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A MACRO MODEL OF THE U.S. LABOR MARKET

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1. Introduction

This paper presents a five-equation postwar quarterly econometric model of the U.S. labor market. These equations describe the demand for man-hours, wage adjustments, and the employment/hours decision in the non-farm private business sector, as well as the total supply of labor when farms, households, governments, and the non-profit sector are considered as exogenous. The simultaneous nature of the relationships between the labor market and other sectors of the economy has been dealt with by the method of two-stage least squares to avoid the necessity of building a complete and probably poorly specified model of the entire economy. For example, prices, while endogenous, are left in the product market, as we reject the wage-price-productivity approach used by Perry (1966) and others.

This model has been developed as part of a continuing project at Princeton University on Systems Analysis and the Labor Market. It is felt that the existing macro-models of the labor market, mostly developed as parts of complete models of the economy, are among the weakest parts of those models. For instance, the largest forecasting errors frequently occur in wages and unemployment.\(^1\) Since these errors also affect estimates of labor compensation, many econometricians have despaired of explaining personal income directly and have fallen back on \textit{ad hoc} equations explaining profits, thus leaving personal income as the residual instead of profits.\(^2\) In this paper, we hope to shed some light on the appropriate specifications of the Phillips curve, labor force participation relations, and the demand for labor equations, and further, to reconcile our results with the theory of labor markets.

\(^1\) For example, the Wharton model: Evans and Klein (1967), the Brookings model: Fromm and Taubman (1968), the Commerce Dept. model: Liebenberg, Hirsch, and Popkin (1966).

The procedures used to estimate the model are outlined in Section 2. The variables of the model, the equations, and the outside instruments used in its estimation are given in Section 3. Each of the substantive equations is explained in Section 4. The procedures used to simulate the model are outlined in Section 5, and the results of simulation are given in Section 6.

2. Method of Estimation

As pointed out above, the equations of the model were estimated by the two-stage least squares procedure. Additional instruments were drawn from the other sectors of the economy to improve the efficiency of the estimates. Since all of the equations are linear in the parameters but nonlinear in the variables, we have followed the method proposed by Chernoff and Rubin (1953) for maximum likelihood techniques and extended by Kelejian (1963) to the two-stage least squares method. This amounts to treating each nonlinear function of one or more variables as a new variable, and proceeding with the resulting model which is linear in parameters and variables.

Autocorrelation has been removed from the equations by using Durbin's (1960) two-step technique, generalized by using two-stage least squares in each step—see Johnston (1963, p. 195).

3. Final Equations and Variables

Behavior Equations

The residuals corresponding to equations (3.2-3.4) were autocorrelated in a first-order autoregressive scheme; those corresponding to equation (3.1) were autocorrelated in a second-order autoregressive scheme. Therefore, the equations given below are the prediction equations--
e.g., the terms $u_{t-j}$ added to equations (3.1-3.4) are the corresponding estimated residuals and their coefficients are the estimated parameters in the autoregressive scheme.

The figures in parentheses beneath the parameter estimates are the absolute values of the corresponding $t$-ratios. Estimates of the parameters in the autoregressive scheme and the corresponding $t$-ratios were obtained from the first stage of Durbin's method generalized to the two-stage least squares procedure. $\bar{DV}$ is the mean of the dependent variable and $d$ is the Durbin-Watson statistic for the equation after the application of Durbin's method. Finally, the $R^2$ statistic and the standard error of estimate, $\sigma_u$, are not corrected for degrees of freedom, and are defined about the original dependent variable (e.g., in 3.1, about $\log L$ instead of $\log L - 1.18 \log L_{-1} + .4 \log L_{-2}$).

