analytical methods for occulter design in §2.4, but focus in particular on numerical optimization methods which provide a greater degree of freedom in designing the apodization profile leading to improved performance. The exposition in §2.1-2.4 follows closely Vanderbei et al [172] and also Cady [19]. Finally, in §2.5 we provide as an illustrative example a novel occulter design for a proposed technology demonstrator mission [119].

2.1 Fresnel propagation for an apodized occulter

Occulters are typically designed as circularly symmetric apodized functions. This is done for a number of reasons. First, designing a single-dimensional apodization is
much more numerically tractable. Second, a simple method of approximating the
apodized design with a binary mask has been developed which we will describe in
this chapter. Finally, analysis and propagation of occulters designed with circular
symmetry is also more numerically tractable and provides a first measure of the per-
formance of the design – later, in Chapters 5 and 6, we will look at a two-dimensional
propagation model which allows breaking the circular symmetry and investigating the
robustness of occulter designs with an emphasis on modeling expected performance
in the laboratory.

To compute the effect on the diffracted electric field to allow for suppression of
the incoming electric field at the telescope pupil, we make use of Fresnel diffraction.
The assumptions and derivation for Fresnel diffraction are outlined in §1.5. Addition-
ally, we will make use of Babinet’s Principle, which states that in the case of scalar
diffraction the sum of two observed fields due to the diffraction of an aperture and
complementary obstruction will be that observed if there is no aperture or obstruc-
tion [13]. Thus, if we let \( E \) denote the electric field in the absence of any aperture
or obstruction, \( E_a \) the diffracted field in the presence of the aperture, and \( E_o \) that in
the presence of the obstruction we have:

\[
E = E_a + E_o. \quad (2.1.1)
\]

Bearing this in mind, to determine the electric field of an apodized occulter, it is eas-
liest to consider its complimentary apodized hole. This arrangement is seen in Figure
2.1(a). We assume that the apodized hole is located at a plane \( P_1 \), has a transmission
profile \( A(r) \), with \( r \) being the radial coordinate across \( P_1 \) and with a maximum radial
extent \( R \). \( A(r) \) is allowed to take on values on the range \([0, 1]\), where 0 means it is
fully opaque and 1 is fully transparent – this is a pure amplitude attenuation with no
phase change allowed. We are interested in computing the diffracted field \( E_{\text{hole}}(\rho) \) at
Figure 2.1: Schematic demonstrating one-dimensional coordinate system and apodization functions for: (a) apodized hole (b) complementary apodized occulter

(a) apodized hole (b) complementary apodized occulter

We recall Equation 1.5.21 which describes Fresnel propagation for cases of circular symmetry as is the case here for the apodized hole. Thus, we can write the Fresnel propagation from an apodized hole that is complementary to the desired apodized occulter as

\[ E_{\text{hole}}(\rho) = E_0 e^{\frac{2\pi i z}{\lambda z}} \int_0^R r e^{\frac{\pi i (r^2 + \rho^2)}{\lambda z}} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) A(r)rdr, \quad 0 \leq \rho \leq \rho_{\text{max}} \quad (2.1.2) \]

where we assume a planar electric field with amplitude \( E_0 \) is incident on the aperture, and \( J_0 \) is the zeroth order Bessel function of the first kind and arises as a result of circular symmetry. We can now apply Babinet’s Principle to obtain the diffracted...
Electric field from an apodized occulter screen:

\[
E_{\text{occ}}(\rho) = E_0 e^{\frac{2\pi i z}{\lambda z}} \left(1 - \frac{2\pi}{i \lambda z} \int_0^R e^{\frac{2\pi i}{\lambda z} (r^2 + \rho^2)} J_0 \left(\frac{2\pi r \rho}{\lambda z}\right) A(r)rdr\right), \quad 0 \leq \rho \leq \rho_{\text{max}}
\]

\[
= E_0 e^{\frac{2\pi i z}{\lambda z}} - E_{\text{hole}}(\rho).
\] (2.1.3)

The geometry of the apodized occulter screen arrangement is shown in Figure 2.1(b). Some occulter designs (e.g., Copi and Starkman [38]) that have been proposed were fully apodized and this propagation would describe the ideal performance of such designs that feature circular symmetry.\(^1\) This performance however is extremely sensitive to phase errors introduced by the surface roughness of an occulter screen manufactured to the necessary dimensions.

### 2.2 Fresnel propagation for a petalized occulter

We can approximate an apodization profile that features a partially transmissive screen using a binary occulter by introducing a set of petals which cover an equal area to the partially transmissive screen. A binary occulter only features open areas

\(^1\) Other designs featured a rectangular geometry [78].
through which light can pass unimpeded and opaque areas which are assumed to completely block light. For the apodized occulter shown in Figure 2.1(b), the corresponding binary mask with 16-petals is shown in Figure 2.2 is created from \( N = 16 \) petals that are defined by turning the apodization profile into the edge of the binary mask, with opaque points on the mask belonging to the set \( S \):

\[
S = \{(r, \theta), 0 \leq r \leq R, \theta \in \Theta(r)\}, \quad (2.2.1)
\]

\[
\Theta(r) = \bigcup_{n=0}^{N-1} \left[ \frac{2\pi n}{N} - \frac{\pi}{N}A(r), \frac{2\pi n}{N} + \frac{\pi}{N}A(r) \right]. \quad (2.2.2)
\]

The petalization process is identical to that for star-shaped coronagraph masks described in Vanderbei et al [175]. The electric field past the petalized occulter can now be written as follows:

\[
E_{\text{bin}}(\rho, \phi) = E_0 e^{\frac{2\pi \rho z}{\lambda z}} \left(1 - \frac{1}{i \lambda z} \int \int_S e^{\frac{\pi i}{2z}(r^2+\rho^2)} e^{-\frac{2\pi i}{2z} r \rho \cos(\theta - \phi)} r dr d\theta \right). \quad (2.2.3)
\]

The Jacobi-Anger expansion is given by the expression:

\[
e^{-i \frac{2\pi r \rho}{\lambda z} \cos(\theta - \phi)} = \sum_{m=-\infty}^{\infty} i^m J_m \left( -\frac{2\pi r \rho}{\lambda z} \right) e^{im(\theta - \phi)}. \quad (2.2.4)
\]

We apply the Jacobi-Anger expansion and separate the angular and radial integrals, to re-write the integral as follows

\[
E_{\text{bin}}(\rho, \phi) = E_0 e^{\frac{2\pi \rho z}{\lambda z}} \left(1 - \frac{2\pi}{i \lambda z} \int_0^R e^{\frac{\pi i}{2z}(r^2+\rho^2)} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) A(r) r dr - \right.
\]

\[\ldots \sum_{k=1}^{\infty} \frac{4\pi}{i \lambda z} \frac{\cos(kN \phi)}{k \lambda z} \int_0^R e^{\frac{\pi i}{2z}(r^2+\rho^2)} J_{kN} \left( \frac{2\pi r \rho}{\lambda z} \right) \frac{\sin(k\pi A(r))}{k \pi} r dr \right). \quad (2.2.5)
\]

where \( 0 \leq \rho \leq \rho_{\text{max}}, \ 0 \leq \phi \leq 2\pi \). This expression can be considered to be the original circularly-symmetric Fresnel propagation from Equation 2.1.3 with the angular dependence at the pupil plane provided by an expansion of higher-order Bessel terms.
Fortunately, the series converges quickly as terms are added. The effect of adding additional petals is to increase the order of terms in the Bessel function expansion and thus improve the accuracy of the approximation particularly at the center of the shadow.

### 2.3 Propagation to the image plane

To proceed to the image plane (P3), we let \((x, y)\) be the Cartesian coordinates across the pupil plane. \(E_{\text{pup}}(x, y)\) describes the electric field at the pupil plane. We define a circular pupil function \(p(x, y)\) with 1 inside the pupil:

\[
p(x, y) = \begin{cases} 
1, & \sqrt{x^2 + y^2} \leq r_a \\
0, & \text{else}
\end{cases}
\]  

where \(r_a\) is the radial extent of the telescope aperture. The focus is set on a star at infinity. Therefore, we Fresnel propagate a distance \(s\) past the lens equal to the telescope’s focal length \(f\). Then the expression for the electric field at the image plane defined by Cartesian coordinates \((\xi, \eta)\), ignoring leading phase terms, becomes:

\[
E_{\text{im}}(\xi, \eta) = \frac{1}{i\lambda s} \int \int_{p(x, y) = 1} E_{\text{pup}}(x, y) e^{-\frac{2\pi i}{\lambda s}(x\xi + y\eta)} dxdy
\]  

This expression can be evaluated via any discrete Fourier transform, such as the matrix Fourier transform [160] described in §1.7.

### 2.4 Design of an apodized occulter profile

Different approaches have been proposed for obtaining an occulter apodization profile. These include choosing an analytical function for the transmission function:

- a polynomial function with an opaque disk in the center as proposed by Copi and
A specific polynomial function apodization is chosen to provide an exact solution on-axis and is then adjusted to provide suppression over a larger area of the shadow and wavelength range. For example, a polynomial function can take the form

\[ A(y) = (35y^4 - 84y^5 + 70y^6 - 20y^7) \]  

where \( a = 0.15 \) is the fraction of the opaque disk such that \( A(r) = 0, r \leq a \) and a maximum radial extent \( R \) with \( A(r) = 1, r \geq R \).

- a hypergaussian as proposed by Cash [30]

\[ A(r) = 1 - e^{\left(\frac{r-a}{b}\right)^n} \]  

with an explicit expression for minimizing the suppression with \( a = b \). For example, the NWO design [31] uses a specific choice of \( R = 25, a = b = 12.5 \), and \( n = 6 \).

The example polynomial functions does not provide sufficient suppression for imaging Earth-like planets, but a generalized search might be able to find such a function. The hypergaussian design does, although a generalized numerical approach can be used to further reduce the radius \( R \) and improve broadband performance [172].

Several formulations have been proposed for optimizing the design of the apodization function for an occulter. For occulter design, we typically compute the electric field due to the occulter at the pupil plane of the telescope using only the zeroth order Bessel function. This is the expression for \( E_{\text{occ}}(r) \) found in Equation 2.1.3. We define a discretized apodization profile \( A(r) \) as the set of decision variables which can be allowed to vary by the optimization procedure – this increases the number of
degrees of freedom compared to analytical designs such as the one above. We seek to minimize the area under the apodization function and thus the size of the occulter screen. Constraints can be written on the electric field $E_{\text{occ}}(\rho)$ to force it to remain at a desired ($10^{-10}$) suppression level:

$$\min \quad \int_0^R A(r)rdr \quad (2.4.4)$$

subj. to : \quad $-10^{-c} \leq \text{Re}(E_{\text{occ}}(\rho; \lambda)) \leq 10^{-c}$ \quad (2.4.5)

$$-10^{-c} \leq \text{Im}(E_{\text{occ}}(\rho; \lambda)) \leq 10^{-c} \quad (2.4.6)$$

We are actually interested in constraining the intensity (i.e., the square of the electric field) to the $10^{-10}$ level, however this is a quadratic problem which is not numerically solvable. Instead we pose linear constraints on the electric field directly as in Equations 2.4.5 and 2.4.6, a more conservative formulation.

The problem as formulated above is infinite dimensional. We apply midpoint discretization for the radial coordinates $r$ and $\rho$ over the desired ranges. The upper bound $\rho_{\text{max}}$ determines the maximum amount of the shadow that has been optimized for suppression. To solve this finite-dimensional, linear programming problem we can make use of a number of numerical packages. We use LOQO, an interior point non-linear programming optimization package developed by R. Vanderbei [170], for all occulter optimization problems formulated throughout this dissertation. LOQO allows nonconvex linear programming by solving a series of quadratic problems obtained through a merit function and a modified descent direction using an interior-point method [174].

The solution of the problem is a set of binary concentric rings, and is similar to the spiderweb coronagraph masks in [176]. We can introduce additional constraints in the form of monotonicity and smoothness on the apodization function $A(r)$ to ensure that the occulter is structurally connected when petalized. A fully opaque central
A disk that can accommodate a spacecraft bus can also be added in the form of an opaque portion of the apodization function up to the desired radius \( a \) of the central disk. These additional constraints are expressed in the expanded linear programming below:

\[
\begin{align*}
\min & \quad \int_0^R A(r) r \, dr \\
\text{subj. to} & \quad -10^{-c} \leq \text{Re}(E_{occ}(\rho; \lambda)) \leq 10^{-c} \\
& \quad -10^{-c} \leq \text{Im}(E_{occ}(\rho; \lambda)) \leq 10^{-c} \\
& \quad A(r) = 1, \ 0 \leq r \leq a \\
& \quad A'(r) \leq 0, \ |A''(r)| \leq \sigma, \ \forall \ 0 \leq r \leq R
\end{align*}
\]  

where \( \sigma \) is the bound on smoothness. The problem is discretized identically to the previous formulation. This basic optimization problem can be reformulated in different ways. For example, the objective function can be written as a suppression minimization function [25]. Additional manufacturing constraints that can be added to this model [19]:

\[
\begin{align*}
\frac{2\pi r A(r)}{N} & \geq \sigma_1, \ a \leq r \leq R \\
\frac{2\pi r (1 - A(r))}{N} & \geq \sigma_2, \ a \leq r \leq R
\end{align*}
\]

where Equation 2.4.12 imposes a minimum petal width \( \sigma_1 \) and Equation 2.4.13 forces a minimum gap width \( \sigma_2 \) between petals. Occulters have also been designed for two-distances to by reducing the optimized wavelength band and moving in to observe the red-end (albeit at a worse inner working angle) [81].

We mention here three recent papers that have provided different design ideas:

- Cady [24] showed that the number of petals necessary can be reduced by introducing asymmetry in the petal shape; this approach can also be used to
introduce additional support elements.

- Shiri and Wasylkiwskyj [148] proposed petalizing an occulter and simultaneously apodizing the tips; it is not clear that this is an improvement on petalized tips, and especially the formulation above which allows for a minimal petal width, as occulters tend to be extremely sensitive to phase errors.

- Flamary and Aime [55] proposed optimizing occulters in the image plane instead of the pupil plane. This approach appears to provide some performance improvements in theory.

2.5 Example of occulter design for a technology demonstrator mission

We now illustrate occulter design theory using an example. We show how an occulter is sized based on the science requirements, and demonstrate the constraints that are applied in the design process.

Due to the long baselines associated with an occulter, it is impossible to physically test a full-scale occulter on the ground. A full-scale mission is a major undertaking, and a technology demonstrator precursor would assist in minimizing risk associated with an occulter mission. A small-scale occulter mission has therefore been proposed to demonstrate the technology underlying occulters, and designed to obtain limited science by targeting specifically Alpha Centauri A [119]. Because Alpha Centauri is a binary system, for this discussion it is assumed that Alpha Centauri B can be blocked either by a second identical occulter, or creating an artificial dark hole in the image plane using a diffractive grid in the pupil created by a deformable mirror [165]. As a small demonstrator mission, it was possible to find an Earth orbit with sufficiently high eccentricity for which sufficient periods of alignment could be controlled over...
Table 2.1: Optimization parameters for occulter optical design for the technology demonstrator mission.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation distance, $z$</td>
<td>670 km</td>
</tr>
<tr>
<td>Maximum radius, $R$</td>
<td>2.5 m</td>
</tr>
<tr>
<td>Opaque disk, $a$</td>
<td>1 m</td>
</tr>
<tr>
<td>Opaque fraction, $a/R$</td>
<td>0.4</td>
</tr>
<tr>
<td>Smoothness, $\sigma$</td>
<td>8.6</td>
</tr>
<tr>
<td>Lower wavelength, $\lambda_{\text{min}}$</td>
<td>400 nm</td>
</tr>
<tr>
<td>Upper wavelength, $\lambda_{\text{max}}$</td>
<td>1000 nm</td>
</tr>
<tr>
<td>Shadow radius, $\rho_{\text{max}}$</td>
<td>0.27 m</td>
</tr>
<tr>
<td>Shadow discretization</td>
<td>500</td>
</tr>
<tr>
<td>Apodization discretization</td>
<td>3000</td>
</tr>
<tr>
<td>Wavelength discretization</td>
<td>7</td>
</tr>
<tr>
<td>Suppression constraint, $10^{-2c}$</td>
<td>$10^{-10}$</td>
</tr>
</tbody>
</table>

one section of the orbit.

The general mission design is described more broadly in [119], and here we focus on the occulter design. Restricting the mission to a search for planetary companions in a nearby system allows oversizing the occulter as the inner working angle can be made much larger. This can be used to bring the occulter to a much closer separation. The primary mission constraint was designing an occulter with a tip-to-tip diameter of 5 m so that it could be fit inexpensively as a secondary payload on an ESPA Grande ring. From the science requirements for imaging the habitable zone an inner working angle of approximately 1 arcsecond was necessary. This allows setting the telescope-occulter separation to 670 km. Additionally, folding the petals required maximizing the fraction of the total covered by the central disk and this constraint was competing with the need to maximize the size of the shadow to increase the diameter of the observing telescope and allow for sufficient alignment tolerance in the shadow.

The final occulter shape design is based on the optimization problem in Equations
Figure 2.3: Performance of technology demonstrator occulter (a) Suppression (log-scale) of shadow intensity at the telescope pupil plane (b) Contrast (log-scale) of residual starlight at the telescope focal plane.

2.4.7-2.4.11. The parameters used in the optimization are summarized in Table 2.1. The desired $10^{-10}$ suppression constraint is in the intensity of the shadow – thus, this corresponds to a $10^{-5}$ amplitude constraint in Equations 2.4.5 and 2.4.6.

The performance of the final occulter design is summarized in Figure 2.3. In Figure 2.3(a), we show the suppression of the shadow at the telescope’s pupil plane. We see that at the center of the shadow the suppression is several orders of magnitude deeper than outside – for planet detection, the telescope must be maintained in this region of the shadow. In Figure 2.3(b), we see the system’s PSF. Compared to a flagship mission (see e.g., THEIA’s PSF in Figure 1.5) the angular separation observable is increased due to the closer geometry of this demonstrator mission – thus, the angular separation at the occulter tips is on the order of 1 as for this technology demonstrator occulter instead of 100 mas.

Finally, in Figure 2.4a, we demonstrate the process through which we pick an appropriate number of petals for an occulter. We compute, using Equation 2.2.5, the suppression profile for occulters with different numbers of petals. In Equation 2.2.5, we use $K = 20$ non-zero Bessel terms in the summation for the petalized cases.
Figure 2.4: Petalization of an apodized design (a) Average suppression across designed dark hole in the shadow for occulter realizations with different numbers of petals and compared to the performance for the pure apodized design (b) Flower-shaped occulter realization with sixteen petals.

which is sufficient for convergence even for the lowest number of petals considered (propagations with higher numbers of petals tend to converge more quickly). We compute the average suppression across the entire designed dark hole for all these cases, and summarize these results in Figure 2.4(a). For comparison, we also plot the suppression performance for the apodized case. We wish to pick the minimum number of petals for which the apodized performance is fully recovered. In this case, sixteen petals is sufficient and in Figure 2.4(b) we show this petalized occulter.
Chapter 3

Scaling of Occulter Designs for Laboratory Verification

Occulters are designed under the assumption of an ideal space environment. However, before building and operating a space occulter mission, it is important to provide experimental verification of the performance of occulter designs. This process is two-pronged: verification of manufacturing tolerances and verification of the validity of the optical models.

First, it must be shown that the occulter shape can be manufactured within tolerances prescribed by numerical models. For this purpose, optical models have been used to determine occulter manufacturing tolerances for which target contrast levels can still be achieved by investigating the effect on performance of various types of shape errors [47, 146, 145]. Subsequently, an individual petal was fabricated and its edges were precisely measured. These precise measurements of the manufactured petal edge were used in optical models to more realistically predict the performance of a full-size occulter [87]. Additionally, four petals were manufactured and the proposed space deployment mechanism was used to align them. Their alignment was measured and used as part of optical models to predict the occulter’s deployed performance.
successfully validating the deployment mechanism [181].

Second, and constituting the main focus of this dissertation, the optical models used to design and predict the performance of the occulter must be experimentally verified as well. A full-scale occulter cannot be directly optically tested because of the long separation distances involved at the space scale. Therefore, to test the performance of an occulter we must scale the occulter to laboratory size while maintaining identical optical performance. Ideally, we wish to experimentally test the performance of an occulter shape that would be flown as a mission. At the same time, we must identify a method to mount the occulter that allows for a valid comparison to a space occulter.

In this chapter, we address the scaling of occulter designs to laboratory dimensions. We discuss the introduction of an outer ring and support struts that enable mounting of the occulter in §3.1. In §3.2, we demonstrate how occulters can be scaled for a different distance while maintaining an identical shadow intensity by maintaining constant Fresnel numbers. In §3.2, we introduce a change of variables that maintains the same shadow intensity but can be re-interpreted to correspond to an occulter illuminated by a diverging beam. A non-dimensionalization of the propagation integrals is presented in §3.4. The design of the current propagation distance-limited occulter testbed is shown in §3.5, and the predicted optical model performance is summarized in §3.6.

## 3.1 Outer ring and struts for mounting the occulter mask

For a space mission, the occulter is surrounded by free space out to infinity. In a laboratory environment, this property of a free space occulter cannot be maintained – there will be a finite limit due to the extent of the laboratory and supports used
to mount the occulter. Thus, an important difficulty with a laboratory experiment is mounting of the occulter mask in a finite laboratory to ensure diffraction is identical with the space occulter. The first occulter experiment undertaken used wires to mount the occulter mask [140]; another experiment used a vacuum tube to better control the air environment of the experiment and also used thinner wires [2, 137, 138]. One possible source of error that may have limited both these experiments is diffracted light from the thin wires used to mount the occulter mask. Our approach is to design an outer ring and support struts to provide a mounting method for the inner occulter. Including the outer ring and struts as part of the design allows us to minimize their diffraction effects [19, 22]. It also mitigates any diffraction effects that arise due to the finite size of the laboratory environment or any collimating optics.

In Figure 3.1(a), we show a sample apodization profile, $A(r)$, featuring an outer ring and one with support struts $bA(r)$. The support struts are obtained by multiplying the apodization profile by a constant factor $0 \leq b \leq 1$, which was set to $b = 0.9$ in this example. Figure 3.1(b) helps visualize the regions of the apodization profile via 16-petal realization. The apodization profile $A(r)$ features an inner occulter shape from 0 to $R_b$ with $R_a$ the extent of the fully opaque inner disk region, and the mono-
tonically increasing region from $R_a$ to $R_b$ representing the petals. From $R_b$ to $R_c$ we have a fully open region – this represents the dark annular working region where a planetary signal would be unaffected by the occulter mask. Finally, from $R_c$ to $R_d$ we have a monotonically decreasing region representing the outer petals.

Using the Fresnel diffraction integral, the electric field at a distance $z$ downstream from this occulter profile with an incident plane, monochromatic wave with wavelength $\lambda$ can be written as

$$E_{\text{mask}}(\rho) = \frac{2\pi}{i\lambda z} e^{\frac{2\pi i}{\lambda z}} \int_0^R e^{\frac{2\pi i}{\lambda z}(r^2+\rho^2)} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) A(r) r dr \quad 0 \leq \rho \leq \rho_{\text{max}} \quad (3.1.1)$$

where $R = R_d$ is the maximum radial extent of the partially-transmissive apodization profile $A(r)$, $\rho$ is the radial distance at the pupil plane, $r$ is the radial distance at the occulter plane, and $J_0$ is the zeroth order Bessel function of the first kind. For simplicity, we have set the planar input beam’s amplitude to unity.

The expression in Equation 3.1.1 is similar to that in Equation 2.1.3 for an occulter surrounded by free-space. Because of the introduction of the outer ring in the region from $R_c$ to $R_d$ the diffraction integral only needs to be applied across regions with some transmission from $R_b$ to $R_d$ and so Babinet’s Principle, which allowed for the integration to infinity around the free-space occulter in Equation 2.1.3, is not needed here.

It has been shown [19] that an outer ring can be designed to allow the electric field across the optimized, dark hole portion of the pupil plane shadow to be negligibly modified compared to the absence of the outer ring. The outer ring is the complement of an occulter whose petalized region starts from a circular disk at $R_c$. To see this,
we re-write Equation 3.1.1

\[ E_{\text{mask}}(\rho) = \frac{2\pi}{i\lambda z} e^{\frac{\pi i \rho^2}{\lambda z}} e^{\frac{2\pi i \rho}{\lambda z}} \left( \int_{R_a}^{R_b} e^{\frac{2\pi i \rho}{\lambda z}} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) A(r) r dr + \int_{R_b}^{R_c} e^{\frac{\pi i r^2}{\lambda z}} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) \frac{2\pi r \rho}{\lambda z} A(r) r dr \right) \quad (3.1.2) \]

where \( 0 \leq \rho \leq \rho_{\text{max}} \). We also write the diffraction contribution due to the inner mask only (i.e., \( A(r) \) up to \( R_b \)) assuming it surrounded by free-space:

\[ E_{\text{inner}}(\rho) = \frac{2\pi}{i\lambda z} e^{\frac{\pi i \rho^2}{\lambda z}} e^{\frac{2\pi i \rho}{\lambda z}} \left( \int_{0}^{R_b} e^{\frac{2\pi i \rho}{\lambda z}} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) A(r) r dr + \int_{R_b}^{\infty} e^{\frac{\pi i r^2}{\lambda z}} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) r dr \right) . \quad (3.1.3) \]

Similarly to Equation 2.1.3, we apply Babinet’s Principle to eliminate the integral to infinity [13] using the complement of the inner occulter’s transmission profile:

\[ E_{\text{inner}}(\rho) = e^{\frac{2\pi i \rho}{\lambda z}} \left( 1 - \frac{2\pi}{i\lambda z} e^{\frac{\pi i \rho^2}{\lambda z}} \int_{0}^{R_b} e^{\frac{2\pi i \rho}{\lambda z}} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) (1 - A(r)) r dr \right) \quad 0 \leq \rho \leq \rho_{\text{max}}. \quad (3.1.4) \]

Next, we compute the difference between the electric field due to the inner mask surrounded by free-space only and that of the full occulter mask featuring an outer ring:

\[ E_{\Delta}(\rho) = E_{\text{inner}}(\rho) - E_{\text{mask}}(\rho) \]

\[ = e^{\frac{2\pi i \rho}{\lambda z}} \left( 1 - \frac{2\pi}{i\lambda z} e^{\frac{\pi i \rho^2}{\lambda z}} \left( \int_{0}^{R_c} e^{\frac{2\pi i \rho}{\lambda z}} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) (r - A(r)) r dr \right) \right) \quad 0 \leq \rho \leq \rho_{\text{max}}. \quad (3.1.5) \]
We define a transmission profile $A_{\text{out}}(r)$ for the outer ring as follows:

$$A_{\text{out}}(r) = \begin{cases} 
1 & : 0 \leq r < R_c, \\
A(r) & : R_c \leq r \leq R_d, \\
0 & : r > R_d.
\end{cases}$$  \hfill (3.1.6)

We re-write Equation 3.1.5:

$$E_\Delta(\rho) = e^{2\pi i z} \left( 1 - \frac{2\pi}{i\lambda z} e^{\frac{\pi i \rho^2}{\lambda z}} \int_0^{R_d} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) A_{\text{out}}(r)r\,dr \right) 0 \leq \rho \leq \rho_{\text{max}}. \hfill (3.1.7)$$

In this form, we notice this expression is identical to Equation 2.1.3. We can therefore write an optimization problem for an occulter extending out to $R_d$ with an opaque circular disk out to $R_c$. The outer ring is the complement of such an occulter, oversized with respect to the inner occulter we wish to test. We can ensure that this outer ring therefore has a minimal effect on the diffraction of the inner occulter by setting the physical extent and target suppression level of its optimized portion of the shadow to be equal or greater than the inner occulter.

