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By combining features of rationing models and hedonic models in a novel way, this paper develops a structural model of categorical labor force behavior to help explain several puzzles in data on unemployment and discouraged workers. It traces the links among minimum wages or other rigidities, hiring and firing decisions by firms, and labor force participation decisions by individuals of differing skill levels. A key comparative static result is that a rise in an effective minimum wage increases the labor force participation of more skilled marginal workers but reduces the participation of less skilled marginal workers.

1. INTRODUCTION

ONE OF THE FUNCTIONS of the equilibrium wage in an unrestricted competitive labor market is the rationing of short-run demand for labor by firms among the individual suppliers of labor. If such rationing by wages is perfect, there is no true unemployment or excess supply, though there may be frictional unemployment. Nonprice rationing also tends to eliminate excess supply when minimum wage laws prevent prices from doing so. Awareness of this rationing tends to discourage entry or continued participation by those who are unlikely to get jobs.²

This paper makes the extreme assumption that a minimum wage law is enforced universally in some region,³ and traces the consequences for participation and demand after workers and firms have adjusted to its presence without migrating. Unemployment would disappear if minimum wage laws were repealed, if the unskilled jobless migrated freely to an uncovered sector, or if firms measured job applicants' skills without error.

Section 2 describes short run equilibrium among heterogeneous workers and firms in the absence of minimum wage laws. Section 3 describes the initial effect of a universal minimum wage law on that equilibrium. Section 4 discusses the new equilibrium after firms and

the population have adjusted their short-run behavior to an established universal minimum wage.

Finally, Section 5 briefly applies this model to several well-known puzzles in categorical labor force data and to unemployment policy.

2. SHORT-RUN EQUILIBRIUM WITH HETEROGENEOUS WORKERS AND FIRMS

Suppose there is a set $\{j\} = (1, \dots, J)$ of firms in some economic region which has a working-age population of some large number, I . Suppose no firm incurs hiring, training, or dismissal costs for any worker and all human capital is entirely general rather than firm-specific. However, suppose different members of the population set $\{i\} = (1, \dots, I)$ have different levels H_i of human capital, or skill. In the short run, these levels are exogenous constants for each person but still choice variables for each firm if the firm can measure them. Let firm j 's assessment of individual i 's skills be H_{ji} . Suppose for the moment that there is no measurement error. That is, suppose $H_{ji} = H_i$ for all i and j .

Let each firm j choose employment level m_j^* , hours $L_n^* = L_{jn}^*$ from each worker $n \in \{i_j^*\} = (1, \dots, m_j^*) \subseteq \{i\}$ with perceived human capital $H_{jn}^* = H_{jn}^*(H_n)$, and amount C_j^* of physical capital services to maximize

$$(2.1) \quad \pi_j^o \equiv PF^j\left(\sum_{n=1}^{m_j^*} L_{jn}^* H_n, C_j^*\right) - rC_j^* - z \sum_{n=1}^{m_j^*} L_{jn}^* H_{jn}^* .$$

In firm j 's production function, $F^j\left(\sum_{n=1}^{m_j^*} L_{jn}^* H_n, C_j^*\right)$, the labor input of

human capital services is measured in efficiency units and has three dimensions. m_j is the number of people the firm employs during a time period. L_{jn} is the fraction of the period employee n works for firm j . H_n is the n^{th} employee's quality, human capital, or skill level.⁴ The firm has three ways of increasing its labor input. It can hire another employee and increase it by $L_{jm_j} H_{m_j}$ efficiency units. It can demand more time from employee n and increase labor input by H_n units per additional unit of time. Or it can upgrade its workforce by replacing worker n with another worker who has more skill.

The productivity of any worker to any firm is his unobservable H_n . However, the firm pays the worker according to its perceptions of his productivity, H_{jn} . Let $H_{j1} \geq H_{j2} \geq \dots \geq H_{jm_j}$ be our convention for numbering the firm's employees during any time period.

First-order conditions are

$$(2.2) \quad \frac{\partial \pi_j^0}{\partial C_j} = PF_2^j \left(\sum_{n=1}^{m_j} L_{jn}^* H_n^*, C_j^* \right) - r = 0,$$

$$(2.3) \quad \frac{\Delta \pi_j^0}{\Delta m_j} = PL_{jm_j}^* H_{m_j}^* F_1^j(.,.) - z L_{jm_j}^* H_{jm_j}^* = 0$$

$$(2.4) \quad \frac{\partial \pi_j^0}{\partial L_{jn}} = PH_n^* F_1^j(.,.) - z H_{jn}^* = 0 \text{ for all } n, \text{ and}$$

$$(2.5) \quad \frac{\partial \pi_j^0}{\partial H_{jn}} = PL_{jn}^* \frac{\partial H_n}{\partial H_{jn}} F_1^j(.,.) - z L_{jn}^* = 0 \text{ for all } n.$$

Because $H_{jn} = H_n$ and $\frac{\partial H_n}{\partial H_{jn}} = 1$, the last three conditions are entirely equivalent. Any combination of employment levels, worker hours, and skill levels which equates the rental rate z of human capital services $L_{jn} H_{jn}$ with the value of the marginal product of human capital services achieves the same optimal value of π_j . Firms are indifferent among such levels and there is no unique optimal sorting of the population set among firms, or one-to-one mapping of $\{i\} = (1, \dots, I)$ into $\{i_j^*\} \times \{j\}$. The optimal strategy for firms is to hire the first job applicant whenever it wants more manhours and to fire an arbitrarily selected worker whenever the firm's demand for manhours falls.

The values of the individual m_j 's, L_{ij} 's and H_i 's which lead to any given level of the labor input are a matter of indifference to firms when there is no minimum wage law. Each firm faces a distribution of human capital levels in those labor force participants who apply for work and who accept wage offers. Any level of human capital is acceptable, no matter how low it is, because an applicant may be offered an arbitrarily low wage zH_i per unit of his time, and because this offer may be adjusted downward at any time. Unless all the determinants of firms' labor demand are completely static and no one quits any firm during the period, the chance that an entering job applicant finds no firm willing to hire him is negligible.⁵ If $H_{ji} = H_{ji}(H_i) = H_i$ for all j , each worker i receives a wage of zH_i for each unit of time L_i he supplies to any employer. Workers are indifferent among employers who permit them to work their optimal amounts of time, $L_i^* \in [0, 1]$.

