A SOLUTION TECHNIQUE FOR RATIONAL EXPECTATIONS MODELS WITH APPLICATIONS TO EXCHANGE RATE AND INTEREST RATE DETERMINATION*

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1. INTRODUCTION

It is well known that the solution paths of perfect foresight or rational expectations models generally have a saddle-point structure. This division between directions of stability and instability corresponds to an economic distinction between variables whose initial values are given and those without such data. The condition that the system be placed on the stable manifold of the saddle point then serves to fix these remaining initial values. Although the requirements of long-run perfect foresight needed to achieve this in practice are stringent, the models may yield useful approximations for determination of asset prices in well-organised markets.

Following Dornbusch (1976), models of exchange rate determination have been built along these lines. The price level changes slowly, and is therefore a variable with a pre-determined initial value. The exchange rate can adjust instantly. Following a sudden unforeseen change in the money supply, the exchange rate jumps to a new level which ensures

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that the resulting path converges to the new equilibrium. Buiter and Miller (1980) have extended this model to examine the effects of a change in the rate of growth of the money supply. They find that the reduction of this rate causes a sudden appreciation of the real exchange rate, i.e. a loss of competitiveness.

This paper develops a simple general method for solving such models, i.e. determining the initial values of the free variables in terms of the others. The method is exact for a linear model; more generally, it serves as an approximation in the neighbourhood of the equilibrium. The main advantage of the method is its ease of solution and interpretation when there is only one free variable, which is the case in most qualitative models constructed to examine particular issues.

The method is then applied to a model of exchange rate determination that is a considerable generalisation of the Buiter-Miller model. In addition to the price level, I allow slow adjustments in the expectations concerning long-run inflation, and the output level. This enables us to examine the sensitivity of the exchange rate jump with respect to the various speeds of adjustment. It is found that finite speeds of adjustment of these further variables lead to a strengthening of the Buiter-Miller result, i.e. the loss of competitiveness following a reduction of monetary growth is much greater.

The final section combines exchange rate determination with that of the long-run interest rate. The latter parallels a model of a closed economy in Blanchard (1978) where the output level was the slow-moving variable. With two free variables, analytical results are harder to obtain, and a numerical solution is reported.
2. A GENERAL LINEAR MODEL

Consider a system of $n$ linear differential equations

$$\dot{x} = Ax$$  \hspace{1cm} (1)

where $x$ is an $n$-vector of deviations of all the variables from their equilibrium levels chosen to be $0$, and $A$ is an $n$-by-$n$ matrix of constant coefficients. If $A$ has distinct eigenvalues, as will be the case generically, then it can be diagonalised by a transformation, i.e. there exists a non-singular matrix $M$ such that

$$MAM^{-1} = \Lambda .$$

where $\Lambda$ is a diagonal matrix with entries $\lambda_i$. Writing this as

$$MA = \Lambda M,$$

we recognise that the entries of $\Lambda$ are the eigenvalues of $A$, and the rows of $M$ are the corresponding left-eigenvectors.

Define the change of variables

$$y = Mx$$  \hspace{1cm} (2)

so that

$$\dot{y} = M\dot{x} = MAX = MAM^{-1}y = \Lambda y$$

i.e.

$$\dot{y}_i = \lambda_i y_i \text{ for } i = 1,2,\ldots,n$$

These have solutions

$$y_i(t) = y_i(0) e^{\lambda_i t} \text{ for } i = 1,2,\ldots,n$$  \hspace{1cm} (3)
Suppose some eigenvalues have positive real parts and others have negative real parts, i.e. there is a generalised saddle point.
Partition the vectors as follows. Let $y^1$ be the sub-vector of $y$ corresponding to the eigenvalues with negative real parts, and $y^2$ that for ones with non-negative real parts. For a stable solution, i.e. one which converges to the equilibrium $x = y = 0$ as $t$ goes to infinity, we see from (3) that the initial values must satisfy

$$y^2(0) = 0 \quad (4)$$

Let $x^1$ be the subvector of variables with known initial values $x^1(0)$, and $x^2$ that of ones with free initial values $x^2(0)$. These are then fixed by the condition that the system be placed on the stable solution into the saddle point. For a determinate system, the dimension of $x^2$ must equal that of $y^2$. Partitioning the transformation matrix conformably, and using (2) and (4), we have

$$\begin{bmatrix} y^1(0) \\ 0 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} x^1(0) \\ x^2(0) \end{bmatrix}$$

