DIFFERENCE IDENTITIES
FOR UNEMPLOYMENT RATES

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Abstract

This paper presents some extremely flexible identities useful in analyzing changes in unemployment rates from month to month, from year to year, and over longer periods. An aggregate unemployment rate change is expressed as a polynomial in labor force stocks and first differences in labor force stocks. Terms of this polynomial are interpreted as the effects of (1) changes in the distribution of the labor force among demographic groups; (2) unexpected changes in labor demand within a demographic group; and (3) unexpected changes in labor supply within a demographic group. A simple extension of the framework shows its relationship to recent work with labor force gross flow data. The framework is applied to the increase in black youth unemployment between 1950 and 1970. Most of it may be attributed to a decline in employment among males in the South.
1. Introduction

Policymakers watch unemployment rates very closely. Changes in unemployment rates from month to month, from year to year, and from decade to decade indicate worsening or improving economic conditions in a very broad fashion. Various detailed explanations of long-run trends in unemployment rates have been offered. For example, Perry (1970) noticed that systematic changes taking place in the demographic composition of the population would make the aggregate U.S. unemployment rate increase even if the unemployment rate within each demographic group stayed the same over time. Cogan (1976), in a paper focusing on employment-population ratios, identifies the disappearance of agricultural employment since World War II as the chief factor in the deterioration of the labor market for black youths. Many other factors, including trends in labor force participation, layoff policies, and discouraged worker behavior can influence unemployment rates.

How much of a given change in an unemployment rate should we attribute to each of several factors? For example, according to decennial census data,¹ the unemployment rate for blacks ages 14 to 24 rose 2.3%, to 14.5%, between 1950 and 1970. How much of this change can we attribute to each of various factors measured
by published data for the two census years? This paper presents
an extremely flexible framework for attacking such questions.
Section 2 develops basic notation for manipulating labor force
stock data. Section 3 presents basic identities for changes in
the unemployment rate within a subgroup. Section 4 extends the
identities used by Perry and others to examine the influence of
changes in demographic composition as well as changes in
within-group unemployment rates. Section 5 extends the analysis
in two important ways. Finally, Section 6 gives an example of
empirical use of the basic framework.
2. Notation

Estimates of labor force variables for various timeperiods are available for the entire group under consideration—for example, the civilian noninstitutional population age 16 and over—and for various subgroups by age, sex, race, region, and so on. Let $G$ be an index set for these subgroups, so that the number of people in the entire group during period $t$ is

$$I(t) = \sum_{g \in G} I^g(t).$$

Three basic labor market states—employment, unemployment, and nonparticipation in the labor force—partition $I$ and each of its subgroups. If within each subgroup we denote the number of people in each of these labor market states by $M^g$, $U^g$, and $N^g$, respectively, we have

$$M^g(t) + U^g(t) + N^g(t) = I^g(t)$$

for the subgroups and

$$M(t) + U(t) + N(t) = I(t)$$
for the entire group. Expressions analogous to (1) hold for $N$, $U$, and $N$, as well as for $I$.

Often we wish to distinguish among substates of the basic labor market states—for example, agricultural and nonagricultural employment; layoff and non-layoff unemployment; or discouraged workers and other nonparticipants in the labor force. If we index the substates of employment with set $J$, the substates of nonparticipation with $K$, and the substates of unemployment with $H$, we have

\[(4) \quad N^g(t) = \sum_{j \in J} N_j^g(t),\]

\[(5) \quad N^g(t) = \sum_{k \in K} N_k^g(t), \text{ and}\]

\[(6) \quad U^g(t) = \sum_{h \in H} U_h^g(t).\]

Dropping the $g$ superscripts from (4), (5), and (6) produces similar expressions which hold for the group as a whole.

Often it will be convenient to have a shorthand expression for the sum of the number of unemployed and the number of employed people in group $g$. Since this sum is the number of participants in the labor force, let

\[(7) \quad P^g(t) = N^g(t) + U^g(t), \text{ and let}\]

\[(8) \quad P(t) = M(t) + U(t)\]
be the number of labor force participants in the group as a whole. Another expression analogous to (1) holds for \( P \), since it holds for the components of \( P \).

