Surfactant and Colloid Self-Assembly Simulations

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Abstract

Surfactants and colloids are ubiquitous in biology, the environment, home and personal care products, and industrial applications. The self-assembly of surfactants into micellar aggregates is crucial to their use in many industrial applications, including: detergency, cosmetics, oil-spill remediation and nanoparticle dispersion. Because of their ubiquity, surfactant and colloid self-assembly are canonical subjects of soft matter study, yet there are concentration effects, for example the free surfactant concentration, that remain difficult to measure and understand. We present our findings on the driving forces behind aspects of self-assembly and methodologies for characterizing the behavior of three self-assembling systems: colloids with short-range attraction long-range repulsion, nonionic surfactants and ionic surfactants. To study molecular details of these systems, we use Monte Carlo and molecular dynamics simulations with implicit-solvent models. We directly compare different methods for calculating the critical micelle concentration in all self-assembling systems. We establish that excluded volume and counterion condensation are the main driving forces for the decrease of free surfactant concentration in nonionic and ionic surfactants, respectively. We compare counterion condensation and mean ionic activity measurements directly to experiments, and address experimental disagreements. For short-range attraction long-range repulsion colloids, conditions are found where large preferred aggregates have no net effect on the pressure, which is strikingly different behavior from surfactant self-assembly. The results in this thesis offer a deeper understanding of the phenomena of low-concentration self-assembly, leading to improved methods for estimating micellar properties over wider concentration ranges. These methods will benefit the development of more accurate molecular models and speed up development of new industrial formulations.
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Chapter 1

Introduction

1.1 Surfactant and colloid uses

Surfactants have been studied extensively in interfacial, colloidal, environmental and biological sciences. Their ubiquity in research and industry is partially due to the early discovery of their self-assembly and interfacial behavior, their ability to model other amphiphiles, such as phospholipids, and their low cost of production from petroleum and plant oils.

Surfactant molecules are employed in life and industry because of their amphiphilic nature, where a solvophobic entity is chemically bonded to a solvophillic entity. Amphiphilicity drives the surfactants to solid-liquid,! liquid-vapor and liquid-liquid interfaces. Once at the interface they often reduce interfacial tension by lowering the surface energy. The ability to go to the water-oil interface and reduce interfacial tension is why surfactants are included in: oil spill remediation, enhanced oil recovery, herbicides, detergents, emulsifiers[1] and firefighting foams[2]. Fabric softeners and cosmetics include surfactants because their adsorption (at the solid-fluid interface) improves particle dispersion and control particle size[1]. Surfactants
increase the wetting and protection of paint[3], and improve ink pigment dispersion stability and reduce flocculation[4].

Surfactants are components in complex mixtures in biology and biotechnology. Surfactants are used to stabilize nanoparticles[5] and form micelles for drug delivery in the pharmaceutical industry[6]. Pulmonary surfactants in human lungs reduce the air-water surface tension to increase the ability of lungs to expand[7]. Bile salt micelles solubilize and transport fats, which is an important part of the digestion and metabolism process[8].

Surfactant micelles are also termed “association colloids”[9]. A colloid is an independent kinetic entity which is considerably larger than solvent molecules. Like surfactants, colloidal suspensions appear in nature and industry. Milk, smoke, fog, ice cream, paint and ruby glass are all examples of colloidal suspensions[9]. Stabilized emulsions, clay platelets, hard gold or silica clusters and soft polymer aggregates are some typical colloid particle examples[10]. Some interesting behavior and applications arise from colloid aggregation and assembly, including: opals, clay swelling in ceramics and ferromagnetic fluids[9].

1.2 Self-assembly

Surfactants are composed of many different types of molecules. Common surfactants are composed of a hydrophobic alkane tail, with different hydrophillic head groups: nonionic, anionic, cationic and zwitterionic. However, surfactants with different architectures[11, 12], functionalities[13–15] and even colloidal systems[16–18] have similar behavior. The similarities can be used to apply knowledge developed in one system to another, but also to find out differences, which can lead to the development of different material properties.
More than a century ago, it was found that surfactants form clusters called micelles\[19\]. Surfactant micellization, drawn in Figure 1.1, is the self-assembly of multiple surfactants into clusters, where the solvophobic parts are unstructured in the center, “the core,” and the solvophillic parts are on the aggregate-solvent interface, “the corona.” Not all surfactant molecules are parts of micellar aggregates in a micellizing system; there is always a concentration of un-micellized, free surfactants. Thermodynamically, micellization occurs because the enthalpic gain of separating the solvophobic parts from the solvent overcomes the entropic loss of restricting surfactants to a micelle\[20–22\]. Micellization occurs at a concentration that is a function of temperature, pressure and solvent type, or composition for a mixed solvent or added salt.

Figure 1.1: Schematic of the self-assembly of free, un-aggregated surfactants into a micelle with free surfactants.

As surfactants are added to micellar solutions, micelles begin to interact. First micelles grow or fuse to make larger or worm-like micelles\[23\]. As the surfactant concentration increases above micellization, macrophase separation occurs. Spherical micelles will adopt regularly spaced positions, making an FCC crystal of micelles for example. Similarly, cylindrical micelles can form a two-dimensional crystal of
columns. At the highest concentrations, surfactants form lamellar sheets[24]. These different phases are also used in applications; for example the lamellar phase surfactants make good lubricants[1].

Micelles do not often self-assemble to form clusters of micelles, whereas colloids regularly aggregate. Tuning the colloid-colloid interaction can improve material properties, such as aggregation, which are important for their application. Surface charges induce repulsion, and adding a polymer that is larger than the solvent but smaller than the particles induces attraction[10]. Tuning the repulsion and attraction can lead to behavior that is similar to micellization[18].

The concentration at which micellization occurs, the critical micelle concentration (cmc), is a key property for many surfactant applications. Figure 1.2 schematically shows transitions in a sample of properties relevant to this thesis. Above the cmc, many properties continue to vary, often at a constant rate. The non-zero slope above the cmc of the osmotic pressure, mean ionic activity or partition function, is sufficient to confirm that micellization is not a thermodynamic phase transition[25]. The change in conductivity impacts the concentration of added components for electrical and battery applications in composite materials[26]. Many nano-technology, home and personal care products are formulated near the cmc because the surface tension is near constant above the cmc[1]. The changes in slope carry fundamental and industrial importance.

Other properties have non-monotonic behavior or a non-constant slope above the cmc. The free surfactant concentration impacts many applications, and its decrease was first proposed in 1935[27]. The free surfactant concentration impacts solubilization[28], nano-composite conductivity[29] and micellar catalysis rate constants[30]. Furthermore, the free surfactant concentration is integral to cmc calculation in simulations[31] and micelle size, known as the preferred aggregation number ($\langle M \rangle$), for experiments[32]. The amount of solute incorporated in the
Figure 1.2: Properties illustrating drastic behavioral changes at the critical micelle concentration. Some properties have a constant change in behavior above the cmc (blue), while others continue to vary, often due to the non-monotonic free surfactant concentration behavior (orange).

solution, known as the extent of solubilization, increases at an increasing rate because the free surfactant concentration decreases[33]. Free surfactants adsorb to solid interfaces, which decreases the free surfactant concentration and reduces the concentration available to adsorb[34]. In ionic surfactant micellization, the decrease of free surfactants is accompanied by an increase in the free counterion concentration. Free counterions compete with reactants in ionic surfactant micellar catalysis, and cause a maximum in the rate constant near the cmc[30]. Even though the free surfactant concentration decrease impacts many industries, it is often ignored and set to be a constant value equal to the cmc. To calculate micelle sizes, experimental techniques need to calculate the micellar concentration. They often do so by calculating it as the difference between the total and free surfactant concentrations, where the free surfactant concentration is assumed equal to the cmc[35–37]. In molecular simulations, the free surfactant concentration and micelle sizes are readily available from molecular positions. The challenge for molecular
simulations, especially with explicit solvent, is that concentrations near the cmc are not computationally tractable\[31\]. Experiments can measure the cmc, but not the free surfactant concentration, whereas simulations know the free surfactant concentration but not the cmc. Therefore theoretical and practical methods to measure the free surfactant concentration are important for industrial applications, and enable the improved estimation of micellar properties for both simulations and experiments.

The micelles that form at and above the cmc are dynamic, with surfactants often joining and leaving\[38\]. Less frequently, micelles break apart or fuse together. Thus equilibrated, micellar systems sample a range of sizes. Micellization includes larger preferential micelles (potentially 30-110 surfactants) that are more probable than intermediate-sized micelles (potentially 10-50 surfactants), as well as preferential small oligomer-sized aggregates (potentially 1-15 surfactants) (see Figure 2.4 as an example). Experimentally these micelle sizes are calculated using many different methods, including: nuclear magnetic resonance\[35\], small-angle neutron scattering\[39\], steady-state\[36, 37\] and time-resolved fluorescence quenching \[37\], specific conductivity \[40\] and time-dependent static light scattering \[41\].

\section{1.3 Methods}

Theory and simulations have been used to observe many micellar properties. Theoretical methods have been successful in describing micellar kinetics and the effect the cmc has on surface tension, activity and osmotic pressure. Micellization theories have treated micelles as a separate “pseudo-phase” and as a reaction equilibrium process, known as mass action law. These theories led to important results, including temperature and chain length dependence of the: cmc\[42, 43\], sphere-to-rod transition\[42\] and surface tension\[44\]. Although approximating micelles as a
separate thermodynamic phase is helpful, they are not a separate phase. Nor are micellar solutions composed of only monomers and single-sized micelles. Some theories approximate the process as $M$ surfactant monomers forming mono-disperse micelles in a stepwise process. Although monomer exchange in and out of micelles is an important dynamic mode, it is accompanied by the slower fission/fusion of micelles\[38\]. Improvements to these theories generally focus on developing expressions for the free energy of micellization\[20–22\]. Models have used empirical relationships\[43\], charge frustration\[45\], Debye-Hückle theory\[46\], mean-field\[47\] and molecular simulations\[48\], among other relationships, to improve free energy of micellization calculations.

Thermodynamic models also helped highlight the molecular nature of equilibrium self-assembly, which encourages the use of other computational techniques. Molecular simulation has been an important tool for studying surfactant micellization for almost four decades\[49\]. Self-consistent\[50\] and single-chain mean field theory\[51\] are methods used to study micellization by minimizing the free energy. Mean field theories have been especially powerful for studying dense solutions and microphase separation\[52\]. Both methods treat inter-molecular interactions as a mean-field, which results in the ability to access long timescales but can represent aggregate structure poorly\[53\]. Monte Carlo (MC) methods have been applied to study self-assembly\[49, 54\] because MC simulations have been successful in modeling phase equilibrium\[55\]. Making non-physical moves, such as transfers in and out of a reservoir, cluster moves and chain regrowths, can allow amphiphiles to self-assemble more tractably than in dynamic simulations. Because of their stochastic nature, MC simulations are able to sample a wide range of phase space, but do not give information on the real dynamics of self-assembly.

Molecular dynamics (MD) and dissipative particle dynamics (DPD) simulations can simulate the dynamics of micelle formation. DPD and MD simulations take
into account inter-molecular interactions explicitly. DPD simulations attain these
timescales more tractably[56–58]. However, DPD simulations do not capture many
aspects of the free energy landscape, such as the structure of hydrated water or
realistic inter-molecular potentials. It is also important to note that many of the
simulation studies of amphiphiles have pre-assembled initial conditions. That is to
say, because of tractability, many simulations of amphiphiles are not of self-assembly
and, therefore, focus on the structure and interaction of aggregates.

Because simulating full self-assembly is a challenge, accurately and reliably mea-
suring the cmc also continues to be a challenge. Micellization properties are sensitive
to model parameters[31, 59], so removing as much uncertainty as possible from the
measurement techniques is essential to model improvement. Model improvement will
add value to molecular simulations for formulation development, performance, and
safety[60]. An important component of developing new models is acknowledging the
concentration dependence of many properties, notably micelle size, shape, concen-
tration and counterion binding. Micellar properties are often approximated to be
constant above the cmc, which can lead to inaccuracies in experimental measure-
ment analysis and model development for simulations. Robust methods to calculate
the cmc at high concentrations reduce those inaccuracies. Direct comparison of
the free surfactant concentration decrease between simulations and experiments is
rare. Comparing simulation and experimental results enables testing hypotheses with
simulations and understanding which physics are important to capture the behavior.
Furthermore, the effect of the added electrolyte on the free surfactant concentration
decrease has not received much attention.

The self-assembly of colloids with competing interactions has frequently been
compared to surfactant micellization[17]. Some studies of colloids with competing
interactions have neglected methods and theory developed for surfactants, and as
a result come to poor conclusions. Therefore the techniques developed for surfac-
tant micellization simulations need to be modified in order to be applied to colloids with competing interactions. The small range of self-assembly and experimentally un-realized high-density phases are challenges for implementing colloids with competing interactions in more industrial applications\cite{61}. Therefore, design rules for identifying different types of self-assembly and extending the regions of stability for self-assembly are important.

1.4 Thesis overview and introduction

In this thesis we use MC and MD simulations to study self-assembly in surfactant solutions. Every data point was initialized with a micellized and a free, gas-like configuration, to ensure that equilibrium behavior is observed. Throughout this thesis, except for ionic surfactants with added salt, simulations are run at concentrations both above and below the cmc. Depending on the model and its cmc, this can require long simulation times or advanced sampling techniques. We use a specific class of molecular model to access low concentrations. The large free energy barriers associated with forming a micellar solution from an un-micellized solution of surfactants can lead to hysteresis, where the system can not freely go from the micellar state to the free state. We use histogram reweighing to bypass the hysteresis of micelle formation in some cases.

We model surfactants and colloids using pair-wise potentials on a three-dimensional lattice and in continuum three-dimensional space. In all of the models, the solvent, most often water or an electrolyte-water mixture, is represented implicitly. Solvent molecules are not present, but their impact on the self-assembling entities is imparted by affecting the pairwise potentials. The models range in complexity, from isotropic spherical colloids, to on-lattice nonionic surfactants, to off-lattice ionic surfactants, as schematically shown in Figure 1.3.
Figure 1.3: Representations of the implicit-solvent models of self-assembling systems studied in this thesis.

These models allow us to address challenging problems and questions in self-assembly science. This thesis, by studying distinct molecules, studies what constitutes micellization. This understanding informs the methods developed to identify and quantify micellization. Self-assembly properties of different surfactant types and environments are investigated. For simulations specifically, these methods will enable the improvement of more complex models and mixtures, and speed up property estimation of those models for predictive studies. Furthermore, this thesis aims to demonstrate the scale, behavior and theory of the free surfactant concentration. The free surfactant concentration is a central topic of this thesis because it impacts properties of interest for various industries, as well as experimental and simulation techniques.

The remaining chapters of the dissertation are based on the following publications:


• A. P. Santos and A. Z. Panagiotopoulos. “Surfactant and salt concentration effects on ionic surfactant micellization,” *in preparation*. [Chapter 4]

I additionally contributed to the following publication during my dissertation work, which is referenced within:


* denotes equal contribution.
Chapter 2

Thermodynamic signatures and cluster properties of self-assembly in systems with competing interactions

2.1 Introduction

Proteins, colloids and functionalized nanoparticles can self-assemble into larger structures of finite size when the effective pair interactions are either highly anisotropic [62–64] or isotropic with competing attractive and repulsive forces at different distance ranges [65–67].

Amphiphiles, surfactants and block copolymers also spontaneously form aggregates, commonly referred to as “micelles,” largely due to the asymmetric nature of the molecules[68–70]. Structural and dynamic properties of systems which self-assemble

into clusters due to competition between short-range attraction and long-range repulsion (SALR) have been the subject of many recent studies[71–83]. Self-assembly arises due to the competition between the energy-driven preference to organize and the associated loss of the entropy in both amphiphilic and SALR systems [65, 82, 84]. Interestingly, it was shown that these two classes of self-assembling systems can be described by the same Landau-Brazovskii functional [84], and thus their mean-field phase diagrams are topologically equivalent at high temperatures. Such extraordinary similarities between SALR and amphiphilic systems provoke questions on whether the two systems remain similar beyond mean-field approximations. Exact results, however, can only be obtained by reducing system dimensionality. One-dimensional (1d) studies of structural, mechanical and thermal properties of SALR and amphiphilic systems [85, 86] support the hypothesis that, qualitatively, the two classes of self-assembling systems are fundamentally similar. In 2d, phase diagrams obtained within the mean-field approximation [82] and by means of molecular simulations [82, 87] differ qualitatively, indicating substantial role of the density fluctuations. Recently, 3d simulations of SALR systems[73] showed formation of microphase structures which commonly appear in experiments[88, 89], simulations[90, 91] and mean-field theory[89, 92] of amphiphilic systems. The directionality of surfactants and block copolymers enables a wide variety of structures in phase diagram form and more exotic morphologies[93]. Beyond clusters and gels, SALR phase behavior has been difficult to explore, and thus the extent to which SALR and surfactant phase diagrams agree has yet to be thoroughly investigated[73]. Truskett and co-workers have used machine learning techniques to design pair-wise SALR interaction potentials that lead to particle self-assembly into target structures[94, 95]. In the present study, we focus on 3d systems for a range of low densities, and show that the process of SALR particle aggregation shares many aspects, but also exhibits key differences, relative to micellization of conventional surfactants.
Although clustering of SALR particles has been extensively investigated \cite{74, 80, 96-100}, the osmotic pressure versus density, $p(\rho)$, an important quantity used to detect and describe aggregation in surfactant systems \cite{101, 102}, has not been previously reported. Here, we calculate $p(\rho)$ for different attraction strengths and compare it with the characteristic behavior of micellizing systems. In such systems, the osmotic pressure follows the ideal-gas law when the concentration of amphiphiles is low and the system consists primarily of monomers. Micelles form above a certain concentration, known as the critical micelle concentration (cmc), at which point the slope of the osmotic pressure, among other thermodynamic properties, significantly changes because the clusters become the independent kinetic entities. The onset of micellization is also associated with the appearance of a separate peak in the mass-weighted distribution of the micelle sizes, the cluster size distribution (CSD), at the preferred aggregation number. The cmc is commonly defined as the concentration at which: the slope of the osmotic pressure changes, a preferred aggregation number appears and a maximum in the free oligomer concentration occurs \cite{70}. Importantly, all these manifestations of structural change appear within a narrow concentration range. In this study, we show cases of SALR systems exhibiting traditional micellizing behavior and cases that differ in remarkable ways from this behavior. Our study shows that, depending on the ratio of attraction and repulsion strength, clustering can be qualitatively very similar to micellization or can show unique thermodynamic properties.

We study two model SALR systems using grand-canonical Monte Carlo (MC) simulations to investigate the pressure and structure, and molecular dynamics (MD) simulations to investigate the dynamics, detailed in section 2.2. Our results show that the response of the pressure in SALR systems can be similar or dissimilar to micellization (section 2.3.1) and that the clusters characteristics have comparable trends with temperature (section 2.3.2) regardless of the potential parameters stud-
ied. We discuss the importance of these results, aiming to understanding SALR self-assembly and interaction potential design in section 2.4.

2.2 Methods

2.2.1 Model

Figure 2.1: The two SALR interaction potentials investigated, with varying attraction strength ($\epsilon = 1.6$: red and $\epsilon = 1.0$: blue) and constant repulsion strength. The inset shows the long-ranged nature of the repulsion and the range of attraction ($r_{\text{att}}$).