We also consider the effect of introducing struts to support the inner occulter to the outer ring. As shown in Figure 3.1, struts are introduced by multiplication of the transmission profile $A(r)$ by a factor $0 < b < 1$. Thus, across the nominally open region from $R_b$ to $R_c$ the struts will cover an area fraction of $1 - b$. We now wish to consider the effect of the struts on the diffraction pattern:

$$E_{\text{mask}}(\rho) = \frac{2\pi}{i\lambda z} e^{\frac{2\pi i z}{\lambda z}} \int_0^R e^{\frac{\pi i (r^2 + \rho^2)}{\lambda z}} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) bA(r)r\,dr \quad 0 \leq \rho \leq \rho_{\text{max}}. \hfill (3.1.8)$$

Where we have let $R = R_d$, the outer extent of the apodization profile. Since $b$ is a constant factor it can be factored out. Thus the introduction of support struts through $b$ affects the diffraction pattern for the apodized profile only as a constant
multiplicative factor.

We also wish to consider its effect on the petalized occulter diffraction and apply the petalization process in §2.2. For an N-fold circularly symmetric binary occulter mask, with N even, the set of points S defines the occulter petals given in \((r, \theta)\), polar coordinates at the occulter plane:

\[
S = \{(r, \theta), 0 \leq r \leq R, \theta \in \Theta(r)\},
\]

\[
\Theta(r) = \bigcup_{n=0}^{N-1} \left[ \frac{2\pi n}{N} - \frac{\pi}{N} bA(r), \frac{2\pi n}{N} + \frac{\pi}{N} bA(r) \right].
\]

The Fresnel integral for this binary occulter mask is therefore:

\[
E_{\text{bin}}(\rho, \phi) = \frac{1}{i\lambda z} e^{\frac{2\pi i}{\lambda z}} \int \int_{S} e^{\frac{\pi i}{\lambda z} (r^2 + \rho^2)} e^{-\frac{2\pi i}{\lambda z} \rho \cos(\theta - \phi)} r dr d\theta, \; 0 \leq \rho \leq \rho_{\text{max}}, 0 \leq \phi \leq 2\pi.
\]

We use the Jacobi-Anger expansion

\[
e^{-i\frac{2\pi \rho}{\lambda z} \cos(\theta - \phi)} = \sum_{m=\infty}^{\infty} i^m J_m \left(-\frac{2\pi \rho}{\lambda z}\right) e^{im(\theta - \phi)}
\]

and separate angular and radial coordinates:

\[
E_{\text{bin}}(\rho, \phi) = \frac{1}{i\lambda z} e^{\frac{2\pi i}{\lambda z}} \int_{0}^{R_d} e^{\frac{\pi i}{\lambda z} (r^2 + \rho^2)} \left( \sum_{m=\infty}^{\infty} i^m J_m \left(-\frac{2\pi \rho}{\lambda z}\right) \right) \times
\]

\[
\ldots e^{-im\phi} \left( \int_{S(r)} e^{im\theta} d\theta \right) r dr, \; 0 \leq \rho \leq \rho_{\text{max}}, 0 \leq \phi \leq 2\pi.
\]
where \(0 \leq \rho \leq \rho_{\text{max}}, ~ 0 \leq \phi \leq 2\pi\). We can evaluate the angular integral over \(S\):

\[
\int_{S(r)} e^{im\theta} d\theta = \sum_{n=0}^{N-1} \int_{\frac{2\pi n}{N}}^{\frac{2\pi n + \frac{\pi}{N} bA(r)}{N}} e^{im\theta} d\theta
\]

\[
= \sum_{n=0}^{N-1} \frac{e^{im\theta}}{im} \left[ \frac{2\pi n + \frac{\pi}{N} bA(r)}{N} - \frac{2\pi n - \frac{\pi}{N} bA(r)}{N} \right]
\]

\[
= \sum_{n=0}^{N-1} \frac{e^{im \frac{2\pi n}{N}} (2 \sin (m\pi bA(r)/N))}{m}
\]

\[
= \begin{cases} 
2\pi bA(r) & : m = 0, \\
\frac{2}{k} \sin (k\pi bA(r)) & : m = \pm kN, \\
0 & : \text{else}
\end{cases} \tag{3.1.14}
\]

Continuing, we pull out \(m = 0, \pm kN\) separately and use the property that Bessel-functions are even

\[
E_{\text{bin}}(\rho, \phi) = \frac{2\pi}{i\lambda z} \left( e^{\frac{2\pi i}{\lambda z}} \int_0^R e^{\frac{2\pi i}{\lambda z} (r^2 + \rho^2)} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) bA(r) r dr + \right.
\]

\[
\left. \sum_{k=1}^{\infty} \frac{4\pi \cos (kN\phi)}{i\lambda z} e^{\frac{2\pi i}{\lambda z}} \int_0^R e^{\frac{2\pi i}{\lambda z} (r^2 + \rho^2)} J_{kn} \left( \frac{2\pi r \rho}{\lambda z} \right) \frac{\sin (k\pi bA(r))}{k\pi} r dr \right)
\]

\[
\tag{3.1.15}
\]

where \(0 \leq \rho \leq \rho_{\text{max}}, ~ 0 \leq \phi \leq 2\pi\). We notice that for higher-order Bessel terms \(b\) is an argument in the summation, which means that introduction of the support struts does affect the diffraction pattern structure. Fortunately, it does not affect the argument of the higher-order Bessel terms \(J_{kn}\). Therefore, if the number of petals is chosen appropriately to not affect the dark shadow for the entire mask without support struts, performance will be similar with the addition of the struts. The shadow outside of the dark hole will feature structural differences.
3.2 Fresnel scaling of occulter designs

To test occulters in the laboratory, their dimensions need to be scaled while maintaining an identical shadow to that expected in space. In this section, we show how we can scale down an occulter from space dimensions to laboratory dimensions for a smaller propagation distance by maintaining constant Fresnel numbers to ensure that the diffraction integral remains identical.

First, we consider the diffraction pattern for an occulter mask with an outer ring, described by Equation 3.1.1, which we reproduce here:

\[
E_{\text{mask}}(\rho) = \frac{2\pi}{i\lambda z} e^{\frac{2\pi i}{\lambda z} \rho^2} \int_{0}^{R} e^{\frac{2\pi i}{\lambda z} r^2} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) A(r) r dr \quad 0 \leq \rho \leq \rho_{\text{max}}. \tag{3.2.1}
\]

The dimensions listed here are as-designed at space scale and include the propagation distance \( z \), the radial coordinates across the occulter mask \( r \) and across the shadow \( \rho \). The maximum radial extent of the mask is \( R \), and the maximum extent of the optimized dark hole is \( \rho_{\text{max}} \).

We define here the two Fresnel numbers: \( F_o = \frac{R^2}{\lambda z} \) is across the occulter, and \( F_s = \frac{\rho_{\text{max}}^2}{\lambda z} \) is across the shadow. We introduce a scaling factor \( s \) that keeps both these Fresnel numbers. The electric field downstream from the occulter can now be re-written as

\[
E'_{\text{mask}}(\rho') = \frac{2\pi}{i\lambda z'} e^{\frac{2\pi i}{\lambda z'} \rho'^2} \int_{0}^{R'} e^{\frac{2\pi i}{\lambda z'} r'^2} J_0 \left( \frac{2\pi r' \rho'}{\lambda z'} \right) A'(r') r' dr' \quad 0 \leq \rho' \leq \rho'_{\text{max}} \tag{3.2.2}
\]

where in the above, we have made the following substitutions for scaled physical dimensions: \( \rho' = \rho/s, \ r' = r/s, A'(r') = A(sr'), \) and \( z' = z/s^2, \ R' = R/s, \) and \( E'_{\text{mask}}(\rho') = E_{\text{mask}}(s\rho') \). Note that the radial dimensions scale linearly with \( s \), whereas the propagation distance changes quadratically which works favourably towards scal-
We compare the scaled diffraction integral in Equation 3.2.2 with the original integral in Equation 3.2.1. These integrals are identical, so both the amplitude and phase of the electric field at the shadow plane will be identical between scaled and space dimensions. However, a side-effect is that the physical inner working angle is larger in the laboratory than it is in space.

### 3.3 Diverging beam scaling

The scaling in §3.2 assumes an ideal, planar input beam. Occulters are very sensitive to phase errors such as any introduced by collimating optics upstream from the occulter mask. A numerical example of the effect of phase errors induced by optical aberrations is given in §6.5.3. To circumvent the need for collimating optics, our experiment is designed using a diverging beam input on the occulter mask which can be spatially filtered [19, 22]. In this section, we show how an occulter mask already scaled for the laboratory propagation distance $z'$ can be re-scaled to correspond to a diverging input beam to maintain an identical shadow intensity.

We introduce a scaling factor $\gamma > 1$ applied as a change of variables. We assume that the diverging beam originates at a distance $h$ upstream from the occulter mask. The radial dimension across the occulter mask then is shrunk by a factor of $\gamma$: $r'' = r'/\gamma$. The radial dimension across the shadow is enlarged by a factor of $\gamma$: $\rho'' = \gamma \rho'$. The propagation distance remains unchanged: $z'' = z'$. Setting $h \rightarrow \infty$ represents the collimated case for which $r' = \rho' = r'' = \rho''$. This change of variables is illustrated in Figure 3.2. We can derive an expression for the scaling factor $\gamma$ using similar triangles.
Figure 3.2: Change of variables resulting in diverging beam scaling. The extremity of the mask under a collimated beam is shrunk by a factor \( \gamma \), whereas the extremity of the shadow is increased by a factor \( \gamma \). The divergence angle is given by the proximity \( h \) of the source and through similar triangles an expression for \( \gamma \) can be found.

as follows:

\[
\frac{r''}{h} = \frac{\rho''}{h + z'} \\
\frac{r'/\gamma}{h} = \frac{\gamma r''}{h + z'} \\
\frac{h + z'}{h} = \gamma^2 \\
\gamma = \sqrt{1 + \frac{z'}{h}}. \tag{3.3.1}
\]

The scaling factor \( \gamma \) can also be written in the following ways: \( \gamma = \sqrt{\frac{z'+h}{h}} = \sqrt{z'(\frac{1}{z'} + \frac{1}{h})} \). Letting \( E''_{\text{mask}}(\rho'') = E'_{\text{mask}}(\rho''/\gamma), A''(\rho'') = A'(\gamma r''), \) and \( R'' = R'/\gamma \):

\[
E''_{\text{mask}}(\rho'') = \frac{2\pi \gamma^2}{i\lambda z'} e^{\frac{2\pi i z'}{\lambda z'}} \times \\
\int_0^{R''} e^{\frac{2\pi i}{\lambda z'} (\gamma^2 r''^2 + \rho''^2/\gamma^2)} A''(r'') J_0 \left( \frac{2\pi r'' \rho''}{\lambda z'} \right) r'' dr'' \quad 0 \leq \rho'' \leq \rho''_{\text{max}}. \tag{3.3.2}
\]

Next, we substitute the \( \gamma \) terms inside the exponential to illustrate the new effective
Fresnel number

\[ E''_{\text{mask}}(\rho'') = \frac{2\pi \gamma^2}{i\lambda z''} e^{\frac{2\pi i}{\lambda}z'} \times \cdots \int_{0}^{R''} e^{\frac{2\pi i}{\lambda}(z' + h + r''^2 + \rho''^2) A''(r'')} J_0 \left( \frac{2\pi r'' \rho''}{\lambda z''} \right) r'' dr'' \]  

(3.3.3)

where \( 0 \leq \rho'' \leq \rho''_{\text{max}} \). We can re-write this equation in the following manner:

\[ E''_{\text{mask}}(\rho'') = \frac{2\pi \gamma^2}{i\lambda z''} e^{\frac{2\pi i}{\lambda}z'} e^{-\frac{2\pi i}{\lambda}r''^2} \times \cdots \int_{0}^{R''} e^{\frac{2\pi i}{\lambda}(r''^2 + \rho''^2)} e^{\frac{2\pi i}{\lambda}h} A''(r'') J_0 \left( \frac{2\pi r'' \rho''}{\lambda z''} \right) r'' dr'' \quad 0 \leq \rho'' \leq \rho''_{\text{max}}. \]

(3.3.4)

This is equivalent to a Fresnel propagation of the rescaled mask with an expanding beam. To see this, we consider the form of an expanding beam:

\[ E''_{\text{in}}(r'') = \frac{1}{\sqrt{h^2 + r''^2}} e^{\frac{2\pi i}{\lambda} \sqrt{h^2 + r''^2}} \]  

(3.3.5)

\[ \approx \frac{1}{h} e^{\frac{2\pi i}{\lambda} h} e^{\frac{2\pi i}{\lambda} r''^2}. \]  

(3.3.6)

Where the paraxial approximation step in Equation 3.3.6 is the second-order Taylor series expansion of the precise form of the spherical input beam in Equation 3.3.5:

\[ \sqrt{h^2 + r''^2} = h + \sqrt{h^2 + r''^2} - h \]  

(3.3.7)

\[ = h + \frac{r''^2}{\sqrt{h^2 + r''^2} + h} \]  

(3.3.8)

\[ \approx h + \frac{r''^2}{2h}. \]  

(3.3.9)

For typical laboratory sizes, we have \( h \gg r'' \) which means the amplitude should be almost constant across the occulter mask. Finally, combine the \( r''^2 \) terms and factor...
out the constant phase terms:

\[ E''_{\text{mask}}(\rho'') = \frac{2\pi\gamma^2}{i\lambda z'} e^{\frac{2\pi i}{\lambda z'} r''^2} e^{-\frac{2\pi}{\lambda z'} \rho''^2} \int_0^{R''} e^{\frac{2\pi i}{\lambda z'} \rho''^2 \left( \frac{1}{h' + \frac{1}{z'}} \right)} A''(r'') J_0 \left( \frac{2\pi r'' \rho''}{\lambda z'} \right) r'' dr'' \]

(3.3.10)

where \( 0 \leq \rho'' \leq \rho_{\text{max}}'' \). We have shown through a series of equalities and changes of variables that Equation 3.3.10 is equivalent to the Fresnel propagation in Equation 3.1.1. When we compare Equation 3.3.10 with a rescaled mask with a diverging input beam, the diffraction integral is identical except for the leading constant amplitude term \( \gamma^2 \) and the leading phase term that varies across the shadow radial coordinate. Normalizing for suppression allows for elimination of the constant amplitude; the phase term, however, implies that whereas the shadow intensity is identical when scaled, the PSF in the image plane of the telescope can vary – this effect is apparent when we consider that the geometrical angles change under the scalings proposed compared to the original space dimensions and thus the spatial provenance of the diffracted light changes.

The equivalent Fresnel number \( F_{\text{eq}} \) across the occulter is now the summation of the rescaled occulter radius and the finite distance of the diverging beam. The equivalent Fresnel number is equal to the Fresnel number of the distance scaled occulter, which maintains a constant Fresnel number to the space occulter:

\[
F_{\text{eq}} = F_{o'} + F_{\text{div}}
\]

\[
= \frac{R'^2}{\lambda} \left( \frac{1}{z'} + \frac{1}{h} \right)
= \frac{R'^2}{\lambda} \frac{1}{z'}
= F_o.
\]

To summarize, the scaling approach we used is to first scale the occulter according
to the laboratory separation distance as shown §3.2, and to then re-scale the occulter mask to account for the diverging beam as shown here. The same scaling applies for the higher-order Bessel term summation in the petalized diffraction integral featuring support struts in Equation 3.1.15.

3.4 Non-dimensionalization of diffraction integrals

We can substitute Fresnel numbers in the diffraction integrals for both the planar input case, and for the diverging input beam to non-dimensionalize these equations. This second method thereby demonstrates the scaleable nature of these diffraction integrals [80].

We recall from Equation 3.1.1 the diffraction integral assuming a planar input wave written with physical dimensions and moving the constant phase factor outside of the integral:

$$E_{\text{mask}}(\rho) = \frac{2\pi}{i\lambda z} e^{-\frac{2\pi i}{\lambda z} \rho^2} \int_0^R e^{\frac{2\pi i}{\lambda z} r^2} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) A(r) r dr \quad 0 \leq \rho \leq \rho_{\text{max}}. \quad (3.4.1)$$

We have two Fresnel numbers: $F_o = \frac{R^2}{\lambda z}$ across the occulter and $F_s = \frac{\rho_{\text{max}}^2}{\lambda z}$ across the shadow. The non-dimensional coordinates are $\hat{r} = r/R$ and $\hat{\rho} = \rho/\rho_{\text{max}}$. We can write the non-dimensionalized version of this equation as

$$\hat{E}_{\text{mask}}(\hat{\rho}) = -2\pi i F_o e^{-\frac{\pi i}{\lambda z} \hat{\rho}^2} \int_0^{1} e^{\pi i F_{s\hat{\rho}}^2} J_0 \left( 2\pi \sqrt{F_o F_s} \hat{\rho} \right) \hat{A}(\hat{r}) \hat{r} d\hat{r} \quad 0 \leq \hat{\rho} \leq 1. \quad (3.4.2)$$

where $\hat{A}(\hat{r}) = A(R\hat{r})$, $\hat{E}_{\text{mask}}(\hat{\rho}) = E_{\text{mask}}(\rho_{\text{max}}\rho)$. Equation 3.4.1 in terms of physical dimensions can be obtained by rescaling back the non-dimensionalized coordinates.

The same non-dimensionalization procedure can also be applied for the diverging beam scaling in Equation 3.3.2. The non-dimensionalized coordinates are in terms of
the diverging scaling factor $\gamma > 1$ with $\tilde{r} = \hat{r}/\gamma$, and $\tilde{\rho} = \hat{\rho}/\gamma$. Performing this change of variables:

$$\tilde{E}_{\text{mask}}(\tilde{\rho}) = -2\pi i \gamma^2 F_o e^{\frac{2\pi i z}{\lambda}} e^{\frac{\pi i F_o \tilde{\rho}^2}{2\gamma^2}} \times$$

$$\cdots \int_0^{\tilde{\rho}} \tilde{\rho}^2 J_0 \left( 2\pi \sqrt{F_o F_s \tilde{r}} \tilde{\rho} \right) \tilde{A}(\tilde{r}) \tilde{r} d\tilde{r} \quad 0 \leq \tilde{\rho} \leq \gamma. \quad (3.4.3)$$

where $\tilde{A}(\tilde{r}) = \hat{A}(\gamma \tilde{r})$, $\tilde{E}_{\text{mask}}(\tilde{\rho}) = \hat{E}_{\text{mask}}(\hat{\rho}/\gamma)$. We substitute $\gamma^2 = 1 + \frac{z'}{h}$ inside the integral and separate exponentials:

$$\tilde{E}_{\text{mask}}(\tilde{\rho}) = -2\pi i \gamma^2 F_o e^{\frac{2\pi i z}{\lambda}} e^{\frac{\pi i F_o \tilde{\rho}^2}{2\gamma^2}} \times$$

$$\cdots \int_0^{\gamma} \tilde{\rho}^2 e^{\frac{\pi i F_o \tilde{\rho}^2}{\gamma^2}} J_0 \left( 2\pi \sqrt{F_o F_s \tilde{r}} \tilde{\rho} \right) \tilde{A}(\tilde{r}) \tilde{r} d\tilde{r} \quad 0 \leq \tilde{\rho} \leq \gamma. \quad (3.4.4)$$

The exponential term we separated above matches the phase factor due to the paraxial approximation of a diverging input beam in Equation 3.4.6 modulo a constant amplitude term and a leading phase factor:

$$E''(r'') = \frac{1}{\sqrt{h^2 + r''^2}} e^{\frac{2\pi i}{\lambda} \sqrt{h^2 + r''^2}} \quad (3.4.5)$$

$$\approx \frac{1}{h} e^{\frac{2\pi i}{\lambda} \sqrt{h^2 \frac{r''}{\lambda}}} \quad (3.4.6)$$

Additionally, non-dimensional diffraction integrals can be used in conjunction with non-dimensional representations of occulter designs to span the parameter space of possible optimized occulters and be quickly adapted for different mission scenarios [20].
3.5 Design of a propagation distance-limited experiment

We present here the optimization problem used to design the inner occulter shape and the outer ring for a laboratory experiment whose total distance is limited to \( \approx 10.5 \) m. For more details about the physical layout of the testbed refer to §4.2.

The design of the testbed is reported in [22], and here we summarize the design and list the optimization parameters. The main approach during the design of this experiment was to oversize the occulter mask to ensure that the effect of fixed manufacturing errors is minimized when the occulter is scaled to the relatively small propagation distance available. Additionally, a fairly large divergence factor \( \gamma = 2.7 \) was chosen to ensure that the extent of the dark hole was sufficient.

We denote the electric field due to the apodized occulter screen as \( E_{\text{occ}} \), and use Babinet’s Principle [13] to facilitate computation of the diffraction integral to infinity. This is the same expression for a space occulter from Equation 2.1.3:

\[
E_{\text{occ}}(\rho) = E_0 e^{\frac{2\pi i z}{\lambda}} \left( 1 - \frac{2\pi}{i\lambda z} \int_{0}^{R} e^{\frac{2\pi i (\rho^2 + r^2)}{\lambda z}} J_0 \left( \frac{2\pi r \rho}{\lambda z} \right) A(r) r dr \right).
\] (3.5.1)

To design the inner occulter screen, we formulate, as first proposed in [25], an optimization problem that minimizes an objective function defined as the suppression. We pose constraints independently on the real and imaginary parts of the electric field at the pupil plane of the shadow. This is a conservative approximation of the full, quadratic problem. The radial apodization profile above \( A(r) \), which is discretized appropriately, represents the set of decision variables which the optimization outputs.
The optimization problem can therefore be written as follows:

\[
\begin{align*}
\min & \quad c \\
\text{subj. to} & \quad -\frac{c}{\sqrt{2}} \leq \text{Re}(E_{\text{occ}}(\rho; \lambda)) \leq \frac{c}{\sqrt{2}} \\
& \quad -\frac{c}{\sqrt{2}} \leq \text{Im}(E_{\text{occ}}(\rho; \lambda)) \leq \frac{c}{\sqrt{2}} \\
& \quad c \geq 0 \\
A(r) &= 1, 0 \leq r \leq a \\
A'(r) &\leq 0, |A''(r)| \leq \sigma, \forall 0 \leq r \leq R.
\end{align*}
\]  

(3.5.2)

Where in the above problem, \( \sigma \) represents the smoothness condition threshold, \( a \) the extent of the opaque central disk, \( c \) is the suppression level, and \([\lambda_{\text{min}}, \lambda_{\text{max}}]\) defines the shadow suppression wavelength band. This formulation with a suppression-minimization objective function is different from the original occulter optimization formulation in [172] where the objective function is an area minimization. Changing the objective function to a suppression minimization leads to faster convergence [19], but a possible downside is that the suppression is not guaranteed, even at the discretized wavelengths and locations in the shadow, to meet the desired suppression level as is the case with the explicit suppression constraint formulation. The output apodization profile \( A(r) \) from this optimization problem is the complement of the inner occulter transmission profile in Figure 3.1. Also, as discussed in §3.1, the same optimization problem can be used with different parameters to obtain the outer ring.
Table 3.1: Summary of optimization parameters for inner and outer occulter shape design for current testbed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Inner Occulter</th>
<th>Outer Occulter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation distance, z</td>
<td>97,000 km</td>
<td>97,000 km</td>
</tr>
<tr>
<td>Maximum radius, $R$</td>
<td>188 m</td>
<td>376 m</td>
</tr>
<tr>
<td>Opaque disk, $a$</td>
<td>103 m</td>
<td>301 m</td>
</tr>
<tr>
<td>Opaque fraction, $a/R$</td>
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<td>0.8</td>
</tr>
<tr>
<td>Smoothness, $\sigma$</td>
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<td>0.2</td>
</tr>
<tr>
<td>Lower wavelength, $\lambda_{\text{min}}$</td>
<td>400 nm</td>
<td>400 nm</td>
</tr>
<tr>
<td>Upper wavelength, $\lambda_{\text{max}}$</td>
<td>1100 nm</td>
<td>1100 nm</td>
</tr>
<tr>
<td>Shadow radius, $\rho_{\text{max}}$</td>
<td>12 m</td>
<td>12 m</td>
</tr>
<tr>
<td>Shadow discretization</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Apodization discretization</td>
<td>4000</td>
<td>4000</td>
</tr>
<tr>
<td>Wavelength discretization</td>
<td>32</td>
<td>32</td>
</tr>
</tbody>
</table>

Table 3.2: Summary of scaled laboratory parameters for the current testbed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Space Design</th>
<th>Collimated Scale</th>
<th>Diverging Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation distance, z</td>
<td>97,000 km</td>
<td>9.1 m</td>
<td>9.1 m</td>
</tr>
<tr>
<td>Distance scale, $s$</td>
<td>1</td>
<td>3260</td>
<td>3260</td>
</tr>
<tr>
<td>Source distance, $h$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Divergence scale, $\gamma$</td>
<td>1</td>
<td>1</td>
<td>2.7</td>
</tr>
<tr>
<td>Inner radius, $R_{\text{inn}}$</td>
<td>188 m</td>
<td>58 mm</td>
<td>22 mm</td>
</tr>
<tr>
<td>Outer radius, $R_{\text{out}}$</td>
<td>376 m</td>
<td>115 mm</td>
<td>44 mm</td>
</tr>
<tr>
<td>Dark shadow radius, $\rho_{\text{dark}}$</td>
<td>12 m</td>
<td>3.7 mm</td>
<td>9.7 mm</td>
</tr>
<tr>
<td>Outer shadow radius, $\rho_{\text{out}}$</td>
<td>376 m</td>
<td>115 mm</td>
<td>310 mm</td>
</tr>
<tr>
<td>Telescope diameter</td>
<td>17 m</td>
<td>5.2 mm</td>
<td>14 mm</td>
</tr>
<tr>
<td>Inner Working Angle</td>
<td>400 mas</td>
<td>1300 as</td>
<td>490 as</td>
</tr>
<tr>
<td>Outer Working Angle</td>
<td>640 mas</td>
<td>2080 as</td>
<td>780 as</td>
</tr>
<tr>
<td>Number of petals, $N$</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Minimum strut width</td>
<td>7.4 m</td>
<td>2260 $\mu$m</td>
<td>850 $\mu$m</td>
</tr>
</tbody>
</table>
In Table 3.1, we summarize the optimization parameters used for the optimization problem. Both the inner occulter and the outer occulter (whose complement becomes the outer ring) are shown across the two different columns. The wavelength range is discretized into 25 nm intervals starting from 400 nm up to 1100 nm, with a specific wavelength added at 633 nm for the monochromatic HeNe laser testing in Chapter 4. The radius cut-off of the outer occulter is set to twice that of the inner occulter. The annular working region is a total radial fraction of 0.25 of the total occulter mask between the outer edges of the inner occulter’s petals and the outer occulter’s inner petal edges.

We also show the scaling of the occulter mask parameters in Table 3.2. In the first column we show the parameters of the occulter mask as designed at space dimensions. We then apply a Fresnel scaling assuming a distance of 9.1 m is available as outlined in §3.2. Finally, we introduce a diverging beam 1.5 m upstream from the occulter mask and apply the divergence scaling for the occulter mask and for the shadow as outlined in §3.3. With both the collimated and then the diverging scaling the physical inner working angle and outer working angle change; henceforth for image plane results at all scales we report the equivalent space angle. The telescope diameter for the diverging scaling is chosen to enlarge with the shadow which will give a better resolving performance than would be the case for the collimated scaling. The minimum strut width is computed for $b = 0.9$, which means that the struts cover 10% of the area in the working annular region.

Figure 3.1 shown earlier in this chapter is the combined apodization profile for the inner occulter and outer ring with the optimization parameters described in Table 3.1. The radial dimension is non-dimensionalized and can be rescaled for any of the scaling cases outlined in Table 3.2. The combined optimization output representing the laboratory mask is shown in Figure 3.3(a) – we also use a simple mask that represents a control mask, whose apodization profile corresponds to a simple circular
Figure 3.3: (a) Optimized and control apodizations (b) Laboratory mask realization using 16-petals

inner occulter and a circular outer edge corresponding to the inner and outer working angles. In Figure 3.3(b), we show a schematic of the binary realization of the occulter mask using 16 petals.