A final bit of notation we will need is for the **backward difference operator**, \( B \):

\[
(9) \quad BX = X(t) - X(t-1),
\]

where \( X \) may be an unemployment rate, any of the labor force variables discussed in this section, or a more complex expression.
3. The Unemployment Rate Within a Subgroup

The unemployment rate for subgroup $g$ during period $t$ is

$$(10) \quad u^g(t) = \frac{U^g(t)}{(M^g(t) + U^g(t))},$$

or, using (8),

$$(11) \quad u^g(t) = \frac{U^g(t)}{P^g(t)}.$$

The change in the unemployment rate from period $t-1$ to period $t$ is simply

$$(12) \quad B u^g = \frac{U^g(t)}{P^g(t)} - \frac{U^g(t-1)}{P^g(t-1)} = \frac{P^g(t-1)U^g(t) - P^g(t)U^g(t-1)}{P^g(t)P^g(t-1)}.$$

Adding and subtracting the term $P^g(t-1)U^g(t-1)$ in the numerator and rearranging gives us

$$(13) \quad B u^g = \frac{P^g(t-1)B U^g - U^g(t-1)B P^g}{P^g(t)P^g(t-1)}.$$
Substituting $BM^g + BU^g$ for $BP^g$ and $M^g(t-1) + U^g(t-1)$ for $P^g(t-1)$ yields, after some simplification,

(14) \[ Bu^g = (p^g(t)p^g(t-1))^{-1}(M^g(t-1)BU^g - U^g(t-1)BM^g). \]

When we wish to distinguish among substates of unemployment and employment in decomposing changes in the unemployment rate, we may substitute into (14) the backward differences of (4) and (6) to get

(15) \[ Bu^g = (p^g(t)p^g(t-1))^{-1} \]
\[ \quad \left( M^g(t-1) \sum_{h \in H} BU^g_h - U^g(t-1) \sum_{j \in J} BM^g_j \right). \]

Taking the backward difference of (2) and rearranging gives us

(16) \[ BU^g = BI^g - BM^g - BN^g. \]

Substituting (16) into (14) and simplifying permits us to eliminate $BU^g$ from our decomposition:

(17) \[ Bu^g = (p^g(t)p^g(t-1))^{-1} \]
\[ \quad \left( M^g(t-1)BI^g - P^g(t-1)BM^g - M^g(t-1)BN^g \right). \]
Substituting the backward differences of (4) and (5) gives us

\[(18) \quad B u^8 = (P^8(t)P^8(t-1))^{-1} \]
\[\quad (M^8(t-1)B I^8)\]
\[\quad - P^8(t-1) \sum_{j \in J} B M^8_j \]
\[\quad - M^8(t-1) \sum_{k \in K} B N^8_k \). \]

Identities (14) and (15) decompose changes in unemployment rates into amounts due to changes in the number of employed and amounts due to changes in the number of unemployed. Identities (17) and (18) decompose unemployment rate changes into amounts due to changes in the number of employed, changes in the number of nonparticipants, and changes in the size of the subgroup. The last identity may be transformed to eliminate the change in subgroup size as a separate explanatory factor. Adding in and then subtracting out some terms in $B I^8$ yields, after simplification,

\[(19) \quad B u^8 = (P^8(t)P^8(t-1))^{-1} \]
\[\quad ( - P^8(t-1)( \sum_{j \in J} (B M^8_j - (M^8(t-1)/I^8(t-1))B I^8_j))) \]
\[\quad - M^8(t-1)( \sum_{k \in K} (B N^8_k - (N^8(t-1)/I^8(t-1))B I^8_k)) \).

The terms after the summation signs represent unexpected in-
creases in the sizes of the substates; the expected increase is simply the fraction last period times the increase in the size of the subgroup.