For each person, this L_i^* is at a saddle point of the Langrangian

$$(2.6) \quad K_i \equiv V_i^i(Y_i, 1 - L_i) + \lambda_i [zL_i H_i + v_i - PY_i] + \tau_i [1 - L_i] .$$

The Kuhn-Tucker conditions are

$$(2.7) \quad \frac{\partial K_i}{\partial Y} \equiv V_1^i - \lambda_i P \leq 0,$$

$$(2.8) \quad \frac{\partial K_i}{\partial L_i} = -V_2^i + \lambda_i zH_i - \tau_i \leq 0,$$

$$(2.9) \quad L_i \geq 0,$$

$$(2.10) \quad \tau_i \geq 0, \text{ and complementary slackness condition}$$

$$(2.11) \quad (-V_2^i + \lambda_i zH_i) L_i + \tau_i (1 - L_i) = 0.$$

From the perspective of the population and from the perspective of the firms, there is no unique optimal sorting of workers and firms. Any sorting which permits worker i to supply L_i^* provides him the same optimal V^{i*} .

The usual Inada conditions on V^i guarantee that some market goods always will be consumed ($Y_i > 0$), implying that the first condition holds as a strict equality. They guarantee also that some non-market time ($1 - L_i$) always will be consumed; this means that $\tau_i = 0$ by the complementary slackness condition. When $L_i^* = 0$, that is, when i chooses not to participate in the labor force, the coefficient of L_i in the complementary slackness condition may be less than zero. Following Heckman (1974), we may treat the distance of this coefficient from zero as the strength of i 's commitment to nonparticipation and write as a description of behavior in a cross section that

$$(2.12) \quad \Pr(L_i^* = 0) = \Pr(-V_2^i + \lambda_i z H_i < 0) = \Pr(z H_i < V_2^i \lambda_i^{-1}) \equiv \Pr(w_i < w_i^R).$$

Equivalently,

$$(2.13) \quad \Pr(L_i^* = 0) = \Pr(z < V_2^i \lambda_i^{-1} H_i^{-1}) \equiv \Pr(z < z_i^R).$$

w_i^R is the reservation wage per unit of labor supply L_i . z_i^R is the reservation rental rate per unit of human capital services $L_i H_i$ which just induces i to enter the labor force. w_i^R is the marginal utility of non-market time at zero labor supply. This marginal utility is converted into the same units as the market rate of return to human capital services by dividing by the marginal utility of income, λ_i .

The expression just derived implies that, other things the same, participation is more likely when the wage is higher, when the marginal utility of nonmarket time is lower, when the shadow value of real income is higher and when the amount of human capital embodied in i is higher.

3. INITIAL IMPACT OF A MINIMUM WAGE WHEN FIRMS HAVE PERFECT INFORMATION ABOUT WORKERS

, Suppose the equilibrium described in the last section is disturbed by the imposition of a universal minimum wage, w . The effect of such a minimum wage law is to supplant the payment of $z H_i$ per unit of L_i with the payment of $\sup(w, z H_i)$ per unit to every worker, unless there is an uncovered sector in which less than w may be paid. The rental rate on human capital services $L_i H_i$ implied by such payments is z for i such that $H_i \geq w/z$ and $w/H_i > z$ for i such that $H_i < w/z$.

3.1. Initial Impact on Individual Behavior

The problem faced by individuals i who have more than the critical

skill level w/z is precisely the same as in the last section. However, for less skilled people zH_i is replaced everywhere by w . The human capital variable H_i disappears completely from the maximization problem faced by the unskilled. The implications for their labor force participation are given by

$$(3.1) \quad \Pr(L_i^* = 0) = \Pr(-V_2^i + \lambda_i w < 0) = \Pr(w < V_2^i \lambda_i^{-1}), \text{ for all } i \text{ such that } H_i < w/z.$$

Regardless of skill level, the reservation wage for the unskilled is the same, $V_2^i \lambda_i^{-1}$. But compared to the situation before the minimum wage was imposed, the less skilled have stronger incentives to enter the labor force; their market wages have increased. These incentives are strongest for the least skilled, that is, for those who without a minimum wage would earn the least per unit of L_i .

3.2. Initial Impact on Firms' Behavior

A minimum wage of w per manhour destroys firms' indifference to workers' skill levels. Every law-abiding firm has a minimum wage constraint placed on its maximization problem.⁶ If all human capital is completely general rather than firm-specific, it responds by upgrading its workforce and by instituting hiring standards. When firms have perfect information about individuals' productivities, they'll practice perfect rationing:

$$(3.2) \quad H_n = H_{jn} \geq w/z \text{ for all } n = 1, \dots, m_j \text{ and for all } j.$$

Net revenues are maximized by never offering employment to anyone who is believed to have less than the critical level of skill. Their

services cost $w/H_1 > z$ per efficiency unit. Among people meeting or surpassing the critical level w/z , firms are indifferent; optimal sorting of workers among firms are not unique.

There is a paradox here. If z doesn't change, the demand for labor in efficiency units does not change. The aggregate supply of workers who have at least the critical level of skill is no larger than it was before the minimum wage was imposed. Yet people who have less than the critical skill level w/z have greater incentive to work than before. The total supply of labor in efficiency units exceeds the total demand, and there is downward pressure on wages offered to the less skilled. But the minimum wage law prevents their wages from falling and employers decrease job offers to the less skilled. Such a labor market might exist for a short time after a minimum wage is first introduced. But how could it persist over time? The behavior the model implies for firms and for unskilled people is mutually inconsistent. Unskilled people continue to offer more than the amount, zero, of labor firms demand from them. It never is in the interests of any firm to hire them.

Using the ideas that workers' skills are unobservable measured by firms only with some random error, and that individuals take the probability of unemployment into account when they decide how much leisure to consume, we may resolve this paradox to a great extent. Firm j would hire applicant i with less actual skill H_1 than the critical

level w/z if the firm's perception H_{ji} of his skills were higher. The unskilled applicant searching among many such firms would have a non-zero chance of finding and even of keeping a job. Individuals' ignorance of job rationing and the implicit assumption that $H_{ji} = H_i$ for all j are the sources of the inconsistency between the collective behavior of workers and firms. Relaxing these assumptions leads us to consider explicitly the relationships among people's levels of human capital, firms' perceptions of these levels, and applicants' chances of finding at least one firm whose perceptions are high enough.

