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**UNEMPLOYMENT, DISEQUILIBRIUM, AND THE SHORT RUN PHILLIPS CURVE:
AN ECONOMETRIC APPROACH***

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ABSTRACT

Unemployment, Disequilibrium, and the Short Run Phillips Curve: An Econometric Approach

The paper specifies a disequilibrium model for the aggregate labor market consisting of demand and supply functions for labor, an adjustment equation for wages as well as for prices, a transactions equation and, finally, an equation that relates measured unemployment to vacancies and to excess demand. The model has a more sophisticated treatment of dynamics than earlier disequilibrium models, and uses measured unemployment as an endogenous variable. Two of the error terms are assumed to be serially correlated and the coefficients are estimated by maximum likelihood. The parameter estimates and the goodness-of-fit are satisfactory and the model's implications for the behavior of several important variables are sensible. Excess demand estimates computed in various ways are reasonable. The model is used to estimate the natural rate of unemployment as well as a short run Phillips curve. Finally, the stability properties of the model are analyzed by considering the eigenvalues of the system; they are found to have moduli less than one.

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1. Introduction

Two major approaches to the study of unemployment have been followed. The equilibrium model assumes that real wage rates instantaneously adjust so as to bring the supply and demand of labor into equality. Unemployment is a consequence of erroneous expectations, and/or intertemporal labor supply substitution, and/or government policies such as unemployment insurance.¹ In contrast, the disequilibrium model allows for the possibility that the real wage may fail to equate the supply and demand of labor. Unemployment occurs when the quantity of labor supplied exceeds quantity demanded at the going real wage. Typically, in these models it is assumed that the observed quantity of labor during any given period is the minimum of the quantity demanded and the quantity supplied.

In the United States, most empirical work has followed the equilibrium paradigm. Part of the reason may be ideological--failure of markets to clear is generally viewed as concomitant with the failure of some agents to optimize, a notion that is heretical according to the neoclassical religion. Certainly, the inability to articulate a compelling choice-theoretic explanation for real wage stickiness is a problem for disequilibrium advocates.² Moreover, disequilibrium models are much more difficult to deal with computationally. Standard econometric software packages generally cannot be used, and difficult nonlinear estimation problems often emerge. As a consequence, disequilibrium models that are quite simple from an economic standpoint are quite complicated computationally. Hence, compared to their equilibrium counterparts, disequilibrium models appear rather unsophisticated.

Proponents of disequilibrium models are apt to point out that despite difficulties in explaining precisely why the labor market does not clear at every moment in time, the real world does seem to be like that, and this fact

should be reflected in economic analysis. As Rees [1970, p. 234] observes,

"Although we know very little about the exact nature of the costs of making wage changes, we can infer that they exist. Wages are, next to house rents, the stickiest general class of prices in the economy, seldom adjusted more frequently than once a year. This stickiness may be reinforced by unionism and collective bargaining, but it was present long before unions arrived."

In this paper we do not attempt to settle the equilibrium versus disequilibrium controversy.³ Our goal is rather to put the two types of models on a more equal standing by estimating an economically richer disequilibrium model than has hitherto been studied.

The model, which is presented in Section 2, relates measured unemployment rates to the "true" excess supply of labor, has relatively sophisticated nominal wage and price dynamics, and allows estimation of the short run inflation-unemployment trade-off. Estimation issues are discussed in Section 3. Among the econometric features of the analysis are the presence of corrections for autocorrelation and the inclusion of non-linearities in the variables. Both of these are difficult to deal with in a disequilibrium context. The model is estimated with annual U.S. data for the period 1929-1979, and the results are analyzed in Section 4. Section 5 concludes with a summary and suggestions for future research.

2. The Model

We consider a system of six equations, one each for the marginal productivity of labor, the supply of labor, the observed quantity of labor, nominal wages, the price level, and the vacancy-unemployment rate relationship. The deterministic version of each equation is discussed in turn. We defer until Section 3 the matter of stochastic specification.

Marginal productivity of labor. Profit maximizing behavior by firms conditional on output leads to a demand function for labor of the form

$$D_t = D(W_t/P_t, Q_t, Q_{t-1}, t) ,$$

where D_t is quantity demanded in year t , (W_t/P_t) is the real wage, Q_t is real output, and t is a time trend which proxies for technical progress. Lagged output is included to allow for the possibility of slow adjustment to changes in the level of output.⁴ D_t is the notional demand in the sense that it is the amount of labor that firms desire to employ at wage (W_t/P_t) --not necessarily the quantity they will end up hiring.

For purposes of estimation, a log-linear approximation (except for t) is employed:

$$\ln D_t = \alpha_0 + \alpha_1 \ln(W_t/P_t) + \alpha_2 \ln Q_t + \alpha_3 t + \alpha_4 \ln Q_{t-1} . \quad (2.1)$$

Formulation (2.1) (or a minor variant) is a common starting point for studying labor markets (see, for example, Lucas and Rapping [1970], Rosen and Quandt [1978], Romer, [1981], Smyth [undated], and Hajivassiliou [1983]).

Nevertheless, ideally one would want to study a multi-market model in which output was treated econometrically as an endogenous variable. This task is beyond the scope of the current study, and for tractability it will be assumed that output is exogenous.

Supply of labor. The total number of manhours supplied in year t depends upon the real net wage, (W_{nt}/P_t) , and the potential labor force, H_t , which is essentially a scale variable to capture the effect of population growth. To the extent that labor supply adjusts slowly to changes in the net wage, its lagged value will also matter. Again assuming a log-linear specification:

$$\ln S_t = \beta_0 + \beta_1 \ln(W_{nt}/P_t) + \beta_2 \ln H_t + \beta_3 \ln(W_{nt-1}/P_{t-1}) \quad (2.2)$$

where S_t denotes notional supply.⁵ The basic theory of labor supply suggests that non-labor income belongs in equation (2.2). However, Romer [1981] points out that unearned income is endogenous in a life-cycle model of labor supply determination, and shows that more sensible results can be obtained when it is omitted. Equation (2.2) is very simple in that it ignores the possible role of intertemporal labor supply substitution. Dealing with this problem rigorously requires careful modelling of future wage expectations, a task that is beyond our scope. (See Altonji [1982].) Note, however, that some simple expectational models imply that the lagged wage as well as its current value appear in the supply equation; this is just the case in equation (2.2).