When only the basic labor market states of employment and nonparticipation are of interest, identity (19) reduces to

\[
(20) \quad B_u^g = (P_u^g(t)P_u^g(t-1))^{-1} \\
( - P_u^g(t-1)(BM_u^g - (N_u^g(t-1)/I_u^g(t-1))BI_u^g) \\
- N_u^g(t-1)(BN_u^g - (N_u^g(t-1)/I_u^g(t-1))BI_u^g) ).
\]

Equations (19) and (20) are basic identities for interpreting changes in unemployment rates. Within a subgroup of the entire population, these identities decompose a change in the unemployment rate from one period to any later period into two parts. The term involving BM encompasses the influence of changes in demand, while the term involving BN captures the influence of changes in supply. The equations adjust both demand and supply influences on unemployment rates for those changes which are merely proportional to population growth within the subgroup.
4. The Aggregate Unemployment Rate

The identities derived in the last section decompose within-group changes in unemployment rates into supply and demand components. However, an aggregate unemployment rate changes not only because the unemployment rates of subgroups change, but also because the distribution of the subgroups changes. In this section, an aggregate unemployment rate change is decomposed into sums of within-group changes and distributional changes.

First, notice that the aggregate unemployment rate is

\[(21) \quad u(t) = U(t)/(M(t)+U(t)) = (P(t))^{-1} \sum_{g \in G} u^g(t) \]

\[= \sum_{g \in G} (u^g(t)/P^g(t))(P^g(t)/P(t)) \]

\[= \sum_{g \in G} u^g(t)w^g(t). \]

That is, the aggregate unemployment rate is a weighted sum of the unemployment rates for the subgroups. The weights are the subgroup proportions of aggregate labor force participation:
(22) \( \hat{w}^g(t) = \hat{p}^g(t)/P(t) = (M^g(t) + U^g(t))/(M(t) + U(t)) \).

Changes in the aggregate unemployment rate are simply the backward differences of the last member of (21):

(23) \( B_u = \sum_{g \in G} (u^g(t)w^g(t) - u^g(t-1)w^g(t-1)) \).

Adding and then subtracting an appropriate term produces

(24) \( B_u = \sum_{g \in G} (u^g(t)w^g(t) - u^g(t)w^g(t-1) + u^g(t)w^g(t-1) - u^g(t-1)w^g(t-1)) \)

\[ = \sum_{g \in G} (u^g(t)Bw^g + w^g(t-1)Bu^g) \).

The last member of (24) decomposes an aggregate unemployment rate change into the sum of two terms. The first term is the effect of changes in the distribution of the population among subgroups; these changes are weighted by current-period unemployment rates for the subgroups. The second term is the effect of within-group changes in unemployment rates; these changes are weighted by shares of aggregate participation last period.

Any of the expressions for \( Bu^g \) derived in Section 3 may be substituted into (24) to yield extremely fine partitions of
changes in aggregate unemployment rates. For example, plugging in expression (20) produces

\[
Bu = \sum_{g \in G} (u^g(t)Bw^g \\
+ w^g(t-1) \\
\quad c_N(E_{t-G}(N^g(t-1)/I^g(t-1))B1^g)) \\
+ w^g(t-1) \\
\quad c_N(E_{t-G}(N^g(t-1)/I^g(t-1))B1^g),
\]

where

\[
c_M = -(P^g(t))^{-1}
\]

and

\[
c_N = -(N^g(t-1)/(P^g(t)p^g(t-1)))
\]

are coefficients on the adjusted demand and supply terms in conjunction with the labor force weights from the earlier period.

Expression (25) uses much of the apparatus we've developed so far. It is a powerful tool for disaggregating changes in unemployment rates. If the population being analyzed consists of \(n_G\) subgroups, expression (25) separates a change in the aggregate unemployment rate into \(3n_G\) components. Each subgroup accounts for three of these components. The first term after the summation sign involves \(Bw^g\) and shows in a sense how much of the change in the aggregate unemployment
rate may be attributed to changes in group $g$'s share of the population. If $Bw^g$ had been zero, that is, if the share had not changed, this term would have been zero. For a given value of $Bw^g$, the absolute value of this term is greater the greater is the unemployment rate within subgroup $g$. In other words, (25) confirms our intuition that the relative population growth of a subgroup experiencing a higher unemployment rate affects the aggregate rate more than the relative population growth of a subgroup experiencing a lower unemployment rate.\textsuperscript{5}

The second term of (25) involves $(BM^g-(M^g(t-1)/I^g(t-1))BI^g)$. In the last section, we called this factor the unexpected change in the number of people employed. It shows the change in $M^g$ relative to the change in $I^g$. Since $c_M$ is negative and $w^g(t-1)$ is positive, this term is opposite in sign to $(BM^g-(M^g(t-1)/I^g(t-1))BI^g)$. So, according to this term, if employment within a subgroup increases by more than its share of the growth of the subgroup, the aggregate unemployment rate falls.