In the model studied, particles interact via the Lennard-Jones potential for the short-ranged attraction and the Yukawa potential for long-ranged repulsion,

$$V(r) = 4\epsilon \left[ \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^{6} \right] + \frac{A}{r} e^{-r/\xi},$$

(2.1)

where $\sigma = 1$, $A = 0.5$ and $\xi = 2$; two values of $\epsilon$ were considered: $\epsilon = 1.6$ and $\epsilon = 1.0$ (Figure 2.1). The same form of the SALR potential, but with a much shorter range of attraction, was studied in Refs. [72, 74, 80, 96]. In our case, after shifting the potential by its value at $r_{\text{cut}} = 8\sigma$ to zero, the attraction range ($r_{\text{att}}$) is 2.042$\sigma$ (in the case of $\epsilon = 1.6$) and 1.8096$\sigma$ (for $\epsilon = 1.0$); thus, our model has similar
ranges of attraction to those studied by Charbonneau and co-workers\cite{73, 103}. We chose the two values of $\varepsilon$ so that the second and third virial coefficients have positive and negative values over a significant range of temperatures below the Lennard-Jones critical temperature (Figure 2.2). Results presented in section 2.3 are $k_B T = 0.24 - 0.46$ for $\varepsilon = 1.0$ and $k_B T = 0.58 - 0.79$ for $\varepsilon = 1.6$. For the temperatures studied, $B_2$ changes sign for $\varepsilon = 1.0$ and is negative for $\varepsilon = 1.6$.

![Figure 2.2](image)

**Figure 2.2:** The second ($B_2$) and third ($B_3$) virial coefficients for the two chosen $\varepsilon$ values ($\varepsilon = 1.6$: red and $\varepsilon = 1.0$: blue). The inset shows the curves near $B_3 = 0$.

### 2.2.2 Simulation details

We performed Metropolis Monte Carlo (MC) simulations\cite{104} in the grand canonical ensemble using the Cassandra package\cite{105}, supplemented with a cluster center-of-mass displacement move\cite{106} in order to assist equilibration. Two particles were considered to be part of the same cluster if the distance between them was less than the range of attraction. MC simulations were performed with the following mix of moves: 10% cluster moves, 50% single particle translations and 40% single particles insertions or deletions. Runs consisted of $10^9$ steps after $5 \times 10^8$ equilibration steps. The length of equilibration was determined by the number steps required for two
otherwise equivalent simulations starting from a clustered or un-clustered state to have identical cluster distributions.

Molecular Dynamic (MD) simulations were performed using the HOOMD-blue package in the canonical ensemble [107, 108] with a Nosé-Hoover thermostat coupling constant of 10 and a timestep of 0.01 $\delta t$. The time units are non-dimensional and defined as $\delta t = \sqrt{\frac{\mathcal{M}\mathcal{E}}{\mathcal{D}^2}}$, where $\mathcal{M}$, $\mathcal{E}$ and $\mathcal{D}$ are the scales for for the mass, energy and distance, which are all equal to 1. MD production runs consisted of $5\times10^4 \delta t$ and were initialized from final configurations of the MC runs. Both MC and MD simulations were run in a cubic box, with $L = 17\sigma$. Larger systems, $L = 30\sigma$, were used for some of the structural analysis properties, detailed in the Sec. 2.2. All of the properties described in Sec. 2.2.3 and presented in Sec. 2.3 showed agreement between systems sizes of $L = 17\sigma$ and $L = 30\sigma$.

### 2.2.3 Analysis details

Virial coefficients were calculated using Etomica[109], with an supplementary potential for the SALR system studied. We used the Mayer Sampling Monte Carlo method for calculating higher-order virial coefficients[110]. Single simulations sampling configurations from $10^6$ steps for the two SALR potentials. This method requires the evaluation of a reference system. As in studies from Kofke and co-workers, we use a hard sphere reference state ($\sigma = 1.0$).

Histogram reweighting[102, 111, 112] was used to obtain the osmotic pressure dependence on density, $p(\rho)$, in which the grand canonical partition function is computed in an iterative manner based on an overlap of the energy-density histograms obtained for multiple values of the chemical potential. The resulting osmotic pressure will hitherto be referred to as the “pressure”, since they are equivalent in implicit-solvent systems. Histogram reweighting was only performed for $L = 17\sigma$, however
the pressure from histogram reweighting agrees with the pressure from MD simulations with $L = 30\sigma$.

Static properties of the self-assembled clusters were studied by considering the cluster size distributions (CSDs), density profiles and cluster moments of inertia. The CSDs were calculated using the Hoshen-Kopelman cluster-finding algorithm[113] with the same clustering criterion used for the cluster moves (the distance between particles being less than the range of attraction). Following the convention commonly used in micellization studies, the presented CSDs are the probability distributions of finding a cluster of a given size, $M$, weighted by the cluster size and the average density of the system, i.e. $\rho(M) = \frac{1}{\rho} \sum_m P(M)M$, where $P$ is the probability of finding a cluster of size $M$.

Moments of inertia $I_i$ of clusters of size $M$ about the principal axes were calculated from the inertia tensor[114],

$$I(i, j) = \sum_{l=1}^{M} \begin{cases} \left((i_l - i_{cm})^2 + (j_l - j_{cm})^2 \right) & \text{if } i = j, \\ -(i_l - i_{cm})(j_l - j_{cm}) & \text{if } i \neq j \end{cases}$$

where $i$ and $j$ are x, y, z cartesian coordinates, $i_{cm}$ is the center-of-mass of the cluster in the $i$ direction and $l$ is the particle index. To get the moments of inertia about the principal axes, instead of the x, y and z axes, we use the eigenvalues of the inertia tensor.

The number density of free colloids, $\rho_{\text{free}} = \frac{N_{\text{free}}}{V}$, was calculated as the number density of particles in clusters $M < M_{\text{olig}}$ where $M_{\text{olig}}$ is the least probable cluster size that separates the oligomeric/free clusters from the larger clusters.

The dynamics of clusters were studied by calculating their average life-times. Life-times of clusters were obtained using a methodology similar to that of Viduna et al.[115], where all clusters are tracked during a simulation. The life-time is calculated
as the time, in $\delta t$ units, that separates the birth and death of a cluster size $M$. Whenever $m$ particles join an aggregate of size $M$, with $m = 1, 2, 3, \ldots$, we say an aggregate of size $M + m$ is born and an aggregate of size $M$ dies. Analogously, a detachment of $m$ particles from an aggregate of size $M$ is treated as the death of an aggregate of size $M$, and the birth of an aggregate of size $M - m$. If as many particles leave as join the aggregate of size $M$, then an aggregate of size $M$ is born and another aggregate of size $M$ dies.

### 2.3 Results

#### 2.3.1 Identification and characterization of self-assembly

In this subsection, different types of self-assembly behavior are identified and critical cluster densities are determined from thermodynamic and structural properties. For the more attractive case, with $\varepsilon = 1.6$, the second and third virial coefficients $B_2$ and $B_3$ are negative for all temperatures $k_B T < 1.154$ and $k_B T < 0.957$, respectively. The upper panel of Figure 2.3 shows the pressure, $p$, as a function of the number density, $\rho$, for the $\varepsilon = 1.6$ system. The pressure obeys the ideal-gas law at low densities, but at some higher density that depends on the temperature, the slope of $p(\rho)$ abruptly decreases. This decrease in the slope of the pressure coincides with the formation of clusters. Similar to micellization, when clusters start to form, the CSD changes from monotonic to bimodal, with one local maximum located at $M = 1$ and one at the preferred aggregation size, $\langle M \rangle$ (Figure 2.4). The critical cluster density ($\rho_{ccd}$) can thus be defined as the density at which aggregates start to form (as opposed to a common alternative definition – the density at which as many particles are free as clustered). The preferred aggregation number increases with increasing density, and the location of the local minimum is similar for different values of the density. At densities above the critical cluster density, $p(\rho)$ transitions
Figure 2.3: Pressure vs. density for $\varepsilon = 1.6$ (top), $\varepsilon = 1.0$ (bottom) and the ideal-gas equation-of-state (black dashed line). Different temperatures are drawn as different colors. The arrows mark the critical cluster densities as calculated from the maximum in the free colloid number density ($\rho_{\text{free}}$, see Figure 2.5). For $\varepsilon = 1.0$, the temperatures are colored (from bottom to top) $k_B T = 0.291$, 0.333 and 0.374. For $\varepsilon = 1.6$ the temperatures are colored (from bottom to top) $k_B T = 0.607$, 0.674 and 0.725.

A repulsion-dominated interaction does not prevent clustering in SALR systems. In order to study such a case we set $\varepsilon = 1.0$, for which $B_2 > 0$ when $k_B T > 0.3104$ and $B_3 > 0$ when $k_B T > 0.485$. Panel (d) of Figure 2.4 shows that a clear separate peak in the CSD forms at $k_B T = 0.374$. The preferred aggregation numbers are smaller, compared to the attraction-dominated case, but nevertheless the CSD curves in both cases are qualitatively similar. According to the current understanding of surfactant
micellization, the formation of a separate distribution of clusters, as seen e.g. at $k_BT = 0.374$ and $\varepsilon = 1.0$, should also correspond to a response in the pressure. However, the slope of the pressure for this system is almost the same as the slope of ideal-gas pressure (the green curve at the lower panel of Figure 2.3); thus, the pressure does not respond to clustering.

Although the pressure can have different responses to clustering, the CSDs and the number density of free colloids for both attraction- and repulsion-dominated interacting systems show similar behavior. Upon increasing the number density above the $\rho_{ccd}$, regardless of a change in the pressure slope, the preferred aggregation number shifts to higher cluster sizes and the density of free particles decreases. Figure 2.5 presents the density of free colloids, $\rho_{\text{free}}$, calculated as the number density of particles in clusters $M < M_{\text{olig}}$, where $M_{\text{olig}}$ is the least probable cluster size that separates the oligomeric aggregates from the large clusters. At low densities, where there is no separate distribution in the CSD, $\rho_{\text{free}}$ and $\rho$ are equal. Upon increasing the density, free colloids self-assemble into clusters, and $\rho_{\text{free}}$ exhibits a maximum and then decreases monotonically. The maximum of $\rho_{\text{free}}$ is another way that one can define the critical cluster density. The non-monotonic behavior of $\rho_{\text{free}}$ is common in surfactants [32, 46, 70], although it is often assumed to constant above the ccd.

Critical cluster densities calculated as the: $i$) density at which a local maximum in the CSD occurs, $ii$) density at which the curvature of $p(\rho)$ is maximized and $iii$) the maximum of $\rho_{\text{free}}$, are presented in Figure 2.6. For both $\varepsilon$ values, at respectively low temperatures, the three methods give similar values for the critical cluster density. Agreement between the different methods decreases as the temperature increases, as is the case for model implicit-solvent surfactants [70], so that the critical cluster density indicated by the CSD is higher than that of the maximum in the pressure curvature. At higher temperatures, the ccd is larger, and the free cluster and large cluster distributions in the CSD become less distinct; those factors lead to the CSD
forming a separate distribution at large densities, after multiple large clusters have formed. In contrast to micellization, clustering of SALR particles into aggregates with a well-defined size takes place even when there is no significant change in the pressure slope. Thus, the range for which the critical cluster density can be found is much broader if one measures it from structural properties.

Figure 2.4: Cluster size distribution (CSD) weighted by the average density, $\rho$, for $\varepsilon = 1.6$ (upper panels) and $\varepsilon = 1.0$ (lower panels). Panel (a): $k_BT = 0.607$ and $\rho = 0.006, 0.023, 0.041, 0.055$, panel (b): $k_BT = 0.725$ and $\rho = 0.026, 0.031, 0.052, 0.073$, panel (c): $k_BT = 0.291$ and $\rho = 0.008, 0.012, 0.022, 0.032$ and panel (d): $k_BT = 0.374$ and $\rho = 0.031, 0.041, 0.052, 0.064$.

Figure 2.5: Free colloid density as a function of total density for $\varepsilon = 1.6$ and $\varepsilon = 1.0$. The color code is the same as in Figure 2.3. The dashed line shows $\rho = \rho_{\text{free}}$.

2.3.2 Cluster properties

In what follows, we study static and dynamic cluster properties in order to examine possible reasons why, at sufficiently high temperatures, the self-assembled repulsion-
dominated SALR clusters do not influence the pressure. A structural description of the clusters was made by considering their size distribution (Figure 2.4), density profiles (Figure 2.7), shape (Figure 2.8), radii of gyration (Figure 2.12) and radial distribution functions (2.11). Cluster size distributions (Figure 2.4) show that the $\varepsilon = 1.0$ case clusters consist of about half as many particles as the $\varepsilon = 1.6$ case clusters. However, the trends in the distributions are similar with temperature and density. In this section we show that the size of the clusters, measured both by the preferred aggregation number and the radius of gyration ($\langle R_g \rangle$), are the only properties that differ qualitatively between the two potentials. Furthermore, the repulsion-dominated case ($\varepsilon = 1.0$, $k_B T = 0.37$) does not characteristically differ from the lower-temperature behavior.

Although the size of the clusters depends on $T$ and $\rho$, we propose a simple geometric model for predicting the preferred aggregation number of SALR systems, which requires only pair potential information. The size and shape of self-assembled surfactant aggregates can be estimated using the relative length and headsize of the surfactant. Because SALR particle self-assembly is driven not by amphiphile shape,
but by interaction competition, the length-scales are set from the pair potential. The aggregation number is approximated as the surface area of a cluster divided by the area occupied by an interface-occupying particle,

\[ \langle M \rangle = \frac{S A_{\text{cluster}}}{a_{\text{particle}}} = \frac{4\pi (r_{\text{att}})^2}{(r_{\text{well}})^2} = 4\pi \left( \frac{r_{\text{att}}}{r_{\text{well}}} \right)^2 \]  

(2.3)

where the surface area is approximated as the surface area of a sphere with a radius set to the range of attraction distance, and the area of an interface-occupying particle is approximated as a square with length equal to the attraction well depth. The separation of a nearest-clustered-neighbor is approximated as the attractive well location \( r_{\text{well}} \), which is akin to the radius of a surfactant head group. The distance of an interface-occupying particle from the center of the cluster is approximated by the range of attraction \( r_{\text{att}} \), where \( U(r_{\text{att}}) = 0 \), as if “anchored” to the cluster center, in analogy to a surfactant tail length. For the potentials studied here, the prediction is reasonable: \( \langle M \rangle (\varepsilon = 1.6) = 41 \) and \( \langle M \rangle (\varepsilon = 1.0) = 32 \). The simple geometric model also performs quite well for SALR potential systems in the literature. Carbonneau and co-workers\[73\] reported \( \langle M \rangle = 24 - 31 \) at the ccd and \( k_B T = 0.35 - 0.45 \) for a square-well-linear potential; the simple geometric model predicts \( \langle M \rangle = 28 \). The model was able to predict the mean cluster size within 4 - 50 % error for double-yukawa-hard-sphere potentials with attractive \( \varepsilon \) values ranging from 0.9 to 1.5 \[71\]. Bomont et al. showed that SALR cluster shape and size can have non-trivial behavior with small changes to the potential; the simple geometric model cannot capture that behavior, nor trends in \( k_B T \) and \( \rho \), which is also the case for similar amphiphile micelle predictions.

Density profiles (Figure 2.7) within the clusters are qualitatively similar across \( T \) and \( \varepsilon \). The high-density core of the clusters becomes less dense as the temperature is increased, and the different \( \varepsilon \) values are within each others’ fluctuations at their
respectively matched temperature. Within the same $\varepsilon$ value, the density drops at similar distances from the center-of-mass, but with different slopes; lower-temperature clusters have a sharper transition from the cluster to solution. The high-density core has a similar size for both $\varepsilon$ values, near $r = 1.5\sigma$, however the lower-density corona and overall size are larger for the $\varepsilon = 1.6$ cases. Since the densities of the repulsion-dominated case ($\varepsilon = 1.0$, $k_B T = 0.374$) and the matched-temperature case ($\varepsilon = 1.6$, $k_B T = 0.725$) are similar, the internal structure of the clusters do not explain the change in the pressure behavior. The differences between the $\varepsilon = 1.0$ and $\varepsilon = 1.6$ density profiles is due to the different length-scales where attraction and repulsion compete, as in the simple geometric model. The effective potential on the interfacial particles, the distance of which from the center-of-mass determines the size, is due to the attractive near-neighbors, the repulsive far-neighbors and the effective repulsive out-of-cluster environment.

![Figure 2.7: Density profiles within a cluster as a function of distance from the cluster center-of-mass. The inset shows the x-axis normalized by the radius of gyration of the preferred aggregation number for each $\varepsilon$ value and temperature. Densities are averaged over clusters of sizes $M > M_{\text{olig}}$. All data is from MD simulations with $L = 30\sigma$ and $\rho = 0.063$. The color code is the same as in Figure 2.3.](image)

As the temperature increases for both values of $\varepsilon$, the CSDs show less defined free and large cluster populations, and the density profiles show that the cluster interface
becomes less sharp. The shape of the clusters could be quite different between the two cases. Figure 2.8 shows that clusters become less spherical as the temperature rises, as calculated by the ratios of the principal moments of inertia. The specific trend in the ratios of the principal moments of inertia signify that clusters go from more spherical to more oblate ellipsoidal with increasing temperature. The most important feature in Figure 2.8 is that the trend of the shape with temperature is similar for both $\varepsilon = 1.0$ and $\varepsilon = 1.6$, and that there is no change in the trend at $k_B T = 0.374$ for $\varepsilon = 1.0$ where repulsion-dominated self-assembly begins. Configuration snapshots of the high temperature systems are shown in Figure 2.9, and visually agree with the conclusion made from the quantitative measures. The clusters in the Figure 2.9 seem to be non-convex and relatively ill-defined, especially when compared to the snapshots at lower temperatures included in the electronic supplementary information.

![Figure 2.8: Ratios between moments of inertia for different principle directions, $I_1/I_3$ (solid symbols) and $I_2/I_3$ (open symbols), for both attraction strengths ($\varepsilon = 1.6$: red and $\varepsilon = 1.0$: blue). The ratios were computed for preferred aggregation number sized clusters at each temperature above the $\rho_{ccd}$.](image)

Radial distribution functions were calculated to provide insight into how the key entities of the self-assembled systems, free oligomer-sized and large clusters, interact. The oligomer-oligomer and oligomer-cluster radial distribution functions,
Figure 2.9: Snapshots of configurations for $\varepsilon = 1.0$, $k_B T = 0.374$ (left panel) and $\varepsilon = 1.6$, $k_B T = 0.725$ (right panel). In both cases $\rho = 0.053$ and $L = 17\sigma$. Particles colored gray are part of small, oligomeric aggregates; otherwise particles of matching colors are part of the same larger cluster.

plotted in Figure 2.10, show little difference between different temperatures and $\varepsilon$ values. Oligomer-oligomer radial distributions show that as the temperature increases oligomers are able to approach closer to each other. Similarly, the oligomer-cluster radial distribution functions show that oligomers are able to approach closer to clusters at higher temperatures. Attraction and repulsion between oligomers and clusters are also slightly stronger and longer-ranged at lower temperatures. For the cluster-cluster radial distribution functions (Figure 2.11), the amplitude fades more drastically with temperature. The $\varepsilon = 1.0$ radial distribution functions are shifted to lower values than for $\varepsilon = 1.6$; the inset of Figure 2.11 shows that this shift is due to the different sizes of clusters. Importantly, the radial distribution functions show that there are no qualitative differences in the interactions between different cluster sizes as a function of temperature for the different attraction strengths, and cannot, therefore, explain the repulsion-dominated pressure behavior.

