ESSAYS IN INTERNATIONAL ECONOMICS:
EXCHANGE RATE DISCONNECT AND THE INTERNATIONAL PRICE SYSTEM

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Abstract

The dynamics of import and export prices is crucial for the transmission of shocks across countries and the optimal monetary policy in an open economy. This dissertation consists of three essays studying these issues.

In the first chapter, which is coauthored with Oleg Itskhoki, we propose a dynamic general equilibrium model of exchange rate determination, which simultaneously accounts for all major puzzles associated with nominal and real exchange rates (Meese-Rogoff disconnect puzzle, the PPP puzzle, the terms-of-trade puzzle, the Backus-Smith puzzle, and the UIP puzzle). We propose a small but persistent shock to international asset demand as a driving force and prove it is the only type of shock that can generate the exchange rate disconnect properties. We then show that a model with this financial shock alone is quantitatively consistent with the moments describing the dynamic comovement between exchange rates and macro variables.

The currency in which international prices are set is a factor of fundamental importance in international economics: it determines the benefits of floating versus pegged exchange rates and the spillover effects of national monetary policy on other economies. However, the standard assumption in existing models — that all prices are set in a currency of either the producer or the consumer — is inconsistent with the dominant status of the dollar in global trade and the radical transformation of the price system over history. In the second chapter, I develop a general equilibrium multi-country framework with endogenous currency choice that is consistent with these stylized facts.

The last chapter shows that despite small costs for exporters, the aggregate effects
of currency choice are large. First, I identify a novel source of positive U.S. monetary spillovers on foreign output that outweighs the standard “currency war” effect. Second, I reexamine the optimal monetary policy in an open economy and show that local authorities “lean against the wind” in response to spillovers of U.S. monetary policy, which results in a partial peg to the dollar. The model prediction is consistent with the “fear of floating” and the widespread use of the dollar as an anchor currency seen in the data.
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Chapter 1

Exchange Rate Disconnect

(with Oleg Itskhoki)

1.1 Introduction

*Exchange rate disconnect* is among the most challenging and persistent international macro puzzles (see Obstfeld and Rogoff 2001). The term disconnect narrowly refers to the lack of correlation between exchange rates and other macro variables, but the broader puzzle is more pervasive and nests a number of additional empirical patterns, which stand at odds with conventional international macro models. We define the broader exchange rate disconnect to include:

1.  **Meese and Rogoff (1983a) puzzle**: *nominal* exchange rate follows a volatile random-walk-like process, which is not robustly correlated, even contemporaneously, with macroeconomic fundamentals (see also Engel and West 2005).

2.  **PPP puzzle (Rogoff 1996)**: *real* exchange rate tracks very closely the nominal ex-
change rate at most frequencies and, in particular, exhibits a similarly large persistence and volatility as the nominal exchange rate. Mean reversion, if any, takes a very long time, with half-life estimates in the range of 3-to-5 years, much in excess of conventional durations of price stickiness (see also Chari, Kehoe, and McGrattan 2002a). A related Mussa (1986) puzzle emphasizes a stark change in the properties of the real exchange rate associated with a change in the monetary regime to/from a nominal exchange rate peg (see also Mussa 1990, Monacelli 2004).

3. Terms of trade are positively correlated with the real exchange rate, yet exhibit a markedly lower volatility, in contrast with the predictions of the standard models, suggesting a particular pattern of the law of one price violations (Atkeson and Burstein 2008a). In addition, real exchange rate dynamics at most horizons is largely accounted for by the law of one price violations for tradable goods, while the contribution of the relative non-tradable prices is small (Engel 1999).

4. Backus and Smith (1993) puzzle: the international risk-sharing condition that relative consumption across countries should be strongly positively correlated with the real exchange rates (implying high relative consumption in periods of low relative prices) is sharply violated in the data, with a mildly negative correlation and a markedly lower volatility of relative consumption (see Kollmann 1995 and also Benigno and Thoenissen 2008).

5. Forward premium puzzle (Fama 1984), or the violation of the uncovered interest rate parity (UIP) condition: the UIP prediction that a relatively high interest rate should predict a nominal exchange rate devaluation is violated in the data with an opposite
sign, yet a nearly zero $R^2$ (see also Engel 1996). A related set of puzzles explores further the dynamic comovement between interest rate differential and exchange rate changes, or risk premia (see Engel 2016, Valchev 2016).

We summarize the above puzzles as a set of moments characterizing comovement between exchange rates and macro variables (see Table 1.2), and use them as the quantitative targets in our analysis.

Existing general equilibrium international macro models either feature these puzzles, or attempt to address one puzzle at a time, often at the expense of aggravating the other puzzles, resulting in a lack of a unifying framework that exhibits satisfactory exchange rate properties. This is a major challenge for the academic and policy discussion, since exchange rates are the core prices in any international model, and failing to match their basic properties jeopardizes the conclusions one can draw from the analysis. In particular, would the conclusions in the vast literatures on currency unions, international policy spillovers and international transmission of shocks survive in a model with realistic exchange rate properties? Furthermore, what are the implications of such a model for the numerous micro-level empirical studies that treat exchange rate shocks as a source of exogenous variation?

The goal of this paper is to offer a unifying theory of exchange rates that can simultaneously account for all stylized facts introduced above. We emphasize two main features of our model. First, it relies on a specific driving force (that is, a shock process) behind nominal and real exchange rates, which does not have a simultaneous strong direct effect on consumption, output, prices and interest rates. Second, it relies on a transmission
mechanism, which mutes the effect of volatile exchange rate fluctuations on local prices and quantities. Violation of either of these two properties would break the disconnect. While the literature has provided a lot of empirical evidence on the transmission mechanism, we currently lack direct empirical information on the details of the shock process, which could account for the bulk of exchange rate fluctuations. Under the circumstances, we adopt the following strategy. From the outset, we tightly discipline the transmission mechanism with the empirical estimates from the recent literature. In contrast, we initially impose no restriction on the nature of the shock process, and show theoretically that only one type of shocks can produce the exchange rate disconnect properties in general equilibrium.

In particular, as a diagnostic tool for shock selection, we consider a near-autarky behavior of the economy, and require that the shock process produces a volatile exchange rate behavior with a vanishing effect on the economy’s quantities, prices and interest rates, as the economy becomes closed to trade. Indeed, in the limit of the closed economy, any exchange rate volatility (real or nominal) should be completely inconsequential for allocations. Not surprisingly, productivity and monetary shocks, as well as the majority of other shocks, violate this intuitive requirement. We show that the one shock that satisfies this requirement, and additionally produces the empirically relevant signs of co-movement between exchange rates and macro variables (consumption and interest rates), is the shock to the international asset demand.¹ We then demonstrate how this shock can have a variety of microfoundations in the financial market, including noise trading.

¹An exogenous foreign asset demand shock has been widely used in the portfolio models of the exchange rate (e.g. Kouri 1976, Blanchard, Giavazzi, and Sa 2005), and it is also isomorphic to a UIP shock, as for example in Devereux and Engel (2002a), Kollmann (2005a) and Farhi and Werning (2012).
with limits to arbitrage (e.g., Jeanne and Rose 2002), heterogeneous beliefs (e.g., Bacchetta and van Wincoop 2006) and financial frictions (e.g., Gabaix and Maggiori 2015), as well as time-varying risk premium (e.g., Alvarez, Atkeson, and Kehoe 2009, Colacito and Croce 2013, Farhi and Gabaix 2016).

Further, we show that the model with a single financial shock is consistent both qualitatively and quantitatively with the exchange rate disconnect properties. In particular, small persistent shocks to international asset demand result in a volatile random-walk-like behavior of both nominal and real exchange rates. As the economy becomes closed to international trade, this shock still generates volatile exchange rate fluctuations, which however have a vanishingly small effect on the rest of the economy. Furthermore, the transmission mechanism in the model ensures that exchange rates exhibit empirically relevant comovement properties with macro variables, even when the economy is open to international trade in goods and assets. In particular, the transmission mechanism features four realistic ingredients:

1. significant home bias in consumption, consistent with the empirical trade shares in GDP, which limits the effects of expenditure switching on aggregate consumption, employment and output;

2. pricing to market and law of one price violations due to strategic complementarities in price setting, which limit the response of prices (terms of trade) to exchange rate movements;

3. weak substitutability between home and foreign goods, which limit the extent of expenditure switching conditional on the terms of trade movements; and
4. *monetary policy* that stabilizes domestic inflation (as opposed to a nominal exchange rate peg).

Interestingly, nominal rigidities are not an essential part of the transmission mechanism for generating a disconnect behavior, and therefore we omit them in the baseline model. Later, we generalize our analysis to an environment with nominal stickiness and conventional Taylor rules, and show the robustness of the quantitative properties of the model.

Furthermore, the results of a single-shock baseline model are robust to the introduction of additional shocks, including productivity and monetary shocks. We calibrate a multi-shock version of the model to match the weak correlations between exchange rates and macro variables. Using this calibrated model we conduct a variance decomposition of the equilibrium exchange rate volatility into the contribution of various types of shocks, and find that financial shock still accounts for the bulk of its variation, while both monetary and productivity shocks play limited roles. Since the structure of our model is rather standard, the transmission mechanism for monetary and productivity shocks is not different from a conventional international macro model. What sets our model apart, however, is the emphasis that monetary and productivity shocks cannot be the key drivers of the exchange rate, if the model is to feature the disconnect properties. At the same time, conventional shocks are still central in shaping the dynamics of other macroeconomic variables, resulting in no trade-off for our model in fitting the standard international business cycle moments (as in e.g. Backus, Kehoe, and Kydland 1992, 1994).

The tractability of the baseline model allows us to solve it in closed-form and emphasize four novel mechanisms. The first mechanism is the exchange rate determination,
which emphasizes the interplay between equilibrium forces in the financial and goods markets. A small persistent increase in demand for foreign assets results in a sharp depreciation of the home currency and a slow but persistent appreciation thereafter in order to ensure equilibrium in the asset markets. Intertemporal budget constraint requires that these future appreciations are balanced out by an unexpected depreciation on impact. The more persistent is the shock, the larger is the initial depreciation, and thus the closer is the behavior of the nominal exchange rate to a random walk. Indeed, in our calibration, the equilibrium exchange rate is indistinguishable from a random walk in finite samples.

The second mechanism concerns the real exchange rate, and in particular the PPP and related puzzles, which are often viewed as the prime evidence in support of long-lasting real effects of nominal rigidities (as surveyed in Rogoff 1996). The alternative interpretation in the literature is that, given the moderate empirical durations of nominal prices, sticky price models are incapable of generating persistent PPP deviations observed in the data (see Chari, Kehoe, and McGrattan 2002a). Both of these views adopt the baseline assumption that monetary shocks are the main drivers of the nominal exchange rate, and that nominal rigidity is the key part of the transmission mechanism into the real exchange rate. We suggest an entirely different perspective, which deemphasizes nominal rigidities, and instead shifts focus to the nature of the shock process. We argue that the behavior of the real exchange rates — both in the time series (PPP puzzle) and in the cross-section (see e.g. Kehoe and Midrigan 2008) — is not evidence in favor or against sticky prices, but is instead evidence against monetary shocks as the key source of exchange rate fluctuations. In contrast, we show that financial shocks drive both nominal and real exchange rates in concert, resulting in volatile and persistent behavior for both variables, thus reproducing
the PPP puzzle. The only two relevant ingredients of the transmission mechanism for this result are the monetary policy rule, which stabilizes domestic inflation, and the home bias in consumption, which limits the response of consumer prices to exchange rate.

The third mechanism addresses the Backus-Smith puzzle, namely the comovement between consumption and the real exchange rate. Our approach crucially shifts focus from risk sharing (in the financial market) to expenditure switching (in the goods market) as the key force shaping this comovement. We show that expenditure switching robustly implies a negative correlation between relative consumption and the real exchange rate, as is the case in the data. Intuitively, an exchange rate depreciation increases global demand for domestic goods, which in light of the home bias requires an increase in domestic production and a reduction in domestic consumption. We show that this force is present in all models with expenditure switching and goods market clearing, yet it is usually dominated by the direct effect of shocks on consumption. With financial shock as the key source of exchange rate volatility, there is no direct effect, and expenditure switching is the only force affecting consumption, resulting in the empirically relevant direction of comovement.\footnote{The analytical tractability of our model allows us to establish the relationship between our results and those in the earlier Backus-Smith puzzle literature, in particular in Corsetti, Dedola, and Leduc (2008).} Our transmission mechanism with substantial home bias and low pass-through into prices and quantities ensures that the movements in consumption are very mild, much smaller than those in exchange rates, as is the case in the data.

Lastly, we provide an explicit microfoundation of the financial shock in an extension of the model, in which risk-averse arbitrageurs intermediate international financial transactions and require a risk premium proportional to the size of their currency exposure.
Without compromising the model’s ability to match the main exchange rate moments, this extension results in an endogenous feedback from the net foreign asset position of the country into the risk premium, and allows the model to reproduce the non-monotonic dynamic comovement between UIP deviations and interest rates, emphasized recently by Engel (2016) and Valchev (2016). The model further implies that a policy commitment to an exchange rate peg has a coordination effect on the arbitrageurs, encouraging them to take larger positions and endogenously suppressing the volatility of the UIP deviations (as in Jeanne and Rose 2002). We show that this mechanism is important to account for an additional set of stylized facts associated with a switch in the monetary regime to/from an exchange rate peg, to which we collectively refer as the Mussa puzzle. As with the PPP puzzle, our explanation here emphasizes the nature of the shock driving the exchange rate and the monetary policy rule, rather than nominal rigidities (cf. Monacelli 2004, Kollmann 2005a).

The rest of the paper is organized as follows. In Section 2.2, we describe the modeling framework and prove that the international asset demand shocks is the only shock consistent with the exchange rate disconnect properties. We also discuss in this section various microfoundations for the origin of this financial shock. Section 1.3 then explores the qualitative and quantitative properties of the model with the financial shock alone, addressing in turn all of the exchange rate disconnect puzzles outlined in the beginning of this introduction. Along the way, we also provide a discussion of the relationship of our results to the existing literature. Section 2.4 then describes a number of extensions, including a full-fledged model with nominal rigidities and conventional Taylor rules, as well as a model with an explicit financial sector with noise traders and limits to arbitrage. In this section,
we also allow for multiple sources of shocks and provide a variance decomposition of the exchange rate volatility into the contribution of these various shocks. Lastly, this section addresses the Mussa puzzle and the Engel risk premium puzzle. Section 3.4 discusses the implication of our results and concludes, while the appendix provides detailed derivations and proofs, as well as a number of additional extensions and results.

1.2 Modeling Framework and Shocks

We start with a flexible modeling framework that can nest most standard international macro models, which allows us in what follows to consider various special cases and extensions. There are two countries, home (Europe) and foreign (US, denoted with a *). Each country has its nominal unit of account, in which the local prices are quoted. In particular, the home wage rate is $W_t$ euros and the foreign wage rate is $W_t^*$ dollars. The nominal exchange rate $\mathcal{E}_t$ is the price of dollars in terms of euros, hence an increase in $\mathcal{E}_t$ signifies a nominal devaluation of the euro (the home currency). We allow for a variety of shocks hitting the economy, and proxying in some cases for unmodelled market imperfections. We then explore which of these disturbances can account for the exchange rate disconnect, as we formally define it below in Section 1.2.2.
1.2.1 Model setup

**Households**  A representative home household maximizes the discounted expected utility over consumption and labor:

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t e^{\chi_t} \left( \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{e^{\kappa_t}}{1+1/\nu} L_t^{1+1/\nu} \right),
\]

(1.1)

where \((\chi_t, \kappa_t)\) are the utility shocks, \(\sigma\) is the relative risk aversion parameter, \(\nu\) is the Frisch elasticity of labor supply, and our results are robust to alternative utility specifications, including the GHH utility without income effects on labor supply (see Appendix A.2.7). The flow budget constraint is given by:

\[
P_tC_t + \frac{B_{t+1}}{R_t} + \frac{B^*_t}{e^{\psi_t} R^*_t} \leq B_t + B^*_t \mathcal{E}_t + W_t L_t + \Pi_t - T_t + \Omega_t,
\]

(1.2)

where \(P_t\) is the consumer price index, \((B_t, B^*_t)\) are the quantities of the home and foreign bonds paying out next period one unit of the currency of the issuing country, and \((R_t, R^*_t)\) are their discounts (i.e., \(1/R_t\) and \(1/R^*_t\) are their prices); \(\Pi_t\) are the dividends and \(T_t\) are lump-sum taxes. Lastly, \(\psi_t\) is a wedge between the effective return on foreign bonds for the home households and the foreign interest rate, driven by shocks in the international asset market and with the resulting profits of the financial sector \(\Omega_t = (e^{-\psi_t} - 1) \frac{R^*_t}{R^*_t} \mathcal{E}_t\) reimbursed lump-sum to the households.\(^3\)

The households are active in three markets. First, they supply labor according to the

\(^3\)We adopt the assumption that a risk-free bond is the only internationally-traded asset because we rely on a log-linearization for the analytical solution of the model.
standard static optimality condition:

\[ e^{\kappa_t} C_t^\sigma L_t^{1/\nu} = \frac{W_t}{P_t}, \tag{1.3} \]

where the preference shock \( \kappa_t \) can be alternatively interpreted as the labor wedge, playing an important role in the closed-economy business cycle literature and capturing the departures from the neoclassical labor market dynamics due to search frictions or sticky wages (see e.g. Shimer 2009). In addition, we denote \( W_t \equiv e^{w_t} \) and interpret \( w_t \) as the shock to the nominal value of the unit of account, which captures monetary shocks in our framework.\(^4\)

Second, the households choose their bond positions according to the dynamic optimality conditions:

\[ 1 = R_t E_t \Theta_{t+1} \quad \text{and} \quad 1 = e^{\psi_t} R_t^* E_t \left\{ \frac{\Theta_{t+1}}{\bar{E}_t} \right\}, \tag{1.4} \]

where \( \Theta_{t+1} \equiv \beta \Delta \chi_{t+1} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \)

is the stochastic discount factor. The time preference shock \( \chi_{t+1} \), as in Stockman and Tesar (1995), affects the consumption-savings decision and acts as an intertemporal demand shifter. The \( \psi_t \) shock acts instead as a demand shifter for the foreign-currency bond, and in Section 1.2.3 we discuss a number of microfounded financial models which result in a similar reduced form as (1.4).

\(^4\)In Section 1.4.1 we provide an explicit model with nominal wage stickiness, local-currency price stickiness and a conventional Taylor rule, which offers an example of one typical source of \( (w_t, \kappa_t, \mu_t, \eta_t) \) shocks (with \( \mu_t \) and \( \eta_t \) defined below).
Lastly, the households allocate their within-period expenditure between home and foreign goods:

\[ P_tC_t = P_{Ht}C_{Ht} + P_{Ft}C_{Ft}, \]

and we assume the good demand is homothetic and symmetric, and given by:

\[
C_{Ht} = (1 - \gamma)e^{-\gamma \xi_t} h \left( \frac{P_{Ht}}{P_t} \right) C_t \quad \text{and} \quad C_{Ft} = \gamma e^{(1 - \gamma) \xi_t} h \left( \frac{P_{Ft}}{P_t} \right) C_t, \tag{1.5}
\]

where \( \xi_t \) is the relative demand shock for the foreign good (as in Pavlova and Rigobon 2007) and \( \gamma \) captures the home bias, which can be due to a combination of home bias in preferences, trade costs and non-tradable goods (see Obstfeld and Rogoff 2001). Note that the demand for the foreign good collapses to zero as \( \gamma \to 0 \). The function \( h(\cdot) \) controls the curvature of the demand schedule, and satisfies \( h'(\cdot) < 0 \) and \( h(1) = 1 \).

We denote its point elasticity by \( \theta \equiv -\frac{\partial \log h(x)}{\partial \log x} \bigg|_{x=1} \). The demand formulation in (1.5) emerges from a homothetic and separable Kimball (1995) demand aggregator, as we show in Appendix A.1.2, where we also derive an explicit expression for the price index \( P_t \).\(^5\) In our analysis, we focus on the behavior of the economy around a symmetric steady state and make use of the following three properties of this demand system:

**Lemma 1.1 (Properties of Demand)** In a symmetric steady state with \( \xi = 0 \) and \( P = P_H = P_F \):

(i) The expenditure share on foreign goods (the foreign share for brief), defined as \( \frac{P_{Ft}C_{Ft}}{P_tC_t} \), equals the home bias parameter \( \gamma \).

\(^5\)Note that the conventional CES demand is nested as a special case with \( h(x) = x^{-\theta} \), yet we adopt a more general demand formulation, which allows to accommodate variable markups.
(ii) The log-linear approximation of the consumer price index around the steady state is
given by (where small letters denote log deviations from the steady state):

\[ p_t = (1 - \gamma) p_{Ht} + \gamma p_{Ft}. \quad (1.6) \]

(iii) The log-linear approximation to demand (1.5) around the steady state is given by:

\[ c_{Ht} = -\gamma \xi_t - \theta (p_{Ht} - p_t) + c_t \quad \text{and} \quad c_{Ft} = (1 - \gamma) \xi_t - \theta (p_{Ft} - p_t) + c_t. \quad (1.7) \]

Therefore, \( c_{Ft} - c_{Ht} = \xi_t - \theta (p_{Ft} - p_{Ht}) \), and the elasticity of substitution between
home and foreign goods, defined as \( \frac{\partial \log(C_{Ft}/C_{Ht})}{\partial \log(P_{Ht}/P_{Ft})} \), equals the point elasticity of the
demand schedule \( \theta \).

We show below that the values of \( \gamma \) (trade openness) and \( \theta \) (elasticity of substitution
between home and foreign goods) play a central role in the quantitative properties of the
transmission mechanism.

**Production and prices** Output is produced by a given pool of identical firms according
to a Cobb-Douglas technology in labor \( L_t \) and intermediate inputs \( X_t \):

\[ Y_t = e^{a_t} L_t^{1-\phi} X_t^\phi, \quad (1.8) \]

where \( a_t \) is the productivity shock and \( \phi \) is the elasticity of output with respect to interme-
diates, which determines the equilibrium expenditure share on intermediate goods. The
presence of intermediates is not essential for the qualitative results, however, is needed to
properly capture the degree of trade openness in our calibration. For analytical tractabil-
ity, we focus on a constant-returns-to-scale production without capital, and show the robustness of our results to an extension with capital and adjustment costs in Appendix A.2.7.

Intermediates are the same bundle of home and foreign varieties as the final consumption bundle, and hence their price index is also given by $P_t$. Therefore, the marginal cost of production is:

$$MC_t = e^{-\alpha t} \left( \frac{W_t}{1 - \phi} \right)^{1 - \phi} \left( \frac{P_t}{\phi} \right)^{\phi},$$

and the firms optimally allocate expenditure between labor and intermediates according to the following input demand conditions:

$$W_t L_t = (1 - \phi)MC_t Y_t \quad \text{and} \quad P_t X_t = \phi MC_t Y_t. \quad (1.10)$$

The expenditure on intermediates $X_t$ is further split between the domestic and foreign varieties, $X_{Ht}$ and $X_{Ft}$, in parallel with the household consumption expenditure in (1.5). The total production of the domestic firms is divided between the home and foreign markets, $Y_t = Y_{Ht} + Y_{Ht}^*$, resulting in profits that are distributed to the domestic households:$^6$

$$\Pi_t = (P_{Ht} - MC_t) Y_{Ht} + (P_{Ht}^* \varepsilon_t - MC_t) Y_{Ht}^*. \quad (1.11)$$

$^6$We assume no entry or exit of firms, as our model is a medium-run one (for the horizons of up to 5 years), where empirically extensive margins play negligible roles (see e.g. Bernard, Jensen, Redding, and Schott 2009).
We postulate the following price setting:

\[ P_{Ht} = e^{\mu t} MC_t^{1-\alpha} P^{\alpha}_t, \quad (1.12) \]
\[ P^*_H = e^{\mu t + \eta_t} \left( \frac{MC_t}{E_t} \right)^{1-\alpha} P^{*\alpha}_t, \quad (1.13) \]

where \( \alpha \in [0, 1) \) is the strategic complementarity elasticity, \( \mu_t \) is the markup shock, and \( \eta_t \) is the law of one price (LOP) shock. Given these prices, the firms satisfy the resulting demand in both markets. Equations \((1.12)-(1.13)\) are \textit{ad hoc} yet general pricing equations, as the markup terms (together with a flexible choice of \( \alpha \)) allow them to be consistent with a broad range of price setting models, including both monopolistic and oligopolistic competition models under both CES and non-CES demand. Furthermore, if the time path of \((\mu_t, \eta_t)\) is not restricted, these equations are also consistent with dynamic price setting models, and in particular the sticky price models (with either producer, local or dollar currency pricing).\(^7\)

Strategic complementarities in price setting \((\alpha > 0)\) reflect the tendency of the firms to set prices closer to their local competitors, a pattern which is both pronounced in the data and emerges in a variety of models (see Amiti, Itskhoki, and Konings 2016a), and we emphasize in our analysis below its role for the international transmission of shocks.

Appendix A.1.2 discusses a model of Kimball (1995) demand that is simultaneously consistent with both our choices of elasticity of substitution \( \theta \) and strategic complementarity

\(^7\)Note that \( \eta_t \) can stand in for a trade cost shock, which plays a central role in the recent quantitative analyses of Eaton, Kortum, and Neiman (2015) and Reyes-Heroles (2016). A combination of \( \eta_t \) and \( \xi_t \) can also stand in for a world commodity price shock, acting as a wealth transfer between countries. These shocks are an important source of volatility for the commodity-exporting countries such as Canada, Australia, South America, Brazil and Chile (see e.g. Chen and Rogoff 2003).
elasticity \( \alpha \). Lastly, we note that the violations of the law of one price:

\[
Q_{H} \equiv \frac{P_{H}^* \mathcal{E}_{t}^*}{P_{H}^*} = e^{\eta_{t}} Q_{t}^* , \quad \text{where} \quad Q_{t} \equiv \frac{P_{t}^* \mathcal{E}_{t}^*}{P_{t}} ,
\]

(1.14)

arise either due to the LOP shock \( \eta_{t} \) (capturing, for example, local currency pricing) or due to \( \alpha > 0 \) (capturing pricing-to-market). The \textit{real exchange rate} \( Q_{t} \) reflects the differences in the price levels across the two markets.

**Government** uses lump-sum taxes to finance an exogenous stochastic path of government expenditure \( G_{t} \equiv e^{g_{t}} \), where \( g_{t} \) is the government spending shock. For simplicity, we assume that government expenditure is allocated between the home and foreign goods in the same way as the final consumption in (1.5). The government then taxes households \( T_{t} = P_{t} e^{g_{t}} \) to run a balanced budget, which in view of Ricardian equivalence is without loss of generality.

**Foreign** households are symmetric, except that the home (euro) bonds are not available to them, and their budget constraint is given by:

\[
P_{t}^{*} C_{t}^{*} + \frac{B_{t+1}^{*F}}{R_{t}^{*}} \leq B_{t}^{*F} + W_{t}^{*} L_{t}^{*} + \Pi_{t}^{*} + T_{t}^{*} ,
\]
where $B_t^{*F}$ are the holdings of the foreign (dollar) bond by foreign households.\footnote{We consider this asymmetric formulation between home and foreign for simplicity, and provide a symmetric version of the model with a financial sector in Section 1.4.2.} The optimal savings decision of the foreign households is characterized by the Euler equation:

$$1 = R_t^* \mathbb{E}_t \Theta_{t+1}^*, \quad \text{where} \quad \Theta_{t+1}^* \equiv \beta e^{\Delta \chi_{t+1}} \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*}. \quad (1.15)$$

The foreign households supply labor and demand home and foreign goods according to the optimality condition parallel to (1.3) and (1.5) respectively. In particular, the goods demand by the foreign households is given by:

$$C_{Ht}^* = \gamma e^{(1-\gamma) \xi_t^*} h \left( \frac{P_{Ht}^*}{P_t^*} \right) C_t^* \quad \text{and} \quad C_{Ft}^* = (1-\gamma) e^{-\gamma \xi_t^*} h \left( \frac{P_{Ft}^*}{P_t^*} \right) C_t^*, \quad (1.16)$$

where $\xi_t^*$ is the foreign demand shock for home goods. Lastly, the foreign firms are also symmetric, demand foreign labor and a composite intermediate good, and charge prices according to the counterparts of (1.12)–(1.13) with their own markup and LOP shocks $\mu_t^*$ and $\eta_t^*$, as we detail in Appendix A.1.3.

**Equilibrium conditions** ensure equilibrium in the asset, product and labor markets, as well as the intertemporal budget constraints of the countries. The labor market clears when $L_t$ is consistent simultaneously with labor supply in (1.3) and labor demand in (1.10), and symmetrically for $L_t^*$ in foreign. The goods market clearing requires $Y_t = Y_{Ht} + Y_{Ht}^*$,
Table 1.1: Model parameters and shocks

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_t$ (nominal wage rate)</td>
<td>$\beta = 0.99$ (discount factor)</td>
</tr>
<tr>
<td>$\alpha_t$ (productivity shock)</td>
<td>$\sigma = 2$ (relative risk aversion)</td>
</tr>
<tr>
<td>$g_t$ (government spending shock)</td>
<td>$\nu = 1$ (Frisch elasticity of labor supply)</td>
</tr>
<tr>
<td>$\chi_t$ (intertemporal preference shock)</td>
<td>$\gamma = 0.07$ (foreign share (home bias) parameter)</td>
</tr>
<tr>
<td>$\kappa_t$ (labor wedge)</td>
<td>$\theta = 1.5$ (elasticity of substitution)</td>
</tr>
<tr>
<td>$\mu_t$ (markup shock)</td>
<td>$\alpha = 0.4$ (strategic complementarity elasticity)</td>
</tr>
<tr>
<td>$\eta_t$ (law-of-one-price shock)</td>
<td>$\phi = 0.5$ (intermediate share)</td>
</tr>
<tr>
<td>$\xi_t$ (international good demand shock)</td>
<td>$\rho = 0.97$ (persistence of the shock)</td>
</tr>
<tr>
<td>$\psi_t$ (financial (international asset demand) shock)</td>
<td></td>
</tr>
</tbody>
</table>

Note: The left panel summarizes the shocks to the home economy, with foreign facing a symmetric set of shocks, apart from $\psi_t^*$, which with our structure is equivalent to $\chi_t^*$. The right panel reports the baseline parameter values used in Sections 1.3–2.4.

where

$$Y_{Ht} = C_{Ht} + X_{Ht} + G_{Ht} = (1 - \gamma)e^{-\gamma \xi_t}h \left( \frac{P_{Ht}}{P_t} \right) [C_t + X_t + G_t],$$  \hspace{1cm} (1.17)

$$Y^*_{Ht} = C^*_{Ht} + X^*_{Ht} + G^*_{Ht} = \gamma e^{(1-\gamma) \xi_t}h \left( \frac{P^*_{Ht}}{P^*_t} \right) [C^*_t + X^*_t + G^*_t],$$  \hspace{1cm} (1.18)

and symmetric conditions hold for $Y_{Ft} + Y^*_{Ft} = Y^*_t$. The bonds market clearing requires $B_t = 0$ for the home-currency bond, as it is in zero net supply and not traded internationally, and $B^*_t + B^{*F}_t = 0$ for the foreign-currency bond, which is in zero net supply internationally.

Lastly, we combine the household budget constraint (1.2) with profits (1.11) and taxes to obtain the country budget constraint:

$$\frac{B^*_{t+1} \mathcal{E}_t}{R^*_t} - B^*_t \mathcal{E}_t = NX_t, \quad \text{where} \quad NX_t = \mathcal{E}_t P^*_{Ht} Y^*_{Ht} - P_{Ft} Y_{Ft}. \hspace{1cm} (1.19)$$

$NX_t$ is net exports of the home country (in home currency). Note that the relative price
at which the home country exchanges its exports for imports is the terms of trade:

\[ S_t \equiv \frac{P_{Et}}{P_{Ht}^*E_t}. \] (1.20)

This completes the description of the model environment and the equilibrium system, which we also summarize in Appendix A.1.3.

**Shocks** are summarized in Table 1.1, along with the parameters of the model and their baseline values, which we use in Sections 1.3 and 2.4 for quantitative evaluation of the model. In general, we allow shocks to follow arbitrary joint stochastic processes with unrestricted patterns of cross-correlations. In this sense, our shocks are not primitive innovations, but rather disturbances to the equilibrium conditions of the model, akin to Chari, Kehoe, and McGrattan (2007) *wedges*. We use them differently, however. Instead of accounting for the sources of variation in the macro variables, we prove two theoretical results characterizing which subsets of disturbances can and cannot result in an equilibrium *disconnect* behavior of the exchange rates, as defined below.

### 1.2.2 Disconnect in the limit

This section uses the general modeling framework to prove two propositions, which narrow down the set of shocks that can be consistent with the empirical exchange rate disconnect properties. In particular, we study the behavior of the equilibrium system around

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9For example, Eaton, Kortum, and Neiman (2015) is a recent study, which uses wedge accounting in the international context. Our approach differs in that we do not attempt to fully match macroeconomic time series, but instead focus on a specific theoretical mechanism, which accounts for a set of exchange rate disconnect moments within a parsimonious model. This is also what sets our paper apart from the international DSGE literature following Eichenbaum and Evans (1995).
the autarky limit, which we use as a diagnostic device. The autarky limit (as foreign share \( \gamma \to 0 \)) is interesting for two reasons:

1. Autarky (\( \gamma = 0 \)) offers a model of *complete exchange rate disconnect*. When countries are in autarky, the nominal exchange rate is of no consequence, and can take any values as an outcome of arbitrary sunspot equilibria. Therefore, it can be arbitrary volatile, yet have no relationship with any macro variables in the two economies (the Meese-Rogoff puzzle). Since price levels do not respond to this volatility, the real exchange rate comoves perfectly with these nominal exchange rate shocks, and as a result can exhibit arbitrary persistence (the PPP puzzle). This is possible because in autarky the real exchange rate does not affect allocations.

2. Furthermore, away from autarky, the response of macro variables to exchange rate tends to increase together with the degree of openness \( \gamma \), resulting in more volatile and less disconnected macroeconomic behavior (see Appendix Figure A2 for illustration). Therefore, if the economy does not exhibit exchange rate disconnect properties near autarky (for \( \gamma \approx 0 \)), it is unlikely to feature them away from autarky (for \( \gamma \gg 0 \)).\(^\text{10}\) In addition, \( \gamma \approx 0 \) is not an unreasonable point of approximation from an empirical perspective, as we discuss in Section 1.3.

We now extend the autarky logic to study circumstances under which a near-closed economy features a *near-complete exchange rate disconnect*. While such continuity requirement may appear natural as the equilibrium dynamics is continuous in \( \gamma \), it nonetheless offers a sharp selection criterion for exogenous shocks. This is because a limiting economy with

\(^\text{10}\)The empirical literature finds that more open economies have less volatile exchange rates, even after controlling for country size and other characteristics (e.g., Hau 2002), which is a pattern reproduced by our model (see Section 1.3.1).
\( \gamma > 0 \) acts as a refinement on equilibria when \( \gamma = 0 \), as it rules out the sunspot equilibria with volatile exchange rate dynamics.

We start by formalizing the notion of exchange rate disconnect in the autarky limit:

**Definition 1.1 (Disconnect in the limit)** Denote with \( Z_t \equiv (W_t, P_t, C_t, L_t, Y_t, R_t) \) a vector of all domestic macro variables (wage rate, price level, consumption, employment, output, interest rate) and with \( \varepsilon_t \equiv V'\Omega_t + V^*\Omega_t^* \) an arbitrary combination of shocks \( \Omega_t = \{w_t, \chi_t, \kappa_t, a_t, g_t, \mu_t, \eta_t, \xi_t, \psi_t\} \). We say that an open economy (with \( \gamma > 0 \)) exhibits exchange rate disconnect in the autarky limit, if the impulse responses have the following properties:

\[
\lim_{\gamma \to 0} \frac{dZ_{t+j}}{d\varepsilon_t} = 0 \quad \text{for all} \quad j \geq 0 \quad \text{and} \quad \lim_{\gamma \to 0} \frac{d\varepsilon_t}{d\varepsilon_t} \neq 0. \tag{1.21}
\]

A corollary of condition (1.21) is that \( \lim_{\gamma \to 0} [d \log E_{t+j} - d \log Q_{t+j}] / d\varepsilon_t = 0 \) for all \( j \geq 0 \).

In words, a model, defined by its structure and the set of shocks, exhibits exchange rate disconnect in the autarky limit if the shocks have a vanishingly small effect on the macro variables, yet result in a volatile equilibrium exchange rate, as captured by the two conditions in (1.21). This property captures the exchange rate disconnect in its narrow Meese-Rogoff sense. However, as the corollary points out, this property also implies the PPP-puzzle behavior for the real exchange rate, which in the limit comoves one-for-one with the nominal exchange rate.

Definition 1.1 immediately allows us to exclude a large number of candidate shocks:

**Proposition 1.1** The model of Section 1.2.1 cannot exhibit exchange rate disconnect in the autarky limit (1.21), if the combined shock \( \varepsilon_t \) in Definition 1.1 has a weight of zero on the subset of shocks \( \{\eta_t, \eta_t^*, \xi_t, \xi_t^*, \psi_t\} \).
In other words, this proposition states that the shocks in $\Omega_t \equiv \{w_t, \chi_t, \kappa_t, a_t, g_t, \mu_t\}$ together with their foreign counterparts, in any combinations and with arbitrary cross-correlations, cannot reproduce an exchange rate disconnect property even as the economy approaches autarky. We provide a formal proof in Appendix A.1.4, yet the intuition behind this result is straightforward: Any of the shocks in $\Omega_t$ will have a direct effect on real allocations, prices, and/or interest rates, and thus cannot result in a volatile exchange rate without having a direct effect on the macro variables of the same order of magnitude.\(^{11}\)

Therefore, as an economy subject to these shocks approaches autarky, the disconnect property (1.21) is necessarily violated. The proof of this proposition does not rely on the international risk sharing condition, and therefore this result is robust to the assumption about (in)completeness of the international asset markets.

Proposition 1.1 can be viewed as pessimistic news for both the International RBC and the New Open Economy Macro (NOEM) models of the exchange rate. It does not imply, however, that productivity cannot be an important source of exogenous shocks. Instead, it suggests that productivity shocks $a_t$ are unlikely to be the dominant drivers of exchange rate movements if the model is to exhibit exchange rate disconnect.\(^{12}\) The same applies to monetary shocks in a model with nominal rigidities, which we study in detail in Sec-

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\(^{11}\)Intuitively, the unit of account $w_t$ shocks result in wage inflation, the markup $\mu_t$ shocks result in price inflation, the labor wedge $\kappa_t$ shocks result in changes in either employment or consumption, the productivity $a_t$ shocks result in changes in either employment or output, the government spending $g_t$ shocks result in changes in either consumption or output, and the intertemporal preference shocks $\chi_t$ result in changes in the interest rate (see illustration in Figure A2). The formal proof in Appendix A.1.4 establishes further that no combination of these shocks can be consistent with the disconnect property (1.21).

\(^{12}\)Productivity shocks can have two additional indirect effects, either acting as news shocks about future productivity or by affecting the risk premium (e.g., rare-disaster or long-run-risk shocks). As the direct effect of the productivity shock becomes vanishingly small relative to its indirect effects, this shock becomes similar to an Euler-equation shock in that it does not affect the static equilibrium conditions (see Appendix A.1.7). Proposition 1.1 nonetheless applies as long as the direct effect of the productivity shock is non-trivial.
We view Proposition 1.1 as a diagnostic tool suggesting that the shocks in $\Omega_t^\varnothing$ are unlikely to be successful at reproducing the empirical exchange rate behavior even away from the autarky limit. Therefore, we should first explore the other three types of shocks — namely, the LOP deviation (or trade cost) shock $\eta_t$, the international good demand shock $\xi_t$, and/or the financial shock $\psi_t$ — as the likely key drivers of the exchange rate dynamics. The distinctive feature of these shocks is that they affect the equilibrium system exclusively through the *international equilibrium conditions*: $\psi_t$ affects international risk sharing (see (1.39) below), while $\eta_t$ and $\xi_t$ affect the country budget constraint (1.19) through their impact on export prices (1.13) and export demand (1.18) respectively.\(^\text{13}\) The impact of shocks to these equilibrium conditions on the macro variables is vanishingly small as the economy becomes closed to international trade in goods and assets, yet such shocks can have substantial effect on the equilibrium exchange rates and terms of trade even when $\gamma$ is close to zero.

Proposition 1.1 does not allow us to discriminate between the remaining three types of shocks, as they all satisfy the autarky-limit disconnect condition (1.21). Yet, these shocks differ in the implied comovement between exchange rates and macro variables, which we now use as a further selection criterion. In particular, we explore the comovement between the exchange rates and respectively terms of trade, relative consumption, and the interest rate differential, near the autarky limit (as $\gamma \to 0$). Since these shocks are already consistent with the Meese-Rogoff and the PPP puzzles by virtue of Proposition 1.1, the

\(^{13}\)The $\xi_t$ and $\eta_t$ shocks are additionally featured in the goods market clearing (1.17)–(1.18) and in the price level (1.6), but in both cases their effect on these conditions is proportional to trade openness $\gamma$, and thus vanishes in the autarky limit.
additional moments correspond to the three remaining exchange rate puzzles. We prove the following result (see Appendix A.1.4):

**Proposition 1.2** Near the autarky limit (for \( \gamma \to 0 \)), the international asset demand shock \( \psi_t \) is the only shock in \( \{ \eta_t, \eta_t^*, \xi_t, \xi_t^*, \psi_t \} \) that simultaneously and robustly produces:

(i) a positive correlation between the terms of trade and the real exchange rate;

(ii) a negative correlation between relative consumption growth and real exchange rate depreciation;

(iii) deviations from the UIP and a negative Fama coefficient.

The main conclusion is that both the LOP deviation (trade cost) shock \( \eta_t \) and the international good demand shock \( \xi_t \) produce the counterfactual comovement between the exchange rates and respectively relative consumption (the Backus-Smith puzzle) and interest rate differential (the Forward Premium puzzle). The financial shock \( \psi_t \) is instead consistent with both of these empirical patterns, as we explain in detail in Section 1.3.

To summarize, Propositions 1.1 and 1.2 explain why most shocks have a hard time at reproducing the empirical exchange rate properties, and hence why these properties are labeled as puzzles in the literature. These propositions favor the financial shock \( \psi_t \) as the likely shock to generate exchange rate disconnect in an equilibrium model. While these propositions are concerned with the autarky limit, the continuity of the model in trade openness \( \gamma \) suggests that the near-disconnect properties should hold for \( \gamma > 0 \), but small. In Sections 1.3, we explore the properties of the model with the \( \psi_t \) shock alone, away from the autarky limit, and in Section 2.4 we bring back the additional shocks.
1.2.3 Models of financial shock $\psi_t$

Since Propositions 1.1 and 1.2 favor the international asset demand shock $\psi_t$ as the likely source of the exchange rate disconnect, we discuss here a number of microfoundations for the origins of this shock. In view of the parity condition between home and foreign bonds (arising from (1.4) upon log-linearization), the $\psi_t$ is commonly referred to as the UIP shock:

$$i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} = \psi_t,$$

(1.22)

where $i_t - i^*_t = \log R_t - \log R^*_t$ and $e_t = \log \mathbb{E}_t$. It follows that the (uncovered) interest rate parity $i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1}$ deviates from zero by the magnitude of the financial shock $\psi_t$, which may have a number of origins explored in the macro-finance literature (see also Cochrane 2016):

1. Exogenous preference for international assets, where $\psi_t$ is an ad hoc shock to the utility from holding the foreign bond, as a result of which domestic households are willing to hold the foreign asset with a negative excess return (UIP deviation), as in Dekle, Jeong, and Kiyotaki (2014). This approach is closely related to the non-optimizing portfolio balance models of the 1970-80s (e.g. Kouri 1976, 1983, Branson and Henderson 1985), revived recently by Blanchard, Giavazzi, and Sa (2005) and Gourinchas (2008).

2. Noise traders and limits to arbitrage in currency markets, as in Jeanne and Rose (2002), where all international trade in assets needs to go through risk-averse intermediaries, who are willing to be exposed to the currency risk only if they are offered a sufficient compensation in the form of a positive expected return. A noise trader
shock $\psi_t$, thus, needs to be accommodated by a UIP deviation. In Section 1.4.2, we explore a general equilibrium version of this model and its implications for exchange rate disconnect and additional exchange rate puzzles.

3. A related class of models relies on financial frictions to generate upward sloping supply in the currency market, where $\psi_t$ represents shocks to the risk-bearing capacity of the financial sector, for example a shock to the net worth of the financial intermediaries limiting the size of the positions that they can absorb (see e.g. Hau and Rey 2006, Brunnermeier, Nagel, and Pedersen 2009, Gabaix and Maggiori 2015, Adrian, Etula, and Shin 2015).

4. Incomplete information, heterogeneous beliefs and expectational errors in the currency market, as in Evans and Lyons (2002), Gourinchas and Tornell (2004) and Bacchetta and van Wincoop (2006), where $\psi_t$ represents the deviations from the full-information rational expectations.

5. Time-varying risk premia models, where $\psi_t$ corresponds to shocks to the second moments of the stochastic discount factor, such as rare disasters (Farhi and Gabaix 2016), long-run risk (Colacito and Croce 2013), or habits (Verdelhan 2010). For convenience, these models typically assume complete markets, while an alternative approach relies on modeling segmented markets, where the SDF shocks emerge from the participation margin (see e.g. Alvarez, Atkeson, and Kehoe 2009).

Given our focus on the set of moments characterizing the empirical exchange rate disconnect, all the approaches to modeling the financial shock $\psi_t$ listed above are isomorphic, as they all result in a version of equation (1.22), and hence we cannot distinguish between
them. The specific models of $\psi_t$ can be discriminated based on a richer set of asset market moments, e.g. a term structure of carry trade returns or a comovement of exchange rate with returns across various asset classes (e.g., see Farhi et al. 2009 and Lustig and Verdelhan 2016b). In particular, in Section 1.4.2 we address an additional Engel (2016) risk premium puzzle in the context of a specific financial model of the $\psi_t$ shock that we adopt.

Note that while our model relies on a violation of the UIP, it is silent in general whether the covered interest rate parity (CIP) is violated simultaneously. The exogenous asset demand and financial friction models typically imply CIP violations. In contrast, CIP holds in the risk-premia-based models, and in particular it holds in the limits-to-arbitrage model that we study in Section 1.4.2.

1.3 Baseline Model of Exchange Rate Disconnect

Our baseline model features a single shock — the financial shock $\psi_t$ — and the transmission mechanism, which emphasizes home bias in expenditure (low $\gamma$), strategic complementarities in price setting ($\alpha > 0$), and weak substitutability between home and foreign goods ($\theta > 1$, but small). The other parameters, including the intertemporal elasticity of substitution (IES) and the elasticity of labor supply, prove to be less consequential for the results, as we discuss below. Even more surprisingly, nominal rigidities turn out to be of little importance for generating the quantitative exchange rate disconnect properties in response to a $\psi_t$ shock. Therefore, our baseline model does not feature any nominal rigidities, with both wages and prices set flexibly. We further assume that monetary au-

\[14\] See e.g. Du, Tepper, and Verdelhan (2016) and Jiang, Krishnamurthy, and Lustig (2017) for the empirical analysis of CIP violations over time.
authorities adopt policy rules that fully stabilize local wage inflation at zero, that is $W_t \equiv 1$ and $W_t^* \equiv 1$.\textsuperscript{15}

The model of this section is analytically tractable (upon log-linearization), and all our results can be easily obtained with pen and paper, which allows us to fully explore the intuition behind various mechanisms. At the same time, we emphasize the quantitative objective of this section. That is, our goal is to establish whether a simple one-shock model can be \textit{quantitatively} consistent with a rich set of moments describing the comovement between exchange rates and macro variables. In doing so, we tie our hands from the start, and calibrate the parameters of the model on which we have direct and reliable empirical evidence. In particular, we set $\gamma = 0.07$ to be consistent with the 0.28 trade (imports plus exports) to GDP ratio of the United States, provided the intermediate input share $\phi = 0.5$.\textsuperscript{16} We further use the estimate of Amiti, Itskhoki, and Konings (2016a) of the elasticity of strategic complementarities $\alpha = 0.4$, which is also in line with much of the markup and pass-through literature and corresponds to the own cost shock pass-through elasticity of $1 - \alpha = 0.6$ (see survey in Gopinath and Itskhoki 2011a). In contrast, the value of the elasticity of substitution between home and foreign goods $\theta$ is more contested.

We follow here the recent estimates of Feenstra, Luck, Obstfeld, and Russ (2014) and set

\textsuperscript{15}In Section 1.4.1, we extend our analysis to allow for nominal rigidities, conventional Taylor rules and multiple sources of shocks. We show, in particular, that our results are \textit{not} very sensitive to the extent of nominal rigidities in wage and price setting, as long as monetary authorities follow rules that target local price or wage inflation.

\textsuperscript{16}This value of the trade-to-GDP ratio is also characteristic of the other large developed economies (Japan and the Euro Zone). Appendix A.1.3 derives the relationship between the value of the trade-to-GDP ratio and the value of $\gamma$ (steady state imports-to-expenditure ratio), which we set to be four times smaller. Intuitively, imports in a symmetric steady state are half of total trade (imports plus exports), and GDP (final consumption) is about one half of the total expenditure with the other half allocated to intermediate inputs ($\phi = 0.5$). This value of $\phi$ is consistent with both aggregate input-output matrices and firm level data on intermediate expenditure share in total sales. The decomposition of gross exports for the U.S., the E.U. and Japan in Koopman, Wang, and Wei (2014) suggests this proportion holds for trade flows as well.
\[ \theta = 1.5, \text{ which is also the number used in the original calibrations of Backus, Kehoe, and Kydland (1994) and Chari, Kehoe, and McGrattan (2002a).}^{17} \]

For concreteness, we assume the financial shock \( \psi_t \) follows an exogenous AR(1) process:

\[ \psi_t = \rho \psi_{t-1} + \epsilon_t, \quad (1.23) \]

with persistence \( \rho \in [0, 1] \) and variance of innovations given by \( \sigma^2_\epsilon \). For our quantitative analysis, we assume that \( \psi_t \) shocks are small, but persistent. While \( \psi_t \) shocks are not directly observable, the model implies that \( \rho \) equals the equilibrium persistence of the interest rates. We, therefore, set \( \rho = 0.97 \). For the remaining parameters, we set the relative risk aversion \( \sigma = 2 \), the Frish elasticity of labor supply \( \nu = 1 \) and a quarterly discount factor \( \beta = 0.99 \), as we summarize in Table 1.1.\(^{18} \)

**1.3.1 Equilibrium exchange rate dynamics**

The model admits a convenient recursive structure, which allows us to solve for the relationship between exchange rate and macro variables (prices, quantities and interest rates) using static equilibrium conditions, as we do in turn in Sections 1.3.2–1.3.5 and Appendix A.1.5. We start, however, with the characterization of the equilibrium exchange rate dynamics.

The equilibrium exchange rate process is shaped by the interplay between the country

\(^{17}\)The *macro* elasticity of substitution between the aggregates of home and foreign goods is indeed the relevant elasticity for our analysis, while the estimates of the *micro* elasticity at more disaggregated levels are typically larger (around 4). The quantitative performance of our model does not deteriorate significantly for elasticities of substitution as high as 3.

\(^{18}\)We show robustness to alternative parameter values in Appendix A.1.5. Furthermore, our qualitative results require only a weak parameter restriction, easily met in conventional calibrations, as we discuss in Appendix A.1.5.
budget constraint (1.19) and international asset market equilibrium, as captured by the UIP condition (1.22). Using the static equilibrium conditions, we solve for the equilibrium relationships between net exports, interest rates and exchange rate, which allows us to rewrite (1.22) and the log-linearized version of (1.19) as:

\[ E_t \Delta e_{t+1} = -\frac{1}{1 + \gamma \lambda_1} \psi_t, \]  
\[ \beta b^*_{t+1} - b_t^* = nx_t = \gamma \lambda_2 e_t. \]  

The coefficients \( \lambda_1 \) and \( \lambda_2 \) depend on the structural model parameters, other than \( \beta \) and \( \rho \). We have \( \lambda_1, \lambda_2 > 0 \) under mild parameter restrictions, such as the Marshall-Lerner condition, which we assume are satisfied (see Appendix A.1.5). Equation (1.25) reflects simply that net foreign assets accumulate with trade surpluses, which in turn increase with exchange rate depreciations, and more so the more open is the country to international trade. Equation (1.24), in turn, suggests that a financial shock \( \psi_t > 0 \) — e.g., an increase in demand for foreign assets — requires an expected appreciation of the home currency to balance the depressed relative demand for domestic bonds. We solve then the dynamic system (1.24) and (1.25), together with the shock process (1.23) to obtain:

**Proposition 1.3 (Equilibrium exchange rate process)** *In the baseline model with the* \(^{19}\) *in Appendix A.1.3, we log-linearize the equilibrium system around a symmetric steady steady, and by default use small letters for log deviations. Since the NFA position \( B^*_t \) and net exports \( NX_t \) are zero in a symmetric steady state, we denote \( b^*_t \equiv \frac{\bar{B}_t}{\bar{P}_H} \bar{Y}^*_t \) and \( nx_t \equiv \frac{1}{\bar{P}_H} \bar{Y}^*_t \). \(^{20}\) *Observe from (1.22) that \( \psi_t > 0 \) must be accommodated either by \( i_t - i^*_t > 0 \), or by \( E_t \Delta e_{t+1} < 0 \) (expected appreciation), both of which moderate the increased demand for the foreign bond. We show in Section 1.3.5 that both effects occur in equilibrium, with the relative importance of the interest rate adjustment decreasing in country openness. In the autarky limit, the interest rates do not move, and we have \( E_t \Delta e_{t+1} = -\psi_t \), consistent with (1.24) as \( \gamma \rightarrow 0 \)."
financial shock $\psi_t \sim AR(1)$ with persistence $\rho$ and innovation $\varepsilon_t$, the equilibrium nominal exchange rate $e_t$ follows an $ARIMA(1,1,1)$, or equivalently $\Delta e_t \sim ARMA(1,1)$, with an $AR$ root $\rho$ and a non-invertible MA root $1/\beta$:

$$\Delta e_t = \rho \Delta e_{t-1} + \frac{1}{1 + \gamma \lambda_1} \frac{\beta}{1 - \beta \rho} \left( \varepsilon_t - \frac{1}{\beta} \varepsilon_{t-1} \right).$$ \tag{1.26}$$

This describes the unique equilibrium exchange rate path, and non-fundamental solutions do not exist.

An analytical proof of this proposition in Appendix A.1.5 relies on the Blanchard and Kahn (1980) solution method, while here we offer a more intuitive explanation. Asset market equilibrium condition (1.24) determines the expected path of the future exchange rate changes, as emphasize by Engel and West (2005), and can be iterated forward to obtain:

$$e_t = \lim_{T \to \infty} \left\{ E_t e_{t+T} + \frac{1}{1 + \gamma \lambda_1} \sum_{j=0}^{T} E_t \psi_{t+j} \right\} = E_t e_\infty + \frac{1}{1 + \gamma \lambda_1} \frac{\psi_t}{1 - \rho}. \tag{1.27}$$

In contrast to Engel and West (2005), however, the general version of the risk sharing (or UIP) condition features no discounting, and therefore admits a multiplicity of no-bubble solutions parametrized by the long-run expectation $E_t e_\infty$. Nonetheless, the equilibrium path of the exchange rate is uniquely pinned down by the intertemporal budget constraint, which obtains from (1.25) by iterating forward and imposing the No Ponzi Game Condition on net foreign assets $\lim_{T \to \infty} \beta^T b^*_{t+T+1} = 0$. This pins down the only budget-consistent long-run expectation, and hence the instantaneous level of the exchange rate
Indeed, any deviation from these values of $e_t$ and $E_t e_\infty$ would shift the whole path of the exchange rate, and hence all trade surpluses on the right-hand side of (1.25), violating the intertemporal budget constraint. This general equilibrium discipline on the exchange rate determination is what distinguishes our solution from that in Engel and West (2005), as we discuss in detail in Appendix A.1.6.

An increase in demand for foreign assets (i.e., $e_t > 0$ in (1.26)) results in an instantaneous depreciation of the home currency, while also predicting an expected appreciation according to (1.24), akin to the celebrated overshooting mechanism of Dornbusch (1976). This exchange rate path ensures both equilibrium in the financial market (via expected appreciation) and balanced country budget (via instantaneous depreciation). Lastly, note that the equilibrium exchange rate process (1.26) is shaped by parameters $\rho$ and $\beta$, and depends on the other parameters of the model only through the proportional volatility scaler $1/(1 + \gamma \lambda_1)$. In particular, since $\lambda_1 > 0$, the exchange rate volatility increases as the economy becomes more closed to international trade, and it is maximized in the autarky limit.

Iterating (1.25) forward and applying the NPGC, we obtain $b^*_t + \gamma \lambda_2 \sum_{j=0}^{\infty} \beta^j e_{t+j} = 0$, which a fortiori holds in expectations. Combining this with (1.27), which implies $E_t e_{t+j} = E_t e_\infty + \frac{1}{1 + \gamma \lambda_1} \frac{1}{1 - \rho} \beta^j \psi_t$, we obtain the solution for the level of exchange rate as a function of the shock $\psi_t$ and the state variable $b^*_t$ (the predetermined NFA position):

$$e_t = \frac{1}{1 + \gamma \lambda_1} \frac{\beta}{1 - \beta \rho} \psi_t - \frac{1}{\gamma \lambda_2} (1 - \beta) b^*_t.$$  

Lastly, combining with (1.25), we express $(\Delta e_t, \Delta b^*_{t+1})$ as a function of exogenous innovations to the financial shock $\psi_t$. 

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Proposition 1.4 (Near-random-walk behavior) The equilibrium exchange rate process (1.26) becomes indistinguishable from a random walk as $\beta \rho \rightarrow 1$. In particular, the following properties hold:

1. The autocorrelation of exchange rate changes becomes arbitrary close to zero:

$$\lim_{\beta \rightarrow 1} \frac{\text{cov}(\Delta e_{t+1}, \Delta e_t)}{\text{var}(\Delta e_t)} = \frac{1 - \rho}{2} \rightarrow 0. \quad (\rho \rightarrow 1)$$

2. The contribution of the predictable component to the variance of $\Delta e_{t+1}$ shrinks to zero, or equivalently the unpredictable component (innovation) fully dominates the variance of $\Delta e_{t+1}$:

$$\lim_{\beta \rightarrow 1} \frac{\text{var}(\Delta e_{t+1} - E_t \Delta e_{t+1})}{\text{var}(\Delta e_{t+1})} = \frac{1 + \rho}{2} \rightarrow 1. \quad (\rho \rightarrow 1)$$

3. The volatility of the exchange rate innovation (surprise) becomes unboundedly large relative to the volatility of the financial shock:

$$\lim_{\beta \rightarrow 1} \frac{\text{var}(\Delta e_{t+1} - E_t \Delta e_{t+1})}{\text{var}(\psi_{t+1})} = \frac{1}{(1 + \lambda \gamma)^2} \frac{1 + \rho}{1 - \rho} \rightarrow \infty. \quad (\rho \rightarrow 1)$$

The results in Proposition 1.4 derive directly from the exchange rate process in (1.26), and we provide the formal algebra behind them in Appendix A.1.5. A simple way to see why the exchange rate process in (1.26) approaches a random walk is to rewrite it using

---

Note that we first take the $\beta \rightarrow 1$ limit and then consider the $\rho \rightarrow 1$ limit. This is because for $\beta < 1$ and $\rho = 1$ the unconditional second moments are not well-defined for $\Delta e_t$, as it becomes an integrated process. This sequential limit provides a good quantitative approximation to our baseline case in which $\rho = 0.97 < \beta = 0.99$. For the last two moments, we assume that $\{\varepsilon_t, \varepsilon_{t-1}, \ldots\}$ is part of the information set at time $t$ when constructing $E_t \Delta e_{t+1}$, even though the MA root of the $e_t$ process is non-invertible, hence offering a conservative lower bound for our results.
the lag operator:

\[(1 - \rho L) \Delta e_t = \frac{1}{1 + \gamma \lambda} \frac{\beta}{1 - \beta \rho} (1 - \beta^{-1} L) \varepsilon_t. \tag{1.28}\]

As both \(\beta\) and \(\rho\) become close to 1, the lag operators on the two sides of the equation cancel out, and the process converges to a random walk. Intuitively, a more persistent financial shock results in a more persistent expected appreciation with a larger effect on the cumulative discounted value of future net exports, especially when \(\beta\) is high. Therefore, when \(\beta\) and \(\rho\) are both large, a given shock \(\varepsilon_t\) requires a large contemporaneous jump in the exchange rate \(e_t\) to balance the intertemporal budget constraint of the country. In other words, the innovation to \(e_t\) is large relative to the innovation to \(E_t \Delta e_{t+1}\), and hence the unexpected component of the exchange rate changes dominates the expected component, making the process more like a random walk.

We now explore the extent to which Proposition 1.4 provides an accurate approximation of the exchange rate properties away from the \(\beta \rho \rightarrow 1\) limit. In particular, Figure 1.1 plots the impulse response of the exchange rate (left panel) and the finite-sample variance decomposition of exchange rate changes (right panel) for our baseline case with \(\beta = 0.99\) and \(\rho = 0.97\). The left panel illustrates that in response to a financial shock \(\varepsilon_t\) the model produces a large depreciation \(\Delta e_t > 0\) on impact, followed immediately by small and persistent expected appreciations in all future periods \(E_t \Delta e_{t+j} < 0\) for \(j \geq 1\), with \(\Delta e_t\) about 25 times larger than \(E_t \Delta e_{t+1}\) in absolute value. The right panel further shows that the unexpected component dominates the variance of the exchange rate at all horizons. In the limit, the expected future appreciations become arbitrary small relative to the size...
Figure 1.1: Impulse response and variance decomposition of $\Delta e_t$

Note: baseline model with $\rho = 0.97$ and $\beta = 0.99$. Left panel: impulse response to $\varepsilon_0$ of $\Delta e_t$ (solid blue) and $e_t$ (dashed red, equal to the area under the blue curve). Right panel: the contribution of the unexpected component ($\Delta k e_{t+k} - \mathbb{E}_{t-1} \Delta k e_{t+k}$) to the variance of the exchange rate change $\Delta k e_{t+k} \equiv e_{t+k} - e_{t-1}$ at various horizons $k \geq 0$ (the $k = 0$ case corresponds to the second moment in Proposition 1.4). The shaded area is the 95% bootstrap confidence interval in a sample of 120 quarters.

of the devaluation on impact, and thus the impulse response converges to that of a white noise for $\Delta e_t$ (or equivalently, random walk for $e_t$).\textsuperscript{23} In our baseline calibration, the autocorrelation of $\Delta e_t$ has a median estimate of $-0.02$ and is not statistically different from zero in 30-year-long samples (see Appendix Table A1).\textsuperscript{24}

To summarize, we find that with a conventional value of the discount factor $\beta$ and the value of $\rho = 0.97$, the model reproduces the volatile near-random-walk behavior of the nominal exchange rate observed in the data.

\textsuperscript{23}There exists empirical evidence on the departure of the exchange rate process from a pure random walk (see e.g. Lustig, Stathopoulos, and Verdelhan 2016, Eichenbaum, Johannsen, and Rebelo 2017, as well as our discussion in Section 1.4.2).

\textsuperscript{24}The near-random-walk behavior of the exchange rate is a joint equilibrium outcome with a persistent evolution of the net foreign asset position $b^*_t$, which follows an AR(1) process in first differences. Nonetheless, the response of $b^*_t$ to the financial shock is quantitatively muted when the economy is relatively closed to international trade (i.e., $\gamma$ is low). See Appendix A.1.5.
1.3.2 Real exchange rate and the PPP puzzle

We next explore the equilibrium dynamics of the real exchange rate (RER) and the associated purchasing power parity (PPP) puzzle, which we broadly interpret as the close co-movement between the nominal and the real exchange rates, and a volatile near-random-walk behavior of both variables. As emphasized by Rogoff (1996), the high volatility of RER is at odds with the productivity shocks, including shocks to the relative productivity of non-tradables, while the high persistence of RER (3–5 year half-lives) is at odds with the monetary shocks given conventional price durations.

We start by rewriting the definition of RER in (1.14) in logs as:

\[ q_t \equiv e_t + p_t^* - p_t. \]  

(1.29)

Combining the definition of the price level (1.6) with the price setting equations for domestic and foreign goods, we obtain (see Appendix A.1.3):

\[ p_t = \left( w_t - \frac{1}{1-\phi} a_t \right) + \frac{1}{1-\phi} \frac{\gamma}{1-2\gamma} q_t, \]  

(1.30)

where \((w_t - \frac{1}{1-\phi} a_t)\) is the domestic nominal unit labor cost and \(\frac{1}{1-\phi} \frac{\gamma}{1-2\gamma} q_t\) captures the effect of the cost of foreign goods (both final and intermediate) on the price of the domestic consumption bundle.²⁵

A long tradition in international macro literature models the real exchange rate as the

²⁵Note that the expression for \(p_t\) does not depend on the strategic complementarity elasticity \(\alpha\), which controls the extent of exchange rate pass-through. With firms symmetric in \(\alpha\), low pass-through into import prices is exactly offset by the domestic firm price adjustment (for further analysis see Burstein and Gopinath 2012, Amiti, Itskhoki, and Konings 2016a).
relative price of non-tradable goods (e.g., the Balassa-Samuelson effect). This approach is, however, at odds with the empirical decomposition in Engel (1999) that finds a negligible role for the relative non-tradable prices in shaping the dynamics of the real exchange rate. We, therefore, abstract from explicitly modeling the relative prices of non-tradables. As a result, the price level in (1.30) depends on the overall degree of home bias $\gamma$, but not on how the domestic expenditure is split between tradables and non-tradables.

