The Formulation of the Dependent Variable in the Wage Equation

In 1959 Dicks-Mireaux and Dow (1959) set forth what might be termed an annual model of discrete wage adjustments. Essentially, their model could be considered as a method of accounting for the discrete nature of wage adjustments when explaining four quarter percentage changes in the aggregate wage level. In particular, under certain assumptions, it was shown that four quarter percentage changes in the aggregate wage level can be expressed as a function of regressors which are four quarter moving averages. The basic assumptions of their model were that wage agreements are negotiated once every four quarters and that one-fourth of all such adjustments take place in each quarter. Further application of their model of aggregation were given by Klein and Ball (1959) and Perry (1964, 66).

The purpose of this note is to demonstrate that the assumptions necessary for the minimization of the aggregation bias associated with the four quarter formulation of the wage variable are strong enough to enable us to employ a much less restrictive model. In particular, it is suggested that the wage variable be formulated as a one quarter percentage change regardless of how frequently wage agreements are negotiated.

Assume that wage agreements are, on the average, adjusted once every j quarters so that over any consecutive j quarter period all wage agreements are, at least, considered for adjustment. In the analysis below it is assumed that the quarterly time periods are numbered sequentially and indexed by the subscript t. Consider now the relation

\[ W_t = \alpha_1 w_1(t) + \ldots + \alpha_j w_j(t), \quad t = 1, \ldots, n, \]

where \( W_t \) is the average wage level at time \( t \); \( \alpha_i \), \( i = 1, \ldots, j \), is the percentage of total wage agreements which are adjusted in time periods \( i \),
\[ j + i, 2j + i, \text{etc.}; \ w_i(t) \text{ is the corresponding average wage level at time } t \]
for the quarterly group of workers whose wage agreements are adjusted in time
periods \( i, j + i, 2j + i, \text{etc.} \). The assumption that all wages are adjusted over
a \( j \) quarter period implies

\[
(2) \quad \sum_{i=1}^{j} \alpha_i = 1.
\]

The problem is to derive the wage equation corresponding to a \( j \)
quarter percentage change in \( W_t \) given the wage equations corresponding to the
quarterly percentage changes in the component wage series. The procedure is
to express the \( j \) quarter percentage change in \( W_t \) as a function of the quarterly
percentage changes in its component series and then aggregate the component wage
equations.

The absolute change in \( W_t \) over a \( j \) quarter period can be expressed as
the sum of the corresponding quarterly changes:

\[
(3) \quad W_t - W_{t-j} = (W_t - W_{t-1}) + (W_{t-1} - W_{t-2}) + \ldots + (W_{t-j+1} - W_{t-j}).
\]

The quarterly change in \( W_t \) is proportional to the change in the wage level of
those workers whose wages were adjusted in that quarter. In particular

\[
(4) \quad W_t - W_{t-1} = \alpha_{[t]} \Delta W_{[t]}(t),
\]

where \( \Delta W_{[t]}(t) = W_{[t]}(t) - W_{[t]}(t-1) \), where \( [t] \) is a function of \( t \) such that
\( [t] = i \) when \( t = Kj + i, K = 0, 1, \ldots \). Substituting (4) into (3) we have

\[
(5) \quad W_t - W_{t-j} = \sum_{i=0}^{j-1} \alpha_{[t-1]} \Delta W_{[t-1]}(t-i).
\]

Equation (5) simply demonstrates that the \( j \) quarter absolute change in \( W_t \) is
a weighted average of the quarterly changes in the component wage series.
Dividing both sides of (5) by \( W_{t-j} \) and defining, in general \( \omega_{t-j} = \frac{\Delta W(t)}{W_{t}(t-1)} \)

we have from (5)

\[
W_t - W_{t-j} = \frac{\sum_{i=0}^{j-1} \gamma_{[t-i]}(t-i)w_{[t-i]}(t-i)}{W_{t-j}},
\]

where \( \gamma_{[t-i]}(t-i) = \alpha_{[t-i]}w_{[t-i]}(t-i-1) \). We see, therefore, that the j quarter percentage change in \( W_t \) can be expressed as a weighted sum of the quarterly percentage changes in its component wage series. In order to show that the weights sum to unity we note, in general,

\[
\omega_{[t-i]}(t-i-1) = \omega_{[t-i]}(t-j)
\]

because the average wage series of a quarterly group of workers would only be subject to change in those quarters that the corresponding wage agreements are negotiated. We have therefore, from (7) and (1)

\[
\sum_{i=0}^{j-1} \gamma_{[t-i]}(t-i) = 1
\]

Therefore, with obvious notation, if the wage equations corresponding to the component wage series are of the form \(1/\),

\[
\omega_{[t]}(t) = b_0 + b_{1}X_{t1} + \ldots b_{m}X_{mt} + \epsilon_{[t]}(t),
\]

the wage equation corresponding to the j quarter percentage change in \( W_t \) is, from (6) and (8), of the form

\[
1/\text{Notice the restrictiveness of this assumption. Not only are the j wage equations assumed to have the same parameters but also the same series as regressors. For instance, if } X_{1}\text{ represents profits, then a formulation such as (9) implies that all quarterly groups of workers adhere to the same profit series.}
\[ W_t - W_{t-j} = b_0 + b_1 \overline{X}_{Lt} + \ldots + b_m \overline{X}_{mt} + \varepsilon(t), \]

where \( \overline{X}_{Lt} = \sum_{i=0}^{j-1} X_{Lt-i} \gamma_{[t-i]}(t-i), \) \( L = 1, \ldots, m, \) and \( \varepsilon(t) = \sum_{i=0}^{j-1} \varepsilon_{[t-i]}(t-i). \)