\begin{align*}
(3.1) \quad \log L &= -.05 - .0026t - .364 \log W + .557 \log Q + .223 \log Q_{-1} \\
&\quad + .092 \log Q_{-2} + 1.16 \sigma_{-1} - .40 \sigma_{-2} \\
&\quad (2.0) (1.9) (8.3) (5.3) \\
\sigma_u &= .0101, d = 2.27, 19481-19654, \bar{DV} = 4.555, R^2 = .98.
\end{align*}

\begin{align*}
(3.2) \quad \log (E/H) &= \log (\bar{E}/\bar{H}) - .044 - .003DC + .001t + .293 \log (L/\bar{L}) \\
&\quad + .384 \log (L/\bar{L})_{-1} + .052 \log (L/\bar{L})_{-2} + .72 \sigma_{-1} \\
&\quad (1.9) (11.0) (3.5) (4.2) (1.0) (8.8) \\
\sigma_u &= .0053, \bar{DV} = 3.1, d = 1.84, R^2 = .97, 19493-19652.
\end{align*}
(3.3) \[
\frac{LF_2}{P_2} = 3012.6 - 2.96DC - .960t + .338(ET/LF)_{-1} - .282EM/EP \\
\quad - .519DLY/A + .55u^{\Delta} \\
\quad (1.1)\quad (1.0)\quad (3.9)\quad (1.9) \\
\quad (1.9)\quad (5.2)^{1}
\]

\[ \sigma_u = 34.9, \quad d = 1.73, \quad 19493 - 19652, \quad \overline{DV} = 4425.4, \quad R^2 = .48. \]

(3.4) \[
\frac{LF_1}{P_1} = 9415.9 - 36.2DC + .948t - .081(ET/LF)_{-1} \\
\quad + .377EM/EP + .275u^{\Delta} \\
\quad (2.5)\quad (2.6)\quad (1.3) \\
\quad (3.8)\quad (2.2)^{1}
\]

\[ \sigma_u = 26.3, \quad d = 1.85, \quad 19493 - 19652, \quad \overline{DV} = 9712.5, \quad R^2 = .47. \]

(3.5) \[
\frac{w-w_{-1}}{v_{-1}} = .01299 + .00165D - .00114U - .0036(U-U_{-1}) \\
\quad + .1006(MPL-MPL_{-1})/MPL_{-1} + .6172(p-p_{-1})/p_{-1} \\
\quad (4.6)\quad (2.4)\quad (2.3)\quad (2.6) \\
\quad (2.6)\quad (5.0)
\]

\[ \sigma_u = .0035, \quad d = 2.19, \quad 19483 - 19654, \quad \overline{DV} = .0113, \quad R^2 = .68. \]

**Identities**

(3.6) \( L = EH \)

(3.7) \( ET = E + EN \)

(3.8) \( LF = ET + UN \)

(3.9) \( U = UN/(LF-AF) \)

**Variables Appearing in the Model**

Current values of variables prefixed with a star, *, were treated as endogenous in the estimation procedure. All other variables were taken as predetermined.
With two exceptions, all the variables listed below are seasonally adjusted. Further, all variables except wages, prices, employment, and ratio variables are expressed as annual rates. Data from the CMP accounts are measured in billions of constant dollars, while employment, population, and labor force data are measured in thousands. The labor demand side uses nonfarm private business sector employment, manhours, and compensation data compiled by the Bureau of Labor Statistics from the basic establishment-based series published in Employment and Earnings. The labor supply side uses labor force survey-based data on total economy employment, unemployment, and labor force. The difference between the two sets of data is treated here as exogenous (see identity (3.7)).

D = Variable reflecting distributional changes in output between one-digit industries, see text.

DC = 0 or 1 if before or after April 1962 revision of household survey reflecting results of 1960 population census.

*DLY/A = Ratio of labor compensation less taxes plus transfers to household wealth.

*E = Nonfarm private business sector employment, establishment data.

E = Trend in E at high employment, see text.

EM/EP = Ratio of manufacturing employment to total private employment.

EN = Employment in Gov't, households and institutions, armed forces, and on farms, plus coverage and statistical differences between establishment and labor force survey data.

*ET = Total employment including armed forces, labor force survey data.

*H = Nonfarm private business sector hours per man-year, establishment data.