Combining (1.30) and its foreign counterpart for $p_t^*$ with the definition of $q_t$ in (1.29), we solve for the relationship between nominal and real exchange rates:

$$
\left[ 1 + \frac{1}{1-\phi} \frac{2\gamma}{1-2\gamma} \right] q_t = e_t + \left( (w_t^* - w_t) - \frac{1}{1-\phi} (a_t^* - a_t) \right).
$$

Intuitively, when $\gamma$ is small, the real exchange rate approximately equals the differential unit labor costs in the two countries. In contrast, as the home bias disappears ($\gamma \to 1/2$), we have $q_t \to 0$, and the purchasing power parity holds in the limit. Recall that in our baseline model we switch off productivity shocks (that is, $a_t - a_t^* \equiv 0$) and assume that monetary policy fully stabilizes wage inflation (namely, $w_t = w_t^* \equiv 0$), and therefore (1.31) implies the following result:

**Proposition 1.5 (Real exchange rate)** In the baseline model, the relationship between the real exchange rates $q_t$ and the nominal exchange rate $e_t$ is given by:

$$
q_t = \frac{1}{1 + \frac{1}{1-\phi} \frac{2\gamma}{1-2\gamma} } e_t.
$$

Therefore, $q_t$ is perfectly correlated with $e_t$, follows the same ARIMA process with a propor-
tionally smaller innovation, and \( \lim_{\gamma \to 0} (q_t - \epsilon_t) = 0 \). The estimated AR(1) coefficient for \( q_t \) increases towards 1 with sample size, and hence the corresponding half-life estimate increases without bound.

Thus, in response to a financial shocks \( \psi_t \), the nominal and real exchange rates are perfectly correlated, equally persistent, and the real exchange rate is somewhat less volatile:

\[
\frac{\text{std}(\Delta q_t)}{\text{std}(\Delta \epsilon_t)} = \frac{1}{1 + \frac{1}{1 - \rho} \frac{2 \gamma}{1 - 2 \gamma}} < 1,
\]

with this gap vanishing as the economy becomes closed to international trade (\( \gamma \to 0 \)). Quantitatively, under our baseline parameterization with \( \gamma = 0.07 \), the real exchange rate is three-quarters as volatile as the nominal exchange rate. We show in Section 1.4.1 that with nominal rigidities this relative volatility is close to 1, as in the data. However, we emphasize here that even the simple version of our model without any price or wage stickiness can account for the bulk of the empirical properties of the real exchange rate. In particular, if one were to fit an AR(1) process for the real exchange rate, as is conventionally done in the PPP puzzle literature surveyed by Rogoff (1996), one would be challenged to find evidence of mean reversion and would infer very long half-lives for the real exchange rate process. In Appendix Figure A3, we show that under our baseline parametrization with \( \rho = 0.97 \), the model reproduces the 3-to-5 year half-lives of the real exchange rate in finite (30-year-long) samples.\(^{26}\)

\(^{26}\)While real exchange rate follows an integrated ARIMA(1,1,1) process in the baseline model, Section 1.4.2 offers an extension with a financial sector, in which it follows a mean-reverting ARMA(2,1), yet the two are indistinguishable in finite samples.
close comovement between the nominal and the real exchange rates. Furthermore, this is achieved without any reliance on nominal rigidities. A natural question then is why the PPP puzzle posed such a challenge to the literature? Equations (1.29)–(1.31) offer an explanation. The close comovement between \( q_t \) and \( e_t \) suggests that the price levels \( p_t \) and \( p_t^* \) should, in turn, move little with the nominal exchange rate \( e_t \) (see (1.29)). The PPP puzzle literature has largely focused on one conceptual possibility, namely that price levels move little due to nominal rigidities, assuming monetary shocks to be the main drivers of the nominal exchange rate. The issue with this approach is that monetary shocks necessarily imply cointegration between relative nominal variables \( (w_t - w_t^*) \) and \( e_t - (p_t - p_t^*) \), which results in mean reversion in the real exchange rate \( q_t \). The speed of this mean reversion is directly controlled by the duration of nominal price stickiness, which is empirically insufficient to generate long half lives, characteristic of the real exchange rate.

We focus on the other conceptual possibility, namely that prices are largely disconnected from exchange rates, or in other words the low exchange rate pass-through into CPI inflation even in the long run, due to substantial home bias (small \( \gamma \); see (1.30)), as is the case empirically. Importantly, this mechanism requires that the main drivers of the exchange rate are not productivity or monetary shocks, which drive a wedge between nominal and real exchange rates independently of the extent of the home bias, as reflected by \( (a_t - a_t^*) \) and \( (w_t - w_t^*) \) terms in (1.31). Instead, the shock we focus on is the financial shock \( \psi_t \), and it drives no wedge between nominal and real exchange rates, even in the long-run. Home bias is thus the only relevant part of the transmission mechanism, leaving nominal rigidities, real rigidities \( (\alpha) \), or the extent of expenditure switching \( (\theta) \) largely
The mechanism does rely, however, on the monetary policy rule. We show in Section 1.4.1 that our results are robust to conventional Taylor rules, but are sensitive to a switch to an exchange rate peg.

### 1.3.3 Exchange rates and the terms of trade

In this section we explore the joint properties of the equilibrium (CPI-based) real exchange rate, terms of trade and producer prices. As emphasized by Atkeson and Burstein (2008a), the conventional models imply a counterfactually volatile terms of trade and producer prices relative to consumer prices. In the data, consumer- and producer-based real exchange rates are equally volatile, while the terms of trade are substantially more stable — about two-to-three times less volatile than the real exchange rate.

The results in this section follow from two equilibrium relationships between real exchange rates (RER) and the terms of trade (ToT):

\[
q_t = (1 - \gamma)q^P_t - \gamma s_t, \hspace{1cm} (1.33)
\]
\[
s_t = q^P_t - 2\alpha q_t, \hspace{1cm} (1.34)
\]

where \(s_t = p_{Ft} - p^*_Ht - e_t\) is the log of the ToT in (1.20) and \(q^P_t \equiv p^*_Ft + e_t - p_Ht\) is the log producer-price RER. Intuitively, (1.33) reflects that the relative consumer prices \(q_t\) differ from the relative producer prices \(q^P_t\) by the relative price of imports \(s_t\). Equation (1.34), in turn, states that the terms of trade reflect the relative producer prices.

\(^{27}\) As a result, our model is also consistent with the Kehoe and Midrigan (2008)’s finding of the missing correlation between price durations and RER persistence across sectors, which is evidence against monetary shocks as the key driver of the nominal exchange rate rather than against sticky prices as the transmission mechanism.
justed for the law of one price deviations of exports (recall from (1.14) that the log LOP deviation equals $\alpha q_t$).

Conventional models, without strategic complementarities ($\alpha = 0$), and thus without LOP deviations, imply that the ToT equal producer RER, and both are more volatile than the consumer RER:

$$s_t = q_t^P = \frac{1}{1 - 2\gamma} q_t.$$

Intuitively, consumer prices are less volatile than producer prices as they smooth out the relative price fluctuations by combining home and foreign goods into the consumption bundle (i.e., a diversification argument). This is, however, empirically counterfactual, and as explained by Atkeson and Burstein (2008a) is not necessarily the case in the models with pricing to market (PTM), arising, for example, from strategic complementarities in price setting ($\alpha > 0$). Combining (1.33) and (1.34) together, we arrive at:

**Proposition 1.6 (Real exchange rate and the terms of trade)** The real exchange rates and the terms of trade are linked by the following equilibrium relationships:

$$q_t^P = \frac{1 - 2\alpha \gamma}{1 - 2\gamma} q_t \quad \text{and} \quad s_t = \frac{1 - 2\alpha (1 - \gamma)}{1 - 2\gamma} q_t. \quad (1.35)$$

The empirical patterns $\text{std}(\Delta s_t) \ll \text{std}(\Delta q_t) \approx \text{std}(\Delta q_t^P)$ and $\text{corr}(\Delta s_t, \Delta q_t) > 0$ obtain when strategic complementarities in price setting are significant, but not too strong: $\frac{\gamma}{1 - \gamma} \ll \alpha < \frac{1}{2(1 - \gamma)}$.

When $\alpha > \frac{\gamma}{1 - \gamma}$, the model reproduces the empirically relevant case, in which RER $q_t$ is considerably more volatile than the ToT $s_t$. Pricing to market ($\alpha > 0$) smoothes out
the response of ToT to changes in the producer prices \( q_t^P \), and hence makes ToT less volatile, as export prices mimic the local competition. Very strong PTM, \( \alpha > \frac{1}{2(1-\gamma)} \), just like local currency pricing (LCP), can turn the positive correlation between ToT and RER into negative, which is empirically counterfactual, as emphasized by Obstfeld and Rogoff (2000). For the intermediate values, \( \alpha \lesssim 0.5 \), the performance of the model is good on both margins. Additionally, a low value of \( \gamma \) ensures that \( \text{std}(\Delta q_t) \approx \text{std}(\Delta q_t^P) \), as is the case empirically. Under our baseline parameterization with \( \gamma = 0.07 \) and \( \alpha = 0.4 \), the terms of trade are about a third as volatile as the real exchange rate, yet still positively correlated, exactly as in the data.\(^{28}\) We further explore these quantitative implications of the model in Appendix Figure A4.

1.3.4 Exchange rate and consumption: the Backus-Smith puzzle

We now study the relationship between aggregate consumption and the real exchange rate, which in our model is shaped by the product and factor markets, and in particular the expenditure switching forces, as opposed to the risk sharing in the financial market emphasized by the Backus-Smith condition.

First, we combine labor supply (1.3) and labor demand (1.10) to express labor market clearing as:

\[
\sigma \nu c_t + y_t = -\frac{\nu + \phi}{1 - \phi} \frac{\gamma}{1 - 2\gamma} q_t, \tag{1.36}
\]

where the real wage \((w_t - p_t)\) on the right-hand side is expressed using (1.30) as a de-

\(^{28}\)As in Atkeson and Burstein (2008a), the lower volatility of the terms of trade results from the markup adjustment by exporting firms. We check in our quantitative analysis that this does not imply counterfactually volatile aggregate profits.
creasing function of $q_t$. Indeed, real depreciation (high $q_t$) depresses real wage through the increased cost of foreign goods. Equation (1.36) characterizes the locus of allocations consistent with equilibrium in the labor market: low real wage and high consumption reduce labor supply and, hence, output $y_t$.

Second, we look at the product market clearing condition, which derives from (1.17)–(1.18), and upon log-linearization results in:

$$y_t = (1 - \phi) \left( (1 - \gamma)c_t + \gamma c_t^* \right) + \phi \left( (1 - \gamma)y_t + \gamma y_t^* \right) + \gamma \left[ 2\theta(1 - \alpha) \frac{1 - \gamma}{1 - 2\gamma} - \phi \right] q_t. \quad (1.37)$$

This equation characterizes the locus of allocations consistent with equilibrium in the product market. Note that in the closed economy ($\gamma = 0$), we simply have $y_t = c_t$, which holds in log deviations independently of the intermediate share $\phi$. In open economy, home production $y_t$ is split between final consumption and intermediate use at home and abroad, as reflected by the first two terms on the right-hand side of (1.37). The remaining term in the real exchange rate $q_t$ combines the positive effect of expenditure switching from foreign to home goods and the negative effect of substitution away from the intermediate inputs towards local labor. In particular, the expenditure switching effect acts to increase demand for domestic output $y_t$ when home exchange rate depreciates ($q_t$ increases), and this effect is shaped by $\theta(1 - \alpha)$, the product of the exchange rate pass-through into prices $(1 - \alpha)$ and the elasticity of substitution $\theta$.

Now combining (1.36)–(1.37) together with their foreign counterparts, we can solve

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29To derive (1.37), we solve out the real wage and relative prices as a function of the real exchange rate. In particular, in Appendix A.1.5 we show that the exchange rate pass-through into relative prices is determined by $(1 - \alpha)$, as $p_{Ht} - p_t = -(1 - \alpha) \frac{\gamma}{1 - 2\gamma} q_t$ and $p_{Ht}^* - p_t^* = -(1 - \alpha) \frac{1 - \gamma}{1 - 2\gamma} q_t$.  

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for the equilibrium relationship between the relative consumption $c_t - c_t^*$ and the real exchange rate $q_t$ (see Appendix Figure A1 for a simple geometric solution):

**Proposition 1.7 (Backus-Smith resolution)** In the baseline model with financial shocks $\psi_t$ only, the relative home consumption declines with real depreciation, according to the following equilibrium relation:

$$c_t - c_t^* = -\gamma \kappa_q q_t, \quad \text{where} \quad \kappa_q \equiv \frac{2\theta(1-\alpha)^{1-\gamma}(1-2\gamma) + \nu + \nu(2\gamma)}{(1-\phi)[1 + \sigma\nu(1+\frac{2\gamma}{1-\phi})]} \frac{2}{1-2\gamma} > 0. \quad (1.38)$$

The effect of the real exchange rate on the relative consumption vanishes as the economy becomes closed to international trade ($\gamma \to 0$). For $\gamma > 0$, the elasticity of the relative consumption with respect to real exchange rate increases (in absolute terms) in the strength of the expenditure switching effect, $\theta(1-\alpha)$.

It follows from Proposition 1.7 that our baseline model robustly reproduces a negative correlation between relative consumption and real exchange rate, both in levels and in growth rates. That is, our model predicts that consumption is low when prices are low, in relative terms across countries. This violates efficient international risk sharing, predicted by the celebrated Backus-Smith condition, yet is consistent with the patterns in the data (see cross-country estimates in Benigno and Thoenissen 2008). Furthermore, this property of our model stands in stark contrast with predictions of both productivity-driven IRBC models and monetary-shock-driven New Keynesian (NOEM) models, even when those models feature incomplete asset markets.

What is most striking about this result is that we have derived (1.38) using solely labor and product market clearing conditions, which are completely ubiquitous in international
general equilibrium models. Indeed, the negative relationship between consumption and real exchange rate is a robust feature of the expenditure switching mechanism. Real exchange rate depreciation switches expenditure towards home goods, and in order to clear the markets home output needs to rise and home consumption, in view of the home bias, needs to fall. A natural question then is what makes our model different?

There are two key features of our model that allow it to produce the empirical negative correlation between consumption and the real exchange rate. First, we shift the determination of consumption from asset to product markets. Indeed, in complete market models (with CRRA utility), partial equilibrium in the asset market requires a positive correlation between relative consumption and real exchange rate: \( \sigma (c_t - c^*_t) = q_t \), as an outcome of the optimal international risk-sharing. Our model instead features incomplete markets and, more importantly, a risk-sharing shock \( \psi_t \), which implies the following relationship between consumption growth and real appreciation in expectations:

\[
E_t \left\{ \sigma (\Delta c_{t+1} - \Delta c^*_{t+1}) - \Delta q_{t+1} \right\} = \psi_t. \tag{1.39}
\]

Furthermore, the equilibrium relationship between consumption and exchange rate (1.38) is fully determined in product market, without reference to asset market and risk sharing.\(^{30}\)

Second, our model ensures that the key force shaping the comovement between con-

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\(^{30}\)Asset markets matter in general equilibrium, as both consumption and exchange rate dynamics need to be consistent with asset market clearing and equilibrium interest rates (see Section 1.3.5). Equation (1.38), however, is a static equilibrium condition, which holds state-by-state and hence determines the correlation between consumption and real exchange rate independently of their dynamic processes. This is possible because the financial shock \( \psi_t \) does not directly affect the goods market, and thus is absent from (1.38).
sumption and the real exchange rate is expenditure switching, emphasized in equation (1.38). This is in contrast with the IRBC and NOEM models, where real depreciation is associated with increased supply of domestic output—either due to a high productivity shock or a stimulating effect of a monetary easing shock—while expenditure switching is only a byproduct. As a result, real depreciation in these models is associated with an empirically counterfactual domestic consumption boom. The mechanism in our financial model is different. A real depreciation is not caused by increased supply of domestic goods, but instead by increased demand for foreign assets. Therefore, the only effect on the real economy is indirect, induced by expenditure switching arising from a real depreciation, which causes a decline in real wages and consumption.\footnote{While we solved for equilibrium consumption given the exchange rate, one can also do the reverse. Relationship (1.38) can then be interpreted as a Keynes transfer effect: a financial shock makes home households postpone their consumption resulting in a lower relative demand for home goods, which requires an exchange rate depreciation to clear the goods market (see e.g. Pavlova and Rigobon 2008, Caballero, Farhi, and Gourinchas 2008).}

Lastly, we comment on the quantitative implications of Proposition 1.7. A salient feature of the data is a much greater volatility of the exchange rate relative to other macroeconomic variables, and in particular consumption. From (1.38), we see that the relative volatility of consumption tends to zero as the economy becomes closed to international trade ($\gamma \to 0$). Outside this limit, for our baseline parameterization $\frac{\text{std}(\Delta c_t)}{\text{std}(\Delta q_t)} = 0.15$. That is, consumption is about 6 times less volatile than the real exchange rate, in line with the empirical magnitudes (for example, Chari, Kehoe, and McGrattan (2002a) target a ratio of 5).\footnote{We further find that this quantitative performance of the model is not very sensitive to the specific values of the relative risk aversion $\sigma$ and Frish elasticity of labor supply $\nu$, but is mostly sensitive to trade openness $\gamma$ and the combined expenditure switching elasticity $\theta(1 - \alpha)$, as we illustrate in the Appendix Figure A4 and Appendix A.1.5.} We conclude that the model is not only consistent with the negative cor-
relation between consumption and real exchange rate, but also reproduces quantitatively their relative volatilities.

**Alternative mechanisms in the literature** Naturally, all explanations of the Backus-Smith puzzle must relax the straitjacket of the international risk-sharing condition $\sigma(c_t - c_t^*) = q_t$, either by assuming incomplete markets (e.g. Corsetti, Dedola, and Leduc 2008, Benigno and Thoenissen 2008), or by departing from separable CRRA utility (e.g. Colacito and Croce 2013, Karabarbounis 2014). The analytical tractability of our model, coupled with the conventional product and labor market structure, allows us to shed light on the mechanisms in other papers in relationship to our mechanism.

For concreteness, we focus on the model with productivity shocks only, as is the case in much of the Backus-Smith puzzle literature. This case results in the following equilibrium relationship:

$$c_t - c_t^* = \kappa_a (a_t - a_t^*) - \gamma \kappa_q q_t, \quad (1.40)$$

where $\kappa_q > 0$ is given in (1.38) and $\kappa_a > 0$ is derived in the appendix (see (A49)). This is the sense in which the same expenditure switching effect of the real exchange rate on consumption is still present in the models with other shocks, but these shocks also have a direct effect in product and labor markets. Importantly, $\kappa_a$ does not go to zero with $\gamma \to 0$, and therefore, the direct effect always dominates the expenditure switching effect when economies are relatively closed to international trade. While our mechanism in (1.38) emphasizes the expenditure switching as the key source of the relationship between consumption and real exchange rate, other papers have it only as a feedback mechanism, typically not strong enough to overturn the direct effects of the product market shocks.
Equilibrium relationship (1.40) makes clear the two possible ways in which the Backus-Smith puzzle can be resolved. First, this occurs when exchange rate dynamics is shaped by a shock with a small direct effect on consumption relative to its indirect (expenditure switching) effect through exchange rate. Financial shocks emphasized in this paper, as well as news shocks about future productivity or long-run risk shocks in Colacito and Croce (2013) operate in this fashion. Second, if the direct effect is strong and consumption increases with productivity, the Backus-Smith puzzle can be resolved if relative prices also increase with productivity (i.e., $\partial q_t/\partial a_t < 0$). This may occur due to Balassa-Samuelson forces (e.g., Benigno and Thoenissen 2008), persistent productivity growth rates and/or low elasticity of substitution between home and foreign goods ($\theta < 1$), as in Corsetti, Dedola, and Leduc (2008). These alternative mechanisms are, however, at odds with other exchange rate puzzles, including Meese-Rogoff and PPP puzzles discussed above. See Appendix A.1.7 for two illustrations and a further discussion.

1.3.5 Exchange rate and interest rates: the UIP Puzzle

Finally, we explore the equilibrium properties of interest rates, and in particular the UIP puzzle. Home and Foreign Euler equations for local bonds in (1.4) and (1.15) result together in:

$$i_t - i^*_t = -\gamma \lambda_1 \mathbb{E}_t \Delta e_{t+1},$$  (1.41)

where coefficient $\lambda_1 > 0$ under a mild parameter restriction (Appendix A.1.5 provides a derivation, which makes use of the equilibrium relationships between consumption and prices and the exchange rate). Combining (1.41) with the UIP condition (1.22) yields both
the expression for the expected devaluation (1.24) used in Section 1.3.1 and the following solution for the equilibrium interest rate differential:

\[ i_t - i_t^* = \frac{\gamma \lambda_1}{1 + \gamma \lambda_1} \psi_t. \]  

(1.42)

A demand shock for foreign bond \( \psi_t \) raises the interest rate differential \( i_t - i_t^* \) to equilibrate the asset market. Further, (1.42) implies that the interest rate differential, like \( \psi_t \), follows an AR(1) process with persistence \( \rho \), and in addition with volatility declining towards zero in the closed economy limit (\( \gamma \to 0 \)).

Making use of our characterization in Proposition 1.3, we now study the joint properties of the interest rates and the nominal exchange rate (with similar relationships also holding in real terms):

**Proposition 1.8 (Exchange rate and interest rates)** The Fama regression, i.e. the projection of the exchange rate change \( \Delta e_{t+1} \) on the interest rate differential \( (i_t - i_t^*) \), has a negative coefficient \( \beta_F = -1/(\gamma \lambda_1) < 0 \). Furthermore, around the \( \beta \rho \to 1 \) limit:

(i) the \( R^2 \) in the Fama regression becomes arbitrary small;

(ii) the volatility of \( (i_t - i_t^*) \) relative to \( \Delta e_{t+1} \) becomes arbitrary small;

(iii) the persistence of \( \Delta e_{t+1} \) relative to \( (i_t - i_t^*) \) becomes arbitrary small;

(iv) the Sharpe ratio of the Carry trade\(^{33}\) becomes arbitrary small.

Proposition 1.8 suggests that our model provides a good approximation to the observed

---

\(^{33}\)A carry trade is a zero-capital investment strategy, which shorts the low interest rate currency and longs the high interest rate currency. For concreteness, following Lustig and Verdelhan (2011), we focus on a strategy with an intensity (size of the positions taken, \( x_t \)) proportional to the expected return, i.e. \( x_t = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \psi_t \) (see Appendix A.1.5).
empirical patterns. As in the data, positive interest rate differentials predict expected exchange rate appreciations — a pattern of the \textit{UIP deviations} known as the \textit{Forward Premium puzzle} (Fama 1984). This result follows directly from (1.41), as a $\psi_t > 0$ shock results both in a positive interest rate differential and an expected appreciation of the home currency.

At the same time, the predictive ability of the interest rate differentials for future devaluations is very weak in the data (see e.g. Valchev 2016), and our model captures this with a vanishingly small $R^2$ in the Fama regression as the $\psi_t$ shocks become more persistent. Recall from Proposition 1.4 that in this case the unexpected changes dominate the dynamics of the exchange rate, while the expected changes play a vanishingly small role. Under our baseline parameterization, the $R^2$ in the Fama regression is below 0.05, in line with the data. In addition, the interest rate differentials, unlike exchange rate changes, are very smooth and persistent in the data — another disconnect pattern captured by our model. For example, under the baseline parametrization, the model produces interest rate differentials that are less than one-tenth as volatile as the exchange rate changes (see Appendix Table A1). Lastly, the UIP shock in our model does not result in counterfactually large returns on the Carry trade. In Appendix Figure A5, we illustrate the small-sample properties of the Fama regression and the carry trade returns, and show that the associated Sharpe ratio varies between 0.15 and 0.3, in line with the empirical patterns.\footnote{The unconditional Sharpe ratio of the carry trade in the data is about 0.5, but at least half of it comes from the cross-sectional country fixed effects not modeled in our framework, which instead focuses on the time-series properties. Our empirical target for the Sharpe ratio of 0.2 corresponds to the “forward premium trade” in Hassan and Mano (2014).}

To summarize the results of Section 1.3, our baseline model with a simple transmission mechanism and a single financial exchange rate shock is consistent, both qualitatively and
quantitatively, with a rich set of moments describing the dynamic comovement between exchange rates and macro variables. Many of these moments correspond to the long-standing puzzles from the point of view of the conventional international-macro models, including the PPP puzzle and the Backus-Smith puzzle. The financial shock $\psi_t$ admits a number of micro-foundations, yet is not directly observable in the data. When $\psi_t$ is assumed to follow a persistent AR(1) process with small innovations, the model reproduces both the empirical Meese-Rogoff exchange rate dynamics and the comovement properties between exchange rates and interest rates, including the Forward Premium (UIP) puzzle.

1.4 Extensions

This section considers two main extensions to our analysis. Section 1.4.1 generalizes the baseline model to a full-fledged monetary model with nominal stickiness and a Taylor rule. In the context of this extension, we discuss the properties of the model with multiple shocks, as well as the Mussa puzzle associated with the switch between monetary regimes. Section 1.4.2 extends the baseline model to feature an explicit financial sector with risk-averse intermediaries and noise traders to shed light on a number of issues, including the recent Engel (2016) puzzle. Additional extensions, including a model with capital and a full international busyness cycle calibration, are considered in Appendix A.2.7.

1.4.1 A monetary model with nominal rigidities

We now consider the robustness of our findings in Sections 2.2 and 1.3 in a fully specified monetary model with nominal rigidities, arguably a salient feature of the real world. First,
we demonstrate that nominal shocks \textit{per se} cannot reproduce the empirical exchange rate behavior, as suggested by Proposition 1.1. Second, we show that the financial shock $\psi_t$ has similar quantitative properties in the monetary model, as in our baseline model of Section 1.3, despite a different transmission mechanism for the interest rates. Third, we study a calibrated multi-shock model to quantify the contribution of monetary and productivity shocks to the exchange rate volatility. Lastly, we discuss the robustness of the results to various policy rules and the associated Mussa puzzle.

We introduce nominal rigidities as in the standard New Keynesian model (see e.g. Woodford 2003), while leaving the structure of international financial markets as in the baseline model. We focus on a cashless-limit economy and abstract from ZLB, commitment problems and multiplicity of equilibria. In particular, the nominal interest rate is set by a central bank according to a conventional Taylor rule:

$$i_t = \rho_m i_{t-1} + (1 - \rho_m) \delta_\pi \pi_t + \varepsilon_t^m,$$

where $\rho_m$ and $\delta_\pi$ are parameters, $\pi_t$ is the CPI inflation rate and $\varepsilon_t^m$ is an exogenous monetary shock.

Firms are subject to a Calvo price-setting friction with the probability of price adjustment equal to $1 - \lambda_p$. We maintain the assumption that the desired price depends on both own marginal cost and competitor prices, as in (1.12) with the exogenous markup shock shut down. We further assume that exporters set prices in the local currency (LCP), as for example in Chari, Kehoe, and McGrattan (2002a) and Devereux and Engel (2002a). Households set wages and are also subject to the Calvo friction with the probability of
wage adjustment equal to \(1 - \lambda_w\), as described in Gali (2008). Appendix A.1.8 provides a full description of the model with the characterization of its solution, as well as several extensions with PCP price stickiness and alternative Taylor rules.

To calibrate the model, we keep the same values of the parameters as in the baseline case discussed in Section 1.3. The prices are assumed to adjust on average once a year, i.e. \(\lambda = 0.75\) (Nakamura and Steinsson 2008). For wages, we set \(\lambda_w = 0.85\) corresponding to a longer expected wage duration equal to 1.5 years. For the inflation response elasticity in the Taylor rule we use the estimates from Clarida, Gali, and Gertler (2000), namely \(\delta_\pi = 2.15\), which satisfies the Taylor principle. Following the literature, we set the interest rate smoothness parameter \(\rho_m = 0.95\) to match the empirical persistence of the interest rate, and we assume that the monetary shocks \(\varepsilon_t^m\) follow an iid process.

We summarize the results of our analysis in Table 1.2, where we contrast the moments in the data and in various versions of the model. Panel A considers various single-shock models. We first report the moments in two versions of the model with the financial shock \(\psi_t\) alone. Specifically, column 1 reports the moments from a model without nominal rigidities \((\lambda_p = \lambda_w = 0)\), but subject to the Taylor rule (1.43), which is the only departure from our benchmark model of Section 1.3. Column 1 reconfirms our findings that the model captures well the quantitative behavior of the exchange rates, both nominal and real, including their persistence, volatility relative to other macro variables, as well as the direction of comovement with consumption and interest rates.

The model in column 2 of Table 1.2 features additionally wage and price stickiness, as discussed above, while maintaining \(\psi_t\) as the only shock. We see that the introduction of nominal rigidities has very little effect on the quantitative properties of the model, even
Table 1.2: Quantitative properties and comparisons across models

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>A: SINGLE-shock models</th>
<th>B: MULTI-shock models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1) (2)</td>
<td>(3) (4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(5) (6)</td>
<td>(7)</td>
</tr>
<tr>
<td>(\rho(\Delta e))</td>
<td>0.00</td>
<td>-0.02 -0.03 -0.05 0.00</td>
<td>-0.03 -0.02 -0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.09) (0.09) (0.09)</td>
<td>(0.09) (0.09) (0.09)</td>
</tr>
<tr>
<td>(\rho(q))</td>
<td>0.95</td>
<td>0.93 0.91 0.84 0.93</td>
<td>0.93 0.93 0.93</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.04) (0.05) (0.04)</td>
<td>(0.04) (0.04) (0.04)</td>
</tr>
<tr>
<td>(\sigma(\Delta q)/\sigma(\Delta e))</td>
<td>0.99</td>
<td>0.79 0.97 0.97 1.64</td>
<td>0.98 0.94 0.76</td>
</tr>
<tr>
<td>(\text{corr}(\Delta q, \Delta e))</td>
<td>0.98</td>
<td>1 1 0.99 0.99</td>
<td>1.00 0.97 0.94</td>
</tr>
<tr>
<td>(\sigma(\Delta c−\Delta c^*)/\sigma(\Delta q))</td>
<td>0.20</td>
<td>0.31 0.12 0.52 0.64</td>
<td>0.20 0.30 0.31</td>
</tr>
<tr>
<td>(\text{corr}(\Delta c−\Delta c^*, \Delta q))</td>
<td>-0.20</td>
<td>-1 -0.95 1 1</td>
<td>-0.20 -0.20 -0.22</td>
</tr>
<tr>
<td>(\sigma(\Delta nx)/\sigma(\Delta q))</td>
<td>0.10</td>
<td>0.26 0.17 0.08 0.14</td>
<td>0.32 0.30 0.10</td>
</tr>
<tr>
<td>(\text{corr}(\Delta nx, \Delta q))</td>
<td>(\approx)</td>
<td>1 0.99 1 1</td>
<td>-0.00 -0.00 -0.02</td>
</tr>
<tr>
<td>(\sigma(\Delta s)/\sigma(\Delta e))</td>
<td>0.35</td>
<td>0.23 0.80 0.82 0.49</td>
<td>0.80 0.28 0.23</td>
</tr>
<tr>
<td>(\text{corr}(\Delta s, \Delta e))</td>
<td>0.60</td>
<td>1 -0.93 -0.96 0.99</td>
<td>-0.93 0.97 0.94</td>
</tr>
</tbody>
</table>

Fama \(\beta\)  
\[
\leq 0 \quad -2.4 \quad -3.4 \quad 1.2 \quad 1.4 \quad -0.6 \quad -0.7 \quad -2.8
\]
\((1.7) (2.6) (0.7) (0.5) (1.4) (1.3) (3.5)\)

Fama \(R^2\)  
\[
0.02 \quad 0.03 \quad 0.03 \quad 0.09 \quad 0.00 \quad 0.00 \quad 0.01
\]
\((0.02) (0.02) (0.02) (0.02) (0.01) (0.01) (0.02)\)

\(\sigma(i−i^*)/\sigma(\Delta e)\)  
\[
0.06 \quad 0.07 \quad 0.05 \quad 0.14 \quad 0.21 \quad 0.06 \quad 0.08 \quad 0.03
\]
\((0.02) (0.02) (0.02) (0.02) (0.02) (0.02) (0.02)\)

\(\rho(i−i^*)\)  
\[
0.90 \quad 0.93 \quad 0.98 \quad 0.84 \quad 0.93 \quad 0.91 \quad 0.93 \quad 0.90
\]
\((0.04) (0.01) (0.05) (0.04) (0.04) (0.04) (0.04)\)

Carry \(SR\)  
\[
0.20 \quad 0.21 \quad 0.20 \quad 0 \quad 0 \quad 0.17 \quad 0.19 \quad 0.12
\]
\((0.04) (0.04) (0.04) (0.06) (0.06) (0.06) (0.07)\)

Note: The table reports moments in the data (see details in the Data Appendix A.1.11) and in the simulated data for various specification of the model, as described in the text. \(\rho(\cdot)\) denotes autocorrelation, \(\sigma(\cdot)\) denotes standard deviation. The numbers in brackets report standard deviation across 10,000 simulations with 120 quarters (30 years) each, only for the moments that vary considerably across simulations. In all specifications, model parameters are as in Table 1.1, and columns 2, 3 and 5 additionally feature sticky prices \((\lambda_p = 0.75)\) and wages \((\lambda_w = 0.85)\), while columns 1, 4 and 6–7 have flexible prices \((\lambda_p = \lambda_w = 0)\).

Monetary policy in columns 1–6 follows the Taylor rule (1.43) with parameters given in the text, while in column 7 it fully stabilizes the wage inflation \((W_t = W_t^* = 0)\), as in the baseline model of Section 1.3.

Despite the differences in the transmission mechanism. Indeed, in the baseline model, the interest rate settles down to clear the markets, while in the monetary model the path of the interest rate is chosen by the monetary authority according to the Taylor rule (1.43).

In both models, a \(\psi_t\) shock results in a sharp nominal depreciation, which in turn leads to a mild home inflation as the prices of the foreign goods increase. In a monetary model, the
central bank responds by raising the interest rate, and the households respond by cutting their current consumption expenditures, thus enabling the model to reproduce both the Backus-Smith puzzle and the UIP puzzle. Furthermore, the sluggish price adjustment in the model with nominal rigidities increases the volatility of the real exchange rate and reduces the volatility of consumption and interest rates relative to the volatility of the nominal exchange rate, improving somewhat the fit of the model.\footnote{On the other hand, as pointed out by Obstfeld and Rogoff (2000), models with LCP imply a counterfactual negative correlation between the exchange rate and the terms-of-trade. Matching the empirical positive, yet imperfect, correlation between these variables requires a model with a mixture of price setting patterns (flexible, PCP, LCP), which also allows for DCP (dollar/dominant currency pricing), as emphasized recently by Casas, Diez, Gopinath, and Gourinchas (2016).}

Finally, in column 3 and 4 of Table 1.2, we shut down the financial shock and instead consider two conventional international macro models — a NOEM model with nominal rigidities subject to monetary (Taylor rule) shocks $\varepsilon_t^m$ and also for comparison an IRBC model without nominal frictions subject to productivity shocks $a_t$.\footnote{With our focus on exchange rate moments, we only need to specify the processes for the relative shocks, i.e. $a_t - a_t^*$ in case of the productivity shock, which we assume follows an AR(1) process with persistence $\rho_a = 0.97$.} These models fail on a number of moments and reproduce the familiar exchange rate puzzles. In particular, both of these models cannot reproduce the direction of the comovement between interest rates and exchange rate (the UIP puzzle) and consumption and exchange rate (the Backus-Smith puzzle). Furthermore, the NOEM model is challenged to reproduce the persistence of the real exchange rate, yielding a half life of less than a year (the PPP puzzle), while for the IRBC model matching the volatility of the nominal exchange rate requires incredibly large volatility of the relative TFP shocks across countries, resulting in excessively volatile real exchange rate, consumption and interest rates.
**Multiple shocks and variance decomposition**  A natural deficiency of any one-shock model is that it can only speak to the relative volatilities of variables, while implying counterfactual perfect correlations between them. We consider now two models with multiple shocks to study whether they can successfully reproduce the imperfect, and in general weak, empirical correlations between exchange rates and macro variables. In particular, we extend the NOEM and IRBC models from columns 3 and 4 of Table 1.2 to feature two additional shocks each — the financial shock $\psi_t$ and a demand shock for foreign goods $\xi_t$.\(^{37}\)

Leaving unchanged all parameter values, we only need to additionally calibrate the relative variances of the shocks, with the overall level of variance kept to match the volatility of the nominal exchange rate. We set the relative volatilities of the shock to match the correlations of the real exchange rate with consumption and net exports.\(^{38}\)

From columns 5 and 6 of Table 1.2, we see that these extended NOEM and IRBC models are indeed capable at perfectly matching the weak negative correlation between the real exchange rate and consumption and a close-to-zero correlation between the real exchange rate and net exports, without compromising the fit of the other moments relative to the baseline monetary model in column 2. We conclude that these parsimonious, and rather conventional, models provide a quantitative resolution to a broad class of international macro puzzles assembled under the exchange rate disconnect umbrella. The key to this

\(^{37}\)We have also experimented with a LOP-deviation shock $\eta_t$ and a nearly-equivalent iceberg trade cost shock, but find them largely redundant as long as $\xi_t$ is included, which has superior quantitative properties for the moments we have chosen.

\(^{38}\)In the NOEM model, $\sigma_m/\sigma_\varepsilon = 0.315$, where $\sigma_m = \sigma(\varepsilon_t^m - \varepsilon_{t^*})$ for the Taylor rule shock in (1.43) and $\sigma_\varepsilon = \sigma(\varepsilon_t)$ for the innovation to $\psi_t$ in (1.23). In the IRBC model, $\sigma_a/\sigma_\varepsilon = 2.1$, where relative productivity $(1 - \rho_a L)(a_t - a_t^*) \sim \mathcal{N}(0, \sigma_a^2)$ with persistence $\rho_a = \rho = 0.97$. We do not combine productivity and monetary shocks together in any one specification because their relative roles cannot be identified from the set of exchange rate moments that we focus on. Instead, both NOEM and IRBC models include a foreign good demand shock: $(1 - \rho_L)(\xi_t - \xi_{t^*}) \sim \mathcal{N}(0, \sigma_\xi)$. We set $\rho = 0.97$ and $\gamma\sigma_\varepsilon/\sigma_\varepsilon$ set at 2.7 and 2.4 in the two specifications respectively.
success is the presence of the financial shock, as we now show more formally.

In the context of these two multi-shock models, we carry out variance decompositions in order to assess the relative importance of productivity, monetary and financial shocks for the exchange rate dynamics. This decomposition, reported in Table 1.3, reveals a clear pattern, which echoes our theoretical predictions in Propositions 1.1 and 1.2. First, the financial shock $\psi_t$ plays the dominant role, explaining over 70% of the nominal exchange rate variation in both models. Second, the international shock in the goods market $\xi_t$ also plays an important role, contributing about 20% to the nominal exchange rate volatility, to partly balance out the comovement between exchange rate and macro variables induced by the $\psi_t$ shock. Third, the contribution of productivity and monetary shocks is minimal, never exceeding 10% for either nominal or real exchange rate. Indeed, Proposition 1.1 suggests that both of these shocks, if too important in shaping the exchange rate dynamics, result in conventional exchange rate puzzles.

To be clear, the conventional productivity and monetary shocks are still central for the dynamics of the macro variables such as consumption, employment, output and prices levels. For these variables our model replicates the standard international business cycle

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**Table 1.3: Sources of exchange rate variation**

<table>
<thead>
<tr>
<th>Shocks</th>
<th>NOEM</th>
<th>IRBC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\var(\Delta e_t)$</td>
<td>$\var(\Delta q_t)$</td>
</tr>
<tr>
<td>Monetary (Taylor rule)</td>
<td>$\varepsilon^n_t$</td>
<td>10%</td>
</tr>
<tr>
<td>Productivity</td>
<td>$a_t$</td>
<td>—</td>
</tr>
<tr>
<td>Foreign-good demand</td>
<td>$\xi_t$</td>
<td>19%</td>
</tr>
<tr>
<td>Financial</td>
<td>$\psi_t$</td>
<td>71%</td>
</tr>
</tbody>
</table>

Note: The table reports the variance decomposition for nominal and real exchange rate changes into the contributions of shocks in the two multi-shock models, corresponding to columns 5 and 6 of Table 1.2. The relative volatilities of the shocks are calibrated to match $\text{corr}(\Delta c - \Delta c^*, \Delta q) = -0.2$ and $\text{corr}(\Delta nx, \Delta q) = 0$, with other parameters as in Table 1.1.
patterns emphasized by e.g. Backus, Kehoe, and Kydland (1992, 1994), as we show in Appendix Table A4, which reports a formal BKK-style calibration of a version of our model with capital accumulation. Furthermore, our model of disconnect does not fundamentally change the way in which these conventional shocks affect exchange rates. Instead, it puts an upper bound on how important these shocks can be in shaping the overall unconditional exchange rate dynamics, which we argue must be largely driven by $\psi_t$-like shocks in financial markets.

**Mussa puzzle** Mussa (1986) puzzle refers to the striking discontinuous difference in the behavior of the real exchange rate when the monetary authority switches between a pegged and a floating nominal exchange rate regimes. Indeed, in conventional models the real and nominal exchange rates are shaped by different forces, at least once prices have adjusted, and therefore a change in the monetary regime should not alter the behavior of the real exchange rate in such a fundamental way. After briefly reviewing a larger set of stylized facts, to which we collectively refer as the Mussa puzzle, we turn to our model of exchange rate disconnect to study whether it reproduces these empirical patterns in response to a switch in the monetary regime.

Comparing empirical moments for several developed countries before and after the end of the Bretton Woods system of fixed exchange rates, the literature has emphasized the following stylized facts:

1. The volatility of the *real* exchange rate increased almost as much as the volatility of the nominal exchange rate (Mussa 1986). More precisely, the volatility of the nominal exchange rate was about 8 times smaller under the peg, while for the real
exchange rate it was about 4 times smaller, and the correlation between the two variables was 0.66 under the peg relative to a nearly perfect correlation under the float (Monacelli 2004).

2. At the same time, there was almost no difference in the output or consumption volatilities across the two periods (Baxter and Stockman 1989, Flood and Rose 1995).

3. The Backus-Smith risk-sharing condition and the UIP condition both held better in the data during the Bretton Woods period. Specifically, the data from the peg period exhibits the theoretically-predicted positive signs of the respective correlations (Colacito and Croce 2013).39

We summarize these facts in the Data column of Table 1.4, which also reports the respective moments in our simulated model. Our goal here is not to simply match the moments under the peg, but rather to check whether our quantitative exchange rate disconnect model can simultaneously account for the broad patterns of the Mussa puzzle. To this end, we adopt the extended multi-shock NOEM model from column 5 of Table 1.2, and study two alternative nominal peg scenarios, keeping all other parameters unchanged.

By definition, a nominal peg regime requires a change from the Taylor rule (1.43) to a monetary policy rule that directly stabilizes the nominal exchange rate. We focus on the following policy rule:

\[ i_t = \rho_m i_{t-1} + (1 - \rho_m) \delta e (e_t - \bar{e}) + \varepsilon^m, \tag{1.44} \]

39Similarly, Devereux and Hnatkovska (2014) document that Backus-Smith condition holds better across regions within countries, in contrast with its cross-country violations. Another pattern emphasized by Berka, Devereux, and Engel (2012) is a substantially greater role of the non-tradable (Balassa-Samuelson) component in the RER variation under a nominal peg.
Table 1.4: Mussa puzzle: moments under nominal peg relative to float

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model (1)</th>
<th>Model (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{std}(\Delta e_t))</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>(\text{std}(\Delta q_t))</td>
<td>0.26</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td>(\text{corr}(\Delta q_t, \Delta e_t))</td>
<td>0.66</td>
<td>0.79</td>
<td>0.84</td>
</tr>
<tr>
<td>(\text{std}(\Delta c_t - \Delta c^*_t))</td>
<td>\approx 1</td>
<td>2.63</td>
<td>1.33</td>
</tr>
<tr>
<td>(\text{corr}(\Delta c_t - \Delta c^*_t, \Delta q_t))</td>
<td>&gt;0</td>
<td>-0.63</td>
<td>0.13</td>
</tr>
<tr>
<td>Fama (\beta)</td>
<td>&gt;0</td>
<td>-0.1</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Note: The first four lines report moments under a peg relative to their values under a float, e.g. 0.13 for \(\text{std}(\Delta e_t)\) means that nominal exchange rate is about 8 \((\approx 0.13^{-1})\) times less volatile under the peg. In contrast, the last two lines simply report the moments for the peg regime. The model in both columns corresponds to the multi-shock NOEM model from column 5 of Table 1.2, with the Taylor rule changed from (1.43) to a peg (1.44); column 2 additionally shuts down the \(\psi_t\) shock under the peg.

where \(\bar{e}\) is the level of the peg and \(\delta_e\) is the strength of the peg. We adopt this policy rule in both scenarios, calibrating \(\delta_e\) in each case to exactly match the empirical volatility of the nominal exchange rate during the peg regimes (the first moment in Table 1.4).

Another important difference under the peg is a substantially lower volatility in the UIP deviations (as, for example, documented in Kollmann 2005a), which corresponds to a lower variance of the financial shock \(\psi_t\). Jeanne and Rose (2002) suggest a theoretical limits-to-arbitrage mechanism, in which the variance of \(\psi_t\) endogenously declines when the monetary authority commits to a peg, reducing the risk associated with carry trades and making arbitrageurs less averse to large currency positions, as we discuss in Section 1.4.2.\(^{40}\)

To capture this, we consider two extreme scenarios: in the first scenario in column 1 we keep the variance of all shocks \((\psi_t, \xi_t, \varepsilon^m_t)\) unchanged, while in the second scenario in column 2 we fully shut down the dispersion of \(\psi_t\), leaving the other two shocks unchanged.

\(^{40}\)Other structural interpretations of the \(\psi_t\) shock may suggest further patterns of comovement between \(\psi_t\) and the primitive shocks of the model.
From Table 1.4, we see that under both scenarios, the model captures a sharp reduction in the volatility of the real exchange rate under a nominal peg (in fact, overstating it), as well as a sizable reduction in the correlation between the nominal and the real exchange rates from a nearly perfect correlation under the float. The model under the first scenario (i.e., no reduction in $\psi_t$ shocks) fails, however, on the remaining moments, predicting an increase in the volatility of consumption and no reversal in the signs of the Backus-Smith correlation and the Fama coefficient. In turn, the second scenario, with the financial shock shut down, makes the model consistent with these three moments: in this case, the volatility of consumption changes little across the regime switch and the signs of the Backus-Smith correlation and the Fama coefficient turn to positive under the peg.

We conclude that our model of exchange rate disconnect captures, at least qualitatively, the additional set of empirical patterns associated with a switch to a nominal peg regime, which we collectively labeled as the Mussa puzzle. This, however, requires that a pegged nominal exchange rate endogenously reduces the volatility of the UIP deviation shocks. We now turn to an explicit model of the financial sector, which in particular sheds light on this mechanism.

### 1.4.2 A model with a financial sector

In this section we study an extension of the baseline model from Section 1.3 with an explicit financial sector, which microfounds the upward-sloping supply in the financial market and the financial shock $\psi_t$. The model embeds in general equilibrium the noise trader and limits-to-arbitrage model of De Long, Shleifer, Summers, and Waldmann (1990)
and its adaptation to the exchange rate market by Jeanne and Rose (2002). This extension achieves the following goals. First, it allows us to analyze a symmetric world economy, in which each country offers a bond in its own currency, and all international transactions are intermediated by a financial sector, which is averse to large risky positions. This leads to an upward-sloping supply of international bonds, and as a result the dispersion of the UIP deviation shock and the volatility of the nominal exchange rate are determined endogenously in equilibrium. Second, unlike our baseline, the model with a financial sector is stationary, featuring a unique long-run equilibrium and long-run mean reversion in the real exchange rate. Nonetheless, we show that the small-sample quantitative properties of the two models are nearly indistinguishable. Lastly, in the context of this extension we address an additional set of facts on the comovement between interest rates and exchange rates, emphasized recently by Engel (2016) and Valchev (2016).

We start by briefly describing the extended model, relegating the details to Appendix A.1.9, and then proceed to study its qualitative and quantitative properties. We consider an international financial market with three types of agents trading assets. First, there are home and foreign households that trade their local-currency bond only, taking net foreign positions \( B_{t+1} \) and \( B_{t+1}^* \) respectively. \(^{41}\) Second, there are \( n \) noise traders that take a zero-capital position long \( N_{t+1}^* \) in foreign-currency bond and short \( N_{t+1} = -N_{t+1}^* \mathcal{E}_t \) in home-currency bond, and vice versa when \( N_{t+1}^* < 0 \). We assume

\[
N_{t+1}^* = n \left( e^{\psi t} - 1 \right),
\]

\(^{41}\)For convenience, we now have \( B_{t+1} \) denote the value of home-currency bonds purchased at \( t \) and paying out \( R_t B_{t+1} \) units of home currency at \( t + 1 \). Furthermore, \( B_{t+1}^* \) now refers to the position of the foreign households (as home household no longer hold foreign-currency bonds), and it pays \( R_t^* B_{t+1}^* \) units of foreign currency at \( t + 1 \).
where $\psi_t$ is the noise-trader demand shock for foreign currency, which follows an exogenous AR(1) process (1.23) with zero mean.

Third, intermediation in the financial market is done by a measure $m$ of competitive arbitrageurs that collectively take a zero-capital position $D^*_t+1$ in foreign-currency bond and short $D^*_t+1 = -D^*_t+1 \mathcal{E}_t$ home-currency bonds, again allowing for $D^*_t+1 < 0$. We denote the return on a one-dollar $D^*_t+1$ position by $\tilde{R}^*_t+1 \equiv R^*_t - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$. Each arbitrageur chooses his individual position $d^*_t+1$ to maximize a mean-variance utility of returns, $\mathbb{E}_t \tilde{R}^*_t+1 \cdot d^*_t+1 - \frac{\omega}{2} \text{var}_t(\tilde{R}^*_t+1) \cdot d^*_t+1$, where $\omega$ is the risk aversion parameter. The resulting demand for foreign currency bonds by the financial intermediary sector is then:

$$D^*_t+1 = m \frac{\mathbb{E}_t \tilde{R}^*_t+1}{\omega \text{var}_t(\tilde{R}^*_t+1)}.$$ (1.46)

The financial market clears when the interest rates $R_t$ and $R^*_t$ are such that $B^*_t+1 + N^*_t+1 + D^*_t+1 = 0$ and $B^*_t+1 + N^*_t+1 + D^*_t+1 = 0$, which in particular implies that in equilibrium net foreign asset position of home equals net foreign liabilities of foreign, $B^*_t+1 = -B^*_t+1 \mathcal{E}_t$. Lastly, we assume for concreteness that the profits and losses of the arbitrageurs and noise traders are transferred to the foreign households, and thus their budget constraint becomes:

$$B^*_t+1 - R^*_t-1 B^*_t = N X^*_t + \tilde{R}^*_t (D^*_t + N^*_t),$$

while the budget constraint of the home is $B^*_t+1 - R^*_t B^*_t = N X_t$, where $N X_t = -N X^*_t \mathcal{E}_t$. The rest of the model is unchanged. We again solve the model by log-linearization.
around a non-stochastic equilibrium with \( B = B^* = N^* = D^* = \tilde{R}^* = 0 \) and \( \mathcal{E} = 1 \).

In light of this approximate solution approach, the strong assumptions made above — namely, the quadratic utility over dollar returns and the transfer of profits to foreign households — are without loss of generality.\(^{42}\)

The only equation in the linearized equilibrium system that changes in this extension with a financial sector is the UIP condition (1.22), which now becomes (see Appendix A.1.9):

\[
i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1} \quad \text{with} \quad \chi_1 \equiv \frac{n/\beta}{m/(\omega \sigma^2_e)} \quad \text{and} \quad \chi_2 \equiv \frac{\bar{Y}}{m/(\omega \sigma^2_e)},
\]

(1.47)

where \( \sigma^2_e \equiv \text{var}_t(\Delta e_{t+1}) \) is the variance of the innovation to the nominal exchange rate, which is determined endogenously in equilibrium, yet taken as given by the competitive financial sector. The generalized UIP condition (1.47) derives from the international bond market clearing condition combined with (1.45) and (1.46). It characterizes the excess return on the home bond, which ensures that the arbitrageurs are willing to satisfy the relative demand for foreign bonds by both noise traders (\( \psi_t \)) and households (\( b_{t+1} \)). The net foreign asset position of the home households, \( b_{t+1} \equiv R B_{t+1}/\bar{Y} \), reflects the demand for home-currency bond as a savings vehicle, and hence increases the price of the home bond, or equivalently reduces its relative interest rate.

The crucial difference of the new UIP condition (1.47) from (1.22) is that now it features two sources of UIP deviations — an exogenous shock \( \psi_t \) as before and an endogenous feedback via the state variable \( b_{t+1} \). The elasticities \( \chi_1 \) and \( \chi_2 \) of respectively exogenous

\(^{42}\)Note, in particular, that the profits and losses of the noise traders and arbitrageurs, \( \tilde{R}_t^* (D_t^* + N_t^*) \), is a second order term.
and endogenous sources of UIP deviations decrease in the absorption (risk-bearing) capacity of the financial sector \( m/(\omega \sigma_e^2) \), which in turn depends on the size of this sector \( m \), its risk aversion \( \omega \) and the endogenous volatility of the nominal exchange rate \( \sigma_e^2 \) (cf. Gabaix and Maggiori 2015). As the financial sector becomes larger or less risk averse, with \( \frac{\chi}{m/\omega} \sim \frac{n}{m/\omega} \to 0 \), the model features an undistorted UIP condition in the limit (with \( \chi_1 = \chi_2 = 0 \)). If, instead, \( \chi_1 \propto \frac{n}{m/\omega} \) remains positive in the limit, the model admits the baseline UIP condition (1.22) as a special case.

The other equilibrium relationships of the extended model, apart from the UIP condition (1.47), remain unchanged. This includes the proportional relationship between the nominal and the real exchange rates (1.32), as well as the relationships between exchange rates and respectively terms of trade (1.35), consumption (1.38) and interest rates (1.41). As a result, Propositions 1.5–1.7 still hold in the model with a financial sector. The equilibrium dynamics of the exchange rate (Proposition 1.3) differs, however, as we now characterize:

**Proposition 1.9 (Exchange rate process redux)** In the model with a financial sector and a noise-trader shock \( \psi_t \sim AR(1) \) with persistence \( \rho \) and innovation \( \varepsilon_t \), the equilibrium nominal exchange rate \( e_t \) follows an ARMA(2,1) process with AR roots \( \rho \) and \( \zeta_1 < 1 \) (with \( \zeta_1 \to 1 \) iff \( \chi_2 \to 0 \)) and an MA root \( 1/\beta \):

\[
(1 - \rho L)(1 - \zeta_1 L)e_t = \frac{1}{1 + \gamma \lambda_1} \frac{\beta \zeta_1 \chi_1}{1 - \beta \zeta_1 \rho} (1 - \beta^{-1} L)\varepsilon_t. \tag{1.48}
\]

Provided that \( \frac{n \sigma_e}{m/\omega} \) is large enough, there exists a solution with \( \chi_1 > 0 \) and \( \sigma_e^2 = \text{var}_t(\Delta e_{t+1}) > 0 \), such that \( \sigma_e^2 \) increases in \( \frac{n \sigma_e}{m/\omega} \). There always exists another solution with \( \sigma_e^2 = \chi_1 = 0 \).
A formal proof of this proposition is contained in Appendix A.1.9, which also defines the cutoff value for \( \frac{n\sigma_e}{m/\omega} \) and shows that it tends to zero as \( \beta \rho \to 1 \). In the absence of other shocks, there always exists a zero-variance equilibrium, in which the arbitrageurs coordinate to fully offset the noise-trader shock, as this involves no risk as a matter of a self-fulfilling prophecy. However, for a large enough noise-trader shock \( n\sigma_e \), there also exists a positive-variance equilibrium, in which noise-trader shocks result in a volatile exchange rate. Furthermore, any fundamental shock to current account (e.g., productivity shock) shifts the economy to a positive-variance equilibrium. The government, however, has the ability to commit to peg the exchange rate, providing a coordination device for the financial sector to fully absorb the noise-trader shocks. This justifies why in Section 1.4.1 we considered the case in which a monetary peg was associated with a reduction in the variance of the UIP deviation shock.

The equilibrium exchange rate process (1.48) is stationary, unlike (1.28) in the baseline model. However, the two become indistinguishable as \( \chi_2 \to 0 \) and consequently \( \zeta_1 \to 1 \). We discuss below that quantitatively, under our calibration with \( \chi_2 > 0 \), the two processes are nearly identical in finite samples, even though one is stationary and one is integrated in the long run. The model with a financial sector is stationary around a unique steady state with \( B = B^* = \tilde{R}^* = 0 \). The stationarity of the model emerges from the fact that the financial intermediaries are averse to holding positions that expose them to exchange rate risk and require a premium. As a result, a country-borrower faces a higher interest rate relative to \( 1/\beta \), which provides an incentive to gradually close its NFA

\[ \text{We show in the appendix that the NFA position } b_{t+1} \text{ follows a stationary AR}(2) \text{ process with the same roots } \rho \text{ and } \zeta_1. \]
position. This mechanism offers a microfoundation for the state-dependent borrowing rate often adopted to close small open economy models (see Schmitt-Grohé and Uribe 2003).

Quantitative properties  We now turn to a multi-shock version of the model with a financial sector to study its quantitative properties. As in the earlier IRBC model (in column 6 of Table 1.2), we add a productivity shock $a_t$ and a foreign-good-demand shock $\xi_t$, however, for simplicity instead of a Taylor rule we maintain the assumption that monetary policy stabilizes the nominal value of the wage rate, as in the baseline model. We keep all parameters unchanged as in Table 1.1, and only need to calibrate the new parameter $\chi_2$ and the relative volatilities of the three shocks. As before, we choose the relative volatilities to match the consumption and net export correlation with the real exchange rate.\textsuperscript{44} We set $\chi_2 = 0.01$ to match the impulse response of the UIP deviations in Figure 1.3a, as we discuss below.

We report the model-generated moments resulting from this calibration in the last column of Table 1.2. This column shows that the model with a financial sector is as successful at matching the empirical moments as the other multi-shock models (NOEM and IRBC) discussed above, yet as we discuss next it matches an additional set of moments. In particular, the model still reproduces a near-random-walk exchange rate process, with nominal and real exchange rates nearly perfectly correlated, while consumption remains about six times less volatile than the real exchange rate and weakly negatively correlated with it.\textsuperscript{45}

\textsuperscript{44}The exogenous shock to the UIP is now $\chi_1 \psi_t$ with the coefficient $\chi_1$ not separately identified from the volatility of $\psi_t$. Thus, to match the correlation moments, we now set $\sigma_a/(\chi_1 \sigma_{\epsilon}) = 4$ and $\gamma \sigma_{\epsilon}/(\chi_1 \sigma_{\epsilon}) = 2.6$ (cf. footnote 38).

\textsuperscript{45}The only noticeable difference in the fit of this model from the multi-shock IRBC model is the lower volatility of the real exchange rate, which is due to the difference in the monetary policy rule.
Figure 1.2: The response of exchange rate $e_{t+j}$ to innovation in $i_t - i^*_{t}$

Note: The figure plots two alternative shapes of the exchange rate impulse response to an innovation in the interest rate differential. The construction of the impulse response is explained in Appendix A.1.9. The blue solid line is from the single-$\psi_t$-shock model and the dashed red line is from the multi-shock model, both featuring the financial sector and calibrated as discussed in Section 1.4.2.

The similarity of the fit is, perhaps, not surprising, as the only difference of the model with a financial sector sector is the presence of the endogenous feedback $-\chi_2 b_{t+1}$ in the UIP condition (1.47). Despite the implied qualitative difference resulting in stationarity of the exchange rate, the finite sample properties of the model are almost unchanged. One difference that emerges in the model with a financial sector is that non-financial shocks such as $a_t$ and $\xi_t$ now result in endogenous UIP violations through their effect on the state variable $b_{t+1}$, and hence contribute more to the exchange rate volatility. In particular, our calibration of the model with a financial sector attributes a larger role to the good-market shock $\xi_t$ in comparison with the multi-shock IRBC model.

**Engel puzzle** We now turn to the Engel (2016) puzzle, which concerns the shape of the exchange rate response to the movements in the interest rate differential. The Fama (1984) regression suggests that a positive interest rate differential predicts an exchange
rate appreciation, that is \( \text{cov}(i_t - i^*_t, \Delta e_{t+1}) < 0 \). Engel (2016) argues that most models of the forward premium achieve this with an exchange rate process that depreciates on impact at \( t \) and then gradually appreciates starting at \( t + 1 \). This is indeed the case in our model with a single financial shock \( \psi_t \) following an AR(1) process, as we depict with a solid blue line in Figure 1.2. Engel (2016), however, shows that in the data the exchange rate response appears to be different, with an appreciation on impact at \( t \) followed by further appreciation over some period and an eventual reversal into depreciation (see also Valchev 2016).\(^{46}\) We illustrate this alternative impulse response with a dashed red line in Figure 1.2.

We show here that the calibrated multi-shock version of the model with a financial sector matches the empirical impulse responses of the risk premium in Engel (2016) and of the exchange rate in Valchev (2016). The results are reported in Figure 1.3, in which solid blue lines correspond to the empirical impulse responses and the dashed red lines plot the corresponding responses calculated using simulated data from the model. The calibration of the model is the same that yields moments for column 7 of Table 1.2. In our choice of \( \chi_2 = 0.01 \), which parameterizes the endogenous feedback elasticity in the UIP condition (1.47), we targeted the duration till risk premium becomes negative in the impulse response in Figure 1.3a. Recall that all other parameters were kept unchanged from our calibration of the baseline model in Section 1.3.

Our calibrated model captures well that an increase in the interest rate differential

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\(^{46}\)The main empirical results in Engel (2016) are reported in terms of the impulse response of the UIP deviations, as we reproduce in Figure 1.3a, yet his paper also shows that \( \text{cov}(i_t - i^*_t, e_t) < 0 \) and \( \text{cov}(\Delta i_t - \Delta i^*_t, \Delta e_t) < 0 \), consistent with the shape of the dashed red impulse response in Figure 1.2, but not with the solid blue one. We report these correlations in the data and for different version of the model in Appendix Table A2.
today predicts an increase in the risk premium $\mathbb{E}_t \rho_{t+j}$ on impact, where $\rho_{t+j} \equiv i_{t+j-1} - i^*_t - \Delta e_{t+j}$, which then gradually decreases and turns negative 20 months out (Figure 1.3a). Similarly, it captures an exchange rate appreciation on impact, followed by further appreciation over the next 20 months, which then reverts into an expected depre-
ciation (Figure 1.3b). Indeed, these impulse responses capture the subtle departures of the exchange rate process from a random walk, which are present both in the data, as well as in our model.

The mechanism by which our model reproduces these empirical patterns relies crucially on the endogenous state variable \( b_{t+1} \) in the UIP condition (1.47) in the presence of multiple shocks. In particular, the short run dynamics of the UIP deviations is dominated by the noise-trader shock \( \psi_t \). The other shocks do not have a direct impact on the UIP deviations, but shape them indirectly through their effect on the net foreign assets, which build up gradually over time, resulting in a non-monotonic impulse response in Figure 1.2. Non-monotonicity is not guaranteed in general, but does emerge from the combinations of \( \psi_t \) and \( \xi_t \) shocks, as they move risk premium in opposite directions for a given direction of the interest rate change. We, thus, find that the same mix of shocks that results in the broad exchange rate disconnect properties summarized in Table 1.2 also reproduces the dynamic comovement between interest rate differentials and exchange rate changes documented by Engel (2016) and Valchev (2016).

1.5 Conclusion

We propose a parsimonious general equilibrium model of exchange rate determination, which is simultaneously consistent with a wide array of exchange-rate-related moments, offering a unifying resolution to the main exchange rate puzzles in international macroeconomics to which we collectively refer as the exchange rate disconnect. The model is

[47] Here the model is somewhat off on the timing of the reversal, yet the uncertainty bounds around the exact shapes of these impulse responses are wide enough that the two are not statistically different.
analytically tractable, allowing for a complete closed-form characterization, essential for a transparent exploration of the underlying mechanisms. Beyond reproducing the qualitative patterns in the data, the model also matches quantitatively a rich set of moments describing the comovement between exchange rates and macro variables.