### 3.6 Ideal performance of laboratory occulter

The optimization model in Equation 3.5.2 only accounts for the effect of a circularly symmetric apodized profile. A realistic, manufacturable occulter features a set of petals that represent a binary approximation of the partially transmissive apodized profile. It is important to determine the performance of the binary occulter with the effect of the 16 petals included. This will represent the ideal performance of the occulter against which experimental results are to be compared. We also provide simulation results for a mask with a circular inner occulter as a baseline to show the theoretical performance improvement that the optimized occulter shape provides.
3.6.1 One-dimensional propagation equations

We provide here a summary of all the equations used to obtain the ideal performance of the laboratory occulter. These equations are one-dimensional radial integrals which makes them more numerically tractable but assumes a perfect form of circular symmetry, hence the ideal nature of these simulations.

To propagate the input electric field past the occulter mask to the pupil plane where the camera is located, we apply the petalized version of the Fresnel propagation integral in Equation 3.1.15 but with a general electric field input term included and a finite number $K$ of higher-order terms in the summation:

$$E_{\text{bin}}(\rho, \phi) = e^{\frac{2\pi i}{\lambda z} z(2\pi i \lambda z) \int_0^R E_{\text{in}}(r)e^{\frac{2\pi i}{\lambda z}(r^2+\rho^2)} J_0\left(\frac{2\pi r \rho}{\lambda z}\right) bA(r)rdr} +$$

$$\ldots \sum_{k=1}^{K} \frac{4\pi \cos(kN\phi)}{i\lambda z} \int_0^R e^{\frac{2\pi i}{\lambda z}(r^2+\rho^2)} J_{kN}\left(\frac{2\pi r \rho}{\lambda z}\right) \frac{\sin(k\pi bA(r))}{k\pi} rdr.$$

(3.6.1)

where $0 \leq \rho \leq \rho_{\text{max}}, \ 0 \leq \phi \leq 2\pi$. We can perform this integral by applying any of the physical dimensions corresponding to the three scaling cases outlined in Table 3.2. The sequence converges fairly quickly, and we must ensure that the number of non-zero terms $K$ we add to the summation is sufficient for the number of petals $N$ we use. We can retrieve the apodized mask case for which the occulter is designed by setting $K = 0$.

Next, to describe the propagation from the pupil plane to the image plane we let $(x, y)$ be the Cartesian coordinates across the pupil plane. $E_{\text{pup}}(x, y)$ describes the electric field at the pupil plane and can be obtained by interpolation from $E_{\text{bin}}(\rho, \phi)$. 
We define a circular pupil function \( p(x, y) \):

\[
p(x, y) = \begin{cases} 
1, & \sqrt{x^2 + y^2} \leq r_a \\
0, & \text{else.}
\end{cases}
\]  

(3.6.2)

where \( r_a \) is the radial extent of the imaging lens. Thus, for a lens of \( f = 300 \text{ mm} \) operating at f-stop \( f/22 \), \( r_a = 7 \text{ mm} \). Assuming the imaging optic is a thin lens, to focus on a distance \( d \) in front of the lens we Fresnel propagate a distance \( s \) past the lens which introduces a quadratic phase function. Then we use Equation 2.3.2 to obtain an expression for the electric field at the image plane defined by Cartesian coordinates \((\xi, \eta)\), ignoring leading phase terms, becomes:

\[
E_{\text{im}}(\xi, \eta) = \frac{1}{i\lambda s} \int \int_{p(x,y) = 1} E_{\text{pup}}(x, y) e^{-\frac{\pi i}{\lambda s}(x^2+y^2)} e^{-\frac{2\pi i}{\lambda s}(x\xi+y\eta)} \, dx \, dy.
\]  

(3.6.3)

This expression can be evaluated through any discrete Fourier Transform method, for example using the matrix Fourier transform [160] described in §1.7. We match the experimental setup by selecting the same pixel pitch of the science camera, a radial extent that matches the f-stop, and by choice of the propagation distance \( s \) and focal distance \( d \).

Finally we consider different forms of input beams. For propagations at the space and collimated scaling dimensions we use a simple planar input beam of the form \( E_{\text{in}}(r) = E_0 \), which we set to unity. For simulating the laboratory setup, we start with considering, as in Equation 3.4.5, a spherical input beam of the form:

\[
E_{\text{in}}(r) = \frac{E_0}{\sqrt{h^2 + r^2}} e^{\frac{2\pi i}{\lambda} \sqrt{h^2 + r^2}}
\]  

(3.6.4)

\[
\approx \frac{E_0}{\sqrt{h^2}} e^{\frac{2\pi i}{\lambda} \sqrt{h^2}}.
\]  

(3.6.5)

Equation 3.6.5 assumes the amplitude attenuation for the input beam over the mask.
profile is negligible. A second input beam we consider is a paraxial input beam as in Equation 3.4.6. This specific form of the input is the assumption that allows for making the argument in §3.3 of an identical electric field at the shadow under a diverging input beam:

$$E_{in}(r) = \frac{E_0}{h} e^{\frac{2\pi i h}{\lambda}} e^{\frac{\pi i r^2}{\lambda}}.$$  \hfill (3.6.6)

The paraxial approximation is only valid when the following condition is satisfied [66]:

$$h^3 \gg \frac{\pi}{4\lambda} R^4.$$  \hfill (3.6.7)

It turns out that at the laboratory dimensions for the diverging beam from Table 3.2 the inequality is not satisfied (the left-hand side is on the same order of magnitude and smaller than the right-hand side). Nonetheless, Goodman [66] mentions that this condition is conservative. To verify the validity of this approximation, we simulate the laboratory mask with both input conditions from Equation 3.6.4 and Equation 3.6.6 and numerically determine the discrepancy introduced by violating this condition at the end of §3.6.2. One option we have is to re-scale the experiment favourably by setting \(z = h = 5.3 \text{ m}\) which would satisfy the condition in Equation 3.6.7 by one order of magnitude.

### 3.6.2 Simulation results

Using the equations presented in §3.6.1, numerical simulations are performed at the space and laboratory scale, showing results at both the pupil and image planes. The detrimental effect of the addition of sixteen petals, as used in the laboratory, is demonstrated, and the number of petals required to recover full performance is determined.
Figure 3.4: Pupil plane for occulter at space scale as designed (a) Theoretical suppression at the pupil plane for a circularly symmetric apodization function is $10^{-9.2}$. (b) Theoretical suppression at the pupil plane for a sixteen petal binary occulter mask is $10^{-6.9}$.

We start by comparing the suppression-calibrated shadow intensity of the occulter mask at the designed space dimensions in Figure 3.4. In particular, we simulate the electric field due to the circularly symmetric apodized occulter as designed for an ideal planar input beam in Figure 3.4(a), and we have a mean suppression of $10^{-9.2}$ over a 17-m telescope corresponding to the space-scaled aperture used in the laboratory. The mean suppression over the entire designed dark hole with a diameter of 24-m is actually better at $10^{-9.4}$ – this is because there is a residual Poisson spot at the center of the shadow and which has a smaller impact as we increase the aperture. In Figure 3.4(b), we show the corresponding suppression-calibrated shadow for the occulter mask petalized as in the laboratory with $N = 16$. We observe a significant worsening of the performance compared to the circularly symmetric case as the suppression level is shallower and the size of the dark hole is reduced. The mean suppression over the same 17-m telescope aperture is now reduced to $10^{-6.9}$. Qualitatively, the annular openings are bright and there is a suppression roll-off outside of the physical extent of the outer ring.

In Figure 3.5(a), we illustrate the process of choosing an appropriate number
Figure 3.5: Numerical analysis for (a) Convergence of Bessel terms summation with suppression at the aperture as a function of merit for a 16-petal realization. $K = 20$ and higher reaches the summation limit. (b) Mean suppression across aperture for laboratory mask with different petalizations. $N = 300$ or more petals are necessary to fully recover the apodized performance, but the laboratory mask uses $N = 16$.

of Bessel terms to add to the summation to obtain an accurate measurement of suppression for a $N = 16$ petalized occulter mask such as the computation in Figure 3.4(a). We compute mean suppression across the telescope aperture for a varying number of added Bessel terms. We notice that the suppression stabilizes as the Bessel sum converges after sufficient terms have been added. This demonstrates that choosing $K = 20$ non-zero Bessel terms in the summation should be sufficient for the laboratory mask.

The laboratory mask we use has 16-petals. However, it is interesting to consider the number of petals necessary to recover the apodized suppression performance that the occulter mask was designed for. In Figure 3.5(b), we compute the mean suppression across the telescope aperture for an increasing number of petals. We see that a rather large number of petals, approximately 300, is necessary for the petalized occulter to match the apodized occulter.

We also consider the performance of the occulter mask under scaled laboratory conditions. In Figure 3.6(a), we show the azimuthally averaged suppression for the
original space occulter and for scaled laboratory conditions. We use normalized radius representing a fraction of the entire extent to the outer ring (this increases in the lab with the beam divergence). For a more detailed view, we zoom-in on the center of the dark hole across the telescope aperture in Figure 3.6(b). From this comparison we see that the space and laboratory case for a parabolic input are identical (as expected from the derivation in §3.3). The pure diverging beam, however, exhibits a very small variation at the center of the dark hole out to three significant figures. The large-scale comparison of the shadow is seen in Figure 3.7. In this figure, the residual suppression is shown after subtraction of the shadow intensity resulting due to the application parabolic input from the shadow due to the application of a pure diverging beam input. In both cases, the suppression across the dark hole at the center of the shadow is $10^{-6.9}$.

Next, we consider the PSF formed at the telescope focal plane. In Figure 3.8 the PSF is shown as the telescope aperture is decreased from that corresponding to the lab to smaller apertures down to 2-m diameter. The result of a smaller aperture is a spreading of the diffraction rings. For a more quantitative view, we show a cross-section comparison in Figure 3.9 between the PSF corresponding to the space occulter mask and the laboratory mask corresponding to a parabolic beam. We see that these are practically identical.

Finally, to demonstrate the reduction in diffracted light due to the optimal shape of the occulter, we perform a similar propagation to obtain the PSF of the circular control mask corresponding to the apodization profile shown previously in Figure 3.3(a). Thus, in Figure 3.10, we compare the PSF for the simple circular control mask with the optimized occulter mask with both shown on the same log-scale. We can clearly see from this comparison that the optimized occulter mask is an improvement over a circular disk. In Figure 3.11, we show a cross-section comparison of these PSFs.
Figure 3.6: Comparison of azimuthally averaged suppression curves (a) Suppression curves spanning entire occulter shadow comparing the propagation at space dimensions with that at laboratory scale for a parabolic and diverging input beam. The diverging input beam was decimated by a factor of 5 for display-purposes only. All three curves appear to match on this scale. (b) Same suppression curves zoomed-in close to the aperture showing perfect agreement for the parabolic input with the designed space scale, and a small difference for the diverging input beam. Solid red line indicates physical extent of aperture. We use normalized radius to enable comparison across different physical scales.
Figure 3.7: Residual suppression upon subtraction of the shadow due to the ideal parabolic input beam from a diverging input beam for laboratory dimensions. Suppression for both cases across the telescope-camera pupil is in agreement at $10^{-6.9}$.

Figure 3.8: Contrast-calibrated point-spread functions at space dimensions parametrized by telescope diameter.
Figure 3.9: Contrast comparison for space and laboratory PSFs. The laboratory PSF is computed using the parabolic input beam. Solid red lines indicates inner and outer working angles.
Figure 3.10: Simulation of contrast in the image plane for laboratory scale (a) using the control mask (b) using the occulter mask. The inner and outer red circles denote the annular dark region for both masks.
Figure 3.11: Azimuthal theoretical contrast comparison between control and optimized masks.
Chapter 4

Experimental Results

4.1 Overview of experimental approach

In this chapter, we provide the results of experimental measurements obtained at the Princeton occulter testbed that verify the optical performance of a silicon-etched occulter mask designed through optimization methods described previously in Chapter 2 and scaled to provide an identical intensity pattern at the shadow as shown in Chapter 3.

This chapter is organized as follows. In §4.2, we present more details about the testbed used to verify occulter performance. In §4.3, we take measurements with a lensless camera centered in the shadow and provide suppression-calibrated results which measure the direct intensity attenuation of the shadow. We compare the results obtained from the optimized occulter mask with a baseline measurement derived from a simple control mask that uses an unoptimized, circular occulter shape and demonstrate that the optimized occulter mask presents performance improvements; however, when comparing experimental results with the expected performance of the theoretical model presented in §3.6 the experimental mask does not perform to the full capability of the optimized design. This observation motivates contrast-calibrated
measurements presented in §4.4 taken in the image plane of the camera to obtain spatial information about the provenance of the additional light leakage. We also use an occulter pattern identical to that used for the silicon-etching mask, however, instead of etching a pattern in silicon the pattern is aluminum deposited on a glass substrate. The experimental performance of this glass-substrate mask is limited at a level worse than the control mask by the usage of the glass substrate. In §4.5, we compare in detail the performance of both the occulter mask and the circular control mask with predictions from the theoretical model. We conclude with §4.6 in which we provide results from a new control with an optimized outer ring and circular inner occulter manufactured using a new edge thinning process.

Our experimental approach extends upon the results reported in [19] by also taking suppression measurements in the pupil plane but using different light sources and optical elements to create the divergent beam. Additionally, we use a new approach to identify the limits of the experimental performance by taking contrast measurements in the image plane. Finally, we have manufactured and taken measurements with simple, unoptimized control masks to verify our claim that the optimized occulter mask yields performance improvements.

4.2 Testbed layout and description

The occulter testbed is located at the Gas Dynamics Lab on the university’s Forrestal Campus. It consists of a large enclosure, 40′ × 4′ × 8′ in size, that blocks ambient light. The layout is shown schematically in Figure 4.1. There are two passively isolated Newport optical tables at each end of the enclosure. The larger optical table contains the optics that create an artificial star, which, as explained in §3.3 consists of a diverging beam. The smaller optical table at the opposite end of the enclosure contains the telescope optics for observation of the artificial star. On this table is
a camera mounted on two long-travel stages that provide two degrees of freedom of movement in the lateral and vertical directions.

Two different laser sources are used to create the diverging beam as summarized in Table 4.1. The first source is a single-mode 2 mW HeNe laser operating monochromatically at 633 nm. To create the diverging beam, the beam is passed through a beam expander made of two lenses, and is focused onto a 15 μm diameter pinhole through an \( \lambda/4 \) off-axis parabolic mirror. We use a diverging beam focused onto a pinhole so that the pinhole acts as a low-pass spatial filter that removes high-frequency aberrations arising due to surface error on the optics. The choice of lenses and pinhole affects the divergence angle of the beam and its output amplitude profile. An alternative pinhole coupling arrangement is shown in Figure 4.2. The free space HeNe laser is incident on a fold mirror and the laser beam is then focused onto the pinhole using a 20x microscope objective to remove the low-quality OAP surface from the optical train. Performance was found to be equivalent for both pinhole coupling through the OAP and the microscope objective. The power stability of the laser source was measured and found to vary by 10-20 % over the course of observations. The second source consists of a four channel fibre-coupled laser source. Two channels were used with Fabry-Perot diode laser sources operating at 520 μm and 638 μm, and these were directly coupled onto an appropriate single-mode fibre. The beam divergence for these input sources was higher, resulting in less amplitude variation across the mask.

The diverging beam propagates 1.5 m before encountering the occulter mask which it overfills at the end of the optical table. Baffles are placed around the occulter mask extending to the walls of the enclosure and a further three sets of full wall baffles are located downstream to ensure no light propagates to the camera around the annular mask and to minimize any scattered light. The mask is tilted 5° to eliminate a ghost reflection by directing the back-reflection into a black foil beam dump.
Figure 4.1: Layout of the Princeton Occulter Testbed. On the left-end of the enclosure is the optical table containing the artificial star and on the right-end is the moveable camera-telescope.

Beyond the occulter mask, the beam propagates 9.1 m to the second optical table to the camera. The camera has a tip-tilt mount and is placed on two 300 mm long-travel stages as shown in Figure 4.3; both move perpendicular to the propagation direction, one horizontally and one vertically. Together these stages allow the camera to be precisely aligned in the shadow cast by the occulter and the tip-tilt mount allows centering of the image on the detector. The camera is an astronomical-grade thermoelectrically cooled Starlight Xpress SXV-H9 CCD, which was found to reliably regulate at $-10$ °C below ambient when using a cooling fan to compensate for the stagnant air inside the enclosure. A telephoto lens set at $f = 300$ mm is outfitted via the M42 mount, and the smallest available aperture setting at $f/22$ is used to form a six-bladed iris—at a diameter of 14 mm this is smaller than the designed dark hole under diverging beam scaling. The CCD has $1392 \times 1040$ pixels with pixel pitch of $6.45 \mu m \times 6.45 \mu m$. As a result of scaling to lab size, the spatial resolution is more than an order of magnitude better than expected in space for 18 mas pixels [81]. Additionally, the Fresnel scaling results in a larger geometric angle. The increased spatial resolution in the laboratory allows us to better resolve the PSF and identify any unexpected sources of light.

The two different masks used in the laboratory are shown in Figure 4.4. The occulter mask we use is shown in Figure 4.4(a). It was manufactured at the Microdevices
Lab at JPL using Deep Reactive Ion Etching (DRIE) to etch the open areas of the occulter mask. The silicon wafer is 101.6 mm in diameter and 400 µm in thickness. The DRIE process results in vertical etches of the sidewall and thus two steps were used to thin the sidewall to 50 µm. A 100 nm titanium coating was applied to increase the optical absorption to several orders of magnitude beyond the expected level of light in the shadow of the mask. To establish a baseline level to verify the performance of the occulter mask, a simple control mask was manufactured that consists of a circular occulter with supporting struts to a circular outer edge. The control mask shown in Figure 4.4(b) was manufactured by etching the open areas from a rectangular piece of copper. Copper etching has a decreased edge feature accuracy when compared to the DRIE method used for the optimized occulter mask; however, due to the circular shape the expected level of light in the shadow is greater, significantly relaxing the tolerance on the edge features.

Suppression is measured at the pupil plane of the camera with light allowed to impinge directly on the detector. Suppression is the ratio of the flux in the mask’s shadow to the flux without the mask. This measurement is taken by first centering the camera in the shadow of the occulter mask without the lens attached and taking long exposures; then the mask is removed and short exposures are retaken. The suppression measurement describes the performance of the occulter directly and is decoupled from the telescope. Contrast is measured at the image plane with a camera focused on the point source, and is the ratio between the flux at each pixel in the image formed when the mask is in place and the flux of the peak pixel of the PSF without a mask. Performance depends on aperture size. Measuring both suppression and contrast is useful as it allows for verification of the consistency of the results. Contrast measurements can indicate the limitations of the suppression performance of the occulter mask, as the source of any stray light can be directly observed.
Figure 4.2: Optics arrangement creating the artificial star. The HeNe laser beam is folded and then focused onto a pinhole by a microscope objective. ND filters can be added for calibration frames taken when the artificial star is in direct line-of-sight with the camera to ensure the detector is not saturated.

Figure 4.3: Camera mount providing tip and tilt alignment. Two long-travel stages provide alignment in the plane perpendicular to the propagation direction.
Table 4.1: Summary of monochromatic laser sources used for the experimental testbed. The divergence is reported as the numerical aperture NA = \sin \theta, where \theta is the angle subtended from the propagation distance to a 1/e^2 irradiance fall-off.

<table>
<thead>
<tr>
<th>Laser Source</th>
<th>Wavelength</th>
<th>Typical Power</th>
<th>Output Coupling</th>
<th>Divergence (NA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HeNe</td>
<td>633 nm</td>
<td>2 mW</td>
<td>15 µm pinhole</td>
<td>0.05</td>
</tr>
<tr>
<td>Fabry-Perot</td>
<td>520 nm</td>
<td>10 mW</td>
<td>single-mode fibre</td>
<td>0.10-0.14</td>
</tr>
<tr>
<td>Fabry-Perot</td>
<td>638 nm</td>
<td>15 mW</td>
<td>single-mode fibre</td>
<td>0.14-0.14</td>
</tr>
</tbody>
</table>

Figure 4.4: Laboratory masks used (a) the circular occulter mask fabricated via copper etching, (b) optimized occulter mask fabricated via Deep Reactive Ion Etching (DRIE) on a silicon wafer, and (c) identical occulter mask pattern deposited on a glass substrate.

4.3 Monochromatic suppression results

We report on experimental results obtained by direct measurements of the shadow intensity. As mathematically shown in the scaling discussions in §3.2 and §3.3, the intensity of the diffractive shadow is identical when scaled to the laboratory as the diffraction integral is unchanged from space. Therefore, direct measurements of the shadow intensity are the most representative of the performance of the occulter and are uncoupled from the aperture size and geometric scaling of the occulter.

For measuring the suppression at the pupil plane, the telephoto lens is removed from the camera. The camera is mounted on two 300-mm long travel stages that can scan the shadow. To obtain a mosaic of the shadow as shown in Figure 4.5, a set of
Table 4.2: Summary of suppression measurements across the dark hole region. The Fabry-Perot source corresponds to two separate channels on the fibre-coupled laser source.

<table>
<thead>
<tr>
<th>Mask</th>
<th>Wavelength</th>
<th>Laser Source</th>
<th>Suppression, Log</th>
</tr>
</thead>
<tbody>
<tr>
<td>Occulter</td>
<td>520 nm</td>
<td>Fabry-Perot</td>
<td>-4.74</td>
</tr>
<tr>
<td>Occulter</td>
<td>633 nm</td>
<td>HeNe</td>
<td>-4.82</td>
</tr>
<tr>
<td>Occulter</td>
<td>638 nm</td>
<td>Fabry-Perot</td>
<td>-4.77</td>
</tr>
<tr>
<td>Control</td>
<td>633 nm</td>
<td>HeNe</td>
<td>-2.67</td>
</tr>
</tbody>
</table>

frames collected at 5mm horizontal intervals across the shadow are stitched together with an average value produced over the zones of overlap. A series of frames with different exposures are collected to maintain the camera’s pixels in the linear regime. The actual suppression measurements at the center of the dark shadow in Figure 4.5(a) are obtained using the 633 nm laser. The data set consists of 500 stacked frames of 300 sec exposure, each with a 50 frame median-combined dark at equal exposure subtracted. To obtain a measurement of suppression in the dark hole, 50 calibration frames at 0.05 sec are obtained with the mask moved along the line-of-sight to the pinhole through the annular openings. A mean suppression of $10^{-4.82}$ across the detector is reported by dividing the flux for each pixel in the long-exposure stacked frames by the peak pixel flux when in direct line-of-sight to the pinhole.

Similar measurements are taken at two different wavelengths with two channels of the fibre-coupled laser. The rates of exposure are adjusted to account for the increased power of the source, in particular for the suppression calibration frames which can saturate the detector. For the more powerful 638 nm channel, an ND filter was used. Suppression results are $10^{-4.74}$ for the 520 nm channel and $10^{-4.77}$ for the 638 nm channel. Additionally, whereas the 633 nm suppression calibration was obtained by moving the camera in the shadow until in direct line-of-sight to the pupil, these measurements were obtained by removing the occulter mask and keeping
Figure 4.5: Suppression measurements at the pupil plane using: (a) the optimized occulter mask with mean suppression values at the center of the dark hole for three different wavelengths as follows: $10^{-4.74}$ for 520 nm, $10^{-4.82}$ for 633 nm, and $10^{-4.77}$ for 638 nm, and (b) the circular occulter mask with measured suppression of $10^{-2.67}$ for 633 nm.

To obtain a baseline for the performance of the occulter mask, the suppression is compared to that created by a circular occulter illuminated by the same 633 nm diverging beam. The mosaic corresponding to the circular occulter shadow is shown in Figure 4.5(b). The mean measured suppression over the detector centered on the occulter shadow is $10^{-2.67}$, which represents an improvement of two orders of magnitude for the occulter mask. All the suppression measurements are summarized in Table 4.2.

Whereas the suppression measurements for the optimized occulter mask represent an improvement over the control mask featuring a circular inner occulter, we recall the corresponding result from the theoretical simulation in §3.6 which represents two
orders of magnitude better expected performance than the measurements obtained here. Pupil plane measurements do not provide insight into the source of any errors as any light leakage is spread relatively uniformly across the pupil. Rather, measurements at the pupil only provide an estimate of the total impact of the combined experimental errors on the performance of the occulter mask.

4.4 Monochromatic contrast results

Experimental measurements of occulter performance in the pupil plane have shown limitations on the performance of the occulter mask. To obtain a better estimate of the source of these experimental limits, we perform a new set of measurements in the image plane of the camera. By forming an image from the electric field at the shadow, we can obtain spatial information about the source of any additional light leakage.

To obtain contrast measurements, the camera is centered in the shadow and outfitted with the telephoto lens. Centering is most easily achieved by using the pupil plane mosaic at regions of high signal to estimate the edges of the dark hole. The center of the shadow can then be estimated from opposite edges. Final adjustments can be made using the image plane preview, by applying tip and tilt on the camera mount to locate the PSF on the central part of the detector, and by making small translational adjustments with the travel stages.

Actual combined image-plane results are shown in Figure 4.6 using the monochromatic 633 nm laser source. To calibrate for contrast, the flux in each pixel of the stacked frame is divided by the peak pixel flux when the mask is removed. To prevent overexposure in the calibration frames, two OD2.0 filters were placed in series before the pinhole. The two OD2.0 filters have measured transmissions of $1.00 \pm 0.01\%$ and $1.05 \pm 0.02\%$. The stacked calibration dataset consists of 100 frames each taken
Figure 4.6: Contrast measurements at the image plane (a) using the control mask and (b) using the occulter mask. The inner and outer red circles denote the annular dark region for both masks.
Figure 4.7: Experimental contrast results comparison using the same log stretch for the (a) control mask and the (b) occulter mask. Four equal area regions are shown in yellow. The mean contrast across the control mask strut in Region 1 is $10^{-8.22}$, while in Region 2 which has no strut the mean contrast is $10^{-8.99}$. The mean contrast across the occulter mask strut in Region 3 is $10^{-8.44}$ while in Region 4 which has no strut the mean contrast is $10^{-10.48}$. Region 5 has the brightest outer tip pixel measured at $10^{-6.55}$.

Figure 4.8: Experimental image plane contrast results for occulter pattern deposited on glass using the process in [187].
at 0.01 sec to minimize the effect of power fluctuations in the laser source.

Figure 4.6(a) shows the contrast calibrated image plane measurement of the control mask. The control mask dataset consists of 10,000 frames collected at 0.05 sec exposure. Short exposures were necessary to prevent saturation of the bright features of the image while a large number of frames was used to achieve a longer overall integration time allowing features to be seen in the darkest part of the annular region. In Figure 4.6(b) the contrast calibrated image plane measurement of the optimized occulter mask is shown. The occulter mask dataset consists of 10,000 frames collected at 1 sec exposure.

Figure 4.7 shows two panes that were previously shown separately but are shown together here to allow for easier comparison of the control mask with the occulter mask. In Figure 4.7(a) the same contrast-calibrated image from Figure 4.6(a) has the log-scale restretched to match the experimental occulter mask result in Figure 4.7(b). The mean contrast is calculated in four regions of interest, and it is shown that the struts of the control mask are at a similar level to those of the occulter mask but are less visible than those of the occulter mask. The reason for the reduced visibility of the struts is that the diffraction wings of the stronger inner circular edge limits the contrast in the annular regions of the control mask to a level similar to the struts themselves. For the occulter mask, however, the annular regions are about two orders of magnitude below the level of the struts.