Then

$$M_{21} x^1(0) + M_{22} x^2(0) = 0$$

or

$$x^2(0) = -M_{22}^{-1} M_{21} x^1(0) \quad (5)$$

This is the formula giving the desired initial values of the free variables in terms of those of the pre-determined variables and the parameters of the system. In fact the parameters enter the picture in
a particular way: the rows of the partitioned sub-matrix \((M_{21}, M_{22})\) are simply the eigenvectors of \(A\) corresponding to the unstable eigenvalues.

If there is only one unstable eigenvalue, corresponding to one free variable, the formula is particularly simple. Now \(M_{22}\) is a scalar, and since eigenvectors are determinate only up to a scale factor, we can set it equal to \(-1\). Then \(x^2(0)\) is a simple linear combination of the components of \(x^1(0)\), the weights being the corresponding components of the normalised unstable eigenvector.

An alternative and equivalent formula can be cast in terms of right eigenvectors; see Buiter and Miller (1980, Appendix). That would be simpler for a model with only one pre-determined variable and a number of free ones. In general, either might prove the more convenient.

3. MONETARY POLICY AND EXCHANGE RATES

In this section the above model will be applied to the question of the effect of monetary policy on the exchange rate. The general view underlying the model to be constructed is that asset markets are much more likely to behave like efficient auction markets with rational expectations than the markets for goods and services. Accordingly, the former markets are represented by equations showing instantaneous clearing and perfect arbitrage, while the latter markets are given by differential equations showing adjustment processes.

The notation is as follows. The logarithms of the money supply, the price level, real output, and the exchange rate are respectively denoted by \(m, p, y,\) and \(e\). Correspondingly, in logarithmic terms, \(c = e - p\) provides a measure of the real exchange rate (or competitiveness), and \(\ell = m - p\) measures liquidity. The interest rate is denoted by \(r\), and the expectations of long run inflation (called
core inflation by Buiter and Miller) by $h$.

Monetary policy is given by

$$\dot{m} = g$$  \hspace{1cm} (6)

where $g$ is constant over time. The main comparison exercise will be one where the system is initially in a long run equilibrium corresponding to a certain value of $g$, and suddenly at $t = 0$ a new policy with a lower value $(g - \Delta g)$ is instituted. The slowly-adjusting variables remain at their old equilibrium values immediately after the change. The rapidly-adjusting ones can accommodate the new information and jump to new levels which permit convergence to the new long-run equilibrium. In particular, the impact effect on the exchange rate will be determined in this way.

The interest rate can adjust rapidly to equate demand and supply for money, i.e. to maintain the economy on the LM curve. Choosing a linear demand function in the variables used, and ruling out money-illusion, we have

$$m = p + \alpha y - \beta r$$  \hspace{1cm} (7)

Output adjusts towards an IS curve. Letting aggregate 'reduced form' effective demand depend on the real exchange rate and the real interest rate, we have

$$\dot{y} = \psi \left\{ \gamma (e-p) - \delta (r-\bar{r}) - y \right\}$$  \hspace{1cm} (8)

where $\psi$ is an adjustment coefficient. The change in the price level is given by an expectations-augmented Phillips curve

$$\dot{p} = h + \phi (y - y^*)$$  \hspace{1cm} (9)

where $y^*$ is the normal, long-run equilibrium level of $y$, taken to be
independent of any monetary influences, and $\phi$ is an adjustment coefficient. Finally, expectations concerning the normal or long-run rate of inflation adjust to the rate of monetary growth according to

$$\dot{h} = \xi (g-h)$$

(10)

This can be justified by the argument that it is only persistent observation of the rate of monetary growth that can gradually make the policy credible.

Let the foreign rate of interest $r^*$ be constant for sake of convenience. Assume that domestic and foreign bonds are perfect substitutes, and that the foreign exchange market is perfectly arbitraged. Then

$$\dot{e} = r - r^*$$

(11)

This completes the set of basic equations of the model. It is more convenient as well as instructive to cast them in terms of the variables $\zeta, y, c$, and a new variable defined as $q = h - g$.