With this general equilibrium model, one can reconsider the conclusions of the vast international macro literature plagued by the exchange rate puzzles. In particular, the model reproduces exchange rate disconnect without altering the international transmission mechanism for monetary and productivity shocks, including international spillovers from monetary policy (see e.g. Corsetti, Dedola, and Leduc 2010). This is the case because, conditional on a shock, the model relies on a transmission mechanism with conventional properties. What we emphasize instead is that conventional productivity and monetary shocks cannot be the main drivers of the unconditional behavior of exchange rates.\footnote{Therefore, while we emphasize the same empirical patterns as Alvarez, Atkeson, and Kehoe (2007), our conclusions are not as far reaching as theirs (summarized in their title), suggesting in particular that the unconditional disconnect behavior of the nominal exchange rate may be consistent with the conventional transmission of monetary shocks.}

In contrast, the model likely invalidates the conventional normative analysis in open economies, and in particular the studies of the optimal exchange rate regimes and capital controls. The model emphasizes the role of the shocks in the financial markets, as opposed to monetary and goods market shocks, in accounting for the bulk of the unconditional exchange rate variation. Therefore, a normative analysis must allow for financial shocks, which introduce new tradeoffs for alternative policy options. For example, an exchange rate peg may simultaneously reduce monetary policy flexibility, yet improve international risk-sharing by offsetting the noise-trader risk (cf. Devereux and Engel 2003a). Furthermore, a microfoundation for the financial shock is essential, as it may endogenously in-
teract with the policy.

In addition, our framework can be used as a theoretical foundation for the vast empirical literature, which relies on exchange rate variation for identification (see e.g. Burstein and Gopinath 2012). Similarly, it can serve as a point of departure for the equilibrium analysis of the international price system (see Gopinath 2016, and Chapters 2 and 3 below) and the global financial cycle (Rey 2013). Our model also offers a simple general equilibrium framework for nesting the financial sector in an open economy environment. In particular, it summarizes the macroeconomic relationships in a few simple log-linear equations, which can be combined with richer models of the financial sector (as e.g. Evans and Lyons 2002). We see this as a particularly promising next step in exploring further the nature of the financial shocks and disciplining them with additional moments on the comovement between exchange rates and financial variables.
Chapter 2

International Price System

2.1 Introduction

One of the most striking facts about the “International Price System” is that the overwhelming share of international trade is priced in dollars (Gopinath 2016, Goldberg and Tille 2008). This share is close to one for developing countries, but is also above 50% for such developed countries as Japan, South Korea and France (see Figure 2.1).\(^1\) Since prices remain sticky in the currency, in which they are set, at the horizon of about one year, the dollar currency pricing (DCP) can have important implications for the transmission of shocks across countries and the optimal monetary policy. In particular, a depreciation of dollar exchange rate affects not only the relative prices between the U.S. and the rest of the world, but also the prices between third countries if they are set in dollars. This raises several questions: why do firms choose DCP in the first place rather than using the currency of producer or buyer? Under which conditions can dollar loose its global status?

\(^1\)This fact holds even if one excludes commodities and considers only manufactured goods.
Will another dominant currency replace it in the future or we are moving to the world with multiple regional currencies? The existing models, however, provide little guidance as they assume that exporters use either producer currency pricing (PCP) or local currency pricing (LCP), and usually take the currency choice as exogenous.

This chapter develops a tractable general equilibrium framework with endogenous currency choice that is consistent with the key stylized facts about the use of currencies in the international trade. To this end, I augment a conventional New-Keynesian open-economy model a la Gali and Monacelli (2005) with two additional ingredients. First, rather than taken as exogenous, the currency of invoicing is optimally chosen by the individual exporters to minimize the deviation of the preset price from the optimal level (Engel 2006). The currency choice is therefore determined by price stickiness — the same friction that makes this choice consequential for the aggregate economy in the first place — and is well-defined despite the complete asset markets and the zero transaction costs, because it allows the firm to increase average profits rather than hedge against risk. Second, I add input-output linkages and complementarities in price setting. These price linkages are strong in the data, especially for the large firms that account for most of the international trade (see e.g. Amiti, Itskhioki, and Konings 2014, De Loecker, Goldberg, Khandelwal, and Pavcnik 2016). They have also been used to explain several puzzles in international economics (Itskhoki and Mukhin 2017, Casas, Díez, Gopinath, and Gourinchas 2017, Atkeson and Burstein 2008b, Rodnyansky 2017), and as I show, are crucial to understand firms’ currency choice.

2For example, if an optimal price of $100 holds for the exporter regardless of shocks, invoicing is best done in dollars. Meanwhile, setting the price in euros makes the ex-post price deviate from the optimal level and causes the average profits of the firm to drop.
By combining endogenous currency choice with price linkages across firms, I show that, depending on the parameter values, the model can sustain equilibria with producer, local or vehicle currency pricing. In the limiting case when marginal costs are stable...
and the markups are constant, the firms prefer to set prices in producer currency, which validates the standard assumption of PCP in most of the open economy models (see e.g. Obstfeld and Rogoff 1995, Clarida, Gali, and Gertler 2001, Gali and Monacelli 2005). Despite its prevalence in the theoretical literature, this knife-edge case with no links across firms provides a poor approximation to the data. Allowing for realistic complementarities in price setting, on the other hand, means exporters might choose LCP in order to align their prices with the prices of local competitors. Furthermore, allowing for multiple countries means that the exporters must deal with competitors and suppliers coming from different economies; thus, using a vehicle currency can be an optimal way to synchronize prices across firms (cf. Devereux and Engel 2001, Bacchetta and van Wincoop 2005, Bhattarai 2009).

The use of the dollar as a vehicle currency is driven by three factors in the model: the large share of dollarized economies in global trade, the relatively low volatility of U.S. exchange rate, and the path-dependence in currency choice. Intuitively, the large size of the U.S. market implies that foreign suppliers prefer to use dollars to align their prices with the local competitors. The U.S. exporters then find prices of their intermediate inputs stable in dollars and are more likely to use dollar currency pricing in other markets. This increases the share of dollar-denominated inputs and competing products for the non-U.S. exporters, who then become more inclined to use DCP as well. In addition, other currencies are less suitable for synchronizing prices across exporters because of the relatively high volatility of exchange rates in the respective countries. As more firms switch to the dollar, the incentives for other exporters to use DCP both in the U.S. and other countries become even greater. While the endogenous complementarities in currency choice can
potentially generate multiple equilibria for some values of the fundamentals, they also imply a possible inertia in currency choice. This explains the late transition from the pound to the dollar in the first half of the twentieth century and the dominant status of the dollar since then: the initial vehicle currency can retain its international position despite losing its advantage in terms of economic stability and size.³

The last section of the paper discusses some additional mechanisms that can amplify the private benefits of dollar invoicing. I show that a volatile monetary policy makes the prices of inputs and competing products less stable in producer and local currencies respectively, and further encourages the exporters to set their prices in dollars. I then allow domestic firms to choose optimally the currency of invoicing and show that, while they are less likely to set prices in foreign currency than the exporters, an equilibrium with all firms using DCP can emerge in response to large fundamental shocks, e.g. volatile monetary policy. The complementarities in currency choice imply that the dollarization of emerging economies persists even after inflation is stabilized and contributes to the widespread use of the U.S. dollar in international trade.

There are three main strands in the literature that use different types of frictions to explain the dominant status of the dollar in international trade. First, there is a long tradition in economics, going back at least as far as Krugman (1980), that emphasizes the transaction costs in exchange markets: coordination on a single currency raises the chances of a “double coincidence of wants” (Matsuyama, Kiyotaki, and Matsui 1993) and increases the “thickness” of markets (Rey 2001, Devereux and Shi 2013, Chahrour and

³Following the previous literature (e.g. Matsuyama, Kiyotaki, and Matsui 1993), I focus on the evolution of steady states and abstract from the dynamics between them.
Valchev 2017). These theories, therefore, explain the widespread use of the dollar as a medium of exchange but have little to say about its role as an invoicing currency. Second, the use of the dollar as a unit of account can be due to financial frictions, as the firms try to synchronize the risks on their contracts (Doepke and Schneider 2013) and borrow in a cheaper currency (Gopinath and Stein 2017). While this is a promising direction for future research, there is so far little empirical evidence that financial frictions are significant for the large firms that account for most of the international trade.

This paper belongs to the third strand in the literature, the one that emphasizes the role of nominal frictions (see e.g. Devereux and Engel 2001, Bacchetta and van Wincoop 2005, Bhattarai 2009, Cravino 2014, Goldberg and Tille 2008, Drenik and Perez 2017) and has two advantages over the alternatives discussed in the previous paragraph. First, there exists direct empirical evidence in favor of this mechanism that allows to discriminate it against alternative theories (see Gopinath, Itskhoki, and Rigobon 2010). Second, sticky prices lie at the heart of the New-Keynesian open economy models. It is, arguably, preferable to have a theory where the currency choice is determined by the same type of friction that makes it relevant at the aggregate level.4

4That said, all three types of frictions are likely to be important in practice. It is therefore reassuring that these models broadly agree on the set of fundamentals that determine the firms’ currency choice and imply similar comparative statics.
2.2 Baseline Model

2.2.1 Environment

I start with a simple framework that relies on conventional assumptions in the international macro literature and attains closed-form characterization. Since more than two countries are required for a vehicle currency to be well-defined, I assume there is a continuum of symmetric regions \( i \in [0, 1] \) as in Gali and Monacelli (2005). There is potentially one large economy (the U.S.) that includes regions \( i \in [0, n], n < 1 \), and can also be interpreted as a currency union or a set of dollarized countries. The other regions \( i \in (n, 1] \) are small open economies, each with its own nominal unit of account, in which local wages \( W_{it} \) and prices are expressed. Denote the bilateral nominal exchange rate between regions \( i \) and \( j \) with \( E_{ijt} \), which goes up when currency \( i \) devalues relative to currency \( j \).

In each country, there is a representative household, a local government and a continuum of firms producing different varieties of tradable and non-tradable goods. The tradable sector is characterized by intermediate goods in production, strategic complementarities in price setting and the home bias towards domestically produced goods. The prices are set before the realization of shocks and stay rigid for one period with a given probability. While the structure of the tradable sector is crucial, the other details of the model are less important. I make specific assumptions about preferences, the structure of asset markets and monetary policy to simplify exposition, and discuss below how they can be relaxed. The set of exogenous shocks includes changes in productivity, money supply, government spendings, preferences for imported goods and shocks in financial markets.
Households  A representative household in region $i$ chooses consumption bundle $C_{it}$, supplies labor $L_{it}$, invests in local risk-free nominal bond $B_{it+1}$ and in complete set of internationally traded Arrow securities $D_{it+1}^s$ to maximize expected utility\(^5\)

$$
\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left( \log C_{it} - L_{it} \right)
$$

subject to a sequence of budget constraints:

$$
P_{it} C_{it} + \left( \frac{B_{it+1}}{R_{it}} - B_{it} \right) + \psi_{it} \epsilon_{i0t} \left( \sum_{s \in S_{t+1}} Q_t^s D_{it+1}^s - D_{it} \right) = W_{it} L_{it} + \Pi_{it} - T_{it} + \Omega_{it}, \quad (2.2)
$$

where $P_{it}^C$ is the price index of consumption bundle in country $i$, $\Pi_{it}$ are profits of local firms, $T_{it}$ is the lump-sum tax, $R_{it}$ is the nominal interest rate and $Q_t^s$ is the price of Arrow security that pays one dollar in state $s \in S_{t+1}$ in the next period. I suppress state index $s$ in other variables to simplify the notation. Both prices and returns on the Arrow securities are in dollars — which is without loss of generality because of complete markets — and are converted into local currency with the nominal exchange rate $\epsilon_{i0t}$. I allow for cross-country wedge in asset prices and returns $\psi_{it}$, which can be interpreted as a shock in the local financial markets and might be an important source of exchange rate volatility.\(^6\)

The resulting profits (or losses) of the financial sector $\Omega_{it}$ are reimbursed lump-sum to local households.\(^7\) The assumption of complete asset markets is to simplify exposition

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\(^5\)This functional form has widely been used in macroeconomic literature in a context of both closed and open economy (see e.g. Ball and Romer 1990, Golosov and Lucas 2007, Kehoe and Midrigan 2007) and arises naturally when labor is indivisible (Rogerson 1988, Hansen 1985).

\(^6\)See e.g. Itskhoki and Mukhin (2017), Lustig and Verdelhan (2016a), Devereux and Engel (2002b), Kollmann (2005b).

\(^7\)The profits of financial sector are $\Omega_{it} = (\epsilon_{i0t}^\psi - 1) \epsilon_{i0t} \left( \sum_{s \in S_{t+1}} Q_t^s D_{it+1}^s - D_{it} \right)$. 
and the same results can be obtained in case of one internationally traded bond, as shown in Appendix A.2.4.

Consumption bundle consists of tradable and non-tradable goods combined with Cobb-Douglas aggregator:

\[ C_{it} = \left( \frac{C_{Nit}}{1 - \eta} \right)^{1-\eta} \left( \frac{C_{Tit}}{\eta} \right)^{\eta}. \] \hspace{1cm} (2.3)

**Non-tradable sector** In each country, there is a continuum of monopolistically competitive firms producing different varieties \( \omega \in [0, 1] \) of non-tradable goods using the same production technology:

\[ Y_{Nit}(\omega) = e^{a_{Nit}} L_{Nit}(\omega). \] \hspace{1cm} (2.4)

The individual products are then combined into consumption basket \( C_{Nit} \) with a CES aggregator:

\[ C_{Nit} = \left( \int_0^1 C_{Nit}(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}, \] \hspace{1cm} (2.5)

Firms preset prices in local currency before the realization of shocks and update them afterwards with a probability \( \lambda < 1 \).

** Tradable sector** The tradable sector differs from the non-tradable one in three dimensions. First, production of a continuum of unique tradable products \( \omega \in [0, 1] \) in country \( i \) requires both labor \( L_{Tit} \) and tradable intermediate goods \( X_{it} \):

\[ Y_{it}(\omega) = e^{a_{Tit}} \left( \frac{L_{Tit}(\omega)}{1 - \phi} \right)^{1-\phi} \left( \frac{X_{it}(\omega)}{\phi} \right)^{\phi}, \hspace{1cm} \phi < 1. \] \hspace{1cm} (2.6)
Second, the bundle of tradables used in consumption and production includes both local and foreign varieties, which are combined with a homothetic aggregator:

\[
\Phi \left( \left\{ \frac{C_{jit}(\omega)}{C_{Tit}} \right\}_{j,\omega}, \xi_{it}, \gamma \right) = 1, \tag{2.7}
\]

where \( C_{jit}(\omega) \) denotes consumption of product \( \omega \) from country \( j \) exported to country \( i \), \( \xi_{it} \) is a relative demand shock for foreign versus domestic goods, and the home bias \( 1 - \gamma \) reflects either trade costs or home bias in preferences, \( \gamma \in (0, 1) \). Note that when \( n > 0 \), the home bias is effectively higher for large country: in addition to locally produced goods, a positive fraction of expenditures in \( i \in [0, n] \) is spent on goods produced in other regions of the U.S. The bundle of intermediate goods \( X_{it} \) is defined similarly. I use Kimball (1995) aggregator to specify \( \Phi(\cdot) \) (see (A94) in Appendix A.2.2), which implies that equilibrium prices depend not only on marginal costs of production, but also on prices of competitors. I show this deviation from the CES benchmark is important for firms’ currency choice.

Third, for each country of destination, firms choose the currency of invoicing and preset price in it before the realization of shocks. With a probability \( \lambda \), the price can be updated after the uncertainty is resolved. While any currency can be used for invoicing in the international trade, for legal reasons local firms can set prices only in domestic currency. In Section 2.4.1, I relax this assumption and derive additional results when domestic firms optimally choose the currency of invoicing.

**Government** The regional government collects lump-sum taxes \( T_{it} \) from households to finance expenditures \( G_{it} \equiv e^{g_{it}} \), which for simplicity have the same composition of
products as the consumption bundle. The government runs a balanced budget, which is without loss of generality since Ricardian equivalence holds in the model:

\[ T_{it} = P^C_{it} G_{it}. \] (2.8)

The monetary policy is implemented with the nominal interest rates \( R_{it}. \) To simplify the analysis, I assume in the baseline case that monetary policy rule is such that nominal wages \( W_{it} = e^{w_{it}} \) follow exogenous stochastic process. In particular, the special case of fully stable nominal wages \( w_{it} = 0 \) discussed below approximates closely inflation targeting when non-tradable goods account for most of the consumer basket. I discuss the optimal monetary policy and its interactions with firms’ currency choice in detail in Section 3.3.

**Equilibrium conditions** In equilibrium, labor supply equals total demand of non-tradable and tradable sectors:

\[ L_{it} = \int_0^1 \left( L_{Nit}(\omega) + L_{Tit}(\omega) \right) d\omega. \] (2.9)

Non-tradable goods are sold locally to households and the government:

\[ Y_{Nit}(\omega) = C_{Nit}(\omega) + G_{Nit}(\omega). \] (2.10)

---

8 As is standard in the literature, I focus on the cashless limit and abstract from the potential multiplicity of equilibria.

9 Under log-linear preferences (2.1), this policy coincides with targeting nominal spendings, which is another common assumption in the literature (see e.g. Carvalho and Nechio 2011, Mankiw and Reis 2002).
Similarly, tradable goods are used for final consumption of local households and government, for production in the tradable sector and are exported to other regions:

\[ Y_{it}(\omega) = Y_{iit}(\omega) + \int_0^1 Y_{ijt}(\omega) d\omega, \quad \text{where} \quad Y_{ijt}(\omega) = C_{ijt}(\omega) + G_{ijt}(\omega) + X_{ijt}(\omega) \]  

(2.11)

for all \( i, j, \omega \in [0, 1] \). Finally, the market clearing in local and international asset markets \( s \in S_{t+1} \) implies

\[ B_{it+1} = 0, \quad \int_0^1 D^s_{it+1} d\omega = 0. \]  

(2.12)

**Shocks** I assume that each type of shock can be decomposed into a global component and the country-specific one, e.g., \( g_{it} = \bar{g}_t + \tilde{g}_{it} \) for government spending shock, where \( \tilde{g}_{it} \) are uncorrelated across \( i \). In addition, the volatility of country-specific shocks in the U.S. is potentially lower than in other countries by the factor \( \rho \leq 1 \). This can be rationalized with a better diversification of regional risk in a large economy and weaker granularity forces a la Gabaix (2011), and results in a more stable exchange rate in the U.S. For simplicity, I do not impose any parametric relation between \( n \) and \( \rho \) and treat these parameters as exogenous.

**Definition 2.1** Given shocks \( \{a_{N_{it}}, a_{T_{it}}, w_{it}, \xi_{it}, g_{it}, \psi_{it}\} \), a monopolistically competitive equilibrium is defined as follows: a) households maximize utility over consumption of products, labor supply and asset holdings, b) each firm maximizes expected profits over labor and intermediate inputs, currency of invoicing and prices in each market, taking the decisions of all other firms as given and setting domestic prices in local currency, c) the government collects taxes to satisfy budget constraint (2.8), d) all markets clear according to (2.9)-(2.12).
2.2.2 Firm currency choice

This section describes the currency choice problem of an individual exporter. I derive a sufficient statistics for the optimal invoicing, which depends on both partial equilibrium and general equilibrium variables. The next sections discuss how the latter are determined. To obtain sharp analytical results, I approximate equilibrium conditions around the symmetric steady state (see Appendix A.2.2 for details). I denote log-deviations from the steady-state values with small letters and suppress time subscript for simplicity. The expectations and variances are therefore taken conditional on the information that agents have at the beginning of the period before the realization of shocks.

Let \( \Pi_{ji}(p) \) denote the profit of exporter from \( j \) to \( i \) as a function of price \( p \) expressed in currency of destination.\(^{10,11} \) Define the optimal static price \( \tilde{p}_{ji} \) that maximizes profits in a given state of the world:

\[
\tilde{p}_{ji} = \arg\max_p \Pi_{ji}(p). \tag{2.13}
\]

The firms that can adjust after the realization of shocks set price at \( \tilde{p}_{ji} \). On the other hand, the optimal preset price replicates the average \( \tilde{p}_{ji} \) expressed in currency of invoicing \( k \):

\[
\bar{p}^k_{ji} = \mathbb{E}[\tilde{p}_{ji} + e_{ki}]. \tag{2.14}
\]

The expected value of ex post price \( \bar{p}^k_{ji} + e_{ik} \) is therefore the same for all currencies and the currency of invoicing is not determined. It follows, to solve the currency choice problem,

\(^{10}\)Due to constant returns to scale in production, the marginal costs do not depend on quantity produced and the objective function of a firm is separable across markets. Therefore, exporters choose price and currency of invoicing independently for each destination.

\(^{11}\)I assume that profits are expressed in real discounted units, i.e. \( \Pi_{ji}(\cdot) \) includes stochastic discount factor (SDF). The variation in SDF, however, does not affect the results under the approximation used below.
one needs to use the second-order approximation: while the preset price is chosen to replicate the mean value of the optimal price, the currency choice allows firms to target the second moment of $\tilde{p}_{ji}$ (see Engel 2006, Gopinath, Itskhoki, and Rigobon 2010, Cravino 2014). Note this implies that expected movements in prices and exchange rates are fully absorbed by the preset price and have no effect on the currency choice.

**Lemma 2.1 (Currency choice)** To the second-order approximation, the currency choice problem of exporter is equivalent to choosing the currency $k$, in which the optimal price $\tilde{p}_{ji} + e_{ki}$ is most stable:

\[
\max_{k \in [0,1]} \mathbb{E} \Pi_{ji}(\tilde{p}_{ji}^k + e_{ik}) \Leftrightarrow \min_{k \in [0,1]} \mathbb{E} \left[ \tilde{p}_{ji}^k + e_{ik} - \tilde{p}_{ji} \right]^2 \Leftrightarrow \min_{k \in [0,1]} \mathbb{V} \left[ \tilde{p}_{ji} + e_{ki} \right].
\] (2.15)

As can be seen from the second expression, the optimal currency choice allows firms to mitigate the effect of sticky prices and to bring ex post price $\tilde{p}_{ji}^k + e_{ik}$ closer to the optimal state-dependent value $\tilde{p}_{ji}$. This is achieved by choosing the currency, in which optimal price is most stable. For example, if the desired price is $100 in all states of the world, then setting the price in dollars allows the firm to replicate the flexible-price allocation. Similarly, it is optimal to set price in pound sterling when the optimal price is £100 in all states.

The choice is more nuanced when the optimal price is not fully stable in one currency, e.g. when $\tilde{p}_{ji}$ can be expressed as $50 + £50$. In this case, the firm would ideally...

---

12I use a classical result from portfolio theory established first by Samuelson (1970) and applied recently in a general equilibrium setup by Devereux and Sutherland (2011) to show that the second-order approximation to the profit function and the first-order approximation to all other equilibrium conditions are sufficient to get consistent solution.

13In other words, it is optimal to set price in currency $k$ rather than in currency $h$ if the pass-through of bilateral exchange rate shocks $e_{kh}$ into the desired price $\tilde{p}_{ji} + e_{ki}$ is low: see e.g. Proposition 2 in Gopinath, Itskhoki, and Rigobon (2010).
like to set price in terms of a basket of currencies. As shown in Appendix 2.4.3, under some restrictions on exogenous shocks, firms can perfectly replicate flexible prices when allowed to use currency baskets for invoicing. The predictions of the model, however, are inconsistent with the key stylized facts about the international price system in this case. In particular, the share of dollar cannot exceed the share of the U.S. in global trade when invoicing is continuous. I therefore assume that currency choice is discrete and that individual firms find it suboptimal to use baskets of currencies for invoicing, presumably due to information frictions (see e.g. Sims 2003, Mankiw and Reis 2002). In the spirit of Mankiw (1985), I show in Section 3.2 that small frictions are sufficient to rationalize discrete currency choice and can lead to large aggregate effects.

Notice that the firm’s invoicing problem of choosing a basket of currencies that minimizes (2.15) resembles the classical portfolio problem a la Markowitz (1952). The assumption that currency choice is discrete is an analog of financial frictions that have been used to explain the global status of dollar in asset markets (see e.g. Bruno and Shin 2015, Rey 2015). It is worth emphasizing however, that despite these similarities, invoicing decisions of firms in the model are based on nominal frictions, not financial ones: exporters choose currency of invoicing to bring ex-post prices closer to the optimal level and increase average profits, not to redistribute profits across states to hedge against risk. Abstracting from financial frictions might be a reasonable assumption since most of the international trade is done by large firms, which arguably have a better access to financial markets.

14 Note the currency basket is firm-specific and there is no one-size-fits-all solution like Special Drawing Rights (SDR).
15 This however does not exclude mixed strategies when firms randomize across different currencies.
16 At the same time, the model can be extended to incorporate effects of asset market imperfections on currency choice: e.g. when firms have to borrow in dollars to finance their inputs, the pass-through of dollar shocks into costs and optimal price $\tilde{p}_{ji}$ is high, which makes invoicing in dollars more appealing.
While the previous analysis is based on a one-period version of Calvo (1983) price setting, it also applies to other models of price rigidity. Appendix A.2.6 discusses four alternatives. In particular, I show that the baseline results about currency choice can be derived analytically for the case of multiperiod staggered pricing. Lemma 2.1 remains also valid under Rotemberg pricing with quadratic costs of price adjustment and in a menu cost model with fixed costs of adjustment and idiosyncratic productivity shocks. Finally, I relax the assumption that currency choice is made unilaterally by suppliers and show that the same results can be obtained in a model with bargaining between firms in the spirit of Hart and Moore (2008) (see Appendix A.2.6 for details).

2.2.3 Partial equilibrium

This section derives equilibrium conditions in the tradable sector that determine the optimal prices $\tilde{p}_{ji}$. A constant returns to scale technology ensures that equilibrium prices depend only on the supply side of the economy and can be analyzed separately from the quantities. In contrast to the CES case, Kimball demand implies there are strategic complementarities in price setting across firms, so that the optimal price of an exporter from $j$ to $i$ depends not only on its marginal costs but also on prices of competitors in market $i$:

$$\tilde{p}_{ji} = (1 - \alpha) (mc_j + e_{ij}) + \alpha p_i,$$

(2.16)

where $e_{ij}$ converts marginal costs into the destination currency. Parameter $\alpha$ depends on the curvature of $\Phi(\cdot)$ and is different from demand elasticity. In the limit $\alpha \to 0$, when Kimball aggregator converges to CES, the firms charge a constant markup and cost shocks
are the only source of variation in the desired prices.

The cost minimization problem under constant returns to scale technology implies that marginal costs of production in country \( i \) are a weighted sum of local wages \( w_i \) and prices of intermediates \( p_i \) adjusted for productivity:

\[
mc_i = (1 - \phi)w_i + \phi p_i - a_T i. 
\]  
(2.17)

The first-order approximation to the ideal price index for Kimball aggregator is isomorphic to the CES index:

\[
p_i = (1 - \gamma)p_{ii} + \gamma p^I_i, \quad \text{where} \quad p^I_i = \int_0^1 p_{ji} dj. 
\]  
(2.18)

The aggregate index is therefore the sum of prices of locally produced goods \( p_{ii} \) and imported ones \( p^I_i \) with the weight of the former determined by the home bias \( 1 - \gamma \). Lastly, the bilateral price index depends on prices of adjusting and non-adjusting firms:\(^{17}\)

\[
p_{ji} = \lambda \tilde{p}_{ji} + (1 - \lambda)(\tilde{p}^k_{ji} + e_{ik}). 
\]  
(2.19)

A fraction \( \lambda \) of firms update prices after the realization of shocks and set them at the optimal level \( \tilde{p}_{ji} \). The prices of other firms stay constant in the currency of invoicing \( k \), which means they move one-to-one with exchange rate \( e_{ik} \) in the currency of the customers. The currency choice therefore has the first-order effect on ex-post prices. At the same time,\(^{17}\)

\(^{17}\)To simplify the notation, I assume that all exporters from \( j \) to \( i \) use the same currency of invoicing \( k \). The results in Section 2.3 are however derived for the general case if not noted otherwise.
Lemma 2.1 implies that invoicing decision of an individual firm is determined by optimal price $\tilde{p}_{ji}$, which depends on aggregate price indices $p_i$. Thus, the equilibrium price system can be defined as follows.

**Definition 2.2** Given $\{a_{Ti}, w_i, e_{ij}\}$, the equilibrium international price system consists of price indices $\{p_i\}$ and firms’ currency choice $\{k_{ji}\}$ such that: (a) given invoicing regime, $\{p_i\}$ solve the system (2.16)-(2.19), (b) given prices, $\{k_{ji}\}$ solve problem (2.15).

Out of the three variables that are exogenous to the tradable sector, two — nominal wages and exchange rates — are determined by the general equilibrium forces. The next section therefore solves the general equilibrium block for the second moments of $w_i$ and $e_{ij}$.

### 2.2.4 General equilibrium

Definition 2.2 implies that the only general equilibrium objects that matter for exporters’ currency choice are the second moments of exchange rates, nominal wages, and productivity shocks. This section shows that under the assumptions made in the baseline model, these moments do not depend on the invoicing decisions of firms and therefore, the model attains the block-recursive structure: one can solve for equilibrium currency choice taking the relevant general equilibrium moments as given. Importantly, however, this result does not imply that invoicing decisions of firms have no general equilibrium effects. As Section 3.2 makes clear, the aggregate consumption, output, exports and imports do change with the currency of invoicing even though the equilibrium exchange rates do not.
Lemma 2.2 (Exchange rates) The second moments of equilibrium exchange rates are independent from invoicing decisions of firms.

The result follows from the combination of log-linear utility, complete asset markets and the monetary policy rule that targets nominal wages. While these assumptions are sufficient, they are not necessary for Lemma 2.2 to hold. In particular, Appendix A.2.4 shows the same result can be obtained under arbitrary isoelastic preferences, one internationally traded bond and exogenous interest rate shocks. It also shows that even less stringent assumptions are needed if one restricts the analysis to the equilibria with symmetric invoicing.

Lastly, note that the effect of monetary and productivity shocks on exporters’ currency choice depends on their correlation with nominal exchange rates. Empirically, this correlation is close to zero (Meese and Rogoff 1983b) and therefore, I abstract from monetary and productivity shocks in the benchmark model, i.e. $w_{it} = \alpha_{T_{it}} = 0$. I discuss in detail both shocks in Section 3.3 when analyzing the optimal monetary policy. Section 2.4 provides additional results that emerge in the presence of large monetary shocks, while Section A.2.7 discusses the case of inflation targeting.

2.3 Equilibrium Currency Choice

Throughout the history of modern capitalism, the overwhelming share of global trade has been priced in one currency — first in pound sterling and later in dollars. This section shows that the model is consistent with this observation. In particular, strategic complementarities in currency choice that arise naturally across firms due to input-output and
price-setting linkages, imply that exporters are likely to share the same currency of invoicing. I show next there are two fundamental factors — the volatility of exchange rate and country’s share in global trade — that make some currencies more attractive as vehicle ones. Finally, I combine these two results to analyze transition from one dominant currency to another: as fundamental advantages of pound sterling deteriorate, exporters become more likely to use dollars instead. However, due to strategic complementarities, no firm wants to change the currency of invoicing before other ones do, generating path-dependence in currency choice. This result can account for the delayed transition from pound to dollar in the twentieth century and the wide use of dollar in modern economy despite increasing competition with euro and renminbi.

2.3.1 Why dominant currency?

While it is intuitive that firms might set prices in producer or customer currency, it is not immediately clear why invoicing in a third currency might be optimal. In this section, I show that a vehicle currency equilibrium (VCP) can arise naturally when price linkages across firms from different countries are strong enough. The question which currency is used as vehicle one is discussed in the next section.

According to Lemma 2.1, firms choose the currency of invoicing, in which their optimal price is more stable. The currency choice of individual exporter from \( j \) to \( i \) depends therefore on the properties of its desirable price \( \hat{p}_{ji} \), which is determined by the system of equilibrium conditions in tradable sector (2.16)-(2.19) summarized in Figure 2.2. The optimal price depends on marginal costs and the prices of competitors with the weight
Consider first the conventional case of CES aggregator and no intermediates in production. With no complementarities in price setting under CES demand, the desired price is proportional to marginal costs (see Figure 2.2). The latter depends exclusively on nominal wages, which are by assumption stable in domestic currency. It follows, the optimal price of exporter $\tilde{p}_{ji}$ is constant in producer currency as well and therefore, PCP is always optimal.

---

18 There are three additional parameters that affect currency choice. The frequency of price adjustment $\lambda$ affects the prices of inputs and competing products. The size of the large economy $n$ determines the share of goods in global trade coming from the U.S. The relative volatility of exchange rates $\rho$ affects the probability distribution of $\tilde{p}_{ji}$. 

---
Lemma 2.3 (No price linkages) With no intermediates in production, $\phi = 0$, and CES aggregator, $\alpha = 0$, exporters always choose PCP, and no VCP equilibrium exists.

Thus, the standard assumption of PCP in open economy models with $\phi = \alpha = 0$ and a stabilizing monetary policy (see e.g. Obstfeld and Rogoff 1995, Clarida, Gali, and Gertler 2001, Gali and Monacelli 2005) is internally consistent: the equilibrium would not change if firms were allowed to choose optimally the currency of invoicing. The proposition also implies that price linkages across firms are a necessary condition to rationalize the use of vehicle currencies in global trade.

The next result for autarky limit $\gamma \to 0$ clarifies that it is linkages with firms from the third countries that make vehicle currency more appealing. Notice that the currency choice is well defined for individual exporters of zero mass. Since countries of origin and destination are almost closed, the marginal costs of exporters are stable in producer currency and the prices of competitors are stable in local currency. As a result, depending on the value of $\alpha$, firms choose either PCP or LCP.

Lemma 2.4 (Autarky limit) Near the autarky limit $\gamma \to 0$, exporters choose PCP if $\alpha \leq 0.5$ and LCP if $\alpha \geq 0.5$, and no VCP equilibrium exists.

Figure 2.3a shows equilibria in the autarky limit in the coordinates $\alpha$ and $\lambda$. The equilibrium is unique when tradable sector is almost closed since strategic interactions across exporters disappear as their share in the market converges to zero.\textsuperscript{19} The figure also shows that for any values of other parameters, the existence of PCP (LCP) equilibrium can be

\textsuperscript{19}Here and below I abstract from the knife-edge values of parameters, under which firms are indifferent between two invoicing options.
guaranteed if economies are close to autarky and strategic complementarities in price setting are weak (strong).

On the other hand, when openness of economies $\gamma$ is high, so that significant fraction of suppliers and competitors are coming from the third countries, the optimal price $\tilde{p}_{ji}$ of the exporter is no longer stable in either producer or local currency, and using vehicle currency might be optimal. The prices of inputs and competing products that individual exporter faces in this case depend on invoicing decisions of other firms: e.g. when prices of suppliers and competitors are sticky in dollars, the optimal price of exporter is more stable in dollars as well and DCP is more attractive.\footnote{The empirical evidence suggests that international prices are sticky with the frequency of adjustment of the same order of magnitude as producer and consumer prices (Gopinath and Rigobon 2008).} Interestingly, both input-output and price-setting linkages play important role in generating complementarities in currency choice, there are important differences between the two. A higher share of intermediates in production $\phi$ unambiguously increases the share of foreign suppliers and makes VCP more attractive. In contrast, the effect of complementarities in price setting $\alpha$ on VCP is non-monotonic: the optimal price is more stable in producer currency when $\alpha$ is low and in local currency when $\alpha$ is high. For intermediate values of $\alpha$, however, neither of the two currencies dominates and VCP is more likely.\footnote{The VCP region can however be monotonic in $\alpha$ for some values of parameters.} I summarize comparative static results in the next proposition.\footnote{I use the following definition throughout the paper: the region of equilibrium $Z$ in parameter space is said to be increasing in parameter $x$ if for any $x_2 > x_1$ the set of parameters for which $Z$ exists under $x = x_2$ includes the set for which $Z$ exists under $x = x_1$.}

**Proposition 2.1 (Vehicle currency pricing)** The region of the VCP equilibrium in parameter space is non-empty and is increasing in the openness of economies $\gamma$ and the share of intermediates in production $\phi$, and can be non-monotonic in complementarities in price setting $\alpha$.\footnote{I use the following definition throughout the paper: the region of equilibrium $Z$ in parameter space is said to be increasing in parameter $x$ if for any $x_2 > x_1$ the set of parameters for which $Z$ exists under $x = x_2$ includes the set for which $Z$ exists under $x = x_1$.}
setting $\alpha$.

Interpreting empirical evidence through the lens of the model, one can argue that globalization has contributed to the widespread use of the vehicle currency in the international trade. In particular, the high participation of several Asian countries in global value chains can be interpreted as a rise in $\gamma \phi$, which increases the chances of VCP relative to PCP and LCP. The higher openness $\gamma$ of other countries, including the post-Soviet states, makes the use of vehicle currency in the international trade more appealing as well. Lastly, the model also suggests that the puzzling high share of dollar in imports and exports of such advanced economies as South Korea, Japan and Australia can be due to strategic complementarities in currency choice: with other countries in the region using DCP, it might be optimal for firms in these countries to set prices in dollars as well.

Complementarities in currency choice also imply that multiple equilibria can emerge despite unique currency choice of an individual firm. While the set of potential equilibria
is very rich in a general case, the next proposition shows that uniqueness can be guaranteed when there is only one symmetric equilibrium, in which all exporters choose either PCP, LCP or DCP. Intuitively, the complementarities in currency choice imply that if a given regime is not chosen when all other firms are following it, then it cannot be optimal when only some firms are using it. The complementarities also imply that mixed-strategy equilibria are unstable: for example, if firms are indifferent between DCP and LCP in some market, a small exogenous increase in the share of importers pricing in dollars will make indifferent firms strictly prefer DCP to LCP.\footnote{While complementarities in currency choice cannot be ensured for PCP and LCP in general case, they hold under the values of parameter, under which these equilibria can arise.}

**Definition 2.3** An equilibrium is symmetric if all exporters in the world use either PCP, LCP or the same vehicle currency. The equilibrium is unstable if exogenous perturbation of currency choice of an arbitrarily small fraction of exporters makes a positive mass of other firms to change their invoicing decisions.

**Proposition 2.2** Assume that $n = 0$ and $\rho = 1$. Then

1. at least one symmetric equilibrium always exists,
2. if symmetric equilibrium is unique, then no other equilibria exist,
3. all non-pure-strategy equilibria are unstable.

### 2.3.2 Which currency is dominant?

While the previous section rationalizes the use of a vehicle currency in global trade, it does not tell us which currency plays this role. This section describes two fundamental
advantages that can make dollar pricing more attractive than pricing in any other currency.

To separate fundamental factors from the complementarity motive, I focus on the flexible price limit $\lambda \to 1$, when almost all firms adjust prices ex-post and hence, invoicing decision of a given exporter does not depend on currency choice of other firms and the equilibrium price system is always unique (see Appendix A.2.2 for details). Notice that currency choice is well-defined in the presence of an arbitrary small price stickiness: exporter’s invoicing decision depends only on the states of the world in which price remains unadjusted and has a solution even when the probability of these states converges to zero. This contrasts with the case of fully flexible prices $\lambda = 1$, when currency choice is completely inconsequential and therefore is not determined. I start with the case when no DCP equilibrium exists to outline necessary conditions for dollar invoicing.

**Proposition 2.3 (No-DCP benchmark)** If prices are almost flexible, $\lambda \to 1$, and countries are symmetric, $n = 0$, $\rho = 1$, exporters choose PCP when $\alpha \leq \frac{1}{2-\gamma}$, LCP when $\alpha \geq \frac{1}{2-\gamma}$, and no DCP equilibrium exists.

When countries are symmetric, $n = 0$, the fraction of U.S. products in other markets is trivial relative to domestic ones, and exporters find their marginal costs and competitors’ prices more stable in producer and local currency respectively. Since the dollar exchange rate has also no advantage in terms of second moments, $\rho = 1$, DCP is strictly dominated by PCP and LCP. Figure 2.3b illustrates this result in the coordinates $\alpha$ and $\gamma$. The region of DCP is empty, while the choice between PCP and LCP depends on $\alpha$ and $\gamma$: using local currency is optimal only when complementarities in price setting are strong and the share
of local firms in the destination market is sufficiently high.

I show next that any deviation from the benchmark described in Proposition 2.3 is sufficient to sustain DCP equilibrium for some values of other parameters. To this end, consider two points outside of the admissible range: $\phi = 1$ and $\gamma = \alpha = 1$. In the former case, no labor is used in production and as a result, there is no component in marginal costs of firms that is stable in producer currency. Both marginal costs and prices of competitors are equally stable in any currency in this case, and exporters are indifferent between PCP, LCP, DCP or any other currency of invoicing. Similarly, when $\gamma = \alpha = 1$ the optimal price of firms depends only on price index of other importers in the destination market, which is equally stable in all currencies and makes exporters indifferent between using any currency for invoicing. I argue next any deviation from the conditions of Proposition 2.3 make exporters strictly prefer DCP in the neighborhood of these two points.

**Volatility advantage** Suppose first that countries are symmetric in terms of their size, $n = 0$, but the volatility of dollar exchange rate is lower relative to other currencies because of higher diversification of the U.S. economy and smaller fundamental shocks, i.e. $\rho < 1$. To see the benefits of the DCP in this case, consider two limiting cases from above: when $\phi = 1$ or $\gamma = \alpha = 1$, the prices of suppliers and competitors from all countries have a symmetric effect on the optimal price. Therefore, the exporter would like ideally to set price in terms of fully diversified basket of currencies. This is however not possible because of the discrete nature of the invoicing problem, and firms look for a currency with the lowest idiosyncratic volatility that can replicate most closely this diversified portfolio. If $\rho < 1$, DCP strictly dominates other alternatives.
Away from this limit, there is a trade-off between producer/local currency and dollar: the prices of domestic inputs and local competitors are more stable in the former, while dollar provides a better proxy for prices of goods coming from the third countries. At the same time, DCP strictly dominates any other potential vehicle currency. Figure 2.4a shows equilibria for different values of $\rho$ in the coordinates $\alpha$ and $\gamma$. The line separating PCP and LCP equilibria remains the same as in Figure 2.3b as the value of $\rho$ does not affect the trade-off between producer and local currencies. The region of DCP equilibrium is one point when $\rho = 1$ and increases continuously as dollar volatility goes down. Consistent with the discussion above, DCP equilibrium is more likely for higher import share $\gamma$ and intermediate values of price complementarities $\alpha$, while PCP and LCP are always optimal when import share $\gamma$ is low.

**Proposition 2.4 (Volatility advantage)** Assume $\lambda \to 1$ and $n = 0$. Then as long as dollar has lower volatility than other currencies, $\rho < 1$, the region in the parameter space with DCP as a unique equilibrium is non-empty and increases as $\rho$ goes down.

While this result alone is not sufficient to rationalize the global status of dollar, it explains why the use of currencies with volatile exchange rates in the international trade is very limited, e.g. almost all imports and exports of Latin American and Eastern European countries are invoiced in foreign currencies (Casas, Diez, Gopinath, and Gourinchas 2017). The model shows that the relative volatility is important even when exchange rate shocks are not associated with changes in nominal wages (cf. Devereux and Engel 2001, Bhattarai 2009). Section 2.4.2 shows on the other hand that the effect can be significantly amplified when differences in volatility are due to monetary shocks.
Figure 2.4: Currency choice: flexible price limit

Note: the figure shows equilibria for $\lambda \rightarrow 1$, $\phi = 0.5$ and (a) $n = 0$, different values of $\rho$, (b) $\rho = 1$, different values of $n$. PCP, LCP and DCP denote the regions of the corresponding symmetric equilibria. PCP’ (LCP’) denotes the region where small countries set prices in producer (local) currency when trading with each other and use dollars when trading with U.S.

Large share in global trade  Consider next the case when volatility of exchange rates is the same for all countries, $\rho = 1$, but the U.S. accounts for a non-trivial share of the global trade, i.e. $n > 0$. This implies that a positive fraction of inputs used by firms in small economies are produced in the U.S. In addition, a positive mass of competitors in all markets are coming from the U.S. Both factors increase the chances of DCP equilibrium as $n$ goes up. Figure 2.4b shows the region of DCP equilibrium for different values of $n$: while the set consists of only one point when $n = 0$, the region increases as $n$ goes up. The currency of the large economy strongly dominates any other potential vehicle currency.

Proposition 2.5 (Large economy advantage)  Assume $\lambda \rightarrow 1$. Then as long as the share of the U.S. economy in the international trade is positive, $n > 0$, the region in the parameter space with DCP as a unique equilibrium is non-empty and increases as $n$ goes up.

$^{24}$Note that PCP, LCP and DCP coincide for trade flows between regions within the U.S.
The figure also shows that an equilibrium with asymmetric invoicing can arise when \( n > 0 \).\textsuperscript{25} In particular, firms might choose to use producer currency when trading between small economies, but set prices in dollars when exporting to the U.S. This is because the home bias is larger for the U.S. than for other economies when \( n > 0 \), and more competitors in the destination market have prices stable in local currency, i.e. in dollars. Similarly, exporters from the U.S. have a higher share of their marginal costs stable in dollars and can use DCP even when other firms prefer LCP.

2.3.3 Transition

The previous section argues that both fundamental factors, i.e. volatility and size advantage, and complementarities in currency choice contribute to the dominant status of dollar in today’s world. What happens when these factors work in the opposite direction? This situation has happened to the pound sterling in the twentieth century and might be relevant for the dollar as China overgrows the U.S.

To answer the question, I allow for two large countries, the U.S. and the U.K. (see Figure 2.5a for illustration). The economy starts from the point when the U.K. has a fundamental advantage over the U.S. in terms of economy size or exchange rate volatility, which it gradually losses along the transition path. I make three simplifying assumptions as in Matsuyama, Kiyotaki, and Matsui (1993) and Rey (2001). First, all countries find it optimal to trade either in dollars or in pound sterling. Second, since I am interested in long-run changes in currency choice, the focus is on the evolution of steady state in

\textsuperscript{25}Strictly speaking, the same is true in a model with \( n = 0 \) and \( \rho < 1 \), but since U.S. economy has zero mass, that has no effects at the global level.
response to changes in exogenous parameters, while transition between steady states is ignored. Third, with multiple equilibria in the model, there is a continuum of possible transition paths. For selection, I use the argument in the spirit of evolutionary game theory that most agents follow the rule of thumb that has been used before. This implies that as long as the old equilibrium exists, the firms do not coordinate to jump into the new equilibrium. Therefore, among all possible transition paths, the one with the highest hysteresis is chosen.\footnote{While a dynamic model with staggered pricing can be used to select between "history" vs. "expectations", the equilibrium remains non-unique in general case (see e.g. Matsuyama 1991, Krugman 1991). Alternatively, one can use a global game approach in the spirit of Morris and Shin (2001), but its application in dynamic settings is complex and goes beyond the scope of this paper.}

The next proposition characterizes transition driven by changes in one of the two fundamentals — relative volatility of shocks or relative size of the U.S. keeping the total share of two currency unions in the global economy constant.

\textbf{Proposition 2.6 (Transition)} Let \( T(x) \) denote the threshold of \( \frac{\sigma_{UK}^2}{\sigma_{US}^2} \) or \( \frac{n_{US}}{n_{US} + n_{UK}} \), at which
trade flow $x$ from Figure 2.5a switches from pound to dollar. Then

1. the share of pound in the international trade is decreasing along the transition path,

2. the trade flows switch from pound to dollar in the following order:

   • $T(a), T(b) \leq T(c) \leq T(f), T(g)$
   • $T(a) \leq T(d) \leq T(g)$
   • $T(b) \leq T(e) \leq T(f)$

Thus, as U.K. economy becomes smaller or/and more volatile, the share of pound in the international trade monotonically decreases. Figure 2.5b provides an example of a transition path for changes in union size, while Figure A8 in the Appendix shows transition driven by changes in volatilities. While the fundamental factors do change the equilibrium price system, there is also a path-dependence due to strategic complementarities in currency choice.\(^{27}\) In particular, when the size of U.K. and U.S. is about the same, the share of pound in global trade remains as high as 85%. At the same time, the transition is much faster in the limit of flexible prices $\lambda \rightarrow 1$ with no complementarities in currency choice.

The model has also clear predictions about the order, in which trade flows in the global economy switch from pound to dollar. The trade between the U.S. and small economies is the first to become invoiced in dollars because of the prevalence of U.S. firms with costs stable in dollars. At the second stage, the small economies start using dollar as a vehicle currency when trading with each other, and the trade flows between two unions also change the currency of invoicing. Finally, the trade between the U.K. and small economies

\(^{27}\)The standard caveat that there are also equilibria with fast adjustment applies here as well. See Figure A8 for the lower and upper boundaries on the transition paths.
switches to DCP as well. Note if complementarities in currency choice are strong enough, some flows might remain invoiced in pound even as $n_{UK} \to 0$.

These predictions are broadly consistent with the historical evidence — the transition from pound to dollar was sluggish, followed with the lag after the U.S. overtook the U.K. as the largest economy, and was accelerated by large jumps in pound exchange rate after World War I and in 1931 (Eichengreen 2011). While the invoicing data is scarce for the beginning of the twentieth century, the experience of the Eurozone also fits predictions of the model. In particular, the euro is more commonly used in Eurozone trade with developing countries, much less so in trade with the U.S. and even more rarely as a vehicle currency (Kamps 2006).

2.3.4 Calibration

I next calibrate the model to the data and check whether DCP equilibrium can be supported under reasonable values of the parameters. Despite large differences between countries, industries and firms, I argue that the standard calibration with $\alpha = \phi = 0$ and $\gamma$ close to U.S. import-to-GDP ratio of 0.15 does not provide a good approximation to the real world. In particular, a large fraction of non-tradable goods in GDP masks high import share in tradable (manufacturing) sector, which is about 0.6 for small economies and 0.4 for the U.S. Both firm-level data and the aggregate input-output tables imply that intermediate share in production is around $\phi = 0.5$, while the recent empirical estimates of complementarities in price setting suggest $\alpha = 0.5$ (Amiti, Itskhoki, and Konings 2016b).\footnote{Both $\alpha$ and $\phi$ are higher for large firms that account for most of the global trade (Amiti, Itskhoki, and Konings 2014).}
Figure 2.6: Currency choice under the baseline calibration

Note: the figures show the regions where symmetric PCP, LCP and DCP equilibria can be sustained (there is no symmetric equilibria in the white region). Parameter values are from the benchmark calibration: $\phi = 0.5$, $\lambda = 0.5$, $\rho = 0.5$, $n = 0.3$ and the red star shows the baseline calibration for $\gamma = 0.6$ and $\alpha = 0.5$.

Assuming that one period corresponds to a year, I calibrate $\lambda = 0.5$, so that half of firms update prices by the end of the first year and the remaining ones adjust by the end of the second year. Assuming that the volatility of bilateral exchange rate between developing countries is higher than the volatility of exchange rate between a developing country and the U.S. by 33%, I get $\rho = 0.5$. Finally, I use $n = 0.3$, which is a conservative value relative to the large share of dollarized economies in the world (see Ilzetzki, Reinhart, and Rogoff 2017). Figure 2.6 shows that DCP equilibrium can be sustained under the baseline calibration.

Combining the mechanisms outlined above, this result can be interpreted as follows. Given that the U.S. is the largest economy in the world, the foreign firms selling in the U.S. market compete with a high number of local producers, which set prices in dollars. To avoid losing the market share because of unexpected movements in exchange rates,
foreign firms synchronize their prices with the competitors by using dollar invoicing. The U.S. exporters then find the costs of both labor and intermediate inputs stable in dollars and are more inclined to use DCP in other markets. This increases the share of intermediate inputs and competing products invoiced in dollars that exporters in other economies face. Moreover, the firms that export from one developing country to another often find exchange rates of both countries too volatile to be used for invoicing and hence, are looking for a stable vehicle currency. With both the U.S. and emerging economies using dollars, the firms in developed countries might also find it optimal to switch to DCP. The exporters to the U.S. are then even more likely to set prices in dollars, which further strengthens the initial argument. Finally, while there might be also other equilibria with different dominant currencies, the path dependence in currency choice implies exporters might still use DCP despite the loss of its fundamental advantages relative to the middle of the twentieth century.

2.4 Extensions

This section relaxes three assumptions from the baseline model. First, I allow domestic firms to make optimal currency choice and show that while they are less likely to set prices in dollars than exporters, a persistent DCP equilibrium can emerge once local firms switch to dollar invoicing. Second, I replace wage targeting with two alternative monetary policies — random monetary shocks and inflation targeting — and show that DCP equilibrium survives and can even expand relative to the baseline case. Finally, I consider the continuous currency choice when exporters can set prices in terms of basket of cur-
rencies. The predictions of the model are then largely inconsistent with the key stylized facts about the International Price System, showing that discreteness of currency choice is a crucial feature of the benchmark model.

2.4.1 Dollarization

In contrast to the assumption in the baseline model, it is not uncommon for local firms in developing countries to set prices in dollars (see e.g. Drenik and Perez 2017). I therefore extend the model allowing domestic producers in the tradable sector to choose optimally the currency of invoicing and define the global currency pricing (GCP) equilibrium, in which all firms in tradable sector including domestic ones set prices in dollars. In contrast, in DCP equilibrium only exporters price in dollars, while domestic firms use local currency.

**Proposition 2.7** Assume domestic firms optimally choose the currency of invoicing and \( n = 0 \). Then

1. in the flexible price limit \( \lambda \to 1 \), the region of GCP is the subset of DCP, is non-empty as long as \( \rho < 1 \) and is increasing in \( \gamma, \phi \) and \( \alpha \),

2. in the limit of fully rigid prices \( \lambda \to 0 \), the region of DCP is a subset of GCP.

Consider first the flexible price limit \( \lambda \to 1 \) (see Figure 2.7a). With almost all firms adjusting prices after the realization of shocks, the currency choice of domestic producers has no effect on invoicing decisions of exporters, which remain the same as in the baseline model. Since producer and local currencies coincide for domestic firms, they are less likely
Figure 2.7: The optimal invoicing of domestic firms

Note: figure (a) shows equilibria in the flexible price limit $\lambda \to 1$ and $\rho = 0.5$, while figure (b) shows symmetric equilibria under sticky prices $\lambda = 0.5$ and $\rho = 1$. The grey area is the region of global currency pricing (GCP) equilibrium with all firms including domestic ones using dollar for invoicing. Other parameters: $\phi = 0.5$, $n = 0$.

to use dollar invoicing. The GCP equilibrium is therefore a subset of the DCP equilibrium from the benchmark model. The equilibrium invoicing looks very different when prices are sticky: in the limiting case of fully rigid prices, the DCP region is always a subset of the GCP one. Intuitively, strategic complementarities in currency choice that arise under sticky prices imply it is easier to support equilibrium with all firms invoicing in dollars than the one with only exporters using dollars and domestic firms setting prices in local currency. As Figure 2.7b shows, even incomplete price rigidity is sufficient for GCP region to dominate both DCP and LCP ones.

Thus, the model predicts that while domestic firms are less likely to switch to dollar invoicing than exporters, once they do so — e.g. because of unstable monetary policy discussed below — the DCP equilibrium can be sustained more easily and can persist even after fundamentals turn against dollar. The wide use of dollar in Latin American and some
East European countries contributes therefore to the status of dollar in the international trade.

### 2.4.2 Monetary shocks and inflation targeting

While movements in exchange rates are largely disconnected from monetary shocks for most economies (Meese and Rogoff 1983b), the correlation is much higher for countries with unstable inflation. I therefore relax assumption \( w_i = 0 \) and allow for exogenous stochastic shocks in nominal wages.\(^{29}\)

**Proposition 2.8** Assume that monetary shocks follow random walk and \( n = 0 \). Then

1. if \( \lambda \to 1, \rho < 1 \), DCP is the only possible equilibrium in the limit \( \sigma_m^2 \to \infty \),

2. proportional increase in \( \sigma_m^2 \) in all countries expands the DCP region.

Consider first the limiting case when \( w_i \) is the only shock in the economy and prices are almost flexible \( \lambda \to 1 \). The labor costs are no longer stable in producer currency and as a result, neither are the prices of domestic intermediate goods. At the same time, a positive monetary shock is associated with a one-to-one depreciation of local exchange rate, which implies that nominal wages can actually be more stable in foreign currency than in local one. In particular, as long as \( \rho < 1 \), the volatility of nominal wages in dollars \( w_i + e_{0i} \) is lower than the volatility in producer currency \( w_i \), and firms unambiguously prefer DCP to PCP. A symmetric argument applies to LCP. The DCP is therefore a unique equilibrium for arbitrary values of other parameters and in particular, can be sustained even in the limit of closed economy \( \gamma \to 0 \). This prediction of the model is consistent with

\(^{29}\)I focus on the second rather than first moments of monetary shocks, which complements the effect of inflation rate on currency choice emphasized by the previous literature (see e.g. Drenik and Perez 2017).
Figure 2.8: Currency choice under alternative monetary policies

Note: figure (a) shows equilibria in a model with exogenous monetary and financial shocks. The volatility of financial shocks is normalized to one, while the volatility of nominal shocks is shown in the figure, $\lambda \rightarrow 1$, $\phi = 0.5$, $n = 0$, $\rho = 0.5$. Figure (b) shows symmetric equilibria when monetary authorities in all countries stabilize CPI and the only shocks are financial ones. The parameter values are taken from the baseline calibration.

the wide use DCP during the episodes with high and unstable inflation in Latin American countries in 1980s and in Eastern Europe in 1990s.

More generally, in the presence of other shocks, the higher volatility of monetary shocks increases the correlation between wages and exchange rates and extends the region of DCP (see Figure 2.8a). Importantly, this result holds even when volatility of U.S. monetary shocks increases proportionately with nominal shocks in other countries. In contrast to mechanism outlined in Devereux, Engel, and Storgaard (2004), a higher volatility of monetary shocks makes DCP more appealing to firms not because of increasing volatility of other currencies relative to dollar, i.e. falling $\rho$, but because of lower stability of input and competitor prices in producer and local currencies respectively. The model thus suggests that periods of high global inflation — as the one observed in 1970s — can actually increase the use dollar in international trade despite higher volatility of U.S.
exchange rate.

Finally, I replace the wage targeting from the baseline model with a more realistic assumption that monetary authorities stabilize consumer prices and keep inflation at the zero level. Figure 2.8b is the counterpart of Figure 2.6 and shows the resulting symmetric equilibria under inflation targeting and the baseline calibration (see Appendix A.2.7 for the details). The DCP region shrinks in this case because monetary policy stabilizes relatively more input prices and competitor prices in local currency than in the benchmark model. As a result, PCP and LCP become more appealing and firms are less likely to choose the dominant currency. However, the DCP equilibrium survives under the baseline calibration due to large size of non-tradable sector, which mutes the response of monetary policy to movements in exchange rates. Notice that in a more realistic model with firms heterogeneous in their input basket and the set of competitors, the stabilization effect of inflation targeting on exporters’ optimal price is even smaller and the incentives to use PCP and LCP are lower.

2.4.3 Invoicing in terms of currency baskets

This section extends the baseline model by allowing firms to set prices in terms of an arbitrary basket of currencies. Before studying the implications of this assumption, it is worth discussing its economic interpretation. The correct one is that e.g. Apple sells iPhone 7 in Germany for 500 dollars plus 300 euros plus 200 swiss francs. Moreover, the currency weights might be negative, which means that firms are allowed to make transfers to the customers, e.g. a client pays 1200 dollars for the good and gets back 200 euros as
a discount. On the other hand, the interpretation that firm sets price in dollars, euros and francs with probabilities 50%, 30% and 20% respectively is wrong since ex post pass-through of exchange rate shocks conditional on no price adjustment is discrete in this case. Another wrong interpretation is that firm sells some fraction of products in one currency and some fraction in another currency. If this is the same product, customers will only make purchases using the lowest ex post price. Finally, using different currencies for different products is also a wrong interpretation since profits are separable in products.

The stark implication of continuous currency choice is that despite sticky prices firms can bring their ex post prices significantly closer to the optimal level:

**Lemma 2.5** Suppose that prices are set in terms of basket of arbitrary currencies. Then exporters can achieve the optimal pass-through of exchange rate shocks in every state of the world.

Notice however, that while the pass-through of exchange rates into prices is optimal, the pass-through of other shocks is not. In particular, the pass-through is zero for idiosyncratic productivity shocks and even for aggregate shocks as long as they are uncorrelated with movements in exchange rates. In addition, as long as domestic firms are obliged to set prices in local currency as in the baseline model, the prices of importers and the allocation in tradable sector are different from the flexible-price case.

**Proposition 2.9** Assume $w_i = \alpha_{T_i} = 0$, domestic firms set prices in local currency, while exporters can use arbitrary baskets of currencies for invoicing. Then

1. equilibrium is always unique,
Figure 2.9: Dollar share in a basket of currencies used for invoicing

Note: the figure shows the share of dollars in trade between non-U.S. countries when exporters can use arbitrary baskets of currencies and $n = 0.5$. Parameter values: $\rho = 0.5$, $\phi = 0.5$, $\lambda = 0.5$.

1. the share of dollar in the international trade cannot be higher than $n$,

2. relative dollar volatility $\rho$ has no effect on dollar use in international trade,

3. high price rigidity $1 - \lambda$ decreases the use of dollar and stimulates LCP,

4. the share of dollar increases in $\gamma$ and $\phi$ and might be not monotone in $\alpha$.

The intuition for results 1 and 3 is straightforward: the optimal pass-through of dollar exchange rate depends on the fraction of exporters from the U.S. The share of dollar in the optimal basket is proportional to this pass-through, which is incomplete and therefore cannot be higher than $n$. Thus, the model with complete basket cannot match empirical fact that the share of DCP is much higher than the share of U.S. in the international trade. In addition, the model predicts that the relative volatility of dollar $\rho$ plays no role because it has zero effect on the optimal pass-through of exchange rate shocks. Also, in contrast to the baseline model, higher price rigidity actually reduces the international use of dollar.
This is because lower frequency of price adjustment has direct effect only on domestic producers, while the effect on importers is indirect and decreases the pass-through of exchange rate shocks. Finally, the comparative statics with respect to import share $\gamma$ and intermediate share $\phi$ remain unaffected since their main effect comes from the weights of currencies in the optimal basket. Figure 2.9 provides an illustration of the results, showing the dollar share in trade between third (non-U.S.) countries. The fraction of DCP never exceeds the share of U.S. in global trade $n = 0.5$ and is much lower for most values of $\gamma$ and $\alpha$.

## 2.5 Conclusion

In this chapter, I propose a general equilibrium framework with endogenous currency choice that is broadly consistent with the key stylized facts, including the dominant status of dollar as a vehicle currency in global trade and the delayed transition from pound to dollar in the twentieth century. I use the model to study the effect of monetary policy on firms’ currency choice and the dollarization of economies, while Chapter 3 below uses this setup to analyze the effect of (endogenous) dollar pricing on the international spillovers and the optimal monetary policy.

The tractability of the baseline model allows for several other extensions and applications, which I leave for future research. First, augmenting the model with a more realistic financial sector would allow analyzing the interactions between the dominant status of dollar as a vehicle currency in the international trade and as a reserve currency in global asset markets. The model can also be used to study the denomination of international con-
tracts in dollars in both goods and financial markets (see also Gopinath and Stein (2017), Drenik, Kirpalani, and Perez (2018)).

Second, a quantitative version of the model can be obtained by introducing heterogeneity across countries and industries. That would allow to test the cross-sectional predictions of the model about the currency of invoicing and perform counterfactuals about future changes in the international price system. The analytical model presented in this chapter provides important guidance which dimensions of cross-country differences should be incorporated to analyze exporters’ currency choice. In particular, one needs both input-output linkages and the comovement of exchange rates to calibrate such model. Finally, an extension of the model with heterogenous firms can be used as a basis for the micro-level empirical analysis of exporters’ currency choice.
Chapter 3

Monetary Policy in the Open Economy

3.1 Introduction

The currency in which international prices are set is crucial for the transmission of monetary shocks across countries. In a world with sticky nominal prices and large fluctuations in exchange rates, the exporters’ currency choice determines which relative prices in the global economy remain stable in the medium run and which ones fluctuate one-to-one with exchange rates. The answers to the fundamental questions in international economics can change drastically, depending on what assumptions are made about the firms’ currency choices. In particular, while the classical argument in favor of floating exchange rates (Friedman 1953) holds when the prices are set in the currency of the producer (producer currency pricing, PCP), pegging the exchange rate can be optimal when prices are set in the currency of the consumer (local currency pricing, LCP) (Devereux and Engel 2003b). Similarly, the spillover effects of monetary policy on foreign output, which have been at the center of public debates during the global recession (Bernanke 2017), are
negative under PCP and positive under LCP (Betts and Devereux 2000).