Observed quantity of labor. In an equilibrium model, the observed quantity of labor is determined by the intersection of the supply and demand curves. In a disequilibrium model, this is not so. In conformity with most of the work in disequilibrium theory, we assume that the quantity observed is the minimum of the quantities supplied and demanded at the current wage:

$$L_t = \min (S_t, D_t) . \quad (2.3)$$

Clearly, eq. (2.3) does not describe completely what is presumably a very complicated rationing story, in which some submarkets have excess demand and some have excess supply. (See Hajivassiliou [1983].) However, the simple "min condition" helps keep the problem tractable, and we think that it is a reasonable approximation.⁶

Taken together, eqs. (2.1), (2.2), and (2.3) form the bare bones disequilibrium model. Such equations have been estimated by several investigators, often with the addition of a wage adjustment equation which makes the change in real wages some function of excess demand. (See, for example, Rosen and Quandt [1978].) These studies have demonstrated the

computational feasibility of disequilibrium estimation, and indicated the promise that such models have for explaining the time series data.

Still, these models suffer from several deficiencies: (1) They do not show how official unemployment rates and the history of wage and price changes might affect current nominal wages rates. (2) They assume that product prices are exogenous. In particular, past changes in nominal wages exert no impact on the current price level. (3) They ignore information in official unemployment rates that might be exploited to help estimate the excess demand for labor. The remaining equations of the model remedy these problems.

Nominal wage adjustment. We assume that this year's nominal wage rate depends upon the last two years' nominal wages, upon this year's official unemployment rate (U_t), and upon recent changes in prices:

$$\begin{aligned} \ln W_t = & \gamma_0 + \gamma_1 \ln W_{t-1} + \gamma_2 U_t + \gamma_3 (\ln P_t - \ln P_{t-1}) \\ & + \gamma_4 (\ln P_{t-1} - \ln P_{t-2}) + \gamma_5 \ln W_{t-2} \end{aligned} \quad (2.4)$$

Lagged wages are included because of the possibility that adjustment to new wage levels is sluggish. The presence of the official unemployment rate reflects the possibility that when the labor market is slack (high U_t), then nominal wages will be lower, ceteris paribus, and vice versa. Now, it is well known that unemployment as measured in the official statistical series does not correspond well to the theoretical notion of unemployment as the inability to find work at the going wage. Why not, then, include excess demand, $(\ln D_t - \ln S_t)$, rather than U_t ? The choice of U_t reflects the fact that workers and employers do not know $(\ln D_t - \ln S_t)$; they have to rely on their perceptions of the labor market situation, and these are well-measured by U_t . We, in fact, estimated some models with excess demand instead of official unemployment. Such a model is nonnested

with respect to our canonical model and difficult to compare explicitly. We note, however, that the estimated parameters are somewhat less plausible and the predicted excess demands are substantially less plausible (e.g. the model predicts excess supply in World War II). We infer that depriving the model of data on U_t and of eq. (2.4) seriously weakens its ability to capture the underlying process.

Lagged prices are included because of the expectation that workers' nominal wage requests will depend on the extent of recent price changes. Ceteris paribus, higher prices will result in high nominal wages. (Indeed, in much of the union sector, indexing is formally built into wage contracts.) The particular lag structure embodied in (2.4) was selected after some experimentation with other formulations. As usual, theory does not give much guidance with respect to the pattern of lags, and eq. (2.4) was superior to several alternatives in the sense of leading to the best fit to the data. Interestingly, in the versions with alternative lag patterns, the behavior of the rest of the system did not change substantially. This is very comforting in light of the fact that highly nonlinear systems often generate results that are not robust with respect to minor changes in specification.

Price adjustment. The price level this period depends upon the lagged price level and the recent history of nominal wage changes:

$$\begin{aligned} \ln P_t = & \delta_0 + \delta_1 \ln P_{t-1} + \delta_2 (\ln W_t - \ln W_{t-1}) + \delta_3 (\ln W_{t-1} - \ln W_{t-2}) \\ & + \delta_4 (\ln PF_t - \ln PF_{t-1}) , \end{aligned} \quad (2.5)$$

where PF_t is the price of energy. The logic here is very similar to that of (2.4). The lagged price term reflects sluggishness in the price adjustment process. Lagged nominal wages are included because producers take factor costs into account when setting their prices. (Such behavior is consistent

with, for example, simple mark-up models of pricing behavior.) Energy prices are included to allow for the impact of oil price shocks. There are other variables that one might want to include in a price equation--e.g., exchange rates--but as Gordon [1982] notes, it is difficult to obtain the necessary data for the pre-World War II period. Note that because we do not attempt to model disequilibrium in the goods market, excess demand does not appear in (2.5).

Vacancy-unemployment relationship. Let V_t be the vacancy rate in year t and U_t be the official unemployment rate, both measured as fractions. Ignore for the moment that U_t does not measure correctly the discrepancy between the amount of labor supplied and the amount workers desire to supply at the prevailing wage. Then by definition, $D_t = L_t(1+V_t)$ and $S_t = L_t(1+U_t)$, which imply

$$\frac{D_t}{S_t} = \frac{1 + V_t}{1 + U_t}.$$

Taking logarithms

$$\ln D_t - \ln S_t = \ln(1+V_t) - \ln(1+U_t).$$

If V_t and U_t are fairly small, then a Taylor's approximation gives us

$$\ln D_t - \ln S_t = V_t - U_t. \quad (2.6)$$

Unfortunately, U.S. annual data for the vacancy rate do not exist for our sample period. Pencavel [1974] suggests that the vacancy rate is a stable function of the unemployment rate which can be approximated by the hyperbolic relationship⁷

$$V_t = \lambda_1 / U_t, \quad (2.7)$$

where λ_1 is a parameter. Substituting into (2.6) gives us

$$\ln D_t - \ln S_t = \frac{\lambda_1}{U_t} - U_t \quad (2.8)$$

Eq. (2.8) gives the relationship between the official unemployment rate and the excess demand for labor. It does not hold as an identity because:

(a) U_t measures the "true" unemployment rate with error, (b) Eq. (2.7) holds only as an approximation, and (c) Eq. (2.6) holds only as an approximation.

It is possible that the vacancy unemployment relationship is not constant across time. In particular, changes in unemployment insurance programs may influence the relationship. To allow for this possibility we estimate

$$\ln D_t - \ln S_t = \frac{\lambda_1}{U_t} - U_t + \lambda_2 UI_t \quad (2.9)$$

where UI_t is defined as $\ln [\text{real unemployment benefits} + 0.001]$.

The term 0.001 is arbitrary and is used to account for the fact that unemployment benefits were zero prior to 1938.