Following some simplification using (7), we find

$$\dot{q} = -\xi q$$

(12)

$$\dot{\zeta} = -q - \phi (y-y^*)$$

(13)

$$\dot{y} = \psi \{ \gamma c - (\delta/\beta) (a y - \zeta) + \delta (q+g) + \delta \phi (y-y^*) - y \}$$

(14)

$$\dot{c} = (a y - \zeta) / \beta - r^* - (q+g) - \phi (y-y^*)$$

(15)

This linear system has a long-run equilibrium solution given by

$$h = \dot{p} = \dot{e} = g$$

(16)
\[ y = y^* \]  
(17)

\[ r = r^* + g \]  
(18)

\[ z = \alpha y^* - \beta r^* - \beta g \]  
(19)

and

\[ c = (y^* + \delta r^*) / \gamma \]  
(20)

The long run properties are familiar: money is neutral but not super-neutral. In particular, the equilibrium liquidity level is affected by the rate of growth of the nominal money supply.

In matrix form, we can write the system (12)-(15) as

\[
\begin{bmatrix}
q \\
\xi \\
y \\
c
\end{bmatrix} =
\begin{bmatrix}
-\zeta & 0 & 0 & 0 \\
-1 & 0 & -\phi & 0 \\
\psi & \psi \delta / \beta & -\psi (1+\delta \alpha / \beta - \delta \phi) & \psi \gamma \\
-1 & -1/\beta & \alpha / \beta - \phi & 0
\end{bmatrix}
\begin{bmatrix}
q \\
\xi \\
y \\
c
\end{bmatrix} + \text{constants (21)}
\]

For the IS and LM curves to have the right relative slopes, we need

\[ 1 + \delta \alpha / \beta - \delta \phi > 0 \]  
(22)

This will prove useful later. Such a condition is discussed in greater detail by Bui ter and Miller (1980).

Let \( A \) denote the matrix in (21). We know from the general theory of the previous section that the eigenvalues of \( A \) are going to play a crucial role in the solution. They are to be found from the equation

\[ \det (\lambda I - A) = 0 \]

where \( \lambda \) is the unknown. This simplifies to
\[(\lambda + \zeta)\{\lambda^3 + \lambda^2 \psi \left[ 1 + \delta \alpha / \beta - \delta \phi \right] + \lambda \psi \left[ \gamma \phi - \gamma \alpha / \beta + \phi \delta / \beta \right] - \phi \gamma / \beta \} = 0 \quad (23)\]

Let the expression in the brackets be denoted by \(f(\lambda)\). Then one eigenvalue of the system is simply \(-\zeta\), and the other three are roots of the cubic \(f(\lambda) = 0\).

Observe that \(f(0) = -\phi \gamma / \beta < 0\), and \(f(\lambda)\) tends to infinity with \(\lambda\). Therefore there must be at least one real positive root. But the sum of the roots is \(-\left[ 1 + \delta \alpha / \beta - \delta \phi \right]\), which is negative by (22), and the product of the roots is \(\phi \gamma / \beta\), which is positive. The remaining two roots thus have a positive product and a negative sum. This leaves two possibilities: either they are both real and negative, or they are a pair of complex conjugates with negative real parts. In either case, these two roots contribute directions of stability to the system.

The whole four dimensional system therefore has one direction of instability, i.e. the eigenvector corresponding to the one real positive root, and three directions of stability corresponding to the other three roots. This accords with the fact that there are three slowly-changing variables, \(q\), \(\phi\), and \(y\), and one variable \(c\) capable of sudden jumps.

We can calculate the eigenvector corresponding to any root \(\lambda\) as follows. The row eigenvector must yield zero when multiplied on the right by the matrix \((\lambda I - A)\). With a view to applying the outcome to the formula (5), let us normalise it so that its last component is \(-1\). Let the other three components be \(v_1\), \(v_2\), and \(v_3\). Then we have

\[
\begin{bmatrix}
\lambda + \zeta & 0 & 0 & 0 \\
1 & \lambda & \phi & 0 \\
-\psi \delta / \beta & -\psi / \beta & \lambda + \psi \left[ 1 + \delta \alpha / \beta - \delta \phi \right] & -\psi \\
1 & 1 / \beta & \phi - \alpha / \beta & \lambda \\
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
-1
\end{bmatrix}
= \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}
\quad (24)
\]
Successive substitutions yield

\[ v_1 = -\frac{1}{1-\lambda} \left\{ \frac{1}{\lambda - \delta/\gamma} \right\} / (\lambda + \zeta) \]

\[ v_2 = \frac{1}{\lambda - \delta/\gamma} / \beta \]

\[ v_3 = \lambda / \psi \gamma \]