Because the weights defined in (6) are generally unknown functions of time, the regressors in (10) would be approximated by the simple averages

\[ \overline{X}_{Lt} = \frac{1}{j} \sum_{i=0}^{j-1} X_{Lt-i}, \quad L = 1, \ldots, m. \]

It is clear that a specification error is avoided only if \( \gamma_{[t]}(t) = 1/j, \) for all \( t. \) For this reason, apparently, Dicks-Mireaux and Dow (1959) and Perry (1964), in dealing with the special case of \( j = 4, \) have assumed that wage adjustments are uniformly distributed throughout the year. However, it is clear from (6) that \( \alpha_{[t]} = 1/j \) does not imply \( \gamma_{[t]}(t) = 1/j. \) Therefore, unless further assumptions are considered, a specification error is committed when using the simple averages as regressors in models such as (10). However, if, in fact, wage adjustments are uniformly distributed throughout the \( j \) quarter period, this error may not be serious. The reason for this is that one would expect the ratios in (6) of the component wage series to the aggregate to be close to unity unless the labor markets corresponding to the component wage series were highly demarcated. Therefore, assuming that such demarcations are not very sharp, we may, as a first approximation take \( \gamma_{[t]}(t) = 1/j. \)

The question now arises, however, as to why a \( j \) quarter percentage change in \( W_t \) is considered in the first place. That is, if \( \alpha_{[t]} = 1/j \) and the
ratios of the component wage series to the aggregate are approximately unity, then it can be shown that

\[
(11) \quad \frac{W_t - W_{t-1}}{W_{t-1}} = \frac{1}{j} w_{[t]}(t).
\]

Therefore if the wage equation corresponding to \( w_{[t]}(t) \) is given by (9), the wage equation corresponding to the one quarter percentage change in \( W_t \) is

\[
(12) \quad \frac{W_t - W_{t-1}}{W_{t-1}} = \left( \frac{b_0}{j} \right) + \left( \frac{b_1}{j} \right) X_{1t} + \cdots + \left( \frac{b_m}{j} \right) X_{mt} + \frac{\varepsilon_{[t]}(t)}{j}.
\]

Therefore, if \( j \) is a given constant (implicitly assumed in the formulation of (10)), the only problem associated with the estimation of (12) is a trivial scaling problem. That is, if \( \hat{b}_p \) is the estimate of \( b_p \) then \( j \hat{b}_p \) is the estimate of \( b_p \).

In comparing the formulation of (12) with that given by (10) a number of considerations should be noted. First, the formulation of the dependent variable in (12) (in contrast to that in (10)) does not depend upon a knowledge of \( j \). Therefore, various formulations of the independent variables conditional upon particular values of \( j \) can be considered and compared. For example, if \( X_1 \) represents the change in consumer prices since the time of the last wage adjustment, and if \( j \) is not known, then the various formulations of \( X_1 \) corresponding to \( j = 1, 2, 3, \) etc., can be considered and compared in terms of, say, the corresponding estimate, \( S^2 \), of the variance of the disturbance term. For instance, if the \( S^2 \) statistic corresponding to the formulation of \( X_1 \) conditional upon \( j = 4 \) is significantly lower than that corresponding to the case when \( j = 1 \), and these are the only two values of \( j \) considered, then the estimate of \( j \) is \( j = 4 \). Since the dependent variable in (10) is a function
of j, that formulation of the wage equation precludes such experimentation.

It should also be noted that artificial autocorrelation is not induced by the formulation of the wage estimation as given by (12). On the other hand, since the disturbance term in (10) is a moving average of quarterly disturbance terms, that stochastic element will, in general, be autocorrelated. The consequences of such autocorrelation concerning the efficiency of the parameter estimates, tests of significance, etc. are well known and need not be elaborated here.\(^2\) Another point related to the efficiency of the parameter estimates as derived from models (10) and (12) concerns the fact that the number of observations lost due to the formulation of the dependent variable is a minimum when j = 1. Therefore, the parameter estimates derived from (10) would be based on fewer observations and, hence, less efficient.

A final point to consider in comparing (10) and (12) is that the wage variable used in most wage studies in the U.S. is average hourly earnings. For practical purposes, constant weighted averages of wage rates corresponding to most major sectors of the U.S. economy are not available. The significance of this is that the basic assumption of discrete component wage series (see (1)) which lead to the construction of (10) is no longer tenable. For instance, even if wage rates were fixed, average hourly earnings could vary because of interindustry shifts, occupational shifts, changes in the percentage of hours worked at for premium pay, changes in the productivity of workers employed on a piece rate basis, and number of other considerations. Therefore, when the wage variable used is average hourly earnings, (12) should be considered with j = 1. In brief, it is difficult to conceive of a situation for which model (10) is preferable to (12).

\(^2\)See Goldberger (1964, pp. 231-48).
Bibliography