4. SHORT-RUN EQUILIBRIUM WITH ESTABLISHED MINIMUM WAGES AND JOB RATIONING

When an input price is kept above its equilibrium level, demanders must use some nonprice rationing scheme to distribute the surplus among suppliers. With the passage of time, suppliers learn that they cannot all sell enough of the input to equate its value in alternative uses with its market price. The market price no longer is sufficient to guide suppliers' allocation decisions; a nonprice rationing system supplements rationing by price.

When all human capital is general rather than firm-specific and the transaction costs of hiring and firing workers are zero, the non-price rationing scheme for labor supply we found in the last section to be optimal for firms takes place entirely at the extensive margin rather than the intensive margin. We may distinguish it from other kinds of labor rationing by calling it job rationing. Those who are rationed

into firms' demand are employed, while those who are rationed out are properly "unemployed" rather than "underemployed." If all rationing is job rationing then the employed and the unemployed constitute a partition of all labor force participants at any time. Define a dummy variable.

$$(4.1) \quad M_{ij} \equiv 1 \text{ if firm } j \text{ offers to employ individual job applicant } i, \text{ and} \\ \equiv 0 \text{ otherwise.}$$

These M_{ij} are choice variables for firms. Under the assumptions of the last section we saw that if individual i contacts firm j when the firm has a vacancy.

$$(4.2) \quad M_{ij}^* = 1 \text{ if } H_{ji} \geq w/z \text{ and} \\ M_{ij}^* = 0 \text{ if } H_{ji} < w/z,$$

but that the behavior of firms and the populations was mutually inconsistent if $H_{ji} = H_i$ for all j . The total demand for workers was too low to sustain the larger number of labor force participants drawn in by the minimum wage. We may force such behavior to be mutually consistent by expanding our model to include search by labor force participants and more general relationships between apparent and actual human capital than $H_{ji} = H_i$.

4.1 Individual Behavior

Let

$$(4.3) \quad Y_i \equiv \text{the } i^{\text{th}} \text{ individual's desired consumption of goods during} \\ \text{some brief time period, such as a week,}$$

- (4.4) $L_i \equiv$ the fraction of the period he allocates to work,
(4.5) $S_i \equiv$ the maximum fraction of the period he allocates to
job search,
(4.6) $R_i \equiv 1 - L_i - S_i$,
(4.7) $v_i \equiv$ nonwage income,
(4.8) $H_i \equiv$ i's human capital or skill level, an unobservable
variable,
(4.9) $H_{ji} \equiv$ firm j's perception of i's human capital, $j = 1, 2, \dots, J$,
(4.10) $z \equiv$ the market rental rate on human capital services for
those who find employment at wages above the minimum,
(4.11) $w \equiv$ the universal minimum wage per unit of L_i ,
(4.12) $X_i \equiv$ a vector of i's observable attributes,
(4.13) $M_i \equiv 1$ if i is employed at any time during the period, and
zero otherwise, and
(4.14) $k_i = k_i(H_i, S_i) \equiv$ the proportion of firms i can contact during the
period, a function of i's skill level and time intensity of
search.

The probability, conditional on labor force participation, that i finds employment or remains employed during the period is defined by the job rationing function,

$$\begin{aligned}
 (4.15) \quad g_i &\equiv \Pr(M_i = 1 \mid L_i^* > 0) \\
 &= g(H_i, S_i, X_i, z, w, J) \\
 &= 1 - \prod_{j_i = i_1}^J k_i(H_i, S_i) \Pr(M_{ij_i}^* = 0), \{j_i\} \subseteq (1, \dots, J).
 \end{aligned}$$

The complement of g_i is the probability of unemployment conditional on labor force participation, that is, conditional on $L_i^* > 0$. The notion of minimum-wage induced unemployment used in this paper is in accord with recent observations about the job market and with Rees's (1979, p. 105) definition of "involuntary unemployment." In this paper, i suffers minimum-wage-induced unemployment during a period when (i) $L_i^* > 0$; (ii) none of the firms i contacts during the period offer him a job, that is, $M_{ij}^* = 0$, $j = i_1, \dots, k_i(H_i, S_i)$; and (iii) at the beginning of the period, i believes that there is a significant chance some firm will offer to hire him before the expiration of fraction S_i of the period ($g_i > 0$).

A minimum wage law prevents wages from playing their full allocative role in the labor market. So wages must be supplemented by other dimensions of contracts between buyers and sellers of labor. There are two other dimensions in the very short run, hours and work quality. A second price, called the job rationing function here, supplements the wage in guiding the decisions of individuals. In the absence of specific human capital we may ignore the hours dimension and see that job rationing is the result of profit maximization by firms subject to the information they have about individuals' skills or the quality of work they are capable of doing.⁸ The product of the wage and the job rationing function, rather than just the wage by itself, appears in each individual's budget constraint.

(4.15) asserts that any individual who has the same level of skills H_i and other personal characteristics X_i would have the same ex ante probability of finding a job during the time period if he searched with the same intensity. In other words, rationing by job search and by skill supplements rationing by price in equating the supply of unskilled labor with the demand for unskilled labor. For example, when all firms practice the perfect rationing of expression (3.2) and when a jobseeker searches only among firms which have vacancies, (4.15) reduces to

$$(4.16) \quad g_i = 1, H_i \geq w/z, \text{ and} \\ = 0, H_i < w/z.$$

Pure queueing and optimal rationing subject to imperfect information about H_i lead to other specifications for g_i . Yet the short-run analysis of individual behavior remains the same in all three cases. The impact of differing systems of job rationing on individual behavior is captured entirely by g_i .