The standard assumptions in the existing models are, however, inconsistent with two basic empirical facts about the “International Price System” (Gopinath 2016). First, while most of the theoretical literature has focused on the case of PCP and, to a lesser extent LCP, the empirical evidence shows that, for the bulk of international trade, prices are set in just a few currencies, with dollar being the dominant one (see Figures 2.1) (Goldberg and Tille 2008). This suggests that the transmission of shocks across countries might be more asymmetric than predicted by the existing models. Second, the robust relationship between currency choice and the characteristics of the specific firm, industry and country (Gopinath, Itskhoki, and Rigobon 2010), as well as the radical transformation of the international price system over history (Eichengreen 2011) do not support the standard assumption of exogenous time-invariant invoicing. The models with exogenous currency choice are, therefore, subject to the Lucas critique and can potentially lead to poor policy implications.

In this chapter, I use my model of the international price system from Chapter 2 to re-examine the classical positive and normative questions about the international dimensions of monetary policy. In the spirit of Mankiw (1985), I show that, despite only second-order private gains, the currency choice has first-order aggregate implications. Because of strategic complementarities in currency choice, the dollar currency pricing (DCP) might be almost as good as setting price in terms of an optimal basket of currencies from the point of view of individual exporter when all other firms use dollars (cf. Ball and Romer 1991). At the same time, a small perturbation of the fundamentals that makes firms switch from one invoicing regime to another leads to discontinuous changes in how prices, out-
put, consumption, and trade balance respond to exogenous shocks.

First, I identify a novel source of U.S. monetary spillovers on foreign output that has largely been ignored in the previous debates (see e.g. Bernanke 2017). The stimulating monetary policy in the U.S. increases the aggregate demand and in particular, the demand for imported goods, but also makes U.S. goods cheaper relative to foreign ones because of the exchange rate depreciation. The classical conclusion is that the latter effect is stronger when the prices are set in producer currency and, as a result, the net spillovers are negative under PCP and positive under LCP (see e.g. Betts and Devereux 2000, Corsetti and Pesenti 2005). There is, however, an additional effect under DCP: a depreciation of the U.S. dollar decreases the prices of all internationally traded goods, which translates into lower producer and consumer price indices. As long as the aggregate nominal demand remains unchanged, the fall in prices drives the world consumption upwards (Goldberg and Tille 2009), stimulating production in the global economy. This channel has an unambiguously positive effect on foreign output and outweighs the standard expenditure switching towards U.S. goods under the baseline calibration.¹ At the same time, the depreciation of non-vehicle currencies has no additional positive spillover effects on the other economies and is also less effective in stimulating local output.

Second, I show that the currency choice per se does not invalidate the classical argument in favor of floating exchange rates (Friedman 1953). As has been demonstrated by Devereux and Engel (2003b) in the context of a standard New-Keynesian open economy model, the optimal monetary policy implies floating exchange rates under PCP and

¹In contrast to the effect of dollar depreciation on global trade in Boz, Gopinath, and Plagborg-Møller (2017), the response of global output comes from the general equilibrium effects rather than partial equilibrium expenditure switching and does not depend on substitution between goods.
pegging under LCP. I show, however, that, in a standard model, PCP is the only type of equilibrium invoicing that can arise under the optimal policy, when currency choice is endogenous. The decentralized invoicing decisions are, therefore, efficient in the sense that the first-best allocation can always be implemented by the monetary policy that stabilizes the producer price index (PPI). Though standard in the literature, the assumptions underlying this result are restrictive and are inconsistent with the data.

Third, I argue that, in a more realistic environment, there are complementarities between the firms’ currency choices and monetary policy: the optimal policy under DCP involves a partial peg to the dollar, which, in turn, makes dollar invoicing more appealing to the firms. In particular, when the international prices are set in dollars, the U.S. monetary shocks tend to distort the terms of trade between third countries, and the monetary authorities lean against the wind by partially smoothing out movements in exchange rates against the dollar. The DCP can, therefore, contribute to the “fear of floating” and the widespread use of the dollar as an anchor currency seen in the data (Calvo and Reinhart 2002, Ilzetzki, Reinhart, and Rogoff 2017). At the same time, the resulting lower volatility of the U.S. exchange rate makes the dollar more attractive as a vehicle currency and helps to sustain the DCP equilibrium. The partial peg to the dollar also implies that the monetary policy is positively correlated across countries, which can contribute to the global financial cycle (Rey 2015). Importantly, however, despite worsening the trade-off that the policymakers face, the dollar pricing does not transform the “trilemma” into a “dilemma”: the flexible exchange rates are effective in managing local shocks and, in comparison to fixed ones, allow to achieve higher welfare (see Gopinath 2017).
3.2 Transmission of Monetary Shocks

Using the setup from Section 2.2, I start with showing that despite only second-order effect on firm’s profits, the currency choice has first-order general equilibrium implications. I argue that in the empirically relevant case of DCP, the stimulating effect of exchange rate depreciation on local output is higher in the U.S. and lower in other economies, and that spillover effects of dollar depreciation on foreign output are more positive than predicted by the standard models with PCP and LCP.\(^2\) I use a simple calibration of the model to show that small private costs of currency choice can lead to large differences in business cycles (cf. Mankiw 1985). A tractable general equilibrium block of the model allows me not only to formalize several conjectures about the trade balance adjustment from Gopinath (2016) and Goldberg and Tille (2006), but also to analyze the response of output and consumption to monetary shocks and to identify parameters that determine the sign and the magnitudes of the effects. I show in particular there are significant general equilibrium effects of dollar depreciation on foreign output even in the limiting cases when partial equilibrium ones cancel out. For simplicity of exposition, I focus exclusively on equilibria with symmetric invoicing and unexpected shocks, suppressing the time subscript below.

3.2.1 Local effects

The effect of exchange rate depreciation on trade balance, consumption and output depends on how import and export prices respond to these shocks. As emphasized by the previous literature, the pass-through of exchange rates into customer prices is high under

\(^2\)The words “positive” and “negative” in this section refer to signs of the effects and not to their welfare implications.
PCP and low under LCP, which implies that quantities respond much less under LCP than PCP (see e.g. Betts and Devereux 2000). Relative to this benchmark, invoicing in dollars introduces two types of asymmetries — between export and import prices, and between the U.S. and other economies. In particular, the price response resembles PCP on export side and LCP on import side for the U.S. and the other way around for other countries. Thus, in response to positive monetary shock, the trade balance adjusts more through higher exports in the U.S. and lower imports elsewhere (see Casas, Diez, Gopinath, and Gourinchas 2017).

The total effect is however more than just a convex combination of the two due to input-output linkages. Consider a non-U.S. economy. Relative to LCP case, imported intermediates are more expensive and therefore prices of adjusting exporters fall less, depressing exports even further. Relative to PCP case, a weaker growth in exports implies lower demand for foreign intermediates, which amplifies contraction in imports. In contrast, the U.S. trade balance adjusts relatively more through exports than imports under DCP: the depreciation of dollar has minor effects on import prices and decreases the prices of exports in destination currency.

The differences in trade balance adjustment across countries under DCP translate into the asymmetric response of consumption and output. The depreciation of exchange rate stimulates production more in the U.S. than in other countries because of larger expenditure switching towards exported goods and lower increase in prices of foreign intermediates. At the same time, the lower pass-through of exchange rate shocks into CPI implies that the U.S. enjoys smaller fall in consumption.
Proposition 3.1 (Transmission of monetary shocks) Assume $n = 0$ and DCP. Then relative to the effects of monetary shock in other economies, an expansionary monetary policy in the U.S. implies

1. higher exports and imports,
2. lower inflation and higher output,
3. the same net export.

Interestingly, despite these asymmetries across countries, the elasticity of net export with respect to the trade-weighted exchange rate is the same for all economies including the U.S. — the higher elasticity of exports and the lower elasticity of imports in the U.S. exactly offset each other — which has two important implications. First, even under asymmetric currency choice, the trade-weighted rather than invoicing-weighted exchange rate remains sufficient statistics for net exports. Second, consider the case of incomplete international asset markets when exchange rate adjusts to ensure that trade balance holds. The same elasticity of net exports implies then that response of exchange rate to exogenous shocks is symmetric across countries. Therefore, DCP does not necessarily generate lower (or higher) volatility of U.S. exchange rate.

3.2.2 International spillovers

The last decade has witnessed a lively debate about the spillover effects of the Fed’s monetary policy on other countries (see e.g. Bernanke 2017). On the one hand, easy monetary policy increases demand for both domestic and imported goods, stimulating production in all economies. On the other hand, such policy also leads to depreciation of the national
currency, which can potentially make local goods cheaper relative to foreign ones and have negative spillovers on other economies. The classical result in the literature is that the net effect is negative under PCP and positive under LCP: while the former effect does not depend on currency of invoicing, the latter one is large under PCP and mild under LCP (see e.g. Betts and Devereux 2000, Corsetti and Pesenti 2005). I next show that additional channel with unambiguously positive spillovers arises under DCP that has been largely ignored in the previous literature.\(^3\) To this end, consider the effect of U.S. monetary shock that increases nominal spendings in the U.S. I discuss the difference in spillovers that arises under DCP vs. PCP/LCP. Since the aggregate demand effect is independent from currency of invoicing, I focus below exclusively on the pass-through of dollar shocks into global prices, trade and output.

For any variable \(x_i\), which can denote prices or quantities in country \(i\), define the global counterpart as \(x = \int_0^1 x_i \, di\).\(^4\) Aggregating the import price index (in destination currency) across countries, one obtains:

\[
p' = -\frac{(1 - \lambda)(1 - n)\mu^D}{1 - \lambda(\alpha + (1 - \alpha)\phi)} e_0, \tag{3.1}
\]

where \(\mu^D\) equals one if equilibrium invoicing is DCP and zero otherwise. Thus, even when U.S. accounts for a positive share of the world economy \(n > 0\), the pass-through of dollar exchange rate into aggregate import price index is zero if prices are set not in dollars: depreciation of dollar simultaneously decreases prices of U.S. export and increases

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\(^3\)The important exception is the paper by Goldberg and Tille (2009), which shows in a context of a three-country model that U.S. shocks have larger effect on global consumption under DCP.

\(^4\)Note that all variables including prices are expressed in same units — log-deviations from the steady state values — and can therefore be integrated across countries with different units of account.
U.S. import prices, leaving the global price index unchanged. On the other hand, when prices are sticky in dollars, depreciation of $e_0$ decreases international prices in currency of destination for all importers except the U.S., hence $(1 - n)$ term. A fall in international prices in turn translates into lower price index for tradable goods $p$ and lower CPI $p^C$:

\[ p = \gamma p^I, \quad p^C = \eta p + (1 - \eta)p^N, \]

where $p^N$ is the price index in non-tradable sector and $\eta$ is the share of tradable sector.\(^5\)

The movements in international and domestic prices translate into changes in the volume of global trade $y^I$. The pass-through of prices into quantities can be decomposed into four channels:

\[ y^I = -\theta(p^I - p) + \phi(w - p) + (1 - \eta)(p^N - p) + (w - p^C). \]  \hspace{1cm} (3.2)

The first term corresponds to expenditure switching: a fall in relative price of imported goods $p^I - p$ implies that buyers switch from domestic goods towards internationally traded ones with the effect increasing in elasticity of substitution $\theta$. The second term in (3.2) shows that firms substitute labor with cheaper intermediates in production. Similarly, consumers switch from non-tradables to tradables with the effect proportional to the share of non-tradables in consumption basket $1 - \eta$. Finally, lower prices for tradables decrease CPI, which stimulates labor supply through higher real wages. All these effects work in the same direction and increase global trade in response to dollar depreciation under DCP.

\(^5\)Despite its global implications, the dollar exchange rate $e_0$ is determined solely by U.S. shocks and is independent from invoicing regime according to Lemma 2.2.
even when U.S. share in trade is zero. The prediction of the model is therefore consistent with the growing empirical evidence about the effect of dollar shocks on global trade (see Boz, Gopinath, and Plagborg-Møller 2017).

The increase in global demand for imported products translates into higher output $y$ and consumption $c$ worldwide:\footnote{Under more general preferences, the pass-through of $e_0$ into global consumption depends on all four channels.}

\[
y = \eta \phi (w - p) + (w - p^C) \\
\text{Int. vs. L} + (w - p^C) \\
\text{Real Wage} \quad \text{and} \quad c = (w - p^C).
\] (3.3)

The terms $p^N - p$ and $p^I - p$ cancel out due to aggregation between sectors and countries. In particular, substitution from non-tradable goods for tradables does not affect total output. Similarly, expenditure switching effect increases both exports and imports, with the latter crowding out local production. The net effect is therefore, zero and does not depend on the elasticity of substitution $\theta$. The two remaining effects — firm substitution towards intermediate goods and labor supply effect — however boost global production in response to dollar depreciation under DCP. The effect is stronger when the pass-through of dollar shocks into producer price index in tradable sector $p$ and consumer price index $p^C$ is high.\footnote{In particular, it can be close to zero if one assumes that both firms and households buy products from a wholesale/retail sector with very sticky prices.}

**Proposition 3.2 (International spillovers)** Relative to PCP/LCP benchmark, the dollar invoicing implies that expansionary monetary policy in the U.S.

1. increases the volume of international trade $y^I$,
2. increases the global output $y$ and consumption $c$, with the effect independent from elasticity $\theta$,

3. decreases CPI of other economies and boosts consumption and production if $\theta n$ is low.

How is the global output $y$ divided between the U.S. and the rest of the world? Under both PCP and LCP, the depreciation of $e_0$ leaves $y$ unchanged and decreases the relative prices for U.S. goods. It follows that expenditure switching towards U.S. output shrinks production in other economies (see Appendix A.3.1 for details). The negative spillovers are therefore larger when U.S. share in world trade $n$ and demand elasticity $\theta$ are higher. Under DCP, on the other hand, depreciation of dollar increases global output $y$. In the limit $n = 0$, the whole “pie” goes to the RoW and spillover effects are unambiguously positive. Intuitively, expenditure switching towards U.S. goods has zero effect when $n = 0$. At the same time, lower international prices boost trade between non-U.S. countries and stimulate production through general equilibrium effects. When $n > 0$, there are both positive spillovers from trade between third countries and negative ones from trade with the U.S., so that the sign of the net effect depends on parameter values.\(^8\)

Thus, the spillover effects of dollar depreciation on foreign output can be positive even when monetary authorities are constrained by the zero lower bound and cannot stimulate the aggregate demand. This contrasts with the conclusions of the previous literature that depreciation of exchange rate in this case is a zero-sum beggar-thy-neighbor policy that exports recession to other countries and can potentially lead to “currency wars” (see e.g.

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\(^8\)The small previous literature that studied transmission of shocks under DCP has mostly assumed only two countries (see e.g. Canzoneri, Cumby, Diba, and López-Salido 2013, Corsetti and Pesenti 2007). In this case, all imports of the RoW come from the U.S., so that effectively $n = 1$ and as expression (3.1) shows, there are no positive spillovers: depreciation of dollar generates expenditure switching exclusively towards U.S. goods instead of exports of other countries.
On the other hand, the appreciation of dollar can have negative effect on other economies if their output is already inefficiently low. At the same time, the devaluation of non-vehicle currencies leads to standard expenditure-switching effect and is closer to the beggar-thy-neighbor benchmark.

### 3.2.3 Private costs vs. aggregate effects

While the model is intrinsically stylized and abstracts from both cross-country heterogeneity and several ingredients from the DSGE literature (e.g. capital, habit formation, wage rigidity, etc.), it might still be informative to put some numbers on the effects outlined above and to compare private costs with aggregate effects. I use the same values of $\alpha$, $\gamma$, $\phi$, $\lambda$, $n$ and $\rho$ as in Section 2.3.2. In addition, the share of tradable sector $\eta = 0.15$ is calibrated to the share of manufacturing in global GDP and the elasticity of substitution between goods $\theta = 2$ is close to the numbers used in the previous literature (see e.g. Chari, Kehoe, and McGrattan 2002b, Backus, Kehoe, and Kydland 1994, Feenstra, Luck, Obstfeld, and Russ 2014).

Table 3.1 shows the medium-run effects of a monetary shock that increases nominal spendings by 10%. The first three columns correspond to the U.S. monetary shock, while the next ones show effect of monetary expansion in another country. Despite large share of the U.S. in global economy, $n = 0.3$, the results from Proposition 3.1 hold: under DCP, the stimulating monetary policy is significantly more efficient in the U.S. than in other countries: the GDP increases by around 5.5% in the U.S. and 4.8% in other economies. Dollar depreciation also implies lower inflation, giving more room for stimulating monetary policy in other countries.
Table 3.1: Local and spillover effects of monetary shocks

<table>
<thead>
<tr>
<th></th>
<th>U.S. shock</th>
<th>Non-U.S. shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DCP</td>
<td>PCP</td>
</tr>
<tr>
<td>$y_T$</td>
<td>11.06</td>
<td>11.27</td>
</tr>
<tr>
<td>$gdp$</td>
<td>5.52</td>
<td>5.17</td>
</tr>
<tr>
<td>$c$</td>
<td>5.41</td>
<td>4.91</td>
</tr>
<tr>
<td>$y_T$</td>
<td>6.24</td>
<td>-1.05</td>
</tr>
<tr>
<td>$gdp$</td>
<td>0.69</td>
<td>0.12</td>
</tr>
<tr>
<td>$c$</td>
<td>0.74</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Note: the table shows the percentage change in production of tradables, GDP and consumption of U.S. and other countries in response to a local and a foreign monetary shock that increases domestic nominal spendings by 10%.

Consistent with the results from the previous literature, the spillovers of U.S. shock on foreign production in tradable sector is negative under PCP and positive under LCP. The positive effect is however 5 times higher under dollar invoicing.\(^\text{10}\) As a result, foreign GDP and consumption increase respectively by 0.69\% and 0.74\% when prices are set in dollars.

Lastly, I compare these effects with the private costs of currency choice. To this end, I calibrate the standard deviation of the bilateral exchange rate between non-U.S. countries to 0.15 and assume that it is driven by financial shocks. I then calculate losses for an individual exporter of using dollar pricing instead of the optimal basket of currencies keeping the aggregate DCP equilibrium constant (see Section 2.2.2). The aggregate costs across all exporters are only 0.02\% of the global GDP, which is more than one magnitude lower than the spillover effects discussed above. The result resembles the classical argument.

\(^{10}\)In contrast to the conventional model, the spillovers on foreign GDP are positive under PCP because of high share of intermediate goods: dollar depreciation decreases costs of inputs in other countries and stimulates production and consumption. See Rodnyansky (2017) for the empirical evidence in favor of this mechanism.
ment of Mankiw (1985) and Ball and Romer (1990) that small menu costs can lead to large business cycles. In case of open economy, there is however an additional dimension as exporters choose in which currency to set their prices. These decisions are based on the second-order effects on firm’s profits (Lemma 2.1), but have first-order implications for the transmission of monetary shocks within and across countries. The complementarities in currency choice play the same role as real rigidities in a menu cost model and amplify the difference between private and aggregate effects.

### 3.3 Optimal Monetary Policy

The optimal monetary and exchange rate policy is one of the central questions in the international economics. Should the policy focus on inflation targeting and output stabilization as in the closed economy, or movements in exchange rates can be a separate concern for policymakers? Under which conditions is it optimal to peg exchange rate rather than let it float? Which price index is the relevant policy target — consumer prices (CPI) or producer prices (PPI)? While the previous literature has shown that the answers to these questions depend crucially on invoicing of international trade, firms’ currency choice has mostly been taken as exogenous. The results are therefore potentially subject to Lucas critique: the models ignore the fact that firms might change their invoicing decisions in response to monetary policy. In addition, the literature has predominantly focused on PCP and LCP rather than a more empirically relevant case of DCP.

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This section fills in this gap. I first augment a conventional open-economy model from the previous normative literature with the endogenous currency choice and show that the first-best allocation can be implemented by the optimal policy that targets PPI. The individual invoicing decisions generate no inefficiencies in this case and do not alter the classical argument for floating exchange rates. While standard in this literature, the assumptions underlying this result are restrictive and are inconsistent with the data. I then relax them and study the optimal policy under the alternative assumptions about the cooperation across countries and the government ability to commit to a given policy.

3.3.1 Efficient benchmark

In general case, when nominal prices are sticky, the relative international prices get distorted and the equilibrium allocation is not efficient. However, as has been famously argued by Milton Friedman, the floating exchange rates can mitigate these distortions as they allow relative prices across countries to adjust even if nominal prices remain fully rigid: “It is far simpler to allow one price to change, namely, the price of foreign exchange, than to rely upon changes in the multitude of prices that together constitute the internal price structure” (Friedman 1953). This argument has been formalized fifty years later by Devereux and Engel (2003b), who showed that the first-best allocation can be implemented with the optimal monetary policy under floating exchange rates if firms set prices in producer currency. Under LCP, on the other hand, the efficient allocation cannot be achieved and keeping exchange rates constant might be optimal.

I use the model with endogenous currency choice to reexamine conclusions of this lit-
erature. To identify the externalities coming from invoicing decisions of firms and make results directly comparable to the previous literature, I start with the case when price rigidity is the only source of distortions in the economy. In particular, the international asset markets are complete, there is only one sector in the economy, the steady-state markup arising from monopolistic power of firms is eliminated with a fixed subsidy, monetary policy is cooperative across countries and there is no lack of commitment (see Appendix A.3.2 for details).\footnote{The result holds for general isoelastic preferences, an arbitrary frequency of price adjustment \( \lambda \) and all exogenous shocks except for financial ones and markup shocks. The assumption that prices are flexible after one period is not important: the same result can be derived under Calvo pricing.}

**Proposition 3.3** Assume (i) no complementarities in pricing, \( \alpha = 0 \), (ii) full commitment, (iii) cooperative policy across countries. Then efficient allocation can be implemented by the optimal monetary policy that allows for floating exchange rates and stabilizes producer prices (PPI). The equilibrium invoicing is PCP.

One way to interpret the optimal policy is to note that implementation of the first-best allocation requires that the planner replicates the corresponding relative prices. Since nominal prices of goods are sticky, the optimal monetary policy keeps PPI fully stable and makes sure that other prices — nominal wages, interest rates and exchange rates — adjust to replicate optimal relative prices. Under PCP, movements in exchange rates guarantee that product prices in customers’ currency adjust optimally in response to shocks even though prices remain fully stable in currency of producer. This summarizes the logic behind the result from \textit{Devereux and Engel} (2003b).

In contrast to their setup, however, the model with endogenous currency choice generates an additional constraint on the planner’s problem. The key insight of Proposition 3.3
is that this constraint is not binding at the optimum: the firms always choose PCP under the optimal monetary policy. The assumption $\alpha = 0$ implies that producer prices are proportional to their marginal costs. As a result, the monetary policy that targets PPI also stabilizes marginal costs and the optimal price of exporters in producer currency, which means that firms unambiguously prefer PCP. Importantly, however, while PCP constraint is not binding under the optimal policy, the same statement does not hold globally. In other words, condition $\alpha = 0$ alone is not sufficient to guarantee PCP equilibrium — depending on parameter values, DCP or multiple equilibria can arise. The fact that planner can commit to target PPI even in off-equilibrium states of the world, in which firms set prices in dollars, is important to implement the first best.

Proposition 3.3 implies therefore that decentralized currency choice per se does not generate additional inefficiencies. This contrasts with the conclusion of the previous literature that LCP is an important source of distortions in the global economy. In particular, the proposition shows that the analysis of the optimal policy under exogenous LCP and DCP is subject to Lucas critique: it is not possible to sustain such equilibria under the optimal policy without some additional assumptions.

### 3.3.2 Discretionary policy

While Proposition 3.3 provides an important benchmark that clarifies the effect of endogenous currency choice on the optimal policy, the underlying assumptions are hardly realistic. In particular, as has been discussed above, the complementarities in price setting are strong in the data and play the key role in firms’ currency choice. The previous nor-
mative literature has largely ignored price complementarities and provides little guidance about their effect on the optimal policy even when currency choice is exogenous. This section fills in this gap. In addition, I assume that monetary policy is discretionary, i.e. it is chosen after the realization of shocks and takes the ex-ante currency choice of exporters as given. In other words, the planner cannot make a credible threat to punish firms if they deviate from a given invoicing. I therefore solve for the Nash equilibrium, in which firms simultaneously choose the currency of invoicing, taking into account that the monetary policy is determined by the aggregate currency regime.  

To simplify the analysis and obtain sharp analytical results, I follow Devereux and Engel (2003b) and assume fully sticky prices, $\lambda = 0$, symmetric countries, $n = 0$, symmetric invoicing and only productivity shocks $a_i$ (see Appendix A.3.2 for details). 

**Proposition 3.4 (Discretionary policy)** Under the optimal discretionary policy, 

1. exchange rates are more flexible under PCP than under LCP, and are the same under DCP as PCP except for fully stabilized U.S. exchange rate:

$$
e_{ij}^{LCP} = \frac{(1 - \gamma)(a_i - a_j)}{1 - (1 - \gamma)\phi}, \quad e_{ij}^{PCP/DCP} = \frac{a_i - a_j}{1 - (1 - \gamma)\phi}, \quad e_{ij}^{DP} = \frac{a_i}{1 - (1 - \gamma)\phi},
$$

2. the regions of PCP and LCP do not overlap and cover the whole parameter space and the region of DCP is non-empty: $\alpha \leq \frac{1}{2(1 - \gamma)}$ for PCP, $\alpha \geq \frac{1}{2(1 - \gamma)}$ for LCP, and $\frac{1}{2} \leq \alpha \leq \frac{1}{2(1 - \gamma)}$ for DCP,

3. when multiple equilibria coexist for given parameter values, the welfare can be ordered...
as follows: \( W^{DCP} \geq W^{LCP} \) and \( W^{DCP} \leq W^{PCP} \).

Consider first the classical case of PCP and LCP. When nominal prices are rigid, the relative prices do not adjust in response to productivity shocks and the equilibrium allocation is inefficient without government intervention. The monetary policy then stimulates local demand and depreciates exchange rate in response to positive productivity shock. Under PCP, this makes both local and foreign consumers switch to goods produced in country \( i \) increasing the efficiency of the allocation. When prices are sticky in local currency, on the other hand, there is no expenditure switching and the monetary policy can only affect local demand. Since monetary expansion increases demand for all goods, including imported ones, the global planner has to trade off local benefits with negative spillovers on other countries. The optimal response is therefore proportional to the share of local goods \( 1 - \gamma \) and is zero in the limit with no home bias \( \gamma \to 1 \). The implied exchange rates are fully fixed in this case, \( e_i \to 0 \), which resembles the second key result from Devereux and Engel (2003b).\(^\text{14}\)

Interestingly, the regions of PCP and LCP equilibria do not overlap and cover the whole parameter space under the optimal policy (see Figure 3.1a) and are exactly the same as in the flexible-price limit with no productivity shocks in Figure 2.3b. Intuitively, this is because in all three cases the labor wedge is equal zero: in Section 2.3.2 this comes from stable productivity and nominal wages, under PCP implementing the optimal real wage is sufficient to eliminate other wedges as well due to flexible exchange rates, while under LCP the planner cannot affect other distortions in any case. As a result, the relative

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\(^\text{14}\)Importantly, I show in Appendix A.3.2 that the second-order approximation to the planner’s objective function that is derived from market clearing and risk-sharing conditions, does not depend on \( \alpha \) and is the same as for CES aggregator.
volatility of prices and marginal costs expressed in the same currency is constant across three regimes. Since this is a sufficient statistic for exporters’ choice between producer and local currency, the regions of equilibria are the same under flexible prices, PCP and LCP. Notice that LCP equilibrium disappears as the home bias converges to zero, $\gamma \to 1$, which implies that the region where fixed exchange rates are optimal consists of only one point $\alpha = \gamma = 1$.

Turning to the case of dollar pricing, the optimal monetary policy is much closer to the one under PCP than under LCP. When sticky in dollars, the prices of all imported goods move together and it is impossible to generate expenditure switching towards products with lower costs of production. In contrast to LCP case, however, the relative demand for home vs. foreign goods does depend on exchange rates, and the planner finds it optimal to follow the same policy as under PCP. In addition, the planner fully stabilizes dollar exchange rate because the losses from the suboptimal demand for U.S. goods are infinitely small when $n = 0$, while distortions coming from fluctuations in prices invoiced in dollars are large. Because of the intermediate degree of expenditure switching, the implemented allocation under DCP is less efficient than under PCP, but is more efficient than under LCP.

In contrast to PCP and LCP, dollar pricing generates strategic complementarities between firms’ invoicing decisions and the monetary policy. When firms choose PCP/LCP, the policy is symmetric across countries and dollar has no volatility advantage given the same volatility of productivity shocks in the U.S. as in other economies. This policy therefore provides no incentives for firms to set prices in dollars. On the other hand, the planner optimally sets $e_0 = 0$ when firms choose DCP no matter how volatile the productivity
Figure 3.1: Currency choice under the optimal discretionary monetary policy

Note: the figure shows the equilibrium invoicing under the optimal discretionary monetary policy given the following values of parameters: $\lambda = 0$, $\phi = 0.5$, $n = 0$, $\theta = 2$. Plot (a) assumes cooperative policy, while plot (b) assumes that U.S. economy is closed and the monetary policy there is chosen independently from other countries with $\sigma_{a_0}^2 = \sigma_{r}^2$. The solid line shows the boundary between PCP and LCP, while the dashed one shows the boundary of DCP region.

shocks in the U.S. are. The lower volatility of dollar makes it a more appealing vehicle currency and stimulates firms to choose DCP.

3.3.3 Non-cooperative policy

As Section 3.2 suggests, there might be significant spillover effects from U.S. policy on other countries when international prices are set in dollars. These spillovers are fully offset by the global planner, which minimizes the volatility of dollar exchange rate under DCP. This section relaxes the assumption of a cooperative monetary policy and derives the optimal response of other countries to U.S. monetary shocks. Since solving for non-cooperative policy is analytically challenging and usually requires restrictive assumptions on parameter values (see e.g. Benigno and Benigno 2003, Corsetti and Pesenti 2001, Farhi and Werning 2012), I consider the limiting case when U.S. economy is closed, $\eta \to 0$, and
the monetary policy of all other countries remains cooperative. This captures the fact that the optimal policy in the U.S. is more inward-looking than in small economies: in the closed economy limit, it is independent from the global trade invoicing and the policy in other countries.

**Proposition 3.5 (Non-cooperative policy)** Under the optimal non-cooperative discretionary policy,

1. exchange rates between non-U.S. countries are freely floating under both DCP and PCP, and depend only on relative productivities between countries: \( e_{ij} = \frac{a_i - a_j}{1 - (1 - \gamma) \varphi} \),

2. under DCP, monetary policy in all countries comoves positively with the U.S. one and partially smooths out exchange rates against dollar relative to PCP case,

3. DCP region is non-empty even when volatility of \( a_i \) is the same across countries.

The optimal policy in the U.S. adjusts aggregate demand in response to local productivity shocks and achieves efficient allocation within the country. When international prices are set in producer or local currency, the U.S. policy has zero effect on other countries and the equilibrium exchange rates are the same as under full cooperation. Thus, both PCP and LCP equilibria remain the same as described in Proposition 3.4. In contrast, under DCP, there are two types of shocks that policymakers face — local changes in productivity and external movements in terms-of-trade driven by fluctuations in dollar exchange rate. The optimal response to the former is the same as before: while incomplete, the expenditure switching between domestic and imported goods allows to reallocate global demand towards products with lower costs. As a result, the bilateral exchange rates between non-U.S. countries remain the same as under cooperative policy.
The fluctuations in terms-of-trade between third countries that come from movements in dollar exchange rate, on the other hand, are distortionary as they do not reflect relative productivities of the economies. The optimal policy therefore “leans against the wind” and partially offsets movements in $\epsilon_0$, which implies that bilateral exchange rates against dollar are less volatile under DCP than under PCP.\footnote{This result contrasts with the conclusion of Goldberg and Tille (2009) that monetary policy of periphery countries should focus exclusively on local shocks as the latter model does not take into the account losses from price dispersion. The analysis also complements the general result from Casas, Diez, Gopinath, and Gourinchas (2017) that the optimal policy targets price misalignments under DCP.} While exchange rate stabilization allows bringing relative prices across countries closer to the efficient level, such policy is costly as it distorts relative prices within countries. As a result, U.S. shocks are only partially offset under the optimal policy, and the equilibrium exchange rate is neither floating nor fixed. This prediction of the model is consistent with the empirical fact that more than 70\% of countries follow managed float regime ("crawling peg", "dirty float") and use dollar as an anchor currency in their exchange rate policy (Ilzetzki, Reinhart, and Rogoff 2017, Calvo and Reinhart 2002). Proposition 3.5 also contributes to the recent debate about implications of dollar invoicing for the “trilemma”: while the trade-off is worsened by DCP relative to PCP benchmark, the flexible exchange rates still allow to achieve higher welfare than the fixed ones (see Bernanke 2017, Gopinath 2017).

The fact that all economies respond to movements in U.S. exchange rate also implies that monetary policy is correlated across countries despite the assumption that fundamental shocks are purely idiosyncratic. An expansionary monetary policy in the U.S. leads to depreciation of dollar exchange rate and makes central banks in other countries to ease their policy as well. This is consistent with the evidence on the global financial cycles (Rey 2015) and shows that a positive comovement of monetary policy across countries...
can arise not only due to financial linkages, but also because of the dominant status of dollar in international trade (cf. Aoki, Benigno, and Kiyotaki 2016).

Finally, the monetary policy feeds back into firms’ currency choice. Even when volatility of fundamental shocks is the same for the U.S. as for other countries, the optimal policy of pegging exchange rates to dollar implies that dollar is more stable than other currencies and hence, exporters are more likely to use DCP. Thus, the model predicts strategic complementarities between firms’ invoicing decisions and the monetary policy: DCP makes it optimal to peg exchange rates to dollar, which in turn increases incentives of exporters to set prices in dollars. Figure 3.1b shows that the resulting region of DCP equilibrium can be large even when the U.S. has no fundamental advantage.16

3.3.4 Ramsey problem

Consider next the Ramsey problem of a global planner, which optimally chooses monetary policy internalizing its effect on firms’ invoicing decisions. The equilibrium currency choice under the optimal policy is shown in Figure 3.2a and can be understood through the lense of Proposition 3.4. Notice first that the ordering of welfare under alternative currency regimes implies that the planner chooses PCP (DCP) when it coexists with DCP (LCP) by switching to the corresponding monetary policy. As explained above, the reason is that PCP allows to achieve more expenditure switching than DCP, which in turn generates more expenditure switching than LCP.

Furthermore, the planner decides to deviate from the optimal ex-post policy and to use

---

16While equilibrium exchange rates and welfare implications depend on the type of exogenous shock, the results about partial peg to dollar, global cycles in monetary policy and non-empty DCP region are robust and hold in particular for financial shocks.
monetary instruments to affect firms’ currency choice. For the intuition, consider the border between PCP and LCP regions in Figure 3.1a. While exporters are indifferent between two invoicing regimes at these values of parameters, there is a jump in social welfare as economy switches from PCP to LCP. The envelope condition at this border implies that for given invoicing regime, small changes in monetary policy have only second-order costs from non stabilizing PPI, but can have first-order benefits as they make firms switch to PCP. In other words, to generate more expenditure switching, it might be optimal to move exchange rates less, but make firms choose producer currency instead of local one. The same logic applies when firms are indifferent between PCP and DCP.

The planner therefore trades off the optimal movements in exchange rates with the pass-through of these shocks into international prices. In particular, to implement the PCP equilibrium, the planner decreases the volatility of exchange rates (see Figure 3.2b). The depreciation of local exchange rate in response to a positive productivity shock allows
firm to decrease its relative price in other markets under PCP. As response of exchange rates to productivity shocks becomes more muted, the firm’s desired pass-through goes up making PCP more attractive.

**Proposition 3.6** The Ramsey allocation can be implemented via off-equilibrium policies.

The implementation of the Ramsey allocation is nuanced in this setting. As shown above, while solution to Ramsey problem is unique, multiple equilibria can emerge under the optimal policy. I therefore use “sophisticated policies” approach, which was first developed by Atkeson, Chari, and Kehoe (2010) to solve the indeterminacy problem in a context of a standard New-Keynesian model. Loosely speaking, I specify the monetary policy out of equilibrium, i.e. conditional on actions of firms, to “punish” the latter for choosing the “wrong” currency. Of course, this requires much commitment on the side of the planner since the optimal ex-post policy is very different from the ex-ante one.

### 3.4 Conclusion

Despite small private costs, the currency choice of exporters has large aggregate effects. This chapter shows that the spillover effects of dollar depreciation on foreign output are more positive when international prices are set in dollars than predicted by the standard models with producer/local currency pricing. A simple calibration of the model suggests that the net effect is positive, which contrasts with the “beggar-thy-neighbour” view of monetary policy.

The optimal policy analysis, on the other hand, shows a close relation between the dominant status of dollar in the international trade and the wide use of dollar as an anchor.
currency in exchange rate policy. To dampen the international spillovers of U.S. monetary policy arising under dollar currency pricing, other countries find it optimal to partially peg their exchange rates to dollar. This decreases the volatility of U.S. exchange rate and makes dollar pricing more attractive. The model therefore rationalizes the wide use of “crawling peg” to dollar in real world and suggests there is a close connection between the evolution of the international price system and the international monetary system.
Appendix A
Supplemental Material

A.1 Appendix for Chapter 1

A.1.1 Additional tables and figures

Figure A1: Illustration: Consumption-RER relationship

Note: Illustration for Section 1.3.4. A RER depreciation ($q_t \uparrow$) has two effects corresponding to equations (1.36) and (1.37) in the text. The latter is the expenditure switching towards domestic goods, which increases $\tilde{y}_t$ given $\tilde{c}_t$. The former is the reduction in the home real wage, which reduces the supply of labor and home goods $\tilde{y}_t$ given $\tilde{c}_t$. The joint equilibrium in the goods and labor market, hence, requires a reduction in $\tilde{c}_t$, with in general ambiguous effect on output $\tilde{y}_t$. 

$\tilde{c}_t = (c_t - \hat{c}_t)/2$

$\tilde{y}_t = (1 - \phi)(1 - 2\gamma) \frac{1 - \phi(1 - 2\gamma)}{1 - \phi(1 - 2\gamma)} \tilde{c}_t + \gamma\kappa_2 q_t$

Goods market clearing

$\tilde{y}_t = \frac{(1 - \phi)(1 - 2\gamma)}{1 - \phi(1 - 2\gamma)} \tilde{c}_t + \gamma\kappa_2 q_t$

Goods market clearing

$\sigma \nu \tilde{c}_t + \tilde{y}_t = -\gamma \kappa_1 q_t$

Labor market clearing

$\sigma \nu \tilde{c}_t + \tilde{y}_t = -\gamma \kappa_1 q_t$

Labor market clearing
Figure A2: Relative impulse responses to shocks as a function of $\gamma$

Note: The figure plots $\frac{\delta z_t}{\delta e_t} = \frac{\partial z_t}{\partial e_t} \frac{\partial e_t}{\partial e_t}$ for three variables $z_t \in \{p_t - p^*_t, c_t - c^*_t, y_t - y^*_t\}$ (relative price level, relative consumption and relative output respectively) and shocks $e_t \in \Omega_t = \{w_t, a_t, g_t, \kappa_t, \mu_t, \chi_t, \eta_t, \xi_t, \psi_t\}$ across models with different home bias parameter $\gamma \in [0, 0.15]$ and the other parameters as in Table 1.2. For three shocks ($\eta_t, \xi_t, \psi_t$), the impulse responses for all three $z_t$ are negligible relative to $e_t$ in the autarky limit ($\gamma \to 0$), and tend to monotonically depart away from zero with $\gamma > 0$. For the other five shocks ($w_t, a_t, g_t, \kappa_t, \mu_t$), the impulse response for at least one $z_t$ is of the same order of magnitude as that for $e_t$, even near $\gamma = 0$. The $\chi_t$ shock is equivalent to $\psi_t$ shock in terms of its effect on prices and quantities, but they differ in their effect on interest rates (not shown). See discussion in Section 1.2.2 and Propositions 1.1–1.2.

Figure A3: Persistence of the real exchange rate $q_t$

Note: Left panel: OLS estimates $\hat{\rho}_q$ from projection $q_t = \rho_q q_{t-1} + \epsilon^q_t$. Right panel: corresponding half-life estimates calculated according to $HL_q = \frac{\log 0.5}{\log \hat{\rho}_q}$. Based on 10,000 simulations with 120 quarters (30 years) each, where the solid lines plot the median estimates and the areas are the 90% bootstrap sets. The dotted lines in the right panel indicate the conventional 3–5 year half life estimates in the data (Rogoff 1996).
Figure A4: Volatility of prices and quantities relative to real exchange rate

Note: The figures plot the regions in the model parameter space resulting in the empirically-relevant volatilities of prices (terms of trade $s_t$, left panel) and quantities (relative consumption $c_t - c_t^*$, right panel) relative to the volatility of the real exchange rate $q_t$. The left panel is in the $(\alpha, \gamma)$-space and the yellow region within dashed lines corresponds to the parameter combinations that result simultaneously in $\text{var}(\Delta s_t) < \text{var}(\Delta q_t)$ and $\text{corr}(\Delta s_t, \Delta q_t) > 0$, while within the solid lines $\frac{\text{std}(\Delta s_t)}{\text{std}(\Delta q_t)} < 0.5$ (refer to Proposition 1.6). The right panel is in the $(\theta(1-\alpha), \gamma)$-space with the yellow region under the solid line resulting in $\frac{\text{std}(\Delta c_t - \Delta c_t^*)}{\text{std}(\Delta q_t)} < 0.5$ (refer to Proposition 1.7). The region above the dashed line corresponds to the possible parameter combinations for oligopolistic competition under CES demand (as in Atkeson and Burstein 2008a), which does not jointly allow for such $\theta$ and $\alpha$ that result in a quantitatively moderate response of consumption.

Figure A5: Fama regression, UIP deviations and Carry trade returns

Note: Monte Carlo study of the baseline model (with parameters from Table 1.1) based on 10,000 simulations of the model with 120 quarters (30 years). The solid lines plot the median estimates across simulations, the areas represent 90% bootstrap sets, and the red dotted lines are the asymptotic values. Panel (a) plots the $\beta$ coefficient from the Fama regression of $\Delta e_{t+1}$ on $(i_t - i_t^*)$, while panel (b) plots the $R^2$ from this regression. Panel (c) plots the unconditional within-sample Sharpe ratio calculated as the coefficient of variation for the carry return $r^C_{t+1} = \psi_t \cdot (i_t - i_t^* - \Delta e_{t+1})$, as defined in (A71).
Table A1: Quantitative properties of the baseline model and robustness

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Baseline</th>
<th>Robustness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(\theta = 2.5)</td>
<td>(\alpha = 0)</td>
</tr>
<tr>
<td>(\rho(\Delta e))</td>
<td>0.00</td>
<td>-0.02</td>
<td>-0.05</td>
</tr>
<tr>
<td>(\rho(q))</td>
<td>0.95</td>
<td>0.93</td>
<td>0.87</td>
</tr>
<tr>
<td>(HL(q))</td>
<td>12.0</td>
<td>9.9</td>
<td>4.9</td>
</tr>
<tr>
<td>(\sigma(\Delta q)/\sigma(\Delta e))</td>
<td>0.99</td>
<td>0.75</td>
<td>0.54</td>
</tr>
<tr>
<td>(\sigma(\Delta s)/\sigma(\Delta q))</td>
<td>0.35</td>
<td>0.30</td>
<td>1.16</td>
</tr>
<tr>
<td>(\sigma(\Delta q^2)/\sigma(\Delta q))</td>
<td>0.98</td>
<td>1.10</td>
<td>1.16</td>
</tr>
<tr>
<td>(\sigma(\Delta c - \Delta c^*)/\sigma(\Delta q))</td>
<td>0.20</td>
<td>0.31</td>
<td>0.42</td>
</tr>
<tr>
<td>(\sigma(\Delta gdp - \Delta gdp^*)/\sigma(\Delta q))</td>
<td>0.25</td>
<td>0.19</td>
<td>0.44</td>
</tr>
<tr>
<td>(\sigma(\Delta nx)/\sigma(\Delta q))</td>
<td>0.10</td>
<td>0.25</td>
<td>0.65</td>
</tr>
<tr>
<td>Fama (\beta_F)</td>
<td>(\leq 0)</td>
<td>-8.1</td>
<td>(4.7)</td>
</tr>
<tr>
<td>Fama (R^2)</td>
<td>0.02</td>
<td>0.04</td>
<td>0.07</td>
</tr>
<tr>
<td>(\sigma(i - i^*)/\sigma(\Delta e))</td>
<td>0.06</td>
<td>0.03</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Carry (SR)</td>
<td>0.20</td>
<td>0.21</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Notes: The baseline model corresponds to the model of Section 1.3 with parameters as in Table 1.1. The table reports robustness analysis with respect to deviations of various parameters from their baseline values. The moments are robust to changes in \(\nu\), \(\phi\), \(\mu\) and \(\beta\) (not reported for brevity). The robustness panel of the table shows only the moments that are sensitive to the change in the parameter values. Data moments and notation as in Table 1.2, and \(HL(\cdot)\) corresponds to the half-life estimate. The dynamic moments are calculated as the median of the in-sample estimates across 10,000 simulations with 30 years (120 quarters) each and the standard deviation across simulations are reported in brackets. The asymptotic values of the estimates are similar to the medians except for \(\rho(q) \to 1\), \(HL(q) \to \infty\) and \(\beta_F \to -4.6\). The five sections in the table correspond to the five puzzles defined in Section 2.1 and addressed in Section 1.3.1–1.3.5 respectively. See Appendix A.1.5 for further discussion.
Table A2: Unconditional correlations between exchange rate and interest rates

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NOEM (1)</td>
<td>IRBC (2)</td>
</tr>
<tr>
<td>( \text{corr}(\epsilon_t, \epsilon_t - \epsilon_t^*) )</td>
<td>-0.09 (0.09)</td>
<td>0.40 (0.34)</td>
</tr>
<tr>
<td>( \text{corr}(\Delta \epsilon_t, \Delta \epsilon_t - \Delta \epsilon_t^*) )</td>
<td>-0.18 (0.05)</td>
<td>0.38 (0.08)</td>
</tr>
</tbody>
</table>

Note: Models: (1) multi-shock NOEM (from column 5 of Table 1.2); (2) multi-shock IRBC (column 6 of Table 1.2); (3) single-\( \psi_t \)-shock model with a financial sector; (4) multi-shock model with a financial sector (column 7 of Table 1.2). See Appendix A.1.9. The numbers in the brackets give standard errors calculated as Newey-West standard errors with 12 lags (months) for the data moments and as standard deviations across 10,000 simulations for the model-simulated moments.
A.1.2 Demand structure

Consider the general separable homothetic (Kimball 1995) demand aggregator $C_t$ defined implicitly by:

$$\Omega_{Ht} \left( \frac{C_{Ht}}{\Omega_{Ht} C_t} \right) + \Omega_{Ft} \left( \frac{C_{Ft}}{\Omega_{Ft} C_t} \right) = 1,$$

(A1)

where $g' > 0$, $g'' < 0$ and $g(1) = g'(1) = 1$ (a normalization), and

$$\Omega_{Ht} \equiv (1 - \gamma)e^{-\gamma \xi_t} \quad \text{and} \quad \Omega_{Ft} \equiv \gamma e^{(1 - \gamma) \xi_t}$$

are the weights which satisfy the required properties. Then expenditure minimization results in the following demand schedules:

$$C_{Ht} = (1 - \gamma)e^{-\gamma \xi_t} h \left( \frac{P_{Ht}}{D_t} \right) C_t \quad \text{and} \quad C_{Ft} = \gamma e^{(1 - \gamma) \xi_t} h \left( \frac{P_{Ft}}{D_t} \right) C_t,$$

(A2)

where $h(x) \equiv g^{-1}(x)$, which implies $h' < 0$ and $h(1) = 1$, and where $D_t$ is the Lagrange multiplier on the aggregator (A1). We can solve for $D_t$ by substituting demand (A2) into the aggregator (A1):

$$(1 - \gamma)e^{-\gamma \xi_t} g \left( h \left( \frac{P_{Ht}}{D_t} \right) \right) + \gamma e^{(1 - \gamma) \xi_t} g \left( h \left( \frac{P_{Ft}}{D_t} \right) \right) = 1,$$

(A3)

and also define the price index in this economy from the expenditure per unit of $C_t$:

$$P_t = \frac{P_{Ht} C_{Ht} + P_{Ft} C_{Ft}}{C_t} = (1 - \gamma)e^{-\gamma \xi_t} P_{Ht} h \left( \frac{P_{Ht}}{D_t} \right) + \gamma e^{(1 - \gamma) \xi_t} P_{Ft} h \left( \frac{P_{Ft}}{D_t} \right).$$

(A4)

**Proof of Lemma 1.1** It is immediate to check from (A3)–(A4), using $g(1) = h(1) = 1$, that when $\xi_t = 0$ and $P_{Ht} = P_{Ft}$, we have $P_t = D_t = P_{Ht} = P_{Ft}$, which corresponds to the symmetric steady state. Using demand (A2), we immediately have that the symmetric
steady state foreign share is:

\[
\frac{P_{Ft}C_{Ft}}{P_{Ht}C_{Ht} + P_{Ft}C_{Ft}}\bigg|_{\xi_t=0, \ p_{Ht}=p_{Ft}} = \frac{\gamma C_t}{(1 - \gamma)C_t + \gamma C_t} = \gamma,
\]

where we again use \(h(1) = 1\). We next use (A3)–(A4) to obtain the log-linear approximation for \(P_t\) and \(D_t\) around the symmetric steady state:\(^1\)

\[
p_t = d_t = (1 - \gamma)p_{Ht} + \gamma p_{Ft},
\]

where \(p_t\) and \(d_t\) are the log deviations from the steady state values. Note that the taste shock \(\xi_t \neq 0\) does not affect the first order approximation to the prices index (due to the way it enters the weights \(\Omega_{Ht}\) and \(\Omega_{Ft}\)). Finally, log-linearizing (A2), we have:

\[
c_{Ht} = -\gamma \xi_t - \theta(p_{Ht} - d_t) + c_t \quad \text{and} \quad c_{Ft} = (1 - \gamma)\xi_t - \theta(p_{Ft} - d_t) + c_t,
\]

where \(\theta \equiv \left. -\frac{h'(x)x}{h(x)} \right|_{x=1} = \left. -\frac{\partial \log h(x)}{\partial x} \right|_{x=1}\). Together with (A5), these expressions results in (1.7). Subtracting, we have \(c_{Ft} - c_{Ht} = \xi_t - \theta(p_{Ft} - p_{Ht})\), which implies that the elasticity of substitution is indeed \(\theta\). ■

**Monopolistic competition and price setting** Consider now a unit continuum of symmetric domestic firms with marginal cost \(MC_t\) and a unit continuum of symmetric foreign firms with marginal cost \(E_tMC_t^*\) monopolistically competing in the domestic market. We generalize the consumption aggregator \(C_t\) to be defined in the following way:

\[
\int_0^1 \Omega_{Ht} g\left(\frac{C_{Ht}(i)}{\Omega_{Ht}C_t}\right) \, di + \int_0^1 \Omega_{Ft} g\left(\frac{C_{Ft}(i)}{\Omega_{Ft}C_t}\right) \, di = 1,
\]

\(^1\)In the CES case, which obtains with \(g(z) = \frac{1}{\theta - 1}(\theta z^{\frac{\theta - 1}{\theta}} - 1)\), we have \(P_t = D_t\), while for a more general demand \(P_t\) and \(D_t\) different by a second order term around a symmetric steady state. Since our analysis relies on the first order approximation to the equilibrium system, we replace \(D_t\) with \(P_t\) in the demand equations (1.5) in the text.
with taste shocks \((\Omega_{Ht}, \Omega_{Ft})\) determined as above by a common home bias parameter \(\gamma\) and a common demand shifter \(\xi_t\) for all varieties \(i \in [0, 1]\). The households choose \(\{C_{Ht}(i), C_{Ft}(i)\}\) to maximize \(C_t\) given prices and total expenditure:

\[
E_t = P_tC_t = \int_0^1 P_{Ht}(i)C_{Ht}(i)di + \int_0^1 P_{Ft}(i)C_{Ft}(i)di. \tag{A7}
\]

This expenditure minimization results in individual firm demand as in (A2). A representative home firm takes \((C_t, P_t, D_t)\) as given and sets its price to maximize profits from serving the domestic market:

\[
P_{Ht}(i) = \arg \max_{P_{Ht}(i)} \left\{ (P_{Ht}(i) - MC_t)(1 - \gamma)e^{-\gamma\xi_t}h \left( \frac{P_{Ht}(i)}{D_t} \right) C_t \right\},
\]

which results in the standard markup pricing rule, with the markup \(\mathcal{M}_t\) determined by the elasticity of the demand curve \(h(\cdot)\). Since all domestic firms are symmetric, we have \(C_{Ht} = C_{Ht}(i)\) and \(P_{Ht} = P_{Ht}(i)\) for all \(i \in [0, 1]\). Similar price setting rule is used by symmetric foreign firms with marginal costs \(\mathcal{E}_tMC^*_t\), and we also have \(C_{Ft} = C_{Ft}(i)\) and \(P_{Ft} = P_{Ft}(i)\) for all \(i \in [0, 1]\). Following the proof of Lemma 1.1, the elasticity of demand in a symmetric steady state equals \(\theta\), and therefore the steady state markup is given by \(\mathcal{M} = \theta/(\theta - 1)\) for both home and foreign firms.

We next take a log-linear approximation to the optimal price \(P_{Ht}\) around the symmetric steady state:

\[
p_{Ht} = -\Gamma(p_{Ht} - p_t) + mc_t,
\]

where we use the approximation \(d_t = p_t\) and \(\Gamma\) denotes the elasticity of the markup \(\mathcal{M}_t\) with respect to the relative price of the firm, evaluated at the symmetric steady state. Note that this equation is the counterpart to (1.12) in the text with \(\alpha \equiv \frac{\Gamma}{1+\Gamma}\) and \(\mu_t = 0\).
Lastly, we provide further details about the primitive determinants of $\theta$ and $\alpha$ (see Amiti, Itskhoki, and Konings 2016a, for a more indepth exposition). Define the demand elasticity function $\tilde{\theta}(x) \equiv -\frac{\partial \log h(x)}{\partial \log x}$, so that $\theta \equiv \tilde{\theta}(1)$. Then the markup function is given by $\tilde{M}(x) \equiv \frac{\tilde{\theta}(x)}{\tilde{\theta}(x) - 1}$, and the elasticity of the markup is given by $\tilde{\Gamma}(x) \equiv -\frac{\partial \log \tilde{M}(x)}{\partial x}$, with $M = \tilde{M}(1)$ and $\Gamma = \tilde{\Gamma}(1)$. Manipulating these definitions, we can represent

$$\tilde{\Gamma}(x) = \frac{\tilde{\epsilon}(x)}{\tilde{\theta}(x) - 1}, \quad \text{where} \quad \tilde{\epsilon}(x) \equiv \frac{\partial \log \tilde{\theta}(x)}{\partial \log x}$$

is the elasticity of elasticity (or super-elasticity) of demand. Therefore, $\Gamma = \frac{\epsilon}{\theta - 1}$, where $\epsilon = \tilde{\epsilon}(1)$, and we further have:

$$\alpha = \frac{\Gamma}{1 + \Gamma} = \frac{\epsilon}{\epsilon + \theta - 1}.$$

To the extent $\epsilon$ and $\theta$ are controlled by independent parameters, we can decouple the elasticity of substitution $\theta$ from the strategic complementarity elasticity $\alpha$. Indeed, $\theta$ is a characteristic of the slope (the first derivative) of demand $h'$, while $\epsilon$ is a characteristic of the curvature (the second derivative) of demand $h''$. Formally, we have:

$$\theta = \left. -\frac{h'(x)x}{h(x)} \right|_{x=1}, \quad \epsilon = \left. \frac{\partial \log \tilde{\theta}(x)}{\partial \log x} \right|_{x=1} = \left[ 1 - \frac{h'(x)x}{h(x)} + \frac{h''(x)x}{h'(x)} \right]_{x=1} = 1 + \frac{h''(x)x}{h'(x)} \right|_{x=1}.$$

We assume that the demand schedule $h(\cdot)$ is log-concave, that is $\epsilon \geq 0$, and therefore $\alpha \in [0, 1)$, since $\theta > 1$ is the second order requirement for price setting optimality. An appropriate choice of $\epsilon$ produces any required value of $\alpha$ for any given value of $\theta$. A suitable parametric example can be found in Klenow and Willis (2006) and Gopinath and Itskhoki (2010a), where $h(x) = [1 - \epsilon \log(x)]^{\theta / \epsilon}$ for some elasticity parameter $\theta > 1$ and super-elasticity parameter $\epsilon > 0$. 
A.1.3 Equilibrium system

We summarize here the equilibrium system of the general model from Section 1.2.1 by breaking it into blocks:

1. Labor supply (1.3) and its exact foreign counterpart.

2. Labor demand in (1.10), used together with the definition of the marginal cost (1.9), and its exact foreign counterpart.

3. Demand for home and foreign goods:

\[ Y_t = Y_{Ht} + Y_{Ht}^* \quad \text{and} \quad Y_t^* = Y_{Ft} + Y_{Ft}^*, \]  

(A8)

where the sources of demand for home good are given in (1.17) and (1.18), and the counterpart sources of demand for foreign good are given by:

\[ Y_{Ft} = \gamma e^{(1-\gamma)\xi_t} h \left( \frac{P_{Ft}}{P_t} \right) [C_t + X_t + e^{\eta t}], \]  

(A9)

\[ Y_{Ft}^* = (1 - \gamma) e^{-\gamma \xi_t} h \left( \frac{P_{Ft}^*}{P_t^*} \right) [C_t^* + X_t^* + e^{\eta_t}], \]  

(A10)

where \( X_t \) and \( X_t^* \) satisfy the intermediate good demand in (1.10) and its foreign counterpart.

4. Supply of goods: given price setting (1.12)–(1.13) and their foreign counterparts given by:

\[ P_{Ft} = e^{\mu_t + \eta_t} (MC_t^* \mathcal{E}_t)^{1-\alpha} P_t^\alpha, \]  

(A11)

\[ P_{Ft}^* = e^{\mu_t} MC_t^{*1-\alpha} P_t^{*\alpha}, \]  

(A12)

\[ ^2\text{Note that the input demand equations (1.10) together with the marginal cost (1.9) imply the production function equation (1.8).} \]
output produced is determined by the demand equation (A8).

Given prices \((P_{HT}, P_{HT}^*, P_{FT}, P_{FT}^*)\), equation (1.6) defines the price level \(P_t\) as a log-linear approximation, and a similar equation defines \(P_t^*\).  

5. Asset demand by home and foreign households (1.4) and (1.15), which can be rewritten as an international risk sharing condition and a no-arbitrage condition:

\[
\mathbb{E}_t \left\{ e^{\psi_t} \Theta_{t+1} \frac{E_{t+1}}{E_t} - \Theta^*_{t+1} \right\} = 0, \tag{A13}
\]

\[
\mathbb{E}_t \left\{ \Theta_{t+1} \left[ e^{\psi_t} R_t \frac{E_{t+1}}{E_t} - R_t \right] \right\} = 0, \tag{A14}
\]

with the stochastic discount factors \(\Theta_{t+1}\) and \(\Theta^*_{t+1}\) defined in the text.

6. Home-country flow budget constraint (1.19), with its foreign counterpart redundant by Walras Law.

**Symmetric steady state**

In a symmetric steady state, \(B^* = B^{*F} = 0\), and the shocks (defined in Table 1.1) take the following values:

\[
\psi = \xi = \xi^* = \eta = \eta^* = \chi = \chi^* = 0,
\]

and we normalize \(W = W^* = 1\) (corresponding to \(w = w^* = 0\)). We let the remaining shocks take arbitrary (zero or non-zero) symmetric values:

\[
a = a^*, \quad g = g^*, \quad \kappa = \kappa^* \quad \text{and} \quad \mu = \mu^*.
\]

\(^3\)Log-linear expression for \(p_t\) in (1.6) can be replaced with two non-linear expressions (A3)–(A4) defining \((P_t, D_t)\), and \(P_t\) should be replaced with \(D_t\) in demand equations (1.17)–(1.18) and (A9)–(A10). The rest of the equilibrium system stays unchanged. However, these adjustments do not have first order consequences, as \(P_t\) and \(D_t\) are the same up to second order terms, and therefore the log-linearized system in Appendix A.1.3 is unchanged.
We start with the equations for prices. In a symmetric steady state, exchange rates and terms of trade are equal to 1:

$$E = Q = S = 1,$$  \hspace{1cm} (A15)

and therefore we can evaluate the prices using the equilibrium conditions described above:

$$P = P^* = P_H = P^*_H = P_F = \left[ \frac{e^{\frac{\mu - \alpha}{1 - \phi}}}{(1 - \phi)^{1 - \phi} \phi^\phi} \right]^{\frac{1}{1 - \phi}},$$  \hspace{1cm} (A16)

with the marginal costs given by

$$MC = MC^* = \frac{e^{-\alpha} P^\phi}{(1 - \phi)^{1 - \phi} \phi^\phi} = \left[ \frac{e^{\frac{\phi \mu - \alpha}{1 - \phi} - \frac{\phi}{1 - \phi} \phi^\phi}}{} \right]^{\frac{1}{1 - \phi}}.$$

Next we use these expressions together with production function, labor demand and labor supply to obtain two relationships for \((C, Y, L)\):

$$L = e^{\frac{\phi}{1 - \phi} \frac{\mu}{1 - \alpha} - \frac{\alpha}{1 - \phi} \phi^{-\frac{\phi}{1 - \phi}} Y,}$$  \hspace{1cm} (A17)

$$C^\sigma L^{1/\nu} = \frac{e^{-\kappa}}{P} = e^{\frac{\alpha}{1 - \phi} - \frac{\mu}{1 - \alpha} - \frac{\mu}{1 - \phi} \phi^{-\frac{\phi}{1 - \phi}} (1 - \phi) \phi^{-\phi}}.$$  \hspace{1cm} (A18)

Substituting prices (and using \(h(1) = 1\)) and intermediate good demand \(X = \phi \frac{MC}{P} Y = e^{\frac{\mu - \alpha}{1 - \alpha} \phi} Y\) into the goods market clearing, we obtain an additional relationship between \(C\) and \(Y\):

$$C + e^\theta = \left[ 1 - e^{\frac{\mu}{1 - \alpha} \phi} \right] Y.$$  \hspace{1cm} (A19)

We further have \(Y = Y^*\), and \(Y_H = Y_F^* = (1 - \gamma)Y\) and \(Y_H^* = Y_F = \gamma Y\).

The asset demand conditions imply that \(R = R^* = 1/\beta\).
Lastly, we define the following useful ratios:

\[ \zeta \equiv \frac{\text{GDP}}{\text{Output}} = \frac{P(C + G)}{P_H Y} = 1 - e^{-\frac{u}{1-\alpha}} \phi, \quad (A20) \]

\[ \gamma \equiv \frac{\text{Import}}{\text{Expenditure}} = \frac{P_F Y_F}{P_H Y_H + P_F Y_F} = \frac{P_F Y_F}{P_H Y} = \gamma, \quad (A21) \]

\[ \text{Import+Export} \quad \frac{\text{GDP}}{\text{GDP}} = \frac{E P^*_H Y^*_H + P_F Y_F}{P(C + G)} = \frac{2\gamma}{\zeta}. \quad (A22) \]

**Log-linearized system**

We log-linearize the equilibrium system (summarized above in Appendix A.1.3) around the symmetric steady state (described in Appendix A.1.3). We split the equilibrium system into three blocks — prices, quantities and dynamic equations — and solve them sequentially, as the equilibrium system is block-recursive.