We can now answer the question concerning the initial response of the exchange rate. When the rate of growth of the money supply is suddenly lowered from \( g \) to \( (g - \Delta g) \), there is by assumption no impact effect on the values of \( q \), \( \ell \), and \( y \). Their new initial values equal the old steady state levels. From (16), (17), and (19) we can calculate their deviations from the new steady state values: these are zero for \( y \), \( \Delta g \) for \( q \), and \(-\beta \Delta g \) for \( \ell \). Let \( \lambda \) be the real positive eigenvalue of the system, and compute the corresponding eigenvector normalised to have the last component equal to \(-1\). Then formula (5) gives \( \Delta c \), the initial deviation of \( c \) from its long-run level: it is found as the sum of the weighted deviations of \( q \), \( \ell \), and \( y \) with respective weights \( v_1 \), \( v_2 \) and \( v_3 \). This works out at

\[ \Delta c = -\frac{(1/\theta - \lambda)}{\lambda + \zeta} \frac{(1/\lambda - \delta/\gamma)}{\beta} \Delta g = \frac{1/\lambda - \delta/\gamma}{\beta} \beta \Delta g \]

or, \( \Delta c = -\theta \Delta g \)  

(26a)

where

\[ \theta = \frac{(1/\lambda - \gamma/\delta)}{\zeta + 1/\beta} \]

(26b)

Since (20) shows that the long-run equilibrium value of \( c \) is unaffected by \( g \), this deviation also equals the size of the impact effect on \( c \).

It is easy to substitute in (23) and verify that

\[ E(\gamma/\delta) = (\gamma/\delta)^2 \frac{1+\psi \gamma/\delta}{\beta} > 0 \]
Hence $\lambda < \gamma/\delta$ and the coefficient $\theta$ is positive, i.e. a lowering of the rate of growth of the money supply causes a sudden loss of competitiveness.

To obtain a rough idea of the magnitude of this loss, the following numerical values of the parameters were chosen: $\alpha = 1$, $\beta = 2$, $\gamma = 0.2$, $\delta = 1$, $\phi = 0.5$, $\psi = \zeta = 1$. The rationale behind the choices was as follows. $\alpha$ is the income elasticity of demand for money. $\beta$ is the semi-elasticity of the money demand with respect to the interest rate, i.e. the full elasticity divided by the interest rate. Values of 0.2 and 0.1 for the respective magnitudes imply a ratio of 2. In symbols, writing $M$ for the demand for money (not in logarithms), we take

$$\beta = \frac{1}{M} \frac{dM}{dr} = \left( \frac{1}{r} \right) \left( \frac{1}{M} \frac{dM}{dr} \right) = \left( \frac{1}{0.1} \right) (0.2) = 2$$

Similar reasoning is applied to $\gamma$ and $\delta$. Writing $Y$ for aggregate demand, $X$ for the demand for exports, and $C$ for the real exchange rate (again, not in logarithms; thus $c = \log C$ etc.), we have

$$\gamma = \frac{C}{Y} \frac{dY}{dC} = \left( \frac{X}{Y} \right) \left( \frac{dY}{dX} \right) \left( \frac{C}{X} \frac{dX}{dC} \right) = (0.2) (1.5) (0.67) = 0.2$$

Finally, writing $I$ for the investment demand,

$$\delta = \frac{1}{Y} \frac{dY}{d(r-p)} = \left( \frac{1}{Y} \right) \left( \frac{dY}{dI} \right) \left( \frac{1}{r-p} \right) \left( \frac{r-p}{I} \frac{dI}{d(r-p)} \right)$$

$$= (0.1) (1.5) \left( \frac{1}{0.05} \right) (0.33) = 1$$

Following Buiter and Miller (1980), I choose $\phi = 0.5$. The other adjustment coefficients $\psi$ and $\zeta$ are each set equal to unity; this corresponds to a mean lag of one year in each of those processes.