The dynamic range of the camera limits the background contrast away from the control mask in regions outside of the cropped image shown. The actual measured background level at sufficiently long exposures with the mask completely blocked off is at a level of $10^{-12}$. This is the best possible achievable contrast in this facility, and further improvements can only be obtained by further reductions of ambient light through increased stray light control, coating of all surfaces with stronger light-absorptive material, and a more light-tight enclosure.
Finally, Figure 4.8 shows the contrast-calibrated image for the optimized occulter pattern deposited on a 6 mm glass substrate. The goal with depositing an occulter pattern on a glass substrate is that, similar to coronagraph designs [28], it is no longer necessary to use a self-standing structure for the mask thus removing the necessity for support struts which were found to glint in the observations above. As a first step, however, an identical pattern was used for comparison to evaluate the suitability of the glass substrate. Unfortunately, the dark annular regions are found to be much brighter than for the binary occulter pattern, and in fact this performs worse than the binary circular occulter pattern. The loss of performance when compared to the identically-shaped optimized occulter is due to internal reflections and wavefront errors from the surface of the glass substrate that dominate any scalar diffraction from the actual occulter pattern. Because of the relatively large size of the pattern deposited on the glass substrate, it is difficult to obtain glass that is either thin or that has a high quality optical surface. Better performance may be obtained in the future using a smaller occulter pattern for which a better quality glass substrate can be obtained. Similar limitations were found in an early version of this experiment in which the occulter mask was also deposited on a wedged glass substrate [120].

In this set of experiments in the image plane, we have compared the optimal occulter mask with the control mask and shown a similar performance improvement to the pupil plane measurements. We have also used an identical occulter optimal shape deposited on glass (for more details about the mask fabrication process on glass see [187]), which we have found to be limited by the optical quality of the glass substrate. These observations will allow us to form and test hypotheses about the nature of the limitations of the experimental performance.
Figure 4.9: Comparison of (a) theoretical and (b) measured PSFs for the circular occulter control mask. A quantitative comparison can be obtained from computing the (a) azimuthal median of both images – the measured median curve is shown in blue with a 95% confidence interval in red, and the theoretical median curve is in black.

Figure 4.10: Suppression mosaic comparison at the pupil plane for the control mask: (a) theoretical (b) measured.
Figure 4.11: Comparison of (a) theoretical and (b) measured PSFs for the optimized occulter control mask. In (b), the set of 16 wedges is superimposed on the PSF to show the regions over which the azimuthal median of the signal for the quantitative comparison in (c) – the measured median curve is shown in blue with a 95% confidence interval in red, and the theoretical median curve is in black.
4.5 Comparison to theoretical model

We have collected experimental measurements both in the pupil and in the image plane. We have compared the performance of an optimized occulter mask with a baseline circular occulter mask. We now compare these experimental results against the performance predictions of the theoretical model presented in §3.6.

The main purpose of the control mask is to act as a baseline against which improvements in performance can be compared. Additionally, usage of a model operating at a lower performance can be used to demonstrate the accuracy of the calibration method for computing performance—that is, that the assumptions behind the numerical propagations to the pupil and subsequent image plane match the experimental results. This verification provides confidence that the performance reported for the actual occulter mask is accurate.

On a qualitative level, for the control mask we compare the contrast-calibrated image-plane theoretical in Figure 3.10(a) with the measurement in Figure 4.6(a) side-by-side in Figure 4.9. The scalar diffraction simulation in Figure 4.9(a) captures all the main features of the experimental image in Figure 4.9(b): two bright inner and outer rings with sixteen divisions at the point of attachment of the support struts, and fine diffraction ringing around the main bright rings. The experimental image does not appear as symmetrical as the theoretical image, and some of the diffraction ringing closest to the image’s background level is not distinguishable. For a quantitative analysis, we provide a cross-section through the image plane and plot the azimuthal median as shown in Figure 4.9(c). We also plot a 95% confidence interval for the median based on percentile populations [76]. The magnitude of the inner peaks are within a factor of 1.06 of each other and for the outer peaks within a factor of 1.56. This relatively close agreement demonstrates the validity of the calibration method and scalar diffraction simulation as we are able to predict contrast of the bright rings. The uniform diffraction rings in the experimental image manifest themselves
as straight edges with short and sharp turns on the azimuthal plot; a few such straight edges can also be seen on the experimental curve on the azimuthal plot but these are relatively smoother.

We also compare the pupil-plane suppression measurements in Figure 4.10, with the theoretical mosaic shown in Figure 4.10(a) and the experimental mosaic shown in Figure 4.10(b). Both exhibit a bright central region, which represents the Poisson spot for a circular occulter, and a fine diffraction ring structure around this central spot. The ring structure contains 16 outward bright wedges corresponding to the support struts. The ring structure around the Poisson spot is more symmetrical for the theoretical case than actually observed experimentally. The decrease in symmetry for the experimental case is to be expected, as it is well known that the shape and intensity of the Poisson spot can change based on phase variations at the input plane [72] – for the copper mask, this would arise from the roughness of the etched features. Nonetheless, the measured suppression is across the physical size of the centered detector which matches the expected theoretical suppression at the $10^{-3}$ level.

Next, we consider the optimized occulter mask and compare the contrast-calibrated image-plane theoretical and experimental images in Figure 4.11. On a qualitative level, we see some significant differences between the theoretical prediction in Figure 4.11(a) and the experimental measurements in Figure 4.11(b). In the diffraction simulation the struts are not expected to be bright, but in the experimental image the struts are glowing. Some tips of the outer petal are brighter than others, which can be attributed to the tilt of the mask to remove the ghost reflection (changing the tilt of the mask predictably changes the bright tips). Lastly there is a glow around the inner mask which is both radially larger and two orders of magnitude brighter than the diffraction analysis predicts. To obtain a quantitative measure of the performance of the optimized occulter mask, we plot a similar cross-section plot using the azimuthal median as shown in Figure 4.11(c). To avoid biases due to the bright
struts and obtain the performance in the darkest portions of the annular region, we choose a set of 16 wedges whose location and size are chosen to avoid the struts—these are shown superimposed as bright regions on the contrast calibrated image in Figure 4.11(b). Thus, the azimuthal median in Figure 4.11(c) is computed using the signal contained within the wedges only and shows an improvement of contrast performance over the control mask of about two orders of magnitude. The median contrast across the wedges at the inner working angle of 400 mas space-equivalent is $10^{-9.98}$ and this improves towards the outer working angle at 638 mas space-equivalent to $10^{-10.60}$. Nonetheless, the optimized occulter mask performs worse than the theoretical diffraction analysis by about two to three orders of magnitude. The bright inner and outer features, which were in close agreement for the control mask are also brighter than expected.

We compare the pupil-plane suppression measurements for the occulter mask in Figure 4.12 in 633 nm incident light. Much of the diffractive structure in the experimental mosaic in the bright annular portions of the mask can also be seen in the theoretical mosaic. The dark hole region from the theoretical model is expected to reach $10^{-6.91}$ suppression, however the measured mosaic remains flat at a measured suppression level of $10^{-4.82}$. Thus, the additional level of light seen in the image plane at the edges of the occulter mask is consistent with the pupil plane suppression measurements. This additional light distributed across the suppression plane results in loss of the high-suppression region, and limits the performance of the occulter mask to only two orders of magnitude better than the circular control mask as opposed to the four orders of magnitude that it is theoretically capable of achieving.

The comparison of the experimental results to the theoretical model has demonstrated very good agreement for the circular occulter mask for both suppression and contrast measurements. This is significant as it increases confidence in the experimental calibration methods which are used identically for the optimized occulter mea-
surements. The experimental results in the pupil plane show performance is about two orders of magnitude worse than the occulter mask was expected to achieve. The image plane results are in agreement at the discrepancy, and illustrate that the main source of light leakage is a relatively uniform glow around the petal edges.

Figure 4.12: Suppression mosaic comparison at the pupil plane for the occulter mask: (a) theoretical (b) measured.
4.6 Contrast measurements for a circular occulter mask with optimized outer ring

We report on the recent manufacture and experimental results of an improved control mask to provide a better baseline for the optimized occulter mask. Throughout this chapter, we have used a control mask with a circular inner occulter and a circular outer ring etched out of copper to act as a baseline for performance of the optimized occulter mask. The copper etching process was chosen for simplicity and reduced cost.

In Figure 4.13, we show an improved mask acting as a baseline that uses the same outer ring shape that we used for the optimized occulter mask. Such a control mask with an identical outer ring to the optimized occulter mask would allow testing for the diffraction effects purely due to the shape of the inner occulter. This can be achieved by manufacturing the mask through the same DRIE process as the optimized occulter mask. An additional benefit from using the DRIE process is that both the control mask and the optimized mask would be made from an identical process and materials, which would ensure that the only difference being tested between the control and the occulter mask would be the shape of the inner occulter.

Compared to the optimized occulter mask for the experiments described previously, this is the first occulter mask manufactured at JPL’s MDL using a multi-layer process to thin the sidewall down to a final 1 µm edge. An identical optimal occulter mask using the same process for a thin sidewall is being manufactured at the time of this writing.

Using this new mask, we take contrast calibrated measurements as shown in Figure 4.14. In Figure 4.14(a), we use a data set consisting of 100 unsaturated frames taken\(^1\) taken

---

\(^1\)This unsaturated set was taken to capture the diffraction peaks and a saturated data set was taken to capture the valleys. However, it was found that there is signal leakage from the saturated pixels to neighbouring pixels – this is why we have taken all data sets using many unsaturated frames to capture as large a dynamic range as possible.
at 0.07 sec exposure. In Figure 4.14(b), we show the theoretical one-dimensional computation for this control mask to compare with the measurement.

For a more quantitative analysis, we refer to the azimuthal median contrast curves in Figure 4.15. Here, we compare the azimuthal median of the measured contrast for the control mask with the optimal outer ring with the azimuthal median of the theoretical curve. We also indicate the 95\% median confidence interval – we note that because of the relatively sparser data set this is a fairly wide interval. Additionally, we reproduce the measured contrast curve for the control mask featuring an outer ring. We note the disappearance of the outer peak due to the outer diffraction ring. However, the contrast across the annular opening is worse when compared to the mask with the circular outer ring. This may be due to the fact that particular data set had many more frames reducing the background level.

Finally, we note that no bright spots are observed across the outer ring when compared to the optimized occulter mask. In Figure 4.7(b), we identified the brightest outer tip to be at a level of $10^{-6.55}$, which is well above the measured median curve in Figure 4.15. The brightest spots seen in the optimized occulter results are along the mask tilt axis, which suggests that the finite mask thickness becomes an issue as the mask is titled. Thus, for this new control mask featuring the same optimized outer ring but manufactured with a better process that reduces the thickness approximately 50-fold, we conclude that it is likely that the reduced finite thickness sidewall results in reduced brightness of the outer tips. The bright petal edges seen in the optimized occulter mask images are below the level of the outer tips and not seen here because of the diffraction roll-off from the inner circular occulter. We will investigate possible sources of this light leakage in the next chapter.
Figure 4.13: Schematic comparison of control masks: (a) Current mask using circular inner occulter and circular outer occulter (b) New mask using circular inner occulter and optimized outer ring (c) Photograph of silicon manufactured control mask with inner circular occulter and optimized outer ring using DRIE process.
Figure 4.14: (a) Measured contrast in the image plane for the new control mask featuring the optimized outer ring. (b) Theoretical contrast in the image plane for the new control mask featuring the optimized outer ring. Inner and outer working angles denoted by red lines.

Figure 4.15: Azimuthal comparison of contrast curves. The measured median for the new control mask with an optimized outer ring is shown in solid blue with the 95% confidence interval denoted in red. The computed theoretical is the dashed blue line. The measured control mask with a circular outer ring is shown in solid black.
Chapter 5

Diffractive Sensitivity Analysis

5.1 Theoretical model and experimental limitations

Previously, in Chapter 4, we have presented experimental results which represent a performance improvement over a circular occulter baseline, but do not reach the full theoretical capability of the designed mask. We wish to identify the limiting factors of the performance of the laboratory mask. For this purpose, we perform a diffractive sensitivity analysis for a number of experimental factors that represent departures from the idealized, petalized version of the occulter mask model described in Chapter 3. By simulating each of these factors independently we can obtain an estimate of the suppression level at which each becomes a limiting factor for the occulter performance. By using realistic laboratory-based measurements for each of these experimental errors we can better model the observed occulter performance by combining the experimental errors in one simulation.

The main weakness of the petalized model presented in Chapter 3 is that it assumes perfect N-fold circular symmetry matching the number of petals N used. However, in reality the manufacturing of the occulter mask introduces straight edges which are approximations of the idealized, symmetric mask. There exists previous
work [19, 21, 47, 146, 145] that estimates the effect of imperfections; most of these are primarily intended for estimating the effect of shape defects of a space occulter as opposed to limitations more typical of a laboratory environment such as non-ideal input beam and wavefront errors. The main approaches for modeling the shadow under non-ideal conditions can be classified as follows:

- radial integration with a Bessel expansion over the azimuthal component, with non-idealizations modeled as part of the Bessel function expansion or analytically estimated [19]
- boundary-diffraction-wave which integrates across any defined occulter shape, but does not allow for modelling of most non-shape errors [21]
- slit-approximation of deviations from the nominal occulter shape as additions or subtractions from the ideal shadow [47]

Here we propose a different approach which is to perform a two-dimensional Fresnel propagation via a Matrix Fourier Transform [160]. This allows for direct modeling of almost any type of non-idealization, but is a fairly computationally expensive method. Additionally, to make this numerically tractable we introduce anti-aliasing at the edges of the occulter mask in order to represent a higher-resolution mask; determining the appropriate ratio of anti-aliasing to numerical sampling requires careful modeling against the baseline performance.

First we consider the various types of experimental errors that we need to model. Hypothesizing possible sources of error is almost impossible from pupil plane measurements where the suppression simply levels off across the dark hole providing no insight as to the cause of the performance degradation. We can, however, use the image plane results in Figure 5.1 to the possible sources of error. The bright edges which are most likely the limiting source of performance may be due to the edge manufacturing process accuracy. There are several brighter points along the edges
Figure 5.1: Image-plane experimental result with a number of possible sources of error identified.

that may be attributable to localized manufacturing defects in the form of over or under-etching. There is a bright glow around the inner mask which is slightly offset, and this may be due to misalignment of the input beam resulting in light leakage. The laser beam itself was idealized in the theoretical model as a spherical input beam with a negligible amplitude attenuation profile, however the laser beam is actually Gaussian and has significant amplitude attenuation over the occulter mask. Along the outer ring, some of the outer petal tips along the tilt axis are significantly brighter than others – an effect which may be due to the tilt of the occulter mask necessary to remove the ghost reflection from the optical axis. These effects are all static, and thus a series of frames taken over a long-term will be nearly identical. There are some background fast-variations, which may be due to atmospheric-induced phase aberrations – these are mostly averaged out from the combined image, but one slower time-varying light streak can be seen across the top of Figure 5.1.
5.2 Two dimensional diffraction model

5.2.1 Fresnel propagations between optical planes

We refer to Figure 5.2 to describe the propagations between the different planes involved in the occulter testbed. We define four planes of propagation: P0 is the plane of the pinhole source (2M × 2N in physical size), P1 is the plane of the occulter mask (2U × 2V in physical size), P2 is the pupil plane where the telescope aperture lies and where we measure the suppression of the shadow (2X × 2Y in physical size), and lastly P3 is the focal plane of the telescope and the plane in which we measure the contrast of the point spread function. Thus, to obtain the point spread function of the occulter testbed we need three separate propagations between these four planes.

The first optical propagation occurs between the artificial star source and the occulter mask. The artificial star source has radius $r_{\text{pin}} = 7.5 \, \mu\text{m}$ diameter pinhole across the plane $P0$ spanned by Cartesian coordinates $(m, n)$ and is defined by the
following circular function:
\[
A_{\text{pin}}(m, n) = \begin{cases} 
1, & \sqrt{m^2 + n^2} \leq r_{\text{pin}} \\
0, & \text{else}.
\end{cases} \tag{5.2.1}
\]

Then the input electric field at P1 spanned by Cartesian coordinates \((u, v)\) can be computed from a two-dimensional Fresnel integral over the pinhole at P0 as follows:
\[
E_{\text{in}}(u, v) = e^{2\pi i h/\lambda} \frac{1}{i\lambda h} e^{\frac{2\pi i}{\lambda h} (u^2 + v^2)} \int_{-N}^{N} \int_{-M}^{M} A_{\text{pin}}(m, n) e^{\frac{2\pi i}{\lambda h} (m^2 + n^2)} e^{-\frac{2\pi i}{\lambda h} (mu + nv)} \, dm \, dn \tag{5.2.2}
\]

where \(h\) is the propagation distance between P0 and P1, \(-M < m < M\), \(-N < n < N\), \(-U < u < U\), and \(-V < v < V\). This expression for \(E_{\text{in}}\) assumes a uniform beam across the pinhole, and we will revisit this assumption introducing a Gaussian profile in §5.3.

Next, we consider the main diffractive propagation past the occulter mask to accurately compute the shadow at P2 spanned by Cartesian coordinates \((x, y)\). We use a 2D mask with transmission profile \(A(u, v)\) that represents the diffractive effect of the occulter mask. The resulting 2D Fresnel propagation integral between the occulter plane at P1 and the telescope’s pupil at P2 is as follows:
\[
E_{\text{pup}}(x, y) = e^{2\pi i z/\lambda} \frac{1}{i\lambda z} e^{\frac{2\pi i}{\lambda z} (x^2 + y^2)} \int_{-V}^{V} \int_{-U}^{U} E_{\text{in}}(u, v) A(u, v) e^{\frac{2\pi i}{\lambda z} (u^2 + v^2)} e^{-\frac{2\pi i}{\lambda z} (ux + vy)} \, du \, dv \tag{5.2.3}
\]

where \(z\) is the propagation distance between P1 and P2, \(E_{\text{in}}\) is the output of the propagation in Equation 5.2.2, \(-X < x < X\), and \(-Y < y < Y\).

Finally, we can propagate from the pupil plane at P2 to the image plane at P3 by introducing a quadratic phase function corresponding to a lens and performing a Fresnel propagation across a distance \(s\) derived from the thin lens equation that takes into account the focal length \(f\) of the lens and the distance \(d\) to the pinhole that the
focus of the lens is set to. This propagation is identical to that described in Equation 2.3.2, which we reproduce here for completeness, ignoring the leading phase factors which need not be carried through as there are no further optical propagations:

\[ E_{\text{im}}(\xi, \eta) = \frac{1}{i\lambda s} \int_{-Y}^{Y} \int_{-X}^{X} E_{\text{pup}}(x, y) A_{\text{pup}}(x, y) e^{\frac{-\pi i}{\lambda s}(x^2 + y^2)} e^{\frac{-\pi i}{\lambda d}(x\xi + y\eta)} \, dx \, dy \]  

(5.2.4)

where \( A_{\text{pup}}(x, y) \) is a circular function of radius matching the camera’s aperture of 7 mm, \( E_{\text{pup}} \) is the output of the propagation in Equation 5.2.3, and \((\xi, \eta)\) are Cartesian coordinates spanning the image plane P3.

To compute a mosaic of the shadow similar to those shown in Figure 4.5, only the result from Equation 5.2.3 is necessary and the appropriate output dimensions for the mosaic are chosen by selecting \( X \) and \( Y \). To compute the point spread function more samples are necessary across the aperture than would typically be computed for a mosaic and instead \( X \) and \( Y \) are chosen only to encompass the camera’s aperture before computing the image plane output in Equation 5.2.4.

To normalize the intensity at the pupil plane in terms of suppression or the intensity at the image plane in terms of contrast, we set \( A(u, v) = 1 \) and obtain \( \hat{E}_{\text{pup}}(x, y) \) as the electric field output of Equation 5.2.3 with no occulter mask in place and also \( \hat{E}_{\text{im}}(\xi, \eta) \) as the corresponding electric field at the image plane from Equation 5.2.4. Then the suppression and contrast metrics are defined as follows:

\[ \text{Supp.}(x, y) = \frac{|E_{\text{pup}}(x, y)|^2}{\max \left| \hat{E}_{\text{pup}}(x, y) \right|^2} \]  

(5.2.5)

\[ \text{Cont.}(\xi, \eta) = \frac{|E_{\text{im}}(\xi, \eta)|^2}{\max \left| \hat{E}_{\text{im}}(\xi, \eta) \right|^2} \]  

(5.2.6)
5.2.2 Numerical implementation

We can perform the Fresnel propagations defined by Equations 5.2.2, 5.2.3, and 5.2.4 in different ways: direct integration, a Fast Fourier Transform (FFT), or a Matrix Fourier Transform. We use direct integration for the one-dimensional propagations described previously in Chapters 2 and 3, but these are slow and impractical to perform across two-dimensions. Consider that Equation 5.2.3 can be re-written as a Fourier Transform of the product of the input electric field, the occulter mask, and a quadratic phase factor:

\[
E_{\text{pup}}(x,y) = \frac{e^{2\pi iz/\lambda}}{i\lambda z} e^{\pi i \frac{\lambda z}{\lambda}}(x^2 + y^2) F \left\{ E_{\text{in}}(u,v) A(u,v) e^{\pi i \frac{\lambda z}{\lambda}(u^2 + v^2)} \right\}
\]  

(5.2.7)

where we have used \( F \) to denote the Fourier transform across \((u, v)\).

For speed, Fresnel propagations evaluated as Fourier Transforms are traditionally performed using FFTs either in spatial or frequency domains [95, 141, 178], but there are some limitations to this approach — in particular, for accuracy this requires padding the input mask by a factor of 3-7 to a total number of samples across each dimension that equals a power of 2. Additionally, the size and possible sampling of the output plane is constrained by the discretization of the input plane. For some methods it is required that the two planes be equal spatially with only a portion of the output plane being numerically accurate. Greater amounts of padding result in a larger portion of the output plane being accurate, however this quickly makes propagations infeasible on a general purpose computing machine due to memory limitations.

The Matrix Fourier Transform method [160], which we previously describe mathematically in §1.7, is particularly well-suited for performing the Fresnel propagation in Equation 5.2.3 for laboratory occulters. This is because for the telescope pupil plane, which represents the propagation output plane, we are often interested in the electric
Table 5.1: Summary of sampling across the different optical planes. The separation is indicated between the planes.

<table>
<thead>
<tr>
<th>#</th>
<th>Optical Plane</th>
<th>Physical Dimension</th>
<th>Number of Samples</th>
<th>Separation</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0</td>
<td>Pinhole Source</td>
<td>15 µm × 15 µm</td>
<td>1000 × 1000</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.5 m</td>
</tr>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>P1</td>
<td>Occulter Plane</td>
<td>88.9 mm × 88.9 mm</td>
<td>16000 × 16000</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td>Pupil Plane</td>
<td>14 mm × 14 mm</td>
<td>1000 × 1000</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.31 m</td>
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</tr>
<tr>
<td>P3</td>
<td>Image Plane</td>
<td>6.6 mm × 6.6 mm</td>
<td>1024 × 1024</td>
<td>N/A</td>
</tr>
</tbody>
</table>

field only across a small portion of the pupil across the detector rather than matching the entire spatial extent of the occulter. Additionally, we only need to perform one propagation to the pupil plane between two planes which are well-defined spatially compared to optical modeling inside a coronagraph where intermediate plane propagations have a low-pass filtering effect [130].

In Table 5.1, we summarize the number of samples and the physical size of each optical plane, as well as the separation distances matching the experimental layout described in Chapter 4. The number of samples across each plane arises due to different considerations: the sampling of pinhole source plane is related to the accuracy of modeling a Gaussian profile across the pinhole source, the number of samples across the occulter plane allow us to tolerance the occulter mask manufacturing process, the number of points across the pupil plane allows accurate representation of the image-plane to the angular extent of the occulter mask, and lastly the image plane sampling is chosen to coincide with the number and physical size of CCD pixels.

5.2.3 Two-dimensional mask model generation

Next, we consider the mask model across the occulter plane $P1$, and discuss how this can be generated with tolerances representing the manufacturing accuracy. Figure
5.3 illustrates the process of generating a two-dimensional occulter mask from the one-dimensional apodization profile.

The mask model is represented by the apodization profile $A(u, v)$ and is part of the input to the Fresnel propagation in Equation 5.2.3. $A(u, v)$ is set to white, or 1, in open areas in which light is allowed to pass and to black, or 0, in areas where the binary mask completely blocks light. To generate a binary mask we follow the procedure:

1. generate an $n \times n$ grid;
2. determine polar coordinates of the midpoint of each grid point;
3. compute the radial and angular coordinates of a petal edge;
4. for each midpoint, compute the angular coordinate in relation to the nearest petal center;
5. determine whether each midpoint is a point on the mask (black) or an opening (white) by comparison to the nearest petal edge angular coordinate.

In Figure 5.3(a), this process is shown graphically. A number of test points $p_1 - p_7$ are chosen that lie on the mask; these test points all have angular coordinates with respect to the petal center that are above the petal edge and are therefore set to black; the entire gray area is above the petal edge represents points on the mask that are fully opaque. Test points $p_8 - p_{12}$ have angular coordinates smaller than the petal edge and are therefore set to white and represent the petal openings; the entire white area below the petal edge represents points not on the mask that are fully transparent. Numerically, an interpolation of the optimized petal edge may be necessary to compare with all midpoints.

To perform a Fresnel propagation for a high resolution occulter mask, we utilize an anti-aliasing technique to reduce the number of grid points necessary to represent...
Figure 5.3: Generation of the two-dimensional apodization profile $A(u, v)$. (a) the one dimensional radial apodization profile is converted to a half-petal edge. Filled test points p1-p7 have angular coordinate $\theta$ greater than the nearest edge point and therefore lie on the mask as does the entire gray area, whereas empty test points p8-p12 have $\theta$ smaller than edge are therefore part of the petal openings as does the entire white area; (b) tip section demonstrating anti-aliasing procedure to obtain a gray approximation at the edges of the mask; (c) resulting 2D mask model using $16,000 \times 16,000$ points, and a $10 \times 10$ anti-aliasing, representing feature accuracy of $\delta R = 0.55 \mu m$. 
the occulter mask. We allow \( A(u, v) \) to take on \textit{gray} values over the range \([0, 1]\) along the edges of the occulter mask in order to approximate a higher-resolution grid than can be numerically tractable to fully simulate as a binary mask. This is achieved by computing a pattern of white and black squares over an area and determining the black fraction of the total area to obtain a gray approximation. An example of this anti-aliasing procedure is shown in Figure 5.3(b).

Thus, a mask with \( n \times n \) samples which uses a \( g \times g \) anti-aliasing approximates a mask with feature size of \( \delta R = 2R/n/g \), where \( R \) is the radius of the mask to the outer edge. Such a mask with feature accuracy \( \delta R = 0.55 \mu m \) is shown in Figure 5.3(c).

### 5.2.4 Comparison to ideal diffraction pattern for square aperture

We also consider whether the theoretical performance is a function of the edge features or if the mask sampling introduces aliasing in the Fresnel propagation. To verify the accuracy of the Fresnel propagation in Equation 5.2.3 and to ensure that an appropriate sampling is chosen we compare the output of the numerical propagator against the idealized Fresnel diffraction from a square aperture at laboratory dimensions (i.e., operating at the same Fresnel number).

