I should like to thank David Bloom for his observations on grievance arbitration in the steel industry, and Mike Abbott and Mike Ranson for several useful discussions. I am also indebted to Edward Lazear and other participants in the Labor Workshop at the University of Chicago.
Abstract

Perhaps the most puzzling aspect of grievance arbitration is the question of why two parties would ever pay a third to redistribute income between them. In this paper labor-management disputes are modelled as the outcome of a bilaterally asymmetric principle-agent relationship, in which neither side can directly observe the inputs of the other. Third party arbitrators are interpreted as ex post signals, whose role in the collective bargain is to force a more efficient equilibrium between the contracting parties. The arbitrator's determination of fact provides a basis for rewards or penalties between the parties that generate incentives for more cooperative behavior. In this light, a characterization of more effective arbitrators is developed, and the use of arbitration as a joint punishment strategy is discussed. Then an extended example is presented and numerically simulated. The simulation results suggest that arbitration can be very effective in increasing the efficiency of the firm in the presence of unobserved inputs from workers and managers.
In contrast to the long tradition of commercial arbitration, the arbitration of disputes in labor management relations is a relatively recent phenomenon.\(^1\) After passage of the War Labor Disputes Act in 1943, however, provisions for third party arbitration of unresolved grievances spread rapidly among collective bargaining agreements. At present, roughly 95 percent of labor contracts specify some form of arbitration procedure.\(^2\) Typically, arbitration is the last step in the dispute resolution process: if bilateral negotiations fail, the grievance is turned over to an impartial arbitrator.\(^3\) Subject to a rather narrow basis for judicial review, the arbitrator's decision is final and binding.\(^4\)

At the essence of arbitration is the question of why two parties would agree to abide by a third party's decision, when between them they possess the information and the legal right to make their own decision.\(^5\) The traditional explanation is that grievances arise in the application of the contract to unforeseen contingencies, over issues where the contract is ambiguous, or even silent.\(^6\) Anticipating these circumstances, both parties agree to limit their behavior ex ante, and prevent the use of disruptive bargaining tactics ex post. Instead, they rely on an arbitrator to resolve the bargaining problems that arise during the course of the agreement, as a consequence of match-specific capital and their inability to write a fully contingent contract. In this interpretation, the arbitrator is retained by the parties to prescribe an appropriate sharing of the rents in the event of a change in their environment.\(^7\) Unfortunately, however, while the legal and professional arbitration literature is concerned mainly with disputes of this nature, the bulk of arbitrators' employment is apparently generated by more mundane and recurring cases - most often discharge and discipline disputes.\(^8\) The persistence of "unresolved grievances" in the application of well established
contract provisions presents a stumbling block for the traditional theory of arbitration: if the contract is complete, and shop-floor precedents are clear, why do such grievances require third-party intervention?

One answer is that the arbitration mechanism fulfills a political goal for one party or the other. During union organizational drives or elections, union officers may seek out grievances to demonstrate the worth of the union, or a particular union administration. Alternatively, union leadership, or company industrial relations personnel, may be unwilling to settle without recourse to the arbitrator. Thus the arbitrator becomes a scapegoat for inevitable but unpopular decisions. Neither of these alternatives offers a long term explanation of the demand for arbitration, however. Parties that pursue grievances indiscriminately have a relatively small chance of success before the arbitrator. Given the costs of arbitration, frivolous grievances impose a heavy tax on union members and the firm. Furthermore, the political consequences of losing a case in arbitration may be no less than the consequences of supporting a similar outcome at the bilateral stage of dispute resolution.

A third interpretation of the arbitrator's role in the collective bargain is that of a signal. In particular, suppose that the output of the firm depends on the inputs of both workers and managers, and assume that neither party can directly observe the other's inputs. In this setting, a disciplinary action is launched when management accuses a worker of shirking, and a grievance occurs when the worker appeals this action and accuses managers of failing to perform their duty. Since neither side can trust the other, ex post verification is impossible, and in the absence of arbitration disputes end in impasse. Furthermore, without monitoring, the parties have no incentive to act cooperatively, and the level of output at the firm is likely
to be suboptimal. On the other hand, if a signal is available on the inputs actually provided by workers and managers, then the outcomes of this signal can be used to structure the rewards to each party and improve the efficiency of the contract. In this context, arbitration permits each party to obtain ex post information on the inputs actually provided by the other, when all that is otherwise jointly observable is the output of the pair.

In contrast to the settlement of grievances arising out of unforeseen contingencies, the arbitration of a wide variety of discharge and discipline cases turns on issues of fact: who did what. In these instances, the arbitrator acts as a lie detector: providing a determination of the extent of liability of each party. The arbitrator's award can be interpreted as a contingent payoff schedule, relating the signal (the arbitrator's determination of fact) to a set of transfers between workers and managers. By appropriate choice of the payoff schedule, each party can be made to work harder on behalf of the pair, and their joint output can be made more nearly optimal. Of course, the third-party status of the arbitrator is essential, since neither side can rely on the other to make a disinterested ruling on behalf of the pair.

This interpretation of the arbitrator's role rationalizes the ongoing demand for arbitration in mature industrial settings, where the contract is complete and precedent is clear. At the same time, it is consistent with several other important features of grievance arbitration, including (1) the fact that most arbitrators are drawn from a select pool of highly paid specialists, (2) the tendency toward permanent umpires in large-scale enterprises, and (3) long-run success rates in arbitration that do not necessarily approach fifty percent. With respect to the first of these, if more experienced arbitrators correspond to more reliable signals, then their
employment in the contract is natural. On the other hand, in settings where
the case load warrants it, the use of a permanent umpire standardizes the
arbitrator's awards and allows the parties to exert a greater degree of
control over the relationship between the arbitrator's perceived signal and
his award. Finally, while the traditional view of arbitration as a mechanism
for splitting rents suggests that each party's success rate before the
arbitrator might approach fifty percent, in the equilibrium of the
non-cooperative game where the arbitrator sits as an ex post signal, the
proportion of grievances sustained depends on the nature of the arbitrator's
signal and the relative efforts of workers and managers in the equilibrium.

The balance of this paper is devoted to a formal model of grievances and
grievance arbitration. The model extends the standard principle-agent
characterization to situations where each party plays the role of agent for
the other.\footnote{The notion of an ex post signal is formalized, and the
equilibrium of the model is analyzed. The potential role of the arbitration
procedure as a joint punishment for unfavorable outcomes is discussed and
compared with the signal processing role of arbitration. Finally, an extended
example is developed and analyzed to aid in the interpretation of the
theory. The paper concludes with several comments on the empirical
implementation of the model, and a discussion of some directions for future
research on grievance arbitration.}

I: A Model of Grievances and Arbitration

In order to focus on the role of arbitrators as signals, it is useful to
eliminate the distinction between union leaders and union members (and thereby
many of the political aspects of arbitration) and consider only a single
worker. At the same time, the industrial relations environment is assumed to be "mature", in the sense that all participants know the distributions of the random variables in the model, and these distributions are unchanged.

Disputes arise as follows: before the state of the world is known, workers and managers choose their inputs the firm, a and b respectively. These inputs, in combination with the realization of uncertainty, generate an output level x. In the study of grievance arbitration, it is particularly convenient to assume that x is dichotomous. If output is satisfactory, no dispute occurs. Otherwise, output is unsatisfactory and each side accuses the other of shirking. For example, suppose that x equals 1 (no dispute) if 
\[ a + b + \varepsilon > 0 , \] and 0 (dispute) if \[ a + b + \varepsilon < 0 , \] where \( \varepsilon \) is distributed as a standard normal variate. More generally, of course, output is a continuous random variable, and higher "output" can be associated with a lower level of dispute activity.