When $L_i^* = 0$, maximum consumption is given by the inequality

$$(4.17) \quad PY_i \leq v_i.$$

When $L_i^* > 0$ and employment is found or kept during the period, the constraint on consumption is

$$(4.18) \quad PY_i \leq \sup([wL_i + v_i], [zL_iH_i + v_i]).$$

When $L_i^* > 0$ and employment is not found or kept during the period, the constraint on consumption is the same as when $L_i^* = 0$. The budget constraint

$$(4.19) \quad PY_i \leq [M_i \sup(wL_i, zL_iH_i) + v_i]$$

reduces to (4.17) with probability one when $L_i^* = 0$, to (4.18) with probability $g(\dots)$ when $L_i^* > 0$ and employment is found or kept, and to (4.17) with probability $1-g(\dots)$ when $L_i^* > 0$ and employment is not found or kept. The presence of the dummy variable M_i is inconvenient. The problem is simplified by assuming certainty equivalence and substituting $g(\dots)$ for M_i in (4.19) as an approximation.⁹ It's convenient at this point also to normalize prices by making P numeraire.

The problem each member of the population faces on this view is to choose L_i , S_i , and Y_i to maximize

$$(4.20) \quad V^i = V^i(Y_i, 1-L_i-S_i)$$

subject to

$$(4.21) \quad Y_i \leq \sup ([wL_i g(H_i, S_i, X_i, z, w, J) + v_i] , \\ [zL_i H_i g(\dots) + v_i])$$

$$(4.22) \quad 1 - R_i \equiv L_i + S_i \leq 1,$$

$$(4.23) \quad L_i \geq 0 \text{ and}$$

$$(4.24) \quad S_i \geq 0 .$$

The constraint qualification condition is satisfied since the opportunity set includes the possibility of simultaneous leisure, work, and search.

The Langrangian is

$$(4.25) \quad K_i \equiv V^i(Y_i, 1 - L_i - S_i) + \lambda_i \{ [wL_i g(H_i, S_i, X_i, z, w, J) + v_i] - Y_i \} + \tau_i (1 - L_i - S_i), \text{ for } i \text{ such that } H_i < w/z, \text{ and}$$

$$(4.26) \quad K_i \equiv V^i(Y_i, 1 - L_i - S_i) + \lambda_i \{ [zL_i H_i g(\dots) + v_i] - Y_i \} \\ + \tau_i (1 - L_i - S_i), \text{ for } i \text{ such that } H_i \geq w/z.$$

For individuals with less than the critical level of skill w/z , the Kuhn-Tucker conditions for an optimum are

$$(4.27) \quad \frac{\partial K_i}{\partial Y_i} \equiv V_1^i - \lambda_i \leq 0,$$

$$(4.28) \quad \frac{\partial K_i}{\partial L_i} \equiv -V_2^i + \lambda_i wg - \tau_i \leq 0,$$

$$(4.29) \quad \frac{\partial K_i}{\partial S_i} \equiv -V^2 + \lambda_i L_i wg_2 - \tau_i \leq 0,$$

$$(4.30) \quad S_i \geq 0,$$

$$(4.31) \quad L_i \geq 0,$$

$$(4.32) \quad \tau_i \geq 0, \text{ and}$$

$$(4.33) \quad (-V_2^i + \lambda_i L_i wg_2 - \tau_i) S_i + (-V_2^i + \lambda_i wg - \tau_i) L_i \\ + \tau_i (1 - L_i - S_i) = 0.$$

Inada conditions on V guarantee that the first condition holds as a strict equality and that τ_i always is zero.

As in previous models, the last condition suggests as a description of labor force participation in a group of the unskilled

$$(4.34) \quad \Pr(N_i | H_i < w/z) \equiv \Pr(L_i^* = 0) = \Pr(-V_2^i + \lambda_i wg < 0) = \Pr(wg_i < V_2^i \lambda_i^{-1}) \\ \equiv \Pr(wg_i < w_i^R),$$

For skilled workers

$$(4.35) \quad \Pr(N_i | H_i \geq w/z) = \Pr(zH_i g_i < w_i^R),$$

In general

$$(4.36) \quad P_N \equiv \Pr(N_i) = \Pr[g_i \sup(w, zH_i) < w_i^R]$$

Even with equilibrium nonprice job rationing, unless the income effect is perverse and strong there is a reservation wage which determine whether an individual wishes to supply any time to the market. This reservation wage is the marginal utility of nonmarket time adjusted in two ways. Dividing by the marginal utility of income converts it from utils to income units. Dividing by the equilibrium conditional probability of finding or keeping a job during the period adjusts it for nonprice rationing present in equilibrium. Unlike the model of the last section, this model has individual participation decisions consistent with the presence of unemployment after a minimum wage has become established in a labor market. Job search which is sure to be unsuccessful does not occur; time is not wasted. If job rationing were accomplished in a completely random way by pure queueing or if there were no alternatives to work, actual job search would have to take place to equate the total marginal cost of work to the artificially high marginal benefit from work. However, to the extent that jobs are rationed among searchers according to their productivities and to the extent that there are alternatives to the labor market, such as school or home activities, awareness of the affects of an established minimum wage discourages participation if substitution effects outweigh income effects.

4.2 Optimal Job Rationing by Firms

Extreme assumptions about firms' perceptions of individuals' skills are instructive. Under perfect rationing, $H_{ji} = H_i$ for all j . Under pure queueing, H_{ji} is completely uncorrelated with H_i . Both these relationships are special cases of more general kinds of relationships between individual i 's skills and firm j 's perceptions of i 's skills.

Every firm j has experience with present and past employees on which it draws in deciding each period whether to hire or keep an individual worker. This experience consists of observations of each of p characteristics. These characteristics might include age, schooling, prior work experience, marital status, race, and sex. Through repeated measurements of actual productivity on the job, firm j may have decided that $H_i > w/z$, $i = 1, \dots, N_{1j}$, and the $H_i < w/z$, $i = N_{1j} + 1, \dots, N_{1j} + N_{0j}$. That is, based on its unique experience, firm j may believe that N_{1j} present and past employees had more than the critical skill level, and that N_{0j} had less. A natural way to think of the use firm j makes of this information is, for $i = N_{1j} + N_{0j} + 1$,

$$\begin{aligned}
 (4.37) \quad \Pr(H_{ji} < w/z) &= \Pr(\log H_{ji} < \log (w/z)) \\
 &= \Pr(X_i' \beta^j + \varepsilon_{ij} < \log (w/z)) \\
 &= \Pr(\varepsilon_{ij} < \log (w/z) - X_i' \beta^j) \\
 &= R^j(\log (w/z) - X_i' \beta^j) ,
 \end{aligned}$$

where $R^j(.)$ is the cumulative distribution function for ε_{ij} , the

subjective component of the firm's assessment of i 's skills. The semi-logarithmic relationship between H_{ji} and observable characteristics stems from the proportionality of H_{ji} to unconstrained wages. The β^j_s , estimated from the perceived performance and characteristics of present and past employees, serve as weights for the characteristics. When these weights are positive, i is more likely to be hired or kept on when the components of X_i are greater.