The principal radii of gyration for different cluster sizes at the higher temperature of interest, plotted in Figure 2.12, demonstrate the effect of cluster size on shape. As clusters grow, their shape changes from more spherical to rod or prolate ellipsoids, for $M > \langle M \rangle$ shown by the inset of Figure 2.12. This type of cluster growth has been observed for model surfactants[51, 116]. This is distinct from the sphere-to-oblate
Figure 2.10: Oligomer-oligomer ($g_{\text{olig-olig}}(r)$, bottom) and oligomer-cluster ($g_{\text{olig-clus}}(r)$, top) radial distribution functions. Calculated using center-of-mass positions of all oligomer-sized clusters of size $M \leq M_{\text{olig}}$ and large clusters of size $M > M_{\text{olig}}$. All data are from MD simulations with $L = 30\sigma$, at $\rho = 0.063$. The color code is the same as in the pressure figure in the main text.

Figure 2.11: Cluster-cluster ($g_{\text{clus-clus}}(r)$) radial distribution functions. Calculated using center-of-mass positions of large clusters of size $M > M_{\text{olig}}$. The inset shows the x-axis normalized by the average radius of gyration of the preferred cluster size. All data are from MD simulations with $L = 30\sigma$ and $\rho = 0.063$. The color code is the same as in the pressure figure in the main text.

behavior observed in the moment of inertia figures with respect to temperature. Similar behavior of $\langle R_g \rangle (M)$ is observed at lower temperatures. For surfactants, it is argued that this is due to a decrease in head-to-tail ratio\cite{21}, and the limit of the distance of the tail-length anchoring to the center. For SALR systems, the effective
range of attraction seems to anchor the particles, and prefer to transition into one single, as opposed to two, elongated axes.

Figure 2.12: Average radius of gyration of clusters with $M$ particles along the principal (closed symbols), secondary (open symbols) and tertiary (stars) axes for $\varepsilon = 1.0, k_B T = 0.374$ (green squares) and $\varepsilon = 1.6, k_B T = 0.725$ (orange circles). The inset shows the same data, with the x-axis normalized by the preferred aggregation number. All data are from MD simulations with $L = 30\sigma$ and $\rho = 0.063$.

The static properties of the clusters for the two considered values of $\varepsilon$ are very similar, with the most prominent difference being the average number of particles which form the clusters. However, in the case of surfactant systems, relatively small micelles can have impact on the pressure slope[117].

The change in pressure upon the self-assembly of clusters is often stated to be caused by a transition from many independent kinetic entities, free particles, to fewer kinetic entities, clusters. Since the transition from pressure-affecting cluster formation to non-pressure-affecting cluster formation is not signaled in the static, structural characteristics of clusters across temperature and $\varepsilon$, we investigated the cluster dynamics. The dynamics of clusters are quantified by the cluster life-time, $\tau$, as a function of the cluster size, $M$, and are presented in Figure 2.13. Cluster life-times are computed as the time, in $\delta t$ units, between two consecutive changes in the cluster size (see Sec. 2.2.3 for details). It is important to note that a majority of

29
Cluster members can stay intact for a long time, but frequent hop events can lower the life-time of the cluster. The mean life-times of the aggregates were calculated at densities well above the critical cluster density. At low temperatures, for both ε values, free particles and preferred-size clusters (most of which are within one standard deviation of a gaussian fit to the CSD) have longer life-times, than aggregates of sizes close to \( M_{\text{olig}} \), defined by the minimum in the corresponding CSD. As the temperature rises the minimum gradually fades. This transition corresponds with the softening of the transition in the pressure.

The life-times for \( \varepsilon = 1.6 \) are generally lower than those for \( \varepsilon = 1.0 \). The fundamental similarities between all the cases become apparent when the temperature, average cluster size and life-time of a monomer are taken into account. In the inset of Figure 2.13 the life-times are collapsed using a normalization, \( \bar{\tau}(M) = \left( \frac{\tau(M)}{\tau(1)} \right) \left( \frac{\rho_{\text{corona}}}{\beta} \right) \), where \( \tau(1) \) is the life-time of a single SALR particle, \( \beta = 1/(k_B T) \) and \( \rho_{\text{corona}} \) is the number density of particles in all the clusters’ corona in the total volume, which are the most probable to hop off the clusters. The number density of particles in the coronas is calculated from the number of clusters in the system \( N_{\text{clus}} \) and the number of particles in the average cluster’s surface \( N_{\text{surface}} \), \( \rho_{\text{corona}} = \frac{N_{\text{clus}} N_{\text{surface}}}{V} \). The density of clusters is approximated as \( N_{\text{clus}}/V = \rho/\langle M \rangle \), and the number of particles at the surface is approximated as the fraction of particles in the volume of a shell with width \( \sigma \) and with the radius equal the radius of gyration \( \langle R_g \rangle \) of the preferred-aggregation-number-sized clusters \( \langle M \rangle \), specifically:

\[
N_{\text{surface}} = \langle M \rangle \frac{V_{\text{corona}}}{V_{\text{cluster}}} = \langle M \rangle \frac{(4\pi/3) (3 \langle R_g \rangle^2 \sigma + \sigma^3/4)}{(4\pi/3) \langle R_g \rangle^3} \sim \frac{3\sigma}{\langle R_g \rangle}. \tag{2.4}
\]
The full normalization is thus

\[
\bar{\tau}(M) = \left( \frac{\tau(M)}{\tau(1)} \right) \left( \frac{\rho \sigma}{\beta \langle R_g \rangle^3} \right) \left( 3 \langle R_g \rangle^2 + \sigma^2/4 \right),
\]

(2.5)

which is only effective at densities well above the ccd. The \( \varepsilon = 1.6 \) life-times collapse on each other better than the \( \varepsilon = 1.0 \) life-times; the difference could be due to the larger radius of gyration for \( \varepsilon = 1.6 \) clusters, making the \( V_{\text{corona}} \) approximation more accurate.

Figure 2.13: The mean life-times of SALR clusters \( (\tau(M)) \), as a function of cluster sizes \( (M) \) in non-dimensional \( \delta t \) units. The inset shows the reduced mean life-times \( (\bar{\tau}(M), \text{see text for normalization}) \) as a function of cluster sizes reduced by the preferred aggregation number \( (M/\langle M \rangle) \). The densities plotted are all above the ccd, specifically: \( \rho(\varepsilon = 1.0, k_B T = 0.291) = \rho(\varepsilon = 1.6, k_B T = 0.607) = 0.053, \rho(\varepsilon = 1.0, k_B T = 0.333) = \rho(\varepsilon = 1.6, k_B T = 0.673) = 0.064 \) and \( \rho(\varepsilon = 1.0, k_B T = 0.374) = \rho(\varepsilon = 1.6, k_B T = 0.725) = 0.077. \) The color code is the same as in Figure 2.3. For \( \varepsilon = 1.0, k_B T = 0.291 \) and \( \varepsilon = 1.0, k_B T = 0.333, \) values of \( \tau(M) \) are not included above certain cluster sizes because \( \rho(M > 55) \) and \( \rho(M > 75), \) respectively, are too small to tractably sample reliably.

2.4 Discussion and conclusions

We have compared static and dynamic properties of SALR particle clusters for two attraction strengths, \( \varepsilon, \) keeping the strength of repulsion constant, that lead to a sign
change in the second virial coefficient. For both cases, the self-assembly of clusters, characterized by a distinct peak in the size distribution, results in a significant change in the slope of the pressure as a function of density at relatively low temperatures. The clusters formed in the more attractive system are about two times larger than those in the more repulsive system. The difference in size follows from differences in the ranges of attraction and repulsion and the attraction-to-repulsion ratio; we have shown that the preferred aggregation number can be approximated using a simple geometric model, similar to that used for surfactants. Apart from the difference in preferred sizes, the structural properties of the clusters in the more attractive and more repulsive cases, specifically the density profiles and cluster shape, are similar, varying with temperature in analogous fashion. An analysis of the particle exchange kinetics shows that at low temperatures, clusters that are more probable to form also have longer life-times. The more repulsive systems generally show longer life-times, however we propose a normalization which takes into account the effect of temperature and cluster sizes. After normalization, the cluster life-times are similar for different values of $\varepsilon$ and $T$.

Larger clusters and free clusters become less distinct as the minimum in the cluster size distribution, which separates clusters and oligomers or monomers, rises at higher temperatures. In both cases, clusters become less spherical and compact at higher temperatures. However, there is a gradual, yet significant change in the pressure’s response to clustering for the two systems as the temperature increases. As the distinction between clusters and free particles weakens, the transition becomes smoother. The distinction between large cluster and free cluster life-times weakens even more drastically. At high temperatures for the more repulsive system, repulsion and attraction become equally competitive and the pressure obeys the ideal-gas law, even at densities well above the critical cluster density. The more attractive system maintains a discernible transition in the pressure, albeit weaker.
than lower-temperature systems. Thus, our study shows that, in the case of clusters formed in repulsion-dominated SALR systems, a separate peak in the cluster size distribution is not sufficient to assure pressure slope changes. On the other hand, attraction-dominated SALR system clusters are qualitatively very similar to surfactant micellar aggregates.

Our study does not identify a significant difference between the structure or dynamics of pressure-affecting clusters and non-pressure-affecting clusters, apart from their size. However the size cannot be the sole explanation, because small SALR clusters effect the pressure slope. Furthermore, in the case of surfactants, even small micelles can influence the pressure slope. For both SALR and surfactant systems pressure response to clustering weakens as the temperature increases. Our results show that SALR particles can cluster even if they are in a repulsion-dominated environment, which is not accompanied by a response in the pressure. Large clusters can possibly exist because of the compensation of the short-range attraction and their lack of directionality. This repulsion-dominated case has similarities with soft and purely repulsive particles[118–121]. Namely, in the soft and purely repulsive systems the pressure slope does not decrease when the system is in the fluid phase and particles gradually cluster upon increasing the density [122]. What distinguishes repulsion-dominated clustering for SALR systems from soft and purely repulsive systems is the formation of a separate and distinct distribution of clusters in the CSD.

The size of and pressure response to SALR cluster self-assembly primarily depends on the ratio between the repulsion and attraction strengths, that often result from electrostatic and depletion interactions, respectively. The attraction-to-repulsion interaction ratio ought to be easily tuned in experiments, but studies of SALR systems suggest the opposite [103, 123], pointing, for example, to correlations in charge redistribution as one of the reasons. Our results indicate that thermodynamic property behavior in SALR systems varies significantly with interaction strength and tempera-
ture. Thus, it is crucial that system design aimed at cluster formation set interactions with care, and not be based solely on the pressure.
Chapter 3

Determination of the critical micelle concentration in simulations of surfactant systems

3.1 Introduction

The formation of long-lived micellar aggregates in surfactant solutions causes a transition in many of their properties, such as the conductivity, surface tension, and osmotic pressure. The concentration of free, unassociated surfactant molecules in solution is of central importance in the theoretical description and physical understanding of micellization [10]. A common assumption is that the concentration of free surfactants is constant above the micellar transition, at which aggregates start forming in solution. This transition concentration is known as the critical micelle concentration (cmc). This approximation of the cmc is used in many experimental techniques – for example, specific conductivity [40], nuclear magnetic resonance [35], time-dependent static light scattering [41], steady-state fluorescence quenching.

and time-resolved fluorescence quenching [37] measurements use models that assume a constant free surfactant concentration to calculate micellar aggregation numbers. Similarly, many simulation studies obtain the cmc from the concentration of free surfactants [56, 124]. Explicit-solvent simulations are often restricted to measuring the cmc this way because low concentrations are not tractable for strongly micellizing systems [57, 124–126].

For ionic surfactants, it is now widely accepted that there are significant changes in the free surfactant concentration as the total surfactant loading is increased above the cmc. This is confirmed by experiments [32, 127, 128], simulations [31, 129–132] and theory [46, 133, 134]. The free surfactant concentration goes through a maximum near the cmc and then decreases at higher overall loadings. Recent simulations of sodium octyl sulfate [31] illustrate how dramatic this effect is: the free surfactant concentration was observed to be ten times lower at a total concentration of 1 M versus its value at 250 mM. The main reason for this decrease in free surfactant concentration for ionic surfactants is the changing ionic strength of the solution at higher loadings [127]. The correction initially proposed for experiments [32, 127] has also been implemented in simulations [31, 46, 125, 132] and justified from theory [46].

The magnitude of the decrease in free surfactant concentration for nonionic surfactants is significantly smaller than for ionic surfactants and still somewhat controversial. Law-of-mass-action theory for micellization predicts a monotonic increase of the free surfactant concentration above the cmc [38, 43, 135]. However, a clear decrease in free surfactant concentration above the cmc has been observed in several simulation studies of nonionic surfactants [51, 129, 136–141]. The main objective of the present work is to clarify the situation with respect to this decrease through the use of large-scale simulations of a simple model nonionic surfactant (also used in several prior studies) and to test the hypothesis that the main contributor to this decrease is the solution volume made inaccessible to free surfactants by micellar
aggregates. The method used to calculate the solution volume made inaccessible to free surfactants is described in Section 3.2.3, and the results are presented in Section 3.3.3. An excluded volume modification to a law-of-mass-action theory is incorporated in Section 3.3.4. A correction to the free surfactant concentration for these volume exclusion effects would lead to reliable methods to obtain the cmc from simulations at high loadings.

There are alternative methods for the calculation of the cmc from simulations that do not rely on the free surfactant concentration. Specifically, at sufficiently high temperatures (or for weakly micellizing systems), the equilibrium aggregation number distribution for micelles can be calculated from the molecular positions that are trivially recorded in simulations. The lowest concentration with a micellar peak in the distribution provides a cmc calculation method [10, 45, 141–144]. The cmc can also be obtained from the osmotic pressure of the solution. This method is occasionally used in experimental studies [129, 145, 146]. In simulations, the osmotic pressure is readily available from the partition function generated via histogram reweighting in the grand-canonical ensemble [25, 147, 148]. In Section 3.3.1 these alternative methods are compared, specifically the osmotic pressure, aggregation number distribution, and free surfactant concentration for a well-controlled model system.

Temperature has a complex impact on micellizing systems [46, 149, 150]. Experimental studies of many different nonionic surfactants, using a variety of techniques, show a minimum in the cmc with respect to temperature [150–154]. The cmc of non-ionic surfactants calculated from implicit-solvent simulations monotonically increases with temperature [25], unless there is a temperature dependent parameter [46, 149]. For both experiments [150, 153] and simulations [25, 155, 156], the transition of the measured properties becomes unclear at high temperatures. Experimental studies of the temperature effect on the cmc typically stay within a temperature range corre-
sponding to strong micellization, since micelles are easier to detect at these conditions [40, 128, 150]. The smaller aggregates that form at higher temperatures generally result in a weak response in techniques such as titration calorimetry [150]. By contrast, it is relatively easy to obtain aggregation data at elevated temperatures from simulations. At high temperatures the mobility is higher and it is easier to overcome free energy barriers to aggregate breakup, while at low temperatures hysteresis between micellar and free states makes equilibration difficult. In prior simulations of nonionic surfactants it has been observed that as temperatures increases, nonionic surfactant aggregates shrink in size until the system no longer has an identifiable micellization transition [25, 136, 137]. Another objective of the present study is to analyze micellar behavior near the upper temperature limit for micellization. Section 3.3.2 addresses this objective.

3.2 Methods

3.2.1 Surfactant Model

Surfactants were modeled using the Larson et al. implicit-solvent lattice model [157]. In this model, space is discretized onto a 3-dimensional simple cubic lattice. The aggregation behavior of this model has been extensively studied by simulations [24, 25, 51, 116, 137, 138, 158–163]. The present study focuses on the first micellar transition to roughly spherical aggregates. We chose to study the $H_4T_4$ surfactant, composed of 4 solvophilic “head” (H) beads and 4 solvophobic “tail” (T) beads, because there are several prior cmc estimates as a function of temperature [25, 51, 147]. Bonds connecting successive surfactant beads can be along vectors (0,0,1), (0,1,1), and (1,1,1) on the lattice, as well as their reflections with respect to the principal axes, resulting in 26 possible directions at distances from 1 to $\sqrt{3}$ in units of the lattice spacing. These directions also define locations for possible interactions of
non-bonded T sites, which have a strength of $\epsilon_{T,T} = -2$ in reduced units. All other interactions are set to zero. Each lattice site can only be occupied by one bead. Empty sites on the lattice can be considered occupied by a monomeric (implicit) solvent. Temperature is defined with respect to the same energy scale as the nearest-neighbor interactions.

### 3.2.2 Monte Carlo Simulations

Monte Carlo (MC) simulations were performed in the canonical ($NVT$) and grand-canonical ($\mu VT$) ensembles. The use of both ensembles served as a consistency check to ensure that the decrease of the free oligomer volume fraction ($\varphi$) observed does correspond to equilibrium behavior. Structural properties were measured using box lengths ($L$) of 40 and 60 lattice sites to test for system-size effects. These “large-scale” simulations are distinct from those used with histogram reweighting for the osmotic pressure calculations, which have smaller system sizes and are described later in this section. Temperatures ($T$) were varied from 6.5 to 10.5 in reduced units. Over this interval, the $\text{H}_4\text{T}_4$ surfactant transitions from strongly micellizing to weakly micellizing to non-micellizing behavior as temperature is increased [25, 147]. Every $NVT$ simulation state point was run with both a preformed micellar initial configuration and with a gas-like initial configuration. Similarly, all $\mu VT$ simulations were run with both high number of surfactants ($N$) (with preformed micelles present) and low $N$ (gas-like) initial configurations. The variations in initial configuration and initial concentration for $\mu VT$ were performed to check for hysteresis effects. Simulations were equilibrated for $10^7$-$10^{10}$ MC steps. Approach to equilibrium was determined by a plateau in the $N$ and the energy ($U$) values. Another condition for equilibrium was the agreement of aggregation number distribution of simulations starting from a micellar and gas-like state. As expected, fewer Monte Carlo steps were needed for simulations initialized with near-equilibrium configurations. Production
data were generated from $10^{10}$ MC steps beyond the equilibration period. The MC move mix was 60% insertions/deletions, 39.5% reptations [164] and 0.5% cluster center-of-mass displacements in the $\mu VT$ ensemble. In the $NVT$ ensemble, the mix of moves was 99.5% reptations and 0.5% cluster center-of-mass displacements. Insertion moves were performed by regrowing the surfactant using configurational-bias [165].

Large-scale ($L = 40$ or 60) simulations at temperatures below 6.5 did not reach equilibrium using the methodology described. This happens despite the large number of MC steps used, because of the very strong hysteresis and low probability of displacing/inserting a surfactant into or out off an existing micelle. The free surfactant concentration is especially sensitive to hysteresis; however, as shown in Section 3.3.3, the free surfactant volume fraction dependence becomes less pronounced at low temperatures.