The technology of the firm is summarized by the effects of changes in a or b on the distribution function of x. Let \( F(x, a, b) \) denote this distribution function, and \( f(x, a, b) \) its associated density. The derivatives \( f_a(x, a, b) \) and \( f_b(x, a, b) \) are assumed to exist, and increases in a or b are assumed to have the effect of shifting the distribution of x to the right. In addition, the support of the distribution of x is assumed to be independent of a and b. This has the implication that no particular realization of x can rule out any input combinations a fortiori. For the example where x is a Probit on the sum of the effort variables a and b, \( f(0, a, b) = N(-a - b) \),
\( f(1, a, b) = 1 - N(-a - b) \), \( f_a(0, a, b) = - n(-a - b) \),
and \( f_a(1, a, b) = n(-a - b) \), where N is the standard normal distribution function and n is the standard normal density.
In addition to $x$, which without loss of generality can be taken to represent the gross income of managers when the output level is $x$, both parties observe a signal or vector of signals $y$, containing possible information on $a$ and $b$. For example, $y$ may summarize an arbitrator's findings on whether $a$ and $b$ actually exceeded some pre-specified levels $a^*$ and $b^*$. The joint density of $x$ and $y$, conditional on $a$ and $b$, is $f(x, y, a, b)$. Again, the support of the distribution of $y$ is assumed to be independent of $a$ and $b$, so that the detection of a signal $y$ does not rule out the possibility of any given pair of inputs $(a, b)$.

Prior to the resolution of uncertainty, workers and managers agree to a wage schedule $s(x, y)$, where $s$ is the net income transfer from managers to workers when $x$, $y$ is observed. In the absence of arbitration, no signal is observed, and the wage is a function only of the level of output, $x$. If the latter is dichotomous, then the wage schedule consists of two parts: a wage in the event of a dispute (if output is low), and a wage otherwise. The arbitration procedure provides additional information on which to base the transfer from managers to workers. Ignoring any uncertainty as to how the arbitrator translates perceived information into awards, the net payment from managers to workers (the gross wage plus the arbitrator's award) becomes a contingent function of the level of dispute activity and the information obtained by the arbitrator.

Given a particular wage schedule, each party then determines an appropriate level of effort. Since neither party can monitor the other's input, each must assume that the other maximizes its own expected utility, subject to $s(x, y)$ and its anticipation of the other's input choice. In other words, given $s(x, y)$, $a$ and $b$ are determined as the Cournot-Nash equilibrium of a two person game. Let
\[ u(s) = \sigma(a) \]

represent the preferences of workers over \((s, a)\) combinations, where \(u\) is concave and increasing and \(\sigma\) is convex and increasing. Symmetrically, let

\[ v(x - s) = \beta(b) \]

represent the preferences of managers, where \(v\) is concave and increasing, and \(\beta\) is convex and increasing. For a given wage schedule \(s(x, y)\) and fixed managerial effort, workers' effort is chosen according to:

\[
(1) \quad \max_{a \in \mathbb{R}} \int_{x, y} u(s) f(x, y, a, b) \, dx \, dy - \alpha(a),
\]

while for given \(s(x, y)\) and fixed worker effort, managers chose \(b\) to solve the program:

\[
(2) \quad \max_{b \in \mathbb{R}} \int_{x, y} v(x - s) f(x, y, a, b) \, dx \, dy - \beta(b).
\]

Assuming that the first order conditions for (1) and (2) are necessary and sufficient, the input pair \((a, b)\) is obtained as the solution to:

\[
(3a) \quad \int_{x, y} u(s) f(s, y, a, b) \, dx \, dy - \alpha'(a) = 0
\]

and
(3b) \[ \int_{\mathbb{R}^2} v(x - s) f_b(s, y, a, b) \, dx \, dy - \beta'(b) = 0. \]

For each feasible wage schedule \( s(x, y) \), there is a Cournot-Nash equilibrium characterized by (3). If workers have a reservation utility requirement, the best equilibrium is the one that maximizes managers' expected utility, subject to equations (3) and the constraint that it provide the minimum utility level for workers. Letting \( \mu_1 \) and \( \mu_2 \) denote the multipliers associated with (3a) and (3b), respectively, and letting \( \lambda \) denote the multiplier associated with the distributional constraint, the optimal wage schedule is obtained by maximizing the Lagrangian expression:

\[
(4) \quad \int_{\mathbb{R}^2} \left\{ v(x - s) - \beta(b) \right\} + \lambda \left\{ u(s) - \alpha(a) \right\} f(x, y, a, b) \, dx \, dy \\
+ \mu_1 \left\{ \int_{\mathbb{R}^2} u(s) f_a \, dx \, dy - \alpha'(a) \right\} \\
+ \mu_2 \left\{ \int_{\mathbb{R}^2} v(x - s) f_b \, dx \, dy - \beta'(b) \right\},
\]

with respect to \( s(x, y) \), \( a \), and \( b \). In general, it is necessary to bound the feasible wage \( s \) at each \((x, y)\) in order to ensure a solution.\(^{21}\)

Maximizing the integrand in (4) at each realization of output \( x \) and the signal \( y \) gives rise to the following first order condition:

\[
(5) \quad v'(x - s) \left[ 1 + \mu_2 \frac{\partial f_b}{\partial s} \right] = \lambda \left[ 1 + \mu_1 \frac{\partial f_a}{\partial s} \right],
\]

provided that \( s \) is interior at \( x, y \). A necessary condition for an interior solution is that at least one of \( v, \alpha \) be strictly concave.

In contrast to the characterization provided by equation (5), it is well known that the optimal payment schedule in the absence of asymmetric information equates the ratio of the marginal utilities of managers and
workers across different realizations of the joint output variable $x$. Incentive considerations introduce a number of distortions into the optimal wage schedule. First, the wage becomes a function of the signal $y$, in general. Second, as an incentive to worker effort, the net wage $s$ will tend to be higher in those states where the inferred level of workers' inputs is higher. Finally, as an incentive to managerial effort, $x - s$ will tend to be higher in those states where the inferred level of managers' inputs is higher. What distinguishes (5) from the characterization of the optimal wage schedule in the absence of unobserved managerial inputs is the fact that the latter two considerations will typically work in opposite directions. As a simple example, suppose that workers and managers are perfectly symmetric, so that $u = v$, $a = b$, and $f_a(x, y, a, b) = f_b(x, y, a, b)$. Then, if it happens that $\lambda = 1$, an optimal wage schedule with asymmetric information is the same as the optimal risk sharing schedule with complete information. Any attempt to rearrange the wage schedule has just as strong a disincentive effect on one party as its incentive effect on the other.

In a bilateral agency problem, the fact that the payout by one party is the payoff to the other complicates the design and interpretation of the contract. Indeed, solutions to the bilateral agency problem with certainty involve breaking the budget-balancing requirement and permitting the parties to throw away output in unfavorable states. The fact that arbitration imposes financial burdens in less desirable states gives the arbitration proceeding some characteristics of a self-imposed joint penalty. On the other hand, with uncertainty in the relation between inputs and outputs, the equilibrium of a bilateral agency game may not require wasted output. These issues are analyzed more carefully in Section III below. For the moment, the imposition of the budget-balance condition in each state is assumed to be
appropriate.