3. Estimation Issues

In this section we discuss the data and outline the estimation procedure. For purposes of reference we restate the model:

$$\ln D_t = \alpha_0 + \alpha_1 \ln (W_t/P_t) + \alpha_2 \ln Q_t + \alpha_3 t + \alpha_4 \ln Q_{t-1} + u_{1t} \quad (3.1)$$

$$\ln S_t = \beta_0 + \beta_1 \ln (W_{nt}/P_t) + \beta_2 \ln H_t + \beta_3 \ln (W_{nt-1}/P_{t-1}) + u_{2t} \quad (3.2)$$

$$\ln L_t = \min(\ln S_t, \ln D_t) \quad (3.3)$$

$$\begin{aligned} \ln W_t = & \gamma_0 + \gamma_1 \ln W_{t-1} + \gamma_2 U_t + \gamma_3 (\ln P_t - \ln P_{t-1}) \\ & + \gamma_4 (\ln P_{t-1} - \ln P_{t-2}) + \gamma_5 \ln W_{t-2} + u_{3t} \end{aligned} \quad (3.4)$$

$$\begin{aligned} \ln P_t = & \delta_0 + \delta_1 \ln P_{t-1} + \delta_2 (\ln W_t - \ln W_{t-1}) \\ & + \delta_3 (\ln W_{t-1} - \ln W_{t-2}) + \delta_4 (\ln PF_t - \ln PF_{t-1}) + u_{4t} \end{aligned} \quad (3.5)$$

$$\ln D_t - \ln S_t = \frac{\lambda_1}{U_t} - U_t + \lambda_2 UI_t + u_{5t} \quad (3.6)$$

the official unemployment rate as a fraction of the civilian labor force.

Stochastic specification and estimation procedure. We assume that the error terms u_{it} ($i=1,\dots,5$) are distributed normally with mean zero and diagonal covariance matrix with elements σ_i^2 ($i=1,\dots,5$) on the main diagonal. For our first set of estimates (Model 1), we assume that $E(u_{it}u_{i\tau}) = 0$ for $i=1,\dots,5$ and all t not equal to τ . In Model 2 this assumption is relaxed and the error terms in equations (3.4) and (3.5) are permitted to have first order serial correlation. Typically, serial correlation is ignored in disequilibrium models because its presence in equations involving latent variables tends to render the likelihood function intractable. (For special exceptions, see Laffont and Monfort [1979], Quandt [1982]). For our case, serial correlation is introduced in the two equations not involving latent variables, which makes the likelihood complicated but not intractable. To our knowledge, such a generalization has not been attempted before. It is noteworthy that neglecting the serial correlation in equations involving latent variables does not interfere with the consistency of the estimates; see Gouriéroux, Monfort and Trognon [1983].

For all cases estimation is by maximum likelihood; the relevant derivations are in Appendix 1. The likelihood functions were maximized numerically using a variety of optimization algorithms.¹⁰

4. Results

Parameter Estimates. The maximum likelihood estimates of the system (3.1) - (3.6) are presented in Table 1. Two models are presented: Model 1 is exactly the model given by (3.1) - (3.6); Model 2 is the same with the further assumption that the error terms in (3.4) and (3.5), u_{3t} and u_{4t} , are serially correlated according to first order Markov processes with

coefficients ρ_3 and ρ_4 respectively. The second column for each model reports 't-values', i.e., the coefficients divided by their asymptotic standard errors.

Consider first the demand equation. There is very little variation in the estimates across models. The value of α_1 implies that the demand elasticity with respect to the real wage is -0.64 to -0.66, estimates within the range reported by Hamermesh [1984] in his survey of labor demand equations. Similarly, the long run output elasticity (found by adding α_2 and α_4) of about 0.80 is quite reasonable. The coefficient of t , α_3 , ranges from 0.0021 to 0.0028, suggesting a very mild positive trend in the demand for labor. In Model 2 all demand coefficients are statistically significant at conventional levels; in Model 1 the t-ratio for α_3 is 1.44.

The supply parameters also show substantial stability across models. In both models, the long run elasticity of labor supply with respect to the after-tax wage, $\beta_1 + \beta_3$, is close to zero and imprecisely estimated. Analyses of time series data have consistently found labor supply elasticities that are small in absolute value. In both models, the elasticity of labor supply with respect to the potential numbers of hours, β_2 , is slightly greater than 0.60, which is lower than one would expect. Even given the fact that the demographic composition of the labor force has changed considerably over time, one still expects that at least roughly, a one percent increase in the potential work force should lead to a one percent increase in labor supply. The reason for the observed result probably lies in the strong collinearity between H_t and W_{nt} . The simple correlation between the two variables is 0.998.¹¹ An ordinary least squares regression of hours worked on H_t alone gives a coefficient on H_t of 0.923 with a

Table 1

Coefficient Estimates

	Model 1		Model 2	
	Coefficients	t-values	Coefficients	t-values
α_0	-2.8290	-7.58	-2.9550	-7.78
α_1	-0.6406	-16.10	-0.6631	-16.36
α_2	0.7096	14.20	0.7130	14.05
α_3	0.0021	1.44	0.0028	1.90
α_4	0.0865	2.27	0.0860	2.24
β_0	1.2324	1.66	1.4031	1.91
β_1	-0.0777	-0.89	-0.0667	-0.78
β_2	0.6300	5.65	0.6065	5.50
β_3	0.0723	1.79	0.0712	1.80
γ_0	0.0713	4.49	0.2869	1.12
γ_1	0.9579	6.06	0.3032	2.11
γ_2	-0.3804	-3.24	-1.6834	-5.44
γ_3	0.7969	3.72	0.3062	1.43
γ_4	0.1458	0.60	0.6300	3.11
γ_5	0.0200	0.13	0.4702	3.41
δ_0	-0.0489	1.35	-0.0683	-1.72
δ_1	1.0103	121.01	1.0147	109.0
δ_2	0.1941	1.42	0.2874	3.36
δ_3	0.2109	2.43	0.1602	2.49
δ_4	0.2367	4.19	0.1785	3.75
λ_1	0.0016	5.97	0.0015	5.62
λ_2	0.0096	1.94	0.0083	1.71

Table 1 (Continued)

ρ_3	-----	-----	0.2486	1.47
ρ_4	-----	-----	0.9916	73.03
σ_1^2	2.19×10^{-4}	4.41	2.22×10^{-4}	4.37
σ_2^2	1.91×10^{-3}	4.29	1.88×10^{-3}	4.29
σ_3^2	9.07×10^{-3}	4.15	1.23×10^{-3}	3.62
σ_4^2	4.84×10^{-4}	2.98	3.92×10^{-4}	4.25
σ_5^2	3.70×10^{-4}	1.18	3.36×10^{-4}	1.07
log L	458.27	-----	472.07	-----

standard error of 0.154.

The parameters of the nominal wage adjustment equation are quite sensitive to whether autocorrelation coefficients are being estimated or not.¹² The coefficient on lagged nominal wages, γ_1 , is in the reasonable range from 0.30 to 0.95. In both models, the coefficient of U_t , γ_2 , is negative, indicating that a higher official unemployment rate is associated with lower nominal wages and conversely. However, with the autocorrelation correction, the absolute value increases substantially (from 0.38 to 1.68), and becomes more significant as well. The positive coefficient γ_3 indicates that increases in prices become translated into higher current nominal wages. The coefficients of the lagged price change, γ_4 , and of the twice lagged wage, γ_5 , are relatively small and insignificant in Model 1; in Model 2 they are both quantitatively large and significant at conventional levels.