Histogram reweighting was used to calculate the thermodynamic properties from $\mu VT$ simulations in small systems with $L = 15$ and $L = 20$, as follows. Over the course of a simulation, the number of surfactants and the system energy are recorded to a histogram from which the probability distribution function, $f(N,U)$, can be calculated. The probability distribution function can be expressed as a function of the microcanonical partition function, $\Omega$, and the grand partition function, $\Xi$, as follows:

$$f(N,U) = \frac{\Omega(N,V,U)}{\Xi(\mu,V,\beta)} \exp(\mu \beta N - \beta U) \quad (3.1)$$

The inverse temperature is $\beta = 1/k_B T$. The Ferrenberg and Swendsen method [166] uses simulations over a range of chemical potentials and temperatures to estimate the partition function within a multiplicative constant, $C$:

$$\Xi(\mu,V,T) = C \sum_{U,N} \exp(S(N,V,U) + \mu \beta N - \beta U) \quad (3.2)$$
The unknown constant can be matched using the ideal gas equation of state at sufficiently low surfactant concentrations. From the grand partition function, the pressure \( P \) can be calculated from the equation:

\[
P = \frac{1}{V\beta} \ln \Xi
\]  

(3.3)

The osmotic pressure (\( \Pi \)) of the surfactant in a two-component system is equivalent to the normalized pressure, \( JP/\epsilon T \), calculated in this procedure, where \( \epsilon \) is the energy scale and \( J \) is the volume of the surfactant. For the model of interest, \( J = 8 \), since the surfactant has 8 total sites, and \( \epsilon = 1 = (2\epsilon_{H,T} - \epsilon_{H,H} - \epsilon_{H,T})/2 \). The change in the response of osmotic pressure to total concentration is an effective method for calculating the cmc, and is one of the many ways it is calculated in experiments [1]. Calculating average cmc’s was done by using 6 different sets of simulation histograms. Uncertainties for the cmc’s from histogram reweighting were obtained by propagating the error from the graphical methods of finding the transition in \( \Pi \), with the uncertainty from the average calculated from the 6 sets. It is imperative that histogram reweighting is done with histograms from simulations that have reached full equilibrium with respect to the number of surfactants in the system; this is significantly easier at high temperatures. Accurate calculation also requires that the low-concentration behavior closely obeys the ideal gas law and that the generated partition function can correctly reproduce average \( N \) and \( U \) from actual simulations, thus confirming that the histogram set that was used for construction of the partition function is internally consistent.

Cluster distributions were calculated for every simulation. A surfactant is considered to be part of a cluster if at least one of its tail beads is a nearest-neighbor of a tail bead belonging to the cluster. This methodology was used previously [25, 51]. The sizes of clusters were tallied over the course of the simulation. The lowest volume
fraction at which there is a separated, polydisperse micellar-sized cluster distribution is one metric by which the cmc is determined. The cmc’s calculated from the aggregation number distributions correspond to a range of values at each temperature, rather than a single value: the upper bound of this range is defined by the lowest $\langle \varphi \rangle$ where a micellar distribution is always observed, and the lower bound is defined as the highest $\langle \varphi \rangle$ where a micellar distribution is never observed, regardless of the initial state.

The local minimum, typically occurring at $M \approx 20$, was identified for each simulation and used to measure the volume fraction of free oligomeric clusters ($\varphi_{\text{olig}}$). The volume fraction of free monomers and oligomeric clusters smaller than the minimum were added to get the $\varphi_{\text{olig}}$ for every simulation and averaged over 600 configurations. The maximum in the $\varphi_{\text{olig}}$ was used to calculate the cmc. Uncertainty in the cmc, as calculated by the maximum in the $\varphi_{\text{olig}}$, originates from the statistical error in the measurement of $\varphi_{\text{olig}}$ over the 600 configurations, and the fluctuation of $\varphi_{\text{olig}}$ near the maximum. To account for the fluctuation, the maximum is calculated by averaging the $\varphi_{\text{olig}}$ values, from both the $NVT$ and $\mu VT$ simulations, within 10% of the highest $\varphi_{\text{olig}}$ recorded at a specific temperature. The 10% threshold was chosen because, for all temperatures, values of $\varphi_{\text{olig}}$ below the 10% threshold are below the maximum with certainty based on the measured $\varphi_{\text{olig}}$. The uncertainty of the cmc is calculated by propagating the uncertainty from $\varphi_{\text{olig}}$ measurements within the 10% range.

3.2.3 Inaccessible Volume Calculation

The volume that is inaccessible to free oligomers is not simply that which is occupied by surfactants. There is an additional inaccessible volume in the vicinity of micelles, where a surfactant present will be considered part of an existing cluster, rather than a free monomer. In the present work, clusters are classified as a micelle or an oligomer
by their aggregation numbers ($M$). The local minimum in the cluster distribution is the cutoff for an aggregate to be considered an oligomer or a micelle ($M_{\text{micelle}}$). For example, in Figure 3.3, at $\mu = -45.5$, the cutoff is 20. Clusters from simulation snapshots were identified using the Hoshen-Kopelman algorithm [113].

For every simulation snapshot, a set of isolated “test” surfactant configurations were generated using the Rosenbluth and Rosenbluth algorithm [167]. The surfactant center-of-mass was calculated for each test configuration as the nearest-integer value. The fraction of configurations that result in overlap or association with an existing micelle was then determined for each lattice site. A site is deemed inaccessible in the following cases: (i) a bead in the monomer and a bead of a surfactant in a micelle co-occupy a site or (ii) the monomer becomes part of the micelle, i.e., a tail bead of the monomer is a nearest neighbor of a tail bead of a surfactant in the aggregate. Case (i) makes the site excluded to the monomer, while case (ii) makes it associated with the micelle. The inaccessible volume is defined as the sum of excluded and associated sites. The inaccessible volume was measured from 600 snapshots of each simulation run. From each snapshot, the inaccessible volume was calculated using 100, 50, 25 and 10 test monomer configurations at every site. We found that using more than 25 test configurations neither changed the average, nor improved the precision. Uncertainties for the inaccessible volume were calculated using Flyvbjerg and Petersen error analysis [168]. Typically 2-8 block transformations were required for the standard deviation to plateau. Larger micellar systems generally required more transformations than smaller systems composed of oligomers.

Figure 1 shows a schematic of this algorithm, projected in two dimensions for simplicity. In the figure, three different monomer configurations are inserted in every site around the trimer aggregate shown. Excluded (case i) and associated (case ii) sites are shaded with different colors. A small aggregate is used here as a clear
demonstration; in practice the inaccessible volume is only calculated for micelles, aggregates of significantly larger \( M \).

Figure 3.1: Visualization of the inaccessible volume calculation method. A two-dimensional schematic is shown for simplicity, but all calculations in this work are for a three-dimensional surfactant model. The center-of-mass of the test configuration is marked by X; different cross-hatchings are used for the “excluded” and “associated” sites, as discussed in the main text. Clear (unhatched) cells correspond to free volume around each aggregate.

3.3 Results and Discussion

3.3.1 Quantitative Measurement of the cmc

The osmotic pressure, \( \Pi \), is a thermodynamic property that can be measured experimentally \([129, 145, 146]\) or from simulations \([25, 147, 148]\) to obtain the cmc. Figure 2 shows how \( \Pi \) varies with volume fraction \( (\varphi) \) at different temperatures for our model surfactant, as obtained from \( \mu VT \) simulations. At low \( \varphi \), \( \Pi \) for all temperatures approaches the ideal-gas-law, manifested by a slope of unity \( (\Pi = \frac{8F}{cT} = \frac{8N}{V} = \varphi) \). The formation of micelles triggers a transition to a lower slope of \( \Pi \) vs \( \varphi \), resulting from the fact that the dominant independent kinetic entities in solution are now mul-
timolecular aggregates. The slopes of the $\Pi$ vs $\varphi$ curves above the cmc increase with temperature. These slopes are not system size dependent, which is a condition for micellization as opposed to a phase transition [25]. At low temperatures the micellar transition associated with the change in slope of $\Pi$ is clear, but at high temperatures the precise volume fraction at which micellization occurs ($\varphi_{\text{cmc}}$) is unclear.

Many possible criteria for identifying the cmc using properties that vary upon micellization have been proposed in the literature [25, 45, 169, 170]. The cmc is frequently defined as the concentration at which micelles begin to form in the system, but another possible definition is to identify it as the concentration at which micelles and free surfactants have equal concentration – the two definitions differ by approximately a factor of two. The former definition is used in the present work. Floriano and co-workers [25] defined the cmc as the point of intersection of the low-concentration and high-concentration limiting lines for the $\Pi$ vs $\varphi$ curves. In later studies, it was found that the location of the maximum in the second derivative of the $\Pi$ vs $\varphi$ curve is a better criterion [130, 155, 156]. In the present study, three
additional metrics were explored in order to determine the optimal approach – all methods produce essentially identical results at low temperatures at which the transition is very sharp, but start diverging at higher temperatures at which the transition is more gradual. We denote by “method I” the identification of the cmc as the concentration of surfactant for which the low-concentration and high-concentration limiting lines intersect [25], “method II” the maximum in the second derivative of $\Pi$ 

$$\left( \max \left| \frac{d^2 \Pi}{d \varphi^2} \right| \right),$$

“method III” the maximum in the curvature ($\kappa$):

$$\kappa = \frac{|d^2 \Pi / d \varphi^2|}{\left[ 1 + \left( d\Pi / d\varphi \right)^2 \right]^{3/2}} \quad (3.4)$$

“Method IV”, by Carpena and co-workers [170], was developed for conductivity experiments and defines a parameter in an integral of a sigmoid function fit for the cmc. It gives values typically lower than method III and higher than methods I and II. Finally, “method V” [169] defines the cmc as the intercept of the low-concentration limit and a horizontal line which intersects the zero in the second derivative of $\Pi$. This method gave cmc values consistently higher than those predicted by the other methods. Regardless of the differences in the magnitude of the cmc’s, all the methods showed the same trend of the cmc with respect to the temperature. However, the magnitude of the cmc’s calculated from Method III are most consistent with the total concentration dependence of the aggregation number distribution and the free surfactant concentration. Thus, method III was therefore the chosen definition of the cmc for $\Pi$ vs $\varphi$ curves. The differences from the earlier reported values are relatively small - for example, at $T = 7$, the earlier estimate of the cmc from [25] is 0.0089, 6% lower than the current value. The cmc volume fraction ($\varphi_{\text{cmc}}$) predicted by method III are shown in Table 3.1 as a function $T$ and box size. The consistency in results for the two system sizes studied is excellent; this shows that $L = 15$ is a sufficient size for accurate determination of cmc’s in this system.
<table>
<thead>
<tr>
<th>$T$</th>
<th>$L = 15$</th>
<th>$L = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.00</td>
<td>0.00190 ± 0.00004</td>
<td>0.0020 ± 0.0001</td>
</tr>
<tr>
<td>6.25</td>
<td>0.00289 ± 0.00007</td>
<td>0.0031 ± 0.0002</td>
</tr>
<tr>
<td>6.50</td>
<td>0.0045 ± 0.0001</td>
<td>0.0047 ± 0.0004</td>
</tr>
<tr>
<td>6.75</td>
<td>0.0066 ± 0.0002</td>
<td>0.0068 ± 0.0008</td>
</tr>
<tr>
<td>7.00</td>
<td>0.0094 ± 0.0002</td>
<td>0.009 ± 0.001</td>
</tr>
<tr>
<td>7.25</td>
<td>0.0132 ± 0.0003</td>
<td>0.013 ± 0.002</td>
</tr>
<tr>
<td>7.50</td>
<td>0.0178 ± 0.0005</td>
<td>0.017 ± 0.002</td>
</tr>
<tr>
<td>7.75</td>
<td>0.0234 ± 0.0008</td>
<td>0.022 ± 0.003</td>
</tr>
<tr>
<td>8.00</td>
<td>0.030 ± 0.001</td>
<td>0.028 ± 0.003</td>
</tr>
<tr>
<td>8.25</td>
<td>0.037 ± 0.003</td>
<td>0.034 ± 0.003</td>
</tr>
<tr>
<td>8.50</td>
<td>0.050 ± 0.001</td>
<td>0.050 ± 0.001</td>
</tr>
<tr>
<td>8.75</td>
<td>0.055 ± 0.003</td>
<td>0.058 ± 0.004</td>
</tr>
<tr>
<td>9.00</td>
<td>0.071 ± 0.003</td>
<td>0.071 ± 0.004</td>
</tr>
</tbody>
</table>

Table 3.1: Critical micelle volume fractions at various box sizes using histogram reweighting. Method III was used to calculate $\varphi_{\text{cmc}}$ from the solution osmotic pressure, $\Pi$.

The aggregation number distribution can also be used to calculate the cmc. A polydisperse distribution of larger aggregates, separate from the free oligomer distribution, is a necessary condition for micellization. The smallest concentration at which there is a minimum in the distribution that separates the oligomeric and micellar distributions is a method for measuring the cmc [142]. Figure 3.3 shows typical cluster distributions for $H_4T_4$, at the same temperature, $T = 8$, but different $\mu$ or $\langle \varphi \rangle$. The distribution at the highest chemical potential shown in the figure, $\mu = -45.5$, clearly indicates that micelles are present in the system. As $\mu$ is lowered the preferred aggregation number ($\langle M \rangle$) shifts to lower values of $M$. At the the lowest $\mu$ shown, -46.1, there is no longer a minimum in the distribution separating oligomers from micelles. Therefore the chemical potential corresponding to the cmc ($\mu_{\text{cmc}}$) is above -46.1 and below -45.9.

Large-scale simulations were performed in both the $\mu VT$ and $NVT$ ensembles, as described in Section 3.2.2. The same behavior shown in Figure 3.3 is seen in $NVT$ simulations, by varying the volume fraction instead of the chemical potential.
Figure 3.3: Volume fraction as a function of the aggregation number from $\mu VT$ simulations at $L = 60$, $T = 8.0$, for different imposed chemical potentials, $\mu$. Volume fractions are shown in Figure 3.4 as filled points of the same color as the corresponding chemical potential curve.

Specifically, the location and height of the micellar distribution agreed at the same $\langle \phi \rangle$. Simulations were run at $L = 40$ and 60 with initial gas-like and micellar configurations. There was not a size effect in the cluster distribution except for small $\phi_{\text{cmc}}$. For example, at $T = 6.5$ the preferred aggregation number is 84 and $\phi_{\text{cmc}} = 0.0045$. Therefore, at least 84 surfactants need to be present in the system to form a micelle of preferred size. In this system, a $L = 40$ and 84 surfactants corresponds to $\phi = 0.0105$, which poses an obvious system size effect; there are not enough surfactants to form a micelle at $L = 40$ for $0.0045 < \phi < 0.105$. Lastly, the aggregation number distribution takes much longer to equilibrate at low temperatures. Inserting a surfactant into a micelle is unlikely at low temperatures and growing a micelle from an oligomeric cluster is even less likely.

The aggregation number distributions were used to calculate the $\phi_{\text{olig}}$ to differentiate oligomeric and micellar aggregates. Figure 3.4 shows the $\phi_{\text{olig}}$ as a function of $\phi$ at $T = 8$. At low volume fraction all of the surfactants are in oligomeric clusters
(\(\varphi = \varphi_{\text{olig}}\)), which corresponds to the ideal-gas behavior seen in the osmotic pressure curves. As mentioned before, and analyzed in Section 3.3.3, \(\varphi_{\text{olig}}\) is non-monotonic above the cmc. The onset of micelle formation causes the \(\varphi_{\text{olig}}\) to deviate from unity, reaching a maximum and then decreases as the \(\varphi\) increases. The effect becomes more drastic at higher temperatures and is also analyzed in section 3.3.3. The cmc is the lowest concentration where the system is not solely composed of oligomeric clusters, corresponding to the smallest concentration at which \(\varphi_{\text{olig}} < \varphi\). The maximum in the \(\varphi_{\text{olig}}\) approximates this concentration which is otherwise difficult to define from the \(\varphi_{\text{olig}}\) curve. The maximum in the \(\varphi_{\text{olig}}\) is justified as a definition of the cmc by comparison to the other cmc calculation methods later in this section. The \(\varphi_{\text{olig}}\) was calculated in both the \(\mu VT\) and \(NVT\) ensembles, and at \(L = 40\) and \(60\). This was done to assess if the decreases in \(\varphi_{\text{olig}}\) above the cmc is equilibrium behavior. The impact of simulation ensemble and system size are discussed in Section 3.3.3.

Figure 3.4 also shows a comparison of cmc methods from the \(\Pi\) curve, aggregation distributions, and the \(\varphi_{\text{olig}}\) curve. Data from \(\Pi\) are in red and \(\varphi_{\text{olig}}\) are in black. Note that \(\varphi_{\text{olig}}\) and \(\Pi\) are on the same scale and are both dimensionless, a convenient result of the definition of \(\Pi\). The color-filled symbols correspond to curves in Figure 3.3. The cmc from the onset of a minimum in the cluster distribution separating oligomeric and micellar aggregates is between \(\langle \varphi \rangle = 0.0309\) (red) and 0.0333 (blue). The cmc, as defined by the aggregation number distribution, is thus within this interval \((0.0309 \leq \varphi_{\text{cmc}} \leq 0.0333)\) and the definitions from the maximum in \(\varphi_{\text{olig}}\) and \(\Pi\) must be within the interval at \(T = 8.0\). The maximum in \(\varphi_{\text{olig}}\) is 0.0323, which is within the interval. Among the many methods that measure the cmc from the maximum in the curvature of the osmotic pressure, method III, agrees well with the cmc calculated from the maximum in \(\varphi_{\text{olig}}\) and the cluster distribution interval.

A comparison of cmc’s obtained from the osmotic pressure, the free oligomer volume fraction, and the aggregation number distributions as a function of temperature
Figure 3.4: Comparison of $\varphi_{\text{cmc}}$ calculation methods at $T = 8.0$. The osmotic pressure at $T = 8.0$ obtained from $\mu VT$ simulations at $L = 15$ (red line) is shown, along with the cmc calculated from $\Pi$ using method III (vertical dashed line). These values are compared to the free oligomer volume fraction, $\varphi_{\text{olig}}$, (circles) from $T = 8.0$ $L = 60$ $\mu VT$ simulations. The colors of the filled symbols correspond to the colors of the state points shown in Figure 3.3. The ideal-gas law is represented by a solid black line with a slope of unity.

is shown in Figure 3.5. Deviations between results obtained through different physical quantities occurs for $T > 8.0$, the “high-temperature” region at which micellization becomes less sharp. This region is the subject of the next section. Qualitatively, it is clear that at the point at which there is a minimum in the aggregation number curves (Figure 3.3), there are already several micellar aggregates in the system. Thus this measure is likely to give a concentration higher than that obtained from the osmotic pressure or the free surfactant concentration, both of which physically approximate the point at which the first aggregate forms in a large system.

Hysteresis at low temperatures is characterized by the large range of the cmc as calculated by the aggregation number. Some simulations at volume fractions near the cmc were hard to equilibrate, specifically, simulations initialized from gas-like and micellar aggregates resulted in different micellar peak heights and locations even after long runs. These effects added largely to the uncertainty of the $\varphi_{\text{cmc}}$ values.
from the maximum in $\varphi_{\text{olig}}$ at $T = 6.5$, which had large fluctuations near the cmc and uncertainty in the $\varphi_{\text{olig}}$ values as calculated from individual configurations. The range of values at higher temperatures in the cmc as calculated from the aggregation number distributions is due to the fact that a finite number of $\langle \varphi_{\text{olig}} \rangle$ were simulated.