The multipliers \( \mu_1 \) and \( \mu_2 \) reflect the marginal value of an additional unit of effort on the part of workers and managers respectively, at the optimum. In general, it need not be true that the optimum is characterized by too little effort on the part of workers or managers. Some insight into this result can be obtained by examining the first order conditions for (4) with respect to the effort variables \( a \) and \( b \). These two equations can be written as:

\[
\begin{bmatrix}
\mu_1 \\
\mu_2 \\
\end{bmatrix} = \begin{bmatrix}
-\int v(x-s)f_a\,dxdy \\
-\lambda \int u(s)f_b\,dxdy \\
\end{bmatrix},
\]

where

\[
H = \begin{bmatrix}
h_{11} & h_{12} \\
h_{21} & h_{22} \\
\end{bmatrix},
\]

\[
h_{11} = \int u(s)f_{aa}\,dxdy - u''(a),
\]

\[
h_{12} = \int v(s)f_{ab}\,dxdy ,
\]

\[
h_{21} = \int u(s)f_{ab}\,dxdy ,
\]

and

\[
h_{22} = \int v(x-s)f_{bb}\,dxdy - \beta''(b).
\]

The diagonal elements \( h_{11} \) and \( h_{22} \) are the second order conditions for (3a) and (3b) respectively, and are necessarily negative. By the same token, the ratio \( -h_{21}/h_{11} \) gives the response of worker effort \( a \) to an increase in managerial effort \( b \), at the Cournot-Nash equilibrium. Typically, this response will be between \(-1\) and \(0\), reflecting the non-cooperative character of the equilibrium and the substitutability of \( a \) and \( b \) in the production of output. A similar argument can be made with respect to the ratio \(-h_{12}/h_{22}\)

Therefore, in well behaved cases, the transpose of \( H \) is a dominant diagonal matrix with all negative elements. On the other hand, the elements of the vector on the right hand side of (6) evaluate the effects of an
increase in effort by one party on the utility level of the other. Suppose that at the optimum, each side would prefer the other to increase effort. Then (6) implies that at least one of \( \mu_1 \) and \( \mu_2 \) is strictly positive.\textsuperscript{26} However, even in this extremely restrictive case, one or the other of \( \mu_1 \) and \( \mu_2 \) may well be negative.

More generally, even if \( a \) and \( b \) are perfect substitutes in production, they will not be perfect substitutes in the generation of signals. For example, suppose that the signal gives an arbitrator's finding of the relative size of \( a \) and \( b \). The wage schedule will tend to offer increased rewards to workers when the arbitrator detects a larger relative contribution by workers. If the optimal wage schedule generates a Cournot-Nash equilibrium with \( a > b \), managers may actually prefer a reduction in worker effort (other things equal) in order to improve their probability of success before the arbitrator. In this situation, an increase in effort by workers may actually force an increase in effort from managers at the Cournot-Nash equilibrium, and the optimum may be characterized by too much effort on the part of managers.

II: The Value of Information

When will the opportunity to observe a given signal \( y \) be a valuable one, and how will the net wage depend on the auxiliary signal? The first of these questions can be readily answered by inspection of equation (5). Suppose that \( f_a(x, y, a, b)/f(x, y, a, b) \) and \( f_b(x, y, a, b)/f(x, y, a, b) \) are independent of \( y \) for all \( x \). Then (5) implies that \( s(x, y) = s^*(x) \) at each \( x \). A sufficient condition for both \( f_a/f \) and \( f_b/f \) to be independent of \( y \) is that the joint density of \( x \) and \( y \) can be factored as:
(7) \[ f(x, y, a, b) = g(x, a, b) \cdot h(x, y) \]

In other words, if \( x \) is a sufficient statistic for \( x \) and \( y \), relative to \( (a, b) \), then the wage schedule is independent of \( y \). Intuitively, if (7) is true, then \( y \) adds no extra information on \( a \) and/or \( b \), given \( x \), and conditioning the wage schedule on the arbitrator's signal simply adds noise.

In the standard principle-agent model, any signal \( y \) can be used to improve the contract if and only if (7) is false. In the bilateral agency model, however, there is an important caveat. Return to the symmetric and balanced example where \( f_a = f_b \), workers' and managers' preferences are identical, and \( \lambda = 1 \). In that case, extra information has no value if it provides symmetric information on \( a \) and \( b \): that is, if \( f_a(x, y, a, b) = f_b(x, y, a, b) \). To see why, observe that with \( \lambda = 1 \), \( \psi_1 = \psi_2 \), and with identical preferences, equations (5) and (6) are satisfied with \( s(x) = \frac{1}{2}x \) whenever \( f_a(x, y, a, b) = f_b(x, y, a, b) \).

In fact, only if the incentive effects on workers and managers exactly offset each other will extra information be worthless when (7) is false. This is summarized by the following propositions:

**Proposition I**

If \( s(x) \) is an optimal wage schedule, unique and interior at each \( x \), and \( y \) is a signal such that (7) is false, and if there does not exist a constant \( \phi > 0 \) such that \( v'(x - s) = \phi u'(s) \) for all \( x \), then the signal \( y \) can be used to improve the efficiency of the contract.

**Proposition II**

If \( s(x) \) is an optimal wage schedule, unique and interior at each \( x \), and \( y \) is a signal such that (7) is false, and if \( f_a(x, y, a, b) \)
# \( f_1(x, y, a, b) \) for \( y \in Y_1(x) \), \( P(Y_1(x)) > 0 \), for a set of \( x \) with positive probability, then the signal \( y \) can be used to improve the efficiency of the contract.

A sketch of the proof of Proposition, and a complete proof of Proposition II, are provided in the Appendix.

Proposition II can be summarized by the statement that asymmetric signals are always valuable. A special case of an asymmetric signal is one that conveys information on only one of the inputs. For instance, if \( y \) is informative for \( a \), but not for \( b \), then

\[
\frac{f_b(x, y, a, b)}{f(x, y, a, b)} = f(x, a, b)
\]

independent of \( y \), and the density function can be factored as:

\[
(f'') \quad f(x, y, a, b) = q(x, a, b) h(x, y, a).
\]

In case the arbitration procedure provides information on only one of the inputs, it is relatively easy to characterize the arbitrator's awards. For instance, if the signal \( y \) only conveys information on workers' effort, and if that level of effort is too low, i.e. \( \mu_1 > 0 \), then \( s(x, y) \) is increasing or decreasing in \( y \) (for each \( x \)) as

\[
f_a/f = h_a(x, y, a)/h(x, y, a)
\]

is increasing or decreasing \( y \). Intuitively, if increasing \( y \) signals more effort on the part of workers, and gives no information on management's efforts, then the net payment to workers is increasing in \( y \).