**Exchange rates and prices** The price block contains the definitions of the price index (1.6) and its foreign counterpart:

\[ p_t^* = \gamma p_{Ht}^* + (1 - \gamma) p_{Ft}^*, \quad (A23) \]

as well as the price setting equations (1.12)–(1.13) and (A11)–(A12), in which we substitute the marginal cost (1.9) and its foreign counterpart and log-linearize to obtain:

\[ p_{Ht} = \mu_t - (1-\alpha)a_t + (1-\alpha)(1-\phi)(w_t - p_t) + p_t, \quad (A24) \]

\[ p_{Ht}^* = \mu_t + \eta_t - (1-\alpha)a_t + (1-\alpha)[(1-\phi)(w_t - p_t) + p_t - e_t] + \alpha p_t^*, \quad (A25) \]

\[ p_{Ft}^* = \mu_t^* - (1-\alpha)a_t^* + (1-\alpha)(1-\phi)(w_t^* - p_t^*) + p_t^*, \quad (A26) \]

\[ p_{Ft} = \mu_t^* + \eta_t^* - (1-\alpha)a_t^* + (1-\alpha)[(1-\phi)(w_t^* - p_t^*) + p_t^* + e_t] + \alpha p_t. \quad (A27) \]
In addition, we use the logs of the definitions of the real exchange rate and terms of trade (1.14) and (1.20):

\[ q_t = p_t^* + e_t - p_t, \]  
(A28)

\[ s_t = p_{Ft}^* - p_{Ht}^* - e_t. \]  
(A29)

First, it is useful to define the log LOP deviations (as in equation (1.14) and in its foreign counterpart):

\[ q_{Ht} \equiv p_{Ht}^* + e_t - p_{Ht} = \eta_t + \alpha q_t, \]  
(A30)

\[ q_{Ft} \equiv p_{Ft}^* + e_t - p_{Ft} = -\eta_t^* + \alpha q_t, \]  
(A31)

where the expression on the right-hand side are obtained by using (A24)–(A27) together with (A28). Then, we combine (A28)–(A29) together with these expressions, to obtain:

\[ s_t = q_P^T - 2\tilde{\eta}_t - 2\alpha q_t, \]  
(A32)

\[ q_t = (1 - \gamma)q_P^T - \gamma s_t, \]  
(A33)

where \( q_P^T = p_{Ft}^* + e_t - p_{Ht} \) is the producer-price-based real exchange rate and we use the tilde notation \( \tilde{x}_t \equiv (x_t - x_t^*)/2 \) for any pair of variables \((x_t, x_t^*)\). Lastly, we solve for \( q_P^T \) and \( s_t \) as function of \( q_t \):

\[ q_P^T = \frac{1 - 2\alpha\gamma}{1 - 2\gamma} q_t - \frac{2\gamma}{1 - 2\gamma} \tilde{\eta}_t, \]  
(A34)

\[ s_t = \frac{1 - 2\alpha(1 - \gamma)}{1 - 2\gamma} q_t - \frac{2(1 - \gamma)}{1 - 2\gamma} \tilde{\eta}_t. \]  
(A35)

Next, we use these solutions together with the expressions for price indexes (1.6) and
\[ p_{Ht} - p_t = -\frac{\gamma}{1 - \gamma}(p_{Ft} - p_t) = \gamma(p_{Ht} - p_{Ft}) = -\frac{(1 - \alpha)\gamma}{1 - 2\gamma}q_t + \frac{\gamma^2 \eta_t - \gamma(1 - \gamma)\eta^*_t}{1 - 2\gamma}, \] 

(A36)

\[ p^*_{Ft} - p^*_t = -\frac{\gamma}{1 - \gamma}(p^*_{Ht} - p^*_t) = \gamma(p^*_{Ft} - p^*_{Ht}) = \frac{(1 - \alpha)\gamma}{1 - 2\gamma}q_t + \frac{\gamma^2 \eta^*_t - \gamma(1 - \gamma)\eta^*_t}{1 - 2\gamma}. \] 

(A37)

Combining these expressions with (A24) and (A26), we can solve for the price levels:

\[ p_t = w_t + \frac{1}{1 - \phi} \left[ \frac{\mu_t - \gamma^2 \eta_t - \gamma(1 - \gamma)\eta^*_t}{1 - 2\gamma} - a_t \right] + \frac{\gamma}{1 - 2\gamma}q_t, \] 

(A38)

\[ p^*_t = w^*_t + \frac{1}{1 - \phi} \left[ \frac{\mu^*_t - \gamma^2 \eta^*_t - \gamma(1 - \gamma)\eta_t}{1 - 2\gamma} - a^*_t \right] - \frac{\gamma}{1 - 2\gamma}q_t, \] 

(A39)

which together allow to solve for the relationship between \( q_t \) and nominal exchange rate \( e_t \):

\[ \left[ (1 - \phi) + \frac{2\gamma}{1 - 2\gamma} \right] q_t = (1 - \phi)e_t - (1 - \phi)2w_t + 2\tilde{a}t - \frac{2\mu_t}{1 - \alpha} + \frac{2\gamma}{1 - 2\gamma} - \frac{\tilde{\eta}_t}{1 - \alpha}. \] 

(A40)

Real exchange rate and quantities The supply side is the combination of labor supply (1.3) and labor demand (1.10) (together with marginal cost (1.9)), which we log-linearize as:

\[ \kappa_t + \sigma c_t + \frac{1}{\sigma} \ell_t = w_t - p_t, \]

(A41)

\[ \ell_t = -a_t - \phi(w_t - p_t) + y_t. \]

(A42)

\(^{4}\)Note from (1.6) that \( p_{Ht} - p_t = \gamma(p_{Ht} - p_{Ft}) \), and we use the following steps to solve for:

\[ p_{Ht} - p_{Ft} = -(p_{Ft} - \hat{p}_{Ht} - e_t) - (p^*_{Ht} + e_t - p_{Ht}) = -(s_t + q_{Ht}) = -(s_t + \alpha q_t + \eta_t), \]

in which we then substitute (A35) to solve out \( s_t \). Similarly, we solve for \( p^*_{Ft} - p^*_t \).
Combining the two to solve out \( \ell_t \), and using (A38) to solve out \((w_t - p_t)\), we obtain:\(^3\)

\[
\nu \sigma c_t + y_t = \frac{1 + \nu}{1 - \phi} a_t - \frac{\nu + \phi}{1 - \phi} \left[ \mu_t - \frac{\gamma^2 \eta^2 - \gamma(1 - \gamma) \eta^*}{1 - 2\gamma} \frac{1}{1 - \alpha} + \frac{\gamma}{1 - 2\gamma} q_t \right] - \nu \kappa_t. 
\]

(A43)

Symmetrically, the same expression for foreign is:

\[
\nu \sigma c_t^* + y_t^* = \frac{1 + \nu}{1 - \phi} a_t^* - \frac{\nu + \phi}{1 - \phi} \left[ \mu_t^* - \frac{\gamma^2 \eta^2 - \gamma(1 - \gamma) \eta^*}{1 - 2\gamma} \frac{1}{1 - \alpha} - \frac{\gamma}{1 - 2\gamma} q_t \right] - \nu \kappa_t^*. 
\]

(A43)

Adding and subtracting the two we obtain:

\[
\nu \sigma \bar{c}_t + \bar{y}_t = \frac{1 + \nu}{1 - \phi} \bar{a}_t - \frac{\nu + \phi}{1 - \phi} \left[ \bar{\mu}_t + \gamma \bar{\eta}_t \right] - \nu \bar{\kappa}_t, \quad \text{(A44)}
\]

\[
\nu \sigma \bar{c}_t + \bar{y}_t = \frac{1 + \nu}{1 - \phi} \bar{a}_t - \frac{\nu + \phi}{1 - \phi} \left[ \bar{\mu}_t - \frac{\gamma}{1 - 2\gamma} \bar{\eta}_t \right] + \frac{\gamma}{1 - 2\gamma} q_t \right] - \nu \bar{\kappa}_t, \quad \text{(A45)}
\]

where \( \bar{x}_t \equiv (x_t + x_t^*)/2 \) and \( \bar{x}_t \equiv (x_t - x_t^*)/2 \) for any pair of variables \((x_t, x_t^*)\).

The demand side is the goods market clearing (A8) together with (1.17)–(1.18), which we log-linearize as:

\[
y_t = (1 - \gamma) y_{Ht} + \gamma y_{Ht}^*, 
\]

\[
y_{Ht} = -\gamma \xi_t - \theta (p_{Ht} - p_t) + \zeta [\xi c_t + (1 - \zeta) g_t] + (1 - \zeta) [(1 - \phi)(w_t - p_t) - a_t + y_t], 
\]

\[
y_{Ht}^* = (1 - \gamma) \xi_t^* - \theta (p_{Ht}^* - p_t^*) + \zeta [\xi c_t^* + (1 - \zeta) g_t^*] + (1 - \zeta) [(1 - \phi)(w_t^* - p_t^*) - a_t^* + y_t^*], 
\]

where \( \zeta \equiv C/(C + G), \) \( \bar{\xi} \equiv P(C + G)/(P_H Y) \), and we used expression (1.10) and (1.9) to substitute for \( X_t \) (and correspondingly for \( X_t^* \)). Combining together, we derive:

\[
y_t - (1 - \zeta) [y_t - 2\gamma \bar{y}_t] - \zeta [c_t - 2\gamma \bar{c}_t] = \gamma \left[ \theta (1 - \alpha) \frac{2(1 - \gamma)}{1 - 2\gamma} - (1 - \zeta) \right] q_t \]

\[+ \zeta (1 - \zeta) [g_t - 2\gamma \bar{g}_t] - \frac{1 - \zeta}{1 - \alpha} [\mu_t - 2\gamma \bar{\mu}_t] + \frac{\theta \gamma (1 - \gamma)}{1 - 2\gamma} \eta_t + \left( \frac{\theta \gamma (1 - \gamma)}{1 - 2\gamma} \right) \eta_t^* - 2\gamma (1 - \gamma) \xi_t, \]

(A46)

where we have slowed out \((w_t - p_t)\) and \((w_t^* - p_t^*)\) using (A38)–(A39) and solved out \((p_{Ht} - \)

\(^3\)A useful interim step is: \( \nu \sigma c_t + y_t = (\nu + \phi)(w_t - p_t) + a_t - \nu \kappa_t. \)
and \( p_t \) and \( (p_H^* - p_t^*) \) using (A36)–(A37). Adding and subtracting the foreign counterpart, we obtain:

\[
\bar{y}_t = \varsigma \bar{c}_t + (1 - \varsigma) \bar{g}_t - \frac{1}{\varsigma} \left( \frac{2 \theta \gamma (1 - \gamma)}{1 - 2 \gamma} - \frac{1 - \varsigma}{1 - \alpha} \right) \bar{\eta}_t, \\
[1 - (1 - 2 \gamma)(1 - \varsigma)] \bar{y}_t = (1 - 2 \gamma) \varsigma \bar{a}_t + (1 - 2 \gamma) \left[ \zeta (1 - \varsigma) \bar{g}_t - \frac{1 - \varsigma}{1 - \alpha} \bar{\mu}_t \right] \\
+ \gamma \frac{1 - \varsigma}{1 - \alpha} \bar{\eta}_t - 2 \gamma (1 - \gamma) \bar{\xi}_t + \gamma \left[ \theta (1 - \alpha) \frac{2 (1 - \gamma)}{1 - 2 \gamma} - (1 - \varsigma) \right] q_t.
\]

An immediate implication of (A44) and (A47) is that \((\bar{y}_t, \bar{c}_t)\) depends only on \((\bar{a}_t, \bar{g}_t, \bar{\kappa}_t, \bar{\mu}_t, \bar{\nu}_t)\) and does not depend on the real exchange rate \(q_t\). In particular, if \(\bar{a}_t = \bar{g}_t = \bar{\kappa}_t = \bar{\mu}_t = \bar{\nu}_t = 0\), then \(\bar{y}_t = \bar{c}_t = 0\). This is the case we focus on throughout the paper, since as we see below the variation in \((\bar{a}_t, \bar{g}_t, \bar{\kappa}_t, \bar{\mu}_t, \bar{\nu}_t)\) does not affect \(q_t\). Combining (A45) and (A48) we can solve for \(\tilde{y}_t\) and \(\tilde{c}_t\). For example, the expression for \(\tilde{c}_t\) is:

\[
\left[ (1 - 2 \gamma) \varsigma (\nu \sigma + \varsigma) + 2 \gamma \nu \sigma \right] \tilde{c}_t = \left[ (1 - 2 \gamma) \varsigma + 2 \gamma \right] \left[ \frac{1 + \nu \bar{a}_t - \nu + \phi}{1 - \phi} \frac{1 - \varsigma}{1 - \alpha} \bar{\mu}_t - (1 - 2 \gamma) \varsigma (1 - \varsigma) \bar{g}_t \right] \\
+ (1 - 2 \gamma) \varsigma \bar{\eta}_t - \gamma \left[ \frac{1 - \varsigma}{1 - \alpha} \bar{\mu}_t - \gamma \right] \left[ \frac{1 - \varsigma}{1 - \alpha} - \frac{\nu + \phi}{1 - \phi} \frac{1 - (1 - \varsigma)(1 - 2 \gamma)}{1 - \alpha} \right] \bar{\eta}_t + 2 \gamma (1 - \gamma) \tilde{\xi}_t \\
- \gamma \left[ \theta (1 - \alpha) \frac{2 (1 - \gamma)}{1 - 2 \gamma} + \nu + \phi \frac{1}{1 - \phi} \frac{1 - \varsigma}{1 - \alpha} \right] q_t.
\]

Lastly, we provide the linearized expression for net exports:

\[
nx_t = \gamma \left( y_H^* - y_F^* - s_t \right),
\]

where \(nx_t = \frac{1}{p_H^*} NX_t\) is linear deviation of net exports from steady state \(NX = 0\) relative to the total value of output. Substituting in the expressions for \(s_t, y_H^*\) and \(y_F^*\), we
obtain:

\[ n_x t = \gamma \left[ \theta (1 - \alpha) \frac{2(1 - \gamma)}{1 - 2\gamma} + \alpha - \frac{1 - \alpha}{1 - 2\gamma} + \frac{2\gamma (1 - \zeta)}{1 - 2\gamma} \right] q_t - 2\gamma \left[ \zeta [\bar{c}_t + (1 - \zeta) \bar{p}_t] + (1 - \zeta) \bar{y}_t \right] + 2\gamma \left[ \frac{1 - \zeta}{1 - \alpha} \bar{\mu}_t - (1 - \gamma) \bar{\xi}_t \right] - 2\gamma \left[ \theta (1 - \gamma) + \frac{1 - \gamma}{1 - 2\gamma} - \frac{1 - \zeta}{1 - \alpha} \right] \bar{\eta}_t. \]  

(A50)

The log-linear approximation to the flow budget constraint (1.19) is given then by:

\[ \beta b^*_t + b^*_t = n_x t, \]  

(A51)

where \( b^*_t = \frac{\bar{\varepsilon}}{\bar{\mu} \bar{y}} B^*_t \) is the linear deviation of the net foreign asset (NFA) position from its steady state value of \( B^* = 0 \) relative to the total value of output (both in foreign currency, using steady state exchange rate of \( \bar{\varepsilon} = 1 \)). Note that the dynamics of \( \bar{\varepsilon}_t \) and \( \bar{R}_t \) has only second order effects on the returns on NFA (and hence drops out from the linearized system), as we approximate around a symmetric steady state with zero NFA position. Equations (A.2.4) is part of the dynamic block.

**Exchange rate and interest rates**

It only remains now to log-linearize the asset demand conditions (1.4) and (1.15), which pins down the equilibrium interest rates, as well as provides an international risk sharing condition:

\[ i_t = \mathbb{E}_t \{ \sigma \Delta c_{t+1} + \Delta p_{t+1} - \Delta \chi_{t+1} \}, \]

\[ i^*_t = \mathbb{E}_t \{ \sigma \Delta c^*_{t+1} + \Delta p^*_{t+1} - \Delta e_{t+1} - \Delta \chi_{t+1} \} - \psi_t, \]

\[ i^*_t = \mathbb{E}_t \{ \sigma \Delta c^*_{t+1} + \Delta p^*_{t+1} - \Delta \chi^*_{t+1} \}, \]

where \( i_t \equiv \log R_t - \log R \) and \( i^*_t \equiv \log R^*_t - \log R^* \). We combine the first two to obtain a no-arbitrage (UIP) condition, the last two to obtain a risk-sharing (Backus-Smith)
condition, and the first with the third to solve for the interest rate differential:

\[ i_t - i_t^* = \mathbb{E}_t \Delta e_{t+1} + \psi_t, \quad (A52) \]

\[ \mathbb{E}_t \left\{ \sigma (\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1} \right\} = \psi_t + \mathbb{E}_t \left\{ 2 \Delta \tilde{\chi}_{t+1} \right\}, \quad (A53) \]

\[ \tilde{i}_t \equiv \frac{1}{2} (i_t - i_t^*) = \mathbb{E}_t \left\{ \sigma \Delta \tilde{c}_{t+1} + \Delta \tilde{p}_{t+1} - \Delta \tilde{\chi}_{t+1} \right\}. \quad (A54) \]

Substituting out \( \Delta c_{t+1} - \Delta c_{t+1}^* = 2 \Delta \tilde{c}_{t+1} \) in (A53) using (A49), we obtain an equation characterizing the expected real depreciation \( \mathbb{E}_t \Delta q_{t+1} \) as a function of exogenous shocks. Together with (A.2.4), in which we substitute (A50), it forms a system of two dynamic equations that describe the equilibrium dynamics of the real exchange rate given the exogenous dynamic processes for the shocks.

### A.1.4 Autarky limit and proofs for Section 1.2.2

**Proof of Propositions 1.1**  The strategy of the proof is to evaluate the log deviations of the macro variables \( z_t \equiv (w_t, p_t, c_t, \ell_t, y_t, i_t) \) from the deterministic steady state (described in Appendix A.1.3) in response to a shock \( \varepsilon_t = V' \Omega_t \neq 0 \). In particular, we explore under which circumstances \( \lim_{\gamma \to 0} z_t = 0 \). It is sufficient to consider the log-linearized equilibrium conditions described in Appendix A.1.3, as providing a counterexample is sufficient for the prove (hence, the focus on the small log deviations is without loss of generality).

Furthermore, the proof does not rely on the international risk sharing conditions, and hence does not depend on the assumptions about the (in)completeness of the international asset markets.

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6We do not impose any restrictions on the process for shocks in \( \Omega_t \), with the exception of the mild requirement that any innovation in \( \Omega_t \) has some contemporaneous effect on the value of shocks in \( \Omega_t \), i.e. we rule out pure news shocks. We discuss examples with specific time series processes for the shocks in the end of this subsection.
To prove the propositions, consider any shock $\varepsilon_t$ with the restriction that

$$\eta_t = \eta^*_t = \xi_t = \xi^*_t = \psi_t \equiv 0.$$  \hspace{1cm} (A55)

We now go through the list of requirements imposed by the first part of the condition (1.21):

1. No wage response $\lim_{\gamma \to 0} w_t = 0$ implies $w_t = 0$, i.e. the unit of account shocks cannot lead to the exchange rate disconnect in the limit.

2. No price level response implies, using (A38) and (A55):

$$\lim_{\gamma \to 0} p_t = w_t + \frac{1}{1 - \phi} \left[ \frac{\mu_t}{1 - \alpha} - a_t \right] = 0,$$

which in light of $w_t = 0$ requires $\mu_t = (1 - \alpha) a_t$, i.e. the markup shocks must offset the productivity shocks to avoid variation in the price level.

When the same requirements are imposed for foreign, it ensures $\lim_{\gamma \to 0} \{q_t - e_t\} = 0$, as immediately follows from the the definition of the real exchange rate $q_t = p^*_t + e_t - p_t$ (see also (A40)).

3. From the labor supply and labor demand conditions (A95)–(A96), no consumption, employment and output response require:

$$\lim_{\gamma \to 0} \left\{ \sigma c_t + \frac{1}{\phi} \ell_t + p_t \right\} = w_t - \kappa_t = 0,$$

$$\lim_{\gamma \to 0} \left\{ y_t - \ell_t + \phi p_t \right\} = a_t + \phi w_t = 0,$$

which then implies $a_t = \kappa_t = w_t \equiv 0$ and by consequence $\mu_t \equiv 0$ from the result above. That is, there cannot be productivity, markup or labor wedge shocks, if the price level, consumption, output and employment are not to respond in the autarky.
4. Rearranging the goods market clearing in the home market (A46), we have:

\[
\lim_{\gamma \to 0} \{ \zeta y_t - \zeta c_t \} = \zeta (1 - \zeta) g_t - \frac{1 - \zeta}{1 - \alpha} \mu_t = 0,
\]

which in light of the above results requires \( g_t \equiv 0 \).

5. Lastly, the home bond demand requires:

\[
\lim_{\gamma \to 0} \{ \sigma \Delta E_{t+1} c_t + \Delta E_{t} p_{t+1} - i_t \} = E_t \Delta \chi_{t+1} = 0,
\]

therefore there cannot be predictable changes in \( \chi_t \) and unpredictable changes in \( \chi_t \) do not affect allocations in a one-period bond economies, hence without loss of generality we impose \( \chi_t \equiv 0 \).

To summarize, the first condition in (1.21) (combined with the absence of \( \eta_t, \xi_t \) and \( \psi_t \) shocks) implies:

\[
w_t = \chi_t = \kappa_t = a_t = \mu_t = g_t \equiv 0,
\]

i.e. no other shock can be consistent with \( \lim_{\gamma \to 0} z_{t+j} = 0 \) for all \( j \geq 0 \), however in the absence of shocks \( \lim_{\gamma \to 0} e_{t+j} = 0 \), violating the second condition in (1.21). A symmetric argument for foreign rules out the foreign counterparts of these shocks. This completes the proof.

Proof of Proposition 1.2. For the proof, we consider the equilibrium system in the autarky limit by only keeping the lowest order terms in \( \gamma \) for each shock or variable.\footnote{For example, consider equation (A40), which we now rewrite as:

\[
q_t - e_t = 2 \left[ \frac{1}{1 - \phi} \left( \tilde{a}_t - \tilde{\mu}_t \frac{1}{1 - \alpha} \right) - \tilde{w}_t \right] + 2 \gamma \frac{1}{1 - \phi} \tilde{\eta}_t,
\]
Throughout the proof we impose \( w_t = \chi_t = \kappa_t = a_t = \mu_t = g_t \equiv 0 \), as well as for their foreign counterparts.

**First**, we consider our three moments of interest when \( \psi_t \) is the only shock, that is we set \( \eta_t = \xi_t \equiv 0 \). For this purpose, it is sufficient to consider the static equilibrium conditions only, as the effect of the \( \psi_t \) shock on the macro variables is exclusively indirect through \( q_t \). Specifically:

1. Consider the near-autarky comovement between the terms of trade and the real exchange rate from (A35):

\[
\lim_{\gamma \to 0} \frac{\text{cov}(\Delta s_t, \Delta q_t)}{\text{var}(\Delta q_t)} = (1 - 2\alpha) > 0 \quad \text{iff} \quad \alpha < \frac{1}{2},
\]

since we have \( \tilde{\eta}_t = 0 \). \( \alpha < 1/2 \) is a necessary parameter requirement for this result, which is borne out in the data, as we discuss in Section 1.3.

2. Consider the near-autarky comovement between the relative consumption and the real exchange rate from (A49), which in the absence of all shocks but \( \psi_t \) simplifies to:

\[
\left[(1-2\gamma)\zeta(\nu\sigma+\varsigma)+2\gamma\nu\sigma\right]c_t = -\gamma \left[\theta(1-\alpha)\frac{2(1-\gamma)}{1-2\gamma} + \frac{\nu + \phi}{1 - \phi} \frac{1}{1 - 2\gamma} - \frac{1 + \nu}{1 - \phi}(1 - \zeta)\right]q_t.
\]

Hence, we have:

\[
\lim_{\gamma \to 0} \frac{1}{\gamma} \frac{\text{cov}(\Delta c_t - \Delta c_t^*, \Delta q_t)}{\text{var}(\Delta q_t)} = -\frac{2}{\zeta(\nu\sigma+\varsigma)} \left(2\theta(1-\alpha) + \zeta\frac{\nu + \phi}{1 - \phi} - (1 - \zeta)\right) < 0,
\]

Note that the gap between \( q_t \) and \( e_t \) is zero-order in \( \gamma \) for shocks \((\tilde{a}_t, \tilde{\mu}_t, \tilde{w}_t)\) and first-order in \( \gamma \) for shock \( \tilde{\eta}_t \).
which is negative for all parameter values since
\[
\frac{\zeta \nu + \phi}{1 - \phi} - (1 - \zeta) = \frac{\zeta \nu + \zeta - (1 - \phi)}{1 - \phi} > 0
\]
as from (A20) \( \zeta = 1 - e^{-\mu/(1-\alpha)}\phi > 1 - \phi \).

3. Consider the near-autarky comovement between the nominal exchange rate and
the nominal interest rate differential (the Fama coefficient) by using (A54), which
we write in the limit as:
\[
i_t - i_t^* = \mathbb{E}_t\{2\sigma \Delta \tilde{c}_{t+1} + 2\Delta \tilde{p}_{t+1}\} = -\frac{2\gamma \sigma}{\zeta (\nu \sigma + \zeta)} \left[ 2\theta (1 - \alpha) - 1 + \frac{\zeta (1 - \zeta/\sigma)}{1 - \phi} \right] \mathbb{E}_t \Delta q_{t+1}.
\]
where we used expression (A49) for \( \tilde{c}_t \) and expressions (A38)–(A39) for \( p_t \) and \( p_t^* \).

Furthermore, (A40) and (A53) imply \( \mathbb{E}_t \Delta e_{t+1} = \mathbb{E}_t \Delta q_{t+1} = -\psi_t \) in the limit and
with \( \psi_t \) shocks only. Therefore, the Fama regression coefficient in the limit is: 8
\[
\lim_{\gamma \to 0} -\gamma \frac{\text{cov} (\mathbb{E}_t \Delta e_{t+1}, i_t - i_t^*)}{\text{var} (i_t - i_t^*)} = \zeta (\nu \sigma + \zeta) \left[ \frac{1}{2\sigma} \frac{\zeta (1 - \zeta/\sigma)}{1 - \phi} \right] < 0,
\]
which is always negative under a mild additional requirement that \( \theta > 1 \) and \( \sigma > 1 \)
(since \( \zeta \leq 1 \) and \( \alpha < 1/2 \), with a necessary condition being substantially weaker.9

This proves the first claim of the proposition that \( \psi_t \) robustly and simultaneously produces
all three empirical regularities in the autarky limit.10

**Second,** recall that the uncovered interest rate parity (A132) implies that the Fama

---

8We make use of the fact that \( \text{cov} (\Delta e_{t+1}, i_t - i_t^*) = \text{cov} (\mathbb{E}_t \Delta e_{t+1}, i_t - i_t^*) \) since \( i_t - i_t^* \) is known at \( t \).

9The Fama coefficient for the real interest rates is always negative without any further parameter
restrictions, as it is proportional to the expression for the Backus-Smith correlation, since the real interest
rate \( r_t \equiv i_t - \mathbb{E}_t \Delta p_{t+1} = \sigma \mathbb{E}_t \Delta \tilde{c}_{t+1} \) in the absence of \( \chi_t \) shocks.

10It is also easy to verify that the dispersion of the (real and nominal) exchange rate is separated from
zero in response to a \( \psi_t \) shock since from (A53) \( \mathbb{E}_t \Delta q_{t+1} = -\psi_t \) and \( q_t \) needs to adjust in response to
\( \psi_t \) to ensure intertemporal budget constraint with net exports following (A50). We show this formally in
Appendix A.1.5 for \( \psi_t \) following an AR(1) process.
regression coefficient:

\[ \beta_F \equiv \frac{\text{cov}(\Delta e_{t+1}, i_t - i_t^*)}{\text{var}(i_t - i_t^*)} = 1 \quad \text{whenever} \quad \psi_t \equiv 0. \]

Therefore, \((\eta_t, \eta_t^*, \xi_t, \xi_t^*)\) shocks that follow any joint process cannot resolve the forward premium puzzle. This is sufficient for the second claim of the proposition that the remaining shocks cannot deliver the empirical comovement for all three moments. Nonetheless, we explore the remaining two moments as well.

**Third**, in the remainder of the proof, we focus on the \(\xi_t\) and \(\eta_t\) shocks (setting all other shocks including \(\psi_t\) to zero), and impose specific time series process for these two types of shocks, which can be viewed as providing counterexamples sufficient for our argument in Proposition 1.2. Specifically, we focus on AR(1) processes for relative shocks:

\[ \tilde{\xi}_t = \rho_\xi \tilde{\xi}_{t-1} + \sigma_\xi \varepsilon^\xi_t, \]

\[ \tilde{\eta}_t = \rho_\eta \tilde{\eta}_{t-1} + \sigma_\eta \varepsilon^\eta_t, \]

with \(\rho_\xi, \rho_\eta \in [0, 1]\) and where \(\varepsilon^\xi_t, \varepsilon^\eta_t \sim iid(0, 1)\). We focus on the zero-order component of the exchange rate dynamics in \(\gamma\), as this component is non-trivial for both \(\tilde{\xi}_t\) and \(\tilde{\eta}_t\) shocks. Therefore, we drop the first and higher order components in \(\gamma\), so that we have \(e_t = q_t\) from (A40) and \(E_t \Delta q_{t+1} = E_t \Delta e_{t+1} = 0\) from (A53) together with (A49). Hence, the dynamics of the exchange rates is a random walk with jumps that satisfy the intertemporal budget constraints. The flow budget constraint (A.2.4) (with net exports (A50), in which we substitute the solutions for \(\tilde{c}_t\) and \(\tilde{y}_t\) from (A45) and (A48)) up to first order terms in \(\gamma\)
is given by:

\[ \beta b^*_t - b^*_t = 2\gamma [\theta q_t - \xi_t - (\theta - 1)\eta_t] , \]

where \( \vartheta \equiv \theta(1-\alpha) - \frac{1-2\alpha}{2} \). Solving this equation forward and imposing \( \lim_{T \to \infty} \beta^T b^*_T = 0 \), we obtain the solution for the equilibrium exchange rate:\(^{11}\)

\[ \Delta q_{t+1} = \frac{1}{\vartheta} \frac{1 - \beta}{1 - \beta \rho_\xi} \sigma_\xi \varepsilon^\xi_{t+1} + \frac{\theta - 1}{\vartheta} \frac{1 - \beta}{1 - \beta \rho_\eta} \sigma_\eta \varepsilon^\eta_{t+1} . \]

We can now calculate the moments using static equilibrium conditions (A35) for \( s_t \) and (A49) for \( \tilde{c}_t \):\(^{12}\)

\[
\lim_{\gamma \to 0} \frac{\text{cov}(\Delta s_t, \Delta q_t)}{\text{var}(\Delta q_t)} = \begin{cases} 
(1 - 2\alpha) > 0, & \text{for } \xi_t \text{ shock} \\
(1 - 2\alpha) - 2 \frac{\text{cov}(\Delta \tilde{c}_t, \Delta q_t)}{\text{var}(\Delta q_t)} & < 0, \text{ for } \eta_t \text{ shock,} \\
\end{cases}
\]

\[
\lim_{\gamma \to 0} \frac{1}{\gamma} \frac{\text{cov}(\Delta \tilde{c}_t, \Delta q_t)}{\text{var}(\Delta q_t)} = \begin{cases} 
\frac{2\theta(1-\alpha) - (1-2\alpha) - \sigma_\xi}{\zeta(\nu\sigma + \zeta)} \left[ \frac{\beta(1-\rho_\xi)}{1-\beta} - \frac{(1-2\alpha) - \zeta(\nu\sigma + \zeta)}{2\theta(1-\alpha) - (1-2\alpha)} \right] > 0, & \text{for } \xi_t, \\
\frac{2\theta(1-\alpha) - \zeta(\nu\sigma + \zeta)}{\zeta(\nu\sigma + \zeta)} \left[ 1 - \frac{\theta - 1}{\vartheta} \frac{1-\beta}{1-\beta \rho_\eta} \right] > 0, & \text{for } \eta_t, \\
\end{cases}
\]

where for the first moment we maintain the assumption that \( \alpha < 1/2 \) and to sign the second moment we use the fact that \( \zeta > 1 - \phi \). To see that the Backus-Smith correlation under \( \xi_t \) shocks can take both signs, it is sufficient to consider the case with \( \rho_\xi = 1 \) (when the correlation is negative) and the case with \( \rho_\xi = 0 \) and \( \beta \approx 1 \) (when the correlation is positive). If \( \beta \geq \rho_\xi \), under our parameterization it is sufficient to have the

---

\(^{11}\)We describe a rigorous solution method in Appendix A.1.5, while here we offer a heuristic argument: the net present value (using \( \beta \) as a discount factor) of any innovation to the right hand side of the flow budget constraint needs to be zero for intertemporal budget balance. Denote \( \varepsilon_t \equiv \Delta q_t \) the (random walk) innovation of the exchange rate. The net present value of the innovation to the budget constraint is therefore \( \sum_{j=0}^{\infty} \beta j [\theta \varepsilon_t - \rho_\xi \sigma_\xi \varepsilon_t + \theta - 1] \rho_\eta \sigma_\eta \varepsilon_t] = 0 \), and solving for \( \varepsilon_t \) from this equation we obtain the expression in the proof.

\(^{12}\)In our calculations, we use the interim results that in response to \( \eta_t \) shocks:

\[
\text{cov}(\Delta q_{t+1}) = \frac{\theta - 1}{\theta(1-\alpha) - \frac{1-2\alpha}{2} \frac{1-\beta}{1-\beta \rho_\eta}} 2 \sigma_\eta^2 \quad \text{and} \quad \text{cov}(\Delta \tilde{c}_{t+1}, \Delta q_{t+1}) = \frac{\theta - 1}{\theta(1-\alpha) - \frac{1-2\alpha}{2} \frac{1-\beta}{1-\beta \rho_\eta}} \sigma_\eta^2,
\]

and similarly for the \( \xi_t \) shock.
quarterly discount factor $\beta > 0.75$ for the sign to be positive (with the calibrated value of $\beta = 0.99$). This shows that the $\tilde{\eta}_t$ shock robustly generates counterfactual comovement with all three macro variables, while the $\tilde{\xi}_t$ shock does not robustly deliver empirically relevant comovement between exchange rates on one hand and interest rates and relative consumption on the other hand. ■

A.1.5 The baseline model of Section 1.3 with $\psi_t$ shock

Consider the log-linearized equilibrium system from Appendix A.1.3, in which we set $w_t = \mu_t = \eta_t = \xi_t = g_t = a_t = \kappa_t = \chi_t = 0$ at Home, and equivalently in Foreign, and also specialize to $\zeta = 1$ and $\zeta = 1 - \phi$ (corresponding to $G = 0$ and $\bar{\mu} = 0$ respectively).

The equilibrium system is block recursive, and we solve it in turn for prices, quantities and equilibrium dynamics, followed by the discussion of interest rates.

Price block We rewrite the solution from Appendix A.1.3 for this special case as follows:\(^{13}\)

\[ s_t = \frac{1 - 2\alpha(1 - \gamma)}{1 - 2\gamma} q_t \quad \text{and} \quad q_t^p = \frac{1 - 2\alpha\gamma}{1 - 2\gamma} q_t, \]
\[ p_t = \frac{1}{1 - \phi} \frac{\gamma}{1 - 2\gamma} q_t \quad \text{and} \quad p_t^* = -\frac{1}{1 - \phi} \frac{\gamma}{1 - 2\gamma} q_t, \tag{A56} \]

and

\[ q_t = \frac{1}{1 + \frac{1}{1 - \phi} \frac{2\gamma}{1 - 2\gamma}} e_t. \tag{A57} \]

\(^{13}\)We also have $p_{HT} - p_t = -\frac{\gamma}{1 - \gamma}(p_{FT} - p_t) = -\frac{(1 - \alpha)\gamma}{1 - 2\gamma} q_t$, and symmetrically in the Foreign market.
Quantity block  Again, rewriting the general solution (A45) and (A48) for this special case, we have:

\[
\nu \sigma \tilde{c}_t + \tilde{y}_t = -\frac{\nu + \phi}{1 - \phi} \frac{\gamma}{1 - 2\gamma} q_t, \quad (A58)
\]

\[
[1 - (1 - 2\gamma)(1 - \zeta)]\tilde{y}_t - (1 - 2\gamma)\zeta \tilde{c}_t = \gamma \left[ 2\theta (1 - \alpha) \frac{1 - \gamma}{1 - 2\gamma} - (1 - \zeta) \right] q_t, \quad (A59)
\]

where we used the fact that now \(\zeta = \frac{c}{C + G} = 1\), and in what follows we also use \(\zeta = 1 - e^{-\frac{\mu}{1 - \sigma}} = 1 - \phi\). (Recall that \(\tilde{c}_t \equiv \frac{1}{2}(c_t - c_t^*)\).)

We solve (A58)–(A59) for consumption and output explicitly:

\[
c_t - c_t^* = -\gamma \kappa_q^c q_t, \quad \kappa_q^c \equiv \frac{2\theta (1 - \alpha) \frac{1 - \gamma}{1 - 2\gamma} + \nu + \frac{\nu + \phi}{1 - \phi} \frac{2\gamma}{1 - 2\gamma}}{1 + \sigma \nu \left(1 + \frac{\nu}{1 - \phi} \frac{2\gamma}{1 - 2\gamma}\right)} \frac{1}{1 - \phi} \frac{2}{1 - 2\gamma} > 0, \quad (A60)
\]

\[
y_t - y_t^* = \gamma \kappa_q^y q_t, \quad \kappa_q^y \equiv \sigma \nu \kappa_q^c \frac{\nu + \phi}{1 - \phi} \frac{2}{1 - 2\gamma} = \frac{\sigma \nu (2\theta (1 - \alpha) \frac{1 - \gamma}{1 - 2\gamma} - \phi) - (\nu + \phi)}{1 + \sigma \nu \left(1 + \frac{\nu}{1 - \phi} \frac{2\gamma}{1 - 2\gamma}\right)} \frac{1}{1 - \phi} \frac{2}{1 - 2\gamma}.
\]

Note that in the text in (1.38) we simply use \(\kappa_q\) for \(\kappa_q^c\).

Using (A50), we can rewrite net exports in this case as:

\[
nx_t = \gamma \left[ 2\theta (1 - \alpha) \frac{1 - \gamma}{1 - 2\gamma} + \alpha - \frac{1 - \alpha}{1 - 2\gamma} + \frac{2\gamma}{1 - 2\gamma} \phi \right] q_t - 2\gamma (1 - \phi) \tilde{c}_t + \phi \tilde{y}_t
\]

\[
= \gamma \kappa_q^{nx} q_t, \quad \kappa_q^{nx} \equiv \left[ 2\theta (1 - \alpha) \frac{1 - \gamma}{1 - 2\gamma} + 2(1 - \gamma) \alpha - 1 - \gamma \kappa_q^y \right] \frac{1}{1 - 2\gamma}. \quad (A61)
\]

Note that \(\kappa_q^y < \kappa_q^c\) and may be negative. Furthermore, \(\kappa_q^{nx} > 0\) iff (after using \(\kappa_q^y\) and simplifying):

\[
2\theta (1 - \alpha) \frac{1 - \gamma}{1 - 2\gamma} \frac{1 + \sigma \nu}{1 + \sigma \nu \left(1 + \frac{\nu}{1 - \phi} \frac{2\gamma}{1 - 2\gamma}\right)} > 1 - 2(1 - \gamma) \alpha - \frac{1 - \gamma}{1 - \phi} \frac{2\gamma}{1 - 2\gamma} \left[ \nu + (1 + \sigma \nu) \phi \right] \frac{1}{1 + \sigma \nu \left(1 + \frac{\nu}{1 - \phi} \frac{2\gamma}{1 - 2\gamma}\right)}. \quad (A62)
\]

Finally, note that we scaled the coefficient \(\kappa_q^c, \kappa_q^y\) and \(\kappa_q^{nx}\) so that they are zero-order in \(\gamma\), that is the limits of these coefficients as \(\gamma \to 0\) are separated from both 0 and \(\infty\).

\[\text{Note that } \phi \tilde{y}_t + (1 - \phi) \tilde{c}_t = \frac{1}{1 - 2\gamma} \tilde{y}_t - \left[ 2\theta (1 - \alpha) \frac{1 - \gamma}{1 - 2\gamma} - \phi \right] \frac{\gamma}{1 - 2\gamma} q_t. \]
Dynamic block  We combine together the international risk-sharing condition \((A53)\) and the flow budget constrain \((A.2.4)\), and use the solution \((A60)–(A61)\) above, to obtain:

\[
\psi_t = E_t \{ \sigma \Delta(c_{t+1} - c^*_t) - \Delta q_{t+1} \} = -(1 + \gamma \sigma \kappa_q^c) E_t \Delta q_{t+1}, \tag{A63}
\]

\[
\beta b^*_t - b^*_t = n x_t = \gamma \kappa_q^{nx} q_t. \tag{A64}
\]

Given the proportional relationship between real and nominal exchange rates \((A57)\), we can equivalently rewrite the dynamic system above in terms of the nominal exchange rate \(e_t\), corresponding to \((1.24)–(1.25)\) in the text, with:

\[
\lambda_1 = \frac{\sigma \kappa_q^c - \frac{1}{1 - \phi} \frac{2}{1 - 2\gamma}}{1 + \frac{1}{1 - \phi} \frac{2}{1 - 2\gamma}} = \frac{1}{1 + \frac{1}{1 - \phi} \frac{2}{1 - 2\gamma}} \left( \frac{1 - \gamma}{1 - 2\gamma} \right) \left[ \frac{2\theta (1 - \alpha)}{1 - 2\gamma} \frac{1}{1 - 2\gamma} \phi \frac{2\gamma}{1 - 2\gamma} \right] - 1, \tag{A65}
\]

\[
\lambda_2 \equiv \frac{\kappa_q^{nx}}{1 + \frac{1}{1 - \phi} \frac{2}{1 - 2\gamma}}. \tag{A66}
\]

Note that \((A62)\) ensures \(\kappa_q^{nx} > 0\), which also implies \(\lambda_2 > 0\). In turn, \(\lambda_1 > 0\) iff:

\[
\sigma \left[ \frac{2\theta (1 - \alpha)}{1 - 2\gamma} \frac{1 - \gamma}{1 - 2\gamma} \phi \frac{2\gamma}{1 - 2\gamma} \right] > 1. \tag{A67}
\]

We discuss sufficient conditions for \((A62)\) and \((A67)\) in Appendix A.1.5. Note, however, that the signs of \(\lambda_1\) and \(\lambda_2\) are inconsequential for the proofs of the main results.

Equations \((A63)–(A64)\) define a dynamic system in \((b_t, q_t)\), where we assume the exogenous shock \(\psi_t\) follows an AR(1) process:

\[
\psi_t = \rho \psi_{t-1} + \varepsilon_t. \tag{A68}
\]

This system can be solved for the equilibrium dynamics of \(q_t\) and \(b_t\) using the Blanchard and Kahn (1980) method.\(^{15}\) We prove:

\(^{15}\)Alternatively, this system can be solved by the method of undetermined coefficients. Rewriting \((A63)\) as \(E_t q_{t+1} = q_t - \psi_t\), it must be that \(q_t = \frac{1}{1 - \rho} \psi_t + m_t\), where \(m_t\) is a martingale with the innovation given
**Lemma A1** The unique non-explosive solution to the dynamic system (A63)–(A64) with (A68) is given by:

\[
\Delta q_{t+1} = \rho \Delta q_t + \frac{1}{1 + \gamma \sigma \kappa_q} \frac{\beta}{1 - \beta \rho} \left( \varepsilon_{t+1} - \frac{1}{\beta} \varepsilon_t \right),
\]

(A69)

\[
\Delta \hat{b}^*_{t+1} = \rho \Delta \hat{b}^*_t + \frac{\gamma \kappa^{nx}_q}{1 + \gamma \sigma \kappa_q} \frac{1}{1 - \beta \rho} \varepsilon_t.
\]

(A70)

**Proof:** We rewrite the dynamic system (A63)–(A64) in matrix form as:

\[
\begin{bmatrix}
q_{t+1} \\
\hat{b}_{t+1}
\end{bmatrix}
= \begin{bmatrix}
1 & 0 \\
1/\beta & 1/\beta
\end{bmatrix}
\begin{bmatrix}
q_t \\
\hat{b}_t
\end{bmatrix}
- \begin{bmatrix}
\psi_t \\
0
\end{bmatrix},
\]

where we use the rescaled variables \( \hat{b}_t = \frac{b_t}{\gamma \kappa^{nx}_q} \) and \( \hat{\psi}_t = \frac{\psi_t}{1 + \gamma \sigma \kappa_q} \) for convenience. Matrix \( A \) has two eigenvalues: 1 and \( 1/\beta > 1 \). The left eigenvector of matrix \( A \) associated with eigenvalue \( 1/\beta \) is \( v = (1, 1 - \beta) \). Premultiplying the system by \( v \) from the left, we have:

\[
\mathbb{E}_t z_{t+1} = \frac{1}{\beta} z_t - \hat{\psi}_t,
\]

where \( z_t \equiv v \begin{bmatrix}
q_t \\
\hat{b}_t
\end{bmatrix} = q_t + (1 - \beta) \hat{b}_t \).

The unique non-explosive (forward) solution of this dynamic equation is:\footnote{The remaining explosive solutions feature \( \lim_{j \to \infty} \beta^j \mathbb{E}_t z_{t+j+1} = \infty \), violating the No Ponzi Game Condition for \( \hat{b}^*_t \).}

\[
z_t = \beta \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \hat{\psi}_{t+j} = \frac{\beta}{1 - \beta \rho} \hat{\psi}_t.
\]

This implies a cointegration relationship for the endogenous variables, \( q_t + (1 - \beta) \hat{b}_t = \frac{\beta}{1 - \beta \rho} \hat{\psi}_t \), which can be used to solve out \( q_t \) in (A64), yielding:

\[
\Delta \hat{b}^*_{t+1} = \frac{1}{1 - \beta \rho} \hat{\psi}_t,
\]

which implies (A70) given the definitions of \( \hat{b}_t \) and \( \hat{\psi}_t \), and using the fact that \( (1 - \rho L) \psi_t = \theta \). That is, \( m_{t+1} + \theta \varepsilon_{t+1} \), where \( \theta \) is the undetermined coefficient. We then use (A64) to find the unique value of \( \theta \) that results in a non-explosive path for \( \hat{b}^*_t \).
where the lag operator $Lx_t = x_{t-1}$ for an arbitrary variable $x_t$.

Next we take the first difference of (A64):

$$\Delta q_t = \beta \Delta \hat{b}_{t+1} - \Delta \hat{b}_t = \frac{\beta}{1 - \beta \rho} \hat{\psi}_t - \frac{1}{1 - \beta \rho} \hat{\psi}_{t-1},$$

where the second equality substitutes in the solution for $\Delta \hat{b}_{t+1}$. Applying the $(1 - \rho L)$ operator on both sides, we obtain equation (A69), since $(1 - \rho L) \hat{\psi}_t = \hat{\psi}_t - \rho \hat{\psi}_{t-1} = \frac{1}{1 + \gamma \sigma \kappa_q^c} \varepsilon_t$. ■

Proof of Proposition 1.3 Lemma A1 implies that the unique (NPG-admissible) solution of the dynamic system results in an ARIMA(1,1,1) process for the exchange rate and an ARIMA(1,1,0) process of the NFA position of the country. Equivalently, the change in the exchange rate $\Delta q_t$ follows an ARMA(1,1) with the AR root $\rho$ and the MA root $1/\beta$, while the change in the NFA $\Delta b_{t+1}^*$ follows an AR(1) process with root $\rho$. Also note that the smaller is $\gamma$, the larger is the response of $q_t$ and the smaller is the response of $b_{t+1}^*$ to the innovation $\varepsilon_t$ of the financial shock $\psi_t$. In light of (A57) and (A65), Lemma A1 implies Proposition 1.3. ■

Proofs for Proposition 1.4 and 1.5 Next we discuss the properties of the equilibrium exchange rate dynamics, which in light of (A57) apply equally to both the nominal and the real exchange rate. Given the solution (A69), we can now characterize the statistical properties of the exchange rate process:

1. Unconditional variance of $\Delta q_t$ can be calculated from (A69) as follows:

$$\sigma^2_{\Delta q} = \rho^2 \sigma^2_{\Delta q} + \frac{1}{(1 + \gamma \sigma \kappa_q^c)^2} \frac{1 + \beta^2}{(1 - \beta \rho)^2} \sigma^2_\varepsilon \frac{1}{(1 + \gamma \sigma \kappa_q^c)^2} \frac{2 \beta \rho}{(1 - \beta \rho)^2} \sigma^2_\varepsilon.$$
where $\sigma_{\Delta q}^2 = \text{var}(\Delta q_{t+1})$. Solving for $\sigma_{\Delta q}^2$ and for $\sigma_{\Delta e}^2$ using (A57) and (A65), we have:

$$\sigma_{\Delta q}^2 = \frac{1}{(1 + \gamma \sigma \kappa_q^2)^2} \frac{1 - 2\beta \rho + \beta^2}{(1 - \beta \rho)^2} \sigma_{\varepsilon}^2,$$

$$\sigma_{\Delta e}^2 = \frac{1}{(1 + \gamma \lambda_1)^2} \frac{1 - 2\beta \rho + \beta^2}{(1 - \beta \rho)^2} \sigma_{\varepsilon}^2,$$

since we have $1 + \gamma \lambda_1 = \frac{1 + \gamma \sigma \kappa_q^2}{1 + \frac{1}{1 - \tau}}$.

Noting that $\text{var}(\psi_{t+1}) = \sigma_{\varepsilon}^2/(1 - \rho^2)$, we further have:

$$\frac{\text{var}(\Delta e_{t+1})}{\text{var}(\psi_{t+1})} = \frac{1}{(1 + \gamma \lambda_1)^2} \frac{1 - 2\beta \rho + \beta^2}{(1 - \beta \rho)^2} \frac{1}{\beta \rightarrow 1} \frac{2}{(1 + \gamma \lambda_1)^2} \frac{1}{1 - \rho},$$

which increases without bound as $\rho \rightarrow 1$.

2. Auto-covariance of $\Delta q_t$ can be similarly calculated using (A69) as:

$$\text{cov}(\Delta q_{t+1}, \Delta q_t) = \rho \sigma_{\Delta q}^2 - \frac{1}{(1 + \gamma \sigma \kappa_q^2)^2} \frac{\beta}{(1 - \beta \rho)^2} \sigma_{\varepsilon}^2 = -\frac{(\beta - \rho)(1 - \beta \rho)}{1 - 2\beta \rho + \beta^2} \sigma_{\Delta q}^2.$$

The autocorrelation of the exchange rate changes (both nominal and real, in light of (A57)) is:

$$\rho_{\Delta e} = \rho_{\Delta q} = \frac{\text{cov}(\Delta q_{t+1}, \Delta q_t)}{\sigma_{\Delta q}^2} = -\frac{(\beta - \rho)(1 - \beta \rho)}{1 - 2\beta \rho + \beta^2} \frac{1}{\beta \rightarrow 1} \frac{1}{2},$$

confirming claim 1 in Propositions 1.4.

3. The variance of innovation of $\Delta q_{t+1}$ is:

$$\text{var}_t(\Delta q_{t+1}) = \text{var}(\Delta q_{t+1} - \mathbb{E}_t\Delta q_{t+1}) = \left(\frac{\beta}{1 - \beta \rho}\right)^2 \sigma_{\varepsilon}^2 \left(1 + \gamma \sigma \kappa_q^2\right)^2,$$

where we assume that the information set at time $t$ includes $\{q_t, q_{t-1}, \ldots, \varepsilon_t, \varepsilon_{t-1}, \ldots\}$. 

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Since \( \Delta e_t \) equals \( \Delta q_t \) scaled by a constant, we further have:

\[
\frac{\text{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})}{\text{var}(\Delta e_{t+1})} = \frac{\text{var}(\Delta q_{t+1} - \mathbb{E}_t \Delta q_{t+1})}{\text{var}(\Delta q_{t+1})} = \frac{\beta^2(1 - \rho^2)}{1 - 2\beta \rho + \beta^2} \xrightarrow{\beta \to 1} \frac{1 + \rho}{2},
\]

confirming claim 2 in Propositions 1.4.

Combining with the result in point 1 above, we have:

\[
\frac{\text{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})}{\text{var}(\psi_{t+1})} = \frac{1}{(1 + \gamma \lambda_1)^2} \frac{\beta^2(1 - \rho^2)}{(1 - \beta \rho)^2} \xrightarrow{\beta \to 1} \frac{1}{(1 + \gamma \lambda_1)^2} \frac{1 + \rho}{1 - \rho},
\]

which tends to infinity with \( \rho \to 1 \), confirming claim 3 in Propositions 1.4. \[\blacksquare\]

4. We now calculate the finite-sample autocorrelation of the real exchange rate in levels, that is the coefficient from a regression of \( q_t \) on \( q_{t-1} \) (with a constant) in a sample with \( T + 1 \) observations. Even though the second moments are not well-defined in population, this finite sample correlation is well-defined. We have:

\[
\hat{\rho}_q(T) = \frac{1}{T} \sum_{t=1}^{T} (q_t - \bar{q})(q_{t-1} - \bar{q}) = 1 + \frac{\frac{1}{T} \sum_{t=1}^{T} \Delta q_t q_{t-1}}{\frac{1}{T} \sum_{t=1}^{T} (q_{t-1} - \bar{q})^2}.
\]

Note that the denominator is positive and finite for any finite \( T \), but diverges as \( T \to \infty \), since \( q_t \) is an integrated process. The numerator, however, has a finite limit (assuming \( \beta, \rho < 1 \), which ensures stationarity of \( \Delta q_t \), and conditioning on the given initial value of the process \( q_0 \)):

\[
p\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \Delta q_t q_{t-1} = \text{cov}(\Delta q_t, q_{t-1}) = \sum_{j=1}^{\infty} \text{cov}(\Delta q_t, \Delta q_{t-j}) = \frac{\text{cov}(\Delta q_t, \Delta q_{t-1})}{1 - \rho} = -\frac{\beta - \rho}{(1 - \rho)(1 - \beta \rho)} \frac{\sigma_q^2}{(1 + \gamma \sigma \kappa_q^e)^2},
\]

where we used the fact that for process (A69) \( \text{cov}(\Delta q_t, \Delta q_{t-j}) = \rho^{j-1} \text{cov}(\Delta q_{t-j+1}, \Delta q_{t-j}) \)
for $j \geq 1$ and the expression for $\text{cov}(\Delta q_t, \Delta q_{t-1})$ obtained above.

To summarize, this analysis implies that the finite sample autocorrelation of $q_t$: (a) tends to 1 asymptotically as samples size increases; and (b) is smaller than 1 in large but finite samples, provided that $\rho < \beta$. This confirms the claims in Proposition 1.5.

\[\blacksquare\]

**Interest rates and Carry trades** Finally, we turn to the properties of the interest rates, which are linked to consumption and prices by (A54). Substituting in the solution for consumption (A60) and prices (A56), we arrive at:

\[i_t - i_t^* = -\left(\sigma \kappa_q^c c - \frac{1}{1-\phi} \frac{2\gamma}{1-\gamma}\right) E_t \Delta q_{t+1} = -\gamma \lambda_1 E_t \Delta e_{t+1},\]

corresponding to (1.41) and with $\lambda_1$ defined in (A65). In this analysis we assume that the parameter restriction (A67) is satisfied, and $\lambda_1 > 0$.\(^{17}\) Combining the expression for interest rate differential with the UIP condition (A132), we obtain expression (1.42) in the text.

Lastly, we define a Carry trade. Consider a trade strategy that invests $x_t \equiv i_t - i_t^* - E_t \Delta e_{t+1}$ in the home bond and sells short $x_t$ units of foreign bond, including the case when $x_t < 0$ (i.e., shorting the home bond and investing in foreign bond in this case). We refer to this strategy as a *Carry trade*. Note that this trade requires zero capital at $t$ and the intensity (exposure) of the trade is proportional to its expected return, which from (A132) equals $x_t = \psi_t$. The return on this trade and the corresponding (unconditional) Sharpe

\(^{17}\)There exists a parallel relationship in real terms (where $r_t \equiv i_t - E_t \Delta p_{t+1}$ denotes the real interest rate):

\[r_t - r_t^* = \sigma E_t \{\Delta c_{t+1} - \Delta c_{t+1}^*\} = -\gamma \kappa_q^c E_t \Delta q_{t+1},\]

with the negative sign independently of the parameter values.
ratio are given by:

\[ r_{t+1}^C = x_t (i_t - i_t^* - \Delta e_{t+1}) \quad \text{and} \quad SR_r^C = \frac{\mathbb{E} r_{t+1}^C}{\text{std}(r_{t+1}^C)}. \]  

(A71)

**Proof of Proposition 1.8**  
First, consider the Fama regression of \( \Delta e_{t+1} \) on \( i_t - i_t^* \). From (1.26), it follows:

\[ \Delta e_{t+1} = \mathbb{E}_t \Delta e_{t+1} + \frac{1}{1 + \gamma \lambda_1} \frac{\beta}{1 - \beta \rho} \varepsilon_{t+1}, \]

since \( \varepsilon_{t+1} \) is the only innovation relative to the information set of time \( t \). Furthermore, from our derivations above we have \( i_t - i_t^* = -\gamma \lambda_1 \mathbb{E}_t \Delta e_{t+1} \), and therefore, we can write the regression as:

\[ \Delta e_{t+1} = -\frac{1}{\gamma \lambda_1} (i_t - i_t^*) + \frac{1}{1 + \gamma \lambda_1} \frac{\beta}{1 - \beta \rho} \varepsilon_{t+1}. \]

Since \( \varepsilon_{t+1} \) is a regression residual (i.e., \( \mathbb{E} \{ \varepsilon_{t+1} | i_t - i_t^* \} = 0 \)), the Fama regression coefficient is given by \( \beta_F \equiv -\frac{1}{\gamma \lambda_1} < 0 \). The \( R^2 \) in this regression is given by the share of the predictable variation in \( \Delta e_{t+1} \), since \( i_t - i_t^* \) absorbs the entire predictable component \( \mathbb{E}_t \Delta e_{t+1} \) (see Proposition 1.4 and its proof for derivation of the variance share of the unpredictable component):

\[ R^2 = \frac{\text{var}(\mathbb{E}_t \Delta e_{t+1})}{\text{var}(\Delta e_{t+1})} = 1 - \frac{\text{var}(\Delta e_{t+1} - \mathbb{E}_t \Delta e_{t+1})}{\text{var}(\Delta e_{t+1})} = 1 - \frac{\beta^2 (1 - \rho^2)}{1 - 2 \beta \rho + \beta^2} = \frac{(1 - \beta \rho)^2}{1 - 2 \beta \rho + \beta^2}. \]

Note that \( \lim_{\beta \to 1} R^2 = \frac{1 - \rho}{2} \), which tends to zero with \( \rho \to 1 \). This proves claim (i).

Using the same arguments, we prove claim (ii):

\[ \frac{\text{var}(\Delta e_{t+1})}{\text{var}(i_t - i_t^*)} = \frac{1}{(\gamma \lambda_1)^2} \frac{\text{var}(\Delta e_{t+1})}{\text{var}(\mathbb{E}_t \Delta e_{t+1})} = \frac{1}{(\gamma \lambda_1)^2} \frac{1}{R^2}. \]

Since \( \gamma \lambda_1 \) is separated from zero when \( \gamma > 0 \) and does not depend on \( \beta \) and \( \rho \), the asymp-
totics of this relative variances is the same as that of $1/R^2$, which goes to infinity as $\beta, \rho \to 1$.

Claim (iii) follows from the fact that $\rho_{\Delta e} = \text{corr}(\Delta e_{t+1}, \Delta e_t) \to 0$ as $\beta, \rho \to 1$ (see Proposition 1.4), while the persistence of $i_t - i^* = \frac{\gamma \lambda_1}{1 + \gamma \lambda_1} \psi_t$ equals $\rho \to 1$.

Lastly, we make use of the definition of the Carry trade return $r^C_{t+1}$ and its Sharpe ratio $SR^C$ in (A71), to prove claim (iv). In particular, we calculate the expected return and the variance of the returns (using the fact that from (A132) we have $i_t - i^* = \Delta e_{t+1} = \psi_t - (\Delta e_{t+1} - E_t \Delta e_{t+1})$):

$$Er^C_{t+1} = E\{\psi_t E_t \{i_t - i^*_t - \Delta e_{t+1}\}\} = E\psi_t^2 = \text{var}(\psi_t) = \sigma^2_{\psi},$$

$$\text{var}(r^C_{t+1}) = E(r^C_{t+1})^2 - (Er^C_{t+1})^2 = E\{\psi_t^2 [\psi_t - (\Delta e_{t+1} - E_t \Delta e_{t+1})]^2\} - \sigma^4_{\psi},$$

$$= E\psi_t^4 + E\{\psi_t^2 \text{var}(\Delta e_{t+1})\} - \sigma^4_{\psi} = 2\sigma^4_{\psi} + \text{var}(\Delta e_{t+1}) \sigma^2_{\psi},$$

where the last line uses the fact that $\text{var}(\Delta e_{t+1}) = E_t \{\Delta e_{t+1} - E_t \Delta e_{t+1}\}^2$ depends only on the parameters and does not depend on $\psi_t$ (i.e., the unexpected component of $\Delta e_{t+1}$ is homoskedastic; see the proof of Proposition 1.4), the fact that $E\{\psi_t^3(\Delta e_{t+1} - E_t \Delta e_{t+1})\} = E\{\psi_t^3 E_t \{\Delta e_{t+1} - E_t \Delta e_{t+1}\}\} = 0$, and lastly that $E\psi_t^4 = 3(E\psi_t^2)^2 = 3\sigma^4_{\psi}$ under the additional assumption that $\varepsilon_t$ is normally distribution (in which case $\psi_t$ is also normal).

With this, we calculate:

$$SR^C = \frac{\sigma^2_{\psi}}{\sqrt{2\sigma^4_{\psi} + \text{var}(\Delta e_{t+1}) \sigma^2_{\psi}}} = \left(2 + \frac{1}{\beta^2 (1 - \rho^2)} \frac{\beta^2 (1 - \rho^2)}{(1 + \gamma \lambda_1)^2} \frac{1}{(1 - \beta \rho)^2} \right)^{-1/2},$$

where we use the expression for $\text{var}(\Delta e_{t+1}) / \sigma^2_{\psi}$ from the proof of Proposition 1.4. Note that $SR^C \to 0$ as $\beta, \rho \to 1$, as $\text{var}(\Delta e_{t+1}) / \sigma^2_{\psi} \to \infty$. ■
Parameter restrictions

The primitive parameters (defined in Table 1.1) can take the following values:

- $\beta \in (0, 1)$ and $\rho \in [0, 1]$, while the model admits solution as long as $\beta \rho < 1$
- $\sigma, \theta > 0$ and $\nu \geq 0$
- $\gamma \in [0, 1/2]$ with $\gamma = 0$ corresponding to autarky and $\gamma = 1/2$ corresponding to no home bias
- $\alpha \in [0, 1)$ with $\alpha = 0$ corresponding to no strategic complementarities and complete pass-through
- $\phi \in [0, 1)$ with $\phi = 0$ corresponding to no intermediate inputs.

In addition, we impose the following sufficient parameter restrictions needed for certain results:

**Assumption A1:** $\alpha < 1/2$.

A1 ensures positive correlation between RER and ToT (Proposition 1.6). A1 is consistent with the range of empirical estimates for the elasticity of strategic complementarities.

**Assumption A2:** $\theta > 1/\sigma$.

Together with A1, A2 implies $2\sigma \theta (1 - \alpha) > 1$, which is a sufficient condition for (A67), ensuring that $\lambda_1 > 0$, i.e. that the nominal interest rate falls with expected depreciation (see (1.41); needed for Proposition 1.8, and useful but not necessary in discussion of Proposition 1.3).\footnote{Condition A2 effectively ensures that nominal and real interest rates move in the same direction, i.e. the expected inflation response does not more than offset the movement in the real interest rate.} Note that here $1/\sigma$ plays the role of IES (elasticity of intertemporal substitution), rather than the income effect in the labor supply (and therefore does not exclude GHH preferences). Empirically, $\theta > 1$ and $1/\sigma \in (1/2, 1)$, so this sufficient condition is met with ease, and the necessary condition (A67) is even further lax.
Assumption A3: \( \theta > \frac{1}{2} + \frac{1}{1-\phi} \frac{\gamma}{1-2\gamma} \).

A3 is a sufficient condition for (A62), a variant of the Marshall-Lerner condition in our general equilibrium model, which ensuring that \( \lambda_2 > 0 \) and \( \kappa_{nx} > 0 \), i.e. that net export improves in response to a devaluation (see (1.25); a useful but not necessary condition for discussion of Proposition 1.3). An alternative necessary condition for (A62) can be written as \( \theta > \frac{1}{2} \frac{1}{1-\gamma} \left[ 1 + 2\gamma \frac{\phi}{1-\phi} \right] \). A necessary condition (A62) is noticeably weaker, and in particular is relaxed when \( \alpha > 0 \). In the limit of autarky (\( \gamma \approx 0 \)), \( \theta > 1/2 \) is both necessary and sufficient, corresponding to the classical Marshall-Lerner condition. Since empirically \( \theta > 1 \) and \( \phi \approx 1/2 \), A3 would be easily satisfied even for countries that are a number of times more open than the United States.

**Quantitative properties and robustness**

We now explore the robustness of our quantitative findings in the baseline model (with \( \psi_t \) shocks only), away from both limits \( \gamma \to 0 \) and \( \beta\rho \to 1 \), and with respect to departures from our baseline parameterization summarized in Table 1.1. The results are reported in Table A1, along with the benchmark empirical moments. The robustness columns report only the moments that are sensitive to the changes in the parameter values.

The robustness analysis suggests that the model requires a high \( \rho \) in order to capture the dynamic properties of the exchange rates, as a lower \( \rho \) results in less persistent exchange rates and in more predictable exchange rate changes. The quantitative success of the model also relies on home bias (a low \( \gamma \)), as when \( \gamma \) is doubled (corresponding to a 60% trade to GDP ratio), the model predicts a considerably more volatile response of the real variables to the real exchange rate, in contrast with the data. Having moderately
low $\theta$ and high $\alpha$ is also important for the fit of the model, while it is more robust with respect to variation in other parameters, including risk aversion $\sigma$ and Frisch elasticity $\nu$ (not reported in the table). Similarly, we check robustness with respect to $\varsigma = C/(C + G)$ and $\bar{\mu}$ (average markup).