![Figure 3.5: Comparison of $\varphi_{\text{cmc}}$ values from the maximum in $\varphi_{\text{olig}}$ (green squares), $\Pi$ (black circles) and the minimum in the aggregation number between oligomeric and micellar distributions (red area) as a function of temperature.](image)

**3.3.2 High-Temperature Limit for Micellization**

The response of properties to the micellar transition becomes less drastic as the temperature increases, which makes the precise determination of a temperature above which there is no micellar transition a challenge. In order to obtain a reliable free energy function from grand canonical simulations of micellizing systems, however, it is convenient to “link” states containing aggregates and states containing only free surfactants and oligomers through simulations at sufficiently high temperatures so that sampling of intermediate occupancy states can be performed reversibly. As
observed already in Section 3.3.1, different methods for estimating the cmc give divergent results at temperatures approaching this limit. Figure 3.2 illustrates that at the highest temperature shown, \( T = 9.0 \), the osmotic pressure curve shows only a gradual change in slope, thus making it difficult to pinpoint the onset of the micellar transition. A comparable difficulty in cmc determination is present when using the aggregation number distributions, as illustrated in Figure 3.6. The distributions are shown at \( \varphi = 0.15 \), a value higher than the extrapolated cmc value at \( T = 9.50 \) based on data at lower temperatures. As seen in the figure, there is no preferential aggregation number at \( T \geq 9.5 \), the distribution is monotonically decreasing and extends to very large clusters. Specifically, the largest aggregate observed for \( T = 9.5 \) at \( \varphi = 0.15 \), the state shown in Figure 3.6, has 2311 surfactants, while at \( T = 9.0 \) the largest aggregate observed has 1403 surfactants – these are outside the range of the \( M \) axis in the figure. This behavior is similar to that observed for homopolymers. Even though there are micelle-sized aggregates at these high temperatures, there is not a preferential value for the aggregation number. At even higher volume fractions, the systems forms percolated (system-spanning) aggregates.
3.3.3 Effect of Inaccessible Volume on the Free Amphiphile Volume Fraction

In Section 3.3.1, the decrease in the free oligomer volume fraction was identified and proposed as one of the methods to calculate the cmc. An analysis of the cause for the decrease is presented in this section. Figure 3.7 is a snapshot of a typical micellized system. From the point of view of the monomers, the volume occupied by micelles is excluded. Because the radius of gyration of the monomers is greater than a single site, a volume near the micelles is also excluded. Additionally, voids within the micelles are not accessible, and neither are sites near exposed tail beads. If a monomer were to become associated with the micellar cluster, it is no longer considered free. For the configuration in Figure 3.7, the inaccessible volume described is considerable, and drawn as a surface around the micelles.

Figure 3.7: Visualization of the inaccessible volume in a typical micellized system (L = 40, T = 7.0 and $\varphi = 0.077$). Solvophobic (orange) and solvophilic (black) beads are represented by spheres. The cyan structure around the micelles contains the volume that is not accessible to oligomers, as calculated by the algorithm presented in Section 3.2.3. The figure was made using PyMol [171].

The average inaccessible volume ($V_i$) was calculated using the method described in Section 3.2.3 as a function of $\langle \varphi \rangle$ and $T$, for $L = 40$ and $L = 60$. A volume fraction of
free oligomers not in the total volume but in the accessible volume may be calculated as $\varphi_{\text{olig,acc}} = \varphi_{\text{olig}} \left( \frac{V}{V-V_i} \right)$, and is presented in Figure 3.8 as open symbols.

Figure 3.8: Volume fraction of free oligomers ($\varphi_{\text{olig}}$, filled symbols) and the volume fraction of free oligomers in the accessible volume ($\varphi_{\text{olig,acc}}$, open symbols) as a function of total surfactant volume fraction, $\varphi$, at various temperatures, from $\mu VT$ simulations with $L = 60$. The black line (of unit slope) represents the ideal gas law. The inset shows data for $T = 6.5$.

The values for both $\varphi_{\text{olig}}$ and $\varphi_{\text{olig,acc}}$ shown in Figure 3.8 are independent of size, ensemble and initial configurations. At the range of temperatures represented in Figure 8, $\varphi_{\text{olig,acc}}$ is nearly constant. At higher temperatures, near and above the upper limit for micellization, $\varphi_{\text{olig,acc}}$ follows the ideal-gas trend until percolation occurs. The constancy of $\varphi_{\text{olig,acc}}$ above the cmc confirms the hypothesis that inaccessible volume is the main cause for the observed decrease in $\varphi_{\text{olig}}$ above the cmc.

There is a slight increase in the effect of inaccessible volume on $\varphi_{\text{olig}}$ at higher temperatures, due to the change in the size and shape of micelles as a function of $\varphi$ and $T$. At low temperatures the micelles are large and dense, whereas at high temperature the solvent penetrates deeper into the micelle core. Micelles at higher temperature are more spread out, thus taking up more accessible volume for the amount of volume excluded explicitly by surfactants in the micelles.

As stated already, the free oligomer or monomer concentration from a single simulation at concentrations above the cmc is often used in simulations to provide
an approximation of the cmc. This has been shown earlier to be a poor approximation for ionic surfactants [31, 46, 125, 132]. Here, we show that this approximation can lead to significant errors for nonionic surfactants as well. For example, at $T = 7.5$, for an NVT simulation at $\varphi = 0.0916$ (approximately five times $\varphi_{\text{cmc}}$), the uncorrected oligomer volume fraction is $\varphi_{\text{olig}} = 0.0146$. However, the actual value is $\varphi_{\text{cmc}} = 0.0182 \pm 0.0005$, as calculated by the maximum in the $\varphi_{\text{olig}}$, and $\varphi_{\text{cmc}} = 0.0178 \pm 0.0006$ from the osmotic pressure. Thus, $\varphi_{\text{olig}}$ underestimates the true cmc by approximately 20%. Large-scale MD simulations of surfactants are often restricted to concentrations much greater than the cmc, so this correction needs to be taken into account for accurate cmc calculations.

### 3.3.4 Micellization theory with excluded volume

Theoretical studies of nonionic surfactant micellization have been able to capture many behaviors, including the: temperature and chain length dependence of the cmc[42, 43], sphere-to-rod transition[42] and surface tension[44], to name a few. Theoretical frameworks conclude that the free nonionic surfactant concentration increases monotonically above the cmc[135]. However, the activity and free surfactant concentration are not necessarily equivalent, which is often assumed in derivations. The $\varphi_{\text{olig, acc}}$ calculated from simulations in the previous section is likely a measure of the activity. Therefore, accounting for the inaccessible volume is required to describe the decrease in the free surfactant concentration.

To support the claim that the inaccessible volume is required to capture the decrease in the free surfactant concentration, we use the Maibaum et al. theory, a free energy and law-of-mass-action theory with empirical relationships for polyethylene glycol-polyethylene oxide (PEG-PEO) surfactants[43]. Different PEG-PEO surfactant architectures are studied and referred to as $C_mE_n$, where $m$ is the number of hydrophobic units and $n$ is the number of hydrophilic units. The model defines a
driving force for micellization, $\beta \Delta G$. An expression for the free-energy density $\beta F = \sum_n \left[ \rho_n \left( \ln(\rho_n a^3) - 1 \right) + \rho_n \beta f_n - \beta \mu n \rho_n \right]$ (3.5), is solved where $a$ is the surfactant size, $\rho_n \left( \ln(\rho_n a^3) - 1 \right)$ is the ideal-gas translational entropy, $f_n$ is the internal free energy of cluster with $n$ surfactants and the total surfactant concentration is $\sum_n n \rho_n$. Monodispersity of the free surfactant and micellar distributions is assumed, i.e. $\rho \simeq \rho_1 + M \rho_M$, where $M$ is the preferred micelle size. We measure a monotonic increase in the free monomer concentration with respect to the overall concentration using the original theory, shown in Figure 3.9.

We use the same framework, but incorporate ideal-behavior deviations in the translational entropy of the aggregates with the excluded volume, $\beta S_n = \ln \left( \frac{\rho_n a^3}{1 - \rho_M \delta a^2} \right) - 1$. The added term assumes that the excluded volume equals the volume occupied by micelles ($n \delta a^2$), which we showed in the previous section to be insufficient to account for the full decrease in free surfactant concentration in simulations. However the goal of using this theory is to show that non-monotonic behavior is predicted by this model with the available parameters. We performed the minimization of the system of equations using a constrained numerical solver (L-BFGS-B) with respect to $\rho_1$ and $M$.

Figure 3.9 shows that this methodology does indeed yield a maximum in the monomer concentration, which is not present in the original model. Furthermore, it shows nearly exact agreement with Maibaum et al. [43] for $\rho_{\text{cmc}}(T)$, $\rho_{\text{cmc}}(m)$, $M(T)$ and $M(m)$. Figure 3.10 demonstrates that instead of setting tail length, $\delta$, to a constant, making it variable to the number of hydrophobic units, $m$, as $\delta(m) = a m$ improves agreement of $M(m)$ with the experimental data, without appreciable change to $\rho_{\text{cmc}}(m)$.
Figure 3.9: Free monomer concentration normalized by the cmc as a function of total $C_mE_6$ concentration without (full lines) and with (dashed lines) the inaccessible volume correction to the Maibaum et al. theory[43]. $C_6E_6$ (red), $C_8E_6$ (green) and $C_{10}E_6$ (blue) surfactants are shown at 298 K.

3.4 Summary and Conclusions

We have developed a method to calculate the volume made inaccessible to free surfactants in micellar solutions. We show that the concentration of free surfactants in the accessible volume is nearly constant above the cmc, whereas the concentration of free surfactants in the total volume decreases above the cmc. We thus conclude that, for nonionic surfactants, the decrease in concentration of free surfactants above the cmc is due to the inaccessible volume. These volume exclusion effects are not accounted for in typical law-of-mass-action theory descriptions of micellization. We incorporate this effect with a modification to an existing thermodynamic model, and find that the excluded volume is sufficient to produce a decrease in the free surfactant concentration above the cmc. Because of the decrease in free surfactant concentration, simulation studies of nonionic surfactants restricted to concentrations far above the cmc need to take into account the inaccessible volume to obtain accurate estimates.
Figure 3.10: $C_mE_6$ preferred aggregation number as a function of $m$ at $T = 298$ K from the Maibaum et al. theory with (blue line) and without (green line) the inaccessible volume correction, and with the tail length being variable with surfactant architecture, $d = f(m)$ (red line)[43], compared with experimental values (black points) measured using: light-scattering (diamond[173]), time-resolved fluorescence quenching (circle[174]) and small-angle neutron scattering (triangle[175] and square[39]).

for the cmc. The impact of the inaccessible volume on ionic surfactants, for which solution ionic strength effects also come into play, will be the subject of future work.

Three methods for measuring cmc’s from simulations of surfactant solutions were analyzed, namely through the maximum in the free oligomer concentration, the osmotic pressure of the solution obtained in grand canonical Monte Carlo simulations, and the presence of a minimum in the aggregate size distributions between oligomeric and micellar distributions. The maximum in the free oligomer concentration, as mentioned, is a result of the inaccessible volume. Because micelles become more compact and smooth at low temperatures, the effect of the inaccessible volume is less drastic, and the maximum in the free oligomer concentration is more difficult to identify. Additionally, at low temperatures it is more difficult to reach equilibrium states, because of the significant barriers to surfactant transfer between free solution into and
the interior of micelles. Reaching equilibrium at low temperature is also a challenge when calculating the cmc from a minimum in the aggregation number distributions, because these distributions cannot be reliably obtained. By contrast, cmc’s calculated from the osmotic pressure using histogram reweighting can be obtained from considerably fewer simulations, using smaller system sizes. With fully equilibrated histograms (which require simulations over a range of temperatures), the cmc at low temperature can be reliably calculated. The main limitation of the osmotic pressure method is that it requires sampling in the grand canonical ensemble, which may not be practical for explicit-solvent realistic surfactant models.

As the temperature increases the cmc definitions begin to deviate from each other, since micellar aggregates become less sharply defined and the aggregate size distributions develop broad tails. These deviations signal the approach of an upper temperature limit above which micellization does not occur. This upper temperature limit for micellization can be obtained from aggregation number distributions well above the extrapolated cmc. For the $\text{H}_4\text{T}_4$ surfactant, we find that this limit occurs approximately at $T = 9.5$. 
Chapter 4

Surfactant and salt concentration effects on ionic surfactant micellization

4.1 Introduction

Many materials relevant for biological and industrial processes rely on self-assembling molecules. Surfactants are examples of such self-assembling molecules. They contain covalently bonded hydrophobic and hydrophilic segments, and are often used because of their ability to micellize, forming well-defined, long-lived aggregates where the hydrophobic parts are in the interior and the hydrophilic parts interface with the solvent. Micellization results in drastic changes in the properties which are leveraged by nature and industry. Surfactants are used in detergency and cosmetics because of their ability to reduce surface tension[1]. The surface tension ceases to decrease when the surfactants micellize. Surfactants are also used to disperse solvophobic molecules for nano-technologies, such as the templated growth of metal nanowires within micelles[176]. The change in the osmotic pressure upon micellization gives
insight into thermodynamics[129] and is used for ultrafiltration[177]. The mean ionic surfactant activity goes from equivalence to the surfactant concentration to being constant at micellization[178]. The onset of micellization, affects the surface tension, micelle size, osmotic pressure and mean ionic activity, among other properties, and occurs at the critical micelle concentration (cmc).

Concentration effects persist above the cmc. The concentration of free, unmicellized surfactants is an example of a property that changes with concentration above the cmc and impacts many industries. Free surfactants in skin care products are a main cause of skin irritation because they adsorb to charged sites in cell walls, and denature proteins[179–184]. Understanding such phenomena helped lead to the development of “tear-free” soap. The cmc has a strong effect on the free surfactant concentration decrease, because the free surfactant concentration is at a maximum at the cmc. Decreased free surfactant concentration and diffusion can improve foam-stabilization[185] and deplete textile wetting[1] because free surfactants are key surface-tension-altering agents. As the free surfactant concentration decreases, the micelle concentration must increase. As the concentration of micelles increases, so does the system’s ability to solubilize added components, such as fragrances and oils[28, 186, 187]. Ionic surfactants are able to disperse functionalized graphene sheets, improving processibility while maintaining high conductivity. Furthermore, the molar conductivity of functionalized graphene sheets is sensitive to the concentration of free surfactants[29]. Beyond the direct impact free surfactant concentration has on many applications, it indirectly affects measurement methodologies.

Simulation and experimental methodologies rely on approximations of the free surfactant concentration to calculate other micellar properties. Experimental techniques often approximate the free surfactant concentration as the cmc – for example, specific conductivity[40], nuclear magnetic resonance[35], time-dependent static
light scattering [41], steady-state[36, 37] and time-resolved fluorescence quenching[37] measurements use models that assume a constant free surfactant concentration to calculate micelle aggregation numbers. However, experiments, simulations and theory have all shown that the free surfactant concentration decreases above the cmc. Electromotive force measurements of ionic surfactant solutions, using electrochemical cells with ion-selective electrodes, provide clear evidence that the free surfactant concentration decreases above the cmc[188]. Certain membranes, often a polymer-gel membrane with ionophores (Bu₃N⁺(C₁₀H₃₃) for example) in a lipophilic solvent such as dicholorobenzene, only allow the charged surfactant to interact with the electrode[188]. The electromotive force measured can be related to a chemical potential using the Nernst equation[178, 189], and shows a clear decrease above the cmc.

Simulations have helped address the cause of the free nonionic surfactant concentration decrease; free nonionic surfactant concentrations have not been measured experimentally. Molecular simulations have shown the free nonionic surfactant concentration decrease to be true for lattice [51, 136–138, 141, 190, 191] and continuum [139, 140] nonionic surfactant models. Previous work showed that the decrease is well-explained by the volume excluded to free surfactants by micelles, and that the free surfactant concentration in the free volume is a good estimate for the cmc at most concentrations[190]. This proved to be true for various surfactant architectures[191].

The decrease of free ionic surfactant concentration has also been studied with simulations and theory. Gunnarson et al. developed a theory which uses mass-action law, and free energy of micellization expressions with Poisson-Boltzmann electrostatics[133]. Jusufi et al. incorporated Debye-Hückel electrostatics[46] into the Maibaum et al.[43] theory for nonionic surfactants, which utilizes ideal-gas entropy and experimental values for oil-water surface tensions. Quina and co-workers[32], developed a semi-empirically derived theory for the relationship between the most
likely number of surfactants in a cluster (the preferred aggregation number, \(\langle M \rangle\)), added salt concentration \((\rho_{\text{salt}})\), surfactant concentration \((\rho)\), free surfactant concentration \((\rho_{\text{free}})\), the cmc \((\rho_{\text{cmc}})\) and the degree-of-counterion association \((\alpha)\). Empirical aspects of the derivation were: (i) the aggregation number dependence on concentration, (ii) the cmc dependence on aggregation number and molecular weight \((\text{MW})\), and (iii) the cmc dependence on added salt. The aggregation number dependence on concentration is observed in small-angle neutron scattering\[192\], time-resolved fluorescence quenching\[193\] and electron paramagnetic resonance\[194\] measurements and is given by: \(\langle M \rangle = \kappa \rho^\gamma\), where \(\kappa\) and \(\gamma\) are parameters fitted to data. The cmc dependence on aggregation number and molecular weight is: \(\langle M \rangle = K_2 \rho_{\text{cmc}}^{K_1 \text{MW}}\), where \(K_1\) and \(K_2\) are fits to data\[195\]. It is also known that the cmc and added salt have the following empirical relationship\[196\]: \(\ln \rho_{\text{cmc}} = -K_3 - K_4 \ln (\rho_{\text{cmc}, \text{salt}=0} + \rho_{\text{salt}})\), where \(K_3\) and \(K_4\) are fitted parameters as well. Additionally, a charge balance on “mobile” species is used, \(\rho_{\text{aq}} = \alpha (\rho - \rho_{\text{free}}) + \rho_{\text{free}} + \rho_{\text{salt}}\), where \(\rho_{\text{aq}}\) is the counterion concentration in the aqueous “pseudo-phase”. Bales incorporated an excluded volume term, \(\frac{1}{1 - V_m \rho}\)[197], where \(V_m\) is the surfactant molar volume. The resulting correction,

\[
\log \rho_{\text{free}} = (1 + \alpha) \log (\rho_{\text{cmc}}(\rho_{\text{salt}} = 0)) - \alpha \log \left(\frac{(1 - \alpha)(\rho - \rho_{\text{free}}) + \rho_{\text{free}} + \rho_{\text{salt}}}{1 - V_m \rho}\right)
\]  

(4.1)

, is tested with molecular simulations in the present work.