The third term of (25) involves $(BN^g-(N^g(t-1)/I^g(t-1))BI^g)$. Since $N^g$ is the complement of labor force participation in subgroup $g$, $(BN^g-(N^g(t-1)/I^g(t-1))BI^g)$ captures changes in labor supply relative to changes in population. Since $w^g(t-1)$ always is positive and $c_N$ always is negative, the third term of (25) is opposite in sign to $(BN^g-(N^g(t-1)/I^g(t-1))BI^g)$. When labor force participation increases by more than its proportionate share of increases in the population, $(BN^g-(N^g(t-1)/I^g(t-1))BI^g)$ is negative and the term describes an increase in the aggregate
unemployment rate. Once more expression (25) accords with our intuition about the sources of changes in aggregate unemployment rates.
5. Two Extensions of the Framework

This section of the paper extends the basic framework to accommodate gross change data on the labor force and to deal with populations divided into nested subgroups.

So far, the identities we have developed require net change data on the labor force rather than the gross change data which is the subject of much recent controversy among labor economists. Yet, as Smith and Vanski (1979, p. 143) and others have shown, the relationship between net and gross changes in the labor force from month to month is quite simple. Let $n_{xy}^8$ be the estimate of the number of people moving from state $x$ to state $y$, with $x$ and $y$ any elements of the set $(M, U, N)$. Then the net change in state $x$ is simply the difference between the number of people entering that state from either of the other two states and the number of people leaving that state for either of the other two states. That is,

\begin{align}
\text{(28)} \quad B_{M}^8 &= n_{UM}^8 + n_{NM}^8 - n_{MU}^8 - n_{NN}^8, \\
\text{(29)} \quad B_{U}^8 &= n_{MU}^8 + n_{NU}^8 - n_{UM}^8 - n_{UN}^8,
\end{align}

and
(30) \( B_N^g = n_N^{gN} + n_U^{gU} - n_N^{gM} - n_U^{gM} \).

Substituting these identities into (14) or (17) produces decompositions of changes in a subgroup's unemployment rate with four times as many explanatory terms as before. These expressions may then be used in the last member of (24) to decompose a change in the aggregate unemployment rate.

Another useful manipulation of the identities allows them to accommodate labor force data for nested subgroups. Suppose the set \( G \) of subgroups may be partitioned further, so that each element of \( G \) is itself a group, \( R^g \). (In the empirical example presented in the next section, \( R^g \) is a region, and each region consists of two sexes.) Then by analogy with (23), within each subgroup \( g \)

\[
(31) \quad Bu^g = \sum_{r \in R^g} (u^r(t)w^r(t) - u^r(t-1)w^r(t-1)),
\]

where

\[
(32) \quad w^r(t) = p^r(t)/p^g(t).
\]

A derivation analogous to that which turned (23) into (24) changes (31) into

\[
(33) \quad Bu^g = \sum_{r \in R^g} (u^r(t)Bw^r + w^r(t-1)Bu^r).
\]

Substituting (33) into the last member of (24) produces a decomposition of aggregate unemployment rate changes into factors associated with nested subgroups:
(35) \[ B_u = \sum_{g \in G} u^g(t)Bw^g \]
\[ + \sum_{r \in R^g} (w^g(t-1)u^r(t)Bw^r \]
\[ + e^r_M(B^r_M - (M^r(t-1)/I^r(t-1))B1^r) \]
\[ + e^r_N(B^r_N - (N^r(t-1)/I^r(t-1))B1^r) \), \]

where

(36) \[ e^r_M = -w^g(t-1)w^r(t-1)p^r(t-1)/(p^r(t)p^r(t-1)) \]

and

(37) \[ e^r_N = -w^g(t-1)w^r(t-1)n^r(t-1)/(p^r(t)p^r(t-1)) \). 