In the price adjustment equation, lagged price has a coefficient of 1.01 in both models. The values of δ_2 and δ_3 are positive in both models (and both statistically significant in Model 2), which suggests that lagged changes in nominal wages have a positive effect on this period's prices. In both models δ_4 is positive and statistically significant, suggesting that even after past wages and prices are taken in account, energy prices have an impact on the price level.

In the vacancies-unemployment relationship, the parameter λ_1 is about 0.0015 in both models. The positive value is expected: When unemployment increases, the vacancy rate decreases. We discuss below whether the magnitude of the estimated λ_1 is sensible. The positive value of λ_2 , which multiplies the unemployment insurance benefits variable, implies that for a constant level of excess demand, when unemployment insurance benefits increase, so does the official unemployment rate.

Comparing Models 1 and 2, it is clear that the standard asymptotic likelihood ratio test rejects Model 1 in favor of Model 2. Model 2 also rejects sparser versions from which, say, δ_4 and λ_2 or δ_4 , λ_2 and α_3 are omitted. Hence, in discussing goodness of fit and other issues below, we concentrate on Model 2.

Before proceeding, we should discuss one potential source of concern over our estimates, namely, the possibility that they are driven by the Great Depression, which occurred in the early years of the sample. To investigate this issue we reestimated the model starting with the year 1938 instead of 1929. The parameter estimates for the 1939-1979 period (which are available upon request to the authors) are substantially the same as those for the 1929-1979 period.

Goodness of Fit. How well does Model 2 "explain" the time series data? To explore this question, we computed for each period the model's prediction for quantity of labor ($\ln L_t$), price level ($\ln P_t$), nominal wage ($\ln W_t$), and official unemployment rate (U_t).¹³ For every variable, we regressed the actual on the predicted value each period, and then computed the R^2 . The results are recorded in Table 2. For all variables, the R^2 s are high. Of course, this observation does not prove that the model is "right." After all, the current values of $\ln W_t$ and $\ln P_t$ depend on their lagged values, and given the high amount of autocorrelation in the data, any macroeconomic model with lagged dependent variables is likely to perform well by this criterion. On the other hand, $\ln L_t$ and U_t are not functions of their past values, yet the fits are comparable those obtained by univariate autoregression models. A simple second order autoregression explains U_t slightly worse than the disequilibrium model ($R^2 = 0.9083$), while a

second order autoregression explains L_t slightly better ($R^2 = 0.9537$).

Table 2

R^2 s for Model 2

<u>Variable</u>			
<u>$\ln L_t$</u>	<u>$\ln P_t$</u>	<u>$\ln W_t$</u>	<u>U_t</u>
0.9421	0.9970	0.9978	0.9167

Excess Demand for Labor and Unemployment Predictions. One of the main reasons for estimating a disequilibrium model of the labor market is to produce estimates of excess demand. The strength of excess demand can be measured in several ways:

(i) $(\ln D_t - \ln S_t)$. The model produces estimates of the logarithms of the notional demand and supply for labor each period. Their difference, the percentage excess demand for labor, provides a measure of unemployment that, in theory, is superior to the official measure. For each period we computed the model's reduced form prediction of excess demand by substituting the appropriate values of the exogenous and lagged endogenous variables into equations (3.1) - (3.6), solving the entire system for the jointly dependent variables, and computing $\ln D_t - \ln S_t$.¹⁴

(ii) Simulated Average $(\ln D_t - \ln S_t)$. In non-linear systems, the predictions obtained simply by substituting exogenous and pre-determined variables may be misleading. Therefore, we performed some stochastic simulations. (See Portes, Quandt, Winter and Yeo [1983].) The simulation strategy was to solve for the jointly determined variables after we

added to each structural equation a normal deviate with the same variance as was estimated for that equation. Repeating this procedure 100 times for each time period, we could obtain the average excess demand for each period over the 100 replications.

(iii) $\Pr(D_t > S_t \mid L_t)$. In some sense, measures (i) and (ii) are point estimates of excess demand. It may be of some interest to know the probability that there was excess demand at all. We therefore compute for each year the probability of excess demand (conditional on the amount of labor.)¹⁵

(iv) Simulated Fraction of Times that $D_t > S_t$. As in (ii) above, we simulated the model 100 times each period, and found the fraction of times that demand exceeded supply.

In Table 3 we display the four measures for each year, as well as the official unemployment rate. As expected, the values of all the indicators in 1932-1940 indicate very substantial excess supplies. More generally, all series tell a very similar story qualitatively. Estimated excess demand is negative in all years except 1943-45, 1953, 1975 and 1979. Simulated average excess demand exhibits a similar pattern, except that it is also positive in 1942, 1946, and 1951-52, but is negative in 1975 and 1979.

In general, the excess demands based on stochastic simulations are to be preferred. When deterministic predictions are made, based on using the expected value of the error term, the asymptotic bias is of the order $O(1)$. The asymptotic bias in the presence of stochastic simulations of errors is of the order $O(1/T)$ (Mariano and Brown [1983]). In addition, the stochastically simulated predictor based on asymptotically efficient estimates has an asymptotic mean squared error which differs from its lower bound by

Table 3

Excess Demand Forecasts

	$\ln D - \ln S$	Simulated Average $\ln D - \ln S$	$\Pr(D > S L)$	Simulated Fraction of Times that $D > S$	Official U
1932	-0.275	-0.197	0.0	0.0	0.236
1933	-0.292	-0.231	0.0	0.0	0.249
1934	-0.233	-0.185	0.0	0.0	0.217
1935	-0.255	-0.243	0.0	0.0	0.201
1936	-0.200	-0.170	0.0	0.0	0.169
1937	-0.185	-0.146	0.0	0.0	0.143
1938	-0.185	-0.143	0.0	0.0	0.190
1939	-0.177	-0.137	0.0	0.0	0.172
1940	-0.146	-0.105	0.0	0.0	0.146
1941	-0.075	-0.022	0.0	0.24	0.099
1942	-0.001	0.065	0.0	0.90	0.047
1943	0.128	0.204	0.14	1.00	0.019
1944	0.171	0.268	1.00	1.00	0.012
1945	0.146	0.254	1.00	1.00	0.019
1946	-0.046	0.046	1.00	0.80	0.039
1947	-0.085	-0.013	0.04	0.33	0.039
1948	-0.025	0.052	0.00	0.88	0.038
1949	-0.063	-0.023	0.10	0.24	0.059
1950	-0.062	-0.035	0.0	0.15	0.053
1951	-0.026	0.003	0.01	0.55	0.033
1952	-0.009	0.018	0.73	0.66	0.030
1953	0.005	0.056	0.87	0.89	0.029
1954	-0.030	-0.004	0.77	0.42	0.055
1955	-0.033	-0.015	0.02	0.31	0.044
1956	-0.035	-0.014	0.09	0.37	0.041
1957	-0.049	-0.043	0.03	0.13	0.043
1958	-0.070	-0.069	0.03	0.01	0.068
1959	-0.060	-0.071	0.0	0.0	0.055
1960	-0.061	-0.054	0.0	0.04	0.055
1961	-0.065	-0.073	0.0	0.0	0.067
1962	-0.056	-0.069	0.0	0.03	0.055
1963	-0.053	-0.068	0.01	0.02	0.057
1964	-0.047	-0.065	0.0	0.03	0.052
1965	-0.033	-0.053	0.02	0.05	0.045
1966	-0.011	-0.027	0.09	0.23	0.038
1967	-0.006	-0.031	0.24	0.17	0.038
1968	-0.005	-0.029	0.23	0.23	0.036
1969	-0.014	-0.050	0.23	0.06	0.035
1970	-0.027	-0.062	0.14	0.01	0.049
1971	-0.036	-0.084	0.0	0.0	0.059
1972	-0.036	-0.085	0.0	0.0	0.056
1973	-0.019	-0.081	0.0	0.01	0.049
1974	-0.011	-0.071	0.01	0.0	0.056
1975	0.014	-0.052	0.0	0.04	0.085