It is well-known that the Fresnel diffraction of a square aperture can be computed from a summation of Fresnel integrals \([66, 74]\). Consider a monochromatic, plane wave of unit amplitude incident on a square aperture of radius \( 2w \). We can write the Fresnel diffraction equation as:

\[
E_{\text{ideal}, \square}(x, y) = \frac{e^{\frac{2\pi i}{\lambda z}}}{i\lambda z} \int_{-w}^{w} \int_{-w}^{w} e^{\frac{2\pi i}{\lambda z}((x-u)^2 + (y-v)^2)} dudv \quad (5.2.8)
\]
which can be separated into two one-dimensional integrals:

\[ E_{\text{ideal},\Box}(x, y) = \frac{e^{\frac{2\pi i z}{\lambda}}}{i} E_{\Box}(x) E_{\Box}(y) \]  

with \( E_{\Box}(x) \) and \( E_{\Box}(y) \) the integrals along each dimension, defined as follows:

\[ E_{\Box}(x) = \frac{1}{\lambda z} \int_{-w}^{w} e^{\frac{\pi i z}{\lambda z} (x-u)^2} du \]  

\[ E_{\Box}(y) = \frac{1}{\lambda z} \int_{-w}^{w} e^{\frac{\pi i z}{\lambda z} (y-v)^2} dv \]

These can be evaluated using the well-known Fresnel integrals \( C(\alpha) \) and \( S(\alpha) \) defined by the expression:

\[ \int_{0}^{\alpha} e^{i\pi \alpha'^2/2} d\alpha' = C(\alpha) + iS(\alpha) \]

\[ = \int_{0}^{\alpha} \cos(\pi \alpha'^2/2) d\alpha' + i \int_{0}^{\alpha} \sin(\pi \alpha'^2/2) d\alpha' \]

We apply the following changes of variables:

\[ \alpha_1 = -\sqrt{2} \left( \sqrt{\frac{w^2}{\lambda z} + \frac{x}{\lambda z}} \right), \quad \alpha_2 = \sqrt{2} \left( \sqrt{\frac{w^2}{\lambda z} + \frac{x}{\lambda z}} \right) \]

\[ \beta_1 = -\sqrt{2} \left( \sqrt{\frac{w^2}{\lambda z} + \frac{y}{\lambda z}} \right), \quad \beta_2 = \sqrt{2} \left( \sqrt{\frac{w^2}{\lambda z} + \frac{y}{\lambda z}} \right) \]

Note that \( \frac{w^2}{\lambda z} \) is the Fresnel number of the aperture, and by choosing \( w = R \), the radial size of the laboratory occulter mask, we are operating at the same Fresnel number. The intensity of the square aperture’s diffraction pattern is then:

\[ I_{\text{ideal},\Box} = \frac{1}{4} \left( (C(\alpha_2) - C(\alpha_1))^2 + (S(\alpha_2) - S(\alpha_1))^2 \right) \times \]

\[ \ldots (\ldots (C(\beta_2) - C(\beta_1))^2 + (S(\beta_2) - S(\beta_1))^2) \]
Approximations of these integrals can be found in Chapter 7 of Abramowitz and Stegun [1]. To numerically compute the diffraction pattern, we use MEX function implementations in MATLAB that enable fast and accurate computations of the Fresnel Sine and Cosine integrals up to 15 significant digits [42]. We compare this idealized diffraction pattern with $E_{\text{pup},\square}(u, v)$ a numerical implementation obtained from Equation 5.2.3, by setting $A(u, v) = E_{\text{in},\square}(u, v) = 1$ and choosing the same output sampling. Then we compute the residual:

$$\Delta I_{\square}(x, y) = I_{\text{ideal},\square}(x, y) - |E_{\text{pup},\square}(x, y)|^2 \quad (5.2.16)$$

In Figure 5.4, we provide the root-mean-square of the residual for different $n \times n$ sampling square masks. A choice of more than 8000 × 8000 points across the square mask provides agreement of approximately 7 significant figures, and demonstrates the accuracy of the Fresnel propagator at laboratory dimensions.

Figure 5.4: Quantification of error in Fresnel numerical propagation by comparison to idealized diffraction pattern for a square aperture.
Figure 5.5: Comparison of ideal model with feature-size limited model (a) Theoretical contrast at the image plane using the 1D binary propagation. Corresponding suppression is $10^{-6.91}$. The inner and outer red circles denote the annular dark region. (b) Theoretical contrast at the image plane using a 2D propagation with high-resolution mask with 0.55 μm feature size. Corresponding suppression is $10^{-6.60}$.

5.3 Modeling experimental errors

We next consider how the experimental errors previously identified in §5.1 can be mathematically modeled in the two-dimensional propagation model introduced in §5.2. The main advantage of this is that errors’ diffractive effects are directly modeled.

5.3.1 High precision model as the base case

To demonstrate the numerical accuracy of this propagation method, we choose a detailed mask as shown in Figure 5.3(c) with $n = 16,000, \ g = 10$, which at laboratory dimensions approximates a feature size of $\delta R = 0.55 \, \mu\text{m}$. We compare the propagation to the image plane for both the binary petalized mask using the one-dimensional Bessel expansion propagation, which represents a mask with perfect feature accuracy, with the performance of the high-resolution mask in Figure 5.5. The maximum theoretical suppression performance across the aperture is $10^{-6.91}$ as reported previously in the simulation corresponding to Figure 3.4b, and the high-resolution mask
attains a $10^{-6.60}$ suppression measurement. This comparison demonstrates that a mask with sufficiently high resolution can match the theoretical performance of the occulter mask as realized with 16-petals and validates the 2D propagation model.

Table 5.5, which can be found at the end of the chapter, summarizes the suppression and contrast performance for simulations with various errors included and varied individually as will be discussed in §5.3. The performance of the high resolution mask described above represents the base case against which we compare the effect of other errors and is summarized across line 1a. We also simulate the performance of the mask when a Gaussian profile is incident across the pinhole instead of a uniform beam; the effect of a focused Gaussian profile at the pinhole is to more evenly distribute the electric field at the occulter plane which is closer to the idealized space case. This results in a very small performance improvement with suppression of $10^{-6.69}$, and the results are summarized across line 1b in Table 5.5.

### 5.3.2 Manufacturing process feature accuracy

When converting the mask’s apodization profile for CAD specifications for etching of the silicon wafer, the output consists of 16 different sets of points with each defining a polygon that represents one petal opening of the occulter mask. The spacing of Table 5.2: Summary of sampling across the occulter plane $P1$ to represent different representative feature accuracies.

<table>
<thead>
<tr>
<th>Feature Size $\delta R$</th>
<th>Number of Samples $n \times n$</th>
<th>Anti-Aliasing $g \times g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.55 $\mu$m</td>
<td>$16,000 \times 16,000$</td>
<td>$10 \times 10$</td>
</tr>
<tr>
<td>0.72 $\mu$m</td>
<td>$12,000 \times 12,000$</td>
<td>$10 \times 10$</td>
</tr>
<tr>
<td>2.41 $\mu$m</td>
<td>$12,000 \times 12,000$</td>
<td>$3 \times 3$</td>
</tr>
<tr>
<td>3.10 $\mu$m</td>
<td>$14,000 \times 14,000$</td>
<td>$2 \times 2$</td>
</tr>
<tr>
<td>5.42 $\mu$m</td>
<td>$16,000 \times 16,000$</td>
<td>$1 \times 1$</td>
</tr>
<tr>
<td>7.22 $\mu$m</td>
<td>$12,000 \times 12,000$</td>
<td>$1 \times 1$</td>
</tr>
</tbody>
</table>
points along the edges represents the accuracy with which the polygon is defined – a small number of points results in longer straight edges which can introduce additional diffraction leakage along the occulter edges. Additionally, there are minimum features introduced by the etching process itself which is tolerated to operate at 1 µm.

For the 2D diffraction model previously described in §5.2, the manufacturing process δR feature accuracy can be controlled by choice of the number of sampling points \( n \) and number of anti-aliasing points \( g \). In Table 5.2, we summarize different mask models generated as described in §5.2.3 used to represent masks with different feature sizes.

Using these mask models across the occulter plane \( P1 \), we perform optical propagations and compare these to the high-resolution mask performance. Figure 5.6 shows the contrast-calibrated image plane results as the manufacturing feature accuracy is worsened. For quantitative results, refer to Table 5.5, where lines 2a-2e represent the corresponding numerical simulations. Suppression performance across the dark hole is reduced from \( 10^{-6.69} \) for the high-resolution mask to \( 10^{-4.39} \) in the worst case tested. The corresponding contrast across the dark annular regions in the image plane is decreased from \( 10^{-13.11} \) to \( 10^{-10.41} \).

Qualitatively we can see in the image plane that the edge diffraction is significantly increased resulting in bright edges across the occulter mask similar to those seen experimentally in Figure 5.1. A limitation of this model of edge diffraction is that there may be some discretization errors arising from converting a 16-fold symmetrical mask to a rectangular grid. These errors are likely the source for the bright points along the struts for the mask models which do not feature anti-aliasing and which are not seen experimentally. Nonetheless, a clear trend in Figure 5.6 is that increased straight edges result in significantly more diffraction across the mask edges than predicted from the perfect petal realization expected from theoretical simulations in Chapter 3.
5.3.3 Edge perturbations

The resulting two-dimensional mask $A(u, v)$ after choice of sampling is symmetrical, except for discretization errors arising from representation of a 16-fold circularly symmetric pattern onto a two-dimensional grid. To remove symmetry and simulate manufacturing variations between different petals of the occulter mask, we consider all gray contour pixels which represent the petal edge across the mask $A(u, v)$ and introduce perturbations at these edge pixels.

To obtain an estimate of the amount of perturbations to realistically introduce in the model, we perform spot microscope imaging of the occulter mask. Figure 5.7(a) represents an optical microscope image taken at the $50\times$ magnification – the highest available on the microscope used. Using the MATLAB Image Toolbox `edge` function, the Canny method [27] is used to extract the edge between the silicon mask and background. The extracted edge is then fitted against the theoretical contour in terms of both location along the petal and orientation as shown in Figure 5.7(b).

![Figure 5.7: (a) Microscope image of a petal tip. (b) Extracted edge comparison with theoretical contour. (c) Deviation between measured edge and theoretical contour, compared to sample simulated edge perturbation.](image)
The deviation between the measured edge and the theoretical contour is obtained for each measured point along the extracted edge – this is shown as the measured deviation curve in Figure 5.7(c) and results in an RMS value of 3.1 µm. To match this measured deviation, we use an edge feature accuracy of $\delta R = 2.42$ µm, by setting $n = 12,000, g = 3$.

We introduce edge perturbations in the mask model by using the MATLAB Image Toolbox function `bwconncomp` to identify a starting point in each of the 16 petal openings and the function `bwtraceboundary` to continuously trace all the gray contour pixels for each of the openings. The output consists of 16 different sets of contour grid points indexed by $k$ across each petal $C^k = \{c^k | 0 < c^k < 1, c \in \mathbb{R}\}$ with each set containing $N_k$ elements representing the number of grid points along the petal contour.

Next, using the procedure described in [82], 16 different stochastic sequences can be added as edge perturbations for each petal contour $E^k = \{e^k | e^k \in \mathbb{R}\}$. Such a stochastic sequence is generated as a set of random Gaussian variables with unity variance representing discrete spatial frequency components up to the Nyquist limit, then multiplied with a $1/f^\alpha$ envelope. The DC component is set to 0, and to represent white noise we set $\alpha = 0$. The sequence is then converted via an FFT, using appropriate zero-padding, to the spatial domain across the entire petal and normalized to a RMS-value matching the measured deviation. Next, we create a new set of contour points $\bar{C}^k = \{ar{c}^k | 0 \leq \bar{c}^k \leq 1, \bar{c}^k \in \mathbb{R}\}$ for each petal $k$ by addition of the generated stochastic sequence and conversion of the RMS error to an appropriate gray-scale value:

$$\bar{c}^k_i = c^k_i + \frac{1}{n} \frac{e^k_i}{\delta R}, \quad \forall i \in \{1 \ldots N_k\} \quad (5.3.1)$$

To avoid unphysical effects, we maintain the condition that $0 \leq \bar{c}^k_i \leq 1$ by setting
any negative values to 0 and any numbers greater than unity to 1. For large RMS sequences this condition can result in variation from the original generated edge perturbation and it may be necessary to use a mask model with fewer $n \times n$ samples. A sample resulting perturbed edge sequence is compared to the measured in Figure 5.7(c) across the petal tips. The perturbed edge sequences around the petal contour grid points $\bar{C}^k$ replace the original petal contour grid points $C^k$ for all 16 petals in the two-dimensional mask model $A(u, v)$.

In Figure 5.8 we show the contrast-calibrated image plane results for mask models
across the occulter plane $P1$ with various amounts of RMS edge perturbations injected as described previously. Qualitatively, we see that with increasing edge perturbations there is increased diffraction along the occulter edges. Diffraction from the edges is also very uniform, which is likely a consequence of the white-noise structure of the injected perturbations. All of these simulations use the high-resolution feature size model with $\delta R = 0.55 \mu m$, and therefore RMS perturbations greater than $7 \mu m$ RMS can saturate. This may result in underestimating the effect on performance for such large injected perturbations. This is a limitation of the model hereby presented, and for greater over- and under-etching errors, it may be necessary to use a mask model with fewer $n \times n$ samples and/or modify the edge perturbation injection to affect neighbouring grid points. The corresponding quantitative results are summarized across lines 3a-3j in Table 5.5.

### 5.3.4 Input beam modeling

We can simulate an imperfect input beam across the occulter mask at $P1$ by introducing a Gaussian beam model across the pinhole, and additionally by displacing the optical axis of the pinhole from the occulter plane to simulate optical misalignment.

To account for the Gaussian beam and optical aberrations across the pinhole, we can modify Equation 5.2.2 which was written for a uniform beam across the pinhole.
by introducing two additional terms inside the integral

\[
E_{\text{in}}(u, v) = \frac{e^{2\pi i h/\lambda}}{i\lambda h} e^{\frac{\pi i}{\lambda h} (u^2 + v^2)} \int_{-N}^{N} \int_{-M}^{M} E_{\text{beam}}(m, n) e^{i\phi_{\text{pin}}(m, n)} \times \\
\ldots A_{\text{pin}}(m, n) e^{\frac{\pi i}{\lambda h} (m^2 + n^2)} e^{-\frac{2\pi i}{\lambda h} (mu + nw)} dmdn
\]

(5.3.2)

where \(\phi_{\text{pin}}(m, n)\) are the phase aberrations introduced by the fold mirror and microscope objective, simulated as pure phase aberrations collocated at the pinhole plane.

A circularly symmetric Gaussian beam can be written at the pinhole plane as:

\[
E_{\text{beam}}(m, n) = e^{-\frac{\pi i}{\lambda} \sqrt{m^2 + n^2}} q
\]

(5.3.3)

The complex beam parameter \(q\) is given by:

\[
\frac{1}{q} = \frac{1}{R} - \frac{i \lambda}{\pi w^2}
\]

(5.3.4)

where in the above \(R\) is the radius of curvature and \(w\) is the laser spot size. ABCD ray transfer matrices can be used to relate the complex beam parameter at the laser focus with propagation through free-space and subsequent focusing through the microscope objective modeled as a thin lens with known focal length [91].

The resulting Gaussian beam profile is shown in Figure 5.9(a) across the 15 µm diameter pinhole with added phase aberrations from the upstream optics of RMS \(\lambda/4\) shown in Figure 5.9(b), which are then Fresnel propagated from the pinhole plane to the occulter plane. Lastly, from the experimental image plane results, we hypothesize that some of the light leakage at the inner ring of the occulter mask is due to misalignment of the input beam. This can be geometrically estimated from pixel measurements to be approximately 4.5 mm in the diagonal direction, which is then applied to the occulter plane as shown in Figure 5.9(c).

In Figure 5.10 we show the contrast-calibrated image plane sensitivity analysis
results. We first introduce a pure beam misalignment in the diagonal direction as shown in Figure 5.10(a). This results in light leakage around the central portion of the mask as seen experimentally, with performance relatively robust until there is a clear misalignment and a large amount of light leaks around the central occulter mask. The quantitative results are summarized across lines 4a-4h in Table 5.5.

Next, we introduce pure phase aberrations across the pinhole plane, with the results shown in Figure 5.10(b) for increasing phase aberrations. These aberrations are produced similar to the procedure described for generating the one-dimensional random edge perturbations in §5.3.3 with two main differences: we generate a two-dimensional aberrated matrix instead of a one-dimensional sequence, and secondly instead of a flat frequency envelope used for white noise we apply coloured noise with a frequency envelope of $1/f^\alpha$ where $\alpha = 3/2$. The generated phase aberrations satisfy the following relation:

$$\sigma_{\text{pin,RMS}} = \sqrt{\int_{-M}^{M} \int_{-N}^{N} \phi_{\text{pin}}(m, n) dm \, dn} \quad (5.3.5)$$
Because of the spatial filtering effect of the pinhole high-frequency phase aberrations from the optics are filtered out and only low-order aberrations such as tip-tilt remain. These can be clearly seen in the image plane results for large aberrations on the order of \( \lambda/4 \) or greater, but these have a small effect on the performance metrics. For example, the worst suppression performance is only \( 10^{-6.53} \) compared to the unaberrated performance of \( 10^{-6.69} \). The quantitative results are summarized across lines 5a-5g in Table 5.5, and the results suggest that the pinhole filters aberrations induced by the upstream optical surfaces very well.

### 5.3.5 Wavefront errors

The laboratory enclosure is operating at room temperature in a non-stabilized environment. As such, it is possible for temperature gradients to develop, which may result in atmospheric effects due to variations in refractive index across the propagation direction. This is a separate effect from aberrations introduced by the surface quality of the optics, the majority of which have been shown in §5.3.4 to be spatially filtered by the pinhole.

Our theoretical simulations have assumed propagation through vacuum, so it is important to assess the effect of this non-idealization. To model the effect of atmospheric turbulence, we introduce stochastic phase errors collocated at the occulter plane. This is different from the surface errors introduced by the upstream optics used to focus the Gaussian beam in §5.3.4, which are applied at the pinhole plane.
Figure 5.12: (a) Experimental contrast at image plane for mask deposited on glass. (b) Diffractive analysis with wavefront errors included.

We consider phase errors at the input plane because this is where occulters are most susceptible to disturbances affecting the formation of the shadow. Conversely, occulters are very resilient to phase errors after the occulter – to illustrate this, we consider that occulters have been proposed to operate in low Earth orbit casting a shadow on the ground with typical atmospheric seeing effects resulting in a leading tilt aberration of the bright portion of the downstream electric field bending towards the dark hole with a net reduction in the shadow size on the order of mm for km-distance propagations through the atmosphere.

Mathematically, to introduce these phase aberrations we modify the propagation from the occulter plane from Equation 5.2.3 to the pupil plane as follows:

\[ E_{\text{pup}}(x, y) = \frac{e^{2\pi i z / \lambda}}{i \lambda z} e^{\frac{\pi i}{\lambda z} (x^2 + y^2)} \int_{-V}^{V} \int_{-U}^{U} E_{\text{in}}(u, v) e^{i\phi_{\text{occ}}(u, v)} \times \]

\[ \ldots A(u, v) e^{\frac{\pi i}{\lambda z} (u^2 + v^2)} e^{-\frac{2\pi i}{\lambda z} (ux + vy)} dudv \]  

(5.3.6)

The phase aberrations \( \phi_{\text{occ}}(u, v) \) are randomly generated similarly to those in §5.3.4, with the number of points matching the dimension of the occulter mask across \( P1 \).
The phase aberrations are generated to maintain the following RMS relation:

\[
\sigma_{\text{occ,RMS}} = \sqrt{\int_{-V}^{V} \int_{-U}^{U} \phi_{\text{occ}}(u,v) du dv}
\]  

(5.3.7)

In Figure 5.11, we present contrast-calibrated image plane results as wavefront aberrations collocated across the occulter plane increase as indicated. We see qualitatively that the occulter performance is highly sensitive especially in the important dark annular region to wavefront aberrations. Conversely, we also see that larger wavefront aberrations are not present at the occulter plane as the dominating limiting factor of the performance from the experimental results appears to be the edge diffraction rather than wavefront errors. Quantitatively, we summarize the performance of the occulter mask under wavefront aberrations across lines 6a-6p in Table 5.5.

One regime in which wavefront errors dominate is the case in which we do not use a fully binary occulter mask. In the experimental results in Chapter 4, we have shown in Figure 4.8 the image plane performance of an identical occulter pattern deposited on a glass substrate. In the open areas, the glass substrate can introduce phase aberrations. Using the model described above we compute the image plane performance for large phase aberrations of \(2 \lambda\) RMS and compare these to the experimental results in Figure 5.12. The re-stretched experimental image is shown in Figure 5.12(a). The simulation including large wavefront errors collocated at the occulter plane is shown in Figure 5.12(b). We note that there are structural differences in the simulated point spread function compared to the experimental. These are likely due to the unmodeled internal reflections. The phase aberration model, however, successfully predicts contrast performance to a level worse than the circular results in §4.4. The circular occulter performance was dominated by edge diffraction at a contrast level of \(\approx 10^{-5.5}\) with the dark annular region reaching \(\approx 10^{-8.5}\). The phase aberration model predicts
that the contrast performance level across the edges is at a level better than the contrast performance across the annular openings. This experiment and simulation demonstrates the sensitivity of occulter designs to input phase aberrations.

### 5.3.6 Mask tilt

The occulter mask is tilted at a $5^\circ$ angle to ensure a ghost reflection does not back-propagate into the occulter plane. Previous tolerancing studies have shown that occulters are tolerant of out-of-plane tilts of less than $10^\circ$ [47]. To model the loss of symmetry due to the mask tilt, a rotation and in-plane projection can be applied for the occulter mask.

We can apply such a rotation using a two-dimensional affine rotation using the `imwarp` function in MATLAB. The resulting mask with a $5^\circ$ rotation is shown in Figure 5.13(a). Figure 5.13(b) shows an exaggerated $25^\circ$ tilt for illustration. We only model the amplitude effect of the mask tilt. A limitation of this model is that Nyquist sampling of the phase ramp necessary to model the phase shift across the mask is numerically intractable at the physical dimensions of the laboratory mask.

In Figure 5.14, we show contrast-calibrated image plane results for increasing mask tilts. We can see that the performance of the mask is very robust to the loss of amplitude symmetry due to the mask tilt. The quantitative results are summarized across lines 7a-7e of Table 5.5, with suppression performance only decreasing to $10^{-6.48}$ for a $20^\circ$ tilt. There is more variation in the tabulated contrast levels across the dark
annular regions, primarily arising due to the change of the image shape from circular to elliptical symmetry.

## 5.4 Diffractive analysis with errors included

In §5.3, we have performed sensitivity analyses across each of the various modeled errors across the optical propagations. Some of these errors may have cross-effects – for example beam misalignment can worsen the diffraction due to non-symmetrical edges.

We combine all the different modeled errors in one optical propagation to realistically assess the diffractive performance of the occulter mask in the laboratory environment. Realistic parameters for the laboratory environment are listed in Table 5.3. The feature and edge accuracy are chosen to match the spot microscope images presented in §5.3.3. The mask tilt is a measured laboratory parameter. The phase aberrations across the pinhole are due to the surface quality of the upstream optics. The beam displacement is measured from the experimental image plane results. Only the wavefront aberrations across the occulter plane are unknown, however we choose a small value as we can infer from comparison of the experimental results to the sensitivity analysis that the wavefront aberrations are not a dominating limiting factor.

From the sensitivity analysis in §5.2.3 and the quantitative results in Table 5.5,
Table 5.3: Summary of realistic error parameters for diffractive simulation of laboratory environment.

<table>
<thead>
<tr>
<th>Error Parameter</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feature accuracy, $\delta R$</td>
<td>$2.41 \mu m$</td>
</tr>
<tr>
<td>Edge perturbations</td>
<td>$3.1 \mu m$ RMS</td>
</tr>
<tr>
<td>Optics aberrations, $\sigma_{pin,RMS}$</td>
<td>$\lambda/4 \approx 160 \text{ nm RMS}$</td>
</tr>
<tr>
<td>Wavefront aberrations, $\sigma_{occ,RMS}$</td>
<td>$3.1 \text{ nm RMS}$</td>
</tr>
<tr>
<td>Diagonal beam misalignment</td>
<td>$4.1 \text{ mm}$</td>
</tr>
<tr>
<td>Mask tilt</td>
<td>$5^\circ$</td>
</tr>
</tbody>
</table>

none of the modeled errors individually results in the loss of performance observed in the laboratory when these error parameters are set in the range expected experimentally as listed in Table 5.3. However, we can conclude that the dominating sources of error are related to edge diffraction, and it is possible that a combination of these errors in particular small optical misalignments with non-ideal edges can reproduce similar performance to that observed in the laboratory.

In Figure 5.15 we compare a contrast-calibrated image plane derived from the simulated optical propagation including all the modeled errors in Figure 5.15(a) with the experimental results in Figure 5.15(b) previously shown in Chapter 4 on the same page for ease of comparison. We can see that the diffractive propagation with errors included approximates the experimental results much better than the N-fold circularly symmetric theoretical model discussed in Chapter 3. In particular, in the image plane simulation the edges of the occulter mask are now bright to a level very similar to the experimental results. There is leakage of light around the central occulter, and diffraction ringing can be seen in portions of the dark annular region. Some differences can also be seen. The most conspicuous difference is that bright tips seen in the experimental results are not replicated in the diffractive simulation – this effect may be due to specular reflection off the finite thickness of the occulter mask’s sidewall and therefore may not be a diffractive effect. Additionally, in the experimental image
Figure 5.15: Comparison of experimental with simulation results (a) Experimental contrast at image plane. Corresponding measured suppression $10^{-4.82}$. (b) Diffractive analysis with experimental errors. Corresponding modeled suppression $10^{-4.85}$. 
plane there are bright points along the edges of the occulter mask, whereas in the
diffra ctive simulation the edges are fairly uniformly bright. This discrepancy may
be due to localized under-etching or over-etching spanning a larger distance, whereas
the diffractive simulation assumed stochastic white noise perturbations from the ideal
contour along the edges of the occulter mask.

For a more quantitative comparison, the azimuthal cross-sections are shown in
Figure 5.16 and we also include the theoretical performance curve from Chapter 3 for
the idealized, perfect 16-fold circular realization of the mask. The peak at the inner
petal edges matches the experimental curve, but the inner ring is wider experimentally
than predicted from the simulation. Additionally the contrast at the outer peak and
across the dark annular openings is about half an order of magnitude worse than
predicted from the diffractive error analysis. The discrepancy at the outer edges
occurs at the bright tips which may be due to the finite thickness of the occulter
mask and which is not modeled.

In Figure 5.17 we present the corresponding suppression mosaics. The experimen-
Figure 5.17: Suppression-calibrated occulter shadow mosaic comparison: (a) diffractive analysis with errors included (b) experimental results.
Figure 5.18: Suppression performance for a Monte Carlo analysis using 100 simulations at 633 nm and the corresponding probability density function. Mean suppression performance is $10^{-4.95}$.

tal results in Figure 5.17(a) are compared to the diffractive simulation with modeled errors in shown in Figure 5.17(b). The diffractive structure expected away from the dark hole region is very similar to that obtained experimentally, but this also applies to the theoretical prediction in 4.10(a). The effect of the simulated diffractive errors in 5.17(a) can be seen by comparing the suppression level across the dark hole region, where we see a uniform light residual. This results in a predicted suppression performance at the pupil plane across the detector that is at a similar level, $10^{-4.85}$, as the experimentally measured suppression at $10^{-4.82}$. By comparison, the theoretical performance without any errors included predicted a suppression level of $10^{-6.90}$.

The combined simulation described above however is only one simulation based on measured or quoted laboratory error parameters. Some of the errors introduced, including the edge perturbations, the optical train aberrations prior to the pinhole, and phase aberrations collocated at the occulter plane are stochastic errors. The occulter performance can vary between simulations. We therefore perform a Monte
Table 5.4: Summary of suppression performance results when comparing the ideal model, the 5th percentile, mean, and 95th percentile of the Monte Carlo simulations with all errors included, and the experimental measurements.

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Carlo assessment [129] using a larger number of simulations with stochastic errors generated randomly for each of the aberrations described above. Such an ensemble of simulations gives a better assessment of the variation of the performance of the occultor than a single simulation. We perform 100 simulations for each of the monochromatic wavelengths (520 nm, 633 nm, and 638 nm) for which the suppression performance was reported in the laboratory results in Chapter 4. In Figure 5.18 we show the suppression results for the set of Monte Carlo simulations at 633 nm that provides an estimate of the expected suppression distribution. The suppression results are placed into ten equally sized bins, with the simulation frequency indicated. The probability density function (PDF) of the expected suppression performance is reconstructed from the simulated suppression results through normal kernel density estimates [126, 136] using the `ksdensity` function in MATLAB’s Statistics toolbox. We can obtain quantitative estimates of the simulated suppression performance by computing the cumulative distribution function (CDF). From the CDF, we can find the suppression at the 5th percentile and 95th percentiles. In Table 5.4, we compare the suppression distributions of the Monte Carlo simulations with the results of the ideal model and with the experimental measurements.