Propositions I and II can also form the basis for a ranking of available
signals (or arbitrators). Consider two signals, $y_1$ and $y_2$. If

\[(8a) \quad f(x, y_1, y_2, a, b) = g(x, y_1, a, b) \cdot h(x, y_1, y_2)\]

one could say the signal $y_2$ is superfluous, given $x$ and $y_1$. On the other hand, if

\[(8b) \quad f(x, y_1, y_2, a, b) \neq g(x, y_2, a, b) \cdot h(x, y_1, y_2)\]

then $y_1$ is valuable, given $x$ and $y_2$. If both (8a) and (8b) are true, then the signal $y_1$ is strictly preferrable to the signal $y_2$. However, whether these ideas can be given empirical content is a difficult question.\(^{29}\)

One particularly simple interpretation of a better arbitrator is that of a less noisy signal. Suppose that $y_1$ is a continuous signal, and suppose that

\[y_2 = y_1 + z \delta,\]

where $z$ is a positive constant, and $\delta$ is distributed independently of $y_1$ with density function $\phi$. Then

\[f(x, y_1, y_2, a, b) = f(y_2 | x, y_1, a, b) \cdot f(x, y_1, a, b)\]

\[= \phi(y_2 - z \delta) \cdot f(x, y_1, a, b),\]

which implies that $y_2$ is a worse signal than $y_1$. Furthermore, any signal derived from $y_1$ is preferred to a similar signal derived from $y_2$.\]
according to the criterion of (8). On this basis, a better arbitrator is associated with a lower value of the constant $z$. However, to the extent that different arbitrators have different skills in determining the contributions of $a$ and $b$ ex post, this characterization of arbitrator "quality" is incomplete.

More generally, a better arbitrator is one who is capable of "finer" judgements. The decisions of a better arbitrator are necessarily more extreme than those of a poorer one. However, the fact that a better arbitrator introduces additional variation into the payoffs of both parties, for a fixed level of output, is more than offset by the improvement in efficiency that the arbitrator's presence induces. In this context, the economically valuable attribute of an arbitrator is his integrity in signal detection, and not the kind of information that he can provide. For example, an arbitrator who is capable of judging only one side of the case (the inputs of workers) may still be quite useful to that side as a means of verifying its effort ex post.

Finally, it is important to recognize that variations in the arbitrator's findings are due entirely to the noise in his signal. Given the net wage schedule, which encompasses both the gross transfer from managers to workers and the arbitrator's award, workers' and managers' effort choices are fixed. Since there is no contingent information available to either party, neither workers nor managers ever deviate from the input choices described by equations (3). In this sense, the fact that the returns of workers and managers depend on the arbitrator's signal is entirely due to the ex ante incentive effects of the presence of the signal. While it would be possible to consider a richer model, in which (for example) workers learn additional information and then make their effort choices, such a model is beyond the
scope of this investigation.

III: Joint Punishment

If the inputs $a$ and $b$ each produce the public good "more output", then one way of eliciting more effort from both parties simultaneously is to threaten joint punishment in the event of an unsatisfactory outcome. For example, both parties could promise to make a contribution to charity in the event of a grievance. An essential aspect of joint punishment is the ability to break the budget balance requirement. Assuming free disposal, however, it is always possible to consider wasting output in selected states. Since arbitration is typically costly, and both parties generally share the cost of the proceedings, one interpretation of arbitration is as a joint punishment strategy.

Consider the equilibrium described by the maximized Lagrangian expression (4). The marginal value of an additional unit of income that accrues to managers in the state described by the output level $x$ and the signal $y$ is

$$\text{(9a)} \quad \nu'(x-s) f(x,y,a,b) + \nu_2 f_b(x,y,a,b),$$

while the marginal value of an additional unit of income that accrues to workers in that state is:

$$\text{(9b)} \quad \lambda u'(s) f(x,y,a,b) + \nu_1 f_a(x,y,a,b).$$

If either of these expressions is positive, income is never wasted in the state $(x,y)$. Since $f(x,y,a,b) > 0$, if $\nu_1$ and $\nu_2$ are positive the only states where budget balance is potentially restrictive are those with both
$f_a(x,y,a,b)$ and $f_b(x,y,a,b)$ negative. Suppose that no signal is observed: then states with $f_b(x,a,b) < 0$ are low output states, since increases in $b$ shift the distribution of $x$ to the right. By the same token, if the distribution of the arbitrator's signal is shifted to the right by managerial effort, then states with $f_b(x,y,a,b) < 0$ are states with a low level of output and/or a low level of the signal $y$.

In general, there may be no states where the parties could be made better off by wasting income. However, if joint punishment is optimal, it is always optimal to punish up to the limits defined by the minimum return to each party.\textsuperscript{32} Thus, if joint punishment is indicated, the payoffs of each party fall into two regimes: in some realizations of $(x,y)$, no income is wasted and the net transfer is characterized by (5); in other realizations of $(x,y)$, each party gives up the most that it can.

The extreme nature of joint punishment, if in fact it is optimal, suggests that arbitration procedures are probably not usefully interpreted as mechanisms for wasting income. At the same time, it is difficult to reconcile several other key features of arbitration with the waste motive. In the first instance, why are arbitrators so carefully selected, if their only role is to consume resources? Secondly, what is the interpretation of the arbitrator's award? Finally, why is a third party needed to preside over the punishment? While each of these aspects of arbitration is inconsistent with the joint punishment motive, each is entirely consistent with the interpretation of arbitrators as ex post signals. In view of this fact, the latter interpretation will be pursued in the remainder of this paper.
IV. An Example

While the analysis of the preceding sections offer a general framework for modelling grievance arbitration, it is none the less too general to address some fundamental questions: under what circumstances is the opportunity for arbitration more valuable; how does the existence of arbitration affect the efforts of workers and managers; how useful is the description of a better arbitrator? This section outlines a simple example of the bilateral agency problem, in which both output and signals are observed as discrete events. The equilibrium of the model is calculated numerically, and some comparative static results are tabulated. The example is intended to be illustrative both of the general analysis of the bilateral agency model, and of its potential application to the grievance arbitration setting.

Assume that a grievance occurs if and only if the sum of the effort variables and a random variable $\epsilon$ is negative, where $\epsilon$ has a standard logistic distribution. Let $x = 0$ indicate a dispute, and the payoff to the pair in the event of a dispute, and let $x = 1$ indicate no dispute, and the payoff in that event. The densities (probabilities) associated with the two events are:

$$ f(0,a,b) = L(-a-b) $$

and

$$ f(1,a,b) = 1 - L(-a-b), $$

where $L$ is the logistic distribution function.

In the absence of an arbitration procedure, $x$ is the only information available, and the parties choose, $s_0$, the wage when there is a dispute, and $s_1$, the wage when there is no dispute. Assume that $a$ and $b$ are each chosen from the interval $(0, \infty)$. Then
\[ f_a(0, a, b) = f_b(0, a, b) = -\ell(-a - b) < 0 \]

and

\[ f_a(1, a, b) = f_b(1, a, b) = \ell(-a - b) > 0 , \]

where \( \ell \) is the logistic density function.