Expression (35) is the basis for the empirical work reported in the next section, and it is discussed further there. However, notice that the same process which permitted one level of nesting of subgroups may be repeated. Just as each member \( g \) of \( G \) may be taken as itself a group, \( R^g \), each member \( r \) of \( R^g \) may be taken as some group \( S^g \). An expression analogous to (35) may be derived for two levels of nested subgroups, and we need not stop there. The process may be repeated indefinitely, limited only by the level of disaggregation available in the labor force data we wish to analyze.
6. An Empirical Application

Published labor force data from the 1950 and 1970 censuses are disaggregated by race, region, and sex. According to these data, the unemployment rate for blacks ages 14 through 24 rose 2.3% over the period, from 12.2% to 14.5%. As a simple illustration of the use of our framework, let us decompose this change in the black youth unemployment rate.

Let \( G = (NE, NC, S, W) \) be the Census regions; let \( g^X \) be the number of black males 14 to 24 in region \( g \), and let \( g^Y \) be the number of females. Thus \( R^{NE} = (NE^X, NE^Y) \), and so on for the other regions. Adapted for this data, Expression (35) becomes

\[
(38) \quad B_u = u^{NE}(70)B_w^{NE} + w^{NE}(50)u^{NEX}(70)B_w^{NEX} \\
+ c_N^{NEX}(BNEX - NEX^{(50)}I^{NEX}(50)BI^{NEX}) \\
+ c_M^{NEX}(BMEX - MEX^{(50)}I^{NEX}(50)BI^{NEX}) \\
+ w^{NEX}(50)u^{NEY}(70)B_w^{NEY} \\
+ c_N^{NEY}(BN^EY - N^EY^{(50)}I^{NEY}(50)BI^{NEY}) \\
+ c_M^{NEY}(BM^EY - M^EY^{(50)}I^{NEY}(50)BI^{NEY})
\]
\[ + u^{NC}(70)Bw^{NC} \]
\[ + w^{NC}(50)u^{NCX}(70)Bw^{NCX} \]
\[ + c_N^{NCX}(BN^{NCX}(50)/INCX(50))BINCX \]
\[ + c_M^{NCX}(BM^{NCX}(50)/INCX(50))BINCX \]
\[ + w^{NC}(50)u^{NCY}(70)Bw^{NCY} \]
\[ + c_N^{NCY}(BN^{NCY}(50)/INCY(50))BINARY \]
\[ + c_M^{NCY}(BM^{NCY}(50)/INCY(50))BINARY \]
\[ + u^S(70)Bw^S \]
\[ + u^S(50)u^{SX}(70)Bw^{SX} \]
\[ + c_N^{SX}(BN^{SX}(50)/ISX(50))BISX \]
\[ + c_M^{SX}(BM^{SX}(50)/ISX(50))BISX \]
\[ + w^S(50)u^{SY}(70)Bw^Y \]
\[ + c_N^{SY}(BN^{SY}(50)/ISY(50))BISY \]
\[ + c_M^{SY}(BM^{SY}(50)/ISY(50))BISY \]
\[ + u^W(70)Bw^W \]
\[ + w^W(50)u^{WX}(70)Bw^{WX} \]
\[ + c_N^{WX}(BN^{WX}(50)/IWX(50))BIWX \]
\[ + c_M^{WX}(BM^{WX}(50)/IWX(50))BIWX \]
\[ + w^W(50)u^{WY}(70)Bw^WY \]
\[ + c_N^{WY}(BN^{WY}(50)/IWY(50))BIWY \]
\[ + c_M^{WY}(BM^{WY}(50)/IWY(50))BIWY \].

The coefficients \(c^n_N\) and \(c^n_N\) were defined in (36) and (37).
Expression (38) is the sum of twenty-eight terms, each of which contains just one backward difference in one of the explanatory variables. There are seven terms for each of the four regions; the first seven pertain to the NE region. The first term involves \( B_{\text{NE}} \), the change in the NE region's share of all blacks 14 to 24. The second involves \( B_{\text{nex}} \), the change in the male cell's share of the NE region. The third involves the unexpected change in nonparticipation among NE black males; the fourth, the unexpected change in employment. Terms five through seven are analogous to terms two through four, but they pertain to NE females rather than NE males.