Thus, the measured suppression fall within the expected distribution of the stochastically modelled errors for all three wavelengths and therefore we conclude that the
modeled suppression errors are at a similar level to what we observe in the laboratory. We refer here again, however, to the contrast curves shown in Figure 5.16. The contrast curves correspond to a stochastic simulation with a similar suppression level. The suppression measurements are dominated by the brightest points at the occulter mask, which are found to be in agreement. Nonetheless, there are some discrepancies across the contrast curves which show that there are some limitations of the error model used. Some further steps for improvement of the error modeling include the following:

- cross-correlation between the mask manufacturing specifications and the mask model
- high-resolution microscope imaging of the entire mask and correlation to mask model
- improved modeling of large, localized under-etch or over-etch defects along the petal edge (e.g. a “chink”)
- assessment of the wavefront quality inside the laboratory environment
- physical modeling of the specular and diffuse reflection at the occulter edges

Most of these proposals refer to improving the fidelity of the mask model to account for actual defects of the manufactured occulter mask, to improve upon the uniform error assumptions. Nonetheless, at an order of magnitude performance assessment, the diffractive error analysis works well in predicting the laboratory performance and identifying limiting factors.

5.5 Mask manufacturing improvements

We have presented a diffractive model in §5.2 for evaluating the effect on performance of various experimental limitations. In §5.3 we have performed a sensitivity analysis to
evaluate the effect of each individual error, and in §5.4 we have combined the modeled errors at realistic levels expected in the laboratory in a simulation that successfully predicts the overall performance of the occulter testbed. Further refinements such as cross-correlation with a fully microscope-imaged mask should improve the agreement of the simulation to the experiment.

This analysis has shown that the main limitation on the performance of the occulter mask arise from edge diffraction due to the manufacturing of the occulter mask used in the testbed. Next, we examine possible improvements on the occulter mask manufacturing process and, in particular, the manufacturing specifications, that would improve the measured performance.

In Figure 5.19, we present the straight line spacing along the petal polygons tabulated across all sixteen different petals. We show these as both functions of radius, and as histogram counting the number of edges falling across different edge spacing sizes. In Figure 5.19(a) and 5.19(b), we show the spacing for the occulter mask used to obtain the experimental results in Chapter 4. Figure 5.19(a) shows the edge spacing as a function of the radial distance from the center of the occulter mask. Figure 5.19(b) is a histogram that groups all edges by size. We see that all edges are greater than 5 µm, which is consistent with the simulated errors introduced in the diffractive simulations in §5.4 (obtained by summing δR and the RMS perturbations). It is also evident from the histogram that the minimum edge spacing dominates the petal polygon, but there are edge spacing that can be as high as four times in spacing. The non-uniformity in edge spacing arises from the conversion of uniform radial sampling of the apodization profile – high-slope regions on the apodization profile result in wider edge spacings.

To mitigate the large edge spacing observed in the mask design specification, we have created a mask specification that increases the number of edges fifty-fold as shown in Figure 5.19(c) and 5.19(d) while maintaining uniform radial sampling of the
Figure 5.19: Occulter mask manufacturing edge spacing: current mask with uniform radial spacing (a) edge spacing as radial function (b) edge spacing histogram; increased resolution with uniform radial spacing (c) edge spacing as radial function (d); increased resolution with non-uniform radial spacing (e) edge spacing as radial function (f)
A downside of this sampling scheme is that it maintains a relatively large variation between the smallest and largest edge features. At the small edge feature size end of the spectrum, the sampling approaches the DRIE beam etching width. An alternative sampling scheme is to remove the uniform radial sampling and to sample between intervals as needed to ensure that the largest edge spacing does not exceed 1 µm. The resulting edge spacing using this scheme is shown in Figure 5.19(e) and 5.19(f). Sample inner petal tip openings are shown in Figure 5.20, with the current edge specification shown in Figure 5.20(a) and the resulting edge specification with increased resolution shown in Figure 5.20(b).

We also consider the finite edge thickness. The current mask has a 50 µm edge thickness which may be responsible for some of the bright tips observed in the image plane experimental results. To reduce the effect of the edge thickness, a new multi-step process has been developed at JPL’s Microdevices Lab to reduce the final edge thickness to 1 µm, which should substantially reduce the area from which specular reflection can occur. A first occulter mask using this process was manufactured, and
Figure 5.21: Theoretical PSF comparison showing improved contrast due to better edge features obtained by maintaining all other experimental errors constant.

Results are reported in §4.6 – it is a new control mask that features a circular occulter with the same optimized outer ring. Results show that the optimized outer ring is below the measured diffraction level due to the inner circular control mask. A second mask that features the same optimized design for the inner occulter and the outer ring but prescribed with the high resolution, uniform radial sampling shown in Figure 5.19(c) and 5.19(d) is currently in fabrication and was not available at the time of this writing.

A simulation of an improved mask with similar laboratory errors but with more precisely manufactured errors is is summarized across line 8b. The mean suppression is expected to improve by almost one order of magnitude to $10^{-5.51}$ and the corresponding contrast in the dark annular region is also expected to improve to $10^{-11.98}$. The theoretical effect of improved edges on the PSF contrast is shown in Figure 5.21. This performance improvement will become more important in the context of the next chapter, where we will examine masks scaled at larger distances.
Table 5.5: Summary of sensitivity analysis results using a monochromatic 633 nm laser source. Mean suppression is reported across the entrance aperture. Mean contrast in the dark annular region is reported over the azimuthal median of 16 equally sized wedges from a space equivalent of 400 mas at the inner petal’s outer tips to 637 mas at the outer petal’s inner tips. Error parameters denoted by † are stochastic and performance may vary between simulations. The base case denoted by ‡ in 1a uses a uniform input beam across the pinhole, whereas all other simulations use a Gaussian profile.

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<td>0</td>
<td>100</td>
<td>0</td>
<td>-3.80</td>
<td>-8.40</td>
</tr>
<tr>
<td>6m</td>
<td>Wavefront Aberr.</td>
<td>0.54</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>150</td>
<td>0</td>
<td>-3.28</td>
<td>-7.88</td>
</tr>
<tr>
<td>6n</td>
<td>Wavefront Aberr.</td>
<td>0.54</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>300</td>
<td>0</td>
<td>-2.94</td>
<td>-7.53</td>
</tr>
<tr>
<td>6o</td>
<td>Wavefront Aberr.</td>
<td>0.54</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>500</td>
<td>0</td>
<td>-2.28</td>
<td>-6.91</td>
</tr>
<tr>
<td>6p</td>
<td>Wavefront Aberr.</td>
<td>0.54</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1200</td>
<td>0</td>
<td>-1.28</td>
<td>-5.90</td>
</tr>
<tr>
<td>7a</td>
<td>Mask Tilt</td>
<td>0.54</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>-6.69</td>
<td>-13.11</td>
</tr>
<tr>
<td>7b</td>
<td>Mask Tilt</td>
<td>0.54</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>-6.69</td>
<td>-13.12</td>
</tr>
</tbody>
</table>

continued ...
<table>
<thead>
<tr>
<th>#</th>
<th>Sensitivity Param.</th>
<th>Feature Size $\delta R$ μm</th>
<th>Edge Pert. $\mu$m RMS</th>
<th>Beam Disp. mm</th>
<th>Opt. Ab. $\mu$m RMS</th>
<th>Wave. Ab. $\mu$m RMS</th>
<th>Tilt °</th>
<th>Supp. Log</th>
<th>Contrast Log</th>
</tr>
</thead>
<tbody>
<tr>
<td>7c</td>
<td>Mask Tilt</td>
<td>0.54</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>-6.68</td>
<td>-13.17</td>
</tr>
<tr>
<td>7d</td>
<td>Mask Tilt</td>
<td>0.54</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>-6.61</td>
<td>-13.17</td>
</tr>
<tr>
<td>7e</td>
<td>Mask Tilt</td>
<td>0.54</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>20</td>
<td>-6.48</td>
<td>-12.93</td>
</tr>
<tr>
<td>8a</td>
<td>Combined Errors</td>
<td>2.41</td>
<td>3.10</td>
<td>4.13</td>
<td>160</td>
<td>3.00</td>
<td>5</td>
<td>-4.85</td>
<td>-10.96</td>
</tr>
<tr>
<td>8b</td>
<td>Combined Errors</td>
<td>0.54</td>
<td>0.30</td>
<td>4.13</td>
<td>160</td>
<td>3.00</td>
<td>5</td>
<td>-5.51</td>
<td>-11.98</td>
</tr>
</tbody>
</table>
Chapter 6

Design of An Occulter Experiment at Flight Fresnel Numbers

6.1 Design of a single-distance flight occulter

In the current occulter testbed whose design is described in Chapter 3, the occulter mask that is used is oversized compared to a realistic design that would be flown in space. The necessity to oversize the design arose due to the limited propagation distance available in the occulter testbed described in Chapter 4 – an oversized design means that when scaled to laboratory size the occulter can fill the silicon wafer, and fixed manufacturing errors on the \( \mu \text{m} \) level represent a smaller percentage of that mask.

However, as discussed in Chapter 3, oversized designs are numerically challenging to optimize. Close to an aperture, the output wavefront is the result of the summation of rapidly changing phases across the aperture. As a result increased sampling is required across both the occulter and the pupil planes. The resulting laboratory design, although optimal in apodized form, demonstrates significant performance degradation when petalized. To recover the optimized apodized performance the design requires
Table 6.1: Summary of operational Fresnel numbers for different occulter designs.

<table>
<thead>
<tr>
<th>Design</th>
<th>Occulter Radius</th>
<th>Separation</th>
<th>Wavelength</th>
<th>Fresnel Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>THEIA</td>
<td>20 m</td>
<td>55,000 km</td>
<td>600 nm</td>
<td>12.1</td>
</tr>
<tr>
<td>O3</td>
<td>15 m</td>
<td>21,000 km</td>
<td>600 nm</td>
<td>17.9</td>
</tr>
<tr>
<td>Tech. Demo.</td>
<td>2.5 m</td>
<td>670 km</td>
<td>600 nm</td>
<td>15.5</td>
</tr>
<tr>
<td>Laboratory</td>
<td>188 m</td>
<td>97,000 km</td>
<td>600 nm</td>
<td>607.3</td>
</tr>
<tr>
<td>New Experiment</td>
<td>21.9 m</td>
<td>55,000 km</td>
<td>600 nm</td>
<td>14.5</td>
</tr>
</tbody>
</table>

In excess of 300 petals, an unrealistic number of petals for manufacturing purposes due to very small required gaps. As outlined previously, with 16-petals the theoretical performance of the laboratory mask is $10^{-6.90}$ suppression compared to the designed $10^{-10}$ suppression. We are however limited at a worse level than this because of manufacturing feature sizes and adding more petals would result in smaller dark annular regions and more stray light from the petal edges.

In Table 6.1 we compare the Fresnel number ($\frac{R^2}{\lambda z}$) for the inner occulter mask used in the laboratory with two design reference missions at 600 nm. THEIA [81] is a flagship-class mission designed for a 4-m telescope, and operating at two distances corresponding to a red-band and a blue-band set of wavelengths to reduce the size of the occulter as much as possible, with 600 nm corresponding to the blue-end observations at the far distance. O3 [84] is a probe-class mission designed for a smaller 1.1 m telescope. Its smaller aperture means that spectras are obtained via filtered photometric bands. The technological demonstrator design is discussed in §2.5, and represents a mission featuring a small 5 m occulter flying relatively close-in at 670 km.

As can be observed from Table 6.1, in terms of Fresnel number the laboratory mask is oversized by a factor of 25-50 compared to typical flight occulter designs. Whereas the design limitations described above for the oversized design can potentially be minimized by relaxation of the tolerances on the size of the occulter shadow
and a reduction of the suppression wavelength band, these measures would represent further departures from verification of a realistic flight occulter. Instead, we wish to experimentally verify the optical performance of an occulter design that would operate at a flight Fresnel number.

To design an apodization profile representative of a flight Fresnel occulter, we follow a similar procedure to that described in Chapter 2 following optimal methods introduced in [172]. We start by writing $E_{\text{occ}}(\rho)$ the electric field at distance $z$ downstream from an occulter with apodization profile $A(r)$ as in Equation 2.1.3 combining the Fresnel propagation past an apodized hole with Babinet’s principle for evaluation of the complementary occulter in free space.

Then the linear programming problem listed in Equation 6.1.1 defines the apodization profile $A(r)$ as the decision variable, that is, the output of the optimization problem. We write the desired suppression level of $10^{-10}$ as a specific constraint on the electric field $E_{\text{occ}}(\rho)$, and impose other constraints in the form of a fully opaque disk,
monotonicity, and smoothness:

\[
\min : \int_0^R A(r) r dr
\]

subj. to:

\[-10^{-c} \leq \text{Re}(E_{\text{occ}}(\rho;\lambda)) \leq 10^{-c}, \ \forall \ 0 \leq \rho \leq \rho_{\text{max}}, \ \forall \lambda_{\min} \leq \lambda \leq \lambda_{\text{max}}\]

\[-10^{-c} \leq \text{Im}(E_{\text{occ}}(\rho;\lambda)) \leq 10^{-c}, \ \forall \ 0 \leq \rho \leq \rho_{\text{max}}, \ \forall \lambda_{\min} \leq \lambda \leq \lambda_{\text{max}}\]

\[A(r) = 1, \ 0 \leq r \leq a\]

\[0 \leq A(r) \leq 1, \ \forall \ 0 \leq r \leq R\]

\[A'(r) \leq 0, \ |A''(r)| \leq \sigma, \ \forall \ 0 \leq r \leq R\]  \hspace{1cm} (6.1.1)

Where in the above problem, \(\sigma\) represents the smoothness condition threshold, \(a\) the extent of the opaque central disk, \(2c\) is the suppression performance level sought in the shadow. The formulation is infinite dimensional. We discretize wavelengths in the interval \([\lambda_{\min}, \lambda_{\max}]\), which defines the shadow suppression wavelength band. We also apply midpoint discretization for the radial coordinates, with \(r\) over the range \([0, R]\) and \(\rho\) over the range \([0, \rho_{\text{max}}]\). The upper radial bound \(\rho_{\text{max}}\) on the shadow in the pupil plane defines the optimized dark hole portion of the shadow.

Let the apodization profile output that corresponds to this optimization problem be \(A_{\text{inn}}(r)\) and its maximum radial extent be \(R_{\text{inn}}\). The inner apodization profile \(A_{\text{inn}}(r)\) is shown in Figure 6.1(a). An identical linear programming optimization can be written for obtaining the apodization profile corresponding to the outer ring of the occulter mask, with the only variation being a different maximum radius parameter \(R\) and a larger central disk \(a\), but operating at the same wavelengths and maintaining the suppression constraints over the same shadow portion. Let the output of this second optimization be denoted by \(A_{\text{out}}(r)\) with a maximum radial extent of \(R\); this apodization profile is shown in Figure 6.1(b). In practice several iterations are necessary to tighten the smoothness constraints while maintaining a minimal radius of the occulter mask.
In Table 6.2, we list the parameters used in each of the optimization problems to obtain the corresponding apodization profiles. The wavelength band \([\lambda_{\text{min}}, \lambda_{\text{max}}]\) is evenly discretized in 100 nm intervals to include the end wavelengths. We use midpoint discretization across the occulter radius \(r\) and across the shadow radius \(\rho\). The outer ring is designed to be twice the radius of the inner occulter, and the inner tips of the outer petals are designed to start at four fifths of the radial distance. The suppression constraint is introduced across each component of the electric field and thus setting \(c = 5\) actually corresponds to a shadow intensity suppression \(10^{-10}\).

By comparison to the optimization problem defined in Equation 3.5.2 for the current occulter testbed, we have modified the formulation of the objective function as a minimization of the area of the apodization profile as opposed to suppression minimization and instead introduce the desired suppression level as a constraint. We also do not specifically introduce manufacturing minimum feature size as an explicit constraint and instead evaluate various mask realization with different minimum features in §6.4. A final manufactured mask design may be further optimized by introducing the minimum feature size constraint in conjunction with an appropriate suppression level constraint.

To obtain an apodization profile \(A(r)\) for the laboratory mask featuring both the inner occulter and the outer ring, we combine the apodization profiles \(A_{\text{inn}}(r)\) and \(A_{\text{out}}(r)\) obtained from the two optimization problems described above in the following manner:

\[
A(r) = \begin{cases} 
  bA_{\text{inn}}, & r \leq R_{\text{inn}} \\
  b(1 - A_{\text{out}}), & R_{\text{inn}} < r \leq R \\
  0, & r > R.
\end{cases}
\]

(6.1.2)

In the above, the multiplicative factor \(b\) is added to create uniform area coverage support struts. The resulting combined apodization profile with struts covering 10%
Table 6.2: Summary of optimization parameters for inner and outer occulter shape design for the new testbed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Inner Occulter</th>
<th>Outer Occulter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation distance, ( z )</td>
<td>55,000 km</td>
<td>55,000 km</td>
</tr>
<tr>
<td>Maximum radius, ( R )</td>
<td>21.9 m</td>
<td>43.7 m</td>
</tr>
<tr>
<td>Opaque disk, ( a )</td>
<td>7.0 m</td>
<td>35.0 m</td>
</tr>
<tr>
<td>Opaque fraction, ( a/R )</td>
<td>0.3</td>
<td>0.8</td>
</tr>
<tr>
<td>Smoothness, ( \sigma )</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Lower wavelength, ( \lambda_{\text{min}} )</td>
<td>400 nm</td>
<td>400 nm</td>
</tr>
<tr>
<td>Upper wavelength, ( \lambda_{\text{max}} )</td>
<td>1000 nm</td>
<td>1000 nm</td>
</tr>
<tr>
<td>Shadow radius, ( \rho_{\text{max}} )</td>
<td>2.2 m</td>
<td>2.2 m</td>
</tr>
<tr>
<td>Shadow discretization</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Apodization discretization</td>
<td>3000</td>
<td>3000</td>
</tr>
<tr>
<td>Wavelength discretization</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Suppression constraint, ( 10^{-2c} )</td>
<td>(10^{-10} )</td>
<td>(10^{-10} )</td>
</tr>
</tbody>
</table>

Figure 6.2: (a) Designed apodization profile including outer ring and struts (b) Binary realization of mask profile.

The final line of Table 6.1 shows the Fresnel number of the inner occulter for the new flight design occulter, and its comparison with the other occulter designs. We
see that at the newly designed dimensions, the occulter operates at a Fresnel number very similar to the THEIA design and represents a more realistic, flight design than the oversized design used in the current propagation distance-limited experiment. The THEIA design is radially more compact than the design presented here; this stems primarily from the two-distance nature of THEIA which is designed for a more compact wavelength band.

6.2 Laboratory scaling

The new occulter testbed will be located in the basement of the Chemistry building \(^1\). The floorspace is under review to meet university codes for usage as an experimental space, and the exact final propagation distance is not currently available. Nonetheless, the total propagation distance for the experiment will increase significantly from \(\sim 10\) m to \(\sim 75 \sim 75 - 100\) m. In the analysis that follows, we will assume that a total propagation distance of 100 m is available for the design reference, but we will also investigate the effect of different distances available on the expected performance of the experiment in terms of fixed manufacturing errors.

Figure 6.3 shows an updated layout for the new experimental testbed. In the cur-

\(^1\)An alternate location is the Princeton University Stadium across the empty east-side upper deck
rent testbed, we use a full-size enclosure (i.e., one that is sufficiently large so that it would not clip the expanding beam) to mimic the space environment by eliminating ambient light and diffraction effects from finite edges. Due to the increased propagation distance in the updated testbed, a full-size enclosure connecting the source and the telescope optical table as used in the current testbed represents a significant structural engineering challenge. A more feasible approach is to connect the two enclosures that contain optical tables at either end via a smaller tunnel of diameter $D$, but this requires careful evaluation of the tunnel sizing as discussed in §6.5. The occulter mask would be placed at a distance $h$ from the artificial source and the camera is located at a distance $z$ from the occulter mask. The total propagation distance available is given by $Z = h + z$.

Following the scaling discussion in Chapter 3, we first scale the design from space separation to laboratory separation by maintaining a constant Fresnel number as mathematically shown in §3.2. We designate the propagation distance for the space design as $z_{\text{space}}$. Then we scale to a new propagation distance $z$ corresponding to lab dimensions by introducing a scaling factor $s = \sqrt{z_{\text{space}}/z}$. The new radius of the mask becomes $R' = R/s$. Thus for the case of a direct spatial scaling with a plane wave input beam, the maximal mask size is achieved by increasing the separation distance $z$ available in the laboratory. For a set total distance available distance $Z$, the source would be set to the minimum distance necessary to achieve a collimated beam and this would in turn maximize $z$. The finite size of the input collimated beam can result in significant variation from the idealized infinite-extent planar input beam which can dominate the performance of the diffractive shadow—this effect is the likely limit on the performance of another occulter testbed [137, 138]. Additionally, any collimating optics used introduce wavefront errors that cannot be eliminated. Occulter performance is highly susceptible to phase errors at the occulter plane, as shown by the experiment with the occulter on a glass substrate in §4.4 and in
simulated sensitivity analysis in §5.3.5,

Therefore, to mitigate the diffractive effects related to a collimated input beam, we design the testbed to operate with a diverging input beam. As with the current testbed we scale the design to account for a diverging input beam at the same separation distance as outlined in §3.3 by maintaining equivalent Fresnel numbers. For an occulter with a collimated input beam increasing the separation distance to the camera increases the mask size; however, for a diverging beam, setting the mask at a smaller distance $h$ from the source results in a larger divergence and we must shrink the occulter mask’s radius by a factor $\gamma = \sqrt{1 + z/h}$ such that $R'' = R'/\gamma$ compared to the equivalent collimated mask $R'$ at the same separation distance $z$. We can show mathematically that maximal occulter radius occurs at $z = h$. Consider the scaled occulter’s radius expressed in terms of $h$ for a fixed total available distance $Z$ in relation to its designed radius $R$ and separation in space $z_{\text{space}}$:

\[
R'' = \frac{R'}{\gamma} = \frac{R/\sqrt{z_{\text{space}}}}{\sqrt{1 + z/h}} = \frac{R}{\sqrt{z_{\text{space}}Z}} \sqrt{Z - h\sqrt{h}} \tag{6.2.1}
\]

To find the maximum we set the derivative with respect to $h$ to zero:

\[
0 = \frac{dR''}{dh} = -\frac{\sqrt{h}}{2\sqrt{Z-h}} + \frac{Z-h}{2\sqrt{h}}
\]

\[
\therefore h = Z/2 \tag{6.2.2}
\]

And we note that $\frac{d^2R''}{dh^2} < 0$, ∀ $0 < h < Z$, thus ensuring $h = Z/2$ is indeed a maximum. This can be verified in Figure 6.4 by plotting the inner scaled occulter’s radius as a function of separation $z$ for a fixed total distance $Z = 100$ m. The
maximum occurs when $z = h = 50$ m as expected.

In Table 6.3, we summarize the scaling of the occulter parameters. We first show the parameters of the designed occulter at space dimensions then scale these parameters to the laboratory. We assume a total distance of 100 m is available and optimally set the divergence to maximize the size of the occulter mask, which assumes a 50 m distance from the source to the mask and a 50 m propagation distance past the mask. The intermediate scaling for the collimated case assumes a 50 m distance scaling for comparison purposes. In the final column, we show the resulting scaled parameters after application of the beam divergence. Scaling for a diverging beam results in shrinking the occulter mask while increasing the size of the shadow. Note, that although the shadow diameter increases with the diverging beam scaling, we do not increase the diameter of the telescope aperture.

In the remainder of this chapter, we will compare the expected performance of the idealized design similar to the analysis in Chapter 3, to the expected performance in the presence of various experimental limitations, some of which were previously
Table 6.3: Summary of scaled laboratory parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Space Design</th>
<th>Collimated Scale</th>
<th>Diverging Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation distance, ( z )</td>
<td>55,000 km</td>
<td>50 m</td>
<td>50 m</td>
</tr>
<tr>
<td>Distance scale, ( s )</td>
<td>1</td>
<td>1050</td>
<td>1050</td>
</tr>
<tr>
<td>Source distance, ( h )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>50 m</td>
</tr>
<tr>
<td>Divergence scale, ( \gamma )</td>
<td>1</td>
<td>1</td>
<td>1.4</td>
</tr>
<tr>
<td>Inner radius, ( R_{\text{inn}} )</td>
<td>21.9 m</td>
<td>20.8 mm</td>
<td>14.7 mm</td>
</tr>
<tr>
<td>Outer radius, ( R_{\text{out}} )</td>
<td>43.7 m</td>
<td>41.7 mm</td>
<td>29.5 mm</td>
</tr>
<tr>
<td>Dark shadow radius, ( \rho_{\text{dark}} )</td>
<td>2.2 m</td>
<td>2.1 mm</td>
<td>3.0 mm</td>
</tr>
<tr>
<td>Outer shadow radius, ( \rho_{\text{out}} )</td>
<td>43.7 m</td>
<td>41.7 mm</td>
<td>57.8 mm</td>
</tr>
<tr>
<td>Telescope diameter</td>
<td>4 m</td>
<td>3.8 mm</td>
<td>3.8 mm</td>
</tr>
<tr>
<td>Inner Working Angle</td>
<td>82.1 mas</td>
<td>85.8 as</td>
<td>60.6 as</td>
</tr>
<tr>
<td>Outer Working Angle</td>
<td>131 mas</td>
<td>137 as</td>
<td>97.4 as</td>
</tr>
</tbody>
</table>

modeled in the sensitivity analysis in Chapter 5. We will quantitatively assess the experimental effect of the following limitations:

- diffractive effect of outer ring and comparison with hard-edged limits;
- limitations due to a collimated input beam;
- performance limits due to mask manufacturing feature accuracy;
- diffractive effect of finite-sized tunnel upstream from the mask (input tunnel);
- diffractive effect of finite-sized tunnel downstream from the mask (exit tunnel).

6.3 Ideal performance

In this section, we consider the performance of the optimized occulter profiles with perfectly symmetrical realizations. For this purpose, we use propagations described earlier in Chapters 2 and 3, using the Jacobi-Anger expansion to obtain an azimuthal distribution for a 16-petal occulter.
We will consider the performance of the inner occulter at designed space dimensions and compare this with the performance at laboratory dimensions. Then we will consider the performance of the corresponding full occulter mask at laboratory dimensions showing the effect on performance through the introduction of the outer ring. Finally, we will assess the performance of the laboratory mask under scaled conditions corresponding to a diverging input beam. It is shown that the suppression performance at the pupil plane is identical under distance and beam divergence scaling, however there are differences in the corresponding point spread functions. Additionally, for the ideal laboratory mask we will assess the chromatic performance at monochromatic wavelengths including those between the optimized wavelengths, and use these results to determine the performance when operating with a broadband input beam.

6.3.1 Inner occulter

We begin by verifying the performance of the inner occulter as designed at space dimensions for an ideal, planar input wave and compare this result with the corresponding performance with distance scaling at laboratory dimensions. In §6.1 we have described the linear program and optimization parameters used to design the space occulter apodization profile. To enable efficient numerical optimization, we use only the first Bessel term in the Jacobi-Anger expansion which describes the circularly symmetric shadow at discrete wavelengths and radial points in the shadow. We wish to verify that the occulter shadow is well-behaved when additional Bessel terms are added, and at wavelengths and points in the shadow that were not specifically optimized.