Table 3 (Continued)

1976	-0.035	-0.111	0.0	0.0	0.077
1977	-0.020	-0.105	0.0	0.0	0.071
1978	-0.012	-0.110	0.01	0.0	0.061
1979	0.023	-0.072	0.05	0.0	0.058

at most 1 divided by the number of replications.

The probabilities of excess demand by the two measures show fairly substantial agreement with the patterns of excess demand and fair agreement with one another. On the whole, the two excess demand measures and the simulated fraction of times that $D_t > S_t$ agree better with one another than any of these agrees with $\Pr\{D_t > S_t \mid L_t\}$. A probable reason for this discrepancy is that the latter measure is conditional on the observed L_t whereas the former three are not. This observation also helps explain why $\Pr\{D_t > S_t \mid L_t\}$ is somewhat more likely to take a value of zero or one than the simulated fraction of times that $D_t > S_t$. Since $\Pr\{D_t > S_t \mid L_t\}$ uses more information in a sense, it provides a sharper discrimination between periods of excess demand and excess supply, whereas a bit more "fuzziness" is observable in the figures in the simulated fraction of times that $D_t > S_t$.

Dynamics and Stability. A question of some interest is whether prices and wages in the model are locally stable. Doubts about the stability of the system are created by the observation that in the price equation, the coefficient on lagged price, δ_1 , exceeds one. (See Table 1.) Of course, with a system of difference equations greater than order one, all parameters must be examined simultaneously.

To begin, since equation (3.6) is nonlinear in U_t , we expand in Taylor series about an arbitrary value U_0 , yielding

$$\ln D_t - \ln S_t = \left[-\frac{\lambda_1}{U_0^2} - 1 \right] U_t + c_{1t},$$

where c_{1t} has different values for different periods but can be treated as a constant in any one period. Solving for U_t , substituting for D_t and S_t from the demand and supply equations, and then substituting the

resulting expression in equation (3.4) yields

$$\begin{aligned} \ln W_t = & \gamma_0 + \gamma_1 \ln W_{t-1} - \gamma_2 \left[\frac{(\alpha_1 - \beta_1)(\ln W_t - \ln P_t) - \beta_3(\ln W_{t-1} - \ln P_{t-1})}{1 + \lambda_1/U_0^2} \right] \\ & + \gamma_3(\ln P_t - \ln P_{t-1}) + \gamma_4(\ln P_{t-1} \ln P_{t-2}) + \gamma_5 \ln W_{t-2} + c_{2t}, \end{aligned} \quad (4.1)$$

where c_{2t} is a constant. Also, we can rewrite equation (3.5) as

$$\ln P_t = \delta_0 + \delta_1 \ln P_{t-1} + \delta_2(\ln W_t - \ln W_{t-1}) + \delta_3(\ln W_{t-1} - \ln W_{t-2}) + c_{3t}, \quad (4.2)$$

where c_{3t} is a constant. Stability requires that the roots of the characteristic polynomial for the system (4.1), (4.2) lie within the unit circle. As we show in Appendix 2, for most plausible values of U_0 the system is not stable in this sense. Instead, a perturbation to the system leads to continual increases in wages and prices on the order of 1 or 2 percent per year. The source of the instability appears to be the coefficient of lagged price in equation (3.5). Because δ_1 exceeds unity (albeit by a slight amount), increases in price are continually magnified, and via equation (3.4), these price changes are translated into increases in nominal wages. It is hard to say whether this aspect of the model is "realistic." Certainly, some observers have characterized the U.S. economy as being subject to "inflationary spirals."

Vacancies. Recall equation (3.6), the relationship between the unobserved vacancy rate (V_t), the observed official unemployment rate (U_t), and unemployment insurance benefits (UI_t). Another indicator of the plausibility of our model is whether the magnitudes of the implied values of the vacancy rate are reasonable. To investigate this issue, we can substitute our estimates of D_t , S_t and U_t into (3.6) to calculate V_t conditional on UI_t .¹⁶ Given the simple inverse relationship posited

between U_t and V_t , we know that V_t will be relatively low during the depression, high during World War II, etc. What is reassuring about the results is that the absolute magnitudes seem quite reasonable, something that is not guaranteed by the mere fact that $\lambda_1 > 0$. Specifically, from 1932 to 1940 the estimated vacancy rate is no more than 1 percent, during 1943-45 it rises as high as 19 percent, and varies between 1.7 and 5.7 percent thereafter.

"Natural Rate of Unemployment". Another important feature of any aggregative model of the labor market is its implications for the "natural rate of unemployment"--the official rate of unemployment that is compatible with constant growth of prices and nominal wages. We impose a constant rate of inflation by requiring $\$nP_t - \$nP_{t-1} = \$nW_t - \$nW_{t-1} = G$ in equations (3.4) and (3.5), where G is a constant. (We also set the rate of change in energy prices equal to G .) Some tedious but straightforward algebra reveals that the official rate of unemployment compatible with these conditions, U_t^N , is the positive root of the equation $AU_t^2 + BU_t + C = 0$, where

$$\begin{aligned} A &= (\alpha_1 - \beta_1)\phi_5 + 1 ; \quad \phi_5 = \gamma_2(1 - \gamma_1 - \gamma_5) ; \\ B &= (\alpha_1 - \beta_1)(\psi_2 - \psi_1) + z_{1t} - z_{2t} ; \quad \psi_2 = [\gamma_0 + (\gamma_3 + \gamma_4 - \gamma_1 - \gamma_5)G]/(1 - \gamma_1 - \gamma_5) ; \\ \psi_1 &= [\delta_0 + (\delta_2 + \delta_3 + \delta_4 - \delta_1)G]/(1 - \delta_1) ; \\ C &= \lambda_1 ; \end{aligned} \tag{4.3}$$

and z_{1t} and z_{2t} are defined as in Appendix 1. Note that U_t^N depends upon variables with time subscripts, and hence varies from year to year.