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1/ The rate of man-days idle, and the discount rate of the Federal Reserve Bank of New York are the two exceptions.
\[ \bar{N} = \text{Trend in N at high employment, see text.} \]

\[ *L = \text{Nonfarm private business sector manhours, establishment data.} \]

\[ \bar{L} = \text{Trend in L at high employment, see text.} \]

\[ *LF = \text{Total labor force including armed forces.} \]

\[ *LF_1 = \text{Total primary labor force including armed forces, males 25-54.} \]

\[ *LF_2 = \text{Total secondary labor force, including armed forces, females 14 and over, males 14-24, 55 and over.} \]

\[ *MPL = \text{Marginal productivity of labor in the nonfarm private business sector, calculated from equation (3.1).} \]

\[ P_1 = \text{Noninstitutional population corresponding to the primary labor force.} \]

\[ P_2 = \text{Noninstitutional population corresponding to the secondary labor force.} \]

\[ *p = \text{Nonfarm private business sector GNP deflator.} \]

\[ *Q = \text{Nonfarm private business sector GNP in 1958 prices.} \]

\[ t = \text{Time trend variable, t = 1 in 19471.} \]

\[ *U = \text{Civilian unemployment rate.} \]

\[ *UN = \text{Civilian unemployment.} \]

\[ *w = \text{Compensation per manhour in the nonfarm private business sector.} \]

**Outside Instruments**

In addition to the variables listed below, lagged endogenous variables were also used as instruments. The functional forms of the instruments were chosen to be comparable to the specifications of the dependent variables.

\[ AF = \text{Armed forces.} \]

\[ DT = \text{Discount rate of the Federal Reserve Bank of New York.} \]

\[ EX = \text{Exports of goods and services, 1958 prices.} \]

\[ G = \text{Gov't purchases of goods and services, 1958 prices.} \]

\[ IR = \text{Gross private domestic investment in residential structures, 1958 prices.} \]

\[ INR = \text{Gross private domestic investment in nonresidential structures and equipment, 1958 prices.} \]

\[ MDI = \text{Man-days idle during quarter} \]
PROFI = Lagged ratio of corporate profits to money GNP of the nonfarm private business sector.

S_1, S_2, S_3 = Seasonal constants

WPF = Wholesale price index for consumer foods.

4. Discussion of Equations

Demand for Labor

In this model the total demand for labor inputs consists of an exogenous demand from farms, government, and households and institutions, and an endogenous demand from the nonfarm private business sector. Within this latter sector we dichotomize the determination of labor input into an equation of demand for manhours and a separate equation to determine the division of manhours between employment and hours of work. In accordance with Nerlove's (1967) conclusion that "... the true structural relationships that belong in macro-econometric models are not in fact production functions, but rather derived demand functions subject to distributed lags," we derived the equation for the demand for manhours by maximizing profits subject to a CES production function.

Consider the function

\[(4.1) \quad Q_t^* = [\alpha_1 (e^{\lambda L_t^*}) \frac{\sigma-1}{\sigma} + \alpha_2 (e^{\mu K_t^*}) \frac{\sigma-1}{\sigma} - h \sigma^{-1} u_t^*] \]

where Q_t, L_t, and K_t are, respectively, real output, manhours, and real capital stocks in the nonfarm business sector at time t (asterisks denote equilibrium values of the variables); u_t is a lognormally distributed disturbance term in production at time t with a mean of unity and a constant variance; \( \sigma \) is the elasticity of substitution; \( h \) is the degree of returns to scale, and \( \lambda \) is the rate of labor augmenting technical progress.
The marginal productivity condition for profit maximization with respect to choice of manhours input is

$$\frac{\partial Q_t^*}{\partial L_t^*} = h \alpha_t e^{\lambda t} (e^{\lambda t} L_t^*)^{-1} \sigma * u_t^* - \frac{1}{\sigma} \left( \frac{1 - \sigma}{\partial h} \frac{g^* - 1}{\partial h} \right) w_t^* = \beta \frac{p_t^*}{p_t} v_t^*.$$  