Notes: (a) In response to $\psi_t$ shock, $c_t$ and $c_t^*$ are perfectly negatively correlated so that $\Delta c_t - \Delta c_t^* = 2\Delta c_t$. Therefore, while $\Delta c_t - \Delta c_t^*$ is about three times less volatile than $\Delta q_t$, $\Delta c_t$ alone is about six times less volatile than $\Delta q_t$. (b) The in-sample Fama regression coefficient is $-8$ with huge variation covering zero within two standard deviations. When $\psi_t$ is combined with other shocks, the median coefficient becomes closer to zero, as in the data (see Table 1.2).

### A.1.6 Relationship to Engel and West (2005)

Consider a simple monetary extension to our baseline model, with an interest-elastic money demand:

$$ m_t - p_t = \sigma c_t - \chi i_t, $$

and an exogenous stochastic money supply process $m_t$, resulting in an endogenous path for nominal wages $w_t$ instead of $w_t \equiv 1$ in the baseline model. Since prices and wages are flexible, this change has no effect on the equilibrium path of real variables, including the real exchange rate, but can be consequential for the path of nominal variables, including the nominal exchange rate.

This model extension corresponds to one of the special cases considered in Engel and West (2005; henceforth, EW), and their other special case with a Taylor rule admits a similar characterization (omitted here for brevity). For concreteness, we focus on three
types of shocks: productivity shock $a_t$, financial (UIP) shock $\psi_t$, and monetary shock $m_t$.

**Engel and West (2005) solution.** EW combine the UIP condition (1.22) with money demand (A72) in both countries and the definition of the real exchange rate (1.29) to obtain the equilibrium dynamic expression for the nominal exchange rate as a function of fundamentals:

$$e_t = \delta E_t e_{t+1} + \delta \psi_t + (1 - \delta) f_t,$$

(A73)

where $f_t \equiv q_t + (m_t - m_t^*) - \sigma (c_t - c_t^*)$ and $\delta \equiv \chi / (1 + \chi)$. Under two alternative assumptions on the equilibrium dynamic processes for $\psi_t$ and $f_t$, EW prove that the equilibrium process for nominal exchange rate $e_t$ that satisfies (A73) must converge to a random walk as $\delta \to 1$. Specifically, the conditions are that either (a) $\psi_t \equiv 0$ and $f_t \sim I(1)$, or (b) that $\psi_t \sim I(1)$ and $f_t \sim I(0)$ or $I(1)$, and then $\text{cov}(\Delta e_t, \Delta e_{t+k}) \to 0$ for all $k \neq 0$ as $\delta \to 1$. Interestingly, what matters for the equilibrium exchange rate process here is the elasticity of money demand $\chi$ (which determines $\delta$), but not the discount factor $\beta$, which we emphasize in Proposition 1.3.

**Equilibrium cointegration.** In general, other equilibrium conditions of the model impose cointegration between fundamentals $(\psi_t, f_t)$ featured in (A73). In particular, the international risk-sharing condition (1.39) can be rewritten as:

$$E_t \Delta \tilde{f}_{t+1} = -\psi_t, \quad \text{where} \quad \tilde{f}_t \equiv q_t - \sigma (c_t - c_t^*),$$

(A74)

so that $f_t \equiv \tilde{f}_t + (m_t - m_t^*)$ and $\tilde{f}_t$ isolates the endogenous fundamentals in $f_t$. Condition (A74) is an equilibrium cointegration relationship between exogenous shock $\psi_t$ and
endogenous variable $\tilde{f}_t$. We can use it to solve out $\psi_t$ in (A73) and rewrite it as:

$$(1 - \delta L^{-1})e_t = (1 - \delta L^{-1})\tilde{f}_t + (1 - \delta)(m_t - m_t^*) \cdot$$

This makes it clear that due to cointegration the endogenous fundamentals also have a forward looking root $\delta$, just like nominal exchange rate, and therefore it cancels out, resulting in the following solution:

$$e_t = \tilde{f}_t + (1 - \delta)\sum_{j=0}^{\infty} \delta^j \mathbb{E}_t\{m_{t+j} - m_{t+j}^*\}.$$ 

Furthermore, consider the case when $m_t = m_t^* \equiv 0$, which results in:

$$e_t = \tilde{f}_t = (1 + \gamma \sigma \kappa q)t - \sigma \kappa a_t - a_t^*,$$

where we used the solution for consumption (1.40) to express $\tilde{f}_t$ as a function of endogenous $q_t$ and exogenous $(a_t - a_t^*)$. Note that in this case the root $\delta$ disappears altogether from the equilibrium characterization of $\{e_t\}$. More concretely, assume $\psi_t \sim AR(1)$ is the only source of shocks, as in the baseline model, and then $e_t = (1 + \gamma \sigma \kappa q)t \sim ARIMA(1, 1, 1)$, as characterized in Lemma A1.19 In this case, the process for $e_t$ does not depend on $\delta$ and convergence to a random walk when $\beta \rho \to 1$, in seeming contrast with the predictions of EW.

Given that our model can be formulated as a special case of the EW representation (A73), a natural question is how can it be that our solution does not satisfy their theorem? The following three special cases explain the source of this discrepancy:

19Note the difference between $e_t = (1 + \gamma \sigma \kappa q)t$ and (1.32), which arises due to the fact that under monetary policy rule $m_t \equiv 0, w_t \neq 0$ in general. Nonetheless, the path of $q_t$ is the same independently of the monetary policy rule (as prices and wages are flexible). Furthermore, $e_t$ and $q_t$ are still proportional to each other, albeit with a different factor of proportionality.
1. A model with monetary shocks \((m_t, m^*_t)\) only. This case features \(\psi_t = \tilde{f}_t \equiv 0\) and \(f_t = m_t - m^*_t\), and the EW theorem applies. The same would be true under an alternative formulation with a Taylor rule and Taylor rule shocks.

2. A model with productivity shocks \((a_t, a^*_t)\) only. This case features \(e_t = f_t = \tilde{f}_t\) and \(\psi_t \equiv 0\), which implies \(\mathbb{E}_t \Delta e_{t+1} = \mathbb{E}_t \Delta f_{t+1} = 0\), that is \(e_t\) follows an exact random walk independently of the value of \(\delta\) or \(\beta\), and hence the EW theorem holds trivially in this case. This case extends to all fundamental shocks as long as \(m_t = m^*_t = \psi_t \equiv 0\), as in this case \(i_t - i^*_t = 0\), and the UIP condition immediately implies \(\mathbb{E}_t \Delta e_{t+1} = 0\), independently of the shock processes and parameters of the model.

3. A model with financial (UIP) shocks \(\psi_t\), and possibly all other shocks. This case features cointegration relationship \(\mathbb{E}_t \Delta \tilde{f}_{t+1} = -\psi_t\). Therefore, if \(\psi_t \sim I(k)\) for some integer \(k \geq 0\), then \(f_t\) is at least \(I(k+1)\), and therefore the conditions of the EW theorem are never satisfied in this case.

To summarize, we find that in the presence of the UIP shocks \(\psi_t\) in our model, the equilibrium cointegration relationship between endogenous and exogenous fundamentals violates the conditions of the EW theorem, explaining why we find that an equilibrium exchange rate process has different properties, and in particular does not depend on \(\delta\), but instead depends on \(\beta\).

**A.1.7 Productivity shocks and the Backus-Smith puzzle**

We consider here alternative mechanisms, which can resolve the Backus-Smith puzzle in a bond-only economy subject exclusively to productivity shocks, following much of the
literature (in particular, Corsetti, Dedola, and Leduc 2008). Using the static equilibrium relationship (1.40), we can express the Backus-Smith correlation as:

\[
\frac{\text{cov}(\Delta c_t - \Delta c_t^*, \Delta q_t)}{\text{var}(\Delta q_t)} = \kappa_a \varrho_{a,q} \frac{\text{std}(\Delta a_t - \Delta a_t^*)}{\text{std}(\Delta q_t)} - \gamma \kappa_q,
\]

where \( \varrho_{a,q} \equiv \text{corr}(\Delta a_t - \Delta a_t^*, \Delta q_t) \). The puzzle persists as the typical calibrations imply \( \varrho_{a,q} > 0 \) and \( \gamma \approx 0 \), resulting in the counterfactually positive correlation between relative consumption growth and real exchange rate depreciation. The two mechanism we discuss below either make \( \varrho_{a,q} < 0 \), or increase \( \text{std}(\Delta q_t) / \text{std}(\Delta a_t - \Delta a_t^*) \) to obtain the empirical negative correlation.

In addition to (1.40), the two dynamic equilibrium conditions are the risk sharing condition (1.39) and the budget constraint (analog of (1.25)), which we reproduce here altogether as the special case of the equilibrium system in Appendix A.1.3 with productivity shocks only:

\[
ct - c_t^* = \kappa_a (at - a_t^*) - \gamma \kappa q t,
\]

\[
\mathbb{E}_t \{ \sigma (\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1} \} = 0,
\]

\[
\beta b_{t+1} - b_t^* = nx_t = \gamma [\lambda q q t - \lambda a (at - a_t^*)],
\]

Further, for simplicity, we consider the case with \( \phi = 0 \), \( \zeta = \zeta = 1 \) and \( \nu = 0 \), and the results generalize immediately beyond this special case. In this case, the coefficients in the system above are given by:

\[
\kappa_a = \lambda_a = \frac{1}{1 - 2 \gamma}, \quad \kappa_q = \frac{4 \theta (1 - \alpha)(1 - \gamma)}{(1 - 2 \gamma)^2}, \quad \lambda_q = \frac{\kappa_q}{2} - \frac{1 - 2(1 - \gamma)\alpha}{1 - 2 \gamma}.
\]

Two noteworthy features of this system are:
1. Both coefficients \( \gamma \lambda_a \) and \( \gamma \lambda_q \) tend towards zero with \( \gamma \to 0 \), while this is the case only for \( \gamma \kappa_q \), and not for \( \kappa_a \). This suggests that the direct effect of productivity on consumption will tend to dominate the expenditure switching effect, when \( \gamma \) is small (that is, economy is sufficiently closed). This constitutes the key challenge for the productivity-based models in obtaining the empirical Backus-Smith correlation.

2. Coefficients \( \kappa_a, \kappa_q, \lambda_a > 0 \), while \( \lambda_q > 0 \) iff a version of the Marshal-Lerner condition holds:

\[
\theta > \frac{1}{2} \frac{1 - 2\gamma}{1 - \gamma} \frac{1 - (1 - \gamma)\alpha}{1 - \alpha},
\]

for which Assumption A3 in Appendix A.1.5 is a sufficient condition.

The fact that \( \lambda_q \) can flip sign (when \( \theta \) is sufficiently low) is one path towards a resolution of the Backus-Smith puzzle. The other possibility relies on reducing the volatility of innovation to productivity (making \( \kappa_a(a_t - a_t^*) \) small in (1.40)), while simultaneously increasing its persistence (to increase the response of \( q_t \) and hence the term \( \gamma \kappa_q q_t \) in (1.40)). We consider these two possibilities in turn.

**Low elasticity of substitution** For simplicity we focus here on the case of a random walk process for productivity, and results generalize outside this case. With \( \Delta(a_t - a_t^*) = \varepsilon_t^a \), the combination of (1.39′) and (1.40) results in \( \mathbb{E}_t \Delta q_{t+1} = 0 \). Intuitively, in response to a permanent shift in productivity, the real exchange rate also shifts permanently. Given a random walk path for both \( q_t \) and \( (a_t - a_t^*) \), the intertemporal budget constraint holds only if \( \lambda_q q_t - \lambda_a(a_t - a_t^*) \equiv 0 \). Therefore, the real exchange rate depreciates with a positive productivity shock iff the Marshal-Lerner condition is satisfied, but it appreciates otherwise. Intuitively, an increase in consumption and import demand from a productiv-
ity shock must be offset by an increase in exports, which requires a depreciation iff the Marshal-Lerner condition holds, and vice versa. Combining this relationship between \( q_t \) and \( (a_t - a_t^*) \) with the solution for consumption, we arrive at:

\[
c_t - c_t^* = \left\{ \frac{1}{1 - 2\gamma} \left[ \frac{2\theta(1-\alpha)(1-\gamma)}{1-2\gamma} - [1 - 2(1 - \gamma)\alpha] \right] - \gamma k_q \right\} q_t.
\]

Therefore, the violation of the Marshal-Lerner condition is sufficient for the negative correlation between \( c_t - c_t^* \) and \( q_t \), but the necessary condition is weaker and is given by:

\[
\theta < \frac{1}{2} \frac{1 - 2(1 - \gamma)\alpha}{1 - \alpha}.
\]

With \( \gamma = 0.28 \) (four times that of the US) and \( \alpha = 0 \), this requires \( \theta < 0.7 \). For smaller \( \gamma \) and larger \( \alpha \), this requirement becomes considerably more strict (e.g., for our baseline values of \( \alpha \) and \( \gamma \), \( \theta < 0.25 \)).

**Persistent productivity shocks** Here we assume that the Marshal-Lerner condition is satisfied and \( \lambda_q > 0 \), and instead consider a case with a persistent process for relative productivity growth rates:

\[
\Delta \tilde{a}_t = \rho_a \Delta \tilde{a}_{t-1} + \varepsilon_t^\rho,
\]

where \( \tilde{a}_t \equiv (a_t - a_t^*)/2 \) and \( \rho_a \in [0, 1) \).\(^{20}\) Combining (1.39') and (1.40), we have in this case:

\[
\mathbb{E}_t \Delta q_{t+1} = \frac{2\sigma \kappa_a}{1 + \gamma \sigma k_q} \mathbb{E}_t \Delta \tilde{a}_{t+1} = \frac{2\sigma \kappa_a \rho_a}{1 + \gamma \sigma k_q} \Delta \tilde{a}_t.
\]

Therefore, a positive productivity growth shock results in an expected appreciation. Combining this with the flow budget constraint, we have a system of dynamic equations, which

\(^{20}\)The results in the stationary AR(1) case are qualitatively similar to the random walk limit with \( \rho_a \).
we solve again using the Blanchard-Kahn method (as in Lemma A1):

$$\Delta q_{t+1} = \frac{\hat{\lambda} - \beta \rho_a \hat{\kappa}}{1 - \beta \rho_a} \left( \Delta \tilde{a}_{t+1} - \frac{(\hat{\lambda} - \hat{\kappa}) \rho_a}{\hat{\lambda} - \beta \rho_a \hat{\kappa}} \Delta \tilde{a}_t \right),$$

where \(\hat{\kappa} \equiv \frac{2 \sigma \kappa_a}{1 + \gamma \sigma \kappa_q}\) and \(\hat{\lambda} \equiv 2 \lambda_a / \lambda_q\). With this solution, we can calculate the Backus-Smith covariance (making use of (1.40)):

$$\frac{\text{cov}(\Delta c_t - \Delta c^*_t, \Delta q_t)}{\text{var}(\Delta q_t)} = \frac{\text{cov}(\Delta \tilde{a}_t, \Delta q_t)}{\text{var}(\Delta q_t)} - \gamma \kappa_q$$

$$= 2 \kappa_a \left( \frac{(1 - \beta \rho_a)[\hat{\lambda}(1 - \rho_a^2) - \hat{\kappa} \rho_a(\beta - \rho_a)]}{[\hat{\lambda}(1 - \rho_a) + \hat{\kappa} \rho_a(1 - \beta)]^2 + \rho_a(2 - \rho_a)(\hat{\lambda} - \hat{\kappa})(\hat{\lambda} - \hat{\kappa} \beta \rho_a)} - \gamma \kappa_q.\right.$$

Around \(\rho_a \approx 0\) (random walk), the Marshal-Lerner condition is sufficient to ensure that this expression is positive (corresponding to the case we considered earlier). However, as \(\rho_a\) increases, this equation switches sign to negative, as in the limit \(\beta \rho_a \to 1, \text{var}(\Delta q_t) / \text{var}(\Delta \tilde{a}_t) \to \infty\). This is an intuitive result: as productivity growth shocks become very persistent, the contemporaneous improvement in productivity is small relative to the cumulative expected improvement, and the RER responds to the cumulative expectation. Therefore, tiny shocks to current productivity act like news shocks about future productivity, and trigger large RER movements. Under these circumstances, indirect expenditure switching effect of RER on consumption can dominate the direct contemporaneous productivity effect in (1.40). Long-run risk shocks in Colacito and Croce (2013) operate in a similar way, yet have an additional risk premia effects in (1.39).

\[\text{To apply the Blanchard-Kahn method, we rewrite the system in matrix form as:}\]

$$E_t \left( \begin{array}{c} q_{t+1} \\ \hat{b}_{t+1} \end{array} \right) = \left( \begin{array}{cc} 1 & 0 \\ 1 & 1 / \beta \end{array} \right) \left( \begin{array}{c} q_t \\ \hat{b}_t \end{array} \right) + \left( \begin{array}{c} \hat{\kappa} \rho_a \Delta \tilde{a}_t \\ -\hat{\lambda} \Delta \tilde{a}_t \end{array} \right),$$

where \(\hat{b}_t \equiv \beta b^*_t / (\gamma \lambda_q)\), and look for the unique stationary solution associated with the explosive root \(1 / \beta\). Note that the coefficients do not depend on \(\rho_a\), which facilitates taking the limits below. Note that, given the productivity process, the RER equilibrium process is still an ARIMA(1,1,1), as in Lemma A1, yet with a different MA root.
point out that persistent growth rate shocks are not necessary per se, as persistent effects to output growth can be obtained from endogenous amplification, such as capital accumulation in Corsetti, Dedola, and Leduc (2008).

A.1.8 Monetary model with nominal rigidities (Section 1.4.1)

We outline the details of the monetary model, adopting a general enough setup to nest several extensions as special cases. In particular, we allow for both nominal wage and price rigidities. As before, we focus on Home and symmetric relationships hold in Foreign.

Households Consider a standard New Keynesian two country model in a cashless limit, as described in Galí (2008). In particular, the aggregate labor input is a CES aggregate of individual varieties with elasticity of substitution $\epsilon$, which results in labor demand:

$$L_{it} = \left( \frac{W_{it}}{W_t} \right)^{-\epsilon} L_t,$$

where

$$L_t = \left( \int L_{it}^{\epsilon-1} \, dt \right)^{\frac{\epsilon}{\epsilon - 1}}$$

and

$$W_t = \left( \int W_{it}^{1-\epsilon} \, dt \right)^{\frac{1}{1-\epsilon}},$$

and the rest of the model production structure is unchanged. The first order conditions of the household optimization result in the New Keynesian IS curve and the UIP condition:

$$E_t \{ \sigma \Delta \tilde{c}_{t+1} + \Delta \tilde{p}_{t+1} \} = \tilde{i}_t, \quad (A75)$$

$$E_t \Delta \tilde{c}_{t+1} = 2 \tilde{i}_t - \psi_t, \quad (A76)$$

where as before we use notation $\tilde{x}_t = \frac{1}{2} (x_t - x_t^*)$.

Households set wages a la Calvo and supply as much labor as demanded at a given wage rate. The probability of changing wage in the next period is $1 - \lambda_w$. The first order
condition for wage setting is:

\[ E_t \sum_{s=t}^{\infty} (\beta \lambda_w)^{s-t} \frac{C_s^{\sigma}}{P_s} W^s \left( \frac{\bar{W}_t^{1+\epsilon/\nu}}{s} - \frac{\kappa \epsilon}{\epsilon - 1} P_s C_s^{\sigma} L_s^{1/\nu} W^s \right) = 0. \]

Substituting in labor demand and log-linearizing, we obtain:

\[ \hat{w}_t = \frac{1 - \beta \lambda_w}{1 + \epsilon/\nu} \left( c_t + \frac{1}{\nu} \ell_t + p_t + \frac{\epsilon}{\nu} w_t \right) + \beta \lambda_w E_t \hat{w}_{t+1}, \]

where \( \hat{w}_t \) denotes log deviation from the steady state of the wage rate reset at \( t \). Note that the wage inflation can be expressed as \( \pi_w \equiv \Delta w_t = (1 - \lambda_w)(\hat{w}_t - w_{t-1}) \). Aggregate wages using these equalities and express the wage process in terms of cross-country differences to obtain the NKPC for wages:

\[ [1 + \beta + k_w] \hat{w}_t - \beta E_t \hat{w}_{t+1} - \hat{w}_{t-1} = k_w \left[ \sigma \tilde{c}_t + \frac{1}{\nu} \tilde{\ell}_t + \tilde{p}_t \right], \tag{A77} \]

where \( k_w = \frac{(1 - \beta \lambda_p)(1 - \lambda_w)}{\lambda_w (1 + \epsilon/\nu)} \).

**Firms**  
Assume that firms set prices a la Calvo with probability of changing price next period equal \( 1 - \lambda_p \). There are two Phillips curves, one for domestic sales \( \tilde{p}_{Ht} \) and one for export \( \tilde{p}_{Ht}^* \). The first order conditions for reset prices in log-linearized form are

\[ \hat{p}_{Ht} = (1 - \beta \lambda_p) E_t \sum_{j=t}^{\infty} (\beta \lambda_p)^{j-t} \left[ (1 - \alpha) (1 - \phi) w_j + \phi p_j \right], \]

\[ \hat{p}_{Ht}^* = (1 - \beta \lambda_p) E_t \sum_{j=t}^{\infty} (\beta \lambda_p)^{j-t} \left[ (1 - \alpha) (1 - \phi) w_j + \phi p_j - e_j \right]. \]
The law of motion for home prices and the resulting NKPC are then:

$$\pi_{Ht} = (1 - \lambda_p) (\hat{p}_{Ht} - p_{Ht-1}) = \frac{1 - \lambda_p}{\lambda_p} (\hat{p}_{Ht} - p_{Ht}),$$

$$[1 + \beta + k_p (\gamma + (1 - \alpha) (1 - \gamma) (1 - \phi))] \bar{p}_{Ht} - \hat{p}_{Ht-1}$$

$$= k_p (1 - \alpha) [-\bar{a}_t + (1 - \phi) \bar{w}_t] - k_p \gamma [1 - (1 - \alpha) (1 - \phi)] \bar{p}_{Ht},$$

where $k_p = \frac{(1 - \beta \lambda_p)(1 - \lambda_p)}{\lambda_p}$. On the other hand, the law of motion for export prices depends on currency of invoicing. Assuming LCP one obtains

$$\pi_{Ht}^* = (1 - \lambda_p) (\hat{p}_{Ht}^* - p_{Ht-1}^*) = \frac{1 - \lambda_p}{\lambda_p} (\hat{p}_{Ht}^* - p_{Ht}^*),$$

$$[1 + \beta + k_p (1 - \alpha \gamma + \gamma \phi (1 - \alpha))] \bar{p}_{Ht} - \hat{p}_{Ht-1}^*$$

$$= k_p (1 - \alpha) [-\bar{a}_t + (1 - \phi) \bar{w}_t - e_t] - k_p (1 - \gamma) [\alpha - (1 - \alpha) \phi] \bar{p}_{Ht},$$

In case of PCP the law of motion of price index and NKPC are

$$\pi_{Ht}^* = (1 - \lambda_p) (\hat{p}_{Ht}^* - p_{Ht-1}^*) - \lambda_p \Delta e_t = \frac{1 - \lambda_p}{\lambda_p} (\hat{p}_{Ht}^* - p_{Ht}^*) - \Delta e_t,$$

$$[1 + \beta + k_p (1 - \alpha \gamma + \gamma \phi (1 - \alpha))] (\bar{p}_{Ht}^* + e_t) - \hat{p}_{Ht-1}^* + e_{t+1} - \hat{p}_{Ht-1}^* + e_{t-1}$$

$$= k_p (1 - \alpha) [-\bar{a}_t + (1 - \phi) \bar{w}_t] + k_p [\alpha(1 - \gamma) + \gamma \phi (1 - \alpha)] e_t - k_p (1 - \gamma) [\alpha - (1 - \alpha) \phi] \bar{p}_{Ht}.$$

**Government policy and shocks** We assume that Central Bank conducts active monetary policy, while the government chooses the fiscal policy (taxes) passively to balance the budget. The monetary policy is represented by a standard Taylor rule:

$$i_t = \rho_m i_{t-1} + (1 - \rho_m) [\delta_\pi \pi_t + \delta_y y_t] + \epsilon_t^m,$$  \hspace{1cm} (A78)
where $\delta_y$ is a coefficient on the output gap, which in the baseline case was absent (i.e., $\delta_y = 0$). The Taylor rules are symmetric in both countries. In the case with exchange rate peg (Table 1.4), we check robustness with asymmetric Taylor rules, where one country follows a conventional Taylor rule (1.43), while the other pegs its exchange rate according to (1.44).

We allow persistence of the interest rate to be different from the autocorrelation of other shocks, which in addition to the $\psi_t$ shock in (1.23) also include foreign-good demand and productivity shocks:

\begin{align}
\tilde{\xi}_t &= \rho \tilde{\xi}_{t-1} + \epsilon_t^\xi, \\
\tilde{a}_t &= \rho \tilde{a}_{t-1} + \epsilon_t^a.
\end{align}

**Market clearing**  The last dynamic equation is the country’s budget constraint:

\begin{equation}
\beta b^*_t + 1 = b^*_t + 2 (\tilde{p}^*_H + \tilde{y}^*_H) + e_t,
\end{equation}

where $b^*_t$ is the net foreign asset position of the Home country. The static part of the model is represented by labor demand and goods market equilibrium conditions:

\begin{align}
\tilde{\ell}_t &= \tilde{y}_t - \tilde{a}_t + \phi ((1 - \gamma) \tilde{p}_H - \gamma \tilde{p}^*_H - \tilde{w}_t) \\
\tilde{y}_H &= -\gamma \tilde{\xi}_t - \theta (\tilde{p}_H + \tilde{p}^*_H) + (1 - \phi) \tilde{c}_t + \phi ((1 - \phi) (\tilde{w}_t - \tilde{p}_t) - \tilde{a}_t + \tilde{y}_t) \\
\tilde{y}^*_H &= -\tilde{y}_H - \tilde{\xi}_t - \theta (\tilde{p}_H + \tilde{p}^*_H) \\
\tilde{y}_t &= (1 - \gamma) \tilde{y}_H + \gamma \tilde{y}^*_H.
\end{align}
The numbered equations above define the system that describes the equilibrium dynamics of the model.

**Robustness** All baseline parameters are as described in the main text and we set the elasticity of substitution between different types of labor to $\epsilon = 4$. Table A3 presents the results from alternative monetary models with multiple shocks, which can be compared to the baseline multi-shock monetary model in column 5 of Table 1.2. In particular, we consider the following alternative specifications, adjusting one feature of the model at a time relative to the baseline:

1. Flexible wages ($\lambda_w = 0$): no noticeable difference

2. Flexible prices ($\lambda_p = 0$): the volatility and correlation of terms of trade and real exchange rate relative to the nominal exchange rate deteriorate somewhat, volatility of consumption goes up, and the role of monetary shocks is larger.

3. Lower persistence in the Taylor rule ($\rho_m = 0.8$): Fama coefficient becomes positive, interest rates become more volatile and less persistent.

4. Expected inflation ($E_t \pi_{t+1}$) instead of $\pi_t$ in the Taylor rule (1.43): no noticeable differences.

5. Positive weight on output gap in the Taylor rule ($\delta_y = 0.2$): no noticeable differences.

6. PCP stickiness instead of LCP (with the same $\lambda_p = 0.75$): correlation between RER and ToT becomes approximately $+1$ instead of $-1$, with few other differences.

In each case, we recalibrate the relative volatilities of the shocks (reported in the last two
Table A3: Monetary model: robustness

<table>
<thead>
<tr>
<th></th>
<th>( \lambda_w = 0 ) (1)</th>
<th>( \lambda_p = 0 ) (2)</th>
<th>( \rho_m = 0.8 ) (3)</th>
<th>( \bar{E}<em>t \pi</em>{t+1} ) (4)</th>
<th>( \delta_y = 0.2 ) (5)</th>
<th>PCP (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho (\Delta e) )</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.04</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
<td>(0.09)</td>
<td>(0.09)</td>
<td>(0.09)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>( \rho (q) )</td>
<td>0.92</td>
<td>0.94</td>
<td>0.93</td>
<td>0.92</td>
<td>0.92</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>( \sigma(\Delta q) \sigma(\Delta e) )</td>
<td>0.99</td>
<td>0.75</td>
<td>0.98</td>
<td>0.98</td>
<td>0.97</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>corr (( \Delta e, \Delta q ))</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( \sigma(\Delta c - \Delta c^*) ) ( \sigma(\Delta q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr (( \Delta c - \Delta c^*, \Delta q ))</td>
<td>-0.20</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
</tr>
<tr>
<td>( \sigma(\Delta n x) ) ( \sigma(\Delta q) )</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
</tr>
<tr>
<td>corr (( \Delta n x, \Delta q ))</td>
<td>-0.00</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
</tr>
<tr>
<td>( \sigma(\Delta s) ) ( \sigma(\Delta e) )</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
</tr>
<tr>
<td>corr (( \Delta s, \Delta e ))</td>
<td>-0.92</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
</tr>
</tbody>
</table>

| Fama \( \beta \) | -0.6 (1.5) |
|                   | -0.0 (1.1) |
|                   | 0.4 (0.5)  |
|                   | -0.8 (1.6) |
|                   | -2.4 (2.7) |
|                   | -0.1 (1.2) |
| Fama \( R^2 \)   | 0.00 (0.01)|
|                   | 0.00 (0.01)|
|                   | 0.01 (0.02)|
|                   | 0.00 (0.01)|
|                   | 0.01 (0.02)|
|                   | 0.00 (0.01)|
| \( \sigma(i-i^*) \) \( \sigma(\Delta e) \) | 0.06 (0.02)|
|                   | 0.08 (0.02)|
|                   | 0.16 (0.03)|
|                   | 0.06 (0.02)|
|                   | 0.05 (0.02)|
|                   | 0.08 (0.02)|
| \( \rho (i-i^*) \) | 0.91 (0.05)|
|                   | 0.88 (0.05)|
|                   | 0.81 (0.06)|
|                   | 0.92 (0.04)|
|                   | 0.96 (0.02)|
|                   | 0.90 (0.05)|
| Sharpe Ratio      | 0.17 (0.06)|
|                   | 0.16 (0.06)|
|                   | 0.19 (0.06)|
|                   | 0.17 (0.06)|
|                   | 0.19 (0.06)|
|                   | 0.16 (0.07)|

Decomposition of \( \text{var}(\Delta e_{t+1}) \):

- Monetary shock, \( \varepsilon_t^m \): 8% 29% 19% 8% 1% 21%
- Foreign-good shocks, \( \xi_t \): 23% 20% 21% 19% 39% 24%
- Financial shocks, \( \psi_t \): 69% 51% 60% 73% 60% 55%

Calibrated variances of the shocks:

\( \sigma_m / \sigma_\varepsilon \) | 0.31 0.64 1.07 0.27 0.1 0.51
| \( \gamma \sigma_\xi / \sigma_\varepsilon \) | 2.6 3.1 2.3 2.7 1.8 3.5

Note: The table reports moments as in Table 1.2 for six alternative specifications of the multi-shock monetary model, as explained in the text. The lower panels report the variances decomposition for the nominal exchange rate into the contribution of the shocks (as in Table 1.3) and the calibrated relative volatilities of the shocks, which are adjusted to match \( \text{corr}(\Delta c - \Delta c^*, \Delta q) = -0.20 \) and \( \text{corr}(\Delta n x, \Delta q) = 0.00 \) (see footnote 38).
lines of Table A3) to still match the correlations between consumption and net exports and the real exchange rate.
A.1.9  A model with a financial sector (Section 1.4.2)

This appendix provides the details for the model of Section 1.4.2. We discuss here the equations that change relative to the baseline model summarized in Appendix A.1.3. This concerns only the blocks 5 and 6, namely the Euler equations and the budget constraints, and additionally it involves the equilibrium (market clearing) conditions for the financial intermediation sector.

We start with the home, which now has the following consolidated budget constraint:

\[ B_{t+1} - R_{t-1} B_t = N X_t, \]

where \( N X_t = \epsilon_t P^* H Y^* H_t - P_{Ft} Y_{Ft} = P_{Ht} Y_{Ht} + \epsilon_t P^* H Y^* H_t - P_{t} C_{t}. \)

Since home households can trade only the home currency bond, their intertemporal optimization is characterized by a single Euler equation:

\[ R_t \mathbb{E}_t \Theta_{t+1} = 1, \quad \text{where} \quad \Theta_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}. \]

The log linearization of these two conditions results in:

\[ \beta b_{t+1} - b_t = n x_t, \quad (A86) \]

\[ i_t = \mathbb{E}_t \{ \sigma \Delta c_{t+1} - \Delta p_{t+1} \}, \]

both equations exactly as before.\(^{22}\) Given that the static relationship in the model are unchanged, we still have as in (1.25) that \( n x_t = \gamma \lambda_2 e_t \) with \( \lambda_2 \) defined in (A66).

The foreign households differ only in that the profits and losses of the noise traders and arbitrageurs are transferred to them, but that constitutes a second order term, which

---

\(^{22}\)There is a slight change in notation to \( b_{t+1} = RB_{t+1}/\bar{Y} \) from \( b^*_t = B^*_{t+1}/\bar{Y} \), since previously the NFA of home was in foreign-currency bonds (while now it is in home-currency bonds), and \( B^*_{t+1} \) used to denote the nominal value of the bond.
vanishes in the log linearization. Specifically, we have the two parallel equations for foreign:

\[
B_{t+1}^* - R_{t+1}^* B_t^* = NX_t^* + \tilde{R}_t (N_t^* + D_t^*), \quad \tilde{R}_t \equiv R_{t+1}^* - R_{t-1} \frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}, \\
R_t^* \mathbb{E}_t \Theta_{t+1}^* = 1, \quad \Theta_{t+1}^* = \beta \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*}.
\]

The log-linearization of the former results in \(\beta b_{t+1}^* - b_t^* = nx_t^*\), since in steady state both \(\tilde{R} = 0\) and \(N^* = D^* = 0\), and hence the transfer term is second order. Also note that this log-linearized equation is equivalent to (A86), since by definition \(NX_t^* = -\mathcal{E}_t NX_t\) and from market clearing \(B_{t+1}^* = -\mathcal{E}_t B_{t+1}\) (see below), and hence we drop it from the equilibrium system (Walras law). The log linearization of the second conditions is, as before:

\[
i_t^* = \mathbb{E}_t \{ \Delta c_{t+1}^* - \Delta p_{t+1}^* \}.
\]

Finally, we turn to the financial market clearing \(B_{t+1} + N_{t+1} + D_{t+1} = 0\) and \(B_{t+1}^* + N_{t+1}^* + D_{t+1}^* = 0\). These conditions imply that, given that noise traders and arbitrageurs both hold zero-capital positions \((N_{t+1} = -\mathcal{E}_t N_{t+1}^* \text{ and } D_{t+1} = -\mathcal{E}_t D_{t+1}^*)\), the net foreign assets of foreign equal the net foreign liabilities of home: \(\mathcal{E}_t B_{t+1}^* = -B_{t+1}\). Therefore, in light of the exogenous demand of noise traders (1.45) and optimal demand of the arbitrageurs (1.46), we have the following market clearing condition:

\[
\frac{B_{t+1}}{\mathcal{E}_t} = n \left( e^{\psi_t} - 1 \right) + m \frac{\mathbb{E}_t \tilde{R}_{t+1}}{\omega \text{ var}_t(\tilde{R}_{t+1})}.
\]

Using the fact that

\[
\tilde{R}_{t+1} = R_t^* \left( 1 - \frac{R_t^*}{\mathcal{E}_t} \right) = R_t^* \left( 1 - e^{\psi_t - i_t^* - \Delta e_{t+1}} \right),
\]

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we rewrite this condition as:

\[ \frac{R^*_t B_{t+1}}{E_t} = R^*_t \left( e^{\tilde{\psi}_t} - 1 \right) + m \frac{\mathbb{E}_t \{ 1 - e^{\tilde{\iota}_t - \tilde{\iota}^*_t - \Delta \epsilon_{t+1}} \}}{\omega \text{var}_t \{ 1 - e^{\tilde{\iota}_t - \tilde{\iota}^*_t - \Delta \epsilon} \}}. \]

Using the facts that in steady state \( R^* = 1/\beta, E = 1, B = 0, \psi = 0 \) and \( \iota - \iota^* - \Delta \epsilon = 0 \), we obtain the approximation:  

\[ \tilde{Y} b_{t+1} = \frac{n}{\beta} \psi_t - \frac{m}{\omega \sigma^2_e}(i_t - i^*_t - \mathbb{E}_t e_{t+1}), \]

where \( b_{t+1} \equiv \frac{R^*_t B_{t+1}}{\tilde{Y}} = \frac{B_{t+1}}{\beta \tilde{Y}} \) and \( \sigma^2_e \equiv \text{var}_t(\Delta e_{t+1}) \), which after rearranging results in the UIP condition (1.47) in the text, reproduced here as:

\[ i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} = \chi_1 \psi_t - \chi_2 b_{t+1}, \tag{A87} \]

where

\[ \chi_1 \equiv \frac{n/\beta}{m/(\omega \sigma^2_e)} \quad \text{and} \quad \chi_2 \equiv \frac{\tilde{Y}}{m/(\omega \sigma^2_e)}. \]

Now combing (A86) and (A87) together with the static equilibrium relationships of net exports and nominal interests with the exchange rate (which are unchanged, and as derived in Appendix A.1.3), we obtain the dynamic equilibrium system:

\[ \mathbb{E}_t \Delta e_{t+1} = -\frac{\chi_1}{1 + \gamma \lambda_1} \psi_t + \frac{\chi_2}{1 + \gamma \lambda_1} b_{t+1}, \tag{A88} \]

\[ \beta b_{t+1} - b_t = \gamma \lambda_2 e_t, \tag{A89} \]

where we again assume that the exogenous shock follows an AR(1):

\[ \psi_t = \rho \psi_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iid(0, \sigma^2_\varepsilon). \tag{A90} \]

\[^{23}\text{This can be viewed as a log-linear approximation under an asymptotics where } \sigma^2_\varepsilon \text{ is first order in the size of the shocks, as for example can be the case when the number of arbitrageurs } m \text{ decreases with the volatility of the shocks.}\]
Note that this system takes the one in the baseline model as a special case as \( \chi_2 \to 0 \), and it generalizes it by allowing for \( \chi_2 > 0 \). Note also the additional complication that the \( \chi_1, \chi_2 \) coefficients depend on the equilibrium volatility of the nominal exchange rate innovations, which needs to be taken into account in the solution.

We now prove a generalization of Lemma A1 in Appendix A.1.5 for the model’s extension with a financial sector:

**Lemma A2** (a) The unique non-explosive solution to the dynamic system (A88)–(A90) is given by:

\[
(1 - \zeta_1 L) e_{t+1} = \frac{1}{1 + \gamma \lambda_1} \frac{\beta \zeta_1}{1 - \beta \zeta_1 \rho} (1 - \beta^{-1} L) \chi_1 \psi_{t+1},
\]

\[
(1 - \zeta_1 L) b_{t+1} = \frac{\gamma \lambda_2}{1 + \gamma \lambda_1} \frac{\chi_1 \psi_t}{1 - \beta \zeta_1 \rho},
\]

where \( \zeta_1 \in (0, 1] \) and \( \zeta_1 \to 1 \) as \( \chi_2 \to 0 \).

(b) There exists a cutoff \( \hat{d} > 0 \), such that for \( \frac{1}{\beta(1+\gamma \lambda_1)} \frac{n \omega \sigma_e}{m} < \hat{d} \), the only equilibrium has \( \sigma_e^2 = \chi_1 = \chi_2 = 0 \), while for \( \frac{1}{\beta(1+\gamma \lambda_1)} \frac{n \omega \sigma_e}{m} > \hat{d} \) there also exists an equilibrium with \( \sigma_e > 0 \) and \( \frac{\partial \sigma_e}{\partial (n \omega \sigma_e / m)} > 0 \).

**Proof:** We define the following normalized variables: \( \hat{\psi}_t \equiv \frac{\chi_1 \psi_t}{1 + \gamma \lambda_1} \) and \( \hat{b}_{t+1} = \frac{\beta b_{t+1}}{\gamma \lambda_2} \). Then we can rewrite (A88)–(A89) in the matrix form as:

\[
\begin{pmatrix}
E_t e_{t+1} \\
\hat{b}_{t+1}
\end{pmatrix} = A
\begin{pmatrix}
e_t \\
\hat{b}_t
\end{pmatrix} + \begin{pmatrix} -\hat{\psi}_t \\
0
\end{pmatrix}, \quad A \equiv \begin{pmatrix} 1 + \kappa & \kappa / \beta \\
1 & 1 / \beta
\end{pmatrix},
\]

where \( \kappa \equiv \frac{\gamma \lambda_2 \chi_2 / \beta}{1 + \gamma \lambda_1} \geq 0 \) with \( \kappa \to 0 \) iff \( \chi_2 \to 0 \). The two eigenvalues of \( A \) are the solutions of \( (1 + \kappa - \zeta_i)(1 / \beta - \zeta_i) - \kappa / \beta = 0 \), and are given by:

\[
\zeta_{1,2} = \frac{(1 + \kappa + \frac{1}{\beta}) \mp \sqrt{(1 + \kappa + \frac{1}{\beta})^2 - \frac{4}{\beta}}}{2}
\]

with the property that \( 0 < \zeta_1 \leq 1 \) and \( \frac{1}{\beta} \leq \zeta_2 < \infty \), with the two equalities obtaining iff \( \kappa \to 0 \) (i.e., \( \chi_2 \to 0 \)). Furthermore, the Vieta’s formulas imply \( \zeta_1 \zeta_2 = 1 / \beta \) and \( \zeta_1 + \zeta_2 = 1 + \kappa + 1 / \beta \), which we conveniently use below.
The left eigenvector of $A$ associated with $\zeta_2 > 1$ is $v_2 = [(\zeta_2 - \frac{1}{\beta}), \frac{\kappa}{\beta}]$. Therefore, the cointegration relationship between the variables is (see proof of Lemma A1):

\[
e_t + \frac{\kappa/\beta}{\zeta_2 - 1/\beta} \hat{b}_t = \frac{1}{\zeta_2 - \rho} \hat{\psi}_t,
\]

or equivalently using the Vieta’s formulas:

\[
e_t + \frac{1}{\beta} (1 - \beta \zeta_1) \hat{b}_t = \frac{\beta \zeta_1}{1 - \beta \zeta_1 \rho} \hat{\psi}_t,
\]

Combining with the second equation of the system, this yields the solution for $\hat{b}_{t+1}$:

\[
\hat{b}_{t+1} = \frac{1}{\beta} \hat{b}_t + e_t = \zeta_1 \hat{b}_t + \frac{\beta \zeta_1}{1 - \beta \zeta_1 \rho} \hat{\psi}_t.
\]

Together with the definitions of $\hat{b}_{t+1}$ and $\hat{\psi}_t$, this yields the solution in Lemma A2. Since $\psi_t$ follows an AR(1), $b_{t+1}$ follows an AR(2) with roots $\zeta_1$ and $\rho$.

Next we combine this solution with the cointegration relationship to obtain:

\[
(1 - \zeta_1 L)e_{t+1} = -\frac{1}{\beta} (1 - \beta \zeta_1) (1 - \zeta_1 L) \hat{b}_{t+1} + \frac{\beta \zeta_1}{1 - \beta \zeta_1 \rho} (1 - \zeta_1 L) \hat{\psi}_{t+1} = \frac{\beta \zeta_1}{1 - \beta \zeta_1 \rho} \left( \hat{\psi}_{t+1} - \frac{1}{\beta} \hat{\psi}_t \right),
\]

and therefore $e_{t+1}$ follows an ARMA(2,1) with AR roots $\zeta_1$ and $\rho$ and an MA root $1/\beta$.

Note that as $\chi_2 \to 0$, we have $\kappa \to 0$, $\zeta_1 \to 1$ and $\zeta_2 \to 1/\beta$, and therefore the stationary solutions AR(2) and ARMA(2,1) become integrated solutions of Lemma A1, namely ARIMA(1,1,0) and ARIMA(1,1,1).

Lastly, we characterize the equilibrium volatility of the innovation of the nominal exchange rate. We have:

\[
\sigma_e^2 = \text{var}_t(\Delta e_{t+1}) = \left( \frac{\beta \zeta_1}{1 - \beta \zeta_1 \rho} \right)^2 \text{var}_t(\hat{\psi}_{t+1}) = \left( \frac{1}{\zeta_2 - \rho} \right)^2 \left( \frac{1}{1 + \gamma \lambda_1} \frac{n/\beta}{m/(\omega \sigma_e^2)} \right)^2 \sigma_e^2,
\]

(A93)
Exchange rate volatility, $\sigma_e$

Figure A6: Equilibrium exchange rate volatility

Note: The figure illustrates the three equilibria (black dots), which exist for $d = \frac{1}{\beta(1 + \gamma \lambda_1)} \frac{m \omega \sigma_e}{m} > \hat{d}$. When $d < \hat{d}$, the only equilibrium is $\sigma_e = 0$. As $\beta \rho \rightarrow 1$, the red convex curve $\zeta_2(\sigma_e) - \rho$ starts at the origin, and the two left equilibria coincide.

where we used Vieta’s formula and the definitions of $\hat{\psi}_t$ as function of the primitive $\psi_t$ with innovation $\varepsilon_t$ with variance $\sigma_e^2$. Note that $\zeta_2$ also depends on $\sigma_e^2$ through $\chi_2$, which determines $\kappa$:

$$
\zeta_2 = \left(1 + \kappa + \frac{1}{\beta}\right) + \sqrt{\left(1 + \kappa + \frac{1}{\beta}\right)^2 - \frac{4}{\beta}},
\quad \kappa = \frac{\gamma \lambda_2 \chi_2}{1 + \gamma \lambda_1},
\quad \chi_2 = \frac{\omega \bar{Y}}{m} \sigma_e^2.
$$

Since $\zeta_2 \rightarrow 1/\beta$ as $\sigma_e \rightarrow 0$, (A93) always has a root $\sigma_e = 0$. Denote

$$
d = \frac{1}{\beta(1 + \gamma \lambda_1)} \frac{m \omega \sigma_e}{m} \geq 0.
$$

There exists a $\hat{d} > 0$ such that if $d < \hat{d}$, then $\sigma_e = 0$ is the only solution of (A93). For $d > \hat{d}$, (A93) has two non-zero solution, which satisfy:

$$
\zeta_2 - \rho = d \cdot \sigma_e.
$$

This is because $\zeta_2 \rightarrow 1/\beta$ as $\sigma_e \rightarrow 0$ and $\zeta_2$ is convex in $\sigma_e$ for $\sigma_e > 0$, and $\hat{d}$ is the unique value at which $d \sigma_e$ and $\zeta_2 - \rho$ are tangent (see Figure A6). One of these two
solutions has the property that \( \partial \sigma_e / \partial d > 0 \), and we select this solution as economically meaningful. One can rationalize this solution as stable by introducing explicit dynamics of entrepreneur’s entry. More importantly, this solution becomes the unique non-zero solution of (A93) in the limit \( \beta \rho \to 1 \), as in this limit \( (\zeta_2 - \rho)|_{\sigma_e=0} \to 0 \), hence \( \hat{d} \to 0 \) and the other non-zero root merges with \( \sigma_e = 0 \) root.

**Proof of Proposition 1.9** follows directly from Lemma A2.

**Figure 1.2** plots impulse responses of the nominal exchange rate \( e_{t+j} \) to the innovation in \( i_t - i^*_t \) for \( j \geq 0 \) obtained from a model with a financial sector, both its single-\( \psi_t \)-shock version and the multi-shock version describe in Section 1.4.2. For the single shock case, we construct this impulse response as \( \frac{\partial e_{t+j}}{\partial \varepsilon_t} / \frac{\partial (i_t - i^*_t)}{\partial \varepsilon_t} \), where \( \varepsilon_t \) is the innovation of the \( \psi_t \) process (1.23), the only source of innovations in this version of the model. Given the closed-form solutions for both the nominal exchange rate and the interest rate differential, this impulse response is analytical. In fact, since (1.42) still holds in this model, we have \( \frac{\partial (i_t - i^*_t)}{\partial \varepsilon_t} = \frac{\gamma \lambda_1}{1 + \gamma \lambda_1} \) and therefore the impulse response of the exchange rate to the innovation in the interest rate is simply the impulse response of exchange rate to \( \varepsilon_t \), as characterized by (1.48), scaled by \( \frac{\gamma \lambda_1}{1 + \gamma \lambda_1} \).

Next, consider the multi-shock version. In this case, the innovation to \( i_t - i^*_t \) comes from a combination of shocks, and we define the impulse response as follows:

\[
IRF_{i_t-i^*_t}^{e_{t+j}} = \sum_{z \in \{\psi, a, \xi\}} \frac{\partial e_{t+j}}{\partial \varepsilon^z_t} \sigma_z \left/ \sum_{z \in \{\psi, a, \xi\}} \frac{\partial (i_t - i^*_t)}{\partial \varepsilon^z_t} \sigma_z \right.
\]

where \( z \) indexes the shocks \( (\psi_t, a_t, \xi_t) \), \( \varepsilon^z_t \) is the innovation of respective shock, and \( \sigma^2_z \) is its variance. Therefore, the standard-deviation-weighted response of \( i_t - i^*_t \) to the innovation...
Figure A7: Impulse response of $\rho_{t+j}$ to $i_t - i_t^*$

Note: this Figure complements Figure 1.3a, and plots the impulse response for the ex post realized UIP deviations $\rho_{t+j} = i_{t+j-1} - i_{t+j-1}^* - \Delta e_{t+j}$, instead of $E_t \rho_{t+j}$, reproducing Figure 4 in Engel (2016), as described below.

Figure 1.3 The empirical impulse response functions in Figure 1.3 are calculated following closely Engel (2016) and Valchev (2016). The data used is for US vs trade-weighted average of Canada, France, Germany, Italy, Japan and UK, using monthly data from 1979:06 to 2009:10 provided by Engel (2016). As a result, the empirical impulse response in Figure 1.3a reproduces exactly that in Figure 2 of Engel (2016), while the empirical impulse response in Figure 1.3b differs slightly from that in Figure 2 in Valchev (2016) due to the difference in the dataset, yet the results are consistent qualitatively. The same procedures are applied to calculating impulse response in the model-generated data.

The impulse response in Figure 1.3a plots $\delta_j$ for $j \geq 1$, which are obtained as coeffi-
cents from the regression:

$$\mathbb{E}_t \rho_{t+j} = \zeta_j + \delta_j (i_t - i^*_t) + u_{t+j},$$

where $\rho_{t+j} = i_{t+j-1} - i^*_t - \Delta e_{t+j}$ is the ex post one-period UIP deviation (risk premium) at $t+j$ and its conditional expectation $\mathbb{E}_t \rho_{t+j}$ is constructed using a VEC model for nominal exchange rate, price differential and nominal interest rate differential between countries, as described in detail in Engel (2016). In Figure A7 we plot a similar impulse for realized UIP deviations $\rho_{t+j}$, that is obtained from the following regression:

$$\rho_{t+j} = \zeta_j + \delta_j (i_t - i^*_t) + u_{t+j},$$

as in Figure 4 in Engel (2016). The impulse response in Figure 1.3b plots $\delta_j$ for $j \geq 0$ from:

$$e_{t+j} - e_t = \zeta_j + \delta_j (i_t - i^*_t) + u_{t+j},$$

where $\delta_0 = 0$ by construction.

**Additional moments** Table A2 reports two correlations $- \text{corr}(e_t, i_t - i^*_t)$ and $\text{corr}(\Delta e_t, \Delta i_t - \Delta i^*_t)$ — calculated both in the data (provided in Engel 2016, as described above) and for different model specifications. In the data, both correlations are mildly negative. We consider four model specifications:

1. multi-shock NOEM (corresponding to column 5 of Table 1.2) — both correlations are positive;

2. multi-shock IRBC (column 6 of Table 1.2) — first correlation is positive, second is negative;
(3) single-$\psi_t$-shock model with a financial sector — both correlations are strongly positive;

(4) multi-shock model with a financial sector (column 7 of Table 1.2) — both correlations are mildly negative, as in the data.

Thus, Table A2 shows how a multi-shock model with a financial sector reproduces the empirical unconditional correlation moments in addition to the projection coefficients reported in Figure 1.3.

A.1.10 Additional extensions

We consider four additional extensions:

1. **Decreasing returns to scale.** For tractability, the baseline model assumes constant returns to scale in production, which allows to solve for prices as a function of exchange rate and exogenous shocks, independently from quantities. We relax this assumption and show that qualitatively decreasing returns to scale act similarly to a higher Frisch elasticity $\nu$, and quantitatively both have only very mild effects on the properties of the model (see Appendix A.1.5). The detailed results are available from the authors upon request.

2. **GHH preferences.** The baseline model adopts the separable constant-elasticity utility in consumption and labor, for which the parameter $\sigma$ acts simultaneously as the inverse intertemporal elasticity of substitution in dynamic decisions and the income effect elasticity in labor supply. We consider instead the GHH preference specification with no income effect on labor supply to explore robustness of our qualitative and quantitative results to this feature of the transmission mechanism.
We show that the results remain robust qualitatively, and the only quantitative differences result in higher volatilities of the interest rates, consumption and output, slightly deteriorating the quantitative performance of the model. The detailed results are available from the authors upon request.

3. **A model with capital.** For simplicity, the baseline model abstracts from capital and dynamic investment decisions, to reduce the state space to a single net foreign asset variable. Below we show the robustness of our conclusions in Sections 1.3–2.4 to the introduction of capital accumulation.

4. **Full international business-cycle calibration** below shows that the ability of our model (with capital and productivity shocks) to match the exchange rate disconnect moments does not compromise its ability to match the conventional BKK-style international business cycle moments.

**A model with capital**

**Setup** We assume that firms rent capital from households, who in turn make the investment decisions. Capital is produced from country-specific consumption good with one period lag and potentially subject to capital adjustment costs. We continue to assume that only foreign bond is traded internationally, while domestic bond and capital stocks are traded only by local agents. Below we formulate optimization problems of home agents and derive equilibrium conditions in log-linear form.
Households The problem of Home household now includes the choice of capital investment:

\[
\max_{\{C_t, L_t, I_t, K_{t+1}, B_{t+1}, B^*_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \frac{I_{t+1}^{1+1/\nu}}{1 + 1/\nu} \right)
\]

s.t. \[
P_t \left( C_t + I_t + \frac{\kappa}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \right) + \frac{B_{t+1}}{R_t} + \frac{B^*_{t+1} \epsilon_t}{e^{\sigma_t R_t}} \leq B_t + B^* \epsilon_t + W_t L_t + P_t R^K t + \Pi_t + T_t,
\]

\[K_{t+1} = (1 - \delta) K_t + I_t,
\]

where \(\kappa\) is adjustment cost parameter and returns on capital \(R^K_t\) are in units of the final good. Labor supply and demand for bonds remain the same as in the baseline model and can be written in linearized form as follows:\footnote{With a slight adjustment in notation, we use \(r_t\) in this section to denote the nominal interest rate, with \(i_t\) now used for the log-deviation of investment.}

\[
\sigma c_t + \frac{1}{\nu} l_t = w_t - p_t,
\]

\[
\mathbb{E}_t \{\sigma \Delta c_{t+1} + \Delta p_{t+1}\} = r_t,
\]

\[
\mathbb{E}_t \{\sigma \Delta c_{t+1} + \Delta p_{t+1}\} = r^*_t + \psi_t.
\]

In addition, there is now an optimality condition for investment:

\[
1 + \kappa \left( \frac{I_t}{K_t} - \delta \right) = \beta \mathbb{E}_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left[ R^K_{t+1} + (1 - \delta) \left( 1 + \kappa \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \right) \right]
\]

\[+ \kappa \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right) \left( \frac{I_{t+1}}{K_{t+1}} - \delta \right)^2,
\]

which equalizes costs of investment with expected future returns and a change in adjustment costs in the future. In steady state, this optimality condition pins down the rate of return on capital: \(\beta \left( R^K + 1 - \delta \right) = 1\), implying \(R^K = \frac{1}{\beta} - (1 - \delta)\). Log-linearizing the capital law of motion and the optimality condition, and denoting \(z_t \equiv \log I_t - \log \bar{I}\), we obtain:
\[ k_{t+1} = (1 - \delta) k_t + \delta z_t, \]
\[ \kappa \delta (z_t - k_t) = -\sigma E_t \Delta c_{t+1} + \beta E_t \left[ \bar{R}^K r^K_{t+1} + \kappa \delta (z_{t+1} - k_{t+1}) \right]. \]

**Firms** The pricing block of the equilibrium system remains unchanged except for the marginal costs. Assume that production function is Cobb-Douglas with a share \( \phi_1 \) spent on intermediates. Out of the remaining \( 1 - \phi_1 \) part, \( \phi_2 \) is the capital share and \( 1 - \phi_2 \) is the labor share. We choose steady state productivity level so that marginal costs and prices are equal 1. Log-linear approximation to the pricing block is then:

\[ p_t = (1 - \gamma) p_{Ht} + \gamma p_{Ft}, \]
\[ p_{Ht} = (1 - \alpha) mc_t + \alpha p_t, \]
\[ p_{Ft} = (1 - \alpha) (mc_t^* + e_t) + \alpha p_t, \]
\[ mc_t = \phi_1 p_t + (1 - \phi_1) \phi_2 r^K_t + (1 - \phi_1) (1 - \phi_2) w_t - a_t. \]

**Market clearing** The market clearing conditions are more involved since we now have an additional market for capital:

- demand for labor: \( w_t + l_t = y_t + mc_t \)
- demand for capital: \( r^K_t + p_t + k_t = y_t + mc_t \)
- goods market equilibrium now includes investment demand and adjustment costs in addition to consumption and intermediates. For example, home demand for domestic goods is:

\[ Y_{Ht} = (1 - \gamma) e^{-\gamma \xi_t} \left[ C_t + X_t + I_t + \frac{\kappa}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \right]. \]
Up to the first order approximation, adjustment costs are equal zero, and therefore:

\[ y_t = (1 - \gamma) y_{Ht} + \gamma y_{Ht}^*, \]
\[ y_{Ht} = -\gamma \xi_t - \theta (p_{Ht} - p_t) + (1 - \phi_1) (\varsigma c_t + (1 - \varsigma) i_t) + \phi_1 (y_t + mc_t - p_t), \]
\[ y_{Ht}^* = (1 - \gamma) \xi_t^* - \theta (p_{Ht}^* - p_t^*) + (1 - \phi_1) (\varsigma c_t^* + (1 - \varsigma) i_t^*) + \phi_1 (y_t^* + mc_t^* - p_t^*), \]

where \( \frac{\xi}{\gamma} = (1 - \phi_1) \varsigma = (1 - \phi_1) \left(1 - \frac{\delta \phi_2}{R \phi_2}\right) \). When capital share in production goes to zero, i.e. \( \phi_2 = 0 \), we get the same market clearing conditions as in the baseline model.

The last dynamic equation is country’s budget constraint:

\[ \beta b_{t+1}^* = b_t^* + \frac{\gamma}{1 - \phi_1} nx_t, \]

where \( b_t^* \) is a net foreign asset position of the Home country and \( nx_t = p_{Ht}^* + y_{Ht}^* + e_t - p_{Ft} - y_{Ft} \) is its net export. Finally, a log-linear approximation to the GDP is

\[ gdp_t = \varsigma c_t + (1 - \varsigma) z_t + \frac{\gamma}{1 - \phi_1} nx_t. \]

**Calibration** All parameters from the benchmark model take the same values. Following the previous literature, we choose capital share in value added to be equal 0.3 and the quarterly depreciation rate of 0.02, which implies steady state capital-to-GDP ratio of 5. Adjustment cost parameter is calibrated together with relative volatilities of the shocks to match the relative volatility of investment in addition to exchange rate correlation with consumption and net exports.
**Effect from capital**  It is convenient to separate the effect of capital on the economy into static and dynamic components. In the extreme case when adjustment costs go to infinity, the capital stock becomes constant. As a result, dynamic effect of capital vanishes. However, the presence of capital in production function implies that the technology exhibits decreasing returns to scale in labor. As we discuss above, decreasing returns to scale have similar implications as a higher Frisch elasticity of labor supply (with mild consequences for the quantitative performance of the model). In addition, when adjustment costs are finite, there is also a dynamic effect of capital coming from the intertemporal investment choice of households and time-varying stock of capital, a new state variable.

**Results**  The process for exchange rate can be derived following the same steps as in the baseline case. The main difference is that we now have two states (NFA and capital) and two controls (exchange rate and consumption). It can be shown that for economically meaningful parameter values, the system has two eigenvalues greater than one (one of which is $1/\beta$ as in the baseline model), one eigenvalue smaller than one and one unit eigenvalue. It follows that each of the state variables follows ARIMA(2,1,1) processes, in contrast with the ARIMA(1,1,0) process for the NFA position in the baseline model.

In turn, exchange rate, being a linear function of the two state variables and financial shock, follows an ARIMA(2,1,2) process. Thus, the introduction of capital as an endogenous state variable increases the order of the stochastic process for exchange rate in the similar way as additional exogenous shocks in the baseline model. Importantly, the process remains integrated and indistinguishable from a random walk in the finite-sample numerical simulations of the calibrated model.
We further show that the introduction of capital does not affect the qualitative or quantitative properties of the model with respect to the behavior or real exchange rate and terms of trade. In particular, the real exchange rate still follows closely the volatile and persistent nominal exchange rate process, with the capital state variable introducing only a mild wedge in the relative dynamics of the two variables.

What concerns the exchange rate correlations with respectively the relative consumption (Backus-Smith) and the relative interest rates (Fama Forward Premium), the results in the model are no longer analytical, yet we show quantitatively that the calibrated model with realistic adjustment costs (calibrated to match the volatility of investment relative to the real exchange rate) is able to match both correlations (despite a somewhat different transmission mechanism for the interest rates). In addition, the model matches the empirical negative correlation between relative investment and exchange rate changes (similar pattern as with consumption), another moment at odds with both productivity and monetary shocks. Further detailed derivations and quantitative results are available from the authors upon request (also see Appendix Table A4).

**International busyness cycle calibration**

We now calibrate the model with capital and productivity shock to match the additional international business cycle moments. As a benchmark, we use the Backus, Kehoe, and Kydland (1994; henceforth, BKK) international RBC model, as well as the set of BKK moments on the comovement of macro variables (GDP, consumption, investment $z$ and net exports), in particular across countries. We maintain all baseline parameters as in Table 1.1, and additionally calibrate the investment adjustment cost parameter $\kappa$, as well as
the second moments of the shocks. In particular, we study three versions of the model:

1. a model with home and foreign productivity shocks \((a_t, a^*_t)\) only, calibrated analogously to the BKK baseline model. In particular, our baseline calibration is consistent with the BKK’s values of the key parameters \(\gamma\) and \(\theta\), while \(\alpha = 0\) in BKK. The only major difference of the BKK model from ours is the completeness of international asset markets, while other differences (non-separable utility, time-to-build, VAR structure for productivity shocks) are of minor quantitative importance.

2. a full model with four shocks \((a_t, a^*_t, \tilde{\xi}_t, \psi_t)\) which extends the multi-shock IRBC model from Table 1.2 to allow for imperfectly correlated country-specific productivity shocks, as in BKK, as well as capital accumulation as described above. The goal of this calibration is to check whether an IRBC model with capital accumulation and financial shocks can successfully match the conventional international business cycle (BKK) moments without compromising its fit of the exchange rate disconnect moments.

3. a model with financial shock \(\psi_t\) only, for comparison.

In each case we calibrate the adjustment cost parameter to match the relative volatility of investment, which is about three times larger than that of GDP. In the BKK replication and in the full multi-shock model we calibrate the correlation between productivity shocks to match the correlation between GDP growth rates across countries. In the world model we additionally calibrate the relative volatility of the shocks to match the correlation of real exchange rate with consumption and net exports, as we did in Table 1.2.