Sanders et al. \[31\] performed atomistic simulations of sodium \(n\)-alkyl sulfates with explicit water and obtained the free surfactant concentration, well above the cmc. They showed the importance of methodology for measuring the cmc, and the sensitivity of the cmc to model types and parameters\[31\]. Similar behavior has been found for lower-cmc surfactants using a coarse-grained surfactant model with molecular dynamics (MD)[125] and dissipative particle dynamics (DPD) simulations[198].
An implicit-solvent model is described in Section 4.2 to test the cmc estimation methodology on another model. Benefits of using an implicit-solvent model include the ability to: (i) simulate concentrations at and below the cmc, as coarse-grained model DPD simulations are now able to do[198] and (ii) use Monte Carlo (MC) simulations in the grand-canonical ensemble to measure the activity and potentially by-pass hysteresis. The ability to measure the mean ionic activity enables a direct comparison to experiments and theories. The implicit-solvent model also allows the direct measurement of the free surfactant concentration, the degree-of-counterion association, excluded volume and the osmotic pressure. Those properties are inputs for a semi-empirical extrapolation to the cmc (Equation 4.1)[127] and are also measurable in experiments. These surfactant concentration dependent properties also depend on added salt. The effect of added salt (NaCl) on the free surfactant concentration, the degree-of-counterion association and ionic activity are investigated in the present work.

Salt added to surfactant solutions affects micellar properties relevant to many products and industries. The cmc decreases logarithmically with the added salt concentration, which is largely attributed to electrostatic repulsion screening[199]. The addition of salt also leads to micellar size increase, shape changes[200] and counterion association changes[41, 201]. The structure of the polyatomic electrolytes can induce structural changes in micelles[202]. Salt added to surfactant formulations induces the viscosity to increase[203], which is why salt is often included in personal care products. Furthermore the electromotive force and conductivity increase with added salt[41, 204, 205]. Solubilization is generally enhanced by the addition of salt, due to the increase of micellar size[206]. The cloud point, an important property in supply-chain and storage stability, can increase or decrease with added salt, depending on the surfactant and salt charge, and surfactant structure[207, 208]. Fine-tuning the phase-inversion temperature can be done by adjusting the electrolyte concentration,
a topic studied for enhanced oil recovery\cite{209} and detergency\cite{210}. As electrolyte is added to surfactant solutions, surfactant adsorption increases at neutral and oppositely charged substrates\cite{211} because the screening length decreases. While adsorption to like-charged substrates decreases as electrolyte is added\cite{34}. Similarly, wetting is improved by the addition of electrolytes\cite{212, 213}. Jusufi et al. studied the effect of different salt concentration on the cmc, preferred aggregation number and degree-of-counterion association for NaDS and dodecyltrimethylammonium chloride, and found that explicit ions are important for obtaining agreement with experiments for the preferred aggregation number, but that the cmc values of a Yukawa-screened implicit-salt model agree with experimental results\cite{214}. The effect of added salt on the degree-of-counterion association, which impacts most all of the applications mentioned here, has been shown to have: no\cite{215}, an increasing\cite{41}, a decreasing\cite{216} or a non-monotonic\cite{217} dependence. This experimental disagreement depends on the property, method and concentration fit ranges.

We investigate the effect of added salt (NaCl) on the surfactant (NaDS) concentration dependence of numerous properties, which, like the ionic activity, has not been done previously for surfactant simulations. The models are described in Section 4.2.1 and less common or new techniques for calculating ionic activity and excluded volume are presented in Section 4.2.3. Because surfactant solutions and formulations often include salt, it is important to understand how it changes the dependence of free surfactant and counterion condensation on the surfactant concentration. The effect of salt and surfactant concentration on micellar properties, the excluded volume and degree-of-counterion association, are shown in Section 4.3.1. In Section 4.3.2 some of the discrepancies in the degree-of-counterion association salt and surfactant concentration dependencies and experimental disagreements are addressed by taking advantage of the ability to measure structural and thermodynamic properties using
simulations of the same model. A cmc extrapolation method and the mean ionic activity coefficients are also presented and discussed in Section 4.3.2.

4.2 Model and Methods

4.2.1 Model

Sodium dodecyl sulfate (NaDS), NaSO$_4$C$_{12}$H$_{14}$, is studied in water at 298 K where the water is treated implicitly with a model developed by Jusufi et al.[155], with and without added model NaCl developed by Lenart et al.[218]. The dodecyl sulfate consists of one head group interaction site for the SO$_4^-$ with all of the negative charge (“HG”), and 12 uncharged united atom carbon interaction sites (“C”). Simulating micellization of surfactants in water, without explicitly including water, requires some additional interaction potentials that implicitly account for the effects of water. Typical models, including this model, employ the 12-6 Lennard-Jones, repulsive WCA[219] and Coulombic potentials. In implicit-solvent simulations of electrolytes it is important to modify the Coulombic potential, shown as Equation 4.2, to account for the change of the dielectric permittivity as a function of the ion distance[218]. Equation 4.3, derived from a mathematical model for the solvent electric field[220], incorporates the distance-dependence of the dielectric permittivity. The dielectric permittivity changes, at room temperature, from 78 ($\epsilon_D(r \to \infty) = \epsilon_r$) to 5.2 ($\epsilon_D(r \to 0) = 5.2$), which is measured from dielectric dispersion Cole-Cole plots[221, 222].

$$V_{qq}(r) = \frac{1}{4\pi\epsilon_0} \frac{q_i q_j}{\epsilon_D(r) r}$$  \hspace{1cm} (4.2)

$$\epsilon_D(r) = \frac{\epsilon_r + 5.2}{2} + \frac{\epsilon_r - 5.2}{2} \tanh \left( \frac{r - r_{me}}{\sigma_h} \right)$$  \hspace{1cm} (4.3)

Recently, modifying the Coulombic potential was shown to be qualitatively necessary for micellization in implicit solvent models[223]. Another important effect of water
is ion hydration, which sets a water molecule-sized depletion region for ion-ion pair distances. Therefore a Gaussian repulsion, shown in Equation 4.4, is placed at the first hydration shell location.

\[ V_{\text{hydr}}(r) = \frac{H}{\sigma_h \sqrt{2\pi}} \exp \left[ -\frac{(r - r_{mh})^2}{2\sigma_h^2} \right] \]  

(4.4)

The parameters for \( V_{\text{hydr}}(r) \) are taken from a fit to the radial distribution functions of explicit water simulations[155, 218]. Lastly, the effect of hydrophobicity is imparted to the surfactant chain by making the terminal bead (“Ct”) have stronger attractive Lennard-Jones interactions (\( \epsilon_{C_t-C_t} \)). The value of the parameter \( \epsilon_{C_t-C_t} \) (4.8 kJ/mol) is bound by the work to create a cavity in water and is fit to match the experimental cmc. The nonbonded interaction parameters between atoms are detailed in Table 4.1.

Intramolecular interactions used in this model are similar to the OPLS alkane model[224]. Nonbonded intramolecular interactions were the same as bonded interactions for sites separated by four or more bonds, those with three or less bonds had no non-bonded interactions. Bond lengths were fixed to 0.153 nm[224]. The harmonic angle parameters were \( \theta = 111^\circ \) and \( k_\theta = 31275.3 \) K[224]. Dihedral torsions are defined by the Ryckaert-Bellmans potential for hydrocarbons, not OPLS[225]. We note that the cmc reported here for NaDS at 298 K does not agree with Jusufi and co-workers original results[155, 214].

### 4.2.2 Simulation Details

Grand-canonical Monte Carlo (GCMC) simulations were performed using Cassandra v1.2[226]. For this chapter the following MC moves were additionally incorporated: pair insertion/deletion and cluster center-of-mass displacement[106]. The hydration (Equation 4.4) and modified Coulombic (Equation 4.2) potentials were also imple-
Table 4.1: Non-bonded interaction parameters at $T = 298$ K for the implicit-solvent united-atom model of sodium dodecyl sulphate, sodium chloride and their cross interactions[155, 218].

<table>
<thead>
<tr>
<th>Group</th>
<th>LJ/ WCA</th>
<th>$\sigma$ (nm)</th>
<th>$\epsilon$ (kJ/mol)</th>
<th>$H$ (kJ nm/mol)</th>
<th>$\sigma_h$ (nm)</th>
<th>$r_{mh}$ (nm)</th>
<th>$r_{me}$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C$_t$ - C$_t$</td>
<td>LJ</td>
<td>0.395</td>
<td>4.80</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>C$_t$ - C</td>
<td>LJ</td>
<td>0.395</td>
<td>0.49</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>C$_t$ - HG</td>
<td>WCA</td>
<td>0.469</td>
<td>2.45</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>C$_t$ - Na</td>
<td>WCA</td>
<td>0.3505</td>
<td>2.45</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>C$_t$ - Cl</td>
<td>WCA</td>
<td>0.4425</td>
<td>2.45</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>C - C</td>
<td>LJ</td>
<td>0.395</td>
<td>0.49</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>C - HG</td>
<td>WCA</td>
<td>0.469</td>
<td>2.45</td>
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<td>C - Na</td>
<td>WCA</td>
<td>0.3505</td>
<td>2.45</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>C - Cl</td>
<td>WCA</td>
<td>0.4425</td>
<td>2.45</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>HG - HG</td>
<td>LJ</td>
<td>0.543</td>
<td>1.239</td>
<td>0.076</td>
<td>0.047</td>
<td>0.780</td>
<td>0.0</td>
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<tr>
<td>HG - Na</td>
<td>LJ</td>
<td>0.536</td>
<td>0.124</td>
<td>0.379</td>
<td>0.034</td>
<td>0.444</td>
<td>0.421</td>
</tr>
<tr>
<td>HG - Cl</td>
<td>WCA</td>
<td>0.5165</td>
<td>2.478</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Na - Na</td>
<td>WCA</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<td>LJ</td>
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<td>0.35262</td>
<td>3.4089</td>
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<td>0.310</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Clusters with 14 or more surfactants were considered micellar aggregates (determined by the minimum in the aggregation number distribution), and were selected for cluster displacements. Ewald sums were used for electrostatics with an accuracy tolerance of 0.00001 and non-bonded interaction cut-off of 3.05 nm. MC simulations were performed with the following protocol: 30% molecule translations, 10% DS regrowths, 5% DS rotations, 5% cluster center-of-mass displacement, 50% ion pair insertions/deletions (insertions and deletions were chosen with equal probability; in added NaCl simulations, NaCl and NaDS pairs were selected indiscriminately).

Large-scale molecular dynamics simulations were run using Gromacs v5.1.1[227]. The simulation time step was 4 fs. The nonbonded vdW and Coulombic interaction potential cut-off values were both 3.05 nm. Bonds were restrained using LINCS[228], with an order 6 and 2 iterations for the constraint coupling matrix. Because these
potentials were not developed for implicit-solvent simulations, double precision was required for energy conservation. Electrostatics were modeled with particle mesh Ewald with an accuracy tolerance of 0.00001 and 0.3 nm Fourier spacing. The distance-dependent dielectric permittivity and the hydration potential were implemented in the Gromacs user tables. Simulations were run in the canonical ensemble. The temperature \((T)\) was held constant using the Nosé-Hoover thermostat with a coupling parameter \((\tau_T)\) of 0.05 ps. Equilibration times were typically 3.5 \(\mu s\), but ranged from 1.2-5.4 \(\mu s\) depending on the salt and surfactant concentration; production runs were 2.4 \(\mu s\).

Agreement between the free surfactant concentration of simulations initialized from dramatically different configurations was the main equilibration criteria. Simulations were initialized from small, \(M \approx 30\), medium, \(M \approx 50\), and large, \(M \approx 70\), micelle configurations, with free surfactants consisting the remainder. Box sizes ranged from 7.1 nm to 39 nm (35 nm for added salt), with 110 surfactants. For no added salt, simulations with constant box sizes of 15.3 nm, 24.3 nm and 35 nm, and 50-1080 surfactant pairs were also run. The fixed box size simulations were additionally initialized with a completely un-micellized configuration.

### 4.2.3 Analysis Details

The free surfactant concentration is used to determine equilibration and its behavior is studied in Section 4.3. Free surfactants can be identified from simulation snapshots. Visual inspection of a single snapshot generally agrees with the size and number of clusters identified by most reasonable cluster criteria. However, small differences in the criteria for identifying clustered surfactants yield large differences in the cluster probability distribution, especially in the micellar part of the distribution. On the other hand, the mass-weighted cluster probability distribution, also known as the aggregation number distribution, is less sensitive. Furthermore, the density of free
surfactants \((M < M_{\text{olig}})\) is also sensitive to the clustering criteria. Near the cmc, the free surfactant concentration is essentially independent of reasonable clustering criteria. However at concentrations above the cmc, it is more dependent – a 1 Å difference in the cut-off can produce qualitatively different behavior for NaDS. The length of an out-stretched surfactant generally sets the limit of a reasonable clustering criteria. A less ambiguous clustering definition for liquid-like clusters is a remaining challenge; therefore self-assembly studies which are concerned with micellar sizes and especially the free surfactant concentration need to test the sensitivity of the clustering criteria.

We tested the following three clustering criteria: (i) setting a minimum surfactant tail center-of-mass separation \(d\) (ii) setting a minimum surfactant tail particle separation distance and (iii) using the Sammalkorpi, Karttunen and Haataja (“skh”) criteria\[229\]. The different criteria have similar qualitative and quantitative behavior. We found that a cut-off less than 1 nm for criteria (i) and \((5.5, 6.5, 7.5)\) for criteria (iii) yields an increase in \(\rho_{\text{free}}\) at high \(\rho\) which is likely an artifact of the clustering criterion significantly overestimating the free surfactant concentration. We chose the smallest cut-off which resulted in a monotonic decrease in \(\rho_{\text{free}}\) below the cmc. Specifically, we used a value of \(d = 1\) nm for the surfactant tail center-of-mass clustering criteria. Three-body distances, \(q_6, q_4\) and surfactant tail angles and dihedrals were investigated as well but did not yield a discriminating value that out-performs the center-of-mass separation criteria.

Aggregation number distributions were calculated by identifying clusters from many simulation snapshots and keeping track of their cluster frequency by size\[25\]. The minimum in the distributions that separates the free and aggregated surfactants \((M_{\text{olig}})\) was used to then calculate the concentration of free oligomers for every simulation snapshot. We set \(M_{\text{olig}} = 14\), this property is not very sensitive to con-
centration, especially compared to temperature, which is why only one value is used throughout.

The degree-of-counterion association can be calculated as the fraction of counte-

\[ r_{\text{cut}}^\alpha \]

er, of a micellized surfactant head group to the surfactants in a micelle of size \( M \), \( N^{\text{Na}}(r_{\text{Na-HG}} < r_{\text{cut}}^\alpha) / M = \alpha \). Figure 4.2, shows the dependence of \( r_{\text{cut}}^\alpha \). When salt is added to the system, there will be a small amount of co-ion (Cl) that will be associated with the micelle, in which case
\[ (N^{\text{Na}}(r_{\text{Na-HG}} < r_{\text{cut}}^\alpha) - N^{\text{Cl}}(r_{\text{Cl-HG}} < r_{\text{cut}}^\alpha)) / M = \alpha \]. The co-ion binding is relatively small and accounting for it does not affect the trends.

An off-lattice version of the excluded volume calculation method developed by Santos et al. [190] was implemented. In the method, a library of free surfactant monomers configurations is generated by running a MC simulation of surfactant regrowths at the corresponding simulation temperature. Then in each snapshot from a simulation (MC or MD), a monomer configuration is selected at random from the library and inserted into the box at a random location. An insertion is determined to be excluded if it meets at least one of the following criteria: (i) becomes part of a micellar cluster based on the same clustering criteria used to identify clusters or (ii) is energetically unfavorable \( \Delta U_{\text{monomer insertion}} > U_{\text{ex}} \), where \( \Delta U_{\text{monomer insertion}} \) is the energy change associated with the monomer insertion and \( U_{\text{ex}} \) is the energy criteria for an insertion to be considered excluded). The excluded volume is calculated as:
\[ \left( \frac{N_{\text{ex insertions}}}{N_{\text{total insertions}}} \right) V = V_{\text{ex}} \]. Salt and counterions were not included in the exclusion. If counterions and coions associated with micelles are included, the excluded volume increases, but within uncertainty.

Many features of the excluded volume calculation method are the same as the lattice method, however the off-lattice version introduces another level of arbitrariness mostly from the clustering criteria and the energetic criteria \( U_{\text{ex}} \). The number of trial insertions \( N_{\text{total insertions}} \) is another property that must be chosen. We selected
$U_{ex} = 100 \text{ kJ/mol}$ and $N_{\text{total insertions}} = 400$, by finding the values at which an increase results in less than a 1% change in \[\frac{N_{ex \text{ insertions}}}{N_{\text{total insertions}}}\] for the more and less dense concentrations tested. As may be expected, the cluster criteria is often the most influential to the excluded volume calculation. The excluded volume parameters were insensitive over the range of surfactant and salt concentrations simulated.

The uncertainty of the free surfactant concentration, the excluded volume and the degree-of-counterion association was estimated using the Flyvbjerg-Petersen group blocking analysis\[168\]. Typically 5-9 block transformations were required for the standard deviation to plateau.

The ionic activities calculated from electrochemical experiments have been beneficial for studying the decrease in the free surfactant concentration\[178\]. Experiments of NaDS with NaCl have been conducted, but the ionic activities were not calculated\[204, 230, 231\]. This may have been due to some of the challenges in converting the electromotive force ($E$) measured in the experiments to the activity ($a$). The electromotive force can be related to the free energy change and thus the activity with the Nernst equation \[189\]:

$$E = E^\dagger + \left(\frac{n_{\text{species}} F}{RT}\right) \ln(\gamma \rho) \quad (4.5)$$

where $\gamma$ is the mean ionic activity coefficient ($a = \gamma \rho$) and $E^\dagger$ is a reference electromotive force. Equation 4.5 assumes that $E$ is proportional the chemical potential, $\mu$, by Faraday’s constant $F$. At concentrations well below the cmc, the ionic activity coefficient can be approximated with Debye-Hückel theory, $\gamma = A \sqrt{\rho}$ where $A = -0.5116 \text{ mM}^{-1/2}$ at 298 K. For $\rho_{\text{NaCl}} = 0 \text{ mM}$, the $E(\rho)$ curves were fit for $\rho \leq 6 \text{ mM}$ NaDS ($R^2 \geq 0.98$).

For getting the single-ion Na activities ($a_{\text{Na}}$) in added NaCl there is the following complication: at $\rho_{\text{NaDS}} = 0$ there is a large concentration of Na, and Debye-
Hückel theory is no longer appropriate because the property of interest is the ionic activity with respect to DS or NaDS concentration. Therefore we used experimental measurements of $a_{Na}$ in NaCl and water at 298 K[232], which can use the Debye-Hückel theory to calculate $E^{\dagger}$ at low $\rho_{NaCl}$. This procedure assumes that 

$$a_{Na}(\rho_{NaCl}) = \lim_{\rho_{NaDS} \to \infty} a_{Na}(\rho_{NaCl}; \rho_{NaDS}).$$

The Vera et al. $a_{Na}$ measurements[232] were fit ($R^2 = 0.992$) with: 

$$\log \rho + A \sqrt{\rho} + B \sqrt{\rho} + C \rho,$$

where $A = -0.5116 \text{ mM}^{-1/2}$, $B = 1.39 \pm 0.03 \text{ mM}^{-1/2}$ and $C = 0.076 \pm 0.002 \text{ mM}^{-1/2}$[233]. This procedure is recommended if $\rho_{NaCl} \geq 50 \text{ mM}$, otherwise Debye-Hückel theory is likely sufficient.