In this equation the first derivative of the production function with respect to $L_t^*$ is set equal to $w_t^*$, compensation per manhour at time $t$, divided by $p_t^*$, the deflator for nonfarm private business sector GDP at time $t$ (again asterisks denote equilibrium values). $\beta$ is a factor reflecting any constant economy-wide tendency for profit maximization to leave the economy short of the perfectly competitive position, such as monopoly power. $v_t$ is a lognormally distributed disturbance term at time $t$ including any random factors such as ignorance, management errors, riots, or other unanticipated factors preventing profit maximizing conditions from being realized. Equation (4.2) can then be solved for the logarithm of equilibrium manhours as

$$\log L_t^* = \log [\beta^{-\sigma} (\kappa \lambda t)^\sigma] - (1 - \sigma) \log w_t^* - \sigma \log \frac{p_t^*}{p_t} \left( \frac{1 - \sigma}{h} \right) \log Q_t^* + \frac{g^* - 1}{h} \log u_t^* - \sigma \log v_t^*.$$  

A number of writers have suggested reasons why observed data on manhours, output, and real wages will not correspond to points on an equilibrium derived demand function like (4.3), in addition to the postulated random disturbances in production and profit maximization. For example Oi (1962), Okun (1963), and Kuh (1965) have emphasized factors which result in costs of changing the level of labor input, such as training costs, severance pay, overtime payments, and costs of rescheduling work. Eisner and Strotz (1963) and Gould (1968) have shown that such costs can be treated
by allowing distributed lags in equations such as (4.3).\(^1\)

It is also likely that observed prices, wages, and output differs from the expected values of these variables, which are more likely to be the relevant factors in management decisions. The familiar adaptive expectations theory suggests that the expected value of a variable can be expressed as a distributed lag of past values.

For these reasons the equilibrium values of the variables in (4.3) are approximated as distributed lags of actual values. That is, for any variable \(X_t^*\), let \(X_t^* = a_0X_{t} + a_1X_{t-1} + a_2X_{t-2} + \ldots\). To avoid statistical problems associated with lagged values of the dependent variable, we in effect solve the resulting difference equation for \(\log L_t\) alone as a function of lagged values of the independent variables. In view of the variety of theoretical reasons for expecting distributed lags, no artificial constraints were imposed on the shapes of the lag distributions. The appropriate length of the distributed lag for each variable was determined empirically by test of significance. Autocorrelation introduced by solving out for the observed dependent variable was removed by a second order autoregressive process estimated by the Durbin method. For a discussion of the problems involved in such models, see Griliches (1967).

The estimated equation is shown as equation (3.1) in section 3 above. Manhours, output, and real compensation per manhour were all treated as jointly dependent variables in the general Keynesian framework.

\(^1\) A different treatment, suggested by Fair (1968), is to compute a measure of excess labor and then to modify the equation to take account of the failure of observed data to correspond to points on the derived demand function.
The (stable) second order autoregressive transformation has removed the autocorrelation, as shown by the Durbin-Watson statistic of 2.27. The distributed lag on output has three significant terms, while the lag distribution on real wages only contains one significant term, the current value. The standard error of estimate of the original dependent variable, log L, is only 0.25% of its mean value. The $R^2$ is .98.

The implications of (3.1) for the parameters of the underlying production function are obtained from the equilibrium form of the equation, that is, adding up the distributed lag terms for a coefficient of .872 on log $Q$, -.364 on log w/p and -.0026 on $t$. Using the transformations implied by the coefficients of equation (4.3), the identifiable parameters are estimated as follows (with asymptotic standard errors in parentheses following): Elasticity of substitution $\sigma = .364 (.19)$. Returns to scale parameter $h = 1.25 (.06)$. Rate of labor augmenting technical progress $\lambda = .0041 (.0014)$, or about 1.6% per year. A one-tailed test rejects the hypothesis that $\sigma = 0$ versus the alternative $\sigma > 0$ at the 95% level of significance, the one-tailed test being appropriate since $\sigma > 0 \ a \ priori$. Likewise the returns to scale parameter is significantly different from unity on a two-tailed test at the 99% level of significance, giving evidence of increasing returns to scale. The rate of labor-augmenting technical progress is also significantly different from zero at the 99% level.