The results of these calibrations are reported in Table A4 and we summarize here the
Table A4: International business cycle calibration

<table>
<thead>
<tr>
<th></th>
<th>BKK with ((a_t, a_t^*)) only</th>
<th>Model with (\psi_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Original Replication</td>
</tr>
<tr>
<td>(\rho(\Delta e))</td>
<td>0.00</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>(\rho(q))</td>
<td>0.95</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>(\text{corr}(\Delta e, \Delta q))</td>
<td>0.98</td>
<td>-0.96</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>(\sigma(\Delta c - \Delta c^*) / \sigma(\Delta q))</td>
<td>0.20</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>(\text{corr}(\Delta c - \Delta c^*, \Delta q))</td>
<td>-0.20</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>(\text{corr}(\Delta nx, \Delta q))</td>
<td>(\approx 0)</td>
<td>-0.80</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Fama (\beta)</td>
<td>(\lesssim 0)</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>(0.6)</td>
<td>(3.2)</td>
</tr>
<tr>
<td>Fama (R^2)</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
</tbody>
</table>

Panel B: International business cycle moments

|                                | BKK with \((a_t, a_t^*)\) only | Model with \(\psi_t\)                           |
|                                | Data          | Original Replication | Multi-shock | \(\psi_t\) only |
| \(\sigma(\Delta c) / \sigma(\Delta gdp)\) | 0.49          | 0.47                 | 0.35        | 0.53             |
|                                | (0.01)        | (0.03)               | (0.03)      | 2.60             |
| \(\sigma(\Delta z) / \sigma(\Delta gdp)\) | 3.15          | 3.48                 | 3.78        | 3.15             |
|                                | (0.03)        | (0.16)               | (0.16)      | 3.15             |
| \(\text{corr}(\Delta c, \Delta gdp)\) | 0.76          | 0.88                 | 0.99        | 0.72             |
|                                | (0.00)        | (0.05)               | (0.05)      | -1               |
| \(\text{corr}(\Delta z, \Delta gdp)\) | 0.90          | 0.93                 | 0.99        | 0.83             |
|                                | (0.00)        | (0.03)               | (0.03)      | -1               |
| \(\text{corr}(\Delta nx, \Delta gdp)\) | -0.22         | -0.64                | -0.52       | 0.26             |
|                                | (0.07)        | (0.09)               | (0.09)      | 1                |
| \(\text{corr}(\Delta gdp, \Delta gdp^*)\) | 0.70          | 0.02                 | 0.31        | 0.70             |
|                                | (0.08)        | (0.05)               | (0.05)      | -1               |
| \(\text{corr}(\Delta c, \Delta c^*)\) | 0.46          | 0.77                 | 0.37        | 0.51             |
|                                | (0.08)        | (0.07)               | (0.07)      | -1               |
| \(\text{corr}(\Delta z, \Delta z^*)\) | 0.33          | 0.18                 | 0.55        | -1               |
|                                | (0.09)        | (0.06)               | (0.06)      |                  |

Panel C: Variance decomposition

Nominal exchange rate, \(\text{var}(\Delta e)\):

<table>
<thead>
<tr>
<th></th>
<th>Productivity shocks, ((a_t, a_t^*))</th>
<th>Financial shock, (\psi_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100%</td>
<td>1%</td>
</tr>
</tbody>
</table>
| Consumption, \(\text{var}(\Delta c)\):

<table>
<thead>
<tr>
<th></th>
<th>Productivity shocks, ((a_t, a_t^*))</th>
<th>Financial shock, (\psi_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100%</td>
<td>77%</td>
</tr>
</tbody>
</table>

Note: The business cycle data and original model moments are from BKK (1994, as well as 1992). The other three columns are from our calibrations, as discussed in the text. In the multi-shock model, we calibrate \(\sigma_a / \sigma_\psi = 5.4\), \(\gamma \sigma_\xi / \sigma_\psi = 2.4\) and \(\text{corr}(\Delta a, \Delta a^*) = 0.94\). Our BKK replication has \(\text{corr}(\Delta a, \Delta a^*) = 0.26\). In the variance decomposition for the multi-shock model, the discrepancy from 100% is the contribution of the international good demand shock \(\xi_t\).
main insights:

1. **BKK replication**: Our replication of BKK with incomplete markets allows to bring the cross-country correlations of consumption and DGP closer to the data, while the fit of all other moments is similar to the original BKK.

The BKK calibration also reproduces the conventional exchange rate puzzles, including Meese-Rogoff, PPP, Backus-Smith, and UIP puzzles.

2. **Multi-shock IRBC**. The multi-shock model is successful at simultaneously matching the international business cycle moments and the exchange rate disconnect moments.

Furthermore, the model with a financial shock is considerably better on cross-country correlation, matching among other things the consumption correlation puzzle — the fact that consumption is less correlated than output — thanks to the risk-sharing shock $\psi_t$. The only moment for which the fit deteriorates is the correlation between net exports and GDP, predicted to be mildly positive versus mildly negative in the data.

All exchange rate disconnect moments are on target as before, with a minor exception of the Fama $\beta$ coefficient. Now it is about 1, but with a very wide confidence interval, which includes both 0 and $-1$.

3. **Single financial shock model**. This specification, while good for exchange rate disconnect moments, predicts the wrong signs for most business cycle moments.

The reason is that $\psi_t$ shock on its own results in counterfactual correlation for the macro variables, but in a full multi-shock model its role in shaping the macro vari-
ables is very modest.

4. **Variance decompositions.** Finally, the bottom panel of Table A4 reports the variance decomposition for nominal exchange rate and consumption (as one example of a macro variable). We see that the financial shock, while being dominant in shaping the exchange rate dynamics (almost 60%), has only a very modest contribution to the dynamics of consumption (16%). At the same time, productivity shock account for 77% of the dynamics of consumption, and for almost nothing in the dynamics of the exchange rate (where instead the international good demand shock $\bar{\xi}_t$ plays a sizable secondary role). This explains why the multi-shock model can simultaneously reproduce both the exchange rate disconnect moments (due to $\psi_t$) and international business cycle moments (due to $(a_t, a_t^*)$).

### A.1.11 Data appendix

Data sources for moments used in Tables 1.2, 1.4 and A1:


2. Moments for terms of trade and producer-price real exchange rate: from Table 1 in Atkeson and Burstein (2008a), based on manufacturing prices and estimated for annual differences and HP-filtered quarterly data, 1975-2006.

3. Moments for consumption, investment and GDP: estimates by the authors. The data is for France, Germany, Italy and Spain from 1973 to 2000, quarterly.\(^{25}\) We

---

\(^{25}\)Our data goes through 2015, but we choose the pre-2000 subperiod to be consistent quantitatively with the moments reported in the earlier literature, as in the more recent period the correlation between relative consumption growth and real exchange rate changes became less negative.
take first log differences for each series, calculate a weighted average across countries and take the difference with the corresponding series for the U.S. The weights are proportional to the PPP-adjusted GDP averaged across years. We prefer first-differenced moments, but the results are robust to HP-filtering.


5. Slope coefficient $\beta$ and $R^2$ in Fama regression: survey by Engel (1996) and recent estimates by Burnside, Han, Hirshleifer, and Wang (2011, Table 1) and Valchev (2016, Table B.1).

6. Volatility and persistence of the interest rate differential: estimates by the authors. Monthly data for the U.S. versus the U.K., France, Germany and Japan from 1979:06 to 2009:10.\textsuperscript{26}

7. Carry trade Sharpe ratio: the estimates for the forward premium trade from Hassan and Mano (2014, Table 2).

8. Profits volatility (omitted from the tables for brevity): estimates by the authors. Quarterly data for the U.S., 1973 - 2015. We divide seasonally adjusted corporate profits (before taxes) by the seasonally adjusted nominal GDP, calculate the standard deviation of the first differences of this series and divide it by the standard deviation of changes in exchange rate.

\textsuperscript{26}the interest rates for individual countries have autocorrelation of $0.97 - 0.99$, while the autocorrelation for the interest rate differentials is lower, at $0.85 - 0.90$. In the model of Section 1.3, $\rho$ corresponds to the persistence of both the level $i_t$ and the differential $(i_t - i_t^*)$. In the multi-shock models of Section 2.4, we set $\rho = 0.97$ to target $\rho(i_t - i_t^*) = 0.90$. 218
A.2 Appendix for Chapter 2

A.2.1 Additional figures

(a) Changes in volatility

(b) Bounds on transition path

Figure A8: Transition from pound to dollar

Note: figure (a) shows transition from pound to dollar as the relative volatility of shocks in the U.K. goes up, while figure (b) shows lower and upper bounds for transition paths, i.e. the slowest and the fastest transition from pound to dollar. The parameter values are $\gamma = 0.6$, $\alpha = 0.5$, $\phi = 0.5$, $\lambda = 0.5$ and $n_{US} = n_{UK} = 0.25$. 
A.2.2 Equilibrium system

The Kimball aggregator for consumption bundle of tradable goods in region $i$ is defined as

$$(1-\gamma)e^{-\gamma \xi_i} \int_0^1 \Upsilon \left( \frac{C_{it}(\omega)}{1-\gamma} e^{-\gamma \xi_i C_{Ti}} \right) d\omega + \gamma e^{(1-\gamma)\xi_i} \int_0^1 \int_0^1 \Upsilon \left( \frac{C_{ji}(\omega)}{(1-\gamma)\xi_i C_{Ti}} \right) d\omega d_j = 1,$$

where $\Upsilon(1) = 1$, $\Upsilon'(\cdot) > 0$ and $\Upsilon''(\cdot) < 0$. I borrow expressions for price index and demand for individual goods under Kimball aggregator from Itskhoki and Mukhin (2017) and Amiti, Itskhoki, and Konings (2016b). The equilibrium system of the model consists of the following blocks:

1. Labor supply and labor demand:

   $$C_{it} = \frac{W_{it}}{P_{it}}, \quad (A95)$$

   $$L_{it} = (1 - \phi) \left( \frac{P_{it}}{W_{it}} \right)^{\phi} Y_{it} \frac{Y_{Nit}}{A_{Nit}}, \quad (A96)$$

2. Demand for non-tradables:

   $$Y_{Nit} = \int_0^1 \left( \frac{P_{Nit}(\omega)}{P_{Nit}} \right)^{-\theta} d\omega (C_{Nit} + G_{Nit}),$$

   where

   $$C_{Nit} + G_{Nit} = (1 - \eta) \frac{P_{it}^C}{P_{it}^N}(C_{it} + G_{it}).$$

3. Price setting in non-tradable sector:

   $$P_{it}^N(\omega) = \begin{cases} 
   \bar{P}_{it}^N, & \text{w/p } 1 - \lambda \\
   \bar{P}_{it}^N, & \text{w/p } \lambda 
   \end{cases}$$

220
where

\[ \tilde{P}_{it}^N = \arg \max_P \left( P - (1 - \tau) \frac{W_{it}}{A_{Nit}} \right) \left( \frac{P}{\tilde{P}_{it}^N} \right)^{-\theta} (C_{Nit} + G_{Nit}), \]

\[ \tilde{P}_{it}^N = \arg \max_P \mathbb{E}_{t-1} \left( P - (1 - \tau) \frac{W_{it}}{A_{Nit}} \right) \left( \frac{P}{\tilde{P}_{it}^N} \right)^{-\theta} (C_{Nit} + G_{Nit}). \]

4. Demand for tradables:

\[
Y_{it} = (1 - \gamma) e^{-\gamma \xi_{it}} \int_0^1 h \left( \frac{D_{it}P_{it}(\omega)}{P_{it}} \right) d\omega (C_{Tit} + X_{it} + G_{Tit}) \\
+ \gamma \int_0^1 e^{(1 - \gamma) \xi_{jt}} \int_0^1 h \left( \frac{D_{jt}P_{ijt}(\omega)}{P_{jt}} \right) d\omega (C_{Tjt} + X_{jt} + G_{Tjt}) d\lambda, \tag{A97}
\]

with intermediate and final demand given by

\[
X_{it} = \phi \left( \frac{W_{it}}{P_{it}} \right)^{1 - \phi} \frac{Y_{it}}{A_{Tit}}, \tag{A98}
\]

\[
C_{Tit} + G_{Tit} = \eta \frac{P_{it}^C}{P_{it}} (C_{it} + G_{it}).
\]

5. Price setting and currency choice in tradable sector:

\[
P_{jit}(\omega) = \begin{cases} \\
\tilde{P}_{jit}^k, & \text{w/p} \ 1 - \lambda \\
\tilde{P}_{jit}, & \text{w/p} \ \lambda \\
\end{cases}
\]

where

\[
\tilde{P}_{jit} = \arg \max_{P} (PE_{jit} - (1 - \tau) MC_{jit}) \gamma e^{(1 - \gamma) \xi_{it}} h \left( \frac{D_{it}P_{it}}{P_{it}} \right) (C_{Tit} + X_{it} + G_{Tit}),
\]

\[
\tilde{P}_{jit}^k = \mathbb{E}_{ikt} \cdot \arg \max_{P,k} \mathbb{E}_{t-1} (PE_{jkt} - (1 - \tau) MC_{jit}) \gamma e^{(1 - \gamma) \xi_{it}} h \left( \frac{D_{it}P_{ikt}}{P_{it}} \right) (C_{Tit} + X_{it} + G_{Tit}),
\]

and marginal costs of production are

\[
MC_{jt} = \frac{1}{A_{Tit}} W_{jt}^{1 - \phi} P_{jt}^{\phi}. \tag{A99}
\]
6. Definition of price indices

\[ P_{it}^C = (P_{it}^N)^{1-\eta} P_{it}^\eta, \]

\[ \int_0^1 \left( \frac{P_{it}^N(\omega)}{P_{it}^N} \right)^{1-\theta} d\omega = 1, \]

\[ (1 - \gamma) e^{-\gamma \xi_{it}} \int_0^1 \int_0^1 \gamma e\left((1-\gamma)\xi_{it} \right) d\omega d\omega + \gamma e\left((1-\gamma)\xi_{it} \right) \int_0^1 \int_0^1 \gamma e\left(\frac{D_{it}}{P_{it}}\right) d\omega d\omega d\omega d\omega = 1. \]

7. Asset demand / risk-sharing:

\[ e^{\Delta \psi_{it+1}} \Theta_{it+1} \frac{E_{0t+1} \delta_{it+1}}{E_{0t}} - e^{\Delta \psi_{0t+1}} \Theta_{0t+1} = 0, \quad (A100) \]

where the stochastic discount factor is defined as \( \Theta_{it+1} = \frac{C_{it} P_{it}^C}{C_{it+1} P_{it+1}^C}. \)

8. Country budget constraint is a side equation under complete markets. The net export expressed in dollar terms is

\[ NX_{it} = \int_0^1 \int_0^1 \left\{ \gamma e\left((1-\gamma)\xi_{it} \right) E_{0it} P_{ijt}(\omega) h\left(\frac{D_{ijt}}{P_{ijt}}\right) \left(C_{Tijt} + X_{ijt} + G_{Tijt}\right) \right. \]

\[ - \gamma e\left((1-\gamma)\xi_{it} \right) E_{0it} P_{ijt}(\omega) h\left(\frac{D_{ijt}}{P_{ijt}}\right) \left(C_{Tit} + X_{it} + G_{Tit}\right) \left. \right\} d\omega d\omega d\omega d\omega = 1. \quad (A101) \]

**Symmetric steady state** Consider symmetric steady state with zero net foreign asset positions and all shocks equal zero:

\[ a_{Nt} = a_{Tt} = w_i = \xi_i = \psi_i = G_i = 0. \]

I assume that production subsidy eliminates monopolistic distortion \( \tau = \frac{1}{\theta}. \) This assumption has no effect on the first-order approximation of the equilibrium system discussed.
below, but is important for the welfare analysis.

The symmetry implies that bilateral exchange rate between any countries is one, $\mathcal{E}_{ij} = 1$, and therefore, the prices for all products equal one as well:

$$P_i = P_{ii} = P_{ji} = P_i^N = P_i^C = 1.$$

Steady-state consumption can then be found from labor supply condition:

$$C_i = 1.$$

Combining market clearing in non-tradable sector

$$Y_{Ni} = C_{Ni} = (1 - \eta) C_i$$

and tradable one

$$Y_i = C_{Ti} + X_i = \eta C_i + \phi Y_i,$$

one can solve for steady state level of labor and output:

$$L_i = (1 - \phi) Y_i + Y_{Ni} = 1,$$

$$Y_i = \frac{\eta}{1 - \phi}.$$

A.2.3 Log-linearized system

I next log-linearize the equilibrium system around the symmetric steady state. It is convenient to split the system into four blocks — prices, quantities, dynamic equations and currency choice, and solve them recursively. The time index is suppressed in static blocks to simplify the notation. Small letters denote log-deviations from the steady state, while
small letters without subscript $i$ denote the global means, i.e. $x \equiv \int_0^1 x_i \, di$.

I decompose bilateral exchange rates into country-specific components: $e_{ijt} = e_{it} - e_{jt}$. Such decomposition is non-unique: intuitively, in a world with $N$ countries, there are only $N - 1$ independent bilateral exchange rates. I therefore normalize the mean of exchange rates across countries to zero, i.e. $\int_1^n e_{it} \, di = 0$. The country-specific exchange rate $e_{it}$ can then be interpreted as an average of bilateral exchange rates against other countries.

To get consistent solution, I use a classical result from portfolio theory established first by Samuelson (1970) and applied recently in a general equilibrium setup by Devereux and Sutherland (2011). In a context of my model, the argument consists of two parts. First, the second-order approximation to the profit function is required to determine the zero-order component of currency choice. From Lemma 2.1, it follows then the first-order approximation to other variables is sufficient to solve for currency choice. Second, the zero-order component of the currency choice from Lemma 2.1 is sufficient to get an accurate first-order solution for other variables. Thus, to get consistent solution, one needs to take the second-order approximation to the profit function and the first-order approximation to all other equilibrium conditions.

**Prices**

The price index for non-tradable goods and consumer price index are

$$ p_i^N = \lambda (w_i - a_{N_i}), \quad (A102) $$

$$ p_i^C = \eta p_i + (1 - \eta) p_i^N. \quad (A103) $$
The price block in tradable sector includes marginal costs of production

\[ mc_i = \phi p_i + (1 - \phi) w_i - a_i , \]  
(A104)

the optimal static price

\[ \tilde{p}_{ji} = (1 - \alpha) (mc_j + e_i - e_j) + \alpha p_i , \]  
(A105)

the import price index and the aggregate price index

\[ p^I_i = \int_0^1 p_{ji} \, dj , \]  
(A106)

\[ p_i = (1 - \gamma) p_{ii} + \gamma p^I_i , \]  
(A107)

and the bilateral price index:

\[ p_{ji} = \lambda \tilde{p}_{ji} + (1 - \lambda) (e_i - e_{kj}) , \]  
(A108)

where \( k_{ji} \) denotes the currency choice of exporters from country \( j \) to \( i \). For future use, define also the export price index as

\[ p^E_i = \int_0^1 p_{ij} \, dj , \]  
(A109)

Assume that domestic firms set prices in local currency and invoicing is symmetric across countries. Combine next equations (A104)-(A108) to solve for \( p_i \):

\[ p_i = \chi e_i - \chi_0 e_0 + \chi w w_i + \chi_{u} w - \chi_a a_i - \chi_a a , \]  
(A110)
where

\[ \chi = \frac{\gamma \left[ \lambda (1 - \alpha) + (1 - \lambda) (\mu P + \mu D) \right]}{1 - \lambda (\alpha + (1 - \gamma) (1 - \alpha) \phi)}, \]

\[ \chi_0 = \frac{\gamma}{1 - \lambda (\alpha + (1 - \gamma) (1 - \alpha) \phi)} \left[ \lambda (1 - \alpha) n + (1 - \lambda) (n \mu P + \mu D) + \frac{\lambda (1 - \lambda) (1 - \alpha) \gamma \mu D (1 - n)}{1 - \lambda (\alpha + (1 - \alpha) \phi)} \right], \]

\[ \chi_w = \frac{\lambda (1 - \gamma) (1 - \alpha) (1 - \phi)}{1 - \lambda (\alpha + (1 - \gamma) (1 - \alpha) \phi)}, \]

\[ \chi_a = \frac{\lambda \gamma (1 - \alpha) (1 - \lambda \alpha) (1 - \phi)}{1 - \lambda (\alpha + (1 - \gamma) (1 - \alpha) \phi)}, \]

\[ \chi_{\bar{a}} = \frac{\lambda \gamma (1 - \alpha) (1 - \lambda \alpha)}{1 - \lambda (\alpha + (1 - \gamma) (1 - \alpha) \phi)} \]

Integrate across countries to obtain the global price index

\[ p = (\chi n - \chi_0) e_0 + (\chi_w + \chi_w) w - (\chi_a + \chi_{\bar{a}}) a, \] (A112)

Finally, solve for import price index

\[ p_i^I = -\lambda [(1 - \alpha) (1 - \phi \chi) n + ((1 - \alpha) \phi + \alpha) \chi_0] e_0 - (1 - \lambda) (n \mu P + \mu D) e_0 \]

\[ + \lambda (1 - \alpha + \alpha \chi) e_i + (1 - \lambda) (\mu P + \mu D) e_i \]

\[ + \lambda \alpha \chi_1 w_i + \lambda [(1 - \alpha) (1 - \phi + \phi \chi_1) + ((1 - \alpha) \phi + \alpha) \chi_2] w \]

\[ - \lambda \alpha \chi_2 a_i - \lambda [(1 - \alpha) (\phi \chi_a + 1) + \chi_a (\alpha + (1 - \alpha) \phi)] a. \] (A113)
and export price index

\[ p_i^E = \lambda [(1 - \alpha + \alpha \chi) n - ((1 - \alpha) \phi + \alpha) \chi_0] \epsilon_0 + (1 - \lambda) (n \mu^P - (1 - n) \mu^D) \epsilon_0 \]

\[ - \lambda (1 - \alpha) (1 - \phi \chi) \epsilon_i - (1 - \lambda) \mu^P \epsilon_i \]

\[ + \lambda (1 - \alpha) (1 - \phi + \phi \chi_w) w_i + \lambda [\alpha \chi_w + ((1 - \alpha) \phi + \alpha) \chi_w] w \]

\[ - \lambda (1 - \alpha)(\phi \chi_a + 1) \epsilon_i - \lambda [(1 - \alpha) \phi + \alpha) \chi_a + \alpha \chi_a] a \]

(A114)

Quantities

The market clearing conditions for labor and goods allow to express consumption, labor
and output as functions of prices and shocks. First, labor supply condition determines
consumption

\[ c_i = w_i - p_i^C. \] (A115)

Second, substitute final demand for tradables

\[ c_{Ti} = p_i^C - p_i + c_i + g_i \] (A116)

and intermediate demand for tradables

\[ x_i = mc_i + y_i - p_i \] (A117)

into the market clearing condition

\[ y_i = (1 - \gamma) y_{ii} + \gamma y_i^E, \] (A118)

where the volume of exports is

\[ y_i^E = \int_0^1 y_{ij} \, dj \] (A119)
and bilateral trade flows are

\[ y_{ii} = -\gamma \xi_i - \theta (p_{ii} - p_i) + (1 - \phi) c_{Ti} + \phi x_i, \quad (A120) \]

\[ y_{ij} = (1 - \gamma) \xi_j - \theta (p_{ij} - p_j) + (1 - \phi) c_{Tj} + \phi x_j. \quad (A121) \]

Integrate across countries, use equation (A115) for consumption as well as equations (A104) and (A103) from price block to solve for global production of tradable goods:

\[ y = (1 + \phi) (w - p) + g - \frac{\phi}{1 - \phi} a. \quad (A122) \]

Substitute this expression back into the market clearing condition of a given country to solve for output:

\[ y_i = \frac{\gamma}{1 - (1 - \gamma) \phi} \left[ (p_i^l - p_i^E) - (p_i - p) \right] + \frac{(1 - \gamma) (1 - \phi^2)}{1 - (1 - \gamma) \phi} (w_i - p_i) + \frac{\gamma (1 + \phi)}{1 - (1 - \gamma) \phi} (w - p) \]

\[ - \frac{\gamma (1 - \gamma)}{1 - (1 - \gamma) \phi} (\xi_i - \xi) + \frac{(1 - \gamma) (1 - \phi)}{1 - (1 - \gamma) \phi} g_i + \frac{\gamma}{1 - (1 - \gamma) \phi} g. \quad (A123) \]

Third, total labor demand

\[ l_i = \eta l_{Ti} + (1 - \eta) l_{Ni} \]

is the sum of demand from tradable sector

\[ l_{Ti} = mc_i + y_i - w_i \]

and non-tradable sector

\[ l_{Ni} = y_{Ni} - a_{Ni}, \]

where market clearing for non-tradable goods implies

\[ y_{Ni} = c_{Ni} = p_i^C - p_i^N + c_i + g_i. \quad (A124) \]
Combine these equations together with tradable output (A123) to solve for labor in terms of prices and shocks:

\[ l_i = (1 - \eta) (p_i^C - (1 - \eta)p_i^N) - (1 - \eta) \eta p_i. \]  
(A125)

Fourth, to the first-order approximation, the aggregate imports and exports of country \( i \) are

\[ im_i = p_i^I + y_i^I, \quad ex_i = p_i^E + y_i^E, \]

where volume of imports is defined as

\[ y_i^I = \int_0^1 y_j \, dj. \]  
(A126)

Use expressions for output (A123), consumption (A115) and bilateral trade flows (A121), (A119) and (A126) to solve for exports

\[ y_i^E = -\theta (p_i^E - p) + (1 - \eta) (p^N - p) + \phi (w - p) + (w - p^C) + g + (1 - \gamma) \xi - \frac{\phi}{1 - \phi} a \]  
(A127)

and imports

\[ y_i^I = \frac{1 - \phi}{1 - (1 - \gamma) \phi} \left\{ -\theta (p_i^I - p_i) + (1 - \eta) (p_i^N - p_i) + \phi (w_i - p_i) + (w_i - p_i^C) ight. \\
+ g_i + (1 - \gamma) \xi_i - \frac{\phi}{1 - \phi} a_i \left. \right\} + \frac{\gamma \phi}{1 - (1 - \gamma) \phi} y_i^E. \]  
(A128)

The linearized equation for net exports is

\[ nx_i = ex_i - im_i + (e_i - ne_0). \]

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Substitute in expressions for exports (A127) and imports (A128) to get

\[ nx_i = (e_i - ne_0) - (p_i - p) + \left[ \frac{(1 - \phi) \theta}{1 - (1 - \gamma) \phi} - 1 \right] \left[ (p'_i - p_i) - (p'_E - p) \right] \]
\[ - \frac{(1 - \phi)}{1 - (1 - \gamma) \phi} \left\{ \phi [(w_i - w) - (p_i - p)] + (1 - \eta) \left[ (p'_i - p'_N) - (p_i - p) \right] + [(w_i - w) - (p'_i - p'_C)] \right\} \]
\[ - \frac{(1 - \phi)}{1 - (1 - \gamma) \phi} \left( \xi_i - \xi \right) - \frac{(1 - \phi)}{1 - (1 - \gamma) \phi} (g_i - g) + \frac{1 - \phi}{1 - (1 - \gamma) \phi} (a_i - a) . \]

(A129)

### A.2.4 Equilibrium exchange rates

There are two types of dynamic equations in the model that pin down equilibrium exchange rates — the Euler equations and countries’ budget constraints. I show in this section that result from Lemma 2.2 can be derived under several alternative sets of assumptions about the structure of asset markets, preferences and monetary policy rule. In all cases, exchange rate shocks are uncorrelated \( \text{corr}(e_i, e_j) = 0 \) for \( \forall i \neq j \) and the relative volatility of exchange rates depends only on volatility of exogenous shocks \( \frac{\nu(e_0)}{\nu(e_i)} = \rho \) for \( \forall i \in \{n, 1\} \).

**Baseline case**

**Proof of Lemma 2.2** When asset markets are complete, the countries achieve full risk-sharing:

\[ \Delta e_{it} - \Delta e_{0t} = (\Delta c_{it} - \Delta c_{0t}) + (\Delta p'_C - \Delta p'_C^0) + (\Delta \psi_{it} - \Delta \psi_{0t}) . \]

Since countries are symmetric ex ante, the budget constraint implies that the same condition holds not only in changes, but also state by state:

\[ e_{it} - e_{0t} = (c_{it} - c_{0t}) + (p'_C - p'_C^0) + (\psi_{it} - \psi_{0t}) . \]

(A130)
Substitute in expressions for consumption (A115) to obtain

\[ e_{it} - e_{0t} = (w_{it} - w_{0t}) + (\psi_{it} - \psi_{0t}) . \]

Integrate the risk-sharing condition across countries from \( n \) to \( 1 \), apply the law of large numbers for uncorrelated shocks and use normalization of exchange rates to get

\[ e_{0t} = \tilde{w}_{0t} + \tilde{\psi}_{0t}, \]

where \( \tilde{s}_{it} \) denotes the country-specific component of shock \( s_{it} \). Substitute this condition back into the previous expression to get for any \( i \in [0, 1] \)

\[ e_{it} = \tilde{w}_{it} + \tilde{\psi}_{it}. \quad (A131) \]

Thus, the second moments of exchange rates are independent from firms’ currency choice.

■

Alternative assumptions

**Proposition A1 (Exchange rates)** Assume that

1. the only internationally traded asset is a risk-free nominal bond denominated in arbitrary currency and that all shocks are integrated of the first order,

2. one of the following conditions is satisfied:
   - preferences are log-linear and monetary policy is set in terms of exogenous shocks in \( W_{it} \),
   - arbitrary isoelastic preferences and monetary policy is set in terms of exogenous shocks in \( R_{it} \),

3. \( \beta \to 1 \) or all exporters in the world use either PCP, LCP or DCP.

Then correlation and relative volatility of exchange rates are independent from firms’ currency choice.
Incomplete markets  Consider the case of incomplete asset markets when only one nominal bond is traded internationally. I assume it pays one dollar in every state of the world, which is without loss of generality under the first-order approximation used to solve the model. The no-arbitrage conditions

\[ \mathbb{E}_t \left\{ e^{\Delta \psi_{it+1} \Theta_{it+1}} \frac{\mathcal{E}_{it+1}}{\mathcal{E}_{0t}} - e^{\Delta \psi_{0t+1} \Theta_{0t+1}} \right\} = 0 \]

can be log-linearized to get the UIP condition with the risk premium \( \varsigma_{it} \equiv \mathbb{E}_t \Delta \psi_{it+1} \):

\[ \mathbb{E}_t [\Delta e_{it+1} - \Delta e_{0t+1}] = \mathbb{E}_t [(\Delta c_{it+1} - \Delta c_{0t+1}) + (\Delta p_{it+1}^{C} - \Delta p_{0t+1}^{C})] - (\varsigma_{it} - \varsigma_{0t}). \]

Substitute the labor supply condition (A115) to get

\[ \mathbb{E}_t [\Delta e_{it+1} - \Delta e_{0t+1}] = \mathbb{E}_t [\Delta w_{it+1} - \Delta w_{0t+1}] - (\varsigma_{it} - \varsigma_{0t}). \]

Integrate across countries from \( n \) to 1, apply the law of large numbers for uncorrelated shocks and use normalization of exchange rates to get

\[ \mathbb{E}_t \Delta e_{it+1} = \mathbb{E}_t \Delta \tilde{w}_{it+1} - \tilde{\varsigma}_{it} \quad (A132) \]

for any \( i \in [0, 1] \).

The intertemporal budget constraint is

\[ \sum_{\tau=0}^{\infty} \beta^\tau N X_{it+\tau} + D_{it} = 0, \]

where \( D_{it} \) denotes country’s debt in dollars. Rewrite it in log-linear form and assume that initial debt is zero, which is without loss of generality since we are interested in the
conditional moments:

\[ \sum_{t=0}^{\infty} \beta^t n x_{it} = 0 \]

This can be decomposed into net export in the first period with sticky prices and in all other periods when prices are flexible:

\[ \sum_{t=1}^{\infty} \beta^t n x_{it} + n x_{i0} = 0. \]

Expression (A129) together with price indices implies that under flexible prices the net export of country \( i \) can be written as

\[ n x_{it}^p = k_e (e_{it} - n e_{0t}) + k_s (s_{it} - n s_{0t}), \quad (A133) \]

where \( s_{it} \) is the vector of shocks and \( (k_e, k_s) \) is a vector of constants independent from firms’ currency choice. Combining the last two expressions, one obtains

\[ \sum_{t=1}^{\infty} \beta^t [k_e (e_{it} - n e_{0t}) + k_s (s_{it} - n s_{0t})] + n x_{i0} = 0. \]

Integrate across countries from \( n \) to 1, apply the law of large numbers and exchange rate normalization to get for any \( i \in [0, 1] \)

\[ \sum_{t=1}^{\infty} \beta^t [k_e e_{it} + k_s s_{it}] + \hat{n} x_{i0} = 0, \]

where \( \hat{n} x_{i0} \equiv n x_{i0} - \int_n^1 n x_{i0} d i \). Rewrite the last equation in terms of initial values and growth rates

\[ \sum_{t=1}^{\infty} \beta^t \left[ k_e e_{i0} + k_s s_{i0} + \sum_{\tau=1}^{t} (k_e \Delta e_{\tau \tau} + k_s \Delta s_{\tau \tau}) \right] + \hat{n} x_{i0} = 0, \]
change the order of summation and substitute in the UIP condition (A132):

$$\beta (k_e e_{i0} + k_s \tilde{s}_{i0}) + \beta E_0 \sum_{t=0}^{\infty} \beta^t \left[ k_e \Delta \tilde{w}_{it+1} - k_e \Delta \tilde{\psi}_{it+1} + k_s \Delta \tilde{s}_{it+1} \right] + (1 - \beta) \tilde{n} \tilde{x}_{i0} = 0. $$

Assume that all shocks are integrated of the first order and take the limit $\beta \to 1$ using the fact coefficients $(k_e, k_s)$ do not depend on $\beta$:

$$e_{it} = -\frac{k_s}{k_e} \tilde{s}_{it} - E_t \sum_{\tau=0}^{\infty} \left[ \Delta \tilde{w}_{it+\tau+1} - \tilde{\varsigma}_{it+\tau} + \frac{k_s}{k_e} \Delta \tilde{s}_{it+\tau+1} \right]. \quad (A134)$$

Since invoicing decisions of exporters have no effect on the coefficients in this expression, the (conditional) second moments of exchange rate are independent from firms’ currency choice.

**Interest rate shocks**  Assume again one internationally traded bond and use the Euler equation for domestic bond to write the no-arbitrage condition as

$$E_t \left\{ \Theta_{it+1} \left[ e^{\Delta \psi_{it+1}} R_{0t} \frac{\mathcal{E}_{i0t+1}}{\mathcal{E}_{i0t}} - R_{it} \right] \right\} = 0. $$

This implies the UIP condition with the risk premium shock:

$$E_t [\Delta e_{it+1} - \Delta e_{0t+1}] = r_{it} - r_{0t} - (\varsigma_{it} - \varsigma_{0t}). $$

If interest rate shocks are exogenous and are the sum of global and country-specific components as other shocks, then using integration across countries and exchange rate normalization, we get

$$E_t \Delta e_{it+1} = \tilde{r}_{it} - \tilde{\varsigma}_{it}. $$
I abstract from the issue of multiple equilibria and take the path of interest rates as given as in Farhi and Werning (2016). Following the same steps as before and taking the limit $\beta \to 1$, the budget constraint of country $i$ together with the UIP condition imply

$$e_{it} = -\frac{k_s}{k_e} s_{it} - \mathbb{E}_t \sum_{\tau=0}^{\infty} \left[ r_{it+\tau} - \bar{r}_{it+\tau} + \frac{k_s}{k_e} \Delta s_{it+\tau+1} \right].$$

This, the second moments of exchange rates are independent from firms’ currency choice. Note that this result holds for arbitrary preferences.

**Symmetric invoicing** I show that the elasticity of net export with respect to trade-weighted exchange rate is the same for all countries under symmetric invoicing and therefore, the expression similar to (A133) holds also in the short-run. As a result, the equilibrium response of exchange rates to local shocks is the same for all countries and the relative volatility of exchange rates depends only relative volatility of exogenous shocks.

**Lemma A3** When all exporters in the world use either PCP, LCP or DCP, the elasticity of net exports with respect to $e_i - ne_0$ is the same for all countries including the U.S.

**Proof** From (A110)-(A114), it follows that $p_i - p$ and $p_i^I - p_i^E$ are proportional to $\chi(e_i - ne_0)$ and

$$\left[ \lambda ((1 - \alpha) (2 - \phi \chi) + \alpha \chi) + (1 - \lambda) (2\mu^F + \mu^D) \right] (e_i - ne_0)$$

respectively. The expression for net exports (A129) implies then that the elasticity of $nx_i$ with respect to $e_i - ne_0$ is the same for all countries. ■
A.2.5 Currency choice (Section 2.3)

Proof of Lemma 2.1 Let $s$ denote the aggregate state of the economy that individual firms take as exogenous. Suppress country indices and take the second-order approximation of the profit function at price $p$ around the state-dependent optimal price $\tilde{p}_{ji}$:

$$
\Pi (p, s) = \Pi (\tilde{p}_{ji}, s) + \Pi_p (\tilde{p}_{ji}, s) (p - \tilde{p}_{ji}) + \frac{1}{2} \Pi_{pp} (\tilde{p}_{ji}, s) (p - \tilde{p}_{ji})^2 + O (p - \tilde{p}_{ji})^3,
$$

The first term on the right hand side does not depend on currency of invoicing. From the first-order condition for optimal price, $\Pi_p (\tilde{p}_{ji}, s) = 0$. Finally, the zero-order approximation,

$$
\Pi_{pp} (\tilde{p}_{ji}, s) = \Pi_{pp} (0, 0) + O (s) < 0,
$$

where $\Pi_{pp} (0, 0)$ denotes the derivative in the deterministic steady state. Therefore, to the second-order approximation, the currency choice problem is equivalent to minimization of $E (p - \tilde{p}_{ji})^2$. Note that only first-order approximation is required for $p$ and $\tilde{p}_{ji}$. In particular, the optimal preset price in currency $k$ is $\tilde{p}_{ji}^k = E (\tilde{p}_{ji} - e_{ik})$, so that ex post price is $p = \tilde{p}_{ji}^k + e_{ik}$. Substitute this expression into the objective function to write the currency problem as

$$
\min_k V (\tilde{p}_{ji} + e_{ki})^2, \quad (A135)
$$

which completes the proof of the lemma. ■

Combining equations (A104)-(A108) and suppressing monetary and productivity shocks, we get the optimal price in terms of currency $k$:

$$
\tilde{p}_{ji} + e_{ki} = e_k - (1 - \alpha)(1 - \chi \phi)e_j - \alpha (1 - \chi) e_i - (\alpha + (1 - \alpha) \phi) \chi_0 e_0. \quad (A136)
$$
It is easy to verify that the aggregate pass-through coefficients \(A_{111}\) are positive and no greater than one, i.e. \(0 \leq \chi, \chi_0 \leq 1\). It follows that the coefficients before \(e_j\), \(e_i\) and \(e_0\) are between 0 and 1 as well. Since exchange rates \(e_i\) are uncorrelated across countries, a firm is more likely to choose the currency with the higher weight in \((A_{136})\). This result underlies the comparative statics analysis below.

**Proof of Lemma 2.3** When \(\alpha = \phi = 0\), we get \(\tilde{p}_{ji} + e_{ki} = e_k - e_j\) and the minimum volatility is attained by setting \(k = j\), i.e. exporters choose PCP.

**Proof of Lemma 2.4** Expression \((A_{111})\) implies that in the autarky limit \(\gamma \to 0\), the pass-through coefficients are \(\chi, \chi_0 \to 0\). Thus, \(\tilde{p}_{ji} + e_{ki} \to e_k - (1 - \alpha)e_j - \alpha e_i\) and \(\nabla (\tilde{p}_{ji} + e_{ki})^2\) is equal \(2\alpha^2 \sigma^2_e\) under PCP, \(2(1 - \alpha)^2 \sigma^2_e\) under LCP and \((\rho + \alpha^2 + (1 - \alpha)^2) \sigma^2_e\) under DCP. Hence, exporters choose \(k = j\) when \(\alpha \leq 0.5\) and \(k = i\) when \(\alpha \geq 0.5\).

**Proof of Proposition 2.1** Consider for example the limit \(\gamma, \alpha \to 1\), so that \(\chi \to \mu^P + \mu^D\), \(\chi_0 \to \mu^D\) and \(\tilde{p}_{ji} + e_{ki} \to e_k - (1 - \chi)e_i - \chi_0 e_0\). Conjecture that other firms choose DCP, so that \(\mu^D = 1\). Hence, \(\tilde{p}_{ji} + e_{ki} \to e_k - e_0\) and the firm finds it optimal to choose \(k = 0\).

The DCP equilibrium can therefore be sustained in the neighbourhood of \(\gamma = \alpha = 1\) when prices are sticky.

Note that both \(\chi\) and \(\chi_0\) are increasing in \(\gamma\) and \(\phi\). In addition, given \(\chi\) and \(\chi_0\), the coefficient before \(e_j\) is decreasing in \(\phi\), while the coefficient before \(e_0\) is increasing in \(\phi\). It follows that higher \(\gamma\) and \(\phi\) decrease the weights of \(e_j\) and \(e_i\) and increase the weight of \(e_0\) in \((A_{136})\), which makes PCP and LCP less likely and raises the chances of DCP. The effect of \(\alpha\), on the other hand, is not monotonic.
Lemma A4  In the flexible-price limit $\lambda \to 1$, the equilibrium exists and is generically unique. The invoicing is symmetric across small countries.

Proof  In the flexible-price limit $\lambda \to 1$, the pass-through coefficients from (A111) converge to $\chi \to \frac{\gamma}{1 - (1 - \gamma)\phi}$ and $\chi_0 \to \frac{\gamma^n}{1 - (1 - \gamma)\phi}$ and do not depend on invoicing decisions of firms. The currency choice problem (A135)-(A136) then has unique solution except for some borderline values of parameters. Finally, since coefficients before exchange rates are the same for exporters from all small economies and the volatility of exchange rates is also the same, the equilibrium invoicing is symmetric across them. ■

Proof of Proposition 2.3  When $n = 0$, the desired price of exporters is

$$\tilde{p}_{ji} + e_{ki} = e_k - \frac{1 - \phi}{1 - (1 - \gamma)\phi} \left[ (1 - \alpha) e_j + \alpha (1 - \gamma) e_i \right]. \quad \text{(A137)}$$

Since volatility of all exchange rates is the same when $\rho = 1$, the exporter chooses between producer and local currency based on their weights in (A137): $k = j$ when $1 - \alpha \geq \alpha (1 - \gamma) \Leftrightarrow \alpha \leq \frac{1}{2 - \gamma}$ and $k = i$ otherwise. ■

Proof of Proposition 2.4  Rewrite for simplicity expression (A137) as $\tilde{p}_{ji} + e_{ki} = e_k - ae_j - be_i$. The volatility (A135) under DCP is then $(\rho + a^2 + b^2)\sigma_\epsilon^2$. Since $\rho$ does not affect volatility under PCP and LCP, lower values of $\rho$ unambiguously increase the chances of DCP. Note that in the limit $\phi \to 1$, we have $a = b = 0$ and under $\rho < 1$ DCP strictly dominates both PCP and LCP. ■

Proof of Proposition 2.5  The desired price in the flexible-price limit with $n > 0$ is

$$\tilde{p}_{ji} + e_{ki} = e_k - \frac{1 - \phi}{1 - (1 - \gamma)\phi} \left[ (1 - \alpha) e_j + \alpha (1 - \gamma) e_i \right] - \frac{\gamma(\alpha + (1 - \alpha)\phi)}{1 - (1 - \gamma)\phi} ne_0.$$
As long as $n > 0$, choosing $k = 0$ is optimal for example in the limit $\phi \to 1$. Moreover, keeping the values of other parameters fixed, higher $n$ increases the relative weight of $e_0$ in the optimal price and therefore, makes DCP more likely.

The proof of Proposition 2.2 requires a few additional lemmas. When $n = 0$ and $\rho = 1$, the currency choice of exporters is based on the following inequalities:

$$PCP \succ LCP \iff (1 - \alpha) \phi \chi + \alpha (2 - \chi) < 1,$$  \hspace{1cm} (A138)

$$PCP \succ DCP \iff (1 - \alpha) \phi (\chi + \chi_0) + \alpha (1 + \chi_0) < 1,$$  \hspace{1cm} (A139)

$$DCP \succ LCP \iff (1 - \alpha) (1 - \phi \chi_0) + \alpha [2 - (\chi + \chi_0)] < 1.$$  \hspace{1cm} (A140)

where $\succ$ stays for “preferred to”. I also denote with $\chi^X$ and $\chi^0_X$ the values of the pass-through coefficients in (A111) under symmetric invoicing $X$.

**Lemma A5** If DCP is preferred to PCP (LCP) under PCP (LCP) price index, then this ordering holds under DCP price index as well. Symmetrically, if PCP (LCP) dominates DCP under DCP price index, then this ordering holds under PCP (LCP) price index as well.

**Proof** Since condition (A139) gets tighter with $\chi$ and $\chi_0$ and $\chi^P = \chi_0^P$, $\chi^P < \chi^D$, the relation $DCP \succ PCP$ for $\chi^P (\chi_0^P)$ implies the same ordering for $\chi^D (\chi_0^D)$. Since condition (A140) is relaxed by higher $\chi$ and $\chi_0$ and $\chi^L < \chi^D$, $\chi^L_0 < \chi^D_0$, the relation $DCP \succ LCP$ for $\chi^L$ & $\chi^L_0$ implies the same ordering for $\chi^D$ & $\chi^L_0$. ■

**Lemma A6** It is impossible that exporter chooses PCP when all others choose LCP and simultaneously chooses LCP when all others choose PCP.

**Proof** Suppose that were the case. Then from (3) $\frac{1 - \phi \chi^P}{2 - \chi^P (1 + \phi)} < \alpha < \frac{1 - \phi \chi^L}{2 - \chi^L (1 + \phi)}$. But this requires $\chi^L > \chi^P$, which is not the case. ■

**Lemma A7** Consider pure-strategy NE with a choice only between PCP and LCP. If symmetric LCP equilibrium does not exist, the only possible pure-strategy NE is symmetric PCP.
Proof Pure-strategy equilibria can be parametrized by cdf $F(\cdot)$ for $\mu^i_P \in [0, 1]$ across countries. PCP is chosen by exporter from country $j$ to country $i$ iff

$$(1 - \alpha) \phi x_j + \alpha (2 - x_i) < 1 \Rightarrow \mu_j < a + b \mu_i$$

for some positive constants $a$ and $b$. Integrating across importers, we then derive the equilibrium condition: $\mu_i = \int_j \mathbb{I}\{\mu_j < a + b \mu_i\} \, dj$, or equivalently

$$\frac{1}{0} \int \mathbb{I}\{z < a + bx\} \, dF(z) = F(a + bx) = x$$

for any $x$ with positive density. Suppose next that symmetric LCP equilibrium does not exist, i.e. $F(a) = 0$ is unattainable. This is possible only if $a > 1$. But then for any $x > 0$ with positive density we have $x = F(a + bx) \geq F(a) = 1$, i.e. symmetric PCP is the only PSE. ■

Proof of Proposition 2.2 (1) Suppose there are no symmetric equilibria for some combination of parameters. Note that since $\chi^P = \chi^D$, it follows from (A138) that the preferences between PCP and LCP should be the same under PCP and DCP price indices. First, suppose that $PCP \succ LCP$ under DCP and PCP. Since there is no PCP equilibrium, we must have $DCP \succ PCP$ under PCP price index. But by Lemma A5, we have $DCP \succ PCP$ under DCP price index as well and hence, DCP equilibrium exists. Second, suppose that $LCP \succ PCP$ under DCP and PCP. Then from Lemma A6, we have $LCP \succ PCP$ under LCP price index. Non-existence of LCP equilibrium requires then $DCP \succ LCP$ under LCP price index. By Lemma A5, $DCP \succ LCP$ under DCP price index as well and hence, we obtain DCP equilibrium. In both cases we arrive to contradiction.
(2) First, suppose that DCP is a unique symmetric equilibrium. Then $DCP \succ LCP$ under LCP and $DCP \succ PCP$ under PCP price index. Since $\chi^i$ and $\chi_0^i$ can get higher as one deviates from symmetric LCP, constraint (A140) implies that DCP dominates LCP in any PSNE. But then $\chi^i$ stays the same and $\chi_0^i$ can only increase relative to symmetric PCP and constraint (A139) implies that DCP dominates LCP in any PSNE as well. Second, suppose that LCP is a unique symmetric equilibrium. Since $\chi^i$ and $\chi_0^i$ can only get lower as one deviates from symmetric DCP, constraint (A140) implies that DCP dominates DCP in any PSE as well. The existence of symmetric LCP requires according to constraint (A138) that $\alpha > \frac{1 - \phi}{2 - \chi^L (1 + \phi)} > \frac{1}{2}$. This implies $\alpha > (1 - \alpha) \phi$, so that constraint (A138) relaxes as $\chi^i$ decreases. Therefore, there can be no PSNE with PCP. Finally, suppose that PCP is a unique symmetric NE. Since $\chi^i$ and $\chi_0^i$ can get only lower than under symmetric DCP, constraint (A139) implies that DCP is dominated by PCP in PSNE. According to Lemma A7, there can also be no PSE with positive measure of LCP.

(3) Suppose there is market $i$, in which a positive mass of importers are indifferent between PCP and DCP and play mixed strategies. Take an arbitrary small share of firms pricing in producer currency and exogenously switch their invoicing into dollars. The coefficient $\chi^i$ does not change, while $\chi_0^i$ increases. Condition (A139) implies that the firms that were indifferent now strictly prefer DCP, while Condition (A140) implies that the share of LCP can only fall. Since firms (endogenously) switch to dollar in response to the perturbation, the initial equilibrium is not stable. Note there are no indirect effects coming from other markets: as country $i$ is infinitely small, the changes in invoicing of its imports or exports has no impact on other countries. A symmetric argument applies for other types of mixed equilibria. ■
Proof of Proposition 2.6  It is convenient to use a slightly different notation than in other sections: two currency unions have masses \( n_1 \) and \( n_2 \) with \( n \equiv n_1 + n_2 \), the relative exchange rate volatility of pound is \( \rho \equiv \frac{\sigma^2}{\sigma_1^2 + \sigma_2^2} \), \( \mu^k_i \) denotes the share of country \( i \) imports invoiced in currency \( k \) (\( \mu^1_i + \mu^2_i = 1 \)). I also define pass-through coefficients as follows:

\[ p_i = \chi_i^0 e_i - \chi_i^1 e_1 - \chi_i^2 e_2. \]

The equilibrium price index is given by

\[
[1 - \lambda (\alpha + (1 - \gamma) (1 - \alpha) \phi)] p_i =
\]

\[
= \gamma [1 - \lambda \alpha] e_i - \gamma \left[ \lambda (1 - \alpha) n_1 + (1 - \lambda) \mu^1_i \right] e_1 - \gamma \left[ \lambda (1 - \alpha) n_2 + (1 - \lambda) \mu^2_i \right] e_2
\]

\[
+ \frac{\lambda \gamma^2 (1 - \alpha) (1 - \lambda) \phi}{1 - \lambda (\alpha + (1 - \alpha) \phi)} \left[ (n_1 \mu^1_i - n_2 \mu^2_i - (1 - n) \mu^1_N) e_1 + (n_2 \mu^1_i - n_1 \mu^2_i - (1 - n) \mu^2_N) e_2 \right].
\]

Vehicle currency 1 dominates vehicle currency 2 for exporter from \( j \) to \( i \) iff

\[
(1 - \alpha) \frac{\text{cov}(\phi p_j + e_1 - e_j, e_1 - e_2)}{\text{var}(e_1 - e_2)} + \alpha \frac{\text{cov}(p_i + e_1 - e_i, e_1 - e_2)}{\text{var}(e_1 - e_2)} < \frac{1}{2}.
\]

Using this formula for each bilateral trade flow, we get:

- **RoW exports to RoW:**

\[
(\alpha + (1 - \alpha) \phi) \chi_2^N + [1 - (\chi_1^N + \chi_2^N) (\alpha + (1 - \alpha) \phi)] \rho < \frac{1}{2},
\]

- **RoW exports to currency unions:**

\[
(1 - \alpha) \phi \chi_2^N + \alpha \chi_2^1 + [(1 - \alpha) (1 - \phi \chi_1^N - \phi \chi_2^N) + \alpha (\chi_0^1 - \chi_1^1 - \chi_2^1)] \rho < \frac{1}{2},
\]

\[
(1 - \alpha) \phi \chi_2^N + \alpha (1 + \chi_2^2 - \chi_0^2) + [(1 - \alpha) (1 - \phi \chi_1^N - \phi \chi_2^N) + \alpha (\chi_0^2 - \chi_1^2 - \chi_2^2)] \rho < \frac{1}{2},
\]

- **Currency union exporting to RoW:**

\[
(1 - \alpha) \phi \chi_2^1 + \alpha \chi_2^N + [(1 - \alpha) \phi (\chi_0^1 - \chi_1^1 - \chi_2^1) + \alpha (1 - \chi_1^N - \chi_2^N)] \rho < \frac{1}{2},
\]

\[
(1 - \alpha) (1 + \phi \chi_2^2 - \phi \chi_0^2) + \alpha \chi_2^N + [(1 - \alpha) \phi (\chi_0^2 - \chi_1^2 - \chi_2^2) + \alpha (1 - \chi_1^N - \chi_2^N)] \rho < \frac{1}{2},
\]

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• One currency union exporting to the other:

\[
(1 - \alpha) \phi \chi^1_i + \alpha \left( 1 + \chi^2_j - \chi^2_0 \right) + \left[ (1 - \alpha) \phi (\chi^1_0 - \chi^1_1 - \chi^1_2) + \alpha (\chi^2_0 - \chi^2_1 - \chi^2_2) \right] \rho < \frac{1}{2},
\]

\[
(1 - \alpha) \left( 1 + \phi \chi^2_j - \phi \chi^2_0 \right) + \alpha \chi^1_i + \left[ (1 - \alpha) \phi (\chi^2_0 - \chi^2_1 - \chi^2_2) + \alpha (\chi^1_0 - \chi^1_1 - \chi^1_2) \right] \rho < \frac{1}{2}.
\]

(1) Note first that without change in currency choice, \(\rho\) has no effect on the global share of pound, while higher \(n_2\) implies a lower one. Next, suppose there is a point, at which the change in currency choice increases the fraction of trade invoiced in currency 1, i.e. there exist trade flow from \(j\) to \(i\) that switches invoicing from 2 to 1. Parameter \(\rho\) is present in only CC block (not in price index). Consider the derivative of the first and the second terms in the CC constraint with respect to \(\rho\)

\[
(1 - \alpha) \left[ \phi \chi^j_2 + \left( 1 - \phi \chi^j_1 - \phi \chi^j_2 \right) \rho - \left( 1 - \phi \chi^j_0 \right) \frac{\text{cov} (e_j, e_1 - e_2)}{\text{var} (e_1 - e_2)} \right],
\]

\[
+ \alpha \left[ \chi^i_2 + \left( 1 - \chi^i_1 - \chi^i_2 \right) \rho - \left( 1 - \chi^i_0 \right) \frac{\text{cov} (e_i, e_1 - e_2)}{\text{var} (e_1 - e_2)} \right] < \frac{1}{2}.
\]

The derivative of each term is clearly positive for all countries except for country 1, for which it is proportional to \(\chi^1_0 - \chi^1_1 - \chi^1_2\). This term, however, is non-negative as well:

\[
\gamma \lambda (1 - \alpha) (1 - n) \left[ \frac{\lambda (1 - \alpha) (1 - \phi) + (1 - \lambda) (1 - \gamma \phi)}{1 - \lambda (\alpha + (1 - \alpha) \phi)} \right]
\]

Thus, as \(\rho\) goes up, all constraints become more binding and (everything else equal) can only decrease the use of pound and \(\mu^1_i\) (and hence, increase \(\mu^2_i\)). It follows that \(\chi^i_0\) is unaffected, \(\chi^i_1\) falls and \(\chi^i_2\) rises. According to currency choice inequality, this tightens constraint for currency 1 even further.

Consider next an increase in \(n_2\), assuming that \(n_1 + n_2\) remains unchanged. Country sizes \(n_i\) are present only in price indices, but not in currency choice inequalities. For given currency choice, \(\chi^1_i\) and \(\chi^2_i\) are monotonic in \(n_1\) and \(n_2\) respectively if \(\mu^1_1 \geq \mu^2_2\) with
derivatives equal to

\[ 1 - \frac{\gamma (1 - \lambda) \phi}{1 - \lambda (\alpha + (1 - \alpha) \phi)} (1 - (\mu^1 - \mu^2)) \]

In this case, \( \chi^1 \) decreases and \( \chi^2 \) increases as \( n_2 \) goes up. The currency choice inequalities then tighten with \( n_2 \). The argument from above shows that endogenous change in invoicing pattern amplifies fall in global share of pound. It remains to show that inequality \( \mu^1 \geq \mu^1 \) indeed holds. The second part of the proposition (proven below) implies that the share of dollar denominated imports from RoW to the first country is not smaller than the one to the second country. Considering only trade between two currency unions we get \( \mu^1 - \mu^1 \geq n_1 - n_1 = 0. \)

(2) Consider an increase in \( n_2 \), which leaves \( n \) unchanged. First, note that price index for any country consists of three terms:

\[ p_i \propto \lambda \gamma (1 - \alpha) \phi \int p_j dj + \lambda \gamma (1 - \alpha) \int (e_i - e_j) dj + (1 - \lambda) \gamma [e_i - \mu^1 e_1 - \mu^2 e_2] \]

The first term is the same for all countries, while the second one does not depend on currency of invoicing. The last term, however, implies that starting from the equilibrium where all global trade (except between members of union 2) is denominated in currency 1, \( \mu^2 \) is positive only for \( i = 2 \). Therefore, \( \chi^2 \) is higher and \( \chi^1 \) is lower for country 2. The currency choice inequalities imply then \( T (b) \leq T (c), T (e) \leq T (f) \) and \( T (a) \leq T (c), T (d) \leq T (g) \). But then \( \chi^2 \) remains not lower than any \( \chi^2 \) after any changes in \( a, b, d, e \). As long as this is the case, all previous inequalities should hold. Thus, they hold for the whole transition path. The symmetric argument can be made for country 1 with higher \( \chi^1 \) and lower \( \chi^2 \) implying \( T (c) \leq T (f), T (b) \leq T (e) \) and \( T (c) \leq T (g), T (a) \leq T (d) \).
The comparative statics for $\rho$ can be made in the similar way: the derivative of the LHS of currency choice inequality with respect to $\rho$ is the same for all countries, so that only levels of $\lambda^j_k$ matter. ■

A.2.6 Alternative models of sticky prices

Calvo pricing

This section shows that the main results about currency choice from Section 2.3 hold under staggered pricing. As before, I abstract from monetary and productivity shocks. In addition, to simplify the analysis, the exchange rates are assumed to follow random walk, which requires under complete markets that the process for $\psi_{it}$ is random walk.

Assume that prices of all firms are set a la Calvo with the probability of adjustment $1 - \lambda$ (note the difference in notation from the baseline model). Start with exporter from country $j$ to country $i$. Since exchange rates follow random walk, the first order approximation to the adjusted price does not depend on the currency of invoicing (see Gopinath, Itskhoki, and Rigobon 2010) and can be written in destination currency as

$$\hat{p}_{jit} = (1 - \beta\lambda) \tilde{p}_{jit} + \beta\lambda E_t \hat{p}_{jit+1},$$

where the optimal static price $\tilde{p}_{jit}$ is the same as in the baseline model. The import price index from $j$ to $i$ aggregates across adjusting and non-adjusting firms

$$p_{jit} = (1 - \lambda) \hat{p}_{jit} + \lambda \left( p_{jit-1} + \mu^P \Delta e_{ijt} + \mu^D \Delta e_{it} - n \Delta e_{0t} + \mu^D \Delta e_{0t} \right).$$

The standard manipulations lead to the NKPC:

$$\pi_{it} = \frac{(1 - \beta\lambda)(1 - \lambda)}{\lambda} (\hat{p}_{it} - p_{it}) + \beta E_t \pi_{it+1} + \gamma \left[ \mu^P \Delta e_{it} - n \Delta e_{0t} + \mu^D \Delta e_{it} - \Delta e_{0t} \right].$$
\[ \bar{p}_{it} = (1 - \gamma) (1 - \phi) \phi p_{it} + \gamma (1 - \alpha) \left( \phi p_{it} + e_{it} - n e_{0t} \right) + \alpha p_{it}. \]

I solve for \( p_{it} \) in two steps. First, denote deviations of local variables from global averages with bars:

\[ -\beta \mathbb{E}_t \bar{p}_{it+1} + \left[ 1 + \beta + (1 - (1 - \gamma) \phi) \kappa \right] \bar{p}_{it} - \bar{p}_{it-1} = \kappa \gamma \bar{e}_{it} + \gamma \left( \mu^P + \mu^D \right) \Delta \bar{e}_{it}, \]

where \( \kappa \equiv \frac{(1 - \beta \lambda)(1 - \lambda)(1 - \alpha)}{\lambda} \) and \( \bar{e}_{it} \equiv e_{it} - n e_{0t} \). Rewrite it in terms of lag operator \( \mathbb{L} \) and factorize applying Vieta’s formula:

\[ \bar{p}_{it} = \varphi \bar{p}_{it-1} + \varphi \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta \varphi)^\tau \left\{ \kappa \gamma \bar{e}_{it+\tau} + \gamma \left( \mu^P + \mu^D \right) \Delta \bar{e}_{it+\tau} \right\}. \]

Since exchange rates follow random walk, we get

\[ \bar{p}_{it} = \varphi \bar{p}_{it-1} + \frac{\varphi \kappa}{1 - \beta \varphi} \gamma \bar{e}_{it} + \gamma \varphi \left( \mu^P + \mu^D \right) \Delta \bar{e}_{it}. \]

Second, integrate across all countries to get the the second-order difference equation for global price index:

\[ -\beta \mathbb{E}_t p_{t+1} + [1 + \beta + (1 - \phi) \kappa] p_{t} - p_{t-1} = -\gamma \mu^D (1 - n) \Delta e_{0t}. \]
Using the same steps as above, obtain solution

\[ p_t = \hat{\phi} p_{t-1} - \gamma \hat{\mu}^D (1 - n) \Delta e_{0t}, \]

\[ \hat{\phi} = \frac{1 + \beta + \hat{\varsigma} \kappa - \sqrt{(1 + \beta + \hat{\varsigma} \kappa)^2 - 4\beta}}{2\beta}, \quad \hat{\varsigma} \equiv 1 - \phi. \]

Finally, back out dynamics of country \( i \) price index from \( p_{it} = \bar{p}_{it} + p_t \).

To solve the currency choice problem, consider without loss of generality the case when initial values of all shocks are zero and the optimal preset prices in any currency is zero as well. The ex post price in period \( t \) conditional on non-adjustment is therefore \( e_{ikt} \) when currency \( k \) is used for invoicing. The second-order approximation to the currency choice problem of exporter from \( j \) to \( i \) is

\[ \min_k \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta \lambda)^t \left( \tilde{p}_{jit} + e_{ikt} \right)^2. \]

Note that the interpretation that firm chooses currency \( k \) to mimic dynamics of the optimal invoicing basket is still valid. It also follows that exporters prefer currency \( k \) to currency \( l \) if

\[ \sum_{t=0}^{\infty} (\beta \lambda)^t \mathbb{E}_0 (\tilde{p}_{jit} - e_{ikt})^2 < \sum_{t=0}^{\infty} (\beta \lambda)^t \mathbb{E}_0 (\tilde{p}_{jit} - e_{itt})^2. \]

Using the fact that exchange rates follow random walk and following the steps from Gopinath, Itskhoki, and Rigobon (2010), the inequality can be rewritten as

\[ (1 - \beta \lambda) \sum_{t=0}^{\infty} (\beta \lambda)^t \frac{\text{cov} (\tilde{p}^k_{jit}, \Delta e_{k0})}{\text{var} (\Delta e_{k0})} < \frac{1}{2}, \]
or after substituting the optimal price as

$$(1 - \beta \lambda) \sum_{t=0}^{\infty} (\beta \lambda)^t \frac{\text{cov} [(1 - \alpha) (\phi p_{jt} - e_{jt}) + \alpha (p_{it} - e_{it}) + e_{kt}, \Delta e_{kt0}]}{\text{var} (\Delta e_{kt0})} < \frac{1}{2}$$

To find covariance terms, I normalize volatilities of non-dollar exchange rates to one and the volatility of dollar to $\rho$, and use Yule-Walker equations to compute autocovariance functions:

$$\text{cov} (\bar{p}_{it}, \Delta e_{i0}) = \varphi \text{cov} (\bar{p}_{it-1}, \Delta e_{i0}) + \frac{\varphi \gamma \kappa}{1 - \beta \varphi} \text{cov} (e_{it}, \Delta e_{i0}) + \gamma \varphi (\mu^P + \mu^D) \text{cov} (\Delta e_{it}, \Delta e_{i0}),$$

$$\text{cov} (\bar{p}_{it}, \Delta e_{00}) = \varphi \text{cov} (\bar{p}_{it-1}, \Delta e_{00}) - \frac{\varphi \gamma \kappa n}{1 - \beta \varphi} \text{cov} (e_{0t}, \Delta e_{00}) - \gamma \varphi n (\mu^P + \mu^D) \text{cov} (\Delta e_{0t}, \Delta e_{00}),$$

$$\text{cov} (p_{it}, \Delta e_{00}) = \phi \text{cov} (p_{it-1}, \Delta e_{00}) - \gamma \varphi \mu^D (1 - n) \text{cov} (e_{0t}, \Delta e_{00}).$$

The resulting IRFs are

$$v_{it} \equiv \text{cov} (p_{it}, \Delta e_{i0}) = \gamma \varphi^{t+1} \left[ \frac{\kappa}{1 - \beta \varphi} + (\mu^P + \mu^D) \right] + \frac{1 - \varphi^t \gamma \varphi \kappa}{1 - \varphi \beta \varphi},$$

$$v_{0t} \equiv \text{cov} (p_{it}, \Delta e_{00}) = -\gamma \varphi^{t+1} \left[ \frac{\kappa n}{1 - \beta \varphi} + n (\mu^P + \mu^D) \right] - \frac{1 - \varphi^t \gamma \varphi \kappa n}{1 - \varphi \beta \varphi} \rho - \gamma \varphi^{t+1} \mu^D (1 - n) \rho,$$

and zero for all other exchange rates. Three inequalities determine invoicing decisions of firms:

$$V^{PCP} < V^{LCP} \iff [(1 - \alpha) \phi - \alpha] \frac{\gamma \varphi}{1 - \beta \lambda \varphi} \left[ (1 - \beta \lambda) (\mu^P + \mu^D) + \frac{\kappa}{1 - \beta \varphi} \right] < 1 - 2\alpha,$$

$$V^{DCP} < V^{PCP} \iff [\alpha \rho n + (1 - \alpha) \phi (1 + \rho n)] \frac{\gamma \varphi}{1 - \beta \lambda \varphi} \left[ (1 - \beta \lambda) (\mu^P + \mu^D) + \frac{\kappa}{1 - \beta \varphi} \right]$$

$$+ (\alpha + (1 - \alpha) \phi) \frac{\gamma \varphi (1 - \beta \lambda)}{1 - \beta \lambda \varphi} \rho \mu^D (1 - n) > \frac{1}{2} (1 + \rho) - \alpha,$$

$$V^{DCP} < V^{LCP} \iff [\alpha (1 + \rho n) + (1 - \alpha) \phi \rho n] \frac{\gamma \varphi}{1 - \beta \lambda \varphi} \left[ (1 - \beta \lambda) (\mu^P + \mu^D) + \frac{\kappa}{1 - \beta \varphi} \right]$$

$$+ (\alpha + (1 - \alpha) \phi) \frac{\gamma \varphi (1 - \beta \lambda)}{1 - \beta \lambda \varphi} \rho \mu^D (1 - n) > \alpha - \frac{1}{2} (1 - \rho).$$

**Proposition A2** All results about the currency choice from the benchmark model remain
true in the multiperiod model:

1. there can be no DCP equilibrium in the closed economy limit \( \gamma \to 0 \),

2. there can be no DCP equilibrium in the flexible price limit \( \lambda \to 0 \) with symmetric countries \( n = 0, \rho = 1 \),

3. the DCP region is increasing in \( \rho \) for \( \lambda \to 0, n = 0 \),

4. the DCP region is increasing in \( n \) for \( \lambda \to 0 \),

5. DCP region is non-empty when prices are sticky \( \lambda > 0 \).

Proof

1. In the limit \( \gamma \to 0 \), the processes for \( \bar{p}_t \) and \( p_t \) have the same AR root, \( \phi \to \hat{\phi} > 0 \).