The most important property of (38) is that each explanatory factor appears just once, and it appears in a separate term. Its coefficient is the effect on the aggregate unemployment rate of a marginal change in it. Removing the influence of any one factor entails setting one term of (38) to zero and adding up the new values of the remaining twenty-seven terms. Should keeping one factor constant have no effect on changes in any of the other factors, the constant factor's term in (38) gives its entire impact on the aggregate unemployment rate.

For example, consider the effect of the NE region's share of the black population 14 to 24. This share rose 6.06% over the period, from 12.35% in 1950 to 18.41% in 1970. Had this share not changed, how much larger or smaller would the aggregate unemployment rate for young blacks have been? Expression (38) provides the answer once some assumption is made about the
relationships of the changes in the twenty-eight variables to each other. Ignoring any impact of a zero change in $w^{NE}$ on any of the other variables, the first term of (38) tells us that the 1970 unemployment rate would have been 0.72% lower. 0.72% is the 1970 NE unemployment rate, 11.96%, times the 6.06% region share change. Table la contains the simple arithmetic for this term and for the twenty-seven other terms of (38).

However, changing the 6.06% shift in $w^{NE}$ to a counterfactual zero requires us to change the shifts in the other regions' shares. Since shares must sum to unity, changes in shares must sum to zero. If we assume that a zero change in $w^{NE}$ is matched by a zero change in all the other region shares, Table la still gives us the answer to our question. However, we must add up all the region terms to reach the conclusion that the black youth unemployment rate would have been 0.41% lower.9

Of course, we cannot quite stop here, either. Had population shifts between the regions not occurred precisely as they did, the unemployment rates within those regions, and the components of those regional unemployment rates, would have been different, too. Identity (20) shows quite precisely how the ability or inability of a region to create jobs as fast as its population grows affects unemployment rates. If nonparticipation grows faster than the population, failure to achieve net increases in employment will be offset somewhat. However, faster population growth makes it more difficult for employment to keep pace and thus tends to increase
### Table 1a
Sources of the Change in the Black Youth Unemployment Rate, 1950 to 1970, Using Decennial Census Data

<table>
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<th>OBS</th>
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<th>REGION</th>
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<th>SEX</th>
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<th>AGE</th>
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### Table 1b
Descriptive Statistics for Amounts Explained by Regions

<table>
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<th>N</th>
<th>MEAN</th>
<th>STANDARD DEVIATION</th>
<th>C.V.</th>
<th>MINIMUM VALUE</th>
<th>MAXIMUM VALUE</th>
<th>RANGE</th>
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<td>0.00000000</td>
<td>0.00000000</td>
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<tr>
<td>SUPPLY</td>
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<td>0.00000000</td>
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</table>

### Table 1c
Descriptive Statistics for Amounts Explained by Cells

<table>
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<th>STANDARD DEVIATION</th>
<th>C.V.</th>
<th>MINIMUM VALUE</th>
<th>MAXIMUM VALUE</th>
<th>RANGE</th>
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</thead>
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<td>0.2685102</td>
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</table>
unemployment rates. Had the South not lost as much population share as it did, we'd expect unemployment rates to have been worse there in 1970. Had the other regions not gained as much population as they did, we'd expect unemployment rates to have been lower there. So the coefficients of Table 1a are missing important interaction effects. ¹⁰

However, even an analysis taking interaction into account would be hard pressed to alter the qualitative content of the extraordinary Southern male coefficients in the table. If nonparticipation among them had not grown faster than the population of Southern black males, the aggregate unemployment rate for blacks 14 to 24 would have been an astounding 24.9% higher. However, the unemployment-rate-lowering effect of this dropping out among Southern black males was more than offset by the failure of job growth to keep up with even their rather anemic net population growth. ¹¹ The net impact on the aggregate unemployment rate of supply and demand for Southern black male labor, after correcting for population share effects, is an increase of 1.96 percentage points. This is 85% of the 2.3% change in the aggregate rate we wish to explain.