The individual's true, unobserved efficiency level, H_i , is related systematically to these same p characteristics by

$$(4.38) \quad \log H_i = X_i' \beta + \theta_i .$$

We may think of the θ_i 's as true ability, or the actual extent to which individuals' productivities to firms differ from group averages. θ_i is fixed for every individual and known to every individual. However, it varies randomly in a group of observationally equivalent people and can't be observed directly by others.

Since

$$\begin{aligned} (4.39) \quad \log H_{ji} &= X_i' \beta^j + \varepsilon_{ij} = X_i' \beta^j + X_i' \beta + - X_i' \beta + \varepsilon_{ij} + \theta_i - \theta_i \\ &= X_i' \beta + X_i' (\beta^j - \beta) + \theta_i + (\varepsilon_{ij} - \theta_i) \\ &= \log H_i + X_i' (\beta^j - \beta) + (\varepsilon_{ij} - \theta_i) , \end{aligned}$$

the measurement error that firm j makes when it decides whether to hire or retain individual i consists of two components. The first component is due to the difference between firm j 's sample of past and present employees and the population as a whole. While on average the true

marginal increase in skill due to characteristic X_k is β_k in the population, firm j acts as if it were β_k^j . This systematic difference could occur because firm j 's employees and job applicants have not been random samples of the population, because of vintage effects, or because of job discrimination against characteristic k .

The second component is the difference between any individual's actual ability, θ_i , and any firm's assessment ε_{ij} of that ability. This difference, the last term of (4.39), may be taken to follow some random distribution with mean zero and to be independent across individuals and across the firm's hiring and firing decisions.

The probability firm j refuses to offer a vacant job to individual i or refuses to continue employing him is

$$(4.40) \quad \Pr(M_{ij}^* = 0) = \Pr(H_{ji} < w/z) = R^j(\log(w/z) - X_i' \beta^j).$$

Substituting (4.40) into (4.15) produces the job rationing function

$$(4.41) \quad g_i \equiv \Pr(M_i = 1 \mid L_i^* > 0) \equiv g(H_i, S_i, X_i, z, w, J) \\ = 1 - \prod_{j_i=1}^{J_{k_i}(H_i, S_i)} R^j(\log(w/z) - X_i' \beta^j), \{j_i\} \subseteq (1, \dots, J).$$

This expression shows the interrelationships among the nonprice variables which must ration jobs when a minimum wage of w prevents prices from doing so.

A. No Minimum Wage

When there is no minimum wage, $w = 0$ and expression (4.41) reduces to unity. The model reduces to the model presented in Section 2 above. There is no unemployment, because $H_{ji} \geq w/z$ for all i and j , and every

individual who decides to participate in the labor force earns a wage based on his apparent productive efficiency. In Figure 1, the curve labeled DD' is the horizontal sum over all j of solutions to (2.1). It is the market demand curve for the services, in efficiency units, of labor. More is demanded at lower rental prices z for units of human capital services. The curve labeled SS' is the horizontal sum over all i of solutions to (2.6). It gives the amount of time L_i each person i wishes to supply to the labor market at each rental rate z . In aggregating these labor supply decisions, curve SS' weights each person's L_i by his skill level H_i . All nonparticipants contribute zero to SS' because for them $L_i^* = 0$. On curve SS' labor force participants are ordered by their reservation rental rates (2.13). The intersection of DD' and SS' gives the equilibrium value of z , the rental price of one efficiency unit of human capital services, before the minimum wage is imposed. All those individuals with reservation rental rates above z^0 choose to remain out of the labor force.

B. Perfect Rationing

When actual productive efficiency H_i is observed directly by firms, the distribution of unobserved heterogeneity θ_i and the distributions of measurement errors $\xi_{ij} - \theta_i$ all are degenerate. So $H_i = X_i' \beta^j = X_i' \beta = H_{ji}$ for all firms j . The cumulative distribution function in (4.41) becomes a step function, with $R^j(\log(w/z) - X_i' \beta^j) = 1$, $H_i < w/z$, and $R^j(\dots) = 0$, $H_i \geq w/z$. The expression for g_i becomes a two-valued function. It takes the value zero when $H_i < w/z$ and is

unity when $H_i \geq w/z$.

The curve which gives the response of firms to the value of z remains DD'. Firms are unconstrained since there is excess supply of labor, not excess demand. However, in the period of adjustment to the minimum wage described in Section 3 above, the labor supply of those who have less than the critical level of skill w/z^0 is increased. Instead of the equilibrium rental rate z^0 , such individuals face a range of rental values $z_i = w/H_i > z^0$. z_i is highest for those with the lowest levels of H_i , and so the least skilled respond to the imposition of the minimum wage by offering the most extra time L_i to the market. At the time the minimum wage is imposed, the weighted labor supply curve shifts rightward from SS' toward XX' for two reasons. First $z_i > z^0$ now exceeds the reservation rental rates of some unskilled individuals who previously were nonparticipants. In other words, those for whom $w > z_i^R H_i > z^0 H_i$ move into the labor force. Second, some of those with reservation rental rates below z^0 now wish to supply more time than they did previously. The new position of the weighted labor supply curve will depend on the distribution of reservation rental rates, on the shapes of the indifference maps of all individuals with less than the critical skill level, and on the correlation between skill levels and reservation rental rates.¹⁰ The net effect of the adjustment to an established minimum wage is to shift the weighted labor supply curve leftward from XX'. The final position will be one like MM'. This curve lies to the left of the pre-minimum wage curve SS' because of the unskilled people who have been discouraged from the labor market by the minimum wage law. After adjustment, all

unemployment is eliminated by perfect rationing. Nonparticipation is increased above the percentage due to high reservation wages by the skill requirements profit maximizing firms are forced to institute. All of those whose $H_1 > w/z'$ and whose reservation wages are low enough earn higher wages $z'H_1$ per period than they did before the minimum wage was imposed, and their individual labor supplies are changed in a predictable way. Those for whom the substitution effect of the wage increase exceeds the income effect work longer hours, and those for whom the opposite is true work shorter hours. The demand for efficiency units of labor services drops from L^0 before the minimum wage to L' after full adjustment to the minimum wage. All the individuals affected by this demand drop are discouraged workers. The number of discouraged workers and the drop in labor demand will be larger the larger the elasticity of substitution between efficiency units of labor services and physical capital, the larger the share of labor expenses in total costs, the larger the elasticity of supply of physical capital, and the larger the elasticity of demand for the output of local industry.