Given the variety of theoretical reasons leading to distributed lags in (4.3) as well as the degree of aggregation involved, one may remain a bit dubious as to how closely the parameter estimates given
above approximate the parameters of a true production function. But other researchers have arrived at generally similar results for aggregative CES production functions.\footnote{See Nerlove (1967), pp. 92-100 for a review of recent estimates of aggregate CES functions. Diwan and David-van de Klundert obtain estimates of .37 and .32 for $\sigma$. Other estimates by Kravis and Kendrick-Sato are biased upward if technical progress is primarily labor augmenting, rather than Hicks neutral. The recent estimates by Bodkin-Klein (1967) of $\sigma = .47$, $h = 1.24$ cover the period 1909-49. Bischoff's (1968) estimate of $\sigma = 1.02$ from a post-Korean War quarterly investment function is notably different, but he assumes $h = 1$, as do all others except Bodkin-Klein. If $h > 1$ and $\sigma < 1$, this leads to an upward bias in the estimate of $\sigma$, since est. \[ (h-1)(1-\sigma) \cdot \text{cov}(\log Q, \log w/p) \] \[ \sigma = \sigma + \frac{h}{\text{var}(\log w/p)} > \sigma \text{ because output and real wages are positively correlated (read the real cost of capital instead of real wages in Bischoff's case).} \]}

Division of Manhours between Employment and Hours

If labor input of manhours in the production function (4.1) is defined as the product of employed men $E$ and hours per man-year $H$, this amounts to the assumption of unitary elasticity of substitution between the two ways of adding to the input of labor services: additional employment with given hours per man-year, and additional hours per man-year with given employment. Thus the optimal provision of any given level of manhours involves balancing the expected relative costs of adding men or adding hours in a cost minimizing framework.

The cost of adding hours with a given number of men is equal to the straight-time hourly wages (including any hourly-based fringes) times the number of men $E$ if hours are less than normal hours $\bar{H}$ (a cycle-corrected trend). But if hours exceed $\bar{H}$, the cost will include the overtime premium $o$, yielding $(s + o)E$. This is modified further by a factor
describing lower efficiency or higher cost when \( H > \bar{H} \), because workers are expected to be less efficient as hours lengthen, in the short run. This factor is approximated as \( (\frac{H}{\bar{H}})^{\alpha} \), giving

\[
(4.4) \quad (s + o)E(\frac{H}{\bar{H}}), \quad \alpha > 0, \quad o = 0 \text{ if } H \leq \bar{H},
\]
as the additional cost of adding hours. Aggregation over differing industries should soften the break at \( \bar{H} \).

Concerning additional employment, an employer's hourly cost is the straight-time hourly wage plus the hourly cost of fringe benefits such as insurance and vacation \( f \), or \( s + f \). The employer also takes on fixed costs in pension liability (depending on eligibility and funding), hiring and training costs, and liability for unemployment compensation (either supplemental benefits or change in tax through experience rating). The significance of these fixed costs varies inversely with the expected duration of the new worker's job. It is assumed that whenever employment is below its trend value \( \bar{E} \), increases in employment are expected to be more permanent than decreases. Conversely, whenever employment is above its trend, decreases in employment are expected to be more permanent than increases. Therefore the expected hourly cost exceeds \( s + f \) when employment \( E \) exceeds trend \( \bar{E} \) and is less than \( s + f \) when \( E < \bar{E} \). The total additional cost of hiring an additional worker, given hours \( H \), may be approximated as

\[
(4.5) \quad (s + f)H(\frac{H}{\bar{H}})^{\beta} \quad 8 > 0.
\]

In effect this converts the uncertain allocation of fixed costs into a certainty equivalent.
The ratio of (4.5) to (4.4) is a ratio of marginal costs and is therefore the slope of an isocost line in \((E, H)\) space, with a slight kink at \(\bar{H}\). Cost minimization implies that this ratio should equal the slope of the isoquant \(EH = L\) for fixed manhours \(L\), which is just \(\frac{dH}{dE} = \frac{H}{E}\). Thus

\[
(4.6) \quad \frac{(s + f)H(E/E)}{(s + o)E(H/H)^\alpha} = -\frac{dH}{dE} = \frac{H}{E}
\]

and cancelling \(H/E\),

\[
(4.7) \quad \frac{E/E}{H/H} = \frac{s + o}{s + f}.
\]