The first order conditions for \( a \) and \( b \), respectively, are:

\[ \ell(-a - b) \left( u(s_1) - u(s_0) \right) - a'(a) = 0 , \text{ or } a = 0 \]

and

\[ \ell(-a - b) \left( v(1 - s_1) - v(-s_0) \right) - b'(b) = 0 , \text{ or } b = 0 . \]

Assuming that \( a'(0) = b'(0) = 0 \), and \( a'', b'' > 0 \), necessary and sufficient conditions for interior solutions are \( 1 > s_1 - s_0 > 0 \). The second order conditions for \( a \) and \( b \), respectively, are

\[ h_{11} = -\ell'(-a - b)Du - a''(a) < 0 \]

and

\[ h_{22} = -\ell'(-a - b)Dv - b''(b) < 0 , \]

where \( Du = u(s_1) - u(s_0) \) measures the workers' utility premium for a favorable output, and \( Dv = v(1 - s_1) - v(-s_0) \) measures the analogous utility premium for managers. The matrix \( H \) of equation (6) is composed of \( h_{11} \), \( h_{22} \), and the off-diagonal elements:

\[ h_{21} = -\ell'(-a - b)Du , \]

and

\[ h_{12} = -\ell'(-a - b)Dv . \]
If $du, dv > 0$, $-h_{21}/h_{11} = da/db \in (-1, 0)$ and $-h_{12}/h_{22} = db/da \in (-1, 0)$, so the Cournot-Nash equilibrium is "stable," with det $H > 0$. At the equilibrium, if managers increase (decrease) their effort by one unit, workers respond by decreasing (increasing) their effort, although by less than one unit.

The first order condition (5) for the wage in the event of a dispute ($s_0$) is:

$$v'(-s_0) - \lambda u'(s_0) = \frac{L(-a - b)}{L(-a - b)} \left\{ \mu_2 v'(-s_0) - \mu_1 u'(s_0) \right\},$$

while the first order condition for the wage in the event of a satisfactory outcome ($s_1$) is:

$$v'(1 - s_1) - \lambda u'(s_1) = \frac{L(-a - b)}{L(-a - b)} \left\{ \mu_2 v'(1 - s_1) - \mu_1 u'(s_1) \right\}.$$ 

It is easy to check that if $\mu_1 = \lambda \mu_2$ at the optimum, then there is no departure from an optimal risk sharing wage schedule. Otherwise, if $\mu_1 > \lambda \mu_2 > 0$, incentive considerations for workers dominate and $s_1 - s_0$ is relatively high, while if $\lambda \mu_2 > \mu_1 > 0$, incentive considerations for managers dominate and the wage differential between the no dispute and dispute states is relatively low.

If both workers and managers are paid more when output is higher, then $du$ and $dv$ are both positive, and each party would prefer the other to work harder, since:
\[ \int u(s) f_b(x) dx = \xi(-a - b) Du > 0, \]

while

\[ \int v(x - s) f_a(x) dx = \xi(-a - b) Dv > 0. \]

In general, however, it does not appear possible to guarantee that each party is rewarded for higher output. In the simulation results reported below, sufficient concavity of the utility functions \( u \) and \( v \) ensures an interior wage schedule and positive utility premiums for the favorable outcome.

Next, consider an arbitration procedure that can only detect the effort of workers. In particular, assume that a signal \( y \) is observed, where

\[
y = 0 \text{ if } a + \delta < \bar{a} \]

and

\[
y = 1 \text{ otherwise,}
\]

and \( \delta \) is distributed as a logistic variate with mean 0 and variance \( \pi^2/3 \). Assume further that \( y \) is only observed in the event of a grievance(\( x = 0 \)). If there are fixed costs in the arbitration procedure, this kind of conditional signal may be optimal.

Under an arbitration scheme, the wage schedule consists of three steps: \( s_{00} \), the wage in the event of a grievance and an arbitration ruling against workers; \( s_{01} \), the wage in the event of a grievance and a ruling against management; and \( s_1 \), the wage in the absence of a dispute. By an argument developed in Section I, it is possible to show \( s_{01} > s_{00} \), provided workers offer too little effort at the optimum, since the signal \( y \) is only informative with respect to \( a \).
For notational convenience, let $\Phi(\bar{a}-a) = P(y=0|x=0)$, and let $\phi(\bar{a}-a)$ represent the associated density. In the presence of the signal $y$, the first order condition for worker effort becomes:

$$
\xi(-a - b) \{u(s_{01}) - \Phi u(s_{00}) - (1 - \Phi) u(s_{01})\}
+ \phi(\bar{a} - a) \xi(-a - b) \{u(s_{01}) - u(s_{00})\} = \alpha'(a) = 0, \text{ or } a = 0.
$$

Comparison with the first order condition in the absence of the signal reveals that the marginal benefits of increased worker effort include both a component for the utility increment between the no-grievance wage and the average wage in the event of a grievance, and a component for the utility increment associated with a positive arbitration result. On the other hand, the first order condition for managerial effort is:

$$
\xi(-a - b) \{v(1 - s_{1}) - \Phi v(-s_{0}) - (1 - \Phi) v(-s_{01})\} = \beta'(b) = 0
$$

or $b = 0$.

Since the arbitration results are independent of management effort, the marginal benefits of $b$ only reflect the difference between utility in the no-grievance state and expected utility in the grievance state. As in the absence of the signal $y$, necessary and sufficient conditions for interior solutions are $Du = u(s_{1}) - \Phi u(s_{00}) - (1 - \Phi) u(s_{01}) > 0$ and $Dv = v(1 - s_{1}) - \Phi v(-s_{0}) - (1 - \Phi) v(-s_{01}) > 0$, or in other words, that both workers and managers receive a higher expected wage in the higher output.
state.

Examination of the elements of the matrix \( H \) reveals:

\[
\begin{align*}
h_{11} &= -\lambda' D_u - \lambda \phi D_u' - \sigma'' - (\lambda \phi + \lambda' \phi') D_u' \\
h_{21} &= -\lambda' D_v - \lambda \phi D_v' \\
h_{12} &= -\lambda' D_v - \lambda \phi D_v' \\
h_{22} &= -\lambda' D_v - \beta''
\end{align*}
\]

where \( D_u' = u(s_{01}) - u(s_{00}) > 0 \) and \( D_v' = v(-s_{01}) - v(-s_{00}) < 0 \) reflect the utility increments for workers and managers respectively, in the event of an arbitration ruling in favor of workers. A sufficient condition for \( \det H > 0 \) is

\[
\lambda(-a - b) \phi(\bar{a} - a) + \lambda\phi(a - b) \phi'(\bar{a} - a) > 0
\]

which also ensures that workers' second order condition is satisfied \( (h_{11} < 0) \), and that workers' response to a unit increase in managerial effort at the equilibrium is to reduce their own effort by something less than one unit. Since the first term in this expression is positive and \( \phi'(\bar{a} - a) > 0 \) if \( \bar{a} - a < 0 \), this condition is equivalent to a condition that \( a \) is not "too far" below \( \bar{a} \), or in other words, that workers' effort is close to the arbitrator's threshold. Because an increase in workers' effort tends to increase their success in arbitration, and therefore increase their wage conditional on a grievance, the reaction of management to an increase in worker effort at the Cournot-Nash equilibrium is not necessarily negative. If the wage premium for a positive arbitration
result in high enough, management may react to increased worker effort by attempting to reduce the probability of a grievance. By the same token, although an increase in management effort is desired by workers (if \( Du > 0 \)), managers may not prefer an increase in worker effort at the optimum.