C. Pure Queueing

When firms are completely unable to observe any components of individuals' skill levels, $\beta^j = 0$, and firms' assessments of individuals skills are completely random and uncorrelated with H_1 . Firms would hire and fire workers entirely on the basis of subjective judgments about their power to contribute to output, but these judgments

would have no systematic relationship to individuals' real productive power. The probability a given firm would refuse to hire a given job applicant and the probability the firm would fire a given employee during any time period would have the same value, $\Pr(H_{ji} < w/z)$. The probability an individual who wishes to work during the period finds at least one firm which will employ him would be the same for all individuals who searched with the same intensity and efficiency. In the simplest case of pure queueing, (4.15) reduces to

$$(4.42) \quad g_i = 1 - \frac{Jk(S_i)}{j_i \prod_{i=1}^j i_1} \Pr(H_{ji} < w/z), \{j_i\} \subseteq (1, \dots, J) \\ = g(S_i).$$

The cumulative distributions R^j in (4.37) are evaluated at a single point, $\log(w/z)$. Everyone has the same probability of getting and holding a job, whether or not his true skill H_i exceeds the critical level w/z . There is true minimum-wage-induced unemployment, but its incidence is symmetric among all groupings of individuals with the same preferences regardless of their skill levels. Everyone who searches with the same intensity expects to spend the same fraction, $(1 - g_j(S_i))$ of any period of labor force participation in unemployment.

Unemployment would be one of the hazards of labor force participation for everyone. On average, fraction g of any period of time during which $L_i^* > 0$ would be spent in state M, while fraction $(1 - g)$ would be spent in state U. Pure queueing results in symmetric unemployment, while perfect rationing results in non-symmetric unemployment during a

transitional period and then in no unemployment at the new equilibrium after adjustment to the minimum wage.

4.3 Equilibrium

In the unconstrained model discussed in Section 2 equilibrium in the labor market was reached when the wage per period received by each worker was zH_i , when each worker supplied L_i^* periods of labor, and when each firm equated the rental rate z of human capital services $L_i H_i$ with the value of the marginal product of human capital services. In the minimum wage models discussed since then, firms have adjusted their demands for labor from individuals only at the extensive margin. A firm refuses to hire or immediately fires any individual for whom H_{ji} falls short of w/z . The firm pays a wage of zH_{ji} to any worker it chooses to employ, though it will get less work than it pays for if $H_i < H_{ji}$. Such a policy has implications for the equilibrium value of z each period. The two "prices" facing each individual each period, z and g_i , are related to each other.

There are many forms this relationship could take. One particularly convenient form stems from the following assumptions about firms' behavior. Suppose risk-neutral firms know that they are making errors in assessing each worker's true skill level, and that they try to counteract the total effect of these errors by maximizing

$$(4.43) \quad E\pi_j \equiv P_j^j \left(\sum_{n=1}^m L_{jn} E(H_n | H_{jn}), C_j \right) - rC_j - z \sum_{n=1}^m L_n H_{jn},$$

subject to the constraint that $H_{jn} \geq w/z$ for all n .¹¹ Suppose further that

the joint distribution of H_n and H_{jn} is bivariate lognormal with correlation coefficient ρ_j and marginals that have means μ_n and μ_{jn} and standard deviations σ_n and σ_{jn} .

In this bivariate lognormal case,

$$(4.44) \quad E\pi_j = PF^j \left(\sum_{n=1}^m L_{jn} \exp \left\{ \mu_n + \rho_j \frac{\sigma_n}{\sigma_{jn}} (\log H_{jn} - \mu_{jn}) + \frac{1}{2} \sigma_n^2 (1 - \rho_j^2) \right\}, C_j \right) - rC_j - z \sum_{n=1}^m L_n H_{jn}.$$

Differentiating with respect to each dimension of labor input gives the optimal number of employees, m_j^* , the optimal amount of time from each employee, $\{L_{jn}^*\}$, and the optimal skill distribution of employees, $\{H_{jn}^*\}$.

These conditions define an individual firm's demand for efficiency units of labor as a function of z , r , w , and the parameters of H_i and H_{ji} . At each level of z , the sum over all J firms of these derived demands gives the demand for efficiency units of labor services. This curve is shown as $D''D'''$ in Figure 1.

The perceptions of individuals of z and of g_i lead them to offer varying amounts of L_i to the market. Weighting their optimal time allocations by their actual skill levels H_i produces a labor supply curve, SS' in Figure 1. The equilibrium conditions for this labor market are simply

$$(4.45) \quad D(z, r, w, \{H_i\}, \{H_{ji}\}) = \int_{-\infty}^{\infty} \int_0^1 L_i H_i g(H_i, S_i, X_i, z, w, J) dS_i dH_i$$

along with expression (4.42) for g_i . Curve MM' in Figure 1 gives the value of the right-hand side of (4.45) for any level of z . The intersection of MM' and $D''D'''$ determines the equilibrium value of z for any

rationing system g (...).

Unemployed labor supply, the distance between MM' and SS' at z^0 , consists of the skill-weighted time allocations of those individuals who don't get hired. Ex post, all the firms these individuals contacted thought their skills were insufficient to justify paying them the minimum wage. However, these individuals took this eventuality into account when they allocated time to the labor market. In order to get the actual amount of skill-weighted labor time on $P''D''$, employers must hire an amount of H_{ji} -weighted labor time which exceeds this level. Even so, more than the amount of skill-weighted labor time which can be hired is attracted to the market. Most of the surplus consists of those whose actual H_i is below the critical level. Some of it, however, consists of individuals whose actual skills justify paying them the minimum wage or more.