With these values, it is found that $\bar{\lambda} = 0.127$, and $\theta = 3.81$. Thus a one per cent lowering of the rate of monetary growth in this
setting causes an immediate 3.81 per cent appreciation of the real exchange rate. In sterling-dollar terms this amounts to roughly 9 cents.

This impact effect is substantially stronger than that found by Buitier and Miller, who in their central case obtain \( \Theta = 2.36 \). Their model and parameter values differ from the ones used here in two respects. In the notation of this paper, they have \( \gamma = \delta = 0.5 \), and fast adjustment in \( q \) and \( y \), i.e. \( \zeta = \psi = \infty \). The first point point cannot account for the difference; in fact if that is the only change made in the above parameter set, we obtain \( \Theta = 3.22 \). Thus, an important source of the difference must lie in the speeds of adjustment. Some simple comparisons show the directions of the effects.

From (23) we see that \( \zeta \) does not affect \( \bar{\lambda} \); thus its only influence on the coefficient in (26) is a direct one. Since \( \bar{\lambda} < 1 / \beta \), we see that increasing \( \zeta \) lowers the coefficient. If \( \zeta \) is raised to infinity while maintaining the other parameters, the coefficient \( \Theta \) becomes 2.86.

The effect of \( \psi \) is on \( \bar{\lambda} \) itself. If we write the function \( f \) showing the argument \( \psi \) explicitly, total differentiation gives

\[
f_\lambda(\bar{\lambda}, \psi) \frac{d\bar{\lambda}}{d\psi} + f_\psi(\bar{\lambda}, \psi) = 0
\]

Since \( \bar{\lambda} \) is the largest real root, and since \( f \) is positive for very large \( \lambda \), we must have \( f_\lambda > 0 \) at \( \bar{\lambda} \). Thus the sign of \( d\bar{\lambda}/d\psi \) is opposite to that of \( f_\psi \) there. From (23) we have

\[
f(\bar{\lambda}, \psi) = \frac{\bar{\lambda}^2}{\lambda^2} [1 + 5a/\beta - 5\psi] + \bar{\lambda} [\gamma \psi - \gamma a/\beta + \psi \delta/\beta] - \phi \gamma / \beta = - \frac{\lambda^3}{\bar{\lambda}^3} / \psi < 0
\]

Therefore \( d\bar{\lambda}/d\psi > 0 \). Raising \( \psi \) then raises \( \bar{\lambda} \), which from (26) is seen
to lower $\theta$.

The differences between the two models can be understood further by examining what happens to the interest rate. From (7), we see that since $m$, $p$, and $y$ do not jump, neither does $r$. However, there is an impact effect on the rate of change of $r$. We find this by differentiating (7) and solving:

$$r = \frac{(\dot{p} - \dot{m} + \alpha y)}{\beta}$$

(27)

Suppose at $t = 0$, $m$ is reduced by one percentage point. Since $h$ and $y$ are both slowly-adjusting, (9) shows that there is no impact effect on $p$, i.e. it remains equal to the old level of $m$ in the first instant. However, we know from (26) that, for the chosen parameter values, $c$ drops at once by 3.81%. From (8), using $\psi = 1$ and $\gamma = .2$, we see that $y$ is suddenly lowered by 0.762 percentage points. Then, using $\alpha = 1$, $\beta = 2$ and substituting in (27), we find that the impact effect on $r$ is a rise of $(1 - 0.762)/2 = 0.119$ units.

In words, what happens is that the loss of competitiveness begins to induce a recession, and this lowers the rate of increase of the demand for money. However, this does not match the drop in the rate of increase of the supply of money. Interest rates therefore begin to be pushed up.

In the Bitter-Miller model, expectations of normal inflation $h$, and therefore from (9) the actual $p$, drop immediately by an amount equal to the reduction in $m$. Then (27) shows that the effect of the induced recession must be such as to begin to reduce the interest rate. This rate is still higher than its ultimate equilibrium value corresponding to the new lower rate of monetary growth, and indeed it is this excess of the interest rate above its long-run level that attracts capital from abroad and causes the initial appreciation of the exchange rate. However, the effect cannot be as strong as it is in the case
where inflationary expectations and the inflation rate are slow to respond, so that the pressure in the money market is stronger and the prospect of abnormally high interest rates is that much greater.