To evaluate the electric field downstream from the inner occulter, we use the Babinet version of the one-dimensional Fresnel diffraction integral including the Jacobi-Anger expansion in Equation 2.2.5 with the series summation of the first twenty
non-zero Bessel terms with sixteen petals (setting $K = 20, N = 16$). We use the apodization profile associated with the inner occulter shown in Figure 6.1(a) sampled with 3000 radial midpoints, generated using the optimization parameters listed in Table 6.2. The space physical dimensions for the optical propagation are listed for the inner occulter in the space design column in Table 6.3. Additionally the wavelength is set at 633 nm to match the monochromatic HeNe laser used for the current experimental laboratory results reported in Chapter 4, and this wavelength lies between the closest optimized wavelengths at 600 nm and 700 nm. The resulting mean suppression over a 4-m telescope centered in the occulter shadow is $10^{-9.92}$, which is only slightly under the $10^{-10}$ suppression constraint imposed in the optimization which represents good performance while being more densely sampled in the shadow and at a different wavelength than the numerical optimization. We also obtain the corresponding point spread function by propagating to the image plane for this telescope aperture using Equation 2.3.2.

Next, we use the same equations and apodization profile but applying the distance scaling for the inner occulter physical dimensions listed under the collimated laboratory case indicated in Table 6.3, and at the same wavelength. The simulation results are shown in Figure 6.5. The suppression-calibrated shadow intensity at the pupil plane is shown in Figure 6.5(a) at laboratory dimensions and the contrast-calibrated point spread function at the image plane is shown in Figure 6.5(b) at the space-equivalent angular separations. To compare the laboratory-dimension simulation with the space simulation we plot azimuthal average curves for the suppression in Figure 6.5(c) and for the contrast in Figure 6.5(d). The suppression curves are identical as expected from the mathematical scaling, with equal mean suppression across the 4 m-equivalent aperture of $10^{-9.92}$. The laboratory case was plotted with markers which were decimated five-fold for display purposes only. The contrast curves are very similar with the laboratory results exhibiting a deeper valley – the mean con-
Figure 6.5: Comparison of performance for inner occulter at designed space dimensions and scaled laboratory dimensions: (a) Suppression at the laboratory pupil plane. (b) Contrast at the laboratory image plane. (c) Azimuthal average across pupil plane for space dimensions with comparison to five-fold decimated (for display purposes only) laboratory case with normalized radial distance across the shadow. Red line indicates extent of telescope aperture. Mean suppression over the telescope aperture for both space and lab cases is $10^{-9.92}$. (d) Azimuthal average across image plane with comparison to space. Solid red line indicates angular extent of outer petal tips and represents the occulter’s inner working angle. Dashed red line indicates where the outer working angle would lie in the presence of the outer ring. Mean contrast in this region of the image plane is $10^{-12.26}$, equal to four significant digits for both cases.
trast across the expected annular working angle region between the outer tips of the inner occulter’s petals and the intended location inner tips of the outer ring’s petals is equal to four significant digits at $10^{-12.26}$.

Thus, we have shown through simulation that the ideal monochromatic performance of a sixteen petal occulter mask is close to its prescribed optimized constraint. Additionally, we have shown that the shadow intensity is identical when the separation distance is scaled from space to laboratory, although the leading term outside the diffraction integral has phase variation that results in small differences in the point spread function.

### 6.3.2 Optimized outer ring

In the previous section, we have verified the performance of the inner occulter when scaled from space dimensions to laboratory dimensions. We now verify the performance of the full occulter mask including the outer ring. To enable comparison with the inner occulter only and thus directly assess the effect of adding the outer ring, we maintain a planar input beam.

To compute the electric field downstream from an occulter mask with an outer ring, we use the Fresnel diffraction integral in Equation 3.1.15 and set the input term to unity. This equation is different from the propagation for the inner occulter only in that it does not make use of the Babinet principle – this is because the radial extent of the occulter mask with an outer ring is finite. We use the laboratory mask apodization profile that combines the inner occulter and the outer ring shown in Figure 6.2(a), with the optimization parameters of the outer ring listed in Table 6.2. The laboratory physical dimensions for the full laboratory mask can be found in Table 6.3 with the scaling applicable for the collimated beam input.

In Figure 6.6, we compare the simulation results using the full mask at laboratory dimensions with those for the inner occulter only which were previously shown in Fig-
Figure 6.6: Effect of outer ring on performance by comparison of azimuthal average suppression and contrast for an inner occulter mask only and a full laboratory mask at laboratory physical dimensions: (a) Azimuthal average across pupil plane with comparison to inner occulter only. Red line indicates telescope aperture extent. (b) Azimuthal average across image plane with comparison to inner occulter only. Red lines indicate angular extent of outer petal tips of inner occulter and inner petal tips of outer ring and define the occulter mask’s inner and outer working angles.

Figure 6.5. We show the azimuthal average curves only as differences are relatively small and difficult to spot in the full two-dimensional shadow and point spread function images. The suppression curves are identical outside the dark hole, but as the suppression level approaches the designed level we see that the mask with the outer ring exhibits a slight worsening of the suppression – the suppression level of $10^{-9.92}$ with the inner occulter only rises to $10^{-9.70}$ with the outer ring added. This is correlated with the contrast curves, where we see that the effect of the ring is to raise contrast slightly at the outer working angle where the outer ring is located. The resulting mean contrast across the working angle region changes from $10^{-12.26}$ to $10^{-11.64}$.

From the analysis above we conclude that the introduction of the outer ring leads to a slight worsening of the ideal performance with only the inner occulter. Figure 6.7 demonstrates the positive effect of using the outer ring to mitigate diffraction effects. For this purpose, we compare the performance of the full occulter mask featuring the outer ring with an occulter mask featuring the same optimized inner occulter shape.
Figure 6.7: Effect of contrast performance for optimized outer ring with a hard circular outer edge: (a) Apodization profile comparison for the optimized mask with an outer ring and a mask with a hard circular outer edge. Red lines indicate radial extent of outer petal tips of inner occulter and inner petal tips of outer ring and define the occulter mask’s inner and outer working angles. (b) Azimuthal average across image plane comparison between optimized outer ring and the hard outer edge. Red lines indicate corresponding annular working region on the point spread function. Mean contrast across the annular working region with the hard circular outer edge is $10^{-4.29}$ compared to $10^{-11.64}$ for the optimized ring.

but using a hard circular outer edge instead of the outer ring. The corresponding apodization profiles for these two different masks is shown in Figure 6.7(a). The azimuthally averaged contrast curves are compared in Figure 6.7(b). We can see clearly that the optimized outer ring significantly improves the contrast performance by several orders of magnitude; by comparison, a hard circular edge is clearly a limiting factor. The mean contrast across the annular working region degrades from $10^{-11.64}$ to $10^{-4.29}$.

The ideal performance is limited by the introduction of the outer ring when compared to an idealized free-floating occulter. However, the introduction of the optimized outer ring provides an important mitigation of the diffraction effects from a hard circular edge and therefore provides an acceptable method for laboratory mounting of the inner occulter.
6.3.3 Diverging beam input

To eliminate wavefront errors associated with collimating optics, our laboratory design uses a diverging input beam which can be spatially filtered. In Chapter 3, we showed mathematically how the shadow can be scaled in the presence of a diverging input beam to maintain an identical intensity pattern. We verify this relationship numerically in this section by applying a diverging input beam and comparing both the resulting pupil and image planes with the collimated case presented in §6.3.2.

To simulate the laboratory setup for a diverging scaling, we use the same Fresnel diffraction integral we used in §6.3.2 for the full occulter mask in Equation 3.1.15. The mask apodization profile with the optimized outer ring in Figure 6.7(a) is identical except a shrinking of the radial axis by a factor of \(\sqrt{2}\) to match the laboratory physical dimensions in Table 6.3. We use the diverging input term in Equation 3.6.4 to account for the diverging input beam.

We summarize the simulation results in Figure 6.8. The suppression-calibrated shadow intensity at the pupil plane is shown in Figure 6.8(a), and the contrast-calibrated point spread function at the image plane is shown in Figure 6.8(b). The pupil plane indicates that the shadow increases in size commensurate with the divergence factor as expected. However, as the shadow increases in size one scaling difficulty that arises is in appropriately scaling the size of the telescope aperture. In Table 6.3, the diameter of the telescope aperture is maintained fixed while the size of the shadow increases to maintain the same relative diffractive aperture performance as expected in space. This will result, however, in a small variation in performance as the same shadow extent is no longer captured by the aperture – in particular, the residual Poisson spot at the center of the shadow increases in size and we expect a slight performance decrease as a result. We will therefore compare both the fixed aperture and an aperture scaled with the shadow to illustrate this effect. For the aperture scaled with the shadow divergence, we expect an increase in performance as
Figure 6.8: Effect on performance of ideal mask under diverging input beam laboratory conditions: (a) Suppression at the laboratory pupil plane for diverging input beam (b) Contrast at the laboratory image plane for diverging input beam maintaining a fixed aperture diameter (c) Azimuthal average suppression across pupil plane with comparison between diverging case to collimated scaling using a normalized radial distance across the shadow (with unity at the tip of the outer ring). The solid red line indicates the extent of the aperture for the diverging beam when maintaining a fixed diameter to the collimated case, with a mean suppression across this aperture of $10^{-9.79}$. The dashed red line indicates the extent of the aperture when scaled with the shadow divergence, with a mean suppression across this aperture of $10^{-9.82}$. (d) Azimuthal average contrast across image plane with comparison between diverging input beam with collimated beam. Two diverging cases are considered: Diverging1 maintains a fixed aperture while Diverging2 scales the aperture with the beam divergence. Solid red lines indicate the annular working region. Mean contrast over the annular region for the two respective diverging cases is $10^{-11.37}$ and $10^{-11.73}$ compared to $10^{-11.64}$ for the collimated case.
the resolving power of the telescope is improved relative to the base collimated case. The increase in residual resolving power is expected to be smaller at the proposed laboratory dimensions than is the case in the current laboratory testbed described in Chapter 4 which features a larger divergence factor.

For a more quantitative summary of the two-dimensional results shown, we obtain azimuthally averaged suppression curves in Figure 6.8(c) and corresponding contrast curves in Figure 6.8(d). In Figure 6.8(c) the suppression curves are normalized across the radial distance to the outer extent of the shadow to enable comparison of two physically different cases with the collimated and diverging input beams. We see that these two curves are identical, demonstrating the validity of divergence scaling. Also indicated in the suppression curve is the radial extent of the fixed and scaled apertures discussed above. The mean suppression across the fixed aperture, denoted by the solid red line, is $10^{-9.79}$, whereas the mean suppression for the scaled aperture denoted by the dashed red line is $10^{-9.82}$. The difference is small, but as expected for the fixed aperture the residual Poisson spot results in a very slight decrease in suppression performance. The scaled aperture performance is identical to the suppression quoted for the collimated case in §6.3.2. In Figure 6.8(d) we compare three contrast curves, one corresponding to the collimated case and two corresponding to the diverging case with the differently scaled telescope apertures. We note that the contrast performance, unlike the suppression performance, is not identical between the diverging and collimated cases. The mean contrast for the fixed aperture is $10^{-11.37}$ which represents a worse contrast performance matching the suppression performance trend due to the increased relative size of the residual Poisson spot. When the aperture diameter is increased from 3.8 mm to 5.4 mm with the shadow divergence, the additional resolving power of the telescope results in a mean contrast over the annular working region of $10^{-11.73}$ which is a better contrast level than $10^{-11.64}$ for the collimated case. In the remainder of this chapter, we have conservatively chosen
to maintain a fixed aperture when scaling laboratory dimensions for the diverging beam. We do note that the curve shape for the collimated case is very similar when the aperture is scaled with the shadow with a better performance due to a tightening of the diffraction wings associated with a larger aperture. Consequently, one option to use the scaled aperture would be to apply a calibration factor that accounts for the better contrast level arising due to the larger aperture to predict the corresponding collimated aperture contrast performance. We can also reduce the resolution in the resulting image to match the unscaled telescope.

Furthermore, although not shown here, we note that when a parabolic approximation is made to the diverging input beam in Equation 3.6.6 the result is identical for both the suppression and contrast curves further verifying the validity of the approximation in §3.3 for the laboratory dimensions considered.

### 6.3.4 Broadband beam input

One of the main advantages of occulters for high-contrast imaging is the relative ease with which they can be optimized to operate over a large wavelength band. As outlined in the Table 6.2, the occulter shadow for this mask design was optimized at 100 nm intervals over the band 400-1000 nm. It is important to numerically validate the performance of the occulter mask at wavelengths between those for which the occulter mask was specifically optimized to determine the suitability of the design for a broadband input and to obtain an estimate of its performance against which experimental results can be compared.

To perform numerical validation of the ideal mask performance for a broadband input, we use the same propagations as described in §6.3.3 when a diverging input beam is applied. We only vary the wavelength in 10 nm intervals over the entire wavelength range for which the occulter was designed to provide optimal suppression.

In Figure 6.9, we show azimuthally averaged suppression curves demonstrating
Figure 6.9: Theoretical suppression performance across broadband wavelengths: (a) Azimuthal average suppression across pupil plane at 50 nm intervals with vertical red line denoting edge of aperture (b) Summary of broadband performance with computed suppression in 10 nm intervals denoted by open circles and optimized wavelengths denoted by asterisk.
Figure 6.10: Theoretical contrast performance across broadband wavelengths: (a) Azimuthal average contrast across pupil plane at 50 nm intervals with vertical red lines denoting annular working region (b) Summary of broadband performance with computed contrast in 10 nm intervals denoted by open circles and optimized wavelengths denoted by asterisk.
the broadband performance of the laboratory mask in the pupil plane. In Figure 6.9(a), suppression curves are plotted at 50 nm intervals which includes the optimized wavelengths and also non-optimized wavelengths lying evenly between the optimized wavelengths. Mean suppression is computed over the telescope aperture whose radial extent is indicated by the solid red line. In Figure 6.9(b), we summarize the mean suppression over the aperture for all the computed wavelengths. A piecewise spline linear interpolation is computed to estimate suppression at wavelengths between the computed cases, and the optimized wavelengths are clearly indicated with asterisks. We notice in general that performance is better for the optimized wavelengths, and particular that for the blue-end wavelengths between 400-500 nm the performance degrades most quickly.

In Figure 6.10, we show azimuthally averaged contrast curves demonstrating the corresponding broadband performance of the laboratory mask in the image plane. Likewise, we show contrast curves at 50 nm intervals in Figure 6.9(a). The mean performance over the annular working region indicated by solid red lines at 10 nm intervals is summarized in Figure 6.10(b). We denote the optimized wavelengths and also compute an estimate of suppression for intermediate wavelengths via interpolation. As with the suppression performance, optimized wavelengths tend to perform better than intermediate wavelengths, with the worst performance degradation occurring at the blue-end.

In Table 6.4, we summarize the expected broadband performance for light. We compute the mean suppression over the individual optimized wavelengths, over the computed 10 nm interval wavelengths, and for the interpolated curves. We find that the mean performance, in particular the suppression, over the 400-1000 nm range over which the occulter was designed is dominated by the degradation in performance at the blue end. All optimized wavelengths satisfy the $10^{-10}$ suppression constraint when the petals are added, including 400 and 500 nm. If we exclude all wavelengths under
Table 6.4: Mean suppression and contrast performance for broadband input beam over the optimized wavelengths only, all computed wavelengths at 10 nm intervals, and the interpolated curves. Filters denoted by † and their wavelengths correspond to settings on the tunable SELECT broadband filter on the Super-K continuum laser.

<table>
<thead>
<tr>
<th>Broadband Input Wavelengths</th>
<th>Broadband Input Filter</th>
<th>Suppression, log</th>
<th>Contrast, log</th>
</tr>
</thead>
<tbody>
<tr>
<td>400-1000 nm</td>
<td>-10.30</td>
<td>-9.24</td>
<td>-9.24</td>
</tr>
<tr>
<td>500-900 nm VIS-nIR†</td>
<td>-10.29</td>
<td>-9.77</td>
<td>-9.76</td>
</tr>
<tr>
<td>450-670 nm VIS(4x)†</td>
<td>-10.29</td>
<td>-8.92</td>
<td>-9.09</td>
</tr>
</tbody>
</table>

500 nm, the performance metrics improve and the expected broadband performance is in much better agreement with the performance over the optimized wavelengths.

We also provide results for two possible bands on the tunable SELECT broadband filter [128] that could be used for the experiment with the Super-K continuum laser assuming even transmission across all wavelengths. The same Super-K continuum laser has been used for broadband testing in the coronagraph laboratory [67].

### 6.4 Manufacturing tolerances

In §6.3, the simulated performance of the occulter mask assumed a perfect realization using sixteen petals. We have numerically verified each step of the scaling process from space dimensions to laboratory dimensions. We will use the results in §6.3.3 using a diverging monochromatic 633 nm input beam on the full laboratory mask with the outer ring and define this as the baseline, ideal performance. We have found in experimental results presented in Chapter 4 from our current oversized testbed that performance is limited by edge diffraction. Subsequently, in the sensitivity analysis of Chapter 5, we have presented a two-dimensional model that allows simulation of edge diffraction effects that represent departures from the idealized, sixteen-fold
realization. In this section, we will apply the model developed in §5.3 to assess an optimistic lower bound on the performance loss that can be expected when edge diffraction is taken into account. We use these results to compute an estimate of the performance variation that can be achieved at different propagation distances. Finally, we perform a similar analysis to obtain a more conservative upper bound estimate when random edge perturbations are added to the mask model. We also compare broadband performance with the ideal simulations in §6.3.4 for different feature sizes.

6.4.1 Feature size accuracy

In Figure 6.11, we briefly summarize the two-dimensional optical model introduced in Chapter 5. P0 is the input plane which spans the artificial star pinhole, P1 is the occulter plane separated by a distance \( h \), P2 is the pupil plane which spans the shadow at a distance \( Z \), and finally P3 is the image plane with the focus of the imaging lens set to the source plane at P0. Propagations between these planes are described Equations 5.2.2, 5.2.3, and 5.2.4 respectively. At the input plane P0 we use the same 15 µm pinhole, but because of the larger separation between P0 and the occulter
Table 6.5: Suppression and contrast performance for different manufacturing feature accuracies for a monochromatic diverging input beam at 633 nm.

<table>
<thead>
<tr>
<th>Lab Feature Size $\delta R$, µm</th>
<th>Mask Model $n \times n$</th>
<th>Space Scale Feature $g \times g$ mm</th>
<th>Suppression Log</th>
<th>Contrast Log</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.8</td>
<td>6000</td>
<td>1</td>
<td>-5.36</td>
<td>-8.04</td>
</tr>
<tr>
<td>7.4</td>
<td>4000</td>
<td>2</td>
<td>-5.52</td>
<td>-8.44</td>
</tr>
<tr>
<td>4.9</td>
<td>6000</td>
<td>2</td>
<td>-5.89</td>
<td>-9.06</td>
</tr>
<tr>
<td>2.0</td>
<td>6000</td>
<td>5</td>
<td>-6.97</td>
<td>-9.83</td>
</tr>
<tr>
<td>1.4</td>
<td>6000</td>
<td>7</td>
<td>-7.90</td>
<td>-10.4</td>
</tr>
<tr>
<td>1.0</td>
<td>6000</td>
<td>10</td>
<td>-8.44</td>
<td>-10.8</td>
</tr>
<tr>
<td>0.5</td>
<td>12000</td>
<td>10</td>
<td>-9.27</td>
<td>-11.2</td>
</tr>
<tr>
<td>0.3</td>
<td>6000</td>
<td>40</td>
<td>-9.49</td>
<td>-11.3</td>
</tr>
<tr>
<td>0.1</td>
<td>6000</td>
<td>100</td>
<td>-9.77</td>
<td>-11.4</td>
</tr>
<tr>
<td>Ideal</td>
<td>N/A</td>
<td>N/A</td>
<td>-9.82</td>
<td>-11.4</td>
</tr>
</tbody>
</table>

plane P1 envisioned for the new testbed than for current testbed the pinhole size can be increased and the amplitude profile at the occulter plane could be relatively more uniform. Additionally, the plane separations are set $h = Z$ to maximize the occulter mask size at P1 for the total available distance.

We use a two-dimensional mask model across P1. The mask model has $n \times n$ samples, and it is allowed to take on gray values at the petal edges. The gray value is computed by $g \times g$ anti-aliasing. Therefore, a mask model with $(n, g)$ and with radius $R$ will therefore be representative of the resulting feature size given by $\delta R = 2R/n/g$.

In Table 6.5, we have listed several mask models and their feature sizes both at laboratory dimensions and at space scale. Using the two dimensional Fresnel propagation equations, we compute the suppression at the pupil plane across the telescope aperture and the corresponding contrast in the annular working region for the different manufacturing feature accuracies. The final listing of the table reproduces the corresponding ideal performance from §6.3.3 assuming no manufacturing errors. We see monotonic worsening of performance from the ideal case as the feature size is increased.
Figure 6.12: Performance of laboratory mask using different mask manufacturing feature accuracies (a) Suppression at the laboratory pupil plane for 0.25 \( \mu \)m features (b) Contrast at the laboratory image plane for 0.25 \( \mu \)m features (c) Suppression at the laboratory pupil plane for 1.96 \( \mu \)m features (d) Contrast at the laboratory image plane for 1.96 \( \mu \)m features (e) Suppression at the laboratory pupil plane for 4.91 \( \mu \)m features (f) Contrast at the laboratory image plane for 4.91 \( \mu \)m features.
Figure 6.13: Performance comparison laboratory mask using different mask manufacturing feature accuracies. Solid red line indicates extent of telescope aperture across which mean suppression is reported. (a) Azimuthally averaged suppression performance (b) Azimuthally averaged contrast performance. Solid red lines indicate annular working region across which mean contrast is reported.
To illustrate the summarized results in Table 6.5, in Figure 6.12 we show the two-dimensional suppression-calibrated pupil and corresponding contrast-calibrated image planes at three representative feature sizes. In Figure 6.12(a) and (b) the results are shown for the 0.25 µm model. This model represents performance close to the ideal but with some visible difference. In the pupil plane, differences are negligible and difficult to spot visually although in the image plane small diffraction features can be seen at the outer ring which are not seen in the ideal case — compare with Figure 6.8(a) and (b). The feature size is increased to 2.0 µm, and we see in the pupil plane an increase in the intensity of the electric field across the shadow’s optimized dark hole in Figure 6.12(c), and a correspondingly worse contrast level in the image plane in 6.12(d) with a prominent enlargement and level increase in the central diffraction lobe with residual features across the dark annular region. Finally, we show the 4.9 µm feature size result and note that the dark hole across the pupil plane in Figure 6.12(e) has disappeared almost entirely, whereas in the image plane in Figure 6.12(f) the intensity of the central lobe has increased while its wings now completely cover the dark annular region.

For a quantitative comparison, we also provide azimuthally averaged suppression and contrast curves in Figure 6.13(a) and (b) respectively. All the cases featured in the two-dimensional plots above are featured and compared to the ideal case. Additionally, we also plot the cases of 0.5 µm and 1.0 µm feature sizes. The half-micron feature size is the smallest achievable resolution in the mask manufacturing process and is therefore likely the indicator for best possible performance. It is possible however that at half-micron features scalar diffraction is no longer fully applicable as the wavelength of light is longer than the feature size. The micron feature size case is instead longer than all wavelengths for which an occulter would be operated at. The 0.3 µm model matches very closely the ideal case which demonstrates the validity of the two-dimensional mask model. As the feature size is increased the performance
for both suppression and contrast predictably worsens.

The most computationally expensive part of this numerical validation process is the creation of the mask models. Because the suppression and contrast performance worsens monotonically with increasing feature sizes we compute estimates of the performance at other feature sizes by applying a piecewise spline interpolation between the computed suppression and contrast results. We can use these interpolated results to obtain an estimate of the performance of the occulter experiment as the total propagation distance is modified resulting in size changes of the occulter mask while manufacturing feature size is maintained fixed. The relative size of the fixed feature sizes decreases as the total propagation distance and corresponding occulter mask radius increases, and, conversely the relative size increases as the mask shrinks when a smaller propagation distance is available.

This process is applied for the suppression performance in Figure 6.14. In Figure 6.14(a), the computed mean suppression results across the telescope aperture are denoted by open circles corresponding to the mask models listed in Table 6.5. The mask models are give us some computed feature sizes and we apply an interpolation to obtain the suppression curve at intermediate feature sizes. Based on the suppression curve as a function of feature size, we estimate the effect of changing the total propagation distance in Figure 6.14(b) while maintaining a fixed manufacturing feature size. The four chosen feature sizes at 100 m total propagation distance correspond to points on the suppression curve in Figure 6.14(a). At other total propagation distances a corresponding smaller or larger feature size on the suppression curve in Figure 6.14(a) is chosen depending on the relative size of the fixed manufacturing error. We notice that the performance is first limited by the manufacturing feature size, and as the total distance increases the expected suppression performance is improved. For example, for a 1 µm feature size at 10 m, the expected performance for this mask is $10^{-6.32}$, which improves for the same feature size to $10^{-8.42}$ if the experiment is
Figure 6.14: Suppression performance from computed manufacturing feature accuracy: (a) Computed suppression at different manufacturing feature size and interpolation for other feature sizes (b) Predicted suppression performance as a function of total propagation distance available with fixed manufacturing feature sizes.
Figure 6.15: Contrast performance from computed manufacturing feature accuracy:
(a) Computed contrast at different manufacturing feature size and interpolation for other feature sizes
(b) Predicted contrast performance as a function of total propagation distance available with fixed manufacturing feature sizes.
In Figure 6.15, we show the corresponding results for the contrast performance. In particular, in Figure 6.15(a) we show the computed mean contrast across the dark annular working region for the mask models in Table 6.5 and show the interpolated contrast curve for intermediate feature sizes. Based on this contrast curve, in Figure 6.15(b) we compute the expected contrast performance as the laboratory mask is rescaled for different total propagation distance. We use the same fixed manufacturing feature sizes as with the suppression curves. For comparison, for a 1 µm at 10 m total propagation distance, the expected performance is $10^{-9.42}$. This contrast performance improves for the same feature size to $10^{-10.83}$ when the mask is rescaled for 100 m total propagation distance.

For all the propagation distances and realistic manufacturing feature sizes considered, the suppression performance is limited by the feature accuracy at more than half an order of magnitude from the ideal case. In Figure 6.16, we therefore consider large-scale sizing of the occulter similar to desert field tests that have been conducted at 2 km separation in the desert [61, 62] or proposed with an airship at 20+ km separations [70] to determine the possible performance improvements that could be achieved at these scales. The radial size of the occulter scaled at km separations is shown in Figure 6.16(a). The scaled suppression performance at these separations is shown for high-resolution feature sizes in Figure 6.16(b), and indeed the expected performance if manufacturing feature sizes can be kept constant better approaches the ideal case. If 0.5 µm can be maintained over a $\approx 100$ mm mask diameter, $\approx 2.5$ km scaling would be sufficient to achieve near ideal performance. However, experimental limitations such as some that are discussed in §6.5 will similarly scale with experiment size and it will be difficult to maintain feature-size limited performance.

Finally, we consider the performance of the feature-size limited laboratory models operating at the expected 100 m total propagation distance when the input light
Figure 6.16: Large-scale occulter sizing and suppression performance (a) Radius of inner occulter sized under same diverging beam scaling with large-scale distance available (b) Expected suppression performance for large-scale occulter and comparison to ideal and model limits.
is broadband. This is interesting to consider not only because an estimate of the broadband performance is the most representative metric for a space mission, but also because the variation between the ideal performance for the optimized wavelengths was better than for most intermediate, non-optimized wavelengths in §6.3.4 by about half an order of magnitude on average for both suppression and contrast. Therefore, it is interesting to determine whether this same difference holds when the performance degrades with feature sizes or whether the optimized wavelength performance will degrade additionally at a similar performance level as the intermediate wavelength. Additionally, the blue-end wavelengths featured significant performance degradation for the computed performance level even above the level with manufacturing errors included for the 633 nm monochromatic input. In this case, it is of interest how the performance metrics compare to the worse ideal level at the blue-end wavelengths when feature sizes are introduced in the model.

We simulate the performance levels as functions of wavelengths with the results shown in Figure 6.17. We consider several different feature size models (0.5 µm, 1.0 µm, 2.0 µm) and compare these with the ideal performance curves previously shown in Figure 6.9 and Figure 6.10.