The amount of employed labor on MM' is the skill-weighted sum of the time allocations of those who succeed in getting employed. Most of these individuals have skills above the critical level. Some of them, however, aren't skilled enough to justify their being paid the minimum wage or more. They are attracted into the labor market by the higher rental rate the minimum wage affords them on their skill, and the imprecision of the rationing system works in their favor.

Finally, all other individuals in the local population comprise nonparticipants at the equilibrium value of z and g (...). Some of these individuals have high levels of skill but high reservation wages

as well. Others have fairly flat indifference curves between goods and leisure but low values of g . They are discouraged workers because they would take minimum wage jobs but believe the chance of being offered such jobs is too low to justify looking for them.

As in previous sections of this paper, the special case of perfect rationing is instructive for understanding the equilibrium diagram. If rationing were perfect, curve $D''D'''$ would shift outward to DD' . Reduced uncertainty about the quality of the labor it hires would lead firms to substitute labor for capital. Further, curve SS' would be shifted leftward to MM' as those who have less than the critical skill level would have no incentive to enter the market. Those who do enter would be certain to find employment. That is, g_i would be one for all those whose L_i 's were positive, and SS' would coincide with MM' .

4.4 Comparative Statics

PROPOSITION 1. Any labor force participant's chance of experiencing minimum-wage-induced unemployment is greater the greater is the ratio of (a) the minimum wage to (b) the rate of return to skill for those who are offered more than the minimum wage. A glance at expression (4.15) suffices to verify this assertion.

$\Pr(M_{ij_i}^* = 0)$

$= \Pr(H_{j_i i} < w/z)$ is greater the greater is w/z . So is the product of a number of such probabilities. But this product is the chance of experiencing unemployment. Formally, define

(4.46) $u_i \equiv 1 - g_i$ = i's chance of experiencing minimum-wage-induced unemployment should he enter the labor force. Then

$$(4.47) \quad \frac{\partial u_i}{\partial (w/z)} = \frac{\partial}{\partial (w/z)} e^{\log(1-g_i)} = (1-g_i) \left(\sum_{j_i=i_1}^{Jk_i(H_i, S_i)} \frac{\partial}{\partial (w/z)} \log(\Pr(H_{j_i i} < w/z)) \right), \quad \text{and}$$

(4.48) $\frac{\partial}{\partial (w/z)} \log \Pr(H_{j_i i} < w/z)$ increases when w/z increases because w/z is a fractile of a distribution.

PROPOSITION 2. Any labor force participant's chance of experiencing minimum-wage-induced unemployment is greater the smaller is the number of firms in his local labor market. Once again, a look at expression (4.15) suffices. Since the log of any probability in the open unit interval is negative,

$$(4.49) \quad \log u = \sum_{j_i=i_1}^{Jk_i} \log (\Pr(M_{ij_i}^* = 0))$$

decreases as more terms are added.

PROPOSITION 3. When firm skill judgments H_{ji} are positively correlated with individuals' actual skills H_i , u_i is greater the lower is H_i . In the bivariate lognormal case, $E(H_{ji}|H_i)$ is $\exp\{\mu_{ji} + \rho_j \frac{\sigma_{ji}}{\sigma_i} (\log H_i - \mu_i) + \frac{1}{2} \sigma_{ji}^2 (1-\rho_j^2)\}$, which falls when H_i falls. The leftward shift of the mean puts more area of the distribution below the w/z fractile.

PROPOSITION 4. An individual is more likely to participate in the labor force the greater is g_i . This follows from a glance at expression (4.36).

PROPOSITION 5. An increase in the minimum wage increases the labor force participation of those individuals with high values of g_i but decreases

the labor force participation of those with low values of g_i . From an argument similar to that given for Proposition 1 we see that

$$(4.50) \quad \frac{\partial g_i}{\partial w} = \frac{\partial}{\partial w} (1-u_i) = - \frac{\partial u_i}{\partial w} =$$

$$- (1-g_i) \frac{J k_i(H_i, S_i)}{\sum_{j_i = i_1} \frac{\partial}{\partial w} \log(\Pr(H_{j_i} < w/z)) < 0.$$

Since

$$(4.51) \quad \frac{\partial}{\partial w} (wg_i) = g_i + w \frac{\partial g_i}{\partial w},$$

the net effect on the slope of the budget constraint depends on whether the positive effect of the first term or the negative effect of the second term is stronger. The second term is close to zero for g_i close to unity, and the second term is close to $-w$ times the sum in (4.50) for g_i close to zero.

PROPOSITION 6. An individual with more than the critical skill level is more likely to participate the greater is the rental rate for skill.

This follows from (4.35).

PROPOSITION 7. If leisure is normal, an individual is more likely to participate the lower is nonwage income. This follows from (4.36).

5. EMPIRICAL AND POLICY IMPLICATIONS

Data on unemployment are hard to interpret because different respondents mean quite different things when they report they are unemployed. (i) Some individuals, those described in this paper, are searching for jobs paying the lowest wage employers can offer. This minimum will be greater than the federal or state minimum wage when firms' wage scales are established by union contract or in response to

the threat of unionization. Because the employers they have contacted don't believe their work is worth the minimum, this group of unemployed get no offers. They would be willing to work for less, but the minimum constrains them from doing so. (ii) Other individuals have unrealistically high wage expectations. They get no job offers at the high wages they demand but they are unwilling to work for less. (iii) Yet other individuals, often subsidized by unemployment compensation payments, engage in a protracted job search, refusing offers until they feel further search would bring no improvement. (iv) Some victims of seasonal and cyclical layoffs make no effort to find interim jobs, because seasonal or cyclical vacations are part of the occupations they chose.

Few surveys of those who report themselves as unemployed have been designed to get at these subtle differences among them. Rosenfeld (1977), an analysis of the best of these surveys, gives strong support for the notion that a substantial part of measured unemployment is of type (i).¹²

So even though it cannot explain all of measured unemployment, the model presented here helps account for several well known features of categorical labor force data. (i) Unemployment rates tend to be inversely related to skill levels. This is Proposition 3 of the last section. (ii) The number of discouraged workers moves procyclically with the number of unemployed. Proposition 4 predicts just such

linkage. (iii) Unemployment rates for blacks are double those of whites and labor force participation rates for black youths are lower than those of whites. The same combination of wage discrimination and lower levels of skills which accounts for black-white wage differentials is linked to job discrimination and unemployment differentials in this paper.