   Therefore, the inequalities reduce to

   \[
   \alpha < \frac{1}{2}, \quad \alpha > \frac{1}{2} (1 + \rho), \quad \alpha < \frac{1}{2} (1 - \rho).
   \]

   The last two expressions imply there are no values of \( \alpha \), for which DCP dominates both PCP and LCP. According to the first inequality, the equilibrium invoicing is PCP if \( \alpha < \frac{1}{2} \) and LCP if \( \alpha > \frac{1}{2} \).

2. In the limit \( \lambda \to 0 \), we obtain \( \kappa \to \infty, \phi, \hat{\phi} \to 0, \kappa \phi \to \frac{1}{1 - (1 - \gamma)\phi} \) and \( \kappa \hat{\phi} \to \frac{1}{1 - \phi} \).

   Add conditions \( n = 0 \) and \( \rho = 1 \) and take the limit in the inequalities:

   \[
   \frac{\gamma [(1 - \alpha) \phi - \alpha]}{1 - (1 - \gamma) \phi} < 1 - 2\alpha \quad \Rightarrow \quad (1 - 2\alpha + \gamma \alpha)(1 - \phi) > 0,
   \]

   \[
   \frac{\gamma (1 - \alpha) \phi}{1 - (1 - \gamma) \phi} > 1 - \alpha \quad \Rightarrow \quad (1 - \alpha)(1 - \phi) < 0,
   \]

   \[
   \frac{\gamma \alpha}{1 - (1 - \gamma) \phi} > \alpha \quad \Rightarrow \quad \alpha (1 - \gamma)(1 - \phi) < 0.
   \]

   Thus, both PCP and LCP strictly dominate DCP. The only two points, for which firms are indifferent between three options are \( \alpha = \gamma = 1 \) and \( \phi = 1 \) as in the
baseline model.

3. Note that $\kappa$, $\varphi$ and $\hat{\varphi}$ do not depend on $\rho$. Therefore, the derivative of the inequalities for DCP vs. PCP/LCP wrt $\rho$ is

$$
(\alpha + (1 - \alpha) \phi) \frac{\gamma \varphi}{1 - \beta \lambda \varphi} \left[ (1 - \beta \lambda) (\mu^P + \mu^D) + \frac{\kappa}{1 - \beta \varphi} \right] 
+ (\alpha + (1 - \alpha) \phi) \frac{\gamma \hat{\varphi} (1 - \beta \lambda)}{1 - \beta \lambda \hat{\varphi}} \mu^D (1 - n) - \frac{1}{2},
$$

which is always negative for $\lambda \to 1$ and $n = 0$.

4. Note that $\kappa$, $\varphi$ and $\hat{\varphi}$ do not depend on $n$. Therefore, the derivative of the inequalities for DCP vs. PCP/LCP wrt $n$ is

$$
(\alpha + (1 - \alpha) \phi) \frac{\gamma \varphi}{1 - \beta \lambda \varphi} \left[ (1 - \beta \lambda) \mu^P + \frac{\kappa}{1 - \beta \varphi} \right] -
(\alpha + (1 - \alpha) \phi) \gamma \mu^D -
(\alpha + (1 - \alpha) \phi) \frac{\gamma \hat{\varphi}}{1 - \beta \lambda \hat{\varphi} - \varphi} \left[ (1 - \beta \lambda) \mu^D, \right.

where $\hat{\varphi} > \varphi$. The derivative is positive in the flexible price limit.

5. Suppose $n = 0$ and $\rho = 1$. Take the limit $\alpha, \gamma \to 1$, which implies $\kappa \to 0$, $\varphi, \hat{\varphi} \to 1$ and show that DCP equilibrium always exists for $\mu^D = 1$.

Rotemberg pricing

I argue next that under the second-order approximation, the currency choice problem of individual firms is the same under Rotemberg pricing as in the baseline model, which relies on Calvo pricing. To simplify notation, I suppress indices of origin and destination below.
There are two steps in firm optimization. In the second one, which happens after the shocks are realized, a firm decides how much to adjust its prices. Taking the second-order approximation of the (static) profit function and assuming quadratic costs of price adjustment, the problem of the firm can be written as

$$\min_p \left\{ \varphi (p - \bar{p})^2 + (p - p_0)^2 \right\},$$

where $\varphi < 0$ is a constant determined at the point of approximation, $\bar{p}$ is the optimal price in a given state of the world, $p_0$ is the value of the preset price, which depends on the value of the exchange rate. The first-order condition implies then that firms chooses a price as a weighted average of the optimal price and the preset price

$$p = \omega \bar{p} + (1 - \omega) p_0,$$

where $\omega = \frac{\varphi}{1 + \varphi}$. Therefore,

$$p - \bar{p} = (1 - \omega) (p_0 - \bar{p}), \quad p - p_0 = -\omega (p_0 - \bar{p}),$$

and hence, the profit function is proportional to $(p_0 - \bar{p})^2$. The first period problem of a firms to choose currency of invoicing is

$$\max \ E \Pi (p, s) \iff \min (p_0 - \bar{p})^2.$$

Thus, the currency choice problem is isomorphic to the one in the benchmark case.

**Menu cost model**

This subsection shows that model predictions remain robust when Calvo pricing is replaced with the endogenous price adjustment. To simplify, I assume as before that prices adjust fully after two periods, while firms optimally choose whether to pay menu costs and to update prices after one period. I use the second-order approximation to firm’s
profit function and the first-order approximation for price indices.\textsuperscript{27} In addition to aggregate exchange rates shocks, firms also experience idiosyncratic productivity shocks, which according to previous studies account for most price adjustments (see e.g. Golosov and Lucas 2007). As in the baseline model, I abstract from monetary and productivity shocks.

I solve the model numerically using the following algorithm. I first guess price function $p_i = p(e_i, e_0)$ for given currency of invoicing. I then estimate deviation of producer’s ex-post price from the optimal level $\bar{p}_{ji}$ in each state of the world and solve for price adjustment decision. Integrating across both idiosyncratic productivity shocks and exchange rates $e_j$, I then update function $p(\cdot, \cdot)$ and iterate this procedure till convergence. Finally, I compute expected profits of a given exporter under alternative invoicing and check whether conjectured currency choice can be sustained in equilibrium.\textsuperscript{28} To implement this algorithm, I use a grid with 31 points for exchange rates and 51 points for idiosyncratic shocks. Following Gopinath and Itskhoki (2010b), I calibrate the standard deviation of productivity shocks to be 5 times larger than the standard deviation of exchange rates.

Figure A9 reproduces two key results from the baseline model in the extension with menu costs. The left plot shows equilibrium invoicing when menu costs are close to zero and dollar has no fundamental advantages. As in Figures 2.3b, the equilibrium is unique for most parameter space and no DCP equilibrium exists. The right figure shows instead

\textsuperscript{27}For the proof that such approximation is consistent see appendix in Gopinath and Itskhoki (2010b).

\textsuperscript{28}As is well known (see e.g. Ball and Romer 1991), there are strategic complementarities in price adjustment decisions, which can lead to multiple equilibria. The initial guess for price function is taken from the baseline model and assumes $\lambda = 0$ for the flexible-price limit and $\lambda = 0.5$ for the baseline calibration. The results remain robust for other initial values.
that the region of DCP is large and close to the one from Figure 2.6 when prices are sticky and countries are asymmetric.

(a) No-DCP benchmark

(b) Baseline calibration

Figure A9: Currency choice in the menu cost model

Note: plot (a) shows DCP region is empty in the limiting case of almost zero menu costs and \( n = 0, \rho = 1 \). Plot (b) shows the region of symmetric DCP equilibrium (other equilibria are suppressed) under the baseline calibration: \( n = 0.3, \rho = 0.5 \) and menu costs are calibrated in such way that the probability of price adjustment is 0.5 for \( \alpha = 0.5, \gamma = 0.6, \phi = 0.5 \).

Model with bargaining

This section outlines a model with bargaining between suppliers and buyers and shows that the same equilibrium as in the baseline model can arise even when prices and invoicing currency are chosen jointly by two firms. The extension is based on Hart and Moore (2008) and Gopinath and Itskhoki (2011b).

The general equilibrium setup is the same as in the benchmark model. The tradable sector is populated by two types of firms. As before, there is a continuum of manufacturing firms producing intermediate goods in each country. In addition, there are wholesale firms, which combine local and imported products using Kimball aggregator and sell output to final consumers and to firms in tradable sector as intermediate inputs. I assume the
most commonly used specification for Kimball demand coming from Klenow and Willis (2007) and use $\Upsilon(\cdot)$ and $h(\cdot)$ below to denote aggregation function and the resulting demand function. Wholesale firms set prices flexibly and charge a constant markup over marginal costs, i.e. demand for their output is

$$Q_i = P_i^{-\zeta} B_i,$$

where $B_i$ is demand shifter taken as given by individual firms and $\zeta > 1$. Elasticity $\zeta$ does not affect optimal price as I show below and therefore, can take arbitrary values. In particular, one can take limit $\zeta \to \infty$ to make wholesale sector perfectly competitive.

Wholesale firms and their suppliers bargain over prices and choose the currency of invoicing before the realization of shocks. After uncertainty is resolved, wholesale firms decide how much inputs suppliers have to deliver. With probability $\lambda$, firms experience large enough idiosyncratic shocks to renegotiate prices ex post. The assumption that contract specifies prices, but not quantities is motivated by the result from the optimal contract literature by Hart and Moore (2008): “The parties are more likely to put restrictions on variables over which there is an extreme conflict of interest, such as price, than on variables over which conflict is less extreme, such as the nature or characteristics of the good to be traded.”

The marginal costs of production for manufacturers are the same as in the baseline model. The price index for bundle of intermediate goods $p_i$ remains also unchanged because of the combination of two assumptions: (i) prices of all wholesale firms are equal in equilibrium due to symmetry, (ii) wholesale firms charge a constant markup over marginal costs. Denote the marginal costs of wholesalers with $R_i$. The profits of wholesale firm for

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The marginal effect of signing a contract with an additional supplier \( j \) on marginal costs of wholesaler \( i \) is equal

\[
dR_i = D_i P_{ji} h \left( \frac{D_i P_{ji}}{R_i} \right) - R_i \Upsilon \left( h \left( \frac{D_i P_{ji}}{R_i} \right) \right).
\]

**Proof** The equilibrium values of \( R_i \) and \( D_i \) are characterized by a system of equations:

\[
\frac{1}{N} \int_0^N \Upsilon \left( h \left( \frac{D_i P_{ji}}{R_i} \right) \right) \, dj = 1,
\]

\[
\frac{1}{N} \int_0^N h \left( \frac{D_i P_{ji}}{R_i} \right) \frac{P_{ji}}{R_i} \, dj = 1.
\]

Take total differential of two equations and use \( x_j \equiv \frac{D_i P_{ji}}{R_i} \) to simplify notation

\[
\Upsilon \left( h \left( x_n \right) \right) \, dn + \int_0^N \Upsilon' \left( h \left( x_j \right) \right) h' \left( x_j \right) x_j \, x_j \, d \log \left( \frac{D_i}{R_i} \right) \, dj = 0,
\]

\[
h \left( x_n \right) x_n \, dn + \int_0^N \left[ h' \left( x_j \right) x_j^2 \, d \log \left( \frac{D_i}{R_i} \right) - h \left( x_j \right) x_j \, d \log R_i \right] \, dj = 0.
\]

Note that \( \Upsilon' \left( h \left( x_j \right) \right) = x_j \) in the first condition from definition of \( h \left( \cdot \right) \) and that \( \frac{1}{N} \int_0^N h \left( x_j \right) x_j \, dj = 1 \) in the second condition according to initial equilibrium system. Using these equalities and substituting the first equation into the second one, we obtain

\[
\, d \log R_i \, dj = \left[ h \left( x_n \right) x_n - \Upsilon \left( h \left( x_n \right) \right) \right] \frac{dn}{N},
\]

which proves the lemma. ■

The benefit of signing a contract for supplier is

\[
\left( P_{ji} - MC^i_j \right) Q_{ji} = \left( P_{ji} - MC^i_j \right) h \left( \frac{D_i P_{ji}}{R_i} \right) R_i^{1-\zeta} \, B_i,
\]

\[\footnote{For simplicity, I assume that demand shifter \( \gamma \) reflects the mass of varieties coming from different countries (extensive margin) rather than the trade flow of a given firm (intensive margin).} \]
where $MC^i_j$ are marginal costs of producer $j$ expressed in currency $i$. Nash bargaining solution can then be obtained from the following problem:

$$\max_{P_{ji}} \left[ (P_{ji} - MC^i_j) h \left( \frac{D_i P_{ji}}{R_i} \right) R_i^{-\zeta} B_i \right]^{1-\tau} \left[ \frac{B_i R_i^{-\zeta}}{\zeta (\zeta - 1)^{\zeta}} \left[ D_i P_{ji} h \left( \frac{D_i P_{ji}}{R_i} \right) - R_i \Upsilon \left( h \left( \frac{D_i P_{ji}}{R_i} \right) \right) \right] \right]^\tau,$$

or equivalently

$$\max_{P_{ji}} (1 - \tau) \log \left[ \frac{R_i x - MC^i_j}{D_i x} \right] h (x) + \tau \log \left[ x h (x) - \Upsilon (h (x)) \right],$$

where $\tau$ denotes the bargaining power of wholesaler and $x \equiv \frac{D_i P_{ji}}{R_i}$. The first order condition is

$$\frac{(1 - \tau) R_i}{D_i x - MC^i_j} + \frac{(1 - \tau) h' (x)}{h (x)} + \frac{\tau \left[ h (x) + x h' (x) - \Upsilon' (h (x)) h' (x) \right]}{x h (x) - \Upsilon (h (x))} = 0.$$

Multiply all terms by $x$, use the definition of $h (x) = \Upsilon'^{-1} (x)$, which implies $\Upsilon' (h (x)) = x$, and definition of $\theta (x) \equiv -\frac{h' (x)}{h (x)}$ to rewrite optimality condition as

$$(1 - \tau) \left[ \frac{P_{ji}}{P_{ji} - MC^i_j} - \theta (x) \right] = \frac{\tau h (x)}{\Upsilon (h (x)) - x h (x)}.$$

Log-linearize equilibrium condition around symmetric deterministic point with all prices being equal $P_{ji} = P = R_i$, $x = D$, $\Upsilon' (1) = D$, $\Upsilon (1) = h (D) = 1$:

$$(1 - \tau) \left[ \frac{P/MC}{(P/MC - 1)^2} (mc^i_j - \tilde{p}_{ji}) - \varepsilon \theta (\tilde{p}_{ji} - p_i) \right] = \tau \frac{\theta D}{(1 - D)^2} \left[ \frac{\theta - 1}{\theta} - D \right] (\tilde{p}_{ji} - p_i),$$

where $\varepsilon \equiv \frac{\partial \log \theta (x)}{\partial \log x}$ and $p_i = r_i$. When suppliers have all bargaining power, $\tau = 0$, the optimal price is exactly the same as in the benchmark case. More generally, since equation is homogeneous in $(\tilde{p}_{ji}, mc^i_j, p_i)$, the optimal price $\tilde{p}_{ji}$ can be written as a weighted sum of marginal costs and local price index as in the baseline model. Moreover, for the aggregator from Klenow and Willis (2007), $D = \frac{\theta - 1}{\theta}$ as in the CES case and therefore, optimal price
does not depend on distribution of bargaining power $\tau$.

**Lemma A9** For Klenow-Willis aggregator, the first-order approximation to the optimal price (2.19) is the same in the model with bargaining as in the baseline model.

Finally, because contract is sticky and can be renegotiated only in extreme states of the world, suppliers and wholesalers choose the currency of invoicing to minimize deviations of ex post price from the optimal one. Under the second order approximation, this implies the same invoicing problem as in the benchmark model:

$$
\min_k \mathbb{E} \left[ \tilde{p}_{ji} - e_{ik} \right]^2
$$

Thus, the equilibrium conditions for marginal costs, price index, optimal price and currency choice are the same to the first order approximation as the ones in the baseline model, and therefore, two models have the same equilibrium.

**A.2.7 Extensions (Section 2.4)**

**Proof of Proposition 2.7** As before, the import price index is

$$
p^I_i = \lambda \left[ (1 - \alpha) (\phi p_i + e_i) + \alpha p_i \right] + (1 - \lambda) \left[ (\mu^P + \mu^D) e_i - \mu^D e_0 \right].
$$

Denote the currency choice of domestic firms with $\hat{\mu}$. Note that PCP and LCP coincide for domestic firms and therefore it is sufficient to focus on $\hat{\mu}^{DCP}$. The price index for local goods is therefore

$$
p^D_i = \lambda \left[ (1 - \alpha) \phi + \alpha \right] p_i + (1 - \lambda) \hat{\mu}^D (e_i - e_0).
$$

Solve for the price index of individual country:

$$
p_i = \frac{\gamma \lambda (1 - \alpha) + \gamma (1 - \lambda) (\mu^P + \mu^D) + (1 - \gamma) (1 - \lambda) \hat{\mu}^D}{1 - \lambda (\alpha + (1 - \alpha) (1 - \gamma) \phi) e_i} - \frac{(1 - \lambda) \left[ \gamma \mu^D + (1 - \gamma) \hat{\mu}^D \right]}{1 - \lambda (\alpha + (1 - \alpha) \phi)} e_0.
$$
In the flexible price limit, the currency of invoicing of both exporters and domestic firms has no effect on equilibrium prices. Therefore, the aggregate price index and the currency choice of exporters remain the same as in the baseline model:

\[ p_i = \frac{\gamma}{1 - (1 - \gamma) \phi} e_i. \]

The currency choice of domestic producers is determined by

\[ \tilde{p}_{ii} + e_{ki} = e_k - \frac{(1 - \phi)(1 - \gamma \alpha)}{1 - (1 - \gamma) \phi} e_i. \]

The volatility of the optimal price expressed in domestic currency and dollars is therefore

\[ V_{PCP/LCP}^{PCP/LCP} = \left[ 1 - \frac{(1 - \phi)(1 - \gamma \alpha)}{1 - (1 - \gamma) \phi} \right]^2, \quad V^{DCP} = \rho + \left[ \frac{(1 - \phi)(1 - \gamma \alpha)}{1 - (1 - \gamma) \phi} \right]^2. \]

It follows that local firms choose DCP if \( \frac{(1 - \phi)(1 - \gamma \alpha)}{1 - (1 - \gamma) \phi} < 1 - \rho \), which is more likely when \( \rho \) is low and \( \gamma \), \( \phi \) and \( \alpha \) are high. In particular, if \( \rho < 1 \), both exporters and domestic suppliers set prices in dollars in two limiting cases: \( \phi \to 1 \) and \( \alpha, \gamma \to 1 \).

Consider next the case with \( \lambda > 0 \). Start with the following observation: in the PCP (LCP) equilibrium in the baseline model domestic firms are also following PCP (LCP). Because of strategic complementarities, this gives these equilibria the highest chances, i.e. if PCP (LCP) equilibrium cannot be sustained when \( \hat{\mu}_{DCP} = 0 \), there is no way to support it with \( \hat{\mu}_{DCP} = 1 \). On the other hand, it might be easier to sustain the DCP equilibrium if domestic firms choose DCP. Indeed, \( \hat{\mu}_{DCP} = 1 \) increases both \( \chi \) and \( \chi_0 \) relative to the baseline model, which makes DCP more attractive for importers. A necessary and sufficient condition to sustain such equilibrium is however that domestic firms choose
DCP. Since $e_i = e_j$ for local firms,

$$\tilde{p}_{ii} + e_{ki} = e_k - [(1 - \alpha)(1 - \phi\chi) + \alpha(1 - \chi)]e_i - (\alpha + (1 - \alpha)\phi)\chi_0e_0.$$  

It follows,

$$V^{PCP/LCP} = [1 - (1 - \alpha)(1 - \phi\chi) - \alpha(1 - \chi)]^2 + (\alpha + (1 - \alpha)\phi)^2\chi_0^2\rho,$$

$$V^{DCP} = [(1 - \alpha)(1 - \phi\chi) + \alpha(1 - \chi)]^2 + [1 - (\alpha + (1 - \alpha)\phi)\chi_0]^2\rho.$$  

DCP dominates local currency if

$$(\alpha + (1 - \alpha)\phi)(\chi + \rho\chi_0) > \frac{1 + \rho}{2},$$

where

$$\chi + \rho\chi_0 = \frac{\gamma\lambda(1 - \alpha) + (1 - \lambda)}{1 - \lambda(1 - \gamma)\phi} + \frac{(1 - \lambda)\rho}{1 - \lambda(\alpha + (1 - \alpha)\phi)}.$$  

Consider the limit of fully rigid prices $\lambda = 0$: $\chi + \rho\chi_0 = 1 + \rho$ and therefore, condition simplifies to $\alpha + (1 - \alpha)\phi > \frac{1}{2}$, which is always satisfied for $\alpha > 0.5$ or $\phi > 0.5$. At the same time, DCP dominates PCP and LCP for importers if

$$(1 - \alpha)\gamma\phi + \alpha(1 + \rho\gamma) > \frac{1 + \rho}{2},$$

$$(1 - \alpha)(1 - \gamma\phi)\rho + \alpha(1 - \gamma)(1 + \rho) < \frac{1 + \rho}{2}.$$  

I argue next that GCP equilibrium exists for these parameters as well. Prove by contradiction. Condition for GCP does not depend on $\gamma$, while conditions for DCP relax as $\gamma$
becomes larger. Therefore, take $\gamma = 1$

$$(1 - \alpha) \phi > \left( \frac{1}{2} - \alpha \right) (1 + \rho),$$

$$(1 - \alpha) \phi > \left( \frac{1}{2} - \alpha \right) - \frac{1}{2\rho}.$$ 

If $\alpha > 0.5$, the GCP equilibrium exists and we arrive to contradiction. If $\alpha < 0.5$, then conditions are relaxed for $\rho = 1$

$$(1 - \alpha) \phi > (1 - 2\alpha),$$

$$(1 - \alpha) \phi > -\alpha.$$ 

The second condition is always satisfied, while the first one implies $\alpha + (1 - \alpha) \phi > 1 - \alpha > 0.5$, so GCP equilibrium exists and we again arrive to contradiction. ■

**Proof of Proposition 2.8** Substitute price index (A110) into the desired price (A136) to obtain

$$\tilde{p}_{ji} + e_{ki} = e_k - (1 - \alpha) [(1 - \phi \chi) e_j - (1 - \phi + \phi \chi_w) w_j] - \alpha [(1 - \chi) e_i - \chi_w w_i]$$

$$- (\alpha + (1 - \alpha) \phi) (\chi_0 e_0 - \chi_w n w_0).$$

Consider first the limiting case when monetary shocks dominate any other shocks in the economy, and according to (A131) and (A134), equilibrium exchange rate is $e_i \approx w_i$. This implies

$$\tilde{p}_{ji} + e_{ki} = e_k - (1 - \alpha) \phi (1 - \chi - \chi_w) e_j - \alpha (1 - \chi - \chi_w) e_i - (\alpha + (1 - \alpha) \phi) (\chi_0 - \chi_w n) e_0,$$
where
\[
1 - \chi - \chi_w = \frac{(1 - \lambda)(1 - \gamma (\mu^P + \mu^D))}{1 - \lambda (\alpha + (1 - \gamma) (1 - \alpha) \phi)} \geq 0,
\]
\[
\chi_0 - \chi_w n = \frac{\gamma (1 - \lambda)}{1 - \lambda (\alpha + (1 - \gamma) (1 - \alpha) \phi)} \left[ (n\mu^P + \mu^D) + \frac{\lambda (1 - \alpha) \phi [n + \gamma (1 - n) \mu^D]}{1 - \lambda (\alpha + (1 - \alpha) \phi)} \right] \geq 0.
\]
In the flexible price limit \(\lambda \to 1\), both coefficients converge to zero and \(\tilde{p}_{ji} + e_{ki} = e_k\).

While firms are indifferent between all currencies when \(\rho = 1\), an arbitrary small volatility advantage is sufficient to guarantee that DCP is used for any values of other parameters.

More generally, \(\text{cov}(w_i, e_i) > 0\) under both complete markets and one internationally traded bond. Therefore, the effective weight of producer and local currency in the optimal price goes down as the volatility of monetary shocks increases, and exporters are more likely to choose DCP.

\[\blacksquare\]

**Proof of Proposition 2.9** Domestic firms set prices in local currency, while importers enjoy the optimal state contingent prices:

\[
p_i = \gamma \frac{(1 - \alpha)}{1 - \gamma \alpha - \lambda (1 - \gamma) (1 - \alpha) \phi} (e_i - n e_0).
\]

All results except for the second one follow immediately from expressions for \(p_i\) and \(\tilde{p}_{ji}\).

Let \(\bar{s}_i\) denote the share of local currency in the optimal basket of exporters. Since \(\chi \leq 1\) and \(\chi_0 = n \chi \leq n\), the dollar share in trade between third countries is \([\alpha + (1 - \alpha) \phi] \chi_0 \leq n\) and the dollar share in the international trade is

\[
\frac{1}{1 - n^2} \left[ (1 - n)^2 \bar{s}_0 + n (1 - n) (\bar{s}_i + \bar{s}_0) + n (1 - n) (\bar{s}_j + \bar{s}_0) \right] = \bar{s}_0 + \frac{n}{1 + n} (\bar{s}_i + \bar{s}_j)
\]

\[= [\alpha + (1 - \alpha) \phi] n \chi + \frac{n}{1 + n} [1 - (\alpha + (1 - \alpha) \phi) \chi] = \frac{n}{1 + n} [\alpha + (1 - \alpha) \phi] \frac{n^2}{1 + n} \chi \leq n.
\]
Inflation targeting

Consider the case when monetary authorities stabilize consumer price index rather than nominal wages. Assuming away productivity shocks in both sectors, we get

\[ p_i^C = \eta p_i + (1 - \eta) p_i^N = \eta (\chi_i - \chi_0 w_0 + \chi_w w_i - \chi_w f) + (1 - \eta) \lambda w_i. \]

Since monetary policy is correlated across countries in this case due to common \( \epsilon_0 \) and \( \omega \) terms, the equilibrium exchange rates are given by

\[ e_i = w_i - w + \psi_i. \]

Substitute this expression into the CPI index:

\[ p_i^C = \eta (\chi (w_i - w + \psi_i) - \chi_0 (w_0 - w + \psi_0) + \chi_w w_i - \chi_w f) + (1 - \eta) \lambda w_i. \]

Integrating this equality across \( i \) and using the policy rule in the U.S. and other countries, obtain the system

\[
\begin{align*}
[\eta (\chi_0 + \chi_w - \chi_w) + (1 - \eta) \lambda] w &= \eta \chi_0 (w_0 + \psi_0), \\
[\eta (\chi - \chi_0 + \chi_w) + (1 - \eta) \lambda] w_0 + \eta (\chi - \chi_0) \psi_0 &= \eta (\chi - \chi_0 + \chi_w) w,
\end{align*}
\]

which can be solved to obtain

\[
\begin{align*}
[\eta (\chi_0 + \chi_w - \chi_w) + (1 - \eta) \lambda - \eta^2 \chi_0 (\chi - \chi_0 + \chi_w) / (\eta (\chi - \chi_0 + \chi_w) + (1 - \eta) \lambda)] w &= \eta \chi_0 w_0 + (1 - \eta) \lambda \psi_0, \\
&= \eta \chi_0 \eta \chi w + (1 - \eta) \lambda \psi_0.
\end{align*}
\]
\[ w_0 = \frac{\eta (\chi - \chi_0 + \chi_w)}{\eta (\chi - \chi_0 + \chi_w) + (1 - \eta) \lambda} w - \frac{\eta (\chi - \chi_0)}{\eta (\chi - \chi_0 + \chi_w) + (1 - \eta) \lambda} \psi_0. \]

Denote solution to this system with \( w = k \psi_0 \) and \( w_0 = k_0 \psi_0 \) and substitute it into CPI of individual country to solve for \( w_i \)

\[ w_i = \frac{\eta \chi_0 (k_0 + 1) + (\chi + \chi_0 + \chi_w) k}{\eta (\chi + \chi_w) + (1 - \eta) \lambda} \psi_0 - \frac{\eta \chi}{\eta (\chi + \chi_w) + (1 - \eta) \lambda} \psi_i \equiv l_0 \psi_0 + l \psi_i. \]

The equilibrium values of exchange rate \( e_i \) then follows from the risk-sharing condition. Given these values, one can then solve the currency choice problem of individual firm that minimizes

\[ \tilde{p}_{ji} + e_{ki} = e_k + (1 - \alpha) (\phi p_j + (1 - \phi) w_j - e_j) + \alpha (p_i - e_i) \]

\[ = e_k - (1 - \alpha) ((1 - \phi \chi) e_j - (1 - \phi + \phi \chi_w) w_j) - \alpha ((1 - \chi) e_i - \chi w_i) \]

\[ - (\alpha + (1 - \alpha) \phi) (\chi_0 e_0 + \chi w_i). \]
A.3 Appendix for Chapter 3

A.3.1 Transmission of shocks

**Proof of Proposition 3.1** Consider a monetary shock \( w_i \). The risk-sharing condition (A131) implies that the depreciation of exchange rate \( e_i \) is the same in all countries. Moreover, the pass-through of \( w_i \) into prices and quantities (A110)-(A123) is independent from currency regime and is the same for all countries when \( n = 0 \). The only difference between the U.S. and other countries is therefore coming from the effect of \( e_i \) on prices and quantities.

Both export and import elasticity with respect to trade-weighted exchange rate \( e_i \) is different for the U.S. than for other countries because of the effect of \( e_0 \) on global economy, which is summarized by the partial elasticity

\[
\frac{\partial e x_i}{\partial e_0} = \frac{\partial i m_i}{\partial e_0} = \left[ (1 - \gamma) (\theta - 1) + \frac{\gamma}{1 + \sigma \nu} \frac{1}{1 - \lambda} (\alpha + (1 - \alpha) \phi) \right] (1 - \lambda) \mu^D,
\]

which is positive under DCP. The effect of \( e_i \) on CPI inflation is given by \( p_i^C = \eta \chi e_i \) for non-U.S. economies and \( p_0^C = \eta (\chi - \chi_0) e_0 \) for the U.S., which implies that inflation is lower in the U.S. From equation (A124), \( e_i \) has no effect on output in non-tradable sector:

\[
y_N = p_i^C - p_i^N + c_i = w_i - p_i^N.
\]

Equation (A123) implies that the relevant price terms in tradable production are

\[
y_i = \frac{\gamma \theta}{1 - (1 - \gamma) \phi} \left[ (p_i^T - p_i^E) - (p_i - p) \right] - \frac{(1 - \gamma) (1 - \phi^2)}{1 - (1 - \gamma) \phi} p_i - \frac{\gamma (1 + \phi)}{1 - (1 - \gamma) \phi} p.
\]

Again, the asymmetries across countries come from the partial derivative with respect to
The stimulating effect on local output is therefore large in the U.S. Finally, Lemma A3 implies that the effect of $e_i$ on net exports of all countries is the same when $n = 0$. ■

Proof of Proposition 3.2 Consider a monetary shock in the U.S. $w_0$. The risk-sharing condition (A131) implies that the depreciation of dollar exchange rate $e_0$ is the same under all invoicing regimes. Moreover, the pass-through of $w_0$ into prices and quantities (A110)-(A123) is independent from currency regime as well. The only difference in international spillovers under PCP/LCP and DCP come from the effect of $e_0$ on foreign prices and quantities. Results (1) and (2) then follow immediately from expressions (3.1)-(3.3). The price index (A110) implies that higher $e_0$ decreases $p_i$ and CPI in other economies, and the foreign consumption increases according to (A115). Finally, consider total production of tradable and non-tradable goods. Equation (A124) implies $e_0$ has no effect on output in non-tradable sector: $y_{Ni} = p_i^C - p_i^N + c_i = w_i - p_i^N$. Equation (A123) implies that the relevant price terms in tradable production are

\[ y_i = \frac{\gamma \theta}{1 - (1 - \gamma) \phi} \left[ (p_i^I - p_i^E) - (p_i - p) \right] - \frac{(1 - \gamma) (1 - \phi^2)}{1 - (1 - \gamma) \phi} p_i - \frac{\gamma (1 + \phi)}{1 - (1 - \gamma) \phi} p, \]

where

\[ (p_i^I - p_i^E) - (p_i - p) = [\chi - \lambda (\alpha \chi + (1 - \alpha) (2 - \phi \chi)) - (1 - \lambda) (2 \mu^P + \mu^D)] n e_0. \]

Thus, when $\theta n \to 0$, the first term drops out and since both $p_i$ and $p$ fall with $e_0$ under DCP, the effect on output is positive. ■
A.3.2 Welfare and policy analysis

**Proof of Proposition 3.3** Assume CES aggregator across tradable products, $\alpha = 0$, and no non-tradable sector, $\eta = 1$. The social planner maximizes the global welfare state by state subject to resource and technology constraints:

$$\max \int_0^1 \left( \log C_i - L_i \right) di$$

s.t. $C_i + X_i + G_i \leq \left[ (1 - \gamma)^{\frac{1}{\theta}} e^{-\frac{\gamma}{\theta} Y_{ii}^{\frac{1}{\theta}}} + \gamma^{\frac{1}{\theta}} e^{\frac{1 - \gamma}{\theta} \int_0^1 Y_j^{\frac{1}{\theta}} dj} \right]^{\frac{\theta}{\theta - 1}}$, $Y_{ii} + \int_0^1 Y_{ij} dj \leq A_i \left( \frac{L_i}{1 - \phi} \right)^{1 - \phi} \left( \frac{X_i}{\phi} \right)^{\phi}$.

The first-order optimality conditions are

$$C_i = \frac{1 - \phi X_i}{\phi L_i}, \quad (A141)$$

$$\left[ (1 - \gamma) e^{-\gamma \xi_i} \frac{C_i + X_i + G_i}{Y_{ii}} \right]^{\frac{1}{\theta}} = \frac{1}{A_i} \left( \frac{1 - \phi X_i}{\phi L_i} \right)^{1 - \phi}, \quad (A142)$$

$$\left( e^{-\xi_i} \frac{1 - \gamma Y_{ij}}{\gamma Y_{ii}} \right)^{-\frac{1}{\theta}} = A_i A_j \left( \frac{X_i}{L_i} / \frac{X_j}{L_j} \right)^{\phi}. \quad (A143)$$

I show next that equilibrium allocation under PCP and the monetary policy that stabilizes marginal costs in every country satisfies these conditions and therefore, is efficient. First, note that with $\alpha = 0$ and constant marginal costs, both adjusting and non-adjusting firms keep their prices constant in producer currency at $P_{ii} = 1$, so that $P_{ij} = E_{jj}$. Second, divide labor demand $(A96)$ by demand for intermediate goods $(A98)$ to get expression for real wage

$$\frac{W_i}{P_i} = \frac{1 - \phi X_i}{\phi L_i}.$$
Substitute it into labor supply to show that optimality condition (A141) is satisfied:

\[ C_i = \frac{W_i}{P_i} = \frac{1 - \phi X_i}{\phi L_i}. \]

Third, using demand for local goods (A97)

\[ Y_{ii} = (1 - \gamma) e^{-\gamma \xi_i} \left( \frac{P_{ii}}{P_i} \right)^{-\theta} (C_i + X_i + G_i), \]

obtain

\[ \left[ (1 - \gamma) e^{-\gamma \xi_i} \frac{C_i + X_i + G_i}{Y_{ii}} \right]^{\frac{1}{\theta}} = \frac{P_{ii}}{P_i}. \]

Combine stable marginal costs condition (A99) together with expression for real wage from above to show

\[ P_i = A_i \left( \frac{W_i}{P_i} \right)^{-\theta} = A_i \left( \frac{1 - \phi X_i}{\phi L_i} \right)^{-\theta}. \]

Together, the last two equation imply that optimality condition (A142) is satisfied.

Fourth, divide demand for local and foreign goods

\[ Y_{ji} = \gamma e^{(1-\gamma)\xi_i} \left( \frac{P_{ji}}{P_i} \right)^{-\theta} (C_i + X_i + G_i), \]

to show

\[ \left( e^{-\xi_i} \frac{1 - \gamma Y_{ji}}{Y_{ii}} \right)^{-\frac{1}{\theta}} = \frac{P_{ji}}{P_{ii}} = \mathcal{E}_{ij}. \]

Substitute expression for \( P_i \) from above into the risk-sharing condition (A100) to get equilibrium exchange rate:

\[ \mathcal{E}_{ij} = \frac{C_i P_i}{C_j P_j} = \frac{A_i}{A_j} \left( \frac{X_i}{L_i} / \frac{X_j}{L_j} \right)^{\phi}. \]

Combining the last two equations, we get optimality condition (A143).

This completes the proof of efficiency of the allocation given that firms use PCP. I
next show using the first-order approximation to the equilibrium system this is indeed
the only equilibrium currency choice. Given $\alpha = 0$, the desired price of exporter from $j$
to $i$ in terms of currency $k$ is

$$\tilde{p}_{ji} - e_{ik} = mc_j + e_k - e_j = e_k - e_j,$$

where the last equality follows from marginal costs targeting. It follows that PCP unam-
biguously dominates any other currency for both exports and domestic firms.

Finally, consider the optimal monetary policy. Complete risk sharing implies $e_{ij} =
 w_i - w_j$. With marginal costs fully stabilized and $\alpha = 0$, the price index is

$$p_i = \gamma \int_0^1 e_{ij} \, dj = \gamma (w_i - w).$$

Substitute this expression into marginal costs to obtain

$$[1 - (1 - \gamma) \phi] w_i = a_i + \gamma \phi w.$$

Integrating across countries and substituting result back into the last equation, we get

$$w = \frac{1}{1 - \phi} a, \quad w_i = \frac{1}{1 - (1 - \gamma) \phi} \left[ a_i + \frac{\gamma \phi}{1 - \phi} a \right].$$

It follows that equilibrium exchange rates are $e_i = \frac{1}{1 - (1 - \gamma) \phi} a_i$.

**Kimball price index** To economize on indices, consider a general price index for Kimball
demand with demand shifters that is determined by the following system:

$$\int_0^1 \gamma_i e^{zi} \, \Phi(h(De^{zi})) \, di = 1,$$

$$\int_0^1 \gamma_i e^{zi} h(De^{zi}) \, e^{zi} \, di = e^d.$$
where \( x_i \) is the log-deviation of \( \frac{\partial P_i}{\partial P} \) from symmetric deterministic point with \( P_i = P \) and some constant \( D, \int_0^1 \gamma_i \, di = 1 \) and \( z_i \) are demand shifters such that \( \int_0^1 \gamma_i z_i \, di = 0 \). Take the SOA to this system. Start with the first equation:

\[
\int_0^1 \gamma_i \left[ \Upsilon (h(D)) + \Upsilon' (h(D)) h' (D) D x_i + \Upsilon (h(D)) \left( z_i + \frac{1}{2} z_i^2 \right) + \Upsilon' (h(D)) h' (D) D x_i z_i + \frac{1}{2} \left( \frac{d \Upsilon' (h(X))}{dX} h'(D) D^2 + \Upsilon' (h(D)) h'' (D) D^2 + \Upsilon' (h(D)) h' (D) D \right) x_i^2 \right] \, di = 1.
\]

From the properties of the functions, we have \( \Upsilon (h(D)) = 1, \Upsilon' (h(D)) = D \) and \( \frac{d \Upsilon' (h(X))}{dX} = \frac{dX}{dX} = 1 \). From the definitions of elasticity and superelasticity of demand:

\[
\theta (X) \equiv -h' (X) \frac{X}{h(X)} \Rightarrow h' (X) = -\theta (X) \frac{h(X)}{X},
\]

\[
\varepsilon (X) \equiv \frac{d \log \left( -h' (X) \frac{X}{h(X)} \right)}{d \log X} = h'' (X) \frac{X}{h'(X)} + 1 + \theta (X) \Rightarrow h'' (X) = \left( \theta (X) + 1 - \varepsilon (X) \right) \frac{\theta (X) h(X)}{X^2}.
\]

Substitute these equalities into the SOA:

\[
\int_0^1 \gamma_i \left[ -\theta D x_i + \frac{1}{2} (-\theta D + (\theta + 1 - \varepsilon) \theta D - \theta D) x_i^2 + z_i + \frac{1}{2} z_i^2 - \theta D x_i z_i \right] \, di = 0
\]

or equivalently,

\[
\int_0^1 \gamma_i \left[ x_i + \frac{1}{2} (1 - \theta + \varepsilon) x_i^2 + x_i z_i - \frac{1}{2 \theta D^2} z_i^2 \right] \, di = 0.
\]

Consider next the second equation of the system determining price indices:

\[
\int_0^1 \gamma_i \left[ h(D) D + (h' (D) D^2 + h(D) D) x_i + \frac{1}{2} (h'' (D) D^3 + 3 h' (D) D^2 + h(D) D) x_i^2 \right] \, di + \int_0^1 \gamma_i \left[ h(D) D \left( z_i + \frac{1}{2} z_i^2 \right) + h'(D) D^2 + h(D) D \right] x_i z_i \, di = D \left[ 1 + d + \frac{1}{2} d^2 \right].
\]
Substitute steady-state values:

\[
\int_0^1 \gamma_i \left[ (1 - \theta) x_i + \frac{1}{2} (1 - \theta)^2 - \varepsilon \theta \right] x_i^2 + \left( 1 - \theta \right) x_i z_i + \frac{1}{2} z_i^2 \right] \, di = d + \frac{1}{2} d^2.
\]

Multiple the first equation by \(1 - \theta\) and subtract from the second one. Assume for simplicity that \(D = \frac{\theta - 1}{\theta}\), which is true for CES and Klenow-Willis aggregator. This helps with demand shifters \(z_i\), but does not matter for price terms:

\[-\frac{1}{2} \varepsilon \int_0^1 \gamma_i x_i^2 \, di = d + \frac{1}{2} d^2.\]

Substitute next the definition of \(x_i\) into the system of equations:

\[
\int_0^1 \gamma_i \left[ (d + p_i - p) + \frac{1}{2} (1 - \theta + \varepsilon) (d + p_i - p)^2 + (d + p_i - p) z_i - \frac{1}{2 (\theta - 1)} z_i^2 \right] \, di = 0,
\]

\[-\frac{1}{2} \varepsilon \int_0^1 \gamma_i (d + p_i - p)^2 \, di = d + \frac{1}{2} d^2.\]

Note that to the FOA \(d = 0\), which implies by substitution that all second-order terms with \(d\) are zero. Under CES assumption, \(\varepsilon = 0\), so that \(d = 0\) to the SOA as well as to the first one.

**Lemma A10** The SOA to the Kimball price index is

\[
\int_0^1 \gamma_i \left[ (p_i - p) + \frac{1}{2} (1 - \theta) (p_i - p)^2 + (p_i - p) z_i + \frac{1}{2 (1 - \theta)} z_i^2 \right] \, di = 0,
\]

\[-\frac{1}{2} \varepsilon \int_0^1 \gamma_i (p_i - p)^2 \, di = d.\]

Consider next the SOA to the relative demand \(V_i \equiv e^{z_i} h(e^{x_i})\):

\[
v_i + \frac{1}{2} v_i^2 = h' (D) D x_i + \frac{1}{2} (h'' (D) D^2 + h' (D) D) x_i^2 + h' (D) D x_i z_i + h (D) \left( z_i + \frac{1}{2} z_i^2 \right)
\]

\[= -\theta x_i + \frac{1}{2} (\theta - \varepsilon) \theta x_i^2 - \theta x_i z_i + z_i + \frac{1}{2} z_i^2.\]
Therefore, using result from Lemma A10
\[
\int_0^1 \gamma_i \left( v_i + \frac{1}{2} v_i^2 \right) \, di = -\theta \int_0^1 \gamma_i \left[ (d + p_i - p) + \frac{1}{2} (\varepsilon - \theta) (d + p_i - p)^2 + (d + p_i - p) z_i - \frac{1}{2} \theta z_i^2 \right] \, di \\
= \frac{\theta}{2} \int_0^1 \gamma_i (p_i - p)^2 \, di - \frac{1}{2} \frac{1}{\theta - 1} \int_0^1 \gamma_i z_i^2 \, di.
\]

**Lemma A11** The sum of SOA of relative demand is
\[
\int_0^1 \gamma_i \left( v_i + \frac{1}{2} v_i^2 \right) \, di = \frac{\theta}{2} \int_0^1 \gamma_i (p_i - p)^2 \, di - \frac{1}{2} \frac{1}{\theta - 1} \int_0^1 \gamma_i z_i^2 \, di.
\]

**Labor market and intermediates** Both labor demand and labor supply equations are exact in logs:
\[
c_i = w_i - p_i, \quad l_i = -\phi(w_i - p_i) - a_i + y_i.
\]

Demand for intermediate goods is also exact in logs
\[
x_i = y_i - a_i + (1 - \phi) (w_i - p_i).
\]

The sum of final and intermediate demand is therefore,
\[
(1 - \phi) \left( c_i + \frac{1}{2} c_i^2 \right) + \phi \left( x_i + \frac{1}{2} x_i^2 \right) = (1 - \phi^2) (w_i - p_i) + \frac{1}{2} [ (1 - \phi) + \phi (1 - \phi)^2 ] (w_i - p_i)^2 \\
- \phi (1 - \phi) (w_i - p_i) a_i + \phi [(1 - \phi) (w_i - p_i) - a_i] y_i + \phi \left( y_i + \frac{1}{2} y_i^2 \right) - \phi \left( a_i - \frac{1}{2} a_i^2 \right).
\]

**Goods market** The market clearing condition in tradable sector of country \( i \) can be written as
\[
Y_i = (1 - \gamma) \int_0^1 e^{-\gamma \xi_i p_i} \left( \frac{D_i P_{ii}(\omega)}{P_i} \right) \, d\omega \left( C_i + X_i + G_i \right) \\
+ \gamma \int_0^1 \int_0^1 e^{(1 - \gamma) \xi_i p_i} \left( \frac{D_j P_{ij}(\omega)}{P_j} \right) \, d\omega \left( C_j + X_j + G_j \right) \, dj \\
\equiv (1 - \gamma) \int_0^1 (V_{ii} C_i + V_{ii} X_i + V_{ii} G_i) \, d\omega + \gamma \int_0^1 \int_0^1 (V_{ij} C_j + V_{ij} X_j + V_{ij} G_j) \, d\omega \, dj.
\]
The SOA to this equation is

\[ y_i + \frac{1}{2} y_i^2 = \left[ (1 - \gamma) \int_0^1 \left( v_{i\omega} + \frac{1}{2} v_{i\omega}^2 \right) d\omega + \gamma \int_0^1 \int_0^1 \left( v_{ij\omega} + \frac{1}{2} v_{ij\omega}^2 \right) d\omega d\omega \right] \\
+ (1 - \gamma) \left[ (1 - \phi) \left( c_i + \frac{1}{2} c_i^2 \right) + \phi \left( x_i + \frac{1}{2} x_i^2 \right) + \left( g_i + \frac{1}{2} g_i^2 \right) \right] \\
+ \gamma \int_0^1 \left[ (1 - \phi) \left( c_j + \frac{1}{2} c_j^2 \right) + \phi \left( x_j + \frac{1}{2} x_j^2 \right) + \left( g_j + \frac{1}{2} g_j^2 \right) \right] d\omega \\
+ \left[ (1 - \gamma) \int_0^1 v_{i\omega} d\omega \left((1 - \phi) c_i + \phi x_i + g_i \right) + \gamma \int_0^1 \int_0^1 v_{ij\omega} d\omega \left((1 - \phi) c_j + \phi x_j + g_j \right) d\omega \right]. \]

Integrate market clearing conditions across countries:

\[ \int_0^1 \left( y_i + \frac{1}{2} y_i^2 \right) d\omega = \int_0^1 \left[ (1 - \gamma) \int_0^1 \left( v_{i\omega} + \frac{1}{2} v_{i\omega}^2 \right) d\omega + \gamma \int_0^1 \int_0^1 \left( v_{ij\omega} + \frac{1}{2} v_{ij\omega}^2 \right) d\omega d\omega \right] d\omega \\
+ \int_0^1 \left[ (1 - \phi) \left( c_i + \frac{1}{2} c_i^2 \right) + \phi \left( x_i + \frac{1}{2} x_i^2 \right) + \left( g_i + \frac{1}{2} g_i^2 \right) \right] d\omega \\
+ \int_0^1 \left[ (1 - \gamma) \int_0^1 v_{i\omega} d\omega + \gamma \int_0^1 \int_0^1 v_{ij\omega} d\omega d\omega \right] \left((1 - \phi) c_i + \phi x_i + g_i \right) d\omega, \]

where I changed the order of integrations.

According to Lemma A11, \((1 - \gamma) \int_0^1 v_{i\omega} d\omega \gamma \int_0^1 \int_0^1 v_{ij\omega} d\omega d\omega d\omega \) is of the second order and therefore, the last term is zero in the SOA. Substitute the result from the proposition into the first term and the expression for consumption and intermediate demand to obtain:

\[ \int_0^1 \left( y_i + \frac{1}{2} y_i^2 \right) d\omega = \int_0^1 \left[ (1 + \phi) (w_i - p_i) + \frac{1}{2} \left[ 1 + \phi (1 - \phi) \right] (w_i - p_i)^2 - \phi (w_i - p_i) a_i + \frac{\theta}{2} \frac{1}{1 - \phi} \sigma_{P_i}^2 \\
+ \phi \left( (w_i - p_i) - \frac{1}{1 - \phi} a_i \right) y_i - \frac{\phi}{1 - \phi} \left( a_i - \frac{1}{2} a_i^2 \right) + \frac{\gamma (1 - \gamma)}{2 (\theta - 1) (1 - \phi)} \xi_i^2 \right] d\omega, \]

where \( \sigma_{P_i}^2 \) denotes dispersion of prices in region \( i \) for brevity.
Loss function  The preferences in country $i$ are given by

$$U_i = \log C_i - L_i.$$ 

The second-order approximation (SOA) to the objective function:

$$U_i = \log C - L + c_i - L \left( l_i + \frac{1}{2} l_i^2 \right).$$ 

Use steady-state values $C = L = 1$ and suppress a constant term:

$$u_i = c_i - l_i - \frac{1}{2} l_i^2.$$ 

Next, substitute in consumption and labor from labor market clearing condition:

$$u_i = (1 + \phi) (w_i - p_i) + \left( a_i - \frac{1}{2} a_i^2 \right) - \frac{1}{2} \phi^2 (w_i - p_i)^2 - \phi (w_i - p_i) a_i$$

$$- \left( y_i + \frac{1}{2} y_i^2 \right) + \left[ \phi (w_i - p_i) + a_i \right] y_i.$$ 

Integrate across countries and use expression for total output from the goods market clearing to see several terms cancel out:

$$u = \int_0^1 \left[ \frac{1}{1 - \phi} \left( a_i - \frac{1}{2} a_i^2 \right) - \frac{1}{2} (1 + \phi) (w_i - p_i)^2 + \frac{1}{1 - \phi} a_i y_i$$

$$- \frac{\theta \sigma^2_{p_i}}{2 (1 - \phi)} - \frac{1}{1 - \phi} \left( g_i + \frac{1}{2} g_i^2 \right) + \frac{\gamma (1 - \gamma)}{2 (\theta - 1) (1 - \phi)} \xi_i \right] \, di.$$ 

Suppress exogenous terms to simplify the expression:

$$u = \int_0^1 \left[ \frac{1}{2} (1 + \phi) (w_i - p_i)^2 - \frac{\theta}{2} \frac{1}{1 - \phi} \sigma^2_{p_i} + \frac{1}{1 - \phi} a_i y_i \right] \, di.$$
The FOA to the output of an individual country in (A123) implies that the price terms in $y_i$ are

$$y_i = \frac{\gamma \theta}{1 - (1 - \gamma) \phi} \left[ (p_i^I - p_i) - (p_i^E - p) \right] + \frac{(1 - \gamma) (1 - \phi^2)}{1 - (1 - \gamma) \phi} (w_i - p_i) + \frac{\gamma (1 + \phi)}{1 - (1 - \gamma) \phi} (w - p).$$

Substitute this equation and change the signs to obtain the loss function:

$$\mathcal{L} = \int_0^1 \left[ \frac{1}{2} (1 + \phi) (w_i - p_i)^2 + \frac{\theta}{1 - \phi} \delta P_i^2 - \frac{1}{1 - \phi} \frac{\gamma}{1 - (1 - \gamma) \phi} \left[ (p_i^I - p_i) - (p_i^E - p) \right] a_i 
- \frac{(1 - \gamma) (1 + \phi)}{1 - (1 - \gamma) \phi} (w_i - p_i) a \right] \, di + \frac{\gamma (1 + \phi)}{1 - (1 - \gamma) \phi} \frac{1}{1 - \phi} (w - p) a. \tag{A144}$$

**Proof of Proposition 3.4** Note that there are no state variables in the model and therefore, the monetary policy affects allocation only in the first period. I therefore focus on one period. The fact that loss function contains only second-order terms implies that the FOA to pricing block and risk-sharing conditions is sufficient. Assuming that invoicing is symmetric across countries and using $\int_0^1 e_i \, di = 0$, the prices are:

$$p_{ji} = (\mu^P + \mu^D) e_i - \mu^D e_0 - \mu^P e_j,$$

$$p_i^I = (\mu^P + \mu^D) e_i - \mu^D e_0,$$

$$p_i^E = -\mu^D e_0 - \mu^P e_i,$$

$$p_i = \gamma (\mu^P + \mu^D) e_i - \gamma \mu^D e_0,$$

$$p = -\gamma \mu^D e_0.$$
In the absence of global shocks, \( e_i = w_i \) from the international risk-sharing. It follows that

\[
\sigma_P^2 = \gamma \int_0^1 p_j^2 \, dj - p_i^2 = \gamma \int_0^1 \left[ (\mu^P + \mu^D) e_i - \mu^P e_0 - \mu^P e_j \right]^2 \, dj - \gamma^2 \left[ (\mu^P + \mu^D) e_i - \mu^D e_0 \right]^2.
\]

Consider first the case of PCP. As long as prices are fully sticky, PCP allows the monetary authoritiesto implement the first-best:

\[
w_i = e_i = \frac{1}{1 - (1 - \gamma) \phi} a_i.
\]

The value of the loss function is then

\[
\mathcal{L}^{PCP} = -\frac{1}{2} \left[ \frac{\gamma (2 - \gamma) \theta}{1 - \phi} + (1 - \gamma)^2 (1 + \phi) \right] \left( \frac{1}{1 - (1 - \gamma) \phi} \right)^2 \sigma_a^2.
\]

The marginal costs are perfectly stabilized and the currency choice is determined by

\[
\tilde{p}_{ji} + e_{ki} = e_k + (1 - \alpha) (mc_j - e_j) + \alpha (p_i - e_i) = e_k - (1 - \alpha) e_j - \alpha (1 - \gamma) e_i
\]

\[
\Rightarrow \alpha \leq \frac{1}{2 - \gamma}.
\]

Suppose firms choose LCP. Then all prices are fully sticky in currency of destination and the loss function simplifies to

\[
\mathcal{L} = (1 + \phi) \int_0^1 \left[ \frac{1}{2} w_i^2 - \frac{1 - \gamma}{1 - (1 - \gamma) \phi} w_i a_i \right] \, di,
\]

and the FOC is

\[
w_i = e_i = \frac{1 - \gamma}{1 - (1 - \gamma) \phi} a_i.
\]

The value of the loss function under the optimal policy is

\[
\mathcal{L}^{LCP} = -\frac{1}{2} (1 - \gamma)^2 (1 + \phi) \left( \frac{1}{1 - (1 - \gamma) \phi} \right)^2 \sigma_a^2.
\]
Thus, firms choose LCP based on

\[ \tilde{p}_{ji} + e_{ki} = e_k + (1 - \alpha) (m c_j - e_j) + \alpha (p_i - e_i) = \frac{1 - \gamma}{1 - (1 - \gamma) \phi} \left[ a_k - \frac{1 - \alpha}{1 - \gamma} a_j - \alpha a_i \right] \]

\[ \Rightarrow \alpha \geq \frac{1}{2 - \gamma}. \]

Assume next that firms choose DCP. Substitute prices into the loss function and exchange rates instead of wages:

\[ L = \int_0^1 \left[ \frac{1}{2} (1 + \phi) \left( (1 - \gamma) e_i + \gamma e_0 \right)^2 - \frac{1}{1 - \phi} \frac{\gamma (1 - \gamma) \theta}{1 - (1 - \gamma) \phi} e_i a_i \right. \]

\[ - \left. \frac{(1 - \gamma) (1 + \phi)}{1 - (1 - \gamma) \phi} \left( (1 - \gamma) e_i + \gamma e_0 \right) a_i + \frac{\gamma (1 - \gamma) \theta}{1 - \phi} \frac{2}{2} (e_i - e_0)^2 \right] \, di. \]

Integrate and use exchange rate normalization to rewrite it as

\[ L = (1 - \gamma) \left[ \frac{\gamma \theta}{1 - \phi} + (1 - \gamma) (1 + \phi) \right] \left[ \frac{1}{2} \int_0^1 e_i^2 \, di - \frac{1}{1 - (1 - \gamma) \phi} \int_0^1 e_i a_i \, di \right] \]

\[ + \frac{1}{2} (1 + \phi) \frac{\gamma^2 e_0^2}{\phi} + \frac{\gamma (1 - \gamma) \theta}{1 - \phi} \frac{e_0^2}{2}. \]

The FOC with respect to \( e_i \) implies

\[ e_i = \frac{1}{1 - (1 - \gamma) \phi} a_i \]

and the optimal value of dollar is \( e_0 = 0 \). The value of the loss function is then

\[ L^{DCP} = -\frac{1}{2} \left[ \frac{\gamma (1 - \gamma) \theta}{1 - \phi} + (1 - \gamma)^2 (1 + \phi) \right] \left( \frac{1}{1 - (1 - \gamma) \phi} \right)^2 \sigma_a^2. \]

Exporters choose DCP based on

\[ \tilde{p}_{ji} + e_{ki} = e_k + (1 - \alpha) (m c_j - e_j) + \alpha (p_i - e_i) = e_k - (1 - \alpha) e_j - \alpha (1 - \gamma) e_i \]

\[ \Rightarrow \frac{1}{2} \leq \alpha \leq \frac{1}{2 (1 - \gamma)}. \]

Thus, for given parameter values, \( L^{PCP} < L^{DCP} < L^{LCP} \). ■
Proof of Proposition 3.5  Consider the case when the monetary policy of U.S. is exogenous. Since U.S. has zero mass, the objective function of the global planner does not change. Since policy across other countries can be correlated in this case, we obtain
\[ e_i = w_i - w \] (including \( e_0 = w_0 - w \)) under exchange rate normalization \( \int_0^1 e_i \, di = 0 \) and \( w = \int_0^1 w_i \, di \).

The optimal policy for the U.S. does not depend on currency choice or monetary policy of other countries and implies \( w_0 = a_0 \) (since there are no intermediate goods in nontradable sector). Under PCP and LCP, the dollar exchange rate plays no role and the optimal policy and currency choice are the same as under cooperative policy. Assume that firms choose DCP. Substitute prices into the loss function and exchange rates instead of wages (note that \( e_0 \) is a function of endogenous \( w \)):

\[
\mathcal{L} = \int_0^1 \left[ \frac{1}{2} (1 + \phi) ((1 - \gamma) (e_i + w) + \gamma w_0)^2 \right. \\
- \left. \frac{1 - (1 - \gamma) \phi}{1 - (1 - \gamma) \phi} \int (1 - \gamma) (e_i + w) + \gamma w_0 \right] \, di.
\]

The FOC with respect to \( e_i \) and \( w \) imply

\[
e_i = \frac{1}{1 - (1 - \gamma) \phi} a_i,
\]

\[
w = \frac{\gamma [\theta - (1 - \phi^2)]}{\gamma \theta + (1 - \gamma) (1 - \phi^2)} w_0,
\]

\[
e_0 = \frac{1 - \phi^2}{\gamma \theta + (1 - \gamma) (1 - \phi^2)} w_0,
\]

\[
w_i = \frac{1}{1 - (1 - \gamma) \phi} a_i + \frac{\gamma [\theta - (1 - \phi^2)]}{\gamma \theta + (1 - \gamma) (1 - \phi^2)} w_0.
\]

The monetary policies \( w_i \) are therefore positively correlated across countries (including the U.S.). In addition, the volatility of exchange rates against dollar are lower under DCP.
than PCP:

\[ e_{i0}^{DCP} = \frac{1}{1 - (1 - \gamma) \phi} a_i - \frac{1 - \phi^2}{\gamma \theta + (1 - \gamma) (1 - \phi^2)} a_0, \]

\[ e_{i0}^{PCP} = \frac{1}{1 - (1 - \gamma) \phi} a_i - a_0. \]

The loss function is

\[ \mathcal{L} = -\frac{1}{2} \frac{1 - \gamma}{1 - \phi} \left[ \gamma \theta + (1 - \gamma) (1 - \phi^2) \right] \left( \frac{1}{1 - (1 - \gamma) \phi} \right)^2 \sigma_a^2 + \frac{1}{2} \frac{\gamma (1 + \phi) \theta}{\gamma \theta + (1 - \gamma) (1 - \phi^2)} \sigma_w^2, \]

which is higher than losses under PCP by the second term. Firms’ currency choice is based on

\[ p_i = \gamma (e_i - e_0), \]

\[ mc_i = \phi p_i + (1 - \phi) w_i - a_i = \frac{\gamma (1 - \phi) [\theta - (1 + \phi)]}{\gamma \theta + (1 - \gamma) (1 - \phi^2)} w_0 = \gamma \frac{\theta}{1 + \phi} - 1 e_0, \]

\[ \tilde{p}_{ji} + e_{ki} = e_k - (1 - \alpha) e_j - \alpha (1 - \gamma) e_i - \gamma \left[ \frac{1}{1 + \phi} \right] e_0, \]

which given uncorrelated exchange rates implies that DCP is always optimal in the limit \( \gamma, \alpha \to 1. \)

Proof of Proposition 3.6  The PCP equilibrium can be implemented when it exists by specifying the following off-equilibrium policy (the case of DCP is similar). Identify an arbitrary trade flow \( ji \) that is not invoiced in producer currency. If the share of \( e_k \) in the optimal basket \( \tilde{p}_{ji} + e_{ki} \) is not equal zero, then make volatility of \( e_k \) infinitely high. This will make it suboptimal to use currency \( k \) for these exporters. It remains to show that the share of \( e_k \) cannot be exactly zero: for any \( k \neq j, \frac{\partial p_{ji}}{\partial e_k} < 1, \frac{\partial (e_k - p_i)}{\partial e_k} < 1 \) and

\[ \frac{\partial (\tilde{p}_{ji} + e_{ki})}{\partial e_k} = 1 - \left[ (1 - \alpha) \phi \left( -\frac{\partial p_{ji}}{\partial e_k} \right) + \alpha \frac{\partial (e_i - p_i)}{\partial e_k} \right] > 0, \]
which completes the proof. ■
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