The methodology used to calculate the mean ionic activity from experiments and simulations is similar. The mean ionic activity can be related to the ion pair chemical potential ($\mu$). The ion pair chemical potential is an input parameter in $\mu VT$ simulations. Therefore the equation of state measured, $\mu(\rho)$, can be used to measure the activity, using:

$$\beta \mu = \beta \mu^{\dagger} + n_{\text{species}} \ln(\gamma \rho) \quad (4.6)$$

where $\beta = 1/(kT)$, $\gamma$ is the mean ionic activity coefficient ($a = \gamma \rho$) and $\mu^{\dagger}$ is the reference chemical potential[233]. As in the experimental system, Debye-Hückel limiting law at low concentration was fit to $\rho \leq 2 \text{ mM}$, below the cmc, and the linear regression gave $R^2 = 0.9999$. GCMC simulations were run at different box sizes, ranging from 6.1 to 51.9 nm where an average of 65 NaDS molecules were present in each box to enable micellization. Simulations were initialized with either an empty box or with a micelle of size $M = 65$. To check the cmc, $\mu_{\text{cmc}}$ was calculated using histogram reweighting with simulations in a $L = 6.1 \text{ nm}$ box with histograms from $T = 298 - 398 \text{ K}$. Results from histogram reweighting can also be used to measure the activity, although are not presented. The lower the electrolyte concentration, the better the fit to Debye-Hückel. In a $L = 6.1 \text{ nm}$ box, the $\mu(\rho)$ does not follow Debye-
Hückel, because there are too few pairs at the relevant concentrations. Therefore
to measure the activity an additional box size of $L = 35$ nm is necessary if doing
histogram reweighting, which if it is a gas-like configuration, is trivial for an implicit-
solvent model.

4.3 Results

4.3.1 Micellar properties

Surfactant micelles occupy volume which is excluded to free surfactants. The ex-
cluded volume is sufficient to capture the decrease of free nonionic surfactants as
the concentration increases above the cmc. The excluded volume has either been
ignored in previous free surfactant correction models, or has been approximated as
the volume occupied by all surfactants. This treatment is independent of micelle
size or salt concentration. For NaDS at 298 K, the excluded volume fraction ($\frac{V_{ex}}{V}$)
calculated rigorously using the method detailed in Section 4.2 is shown in Figure 4.1.
The excluded volume shows a linear dependence with the surfactant concentration.
A linear dependence implies that the micellar behavior does not change with concen-
tration. Analysis of the radii of gyration do not reveal a major shape transition, such
as sphere-to-rod, which could lead to different behavior with salt or surfactant con-
centration. As the salt concentration increases there is a small decrease in $\frac{V_{ex}}{V}$, but it
is relatively insensitive over the calculated surfactant concentration range. Previous
work approximates the excluded volume as the molar volume of the total surfactant
concentration (which was approximated as the anhydrous surfactant density of 1.0
g/mL for NaDS)[31, 197]. This approximation for the excluded volume $V_{ex}$ is not
sensitive to the cmc, shown in Figure 4.1.
The excluded volume fraction reported here is better approximated by the following equation:

\[
\frac{V_{\text{ex}}}{V} = \frac{\left(\frac{\rho_{\text{mic}}}{\left\langle M \right\rangle}\right)^{\frac{4\pi}{3}} R_{\text{ex}}^3}{\rho}
\]  

where \(\rho_{\text{mic}}\) is the density of micellized surfactants, \(\rho_{\text{mic}}\) can be calculated directly from simulations or approximated as \(\rho_{\text{mic}} \sim (\rho - \rho_{\text{cmc}})\). The fact that \(\rho_{\text{free}} \neq \rho_{\text{cmc}}\) does not seem to have a strong effect on \(\frac{V_{\text{ex}}}{V}\) since \(\rho_{\text{cmc}}\) is small. The radius marking inaccessibility to free surfactants around the average micelle, \(R_{\text{ex}}\), can be calculated directly from simulation density profiles. The \(R_{\text{ex}}\) can also be calculated as the surfactant length (\(l_{\text{surf}}\)) to set the micelle boundary, plus separation or hydrated radius of the different ion groups (\(\sigma_{\text{Na-HG}}\)) and half the length of the free surfactant: \(R_{\text{ex}} \sim (1.5l_{\text{surf}} + 2\sigma_{\text{Na-HG}})\). The sum of the volume around an average spherical micelle is multiplied by the number of micelles in the volume, \(\left(\frac{\rho_{\text{mic}}}{\left\langle M \right\rangle}\right)\), which could also be calculated directly from simulations. The equation shown in Figure 4.1 uses \(l_{\text{surf}} = 13.9\,\text{Å}\), which is the structure corresponding to a surfactant with equilibrium angles, and \(\left\langle M \right\rangle = 60\) which is the value at the \(\rho = \rho_{\text{cmc}}\). The simulations do observe growth of \(\left\langle M \right\rangle\) with \(\rho\), as measured by experiments[193], which accounts for the values being less than the fit at higher concentrations. Growth of micelle size with salt concentration also helps explain the general decrease of the excluded volume at higher salt concentrations. We expect that the non-monotonic decrease of with \(\rho_{\text{NaCl}}\) at \(\rho_{\text{NaDS}} = 500\,\text{mM}\) is within the uncertainties.

The micelles which exclude volume also carry a large charge from all the head groups \((qM, \text{where } q \text{ is the surfactant charge, } q = -1 \text{ for dodecyl sulphate})\). Those micelles attract counterions, which associate with micelles. Their association is not simply on or off; there are contact pairs due to vdW attraction and successive shells which correspond to hydration shells. The inset of Figure 4.2 shows different regions which correspond to peaks in the radial distribution function between head groups, and Na and Cl atoms. Because of Coulombic repulsion, association between
Figure 4.1: Sodium dodecyl sulfate excluded volume fraction at 298 K from MD simulations of NaDS at different NaCl concentrations (symbols). The dashed line shows the excluded volume as calculated as the volume occupied by anhydrous surfactant at a certain concentration, $V_{ex} = 0.28838\rho$. The solid line shows $V_{ex} = \frac{4\pi R_{mic}^3}{3(M)_{NaCl}} \rho_{mic}$, see the text for specific values.

HG and Cl atoms is small. A majority of the ions are in the 1st hydration shell ($4.3\,\text{Å} < r_{Na-HG} < 7.0\,\text{Å}$), and is the region which contributes most to the increase with respect to concentration. Contact pairs and second hydration pairs ($7.0\,\text{Å} < r_{Na-HG} < 9.2\,\text{Å}$) contribute a smaller and more constant amount above the cmc. The remainder of $\alpha$ values reported include contact, first hydration and second hydration pairs, $r_{cut}^\alpha = 9.2\,\text{Å}$. The cut-off value for $\alpha$ is less-ambiguous than the clustering criteria. Furthermore, $\alpha$ is much less sensitive to the clustering criteria than the association criteria. Selecting distances based on the radial distribution function is a good protocol to determine the criterion.

The fraction of ions bound to the micelle depends on the structural cut-off used. Figure 4.2 demonstrates that a large fraction of associated ions are in the first hydration shell, and to a smaller extent a secondary peak and the vdW peak. Which
counterion shells are considered associated, likely depends on the technique, which is further discussed in the next Section.

Figure 4.2: Noncumulative degree-of-counterion association for NaDS as a function of concentration and for different Na-surfactant ion head group “HG” distance ranges for NaDS at 298 K without salt. Colors correspond to contact pair ($r_{cut} \leq 4.3$, black), 1st ($4.3 < r_{cut} \leq 7.0$, red) and 2nd ($7.0 < r_{cut} \leq 9.2$, green) hydration shell pairs. The inset shows the radial distribution function for Na-HG and Na-Cl at $\rho_{NaDS} = 500$ mM and $\rho_{NaCl} = 500$ mM at 298 K.

The degree-of-counterion association with added salt is shown in Figure 4.3. The degree-of-counterion association from simulations is zero below the cmc, because there are no micelles to associate with, as shown for the no added salt case. At the cmc there is a sharp increase, followed by a constant, or possibly a maximum, over the range of 1x to 10x the cmc. At about 10x the cmc, $\alpha$ increases on a log scale. Those concentration ranges seem to correspond with the changes in the EMF found in experiments[204]. As the salt concentration increases, we find that $\alpha$ increases monotonically. This can possibly be explained by the higher Na concentration in the free volume, due to the added salt, which raises the concentration, and thus
association. The $\alpha$ collapses with the total electrolyte concentration or ionic strength, $I = \rho_{NaDS} + \rho_{NaCl}$, as shown in the inset of Figure 4.6.

![Graph showing the degree of counterion association as a function of NaDS concentration for different NaCl concentrations at 298 K from MD simulations. The colors correspond to the NaCl concentrations in Figure 4.1. The inset shows the same data with the x-axis being the ionic strength, $I = \rho_{NaDS} + \rho_{NaCl}$.]

Figure 4.3: Degree-of-counterion association as a function of NaDS concentration for different NaCl concentrations at 298 K from MD simulations. The colors correspond to the NaCl concentrations in Figure 4.1. The inset shows the same data with the $x$-axis being the ionic strength, $I = \rho_{NaDS} + \rho_{NaCl}$.

### 4.3.2 Comparison to Experiments

The free surfactant concentration for ionic surfactants has a qualitatively different dependence on total surfactant concentration than nonionic surfactants. Nonionic surfactants have a linear decrease above the cmc\[136, 140, 190\]. For ionic surfactants, at the cmc there is a sharp decrease, followed by a more gradual decrease. When plotted on a log-log scale, as in Figure 4.4, $\rho_{\text{free}}$ is linear with no added NaCl. Experiments have shown that as NaCl is added, the cmc decreases logarithmically\[196\]. Because of the decrease in the cmc, as NaCl is added, the $d\rho_{\text{free}}/d\rho$ decreases as well (the slopes from Evans et al.\[204\] measured from $50 \text{ mM} < \rho < 500 \text{ mM}$ are $d\rho_{\text{free}}/d\rho(\rho_{\text{NaCl}} = 0\text{ mM}) = -0.49 \pm 0.02$ and $d\rho_{\text{free}}/d\rho(\rho_{\text{NaCl}} = 500\text{ mM}) = -0.035 \pm 0.003$). Similarly,
the free nonionic surfactant concentration slope above the cmc, decreases linearly as the cmc decreases\cite{190}. Above the cmc, $\rho_{\text{free}}$ collapses when plotted as a function of the total electrolyte concentration, which agrees with empirical observations\cite{32}.

![Figure 4.4: Free surfactant concentration from MD simulations (symbols) and single-DS ion activities from experiments\cite{204} (lines) of NaDS at different added NaCl concentrations. The colors correspond to the NaCl concentrations in Figure 4.1 with the addition of 10 mM (maroon). The inset shows the same data with the $x$-axis being the ionic strength, $I = \rho_{\text{NaDS}} + \rho_{\text{NaCl}}$.](image)

We measured the free surfactant concentration directly with implicit-solvent surfactant simulations. The excluded-volume contribution to the decease is particularly small, as shown in Figure 4.5. Although at higher temperatures, and thus higher cmcs for this model, the excluded volume has a larger impact. In any case, incorporating the effect of counterion condensation is required to account for the decrease of the free ionic surfactant concentration. Figure 4.5 shows that the correction proposed by Bales and co-workers\cite{32,127} is able to account for a large amount of the decrease, but at higher concentrations ($\rho \sim 100$ mM) the correction decreases, and would estimate the cmc of the model as 2.5 mM, which is an order of magnitude more accurate than using $\rho_{\text{free}} = 0.25$ mM at $\rho = 500$ mM, for example. The cmc
predicted from the mean ionic activity, 3.4 mM, is in agreement with the maximum in $\rho_{\text{free}}$, 3.7\(\pm\) 1.7 mM. The fits shown in Figure 4.5 can be used to determine the cmc. The mean activity is well fit by $a_{\text{NaDS}} = c\rho + d$ where $c = 4.0 \times 10^{-4} \pm 5 \times 10^{-5}$ and $d = 3.45 \pm 0.02$ mM for $\rho > \rho_{\text{cmc}}$. Where $3.45 \pm 0.02$ mM corresponds to the cmc, therefore we can write the equation, as others have as, $a_{\text{NaDS}} = c\rho + \rho_{\text{cmc}}$. For the fit to the free surfactant concentration, $\rho_{\text{free}} = A\rho^b$ where $A = 9 \pm 1$ and $b = -0.53 \pm 0.06$. Where $-b$ can be considered the charge reduced by the micelle size, or $-b = \alpha$. Figure 4.3 shows that 0.53 is in good agreement with the $\alpha$ measured from simulation snapshots for $\rho < 100$ mM. The cmc can be estimated as the concentration at which $\rho_{\text{free}}(\rho_{\text{cmc}}) = \rho$. With the cmc condition, the fit equation can be solved for the cmc as $\rho_{\text{cmc}} = A^{1/(1-b)} = 4.3$ mM, which is within the uncertainty of the value predicted by the maximum in $\rho_{\text{free}}$. If $\rho_{\text{free}}$ is fit over a concentration range of 100 mM $< \rho < 500$ mM, instead of 3.5 mM $< \rho < 500$ mM as is done in Figure 4.5, the fit values are $A = 46 \pm 1$ and $b = -0.83 \pm 0.06$, which shows that depending on the concentration range when measuring $\alpha$.

We found that increasing the NaCl concentration results in a monotonic increase of $\alpha$. Many experimental techniques measure an increase of $\alpha$ with NaCl concentration, as shown in Figure 4.6. However, experimental results from electrochemical and conductivity measurements have also shown non-monotonic dependencies. Furthermore the absolute values measured by different, and within the same, techniques vary from 0.5 to 0.85 at single NaCl concentrations.

One possible explanation for the variability in $\alpha$ measurements is that it is often assumed to be constant with surfactant concentration. Figure 4.3 shows that the $\alpha$ measured structurally is concentration dependent. One could imagine that the concentration range of values used to determine $\alpha$ would impact its measurement. The structural, and three experimental concentration-dependent measures are shown in Figure 4.7.
Simulations can directly measure $\alpha$ from the structure. The trends observed from simulations seem to qualitatively agree with the micellar film and conductivity $\alpha$ measurements, which signal that those experimental techniques likely measure something similar. Measuring $\alpha$ from the electromotive force is fundamentally different. Single-ion activities are calculated from the electromotive force. If single-ion activity of Na is assumed to equal the free Na ions, the fraction with the total concentration can be assumed to be the degree-of-counterion association ($\alpha = a_{Na}/\rho_{NaDS}$). The $\alpha$ measured from the activity follows a logarithmic and qualitatively different increase from the other methods. Changing the cut-off, $r_{cut}^\alpha$, is not sufficient to change the shape of $\alpha(\rho)$ to cause agreement with the structural and activity methods.
Simulations can help clarify the differences in the behavior between the different experimental techniques. Although we were not able to calculate the single-ion Na activity directly, we do have the mean ionic activity and the free surfactant concentration. The free surfactant concentration seems to be in agreement with the single-ion DS activity (see Figure 4.4). Thus, by assuming that the mean ionic activity is the geometric mean of the single-ion activities[178], \( a_{\text{NaDS}} = \sqrt{a_{\text{Na}}a_{\text{DS}}} \), the single-ion Na activity can be approximated as \( a_{\text{Na}} = a_{\text{NaDS}}^2/a_{\text{DS}} \). Figure 4.5 shows the values for \( a_{\text{NaDS}} \) and \( \rho_{\text{DS}} \) measured from simulations. The \( \alpha \) measured from the activity by simulations, shown in Figure 4.7, gives near-quantitative agreement with the \( \alpha \) measured from the activity by electrochemical experiments. Figure 4.7 also shows arrows for \( \alpha \) values measured by methods which calculate a surfactant-concentration
independent value. Among the data presented, there is a general trend in that conductivity measurements agree with the structural simulations while electromotive force and light-scattering measurements give considerably higher values.

Figure 4.7: Sodium dodecyl sulfate degree-of-counterion association at 298 K. Black symbols are from MD simulations. The black line is calculated as $\alpha = 1 - a_{Na}/\rho_{NaDS} = 1 - \frac{a_{Na}}{\rho_{NaDS}a_{DS}}$ with simulation data, where we assume $a_{DS} = \rho_{DS}$. The colored arrows correspond to surfactant-concentration independent measurements using the: ratio of Na and DS activity slopes from electromotive force measurements (red arrows[204, 230]), conductivity slope ratios above and below the cmc (orange arrows[41, 205, 217, 235]), and charge-size ratio measured from mobility (violet arrow[236]), light scattering (cyan arrow[237]) and zeta potentials (blue arrows[235]). The concentration-dependent experimental results come from: conductivity (violet stars[236]), micellar thin film thickness (green crosses[238]), and $\alpha = a_{Na}/\rho_{NaDS}$ measured using electromotive force (red triangles[239], circles[178], diamonds[240], squares[204] and crosses[241]).

The mean ionic activity coefficient, shown in Figure 4.8, agrees with experimental values above and below the cmc, when normalized by the cmc. Below the cmc, the mean ionic activity coefficient obeys the Debye-Hückel theory, $\gamma_{NaDS} = -0.5116 \sqrt{\rho_{NaDS}}$. Above the cmc the mean activity is nearly constant, so that the activity coefficient is $\gamma_{NaDS}/\rho_{NaDS} = 3.68$ mM. These two fits give $R^2 \geq 0.999$. Fits of this kind have been used in experimental studies[178]. The NaCl model parameters
were adjusted to fit the experimental NaCl activity coefficients[218], while the NaDS model parameters were fit to structure and the cmc[214]. Therefore, we believe that matching the electrostatics is the most important criteria for capturing the ionic activity behavior.

Figure 4.8: Sodium dodecyl sulfate mean ionic activity coefficient at 298K with no added NaCl calculated using: GCMC simulations (black circles), and electromotive force experiments (red pluses[188], open circles[239], squares[242], diamonds[242], right-facing triangles[178], left-facing triangles[243], upward-facing triangles[240] and downward-facing triangles[243]). At low concentrations the activity coefficient obeys the Debye-Hückel law (green dot-dashed line). The simulation points are fit will with $\gamma_{\text{NaDS}}/\rho_{\text{NaDS}} = 3.68$ mM (black dashed line).

4.4 Discussion and Conclusions

We studied ionic surfactant and salt concentration dependent behavior in the excluded volume, degree-of-counterion association, the mean ionic activity coefficient and the free surfactant concentration. For the activity, free surfactant concentration and the degree-of-counterion association, we were able to compare directly to exper-
imental results. The implicit-solvent model that we used enabled the study of the mean ionic activity coefficient and the simulation of low concentrations.

The cmc of the model used here disagrees with the experimental values, however phenomenologically it agrees with experimentally observed behavior above and below the cmc. Below the cmc, the mean ionic activity and the free surfactant concentration equal the total surfactant concentration, and the excluded volume and degree-of-counterion association are zero. The mean ionic activity increases linearly above the cmc, and with no added salt is nearly constant above the cmc. Thus the mean ionic activity is a good estimate for the cmc at high concentrations. The near-constancy of the mean ionic activity explains the $1/\rho$ decrease in the mean ionic activity coefficient above the cmc. Below the cmc the mean ionic activity coefficient follows the Debye-Hückel limiting law.