The comparative static propositions of the last section have two kinds of policy implications, depending on the values of ρ_j . Even if all ρ_j are zero, so that employers have no information about the skills of job applicants and unemployment is pure queueing, two kinds of policies may raise an individual's chances of avoiding unemployment or discouragement. First, the number and variety of potential employers within the individual's community may be increased. Second, the minimum wage may be lowered relative to the rental rate on skill paid to those employed at higher wages. Both such policies may be costly to society. Tax incentives and other job-creation programs divert government revenues and are hard to target. Lowering the minimum wage level may reduce the school-leaving age for some short-sighted or poor young people and lower their lifetime contributions to national income.

If the ρ_j are not zero, so that employers are able to use information about job applicants' skills, two additional policies may improve job prospects. Third, individuals' skills may be

improved by vocational training programs and by better-quality schools. Finally, the amount of information about individuals available to potential employers may be increased by programs that subsidize training or temporary work by potential employers without obligating them to hire trainees for long-term jobs. Because it entails committing fewer additional resources than the other three policies, this last course may prove to be more cost effective.

6. SUMMARY

This paper has presented a simple framework for understanding categorical data on labor markets. Expressions have been derived for the determinants of nonparticipation, employment, and minimum-wage-induced unemployment. Their comparative statics have appealing properties. Special attention has been given to the allocative role played by firms' judgments of individuals' skills when wages can't allocate labor unaided by such rationing. Joint distributions of these skill judgments and actual skills of individuals in a local labor market have been used to derive equilibrium job rationing functions. Together with the minimum wage and the rental rate for higher levels of skill, these functions determine the level and composition of a local workforce. Faulty information about individuals' skill, in addition to universal minimum wage laws, is sufficient to explain persistent unemployment. Minimum wage laws alone are not.

There are three major effects of minimum wage laws. First,

the labor supply and labor force participation of a less-skilled individual increase to the extent he believes he won't be rationed and decrease to the extent he believes he will be rationed. Second, the equilibrium rental rate on human capital services increases to the extent that less-skilled individuals withdraw from the labor market into nonmarket activities and physical capital cannot be substituted for labor. Third, equilibrium rationing of employment opportunities takes place for those with low levels of skills.

Special cases of job rationing include perfect rationing, under which firms have full information about individuals' skills, and pure queueing, under which firms have no information. Perfect rationing allows firms to offset the adverse selection of unskilled individuals into the workforce so completely that no unemployment, just discouragement of the marginally skilled, results from a universally enforced minimum wage. Pure queueing allows adverse selection full reign. Like pure queueing, the intermediate case of imperfect information leads to all three kinds of outcomes at the extensive margin of individuals' behavior.

The comparative statics of the model are consistent with the well-known concentration of unemployment among the unskilled, with the direct relationship between unemployment and discouragement, and with black-white unemployment and participation differentials. Four kinds of policies to combat unemployment were identified and compared.

NOTES

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² Pissarides (1976) considers this issue using utility functions defined over net worth rather than consumption and leisure. Clark and Summers (1979) stress the importance of nonparticipation in empirical studies of unemployment.

³ The assumption of universal enforcement of minimum wage laws increasingly is justified for urban industries with heavy concentrations of unskilled workers. Between 1947 and 1974, coverage of retail trade increased from 3% to 63% and coverage of services increased from 19% to 83%. See Welch (1976) pp. 2, 4. Modeling an uncovered sector introduces many complications not present in the simpler case discussed here. The unemployed must have preference but the model discussed here still describes the margin of choice for the unemployed.

⁴ Friedman (1962, p. 202) suggests thinking of the labor input in this fashion as consisting of a certain number of efficiency units of manhours. In local equilibrium at a point in time, each dimension of the labor input will have

the same marginal cost for employers of the same type. Pettengill (1980, 1981) has done empirical work recognizing heterogeneity among workers.

⁵ This probability is discussed at length below; see expression (4.15).

⁶ The constraint may differ across firms. Union contracts or the threat of unionization may impose minimum pay scales much higher than the federal minimum wage; see Demsetz (1961). For example, union helpers and laborers in the building trades in Cleveland in 1978 had to be paid a minimum of \$10.54 per hour, not \$2.65 an hour.

⁷ "... (S)everal studies of unemployed youth (as well as of other unemployed workers) have found that virtually no unemployed young job seeker reports having rejected a job offer during the period of search" (Freeman, 1980, p. 22). An example is Rosenfeld (1977).

⁸ In this paper, a worker changes his contribution to his employer's labor input only by changing the number of hours he works, not by changing the intensity of his work effort. If some workers are shirkers, a minimum wage need not lead to rationing, but rather may lead to a "shock effect" of increased productivity. See Stigler (1946).

⁹ In this context, certainty equivalence prevents temporary illiquidity due to unemployment from affecting the consumption of goods and keeps unemployment from spilling over into disequilibrium in the goods market. Hartley and Revankar (1974) make a similar assumption in their analysis of labor supply at the intensive margin. Marshall (1890) uses the same device in his discussion of occupational choice.

¹⁰ For a fuller discussion of this point, see Cave (1981).

¹¹ In general, taking the expectation in this simple way won't be equivalent to taking the expectation with respect to stochastic output. For a linear production function, the two procedures are equivalent.

¹² 3238 of the 4668 unemployed in the May, 1976 CPS responded to Rosenfeld's supplementary questions. The high nonresponse rate and the low potential for disaggregation indicate a need for more surveys of this kind. Yet the implications of the respondents' answers are clear. Since only 32% were on layoff, and more than 81% of laid-off workers reported some effort to find an interim job, only 6% of the unemployment could have been of type (iv). Since only 10% of those unemployed four weeks or more who had contacted at least one employer reported having refused any job offers, type (iii) accounts for a small proportion also. Type (ii) had limited support as well. 22% of the unemployed were willing to take jobs paying less than the federal minimum wage, then \$2.30 per hour. Another 33% were willing to accept a wage between the minimum and \$2.99. Only 22% reported an acceptance wage of \$4.00 or more.

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FIGURE 1.--Optimal Rationing