For particular choices of the utility functions \( u \) and \( v \), the cost functions \( \alpha \) and \( \beta \), and the endowments of workers and managers, the full solutions of the model with and without arbitration are displayed in Table I. The solutions show a marked tendency for workers to expend more effort than managers: this comes mainly from the assumptions that workers are more risk averse than managers, and that workers' endowments are smaller than managers'. Institution of arbitration procedures results in a substantial reduction in the probability of the low output state: from 44 to 33 percent. The source of this reduction is an increase in worker effort. Not surprisingly, when the arbitrator's signal conveys information on only one party's inputs, arbitration is relatively more successful in increasing that party's efforts. The structure of the wage schedules indicate that in each case both parties are better off in the higher output state, and that under arbitration, workers are better off if their grievance is sustained (i.e., the arbitrator rules in their favor) in the event of a dispute. However, there is still a substantial penalty to workers in the dispute state, regardless of the arbitrator's findings. Finally, a check of the conditions for no wasted output, described by equations (9) in Section III, reveals that there is never any incentive for joint punishment in the event of a dispute. This result seems robust to a wide range of choices of the relevant parameters.

Table II reports the value of arbitration (in approximate income units) under alternative assumptions on workers' and managers' relative risk aversion parameters, and the noise in the arbitrator's signal. The results indicate
<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>SYMBOL</th>
<th>NO ARBITRATOR</th>
<th>ARBITRATOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worker Effort</td>
<td>a</td>
<td>.175</td>
<td>.564</td>
</tr>
<tr>
<td>Manager Effort</td>
<td>b</td>
<td>.001</td>
<td>.001</td>
</tr>
<tr>
<td>Net Wage -</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Dispute</td>
<td>$S_1$</td>
<td>.376</td>
<td>.355</td>
</tr>
<tr>
<td>Dispute</td>
<td>$S_0$</td>
<td>.050</td>
<td>-</td>
</tr>
<tr>
<td>Dispute and Grievance</td>
<td>$S_{00}$</td>
<td>-</td>
<td>-.087</td>
</tr>
<tr>
<td>Denied Grievance Sustained</td>
<td>$S_{01}$</td>
<td>-</td>
<td>.103</td>
</tr>
<tr>
<td>Prob. Dispute</td>
<td>$L(-a-b)$</td>
<td>.44</td>
<td>.33</td>
</tr>
<tr>
<td>Prob. Grievance Sustained</td>
<td>$L(\bar{a}-a)$</td>
<td>-</td>
<td>.36</td>
</tr>
</tbody>
</table>

In the simulations reported in Tables I - III, the utility functions $u$ and $v$ are constant relative risk aversion functions; the cost functions $\alpha$ and $\beta$ are identical, constant elasticity functions of the form $\alpha(a) = \frac{1}{1 + t a}$, with $t = 0.40$; the probability of a grievance is $L(-a-b)$, where $L$ is the standard logistic distribution function; the probability of an arbitration ruling against labor is $L(-a/g)$, where $g$ parameterizes the variance of the arbitrator's signal; output is either 0 or 1; the endowments of workers and managers are 0.50 and 2.0, respectively; and the minimum expected utility of workers is -1.10. For the simulations in Table I, the relative risk aversion parameters of workers and managers are set to 3.0 and 2.0, respectively, and the scale parameter of the variance of the arbitrator's signal ($g$) is set to 1.0.
<table>
<thead>
<tr>
<th>Relative Risk Aversion of Worker:</th>
<th>2.6</th>
<th>2.8</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Risk Aversion of Manager:</td>
<td>1.6</td>
<td>2.7/2.7</td>
<td>6.4/6.2</td>
</tr>
<tr>
<td>Risk</td>
<td>1.8</td>
<td>3.0/2.9</td>
<td>6.5/6.4</td>
</tr>
<tr>
<td>Aversion of Manager</td>
<td>2.0</td>
<td>3.2/3.1</td>
<td>6.5/6.4</td>
</tr>
</tbody>
</table>

1See note to Table I. Table entries are in percentages of the cost of a grievance.
a significant certainty equivalent value of arbitration: from 3 to 9 percent of the cost of a dispute. As expected, a lower variance arbitrator is more valuable to the parties, although even a substantial reduction in the variance of the noise in the arbitrator's signal does not increase the value of arbitration by more than .2 percentage points of the cost of a dispute.  

Perhaps the most interesting aspect of Table II is the fact that arbitration is substantially more valuable when workers are more risk averse. Additional insight into this phenomenon is provided by the results in Table III, which describe worker effort in the presence and absence of arbitration, for alternative choices of the risk parameters. In every case, the institution of arbitration procedures gives rise to an increase in worker effort. For less risk averse managers (the top row of the Table) this increase in worker effort is countered by a decrease in managerial efforts. However in every case, arbitration procedures result in a substantial reduction in the probability of low output. As workers become more risk averse, the consequences of an unfavorable arbitration decision becomes more onerous and workers offer greater effort to avoid such decisions. The reduced probability of the low output state increases the joint expected income of the pair more than enough to compensate workers for their extra effort, and the difference accrues to managers.

The implications of this example for the interpretation of grievance arbitration are several. In the first instance, even with an extremely simple arbitration mechanism, the value of an ex post signal is confirmed. To the extent that arbitrators' information is better than a dichotomous indicator of one party's inputs, one could expect the value of arbitration to be enhanced. Secondly, the example underlines the importance of the risk attitudes of the parties in determining the efficacy of arbitration. On the
<table>
<thead>
<tr>
<th>Relative Risk Aversion of Worker:</th>
<th>2.6</th>
<th>2.8</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Risk</td>
<td>1.6</td>
<td>19.0/4.8*</td>
<td>37.3/7.9*</td>
</tr>
<tr>
<td>Risk</td>
<td>1.8</td>
<td>20.8/5.9</td>
<td>39.0/9.6</td>
</tr>
<tr>
<td>Aversion Manager</td>
<td>2.0</td>
<td>22.6/7.1</td>
<td>40.5/11.3</td>
</tr>
</tbody>
</table>

*Effort variable x100. See Notes to Table I. In these cells indicated *, the corresponding change in managerial effort is negative.
surface, it appears that risk aversion is relatively important in the design of effective incentive schemes. Finally, some doubt is cast on the interpretation of a better arbitrator as a less noisy signal. While less noisy signals are more valuable, other attributes of arbitrators, including reputation and reliability in signal processing, may ultimately emerge as the primary determinants of their relative value. Such considerations are beyond the scope of the present analysis, however.

Conclusions

Grievance arbitration is a complex phenomenon. In many instances, arbitrators to rule on issues that are incompletely spelled out in the labor contract. In many other instances, however, arbitrators rule on questions of fact. In the latter cases, I have suggested that arbitrators be interpreted as lie detectors, whose role is to force a more efficient equilibrium between the inputs of managers and workers, when neither side can directly observe the other's actions. This interpretation justifies the ongoing demand for arbitration services in mature industrial settings, where contract language and precedents are clear.

In contrast to a politically motivated theory of arbitration, the model suggests that arbitrators perform an economic service to the parties, akin to a signal in the principle-agent literature. This role sheds some light on the puzzling question of why two parties would agree to abide by a third party's decision, when between them they can make their own decisions. In essence, bilaterally asymmetric information necessitates a neutral third party as final arbitrator of who did what.

From an empirical point of view, the model focuses attention on the kinds of cases that arbitrators decide. A detailed study of the extent to which arbitrators act as triers of fact, rather than contingent decision makers on
behalf of the parties, would help to assess the relative importance of the model in describing the arbitration process. At the same time, an investigation of the disposition of cases across industries or establishments could provide some evidence on the insights of the theory. According to the model, one would expect relative stability in the proportion of grievances sustained within a certain contract, although not necessarily across contracts. At the present time, there is a dearth of empirical evidence on either of these issues.