A closer look at the change in the unemployment rate among Southern black males is warranted. Between 1950 and 1970, this rate rose 4%, to 11.7%. Identity (20) permits us to decompose this 4% increase. The absolute number of Southern black males rose 317,255, from 891,224 to 1,208,479. Employment actually declined by 85,667, from 511,986. We would have expected an
employment increase of \((511,986 / 891,224)\) times 317,255, or 182,255. The difference, 267,922, is divided by the 1970 labor force, 482,813, to produce a demand coefficient of 0.554919.

Nonparticipation among Southern black males rose by 388,823, from 336,843 in 1950. We would have expected an increase of \((336,843 / 891,224)\) times 317,255, or 119,908.24. In accordance with (20), the difference between actual and expected nonparticipation is multiplied by \(-511,986 / (482,813 \times 554,381)\) to produce a supply coefficient of \(-0.514382\). The sum of the demand and supply coefficients is the 4% increase in the Southern black male unemployment rate. Expression (38) shows how regional and sex weights reduce these large coefficients to produce the Southern black male coefficients in Table 1a for the aggregate unemployment rate.

Cogan (1982) argued that the decline in agricultural employment among Southern black males is crucial in explaining the declining ratio of employment to population among black males. The preliminary results presented here are broadly consistent with his findings and may be viewed as extending them to deal with the rise in the black youth unemployment rate.
7. Summary

We have developed some extremely flexible identities useful in analyzing changes in unemployment rates. These identities are polynomials in labor force stocks and first differences in labor force stocks. Terms of these polynomials are interpreted as the effects of three kinds of explanatory factors. First, changes in the distribution of the labor force among demographic groups, and among subgroups of these demographic groups, might change aggregate unemployment rates even if within-group rates were steady. Second, the failure of employment to grow as fast as the population within a demographic group will increase its unemployment rate and the aggregate rate as well. Third, when members of a demographic group drop out of the labor force more rapidly than the population of that group grows, group and aggregate unemployment rates fall. There is reason to believe that the second and third factors are highly correlated for each group.

We applied the framework to the 2.3% growth from 1950 to 1970 in census unemployment rates for blacks 14 to 24. After correcting for the effects of regional population shifts and of
changes in the male-female labor force ratio within each region, the behavior of Southern black males seemed to account for most of the change in the aggregate unemployment rates. Though our results are preliminary because of interaction effects not taken into account\textsuperscript{12}, they seem to confirm and extend Cogan's (1982) findings on the importance of Southern male employment in explaining aggregate black youth labor market behavior.
Notes


2. Perry (1970) is an influential paper which emphasized this point. See also Flaim (1979).

3. Expression (24) results from adding and subtracting the term

\[ w^8(t) - w^8(t-1). \]

If instead the term

\[ w^8(t-1) - w^8(t) \]

is added and subtracted, a set of identities similar to those presented below will result. However, such identities would use current-period population shares and previous-period unemployment rates as weights wherever the identities presented in this paper use previous-period population shares and current-period unemployment rates.

4. See Section 6 of the paper for some caveats about interaction effects ignored in some interpretations of our identities.

5. For example, in 1970 the Census unemployment rate for blacks 14 to 24 was 19.5% in the West and 12.0% in the Northeast. A given percentage change in labor force share would have had more effect on the aggregate unemployment rate for blacks 14 to 24 if it involved the West than if it involved the Northeast.

6. See Smith and Vanski (1979) for a thorough description of these data.

7. For example, Poterba and Summers (1984) argue that the labor market is more stable than others who have interpreted these data have suggested.

Of course, other patterns could be devised to keep the total share change at zero; all region changes need not have been zero. For example, the North-east's share increase could have been apportioned equally among the other three regions.

See Flaim (1979) for a good discussion of the importance of the interaction between labor force share and unemployment rate.

Cave (1983) gives a theoretical framework for thinking about the influence of unemployment rates on an individual's decision whether to participate in the labor force or drop out. We'd expect decreases in the number of jobs available to be associated strongly with increased dropping out of the labor force because of the discouraged worker effect.

One way to deal with the interaction problem might be to use a time series dataset with more frequent observations than the Census gives us, and to estimate empirical interactions among the explanatory factors using observations for the intervening months or years.
References