Single-ion activities give strong experimental evidence for the decrease in the free surfactant concentration. However, whether single-ion activities can be calculated from the electromotive force measured from selective-ion electrodes has been questioned[244, 245]. Work has been done in the field to develop thermodynamic justifications[246]. These issues were not addressed here, however it is striking that, when differences in the cmc are accounted for, there is near quantitative agreement in the decrease in the single-DS ion activity calculated from the E.M.F. and the free DS concentration calculated from simulation snapshots. Therefore, E.M.F. measurements of surfactant solutions with surfactant-selective electrodes do provide clear evidence of the decrease in surfactant concentration.

The decrease of ionic surfactants fundamentally differs from nonionic surfactant concentration decrease, in that it is not linear, but logarithmic. As salt is added, the cmc decreases. The decrease in the cmc leads to smaller negative slopes of $\rho_{\text{free}}$ above the cmc. The correction developed previously was shown to work well in the initial sharp decrease, however gave under-predictions at higher concentrations. Further
confidence in the correction enables simulations to better predict the cmc of models and non-electrochemical experiments to calculate the free and micellar concentrations more accurately. This confidence will aid in the development of improved models to help search surfactant formulations and architectures for specific applications.

Counterion association was shown to have a large impact on the free ionic surfactant concentration. The degree-of-counterion association is concentration dependent, and is largely due to the binding in the first hydration shell. The degree-of-counterion association increases with both surfactant and salt concentrations. A considerable amount of this increase is accounted for by the increase in ionic strength of the solution. Experimental techniques for measuring the degree-of-counterion association often assume it is independent of surfactant concentration. We took advantage of being able to measure the degree-of-counterion association structurally and thermodynamically to interpret differences between techniques. Criteria which incorporate an additional time-dependent residence criteria would be promising, especially comparing to more dynamic experimental techniques like electrophoresis or the zeta potential. A theroretical prediction for $\alpha(\rho_{NaDS}; \rho_{NaCl})$ behavior is an interesting future challenge. Knowing the surfactant and salt concentration dependence of $\alpha$ will aid in formulation design concerned with micellar charge ($\alpha M$) and the aqueous environment. Furthermore, this study helped interpret other experimental measurements of $\alpha$.

A method was developed to rigorously calculate the excluded volume. Above the cmc, the excluded volume increases, and is well described by a linear fit. However in systems with larger micellar size and shape changes this increase will follow the empirically observed behavior of the preferred aggregation number, where it increases with $\langle M \rangle = A \rho_{mic}^b$, where $b \simeq 0.25$ and $A$ is the $\langle M \rangle$ at the cmc. Calculating the excluded volume simply by the volume occupied by the surfactants under-predicts the excluded volume calculated directly, however, the excluded volume effect on
the free surfactant concentration is considerably low compared to the electrostatic contribution.
Chapter 5

Concluding remarks

Three systems were studied in this dissertation: colloids with competing interactions, nonionic surfactants and ionic surfactants in aqueous media. Self-assembly and concentration effects were the focus across the different chapters. Each chapter is summarized and contextualized first, followed by final remarks on conditions for micellization, the free surfactant concentration decrease, the models and methodology. Lastly, future work based on this dissertation is discussed, including new avenues of research and industrial applications.

5.1 Summary

In this dissertation a boundary condition of directionality in amphiphile self-assembly – systems with isotropic interactions – is systematically studied and tested. Specifically, isotropic short-range attraction and long-range repulsion (SALR) interaction between colloids enables their self-assembly. The distinct and broad distributions of free and clustered colloids, and the decrease in the osmotic pressure-density slope identify a critical clustering density and resemble surfactant micellization. Study of SALR self-assembly should therefore take advantage of the surfactant literature and techniques developed therein. SALR systems can also self-assemble in a way which is
distinct from amphiphile micellization, identified by distinct and broad distributions of free and clustered colloids, without a decrease in osmotic pressure-density slope.

A definitive explanation for the novel behavior remains a challenge; cluster size and shape, the cluster concentration, cluster-cluster interactions and cluster kinetics did not show distinct differences between attraction-dominated and repulsion-dominated systems. A qualitative explanation is that isotropic SALR colloids have less restrictive cluster configurational requirements than surfactants. Surfactants’ amphiphilicity restrict them to the micelle corona, whereas SALR colloids occupy the micelle corona and the core.

Studies of SALR systems should identify if pressure-affecting and non-pressure-affecting behavior occurs. The temperature at which the second virial coefficient changes sign is a good first prediction. However, because of the non-ideality of the aggregation behavior, there is not a guarantee that the system clusters at all. Neither ideal-gas nor second-virial expansion models of a system with free colloids and clusters were able to explain the osmotic pressure behavior. If clustering is known to occur, the preferred aggregation number can be predicted using only the interaction potential: \( \langle M \rangle = 4\pi \left( \frac{r_{\text{att}}}{r_{\text{well}}} \right)^2 \), where \( r_{\text{well}} \) is the attractive well location and \( r_{\text{att}} \) is the range of attraction, defined as \( U(r_{\text{att}}) = 0 \).

The nonionic surfactant model used in this dissertation is also able to capture many features of amphiphile self-assembly with a simple interaction potential, and it has been used extensively in the literature. The establishment of the model made it a good tool to develop a protocol and detailed methodology for calculating the critical micelle concentration and the critical micelle temperature from simulations. A clear definition for critical micelle temperature is the temperature at which no preferential micellar distribution appears, even above the extrapolated cmc. This definition does not negate the possibility of gel formation. For the nonionic surfactant system, there were concentrations at which micelles exist within the gel, defined as a system-
spanning aggregate. The cmc calculated from three properties, namely, the osmotic pressure, free surfactant concentration and aggregation number distributions, agreed with each other at low cmc’s when calculated with the correct criteria. Some of those successful criteria were developed in this dissertation. At higher cmc’s there was disagreement between methods, as there is for surface tension and conductivity cmc measurements. Therefore, it is important to choose self-consistent transition criteria and to compare the cmc’s measured from different properties. This nonionic surfactant study was able to clearly address the cause of the free surfactant concentration decrease and develop protocols for measuring the cmc which benefit studies of surfactant mixtures and more complex models.

Charges in ionic surfactant systems introduce an added level of complexity to the methods developed for, and properties studied in, nonionic surfactants. The model studied herein is more realistic and approximates a specific system: alkyl sulphates with and without added salt, specifically sodium dodecyl sulfate (NaDS) and sodium chloride (NaCl). The ionic surfactant model is off-lattice, unlike the nonionic surfactant model, and has many interaction sites, unlike the SALR colloid model. These differences lead to further challenges in correctly calculating the free surfactant concentration, degree-of-counterion association, the inaccessible volume and the cmc extrapolation.

Overcoming these methodological challenges, the free surfactant concentration was calculated from molecular dynamics simulations. The decrease was characteristically different from nonionic surfactants, being logarithmic instead of linear, due to the electrostatic component. When normalized by the cmc, the free surfactant concentration gave good quantitative agreement with experimental values of the single-ion activity of the dodecyl sulfate ions as a function of total NaDS and NaCl concentrations. Further experimental comparison was accomplished with grand-canonical Monte Carlo simulations to measure the mean ionic NaDS activity. This too, when
normalized by the cmc, gave good agreement with experiments, which highlights the importance of the cmc and its measurement.

The degree-of-counterion association measured structurally by the distance of counterions from the micelle interface in simulations showed a positive dependence on the total NaDS and NaCl concentrations. Experimental calculations of the degree-of-counterion association, depending on the technique, assume that it is: (i) independent of the salt and surfactant concentration, (ii) dependent on only the salt concentration, or (iii) dependent on both the salt and surfactant concentration. Furthermore, the different experimental techniques show different absolute values, ranging from 0.5 to 0.9. Even trends of experimental measurements disagree, showing negative, positive and non-monotonic salt concentration dependencies. By using the same model we help illustrate what these different techniques likely measure.

As in the nonionic surfactant system, the cmc calculated from an extrapolation method was tested. The excluded volume has a negligible effect on the free surfactant concentration decrease, especially for low-cmc state conditions. Instead, the free surfactant concentration is largely dependent on the counterion condensation onto the micelles. The addition of salt leads to a more constant free surfactant concentration, yet this is largely due to the decrease in the cmc and thus the maximum free surfactant concentration.

The ionic surfactant results help resolve disagreements among different experimental techniques for micellar properties, as well as support methods for calculating the free surfactant concentration dependence for simulations and experiments. These findings will lead to better predictions and molecular models.
5.2 Conditions for micellization

The conditions for micellization were investigated in a number of ways. In the non-ionic case, a transition in a thermodynamic property and the preference of larger, micellar aggregates are generally sufficient for micellization. However, there are cases where micellar aggregates are preferred but not detectable. For example, approaching the critical micelle temperature, the cmc measured from the pressure and the aggregation distribution differ. Whether the osmotic pressure or the aggregation number yields the correct cmc for these large cmc’s is up for discussion, but could be resolved by communicating the range of cmc values which correspond with gradual micellization. It is important to note that the preference of micelle-sized aggregates, regardless of their appearance, is necessary but not sufficient for micellization. In the SALR case, large preferential aggregates formed without a perceived thermodynamic response in the osmotic pressure, which is not to say there was not a thermodynamic response.

Like temperature and interaction potential, simulation system size can affect the identification of micellization, and the system size can help distinguish between a phase transition and micellization[203]. The conditions for micellization continue to be ambiguous; novel self-assembling entities continue to be tested, such as SALR colloids, janus particles[247], proteins[18] and surfactants[11].

5.3 Free surfactant concentration decrease

The effective use of surfactants in micellar catalysis[30], nano-composites[29] and solubilization[28], is dependent on the free surfactant concentration. The free surfactant concentration is utilized by simulations to determine the cmc[31], and experiments to determine the preferred aggregation number[32].
The decrease in free surfactant and colloid concentration was observed in all of the systems studied in this dissertation, regardless of system size, thermodynamic ensemble and initial condition. The decrease has been observed in trimer amphiphiles[139], among other self-assembling systems, and in confinement[248]. The scale of the decrease is sensitive to many properties, and depending on the concentration range, temperature, salinity and surfactant, could be negligible (a few percent decrease up to three orders of magnitude above the cmc), or quite drastic (an order of magnitude decrease at less than an order of magnitude above the cmc). The importance of the decrease can be predicted using the tools developed in this dissertation and previously.

Generally, the larger the cmc, the larger the magnitude of the decrease, because at the initial micellization there are more free surfactants. If the cmc is the same, the type of the surfactant head group is most important – whether anionic, cationic, nonionic or zwitterionic. This dissertation demonstrates that the decrease in ionic surfactants is typically stronger than nonionic surfactants. For nonionic surfactants the inaccessible volume is the main cause for the decrease and shows a linear decrease above the cmc. The ionic surfactant concentration decreases exponentially above the cmc due to counterion condensation. The free SALR colloid concentration decreased linearly above the cmc, however the excluded volume was insufficient to quantify the decrease. Within the same head group, the tail architecture effect is smaller and originates from changes in the micellar structure[191].

A new method for calculating the excluded volume was developed in chapter 3 and was modified for off-lattice simulations in chapter 4. The method is not particularly computationally expensive but does require simulation configurations, which can have large data storage requirements. The necessity of the method depends on the system. If the effect is particularly small, as it is in the ionic surfactant case at low cmc values, the improvement is less consequential, and a more simple surfactant
volume or radius of gyration type of approximation would be sufficient. The quality of those approximations is typically not sensitive to temperature and architecture.

It is important to note the two aspects of ambiguity in the technique: the cluster identification criteria and the overlap criteria. In our experience the excluded volume is much more sensitive to the clustering criteria than the overlap criteria. The number of simulation configurations and monomer insertion conformations required depend on the micelle size distribution, state conditions and molecular degrees-of-freedom, but can be reliably determined by observing convergence of the excluded volume for representative conditions.

5.4 Models and methodology remarks

Implicit-solvent models, used throughout this dissertation, have many benefits in simulation. Larger time- and length-scales are made accessible by not having to iterate over solvent molecules. Monte Carlo simulations can perform moves that could have prohibitively low acceptance rates in explicit-solvent models of dense systems. The grand-canonical ensemble, and the properties which are easily accessible from it, such as the chemical potential, are thus a beneficial ensemble for implicit-solvent models.

Of course, micellization is very sensitive to the solvent and its characteristics. Therefore, implicit-solvent models should not be used beyond their parameterized state region, without understanding of the phenomena. Most implicit-solvent models are wholly unable to capture some micellization effects. Special corrections to implicit-solvent models, such as directional interactions[149] or a temperature-dependent parameter[156], can capture the non-monotonic dependence on temperature of the cmc. Due to its more complex nature, the ionic surfactant model used in this dissertation has additional limitations. The model uses intra-molecular interac-
tions developed for explicit-solvent or dense alkane simulations[224, 225]. Molecular
dynamics simulations of the model thus require double-precision in solving the forces
because of the dihedral interaction. Furthermore, coarse-graining the head group
\((\text{SO}_4^-)\) while having a united-atom tail is an uncommon protocol. However, the loss
of specificity from the coarse-grained head group is smaller for an implicit solvent
because the solvent is unstructured. The model places all of the work of cavity
formation onto the terminal tail carbon, which goes against data on the hydration
fraction along micellized surfactant tails. Hydration fraction data show that it is
usually evenly distributed among the last 4-6 carbons[48, 249]. A considerable ef-
fort was made, but is not reported in this dissertation, to develop a model with the
hydrophobic interaction distributed across more tail carbons. This proved largely
unsuccessful because such a model encourages intercalation of tails and favors bi-
layer/disk aggregates over spherical aggregates. Like any model, the limitations and
advantages of implicit-solvent surfactant models should be considered in the context
of each research goal.

Surfactant micellization continues to be a challenge due to the potentially
long equilibration times required to observe self-assembly. Studies which use
pre-assembled micelles to look at structure of aggregates often underestimate the
simulation time required to equilibrate the micellar size distribution or shape[59].
The properties of the models studied in this dissertation equilibrate in the following
order: first, the excluded volume (if properly (un-)micellized), the free surfactant
concentration, the counterion condensation (if ionic) and lastly the aggregation
number distribution.

The sensitivity of many micellar properties to the clustering criteria is often over-
looked. Clustering criteria are often verified by visual inspection of a few snapshots.
However, visual inspection misses quantitative differences which can result in dif-
ferences in behavior. Self-assembly studies should always test the sensitivity of the clustering criteria and parameters.

Histogram reweighting was used throughout the dissertation, and was used largely because of the ability to calculate the equation of state ($\mu(\rho, T)$) and the partition function (often reported as $\Pi(\varphi, T)$ or $P(\rho, T)$). These thermodynamic properties enable comparison to experiments and show the impact of structural and dynamic shifts in self-assembly behavior. Histogram reweighting can also bypass regions of hysteresis between free and micellar states; however, the overall benefit depends on the model and state conditions. It is known that histogram reweighting becomes increasingly difficult with system size – not only do individual simulations take longer, but more histograms are required for overlap. Thus, the overall simulation time, number of simulations and reweighting convergence time all increase. In the ionic surfactant case, especially with added salt, required system sizes and number of histograms were computationally demanding.

To calculate the cmc, three properties were compared for nonionic surfactants (Chapter 2): the aggregation number distribution ($\varphi(M)$), free surfactant concentration ($\varphi_{\text{free}}(\varphi)$) and osmotic pressure ($\Pi$). The concentration at which a second micellar peak appears in the $\varphi(M)$, the maximum in $\varphi_{\text{free}}(\varphi)$ and the maximum in the curvature of $\Pi$ are the definitions which were self-consistent. These methods are consistent in the SALR system and with different surfactant architectures[191].

These methods correspond to defining the cmc as the concentration at which the first micelle appears rather than the concentration at which the free surfactant and micellar concentrations are equal. It may be fair to use the later definition ($\rho_{\text{free}} = \rho_{\text{mic}} = \rho_{\text{cmc}}$) when studying the aggregation number distribution and free surfactant concentration properties; however the cmc it would identify is much higher than the “kink” in the osmotic pressure. Since most experimental techniques calculate the cmc from the osmotic pressure, conductivity or surface tension, which exhibit transitions
at the concentration at which the first micelle appears, I advise simulation studies to implement that definition.

5.5 Future Work

This dissertation poses new questions in addition to inviting applications to more industrially relevant systems. The reason for the free colloid concentration decrease for the short-range attraction long-range repulsion system is an open question. A theoretical explanation or an effective extrapolation to the cmc, as was developed for nonionic and ionic surfactants, would expedite design parameter searches for such colloids. Mixtures of repulsion- and attraction-dominated colloids would be a good test case for studying free colloid decrease as a function of concentration and mixture fraction with mixed clusters. Mixtures of colloids with different interactions and sizes can form eutectics\cite{250}, which could be advantageous from a manufacturing perspective. Mixtures of SALR colloids are of interest for protein therapeutics to see how mixtures affect the the range of conditions for reversible aggregation\cite{251} and could provide a potential explanation for, or aid in, the search for experimental high-density phases\cite{61}.

Solubilization is an area for future development for the nonionic surfactant model system. Adding an otherwise insoluble material to a solvent is an important component of many materials and processes used in industry and nature. From the detergency mechanisms\cite{252}, to the addition of fragrances to soap\cite{253, 254}, to an environmentally beneficial alternative to catalysis\cite{255}, the behavior of surfactants and solvophobic particles influences many industries and applications. The lattice model is able to equilibrate and sample a wide parameter space of surfactant and solubilize structure and concentration. A systematic study of concentration and micelle shape effects on solubilization would help connect molecular behavior, such
as the location of the solubilizate in micelles, to macroscopic behavior, such as the ratio of solubilization[33].

Surfactant self-assembly theory and simulations have prompted the discovery and development of formulations for many industries, including personal and home care. The low-cmc conditions that are typically used in those applications have posed a challenge to the reliable and efficient development of accurate molecular models and the simulation of full micellization, not just a preformed micelle. This dissertation, which builds on a large base of literature, aims to improve awareness for concentration effects and reliable measurement techniques. The development of more realistic explicit-solvent models that accurately calculate the cmc, aggregation number and counterion association is a major future challenge, and the protocols and extrapolation techniques in this dissertation can be applied to address the challenge[31, 125].

The direct comparison of the mean ionic activity from simulations to experiments joins comparisons to surface tension measurements[256, 257]. It would be interesting to investigate the effect of the decrease of the free surfactant concentration on the surface tension above the cmc. Experimental studies have been ambiguous on this topic, and simulations of the same model would be able to make a direct comparison. Calculation of the ionic conductivity and ion diffusion in micellar systems has yet to be studied in molecular simulations of surfactants. Ionic surfactant molecular dynamics simulations could take advantage of the polyelectrolyte and ionic liquid simulation literature and be able to compare to experimentally measured properties[258, 259]. The ionic surfactant research presented in this dissertation can be furthered by the analysis of the effect of changing the coions or counterions by size and valency.

In closing, this dissertation used molecular simulations and models to study the self-assembly of colloids and surfactants. Properties, such as the free surfactant concentration, were measured due to the molecular detail of the models. The molecular detail of the self-assembling system also offered the opportunity to test hypotheses
about the origin of certain behavior, such as the non-pressure-affecting colloid self-assembly. By taking advantage of existing, and developing new, techniques, these measured properties were at times directly compared to experimental values, such as the degree-of-counterion association for ionic surfactants with salt. The techniques and protocols developed herein will hopefully support the future work of the field.
Bibliography