A number of extensions and refinements of the theoretical model are obvious. First, the addition of contingent information would make the ex post signal interpretation of arbitration more appealing. Secondly, the arbitration proceeding itself could be modelled as a second stage of the game, with both parties determining their relative expenditures on representation in the hearing. Each of these extensions could be expected to sharpen the empirical content of the theory, and improve the opportunity for discriminating tests of the model.

Finally, it is interesting to speculate on the future of grievance arbitration, in light of the model and recent judicial decisions on the doctrine of employment at will. Increasingly, the courts have limited the right of non-union employers to make unilateral discharge decisions without proof of just cause.37 Inevitably, these developments have stimulated the demand for incentive mechanisms that augment or replace the threat of discharge, and protect the interests of disciplined employees. On the other hand, given the nature of most jobs, it is notoriously difficult to untangle the efforts of individual employees and assess the liability for unsatisfactory outcomes. According to the interpretation of grievance arbitration in this paper, however, this is precisely the role that third
party arbitration has played in the union sector. Whether arbitration procedures will spread to the non-union sector in the face of widespread pressures for just-cause protection of disciplined employees remains an unanswered question and an important impetus for further research.
Footnotes


2 The Bureau of National Affairs, Contract Clause Finder; Collective Bargaining-Negotiations and Contracts (Washington, D.C., The Bureau of National Affairs, 1970). The number of cases arbitrated has risen steadily in the postwar period. In 1951, the American Arbitration Association (AAA) reported 1,403 requests for arbitration panels; in 1980, the AAA received 17,061 such requests. Roughly two-thirds of these cases eventually went to arbitration.

3 Approximately three quarters of all contracts specify an ad hoc arbitrator selection procedure. In just over 10 percent of all contracts, arbitrators are selected and retained on a more permanent basis. Ad hoc selection appears to be more common among smaller contracts. In the automobile and steel industries, for example, individual arbitrators may have lengthy tenure. See the Bureau of National Affairs, Contract Clause Finder.


5 This quandry was raised by Fischer in a comment on a paper by Hildebrandt; see Ben Fischer, "Comment," in Arbitration and Public Policy;
Proceedings of the Annual Meeting of the National Academy of Arbitrators

6 See for instance Harry Shulman, "Reason, Contract and Law in Labor
to one recent interpretation, the arbitrator is the parties' surrogate
"contract reader," whose role is to fill in gaps in the contract (whether from
omission or ambiguity). See Theodore J. St. Antoine, "Judicial Review of
Labor Arbitration Awards: A Second Look at Enterprise Wheel and its

7 This view is enunciated in O. E. Williamson, M. Wachter and J. Harris,
"Understanding the Employment Relation: the Analysis of Idiosyncratic
Exchange," Bell Journal of Economics, Vol. 6, No. 1 (Spring 1975), pp. 250-
280, and seems to be implicit in much of the legal arbitration literature.
See for instance Shulman, op cit.

8 A 1974 tabulation of cases handled by the federal Mediation and
Consiliation Service reveals that 1009 of 2188 cases were concerned with
discharge or discipline proceedings.

9 According to an official of the United Steelworkers Union, this kind of
behavior was not uncommon during the organizational drives of the late
1930's. See Lester M. Thornton, "Distressed Grievance Procedures and their
Rehabilitation - Discussion", in Labor Arbitration and Industrial Change,
Proceedings of the Annual Meeting of the National Academy of Arbitrators
review of collective bargaining situations where grievance procedures were
seriously overloaded, Ross noted that two of the correlates of a heavy
grievance caseload were fractional strife within the union, and the desire of
the union shop committee for re-election. See Arthur M. Ross, "Distressed

10In a study of grievance arbitration in the railroad industry, Mangum attributed the low success rate of the railway unions (in terms of the proportion of grievances sustained) to the unions' inability or unwillingness to settle short of arbitration. In his words, the union hierarchy was befit with "an unusual propensity for buckpassing." Garth Mangum, "Railroad Grievance Procedures," Industrial and Labor Relations Review, Vol. 15, No. 4 (July 1962), p. 499. An interesting hypothesis is that the political difficulties of settling grievances at the pre-arbitration stage are greatest among the most "democratic" unions, and give rise to relatively low success rates in arbitration for these unions. Along the same lines, at least one observer suggested that the passage of the Landrum-Griffin Act was associated with an increase in the political susceptibility of union leadership, and a concomitant decrease in the extent of pre-arbitration grievance settlement. See the discussion attributed to Charles M. Mason in J. Seidman, ed., Trade Union Government and Collective Bargaining, (New York, Praeger Publishers, 1970) p.106.

11One of Mangum's explanations for the heavy use of grievance machinery in the railroad industry is the fact that the costs of arbitration were paid by the government. See Mangum, op cit.

12As a matter of law, unions are charged with the responsibility of providing the grievant with fair representation in the grievance procedure, and can be sued for breach of contract by an unsatisfied grievant. However, the courts have explicitly recognized that "... a union does not breach its duty of fair representation and thereby open up a suit by the employee for breach of contract merely because it settled the grievance short of
arbitration."  

13According to one arbitrator, "(e)specially in the review of employer disciplinary action, the basic question very frequently is on of credibility of witnesses." See Russell A. Smith, "The Search for Truth-The Whole Truth," in Stern and Dennis, eds., op cit., pp. 40-60.

14One of the undisputed facts concerning arbitration is that the distribution of earnings among arbitrators is highly skewed, and that a relatively small number of arbitrators hear a vast majority of the cases. According to the records of the American Academy of Arbitrators, in 1970 the case distribution of arbitrators in the academy was as follows:

- 0-1 cases: 142 arbitrators
- 2-5 cases: 149 arbitrators
- 6-10 cases: 69 arbitrators
- 11-20 cases: 50 arbitrators
- 21-30 cases: 24 arbitrators
- 31 or more cases: 24 arbitrators.


15See for instance: The Bureau of National Affairs, Contract Clause Finder.

16In some instances unions and firms maintain running box scores on the awards of particular arbitrators. However, the minimum acceptable win/loss quotient is often less than fifty percent, especially on the union side.

17Related work on multilateral principle-agent relations includes B. Holmstrom, "Moral Hazard in Teams," The Bell Journal of Economics, Vol. 13,

18 Note that the signal $y$ is observed ex post.

19 Arbitration is a conditional information system: the signal $y$ is observed only for particular realizations of $x$. Conditional signals present no real problem for the general analysis, and may in fact be optimal, given the costs of observing the underlying signal. See B. Holmstrom, "Moral Hazard and Observability," Bell Journal of Economics, Vol. 10, No. 1 (Spring 1979), p. 87. In a bilateral agency problem, the case for observing the signal only if the outcomes are unfavorable is strengthened by the fact that the expenditure on observing the signal amounts to a penalty on both parties. This point is explored further in Section III below.

20 In general, the Cournot-Nash equilibrium need not exist, or may not be unique. For the most part, these two important issues will be ignored.

21 For example, by restricting $s(x,y) \in [c, d + x]$ as suggested in Holmstrom, "Moral Hazard and Observability". In the labor market context, a natural lower bound on the net wage is 0, given the unenforceability of contracts. If both parties are risk neutral, then the wage at $(x,y)$ is always on the boundary: workers are either "fully compensated" or "dismissed".


23 Instances where the wage is independent of the observed signal are characterized in the next section.

24 See Holmstrom, "Moral Hazard in Teams" for a more complete discussion of the bilateral agency problem under certainty.