Casual observation suggests that the story as it develops here is in conformity with the actual effects of a restrictive monetary policy. It is supported by a recollection of what happens when the change in policy is in the opposite direction. It is when the money supply is expanding fast, but prices are held down by incomes policies or natural sluggishness, that we see phases of dramatic collapse of the exchange rate and an abnormally low interest rate. Finally, it is worth pointing out that these phenomena in terms of growth rates have natural parallels in terms of levels in the work of Dornbusch (1976).

Incidentally, the remaining two eigenvalues of the system become (-0.56 ± 0.28i) for the parameter set tried above. Thus, once the initial level of c is chosen correctly, the solutions in the stable manifold are cyclic. Convergence is fairly slow, with a mean lag of about three years, and the period of the cycles is 2π/0.28 = 22 years. To obtain a better idea of the paths of all the variables, numerical solutions will have to be obtained.

4. EXCHANGE RATES AND INTEREST RATES

It is arguable that a weakness of the model of the previous section lies in its formulation of aggregate demand in equation (8). In so far as a real interest rate affects investment demand and hence aggregate demand, the relevant rate should perhaps be the long real rate, i.e. (R - h) where R is defined as the reciprocal of the price of consol bearing a unit coupon. If we let this replace the short rate \( (r - \bar{p}) \) in equation (8), we will need to supplement the system
(8)-(11), or equivalently (12)-(15), by another equation determining the evolution of $R$.

This is to be found from an arbitrage condition. Equating the yield on the consol to that from holding the same amount of money in short-term bonds, we have

$$1 + d(1/R)/dt = r/R$$

or

$$\dot{R} = R (R - r) \quad (28)$$

In the long-run equilibrium, we will have

$$R = r = r^* + g \quad (29)$$

to replace (18).

Unlike the rest of the system, (28) is not linear. To examine its behaviour in a small neighbourhood of the long-run equilibrium, we can take a linear approximation to the right hand side of (28). After some simplification, this becomes

$$\dot{R} = (r^* + g) (R - r) \quad (30)$$

Integration of this would reveal $R$ to be a forward-looking weighted average of $r$. The system then consists of equations (12), (13), (15), (30) and

$$\dot{y} = \psi \{y_c - \delta (R - \bar{h}) - y\} \quad (31)$$

Taking the variables in the order $q, l, y, c$ and $R$, the matrix of the system can be written as
\[
\begin{bmatrix}
-\zeta & 0 & 0 & 0 & 0 \\
0 & -\phi & 0 & 0 & 0 \\
0 & 0 & -\psi & \psi_Y & -\delta \psi \\
1 & -1/\beta & a/\beta - \phi & 0 & 0 \\
0 & (r^* + g)/\beta & -(r^* + g)a/\beta & 0 & r^* + g
\end{bmatrix}
\]

Analytical results seem unlikely from this. However, some numerical calculations have been carried out. These assume the same basic parameter set as used earlier, augmented by the assumption that \( r^* + g = 0.1 \). Then there are two real positive eigenvalues, approximately equal to 0.062 and 0.292, with corresponding normalised eigenvectors

\[
(0.088, -0.176, 0.037, 0.119, 0.973)
\]

and

\[
(-0.306, 0.343, -0.166, -0.114, 0.865)
\]

There are now two variables capable of jumps, namely \( c \) and \( R \). The coefficients linking them to the deviations of the other variables from their long-run equilibrium values can be found using (5). The impact effect of a 1 percentage point lowering of the rate of growth of the money supply is then found to lower \( c \) below its long-run equilibrium value by 5.29 per cent. Since the long-run equilibrium \( c \) is independent of \( g \), this is also the initial loss of competitiveness. The deviation of \( R \) from its new long-run equilibrium value turns out to be 0.317. However, since the new long-run equilibrium value of \( R \) is one percentage point below the old one; this means that on impact the long rate drops by 0.683 points. This is despite an initial gradual increase in the short rate, and reflects the forward-looking behaviour of the long rate. Later, as inflationary expectations and inflation
rates come down, the short rate will fall. From (28) we can infer that it must fall below the long rate for a while in order to pull the latter down to its new long-run equilibrium level.

I would guess that the initial fall in the long-rate is not a necessary feature. For other parameter values, the deviation of R from its new long-run equilibrium level could exceed 1, thus indicating a rise in its level as the impact effect.

REFERENCES