In some macroeconomic models, U_t^N is required to be independent of the value of G . That requirement is not imposed here, so we compute U_t^N for several values of G . Substituting into (4.3) values of G equal to 0.0 and 0.05, yields average values of U_t^N over our sample period of 7.9 and 1.0

percent respectively. It also turns out that the natural rate declines almost monotonically from the 1930's to the present, suggesting that the unavoidable unemployment cost of price and wage stability is declining. We conjecture that this result is a consequence of the very high rates of unemployment during the 1930's.

A related question is what level of official unemployment would be associated with "true" zero excess demand in the labor market-- $(D_t - S_t) = 0$. To compute this figure, we simply note from equation (3.6) that when $D_t = S_t$, $\frac{\lambda_1}{U_t} - U_t + \lambda_2 UI_t = 0$; for each year find the U_t that satisfies this relationship; and take the average over our sample record. According to this calculation, when official unemployment is about 3.3 percent, the labor market is actually in equilibrium.

"Phillips Curve". What does our model imply about the short run trade-off between official unemployment and wage inflation? Of course, these two variables are jointly determined, so it does not make sense simply to plug in the value of one and find the implied value of the other. Instead, we consider how both would move under alternative aggregate demand policies. Specifically, the exogenous values of the model except output (Q_t) are set equal to their 1979 values. We then substitute a number of hypothetical values for Q_{1979} into the system, some higher than the actual value in 1979, and some lower. For every value of Q_{1979} , the model is solved to find the associated values of U_{1979} and $(\ln W_{1979} - \ln W_{1978})$. The results represent an almost linear relationship characterized by the equation $(\Delta W/W) = -2.05U + 0.20$. To attain a nominal wage growth of only 3.5 percent would require an official unemployment rate of about 8.0 percent. Alternatively, if the official unemployment rate were 3.5 percent, one would

expect nominal wage growth of about 12.8 percent. The (approximate) equations for the Phillips Curves for other years in the 1970's are very similar.

The conceptual experiment behind these calculations concerns the short run trade-off between U and $(\Delta W/W)$. It is now widely agreed that in the long run, the rate of unemployment is independent of the inflation rate. Although we considered imposing this constraint on the data, we ultimately decided that a better strategy would be to let the data determine the coefficients, and to refrain from giving these estimates a long run interpretation.

5. Conclusions

We have specified and estimated an aggregate disequilibrium model of the labor market which allows determination of nominal wages, prices, labor demand and supply, and the official unemployment rate. The parameter estimates are quite reasonable; the system provides a good fit to the data; and its implications for the behavior of several important variables are sensible. Of course, more remains to be done. Two of the more vexing problems in the current model are the exogeneity of output and the absence of any important role for expectations. Work is currently under way to remedy these problems.

This research also demonstrates that despite the fact that the computational burden of estimating disequilibrium models is high, such models can successfully be estimated at reasonable cost. This is true even for systems that go beyond the "bare bones" approach of earlier papers. We hope that this knowledge will encourage other investigators.

Appendix 1. Derivation of Likelihood Functions

1. The Basic Model. For the sake of simplifying the notation, D_t , S_t , w_t , p_t will denote in this appendix the natural logarithm of demand, supply, nominal wage and price, respectively. U_t denotes the measured unemployment rate and z_{1t} , z_{2t} , z_{3t} , z_{4t} , z_{5t} are linear functions of predetermined variables and coefficients. The model can then be written as

$$D_t = \alpha_1 w_t - \alpha_1 p_t + z_{1t} + u_{1t} \quad (\text{A.1})$$

$$S_t = \beta_1 w_t - \beta_1 p_t + z_{2t} + u_{2t} \quad (\text{A.2})$$

$$L_t = \min (D_t, S_t) \quad (\text{A.3})$$

$$w_t = \gamma_2 U_t + \gamma_3 p_t + z_{3t} + u_{3t} \quad (\text{A.4})$$

$$p_t = \delta_2 w_t + z_{4t} + u_{4t} \quad (\text{A.5})$$

$$D_t - S_t = \lambda_1 / U_t - U_t + z_{5t} + u_{5t} \quad (\text{A.6})$$

where

$$z_{1t} = \alpha_0 + \alpha_2 \ln Q_t + \alpha_3 t + \alpha_4 \ln Q_{t-1}$$

$$z_{2t} = \beta_0 + \beta_1 (1 - \theta_t) + \beta_2 \ln H_t + \beta_3 \ln (1 - \theta_{t-1})$$

$$z_{3t} = \gamma_0 + \gamma_1 w_{t-1} - \gamma_3 p_{t-1} + \gamma_4 (p_{t-1} - p_{t-2}) \\ + \gamma_5 w_{t-2}$$

$$z_{4t} = \delta_0 + \delta_1 p_{t-1} - (\delta_2 - \delta_3) w_{t-1} - \delta_3 w_{t-2} \\ + \delta_4 (\ln PF_t - \ln PF_{t-1})$$

$$z_{5t} = \lambda_2 U_t$$

Assuming that $u_{1t}, \dots, u_{5t}$ are jointly normal with mean vector zero and diagonal covariance matrix, the joint probability density function of $(D_t, S_t, w_t, p_t, U_t)$ is

$$\begin{aligned}
f(D_t, S_t, w_t, p_t, U_t) = & \frac{|A_t|}{(2\pi)^{5/2} \sigma_1 \sigma_2 \sigma_3 \sigma_4 \sigma_5} \exp \left\{ -\frac{1}{2} \left[\frac{(D_t - \alpha_1 w_t + \alpha_1 p_t - z_{1t})^2}{\sigma_1^2} \right. \right. \\
& + \frac{(S_t - \beta_1 w_t + \beta_1 p_t - z_{2t})^2}{\sigma_2^2} + \frac{(w_t - \gamma_2 U_t - \gamma_3 p_t - z_{3t})^2}{\sigma_3^2} \\
& \left. \left. + \frac{(p_t - \delta_2 w_t - z_{4t})^2}{\sigma_4^2} + \frac{(D_t - S_t - \lambda_1/U_t + U_t - z_{5t})^2}{\sigma_5^2} \right] \right\}, \quad (A.7)
\end{aligned}$$

where A_t is the Jacobian of the transformation $(\lambda_1/U_t + 1)(1 - \delta_2 \gamma_3) - \alpha_1 \gamma_2 \delta_2 + \alpha_1 \gamma_2 + \beta_1 \gamma_2 \delta_2 - \beta_1 \gamma_2$. The required density is

$$h(L_t, w_t, p_t, U_t) = \int_{L_t}^{\infty} f(D_t, L_t, w_t, p_t, U_t) dD_t + \int_{L_t}^{\infty} f(L_t, S_t, w_t, p_t, U_t) dS_t. \quad (A.8)$$