25 It is easy to show that not both $\nu_1$ and $\nu_2$ are equal to zero, by comparing the wage schedule to a full information schedule.
26 Apply Cramer's rule to (6), noting that the determinant of $H$ is positive under the assumed conditions.

27 See Holmstrom, "Moral Hazard and Observability".

28 Holmstrom, "Moral Hazard in Teams" provides a characterization of valuable signals based on the criterion of (7). However in his model one of the partners is risk neutral and this case is essentially ruled out.

29 Equations (8) impose very strong conditions, in general. For example, suppose that $x$ takes on values 0 or 1 according to whether $a + b + c < 0$. Consider the conditional signal $y_1$: $y_1 = 0$ or 1 according to whether $a + \delta_1 < 0$, where $\delta_1$ is normally distributed with mean 0 and variance $\sigma_1^2$. Consider an alternative conditional signal $y_2$: $y_2 = 0$ or 1 according to whether $a + \delta_2 < 0$, where $\delta_2$ is normally distributed with mean 0 and variance $\sigma_2^2$. Even if $\sigma_2^2 < \sigma_1^2$, conditions (8) are not satisfied. Typically, a better signal than $y_2$ includes $y_2$ with less noise or extra information.

30 Ex post, of course, it is never optimal to carry out the threat of wasting output.

31 While many collective bargaining agreements provide for the union and the firm to split the costs of arbitration, in some contracts the arbitrator is able to charge his fees to the party of his choice. If arbitration is performed by a permanent umpire, then the marginal cost of any particular arbitration proceeding is small. Joint costs may still be incurred, however, if an employee with a pending grievance is precluded from working.

32 This follows directly from the observation that the sign of the marginal value of throwing away a unit of income is independent of the level of income of either party, and depends only on the signs of the expressions $f(x, y, a, b) + u_2 f_b(x, y, a, b)$ and $\lambda f(x, y, a, b) + u_1 f_a(x, y, a, b)$. 


33 This characterization of the arbitrator's signal is simplistic. A more general scheme would allow the arbitrator to observe two signals, based on the latent variables \( a + \delta_1 \) and \( b + \delta_2 \). The arbitrator's award schedule would then consist of four steps, depending on whether one, both, or neither party was "found negligent". In this model, of course, no party is ever "negligent", and variations in the arbitrator's awards are associated with the noise in his signal.

34 Since the model is solved with a constant reservation utility for workers, the value of arbitration is represented by an improvement in the expected utility of managers. This utility premium is divided by managers' marginal utility of income at the no-arbitration solution to give an approximate income equivalent.

35 The arbitrator rules against workers if his signal, composed of the sum of worker effort \( a \) and a logistic variate \( \delta \), falls short of the threshold level \( \bar{a} \). For convenience, \( \bar{a} = 0 \) in the simulations. The probability of a ruling against labor is \( L(-a/g) \), where \( L \) is the logistic distribution function and \( g \) parameterizes the variance in \( \delta \) (the variance of \( \delta \) is \( \pi^2/3 \cdot g^2 \)). For the high variance arbitrator, \( g = 1 \), while for the low variance arbitrator, \( g = 0.60 \).

36 The results in Table III are reported for a high variance arbitrator.

Appendix

Proof of Proposition I (Sketch)

The proof essentially mimics the proof in Holmstrom, pp. 85-86. Under the conditions of the proposition, it is possible to consider a small change in the wage schedule at \( x \) which increases the payoff to one or other of the parties according to the realization of \( y \) while at the same time leaving workers as well off. By choosing the variation to alter the effort of the parties (according to the values of \( u_1 \) and \( u_2 \)), managers can be made strictly better off.

Proof of Proposition II:

According to Proposition I, \( y \) is surely valuable if there does not exist a constant \( \phi > 0 \) such that

\[(*) \quad v(x - s) = \phi u'(s)\]

for all \( x \), where \( s = s(x) \) is the optimal wage schedule in the absence of the signal \( y \). Therefore, assume that \((*)\) is true. Since \((7)\) is false and \( f \neq f_1 \) on \( Y_1(x) \), there exists a region \( Y \in Y_1(x) \), with

\[\int_a^b f(x, y, a, b) dy = f(x, Y, a, b) > 0, \text{ and a region } Y^C \text{ of positive probability such that} \]

\[
\frac{f_a(x, Y, a, b)}{f(x, Y, a, b)} > \frac{f_{a^C}(x, Y^C, a, b)}{f(x, Y^C, a, b)}
\]

and

\[
\frac{f_b(x, Y, a, b)}{f(x, Y, a, b)} < \frac{f_{b^C}(x, Y^C, a, b)}{f(x, Y^C, a, b)}.
\]

Consider a variation in \( s(x) \), \( \delta s(x, y) \), such that
\[ \delta s(x, y) = \delta s > 0 , y \in Y \]
\[ \delta s(x, y) = -\delta s \frac{f(x, Y, a, b)}{f(x, Y^c, a, b)} , y \in Y^c \]
\[ \delta s(x, y) = 0 , \text{ otherwise.} \]

The associated first order variation in workers' utility is
\[ \Delta_1(u(s(x))) \int y \delta s(x, y) f(x, y, a, b) \, dy = 0 , \text{ by construction. On the other hand, the first order variation in managers' utility is} \]
\[ \Delta_2 = -v'(x - s(x)) \int y \delta s(x, y) f(x, y, a, b) \, dy 
+ \mu_1 u'(s(x)) \int a \delta s(x, y) f_a(x, y, a, b) \, dy
- \mu_2 v'(x - s(x)) \int y \delta s(x, y) f_b(x, y, a, b) \, dy. \]

From the first order condition (5) for \( s(x) \), we obtain:
\[ v'(x - s(x)) = \lambda u'(s(x)) \frac{1 + \mu_1 f_a/f}{1 + \mu_2 f_b/f}. \]

If \( f_a/f \) is not independent of \( x \), this implies \( \mu_1 = \lambda \mu_2 \).
Furthermore, \( \phi = \lambda \) by comparison with (*). Therefore, \( \mu_1 u'(s(x)) = \mu_2 v'(x - s(x)) \), and

\[ \Delta_2 = \mu_1 u'(s(x)) \int y \delta s(x, y) \{ f_a(x, y, a, b) - f_b(x, y, a, b) \} \, dy 
= \mu_1 u'(s(x)) \{ \delta s(f(x, Y, a, b) - f_b(x, Y, a, b)) 
- \delta s(f(x, Y^c, a, b)) \{ f_a(x, Y^c, a, b) - f_b(x, Y^c, a, b) \} \} \]
\[= u_1 u'(s(x)) \delta s f(x, Y, a, b) \left( \frac{\frac{f_a(x, Y, a, b)}{f(x, Y, a, b)}}{\frac{f_a(x, Y, a, b)}{f(x, Y, a, b)}} - \frac{f_{\lambda}(x, Y^c, a, b)}{f(x, Y, a, b)} \right) \]

\[+ \frac{f_b(x, Y^c, a, b)}{f(x, Y^c, a, b)} - \frac{f_b(x, Y, a, b)}{f(x, Y, a, b)} \]

Thus \( \Delta_2 > 0 \). Since there is a set of \( x \) with positive probability for which the process can be repeated, the signal \( y \) can be used to improve managers' welfare, leaving workers indifferent.