In Figure 6.17(a), we plot the mean suppression across the dark hole as a function of wavelength. The 0.5 µm shows the optimized wavelengths and also the computed wavelengths in 10 nm intervals, which are the same for the other cases considered, and the spline interpolation describing the broadband performance. We see from comparison of the 0.5 µm and 1.0 µm curves with the ideal curve that the suppression performance tends to level off and thus there is a greater performance degradation for the optimized wavelengths. Conversely, the performance degradation is minimal at the blue-end wavelengths compared to the ideal where the performance level is above the limit due to the feature-size. Finally, we see for the 2.0 µm curve, when the feature size dominates all wavelengths, the suppression level is flat across the entire
Figure 6.17: Broadband performance at several feature sizes. Optimized and computed wavelengths are indicated for the 0.5 \( \mu m \) interpolated curve and are identical for the other cases shown (a) Suppression performance as a function of wavelength (b) Contrast performance as a function of wavelength.
Table 6.6: Mean suppression and contrast performance for broadband input beam with comparison between the ideal and several manufacturing feature sizes. Suppression and contrast are reported for the optimized wavelengths only, for all computed wavelengths, and for all wavelengths including interpolated wavelengths. The ideal results were previously reported in §6.3.4.

<table>
<thead>
<tr>
<th>Broadband Input Wavelengths</th>
<th>Features</th>
<th>Suppression, log</th>
<th>Contrast, log</th>
</tr>
</thead>
<tbody>
<tr>
<td>400-1000 nm Ideal</td>
<td>-10.30</td>
<td>-9.24</td>
<td>-9.24</td>
</tr>
<tr>
<td>400-1000 nm 0.5 µm</td>
<td>-9.22</td>
<td>-8.97</td>
<td>-8.97</td>
</tr>
<tr>
<td>400-1000 nm 1.0 µm</td>
<td>-8.74</td>
<td>-8.63</td>
<td>-8.63</td>
</tr>
<tr>
<td>400-1000 nm 2.0 µm</td>
<td>-7.10</td>
<td>-7.08</td>
<td>-7.08</td>
</tr>
<tr>
<td>500-1000 nm 1.0 µm</td>
<td>-8.78</td>
<td>-8.76</td>
<td>-8.77</td>
</tr>
<tr>
<td>500-1000 nm 2.0 µm</td>
<td>-7.07</td>
<td>-7.06</td>
<td>-7.06</td>
</tr>
</tbody>
</table>

spectrum. For a more quantitative comparison, we summarize the results across the suppression columns of Table 6.6, where we consider the average across the entire 400-1000 nm and also as a separate case ignoring wavelengths below 500 nm. We note that the half-order of magnitude performance increase for the optimized wavelengths disappears and the mean suppression level is limited by the feature-size. Thus, we posit that a similar design constrained at a deeper suppression level of $10^{-11}$ would still be limited at a suppression level similar to this mask. However, it is possible that design improvements could minimize the effect of suppression degradation if this were to be formulated into the optimization problem – similar to minimum gap constraints.

In Figure 6.17(b), we plot mean contrast across the dark annular working region as a function of wavelength. As before, we specifically show for the 0.5 µm case the optimized wavelengths and also the computed wavelengths in 10 nm intervals. For quantitative results we refer to the contrast columns in Table 6.6 for 400-1000 nm and 500-1000 nm, where mean contrast is computed across the optimized wavelengths only and against all computed wavelengths for the different manufacturing feature sizes.
considered. The same trends discussed for the suppression are also evident here—namely, the optimized wavelengths feature a greater performance degradation resulting in a level contrast performance limited by the feature size. One trend that applies to the contrast performance only is that the performance tends to degrade towards the infrared end of the spectrum. This can be explained by the tendency of the residual feature-size limited point spread function to broaden at longer wavelengths resulting in additional light across the intended working annular region.

6.4.2 Edge perturbations

In the previous section, we have considered the performance of mask models created with different feature sizes. These deterministic models are symmetrical, except for small quantization errors arising from translation of a sixteen-fold circularly symmetric pattern to a rectangular grid, and therefore represent an optimistic bound on performance. We now consider the effect on performance due to the loss of symmetry that arises from perturbations at the mask edges that model manufacturing defects, that is differences from the prescribed mask edges. Performance with such stochastic manufacturing defects added represents a conservative bound on performance.

To establish a conservative bound on performance through the introduction of edge perturbations, we consider a model in which we can quantify the amount of perturbations added across the petal edges. We start with the highest accuracy model developed in §6.4.1 which featured $\delta R = 0.1\mu m$ and which most closely approached the ideal performance. We isolate all the mask grid point which represent petal edges and inject white noise perturbations using the same process as described in §5.3.3. The RMS perturbation level added corresponds to the additional desired feature accuracy level compared to the original mask model. We compute points that match the feature sizes of the optimistic, deterministic models we already computed in the previous section.
In Figure 6.18, we show the newly computed conservative, stochastic performance bound. Due to the stochastic nature, we perform ten trials for each considered feature size and report the mean suppression across the dark hole in Figure 6.18(a) and the mean contrast in Figure 6.18(b) across the annular working region. We compute a piecewise spline interpolation for intermediate feature sizes, and we show corresponding interpolations for the best and worst trials to gauge the variability resulting from the injected white noise. We also plot the corresponding optimistic, deterministic performance bound for both suppression and contrast, previously shown in Figure 6.14(a) and 6.15(a) respectively. We expect that the performance loss due to edge diffraction at a prescribed feature size to lie somewhere between the two indicated curves. The relative performance degradation is worst at the small feature sizes where the performance was initially the best: suppression at 0.5 μm degrades from $10^{-9.26}$ for the deterministic model to $10^{-8.42}$ and at 1.0 μm the performance degrades from $10^{-8.12}$ to $10^{-7.23}$, whereas at a larger feature size such as 5.0 μm the suppression degrades more gently from $10^{-5.87}$ to $10^{-5.59}$. Contrast across the working annular region generally shows similar trends as suppression but is not necessarily the same because some of the residual light may be contained inside the inner working angle or outside the outer working angle. At 0.5 μm the mean contrast is $10^{-11.15}$ for the deterministic model and degrades to $10^{-9.85}$; at 1.0 μm the deterministic bound is $10^{-10.87}$ and the stochastic bound is $10^{-9.24}$; finally at 5.0 μm the contrast degrades from $10^{-9.02}$ to $10^{-7.85}$.

We also consider a second model, in which instead of computing a conservative upper bound based on assuming that RMS perturbations match the expected feature size, we add a fixed RMS level of stochastic perturbations on symmetric mask models with varying feature sizes. The computed results are shown in Figure 6.19, with the base deterministic curves shown as the baseline and results with injected perturbations ranging from 0.5 μm to 3.0 μm RMS shown. The stochastic results reported
Figure 6.18: Performance of mask with white noise perturbations injected at the petal edges on the highest accuracy deterministic mask model available ($\delta R = 0.1\mu m$). Injected perturbations correspond to RMS levels indicated by the feature size. Mean results from 10 trials are shown with best and worst cases indicated. (a) Computed suppression at different feature sizes and interpolation for intermediate feature sizes (b) Computed contrast at different feature sizes and interpolation for intermediate feature sizes.
Figure 6.19: Performance of mask with fixed levels of white noise perturbations injected on a corresponding deterministic mask model at the petal edges to reduce symmetry. Stochastic results are mean of 10 trials. (a) Computed suppression at different feature sizes and interpolation for intermediate feature sizes (dashed lines are the corresponding readings from Figure 6.18(a)) (b) Computed contrast at different feature sizes and interpolation for intermediate feature sizes.
are averages of ten trials. We can compare the suppression results in Figure 6.19(a) with the results in the previous stochastic analysis. We see that the suppression levels are fairly flat across the injected perturbations curves and generally match the corresponding levels in 6.18(a). For example, we compare the suppression curve for 0.5 µm with the $10^{-8.42}$ level quoted in the first model and we notice that they are relatively close in agreement so we can obtain similar estimates from the stochastic curve in the first model and assume that the performance level will be relatively flat even if the prescribed mask has a higher resolution. In the contrast curves in Figure 6.19(b), a limitation of this model becomes apparent at lower resolution models as the deterministic and stochastic curves coincide. This is because the number of petal edge grid points is reduced at lower resolution models – the lowest resolution model shown in fact has no anti-aliasing with only black and white grid points and therefore there are no points on which the stochastic perturbations can be injected.

The stochastic analysis presented in this section has provided a more conservative estimate of the performance that can be expected due to feature-size limits by introducing errors in the manufacturing process. Additionally, we have shown that a fixed level of manufacturing errors will result in leveling of performance regardless of the prescribed resolution of the mask. Therefore, to obtain performance closest to the ideal case it is very important that the mask manufacturing process be well understood and controlled.

### 6.5 Analysis of experimental limits

In §6.4, we have examined the limits on performance that can be expected when mask manufacturing tolerances are introduced in the model. We have also examined distance scaling relationships and how increasing total distance available allows higher performance levels for occulters.
Figure 6.20: Layout of the new experimental testbed using different diameters for a tunnel input, a chokepoint, and a final tunnel output.

We are interested in designing the experiment enclosure to ensure that the experiment is feature size-limited rather than limited by the finite size of the experimental enclosure or by optical aberrations. We refer to the proposed experimental testbed layout in Figure 6.20, and we note that experimental testbed features a tunnel through which the diverging optical beam can propagate from one end to the other. The tunnel can actually be sized differently upstream from the mask – we will refer to this as the input tunnel – and downstream from the mask – the exit tunnel. Therefore, we examine the diffractive performance limits introduced by the finite size of the enclosure, considering the diffractive effect of the edges on the electric field at the occulter plane due to input tunnel in §6.5.1 and the diffractive effect of the shadow’s intensity roll-off being cut-off by the edges of the exit tunnel in §6.5.2. Finally, we compute some experimental limitations inherent with a collimated input beam to motivate the experimental design using a diverging beam with an outer ring.

### 6.5.1 Diffraction performance limits due to input tunnel

From a diffractive perspective, the negative effect of the finite-size of the input tunnel is to introduce a hard edge upstream from the occulter mask. Light diffracted by the tunnel edges introduces ringing across the occulter mask that may degrade the
performance of the occulter. A larger tunnel, and minimizing the distance between
the point of first contact of the diverging beam to the tunnel edges will reduce the
ringing across the occulter mask. Therefore, it is important to determine the minimum
diameter $D$ of the tunnel that will not limit performance below the ideal level.

In Figure 6.21, we show the numerical model used to assess the impact of the
finite edges on the performance of the experiment. In Figure 6.21(a), we show the
tunnel modeled at plane $P_0$ as a square aperture with sidelengths equal to the tunnel
diameter $D$ at a distance $h = 50$ m upstream from the occulter mask. We use a colli-
mated scaling to assess the performance of the occulter mask. This is a conservative
scenario for the following reasons:

- the occulter mask is larger radially for the collimated scaling and therefore
captures higher amplitude ringing at the edges than for the mask scaled for the
diverging beam

- at the point of first contact of the diverging beam with the tunnel edges, there
is some amplitude attenuation

- the point of first contact occurs several meters downstream from the pinhole
source rather than the full $h = 50$ m modeled here, and this will provide an
improvement in the amount of ringing.

To assess the performance degradation in the presence of input ringing across the
occulter mask, we use a high-accuracy mask model with $6000 \times 6000$ samples across
and $100 \times 100$ anti-aliasing in the gray areas, which at collimated scale results in a
feature size of $\delta R = 0.14 \mu m$. Assuming a plane wave input beam across the occulter
mask at 633 nm, the mean suppression across the dark hole is $10^{-9.78}$. This is the
model performance limit and is very close to the ideal suppression of $10^{-9.82}$.

To compute the diffractive effect on the occulter input from the tunnel edges,
we use the same model for computing diffraction from a square aperture described
Figure 6.21: (a) Finite tunnel size hard edge diffraction model with high-accuracy occulter mask model at P1 with $6000 \times 6000$ grid points across (b) Ringing input due to 100mm tunnel across corresponding occulter grid points (c) Ringing input due to 500mm tunnel across corresponding occulter grid points.

In §5.2.4 using the Fresnel Sine and Cosine integrals. In that model, we set a 50 m separation between the square aperture and the occulter mask. We sample the square aperture output plane identically to the occulter mask, that is at the midpoints of the occulter mask grid spanning a square area of $83 \times 83$ mm. Two examples of resulting ringing output are shown in Figure 6.21(b) for a 100 mm diameter tunnel and in Figure 6.21(c) for a 500 mm tunnel diameter. The amplitude of the ringing peaks decrease as the tunnel diameter is increased as can be seen from the colourbar axis.

In Figure 6.22, we show some sample propagation results obtained by applying
ringing input to the collimated mask model corresponding to different tunnel diameter input to visualize the effect of the degradation in performance when tunnel effects are modeled. In Figure 6.22(a), we show the pupil plane for a 750 mm tunnel, and in Figure 6.22(b) we show the corresponding point spread function. The camera aperture’s spatial extent is indicated by a white circle across the pupil. Mean suppression across the dark hole is $10^{-9.78}$ and mean contrast is $10^{-11.73}$. These results are identical to the base case without any ringing, so we can conclude that a 750 mm tunnel has a negligible effect on the occulter. Next, we reduce the tunnel diameter to 500 mm and show the pupil plane results in Figure 6.22(c) and the point spread function results in Figure 6.22(d). We can see that the residual point spread function is beginning to rise over the previous level especially at the core and outside the outer working angle we can see also residual light at some of the tips of the outer ring. The suppression degrades to $10^{-9.02}$ and the contrast to $10^{-11.07}$. Finally, for a smaller 300 mm tunnel we show the suppression results in Figure 6.22(e) and the image plane results in Figure 6.22(f). We see a significant rise in the level of the dark hole and much residual light across the annular region. The mean suppression is $10^{-7.74}$ and the mean contrast is $10^{-9.75}$.

Corresponding azimuthally averaged cross-sections of these cases, and also including a simulation of a 100 mm tunnel are shown in Figure 6.23. The cross-sections through the pupil plane are shown in Figure 6.23(a) while those through the image plane are shown in Figure 6.23(b). As the tunnel diameter decreases, the suppression is monotonically worse. The contrast results exhibit some variation in the structure as the residual point spread function changes shape but also overall becomes worse. We summarize the quantitative performance results for all these curves and some additional cases in Table 6.7, and also show the peak-to-mean ringing to show the effect on the input with tunnel diameter.

Finally, to obtain an estimate of the size of the tunnel necessary to ensure no
Figure 6.22: Performance of laboratory mask using different input tunnel sizes (a) Suppression at the laboratory pupil plane for 750mm tunnel (b) Contrast at the laboratory image plane for 750mm tunnel (c) Suppression at the laboratory pupil plane for 500 mm tunnel (d) Contrast at the laboratory image plane for 500 mm tunnel (e) Suppression at the laboratory pupil plane for 300 mm tunnel (f) Contrast at the laboratory image plane for 300 mm tunnel.
Figure 6.23: Performance limited by diffraction ringing at the input plane due to finite diameter tunnel: (a) Azimuthally averaged suppression at different tunnel diameters (b) Azimuthally averaged contrast at different tunnel diameters.
Table 6.7: Summary of diffraction effect on suppression and contrast performance for input tunnels of varying diameters.

<table>
<thead>
<tr>
<th>Tunnel Diameter</th>
<th>Suppression</th>
<th>Contrast</th>
<th>Ringing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inf.</td>
<td>-9.78</td>
<td>-11.73</td>
<td>0%</td>
</tr>
<tr>
<td>1m</td>
<td>-9.78</td>
<td>-11.73</td>
<td>0.18%</td>
</tr>
<tr>
<td>0.75m</td>
<td>-9.78</td>
<td>-11.73</td>
<td>0.79%</td>
</tr>
<tr>
<td>0.6m</td>
<td>-9.59</td>
<td>-11.66</td>
<td>1.48%</td>
</tr>
<tr>
<td>0.5m</td>
<td>-9.02</td>
<td>-11.07</td>
<td>2.98%</td>
</tr>
<tr>
<td>0.3m</td>
<td>-7.74</td>
<td>-9.75</td>
<td>5.21%</td>
</tr>
<tr>
<td>0.15m</td>
<td>-3.48</td>
<td>-5.90</td>
<td>13.78%</td>
</tr>
</tbody>
</table>

For intermediate tunnel diameter that were not explicitly computed, we perform a piecewise spline interpolation for the linear suppression. We also indicate the expected feature-size limited suppression level and also the ideal suppression level which is only slightly below the model limit we used to evaluate the effect of the input tunnel on suppression. The model limit can be seen at the flat end of the interpolated curve for tunnel diameter sizes above 0.65 m. Based on these results, we can make a recommendation that the smallest input tunnel diameter opening should be about 0.75 m, including a 0.1 m tolerance allowance for coaxial alignment.

An analysis of specular reflection across the tunnel has shown that the expected level of light across the occulter mask is at a similar level as the induced ringing for a 0.7 m tunnel [108]. Assuming, in the worst case, that this light is fully coherent and diffracts we expect this based on the tunnel analysis to be well below the 0.5 µm level. However, we can reduce all types of reflections inside the tunnel by increasing the input tunnel diameter beyond the minimum level recommended for diffraction purposes to, for example a larger 1 m diameter, and stopping it down to 0.75 m at regular intervals coinciding with the tunnel sections using baffles to reduce the amount of stray light inside the tunnel [54]. Such baffling would result in better than order of magnitude mitigation of the stray light inside the tunnel [108].
Figure 6.24: Suppression performance dependence on input tunnel diameter with manufacturing feature size performance limits shown.
6.5.2 Diffraction performance limits due to exit tunnel

The exit tunnel can also limit performance, but it does so through a different mechanism than the input tunnel which affects the input beam across the occulter mask. The exit tunnel represents a hard edge that would cut-off the shadow at a potentially high intensity level and therefore induce ringing across the dark hole above the intensity level that the dark hole was optimized for. We therefore have to carefully consider the sizing of the exit tunnel.

Some aspects of the experimental design work favourably towards reducing the effect of an exit tunnel:

- outer ring limits the extent of the beam around the occulter mask so the exit tunnel can be designed to be larger than the extent of the finite-sized occulter mask
- diverging shadow is smallest near the mask and increases in size towards the shadow
- ringing from a finite edge is reduced as the edge is closer to the output plane

Nonetheless, we still need to consider the suppression roll-off downstream from the occulter mask and size the tunnel to ensure that its edges are not illuminated by a significant intensity in the shadow. To demonstrate this effect, in Figure 6.25 we compute the suppression roll-off at different distances downstream from the tunnel. In Figure 6.25(a), we show a cross-section midway down the tunnel of the azimuthally-averaged suppression-calibrated shadow roll-off. We indicate where tunnels with different radii would cut-off the shadow. For example, a tunnel with a 100 mm radius would have a significant suppression level of $10^{-4.56}$ at the edges, whereas a tunnel with 300 mm radius would have a suppression level of $10^{-10.61}$ at the edges. It is important to size the tunnel such that the suppression roll-off is sufficient across the entire tunnel.
length. In Figure 6.25(b), we have computed similar cross-sections to the one mentioned above at 1 m steps downstream from the occulter mask. The results from the cross-section at 25 m is indicated by the dashed vertical line. We see in this figure that the suppression level across the tunnel edges rises as the shadow increases in size due to the beam divergence.

Fortunately, ringing from edges that are closer to the plane of the shadow will be reduced compared to ringing from an edge that is further upstream and closer to the occulter. In Figure 6.26 we quantify this phenomenon and take into account the ringing to obtain a better estimate of the expected level ringing induced across the dark hole. In Figure 6.26(a), we have computed the peak across the entire spatial extent of the dark hole to quantify the ringing for evenly illuminated edges at different separations upstream from the shadow. As mentioned previously, we see that the expected ringing level will decrease as the edges are closer to the designated pupil plane. Finally, in Figure 6.26(b), we combine the computed ringing results in Figure 6.26(a) with the variation in illumination level computed in Figure 6.25(b) due to the beam divergence to obtain an estimate of the suppression at which the exit tunnel will become a limiting factor. We also show the ideal and 0.5 μm feature-size limited performance levels below which we wish to maintain the effect of the tunnel.

We can now provide some guidelines towards the sizing of the exit tunnel to ensure that the suppression shadow remains feature-size limited. As shown in Figure 6.26(b), the 0.5 m diameter tunnel is expected to remain strictly below the ideal level. However, the model presented here assumes propagation between two finite planes and does not take into account the cumulative effect of diffractive-ringing across the tunnel. We therefore recommend that the tunnel be sized larger than 0.5 m, with a 0.6 m diameter tunnel providing more than two orders of magnitude margin. Finally, we mention that it is possible to size the tunnel at a smaller diameter closer to the mask and increase the diameter further downstream from the mask as the shadow
radius increases with divergence.
Figure 6.25: Suppression roll-off at exit tunnel edges (a) Pupil-plane cross-section halfway down the exit tunnel with different tunnel radii indicated as straight lines (b) Computed suppression at different exit tunnel radii edges as a function of distance downstream of the mask with cross-section above indicated by straight line.
Figure 6.26: Ringing due to tunnel-edges (a) Ringing from evenly illuminated edges upstream from the designated pupil plane (b) Suppression-calibrated ringing level that takes into account the suppression roll-off and beam divergence.
6.5.3 Experimental limits of collimated input beam

We also consider the experimental limits introduced by usage of a collimated beam. If we compare the scaling for a collimated input beam to the scaling for diverging input beam in Table 6.3, we note that for the collimated case the radial size of the occulter mask is larger than for the diverging case. Therefore, considering that the performance is expected to be feature size-limited, it would appear that scaling for a collimated scale would be desirable. However, we examine the effect of wavefront-errors introduced by collimating optics aberrations and show how these would dominate performance and the experiment would no longer be feature-size limited.

We perform an analysis similar to that described in §5.3.5. The occulter mask model in §5.3.5 is propagation-distance limited experiment and was however only capable of achieving $10^{-6.60}$ suppression, even for the highest resolution model considered. For the analysis here, we will use the same baseline high resolution mask model we used in §6.5.1 to evaluate the effect of diffraction ringing due to the input tunnel. This high-resolution model was capable of achieving near-ideal performance with a mean suppression across the dark hole of $10^{-9.78}$.

We apply a variety of phase maps across the occulter plane with different RMS levels simulating different levels of surface aberrations induced by the collimating optics. We generate phase maps with a frequency envelope of $1/f^{3/2}$ which was also used as a model for optics aberrations for the coronagraph laboratory [90]. Two sample phase maps are shown in Figure 6.27.

We compare three wavefront phase error input maps with the baseline case featuring no phase errors. In Figure 6.28, we show the suppression and contrast performance degradation as increasing optical aberrations are added across the occulter plane. We see in Figure 6.28(a) and 6.28(b) that even relatively small wavefront errors have a very strong impact on the performance of the occulter mask. Here we have degradation of the suppression across the dark hole to the $10^{-7.85}$ level which is
Figure 6.27: Sample phase maps of optical aberrations applied at occultor plane (a) Phase aberrations with $\lambda/20$ RMS (b) Phase aberrations with $\lambda/100$ RMS. Colour bar is in terms of $\lambda/20$ and $\lambda/100$ respectively.

approximately two orders of magnitude performance loss. We continue with Figure 6.28(c) and 6.28(d), corresponding to $\lambda/300$ nm RMS wavefront errors and we can clearly see the near disappearance of the dark hole in the shadow and a sharp increase in the residual point spread function. Finally, for typical $\lambda/20$ wavefront errors expected for high-quality optical surfaces, the performance in Figure 6.28(e) and 6.28(f) has degraded to a level comparable to a circular occultor with suppression across the dark hole of $10^{-2.98}$.

In Figure 6.29 we compare the same cases with the baseline model quantitatively by computing the azimuthally averaged curves. The suppression curves are shown in Figure 6.29(a) and the contrast curves are shown in Figure 6.29(b).

This analysis of the effect of wavefront errors on the performance of the occultor mask confirms that occulters are extremely sensitive to phase errors. Usage of collimating optics introduces significant errors across the occultor mask that quickly dominate performance. Therefore, our proposed input diverging beam has the advantage of not requiring any optics between the spatially filtering pinhole and the occultor mask. As a final note, the sensitivity of the occultor to phase errors suggests that it is important to characterize the laboratory environment to ensure that signif-
ificant wavefront errors do not arise through the propagation of the beam through the enclosed tunnel environment.
Figure 6.28: Performance of laboratory mask using increasing wavefront phase error maps (a) Suppression at the laboratory pupil plane for $\lambda/5000$ nm RMS (b) Contrast at the laboratory image plane for $\lambda/5000$ nm RMS (c) Suppression at the laboratory pupil plane for $\lambda/300$ nm RMS (d) Contrast at the laboratory image plane for $\lambda/300$ nm RMS (e) Suppression at the laboratory pupil plane for $\lambda/20$ nm RMS (f) Contrast at the laboratory image plane for $\lambda/20$ nm RMS.
Figure 6.29: Corresponding azimuthally averaged performance curves (a) Azimuthally averaged suppression for different wavefront phase error maps (b) Azimuthally averaged contrast for different wavefront phase error maps.
Chapter 7

Conclusions

A full-scale occulter mission will be a major undertaking on the part of NASA, but one that carries the potential to give us more information about worlds in neighbouring solar systems than any other method proposed to date. For this reason, it is important to continue technological demonstrations to examine the feasibility of an occulter mission and seek solutions to any limiting factors that may arise. This dissertation is part of the technological demonstration process necessary to validate the feasibility of occulters.

In Chapter 2, we have reviewed approaches used to design space occulters. We have used as an illustrative example the design of a smaller scale technology demonstrator occulter that could be used to obtain limited science only for the nearest solar systems.

We have shown mathematically how occulters can be scaled to laboratory dimensions in Chapter 3 while maintaining an identical diffraction integral. We have illustrated this relation with numerical examples, including a variety of input beams and scaling scenarios. We have also performed simulations that have shown the expected ideal performance of the occulter in both the pupil and image planes, showing the ideal performance that a petalized occulter can obtain is limited in our current
testbed compared to its designed performance.

In Chapter 4, we have presented experimental results from the Forrestal testbed. We show suppression measurements at the $10^{-5}$ level at the pupil with different wavelengths, and also contrast measurements at the $10^{-10}$ level in the image plane. We demonstrate that the measured optimized mask performance is an improvement over a simple circular control mask. We compare the measured performance with the theoretical model which predicts suppression and contrast levels closer to the $10^{-7}$ and $10^{-12}$ levels and determine that the ideal theoretical performance is not reached.

We have undertaken a sensitivity analysis in Chapter 5 whose goal is to determine the robustness of the optical performance of the occulter mask to the types of errors that can be expected in the laboratory environment. We use this sensitivity analysis to identify limiting factors on the performance of the laboratory occulter, and show that the performance is consistent with feature size-induced limits on performance. An analysis of the mask fabrication process has shown that the prescribed feature sizes can be improved. At the time of this writing, a mask with such improved resolution is being manufactured. It is hoped that experimental measurements of this mask will allow us to approach the ideal performance of the occulter mask by an order of magnitude compared to the current laboratory results.

Ultimately, the Forrestal testbed is limited by the propagation distance available and does not allow testing of realistic flight occulters. For this purpose, a longer-distance testbed has been designed in Chapter 6 and its expected performance analyzed. This testbed is currently under construction and it is hoped that this will provide better performance for realistic, flight-scalable occulter designs.

A second goal of the occulter technological demonstration program is demonstrating our ability to align the occulter and telescope during science observations. I have simulated and designed controllers and estimators for the observation portion of an occulter mission [150, 151, 155] – these work and appear feasible, at least in simula-
tion. They rely on a position sensing scheme based on the expected diffraction pattern at out-of-band wavelengths. The goal is therefore to implement such algorithms using physical sensing in the newly designed occulter testbed. Experimental implementations of occulter alignment algorithms are the next priority after the verification of optical performance. An outcome of the work described in this dissertation is that the optical models necessary for the position sensing scheme are accurate to the level necessary for their implementation.
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