Define further

$$\begin{aligned}
z_{6t} &= L_t - \lambda_1/U_t + U_t \\
z_{7t} &= \alpha_1 w_t - \alpha_1 p_t + z_{1t} \\
z_{8t} &= L_t + \lambda_1/U_t - U_t \\
z_{9t} &= \beta_1 w_t - \beta_1 p_t + z_{2t}
\end{aligned}$$

$$\phi_1^2 = \frac{\sigma_2^2 \sigma_5^2}{\sigma_2^2 + \sigma_5^2}$$

$$\phi_2^2 = \frac{\sigma_1^2 \sigma_5^2}{\sigma_1^2 + \sigma_5^2}$$

$$A_t = \frac{\sigma_5^2 z_{9t} + \sigma_2^2 z_{6t}}{\sigma_5^2 + \sigma_2^2} \quad B_t = \frac{\sigma_5^2 z_{9t} + \sigma_2^2 z_{6t}}{\sigma_5^2 + \sigma_2^2}$$

$$C_t = \frac{\sigma_5^2 z_{7t} + \sigma_1^2 z_{8t}}{\sigma_5^2 + \sigma_1^2} \quad F_t = \frac{\sigma_5^2 z_{7t} + \sigma_1^2 z_{8t}}{\sigma_5^2 + \sigma_1^2}$$

Performing the integrations indicated in (A.7) yields

$$h(L_t, w_t, p_t, U_t) = G_{1t}(G_{2t}G_{3t} + G_{4t}G_{5t}) \quad (A.9)$$

where

$$G_{1t} = \frac{|A_t|}{2\pi\sigma_3\sigma_4} \exp \left\{ -\frac{1}{2} \left[\frac{(w_t - \gamma_2 U_t - \gamma_3 p_t - z_{3t})^2}{\sigma_3^2} + \frac{(p_t - \delta_2 w_t - z_{4t})^2}{\sigma_4^2} \right] \right\}$$

$$G_{2t} = \frac{1}{(2\pi)^{1/2}\sigma_1} \exp \left\{ -\frac{1}{2} \left[\frac{(L_t - z_{7t})^2}{\sigma_1^2} \right] \right\}$$

$$G_{3t} = \frac{1}{(2\pi)^{1/2}(\sigma_2^2 + \sigma_5^2)^{1/2}} \exp \left\{ -\frac{1}{2\sigma_1^2} (B - A_t^2) \right\} \left[1 - \Phi \left(\frac{L_t - A_t}{\sigma_1} \right) \right]$$

$$G_{4t} = \frac{1}{(2\pi)^{1/2}\sigma_2} \exp \left\{ -\frac{1}{2} \left[\frac{(L_t - z_{9t})^2}{\sigma_2^2} \right] \right\}$$

$$G_{5t} = \frac{1}{(2\pi)^{1/2}(\sigma_1^2 + \sigma_5^2)^{1/2}} \exp \left\{ -\frac{1}{2\sigma_2^2} (E_t - C_t^2) \right\} \left[1 - \Phi \left(\frac{L_t - C_t}{\sigma_2} \right) \right]$$

$$\Phi(\omega) = \int_{-\infty}^{\omega} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

The loglikelihood then is

$$L = \sum_t \log h(L_t, w_t, p_t, U_t) . \quad (A.10)$$

2. Autocorrelated Error Terms. We assume that u_{3t} and u_{4t} follow first order Markov processes $u_{3t} = \rho_3 u_{3t-1} + \varepsilon_{3t}$, $u_{4t} = \rho_4 u_{4t-1} + \varepsilon_{4t}$. Hence, denoting by u_i and ε_i the vectors of errors ($i=3,4$), we can write

$$R_i u_i = \varepsilon_i \quad i = 3, 4$$

where

$$R_i = \begin{bmatrix} (1-\rho_i^2)^{1/2} & 0 & \dots & 0 \\ -\rho_i & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & -\rho_i & 1 \end{bmatrix}$$

Transforming from the ε 's to the u 's alters only G_{1t} . In analogy with single equation models, the first term is unchanged except for the introduction of $(1-\rho_i^2)^{1/2}$ into the Jacobian and of $(1-\rho_i^2)$ into the matching term of the exponent. In the other terms each squared residual in the exponent is replaced by the square of that residual minus its lagged value which has been multiplied by the matching ρ_i .

Appendix 2. The Characteristic Equation

Let E be the forward operator such that $Ex_t = x_{t+1}$. Denote the characteristic matrix for system (4.1), (4.2) by M . Then the elements of M are defined as follows:

$$M_{11} = \frac{(1+\gamma_2(\alpha_1-\beta_1))}{1 + \lambda_1/U_0^2} E^2 + \left[-\gamma_1 - \frac{\gamma_2\beta_3}{1 + \lambda_2/U_0^2} \right] E - \gamma_5$$

$$M_{12} = \left(-\gamma_3 - \frac{\gamma_2(\alpha_1-\beta_1)}{1 + \lambda_1/U_0^2} \right) E^2 + \left[\frac{\gamma_2\beta_3}{1 + \lambda_1/U_0^2} + \gamma_3 - \gamma_4 \right] E + \gamma_4$$

$$M_{21} = -\delta_2 E^2 + (\delta_2 - \delta_3) E + \delta_3$$

$$M_{22} = E^2 - \delta_1 E.$$

The characteristic polynomial is found by taking the determinant of the matrix M . The polynomial's roots depend on the assumed value U_0 about which the expansion is taken. They were computed for $U_0 = 0.01, 0.02, \dots, 0.20$. For all of these cases there are three positive roots, one negative root and no complex roots. The maximal modulus increases monotonically from 1.0334 for $U_0 = 0.01$ to 1.0464 for $U_0 = 0.20$. The system is thus locally unstable for most plausible unemployment rates.

Appendix 3. Data

This appendix describes the sources and methods of construction of the variables in the model. Throughout, we abbreviate "National Income and Product Accounts of the United States" as "N.I.P.A.".

L_t , total civilian hours worked per year expressed in billions, is total hours minus hours worked in the military. For 1948-79, the following procedure is used. To find hours worked in the military, compute the ratio of the number of military workers to the total number of government workers (N.I.P.A. 1929-1976, pp. 267-69 and N.I.P.A. 1976-1979, P. 55) and multiply by the number of hours worked by government employees (N.I.P.A. 1929-1976, p. 271 and N.I.P.A. 1976-1979, p. 56). This gives hours worked in the military, which is then subtracted from total hours (N.I.P.A. 1929-1976, p. 271, N.I.P.A. 1976-1979, p. 55).

For 1929-42, comparable data do not exist. Using slightly different methods, Rosen and Quandt [1978] constructed a series on civilian hours extending back to 1929. To splice the two series together, for the period 1948 to 1973 we estimated a regression of the logarithm of civilian hours as calculated above on a time trend and on the logarithm of the Rosen-Quandt measure. The R^2 was 0.992. We then substituted values of the Rosen-Quandt measure from 1929-1947 into the regression equation, and used the fitted values.

W_t , gross hourly nominal wage measured in dollars, is the ratio of total civilian compensation to civilian hours worked. Total civilian compensation is total compensation of employees (N.I.P.A. 1929-1976, pp. 238-42 and N.I.P.A. 1976-1979, p. 52) minus compensation of military employees (Historical Statistics of the United States, p. 235; N.I.P.A.

1929-1976, p. 240-42; N.I.P.A. 1976-1979, p. 52). The computation of civilian hours worked is described above.

Q_t , gross national product in 1972 prices, is from the Economic Report of the President 1982, p. 234, for 1941 through 1979. For previous years, the figures are from Historical Statistics of the United States, p. 224. The figures in Historical Statistics were converted from 1958 dollars to 1972 dollars by using the implicit GNP deflator.

P_t , the consumer price index, is from Historical Statistics of the United States, pp. 210-11 for years prior to 1970; and from Economic Report of the President 1982, p. 291, for years after 1970. $P_{1967} = 100.0$.

H_t , the potential labor force in billions of hours, is the number of civilians between the ages of 16 and 65 multiplied by the average number of hours worked per person. The number of civilians in this age group is calculated by taking the total population between 16 and 65 (Economic Report of The President 1981, p. 263) and subtracting membership in the armed forces (Ibid., 264).

θ_t , the average marginal tax rate, is taken from Barro and Sahasakul [1983, p. 20].

U_t , unemployment as a percentage of the civilian labor force, is from Historical Statistics of the United States (p. 135) for 1929-47, and from Economic Report of the President 1982 (p. 271) for 1948-1979.

PF_t , price deflator for fuel and coal, is from various editions of the N.I.P.A.

UI_t , the average weekly unemployment insurance benefit, is formed by

taking the nominal level of benefits and dividing by P_t . For 1938-76, data are from Handbook of Unemployment Insurance Financial Data 1938-1976, U. S. Department of Labor, Employment and Training Administration, 1948, column 32, p. 174. For 1977-79, updates of the Handbook were used.

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Footnotes

¹See, for example, Lucas and Rapping [1970].

²There have been a number of attempts along these lines. See, e.g. Lindbeck and Snower [1985].

³One reason for the difficulty in settling the question statistically is that sophisticated versions of either model may not be conveniently nested within each other. Rosen and Quandt [1978] examine very simple equilibrium and disequilibrium models, and find that the latter does a better job of explaining the data.

⁴We experimented with lagged quantity of labor demanded and the lagged real wage, but these were statistically insignificant.

⁵We also estimated some versions of (2.2) which included the level of unemployment insurance benefits. (See Smyth [undated].) It was not statistically significant.

⁶An alternative notion of disequilibrium has been suggested by Chow [1977] and implemented by Sarantis [1981]. Here disequilibrium is modelled not by a "min condition", but as a situation in which both prices and quantities adjust slowly each period to their long run values. There is no point in engaging in a semantic discussion of which is the "real" disequilibrium model. Suffice it to say that very different maintained hypotheses are used.

⁷We experimented with the more general formation $V_t = \lambda_1 / (U_t - \lambda_3) + \lambda_4$, and found that it did not significantly increase the explanatory power of the model.

⁸Hajivassiliou [1983, p. 20] argues that in the demand for labor equation, the wage should be deflated by the producers' price index.

However, he finds that the choice has virtually no effect on the parameter estimates.

⁹We also examined a specification in which H_t was simply the number of people between the ages of 16 and 65; no major differences resulted.

¹⁰Numerical optimization was performed with the optimization package GQOPT which contains some ten different algorithms. The ones employed included DFP (Davidon-Fletcher-Powell) and the Hook and Jeeves pattern search algorithm. Principal reliance was placed on GRADX due to Goldfeld, Quandt, Trotter [1966]. GRADX is an algorithm in the class of Newton methods and performs an optimal diagonal correction when the matrix of second partials is not negative definite. Computation was in double precision throughout, and first and second partial derivatives were evaluated directly by numerical differencing.

¹¹However, caution must be exercised, because the diagnosis and consequences of multicollinearity in nonlinear models are not well understood. For example, in linear models multicollinearity often leads to parameter estimates that are large relative to their standard errors. In nonlinear models this need not be the case; it depends on the extent to which the model is locally linear.

¹²Note that when u_{3t} is serially correlated, the constant term in the differenced form of the equation is $\gamma_0(1-\rho_3) = 0.2156$.

¹³This was done by substituting the appropriate values of the exogenous and predetermined variables into (3.1) - (3.6), and solving for the endogenous variables. The value of L_t was the minimum of the S_t and D_t so generated. To solve for U_t we: (i) solve for D_t , S_t , W_t , P_t in terms

of U_t ; (ii) substitute the results for D_t and S_t in the unemployment vacancies; and then (iii) solve the resulting quadratic for U_t .

¹⁴One may also compute $E(\ln D_t - \ln S_t \mid \ln L_t)$. This is likely to be more efficient than using simply $(\hat{\ln D}_t - \hat{\ln S}_t)$ (Goldfeld and Quandt [1981]), but is somewhat complicated to compute in the present model and will not be pursued here.

¹⁵ $\Pr \{D_t > S_t \mid L_t\}$ is computed as the ratio $G_{4t} G_{5t} / (G_{2t} G_{3t} + G_{4t} G_{5t})$, where G_t 's are defined in Appendix 1. One can also compute the unconditional probability that D_t exceeds S_t . This can be obtained from the reduced form as follows. Let $D_t = H_{1t} + v_{1t}$, $S_t = H_{2t} + v_{2t}$ where H_{1t} , H_{2t} depend only on parameters and coefficients and v_{1t} , v_{2t} , are reduced form errors. Then the required probability is $\Pr\{H_{1t} - H_{2t} > v_{2t} - v_{1t}\}$ which can be computed once the estimated parameter values are substituted. However, in some applications these probabilities have been found to be very close to the conditional probabilities. (See Burkett [1981].)

¹⁶When we substitute actual rather than estimated values of the official unemployment rate, qualitatively similar results emerge.