After taking logarithms and using some identities, one finds the following equation for the equilibrium division of manhours into employment and hours of work: \(^1/\)

\[
(4.8) \quad \log \frac{E}{H} = \log \frac{E}{H} + \frac{\alpha - s}{\alpha + s} \log \frac{L}{L} - \frac{2}{\alpha} \log \left(\frac{s + f}{s + o}\right).
\]

In practice, a distributed lag would be expected on the ratio of manhours to trend in (4.8), because an increase in manhours above trend would be met first by raising both hours and employment, and then by raising employment further and cutting hours back, as the new level of manhours becomes regarded as permanent. Since the overtime premium has been set by law at halftime, \(\frac{s + f}{s + o} = 2/3(1 + \frac{f}{g})\). But the ratio of fringes to straight-time pay has behaved just like an increasing time trend over the postwar period; its effect cannot be separated econometrically from any trend-like effects, possibly including other supply-side influences such as the desire for increased leisure. Therefore a time trend has been used in place of \(f/s\), to avoid over-interpretation. \(\bar{E}\) was

\(^1/\)Taking logarithms, (4.7) can be regarded as a linear equation in \(\log \frac{E}{E}\) and \(\log \frac{H}{H}\). This can be solved together with the identity \(\log \frac{E}{E} + \log \frac{H}{H} = \log \frac{L}{L}\) for the two variables \(\log \frac{E}{E}\) and \(\log \frac{H}{H}\). Substituting these results into the identity \(\log \frac{E}{H} = \log \frac{E}{H} + \log \frac{E}{E} - \log \frac{H}{H}\), one obtains (4.8).
computed by subtracting actual employment in the exogenous sectors out of
total potential employment at a 4% unemployment rate, as calculated from
the labor force participation functions discussed below.\(^{1/}\) \( \bar{H} \) is taken as
constant at the mean of \( H \). This is equivalent to setting \( \bar{H} = \bar{H}_0 e^{bt} \), since
taking logarithms puts the trend part of \( \bar{H} \) into the general time trend
in any case. \( \bar{L} \) is defined as \( E / \bar{H} \).

In the estimated version of equation (4.8) in the list of equations
above, a dummy was added to allow for the shift in \( \bar{L} \) that occurred when
Alaska and Hawaii were brought into the Census data underlying \( \bar{L} \) in 1960.
A first order autoregressive transformation with \( \rho = .72 \) sufficed to
eliminate the autocorrelation in the residuals, bringing the Durbin-Watson
statistic to 1.84. The hypotheses about the distributed lag are upheld.
The standard error of estimate is only 0.2% of the mean value of \( \log E/H \),
and the \( R^2 \) about \( \log E/H - \log E/\bar{H} \) is .97.

The Wage Equation

The wage adjustment model put forth by Phillips (1958) and extended
by Lipsey (1960) may be termed a discrete disequilibrium wage adjustment
model. That is, in each period a disequilibrium position is assumed to
exist in the labor market, thus generating a wage adjustment. The rate of
unemployment is used as the measure of the extent of disequilibrium; the
wage adjustment is measured in terms of the percentage change in money
wages. In order to synchronize the wage adjustment with the corresponding
disequilibrium position (rate of excess demand), wages are implicitly
assumed to be adjusted at discrete points in time — namely, at the center

\(^{1/}\) For the technique, see the section on labor supply below.
of our observations on unemployment. Therefore, one imagines a continuous build-up of excess demand pressures during the time period between the points of observation and then the corresponding wage adjustment which partially relieves the pressures.

The relevance of such a discrete model of wage adjustments is questionable. For instance, assume that employers learn to anticipate potentially disruptive conditions and therefore eliminate them by making appropriate wage adjustments. In such a situation, wages would be adjusted in response to changes in the determinants of the demand and supply of labor -- e.g., in response to changes in the price level. Then, the rate of excess demand would be zero in each period over time. The model linking wage adjustments to the rate of excess demand would therefore imply that wage changes are zero over time. However, classical economic theory suggests that under such conditions the time path of wages would be determined by the time path of the value of marginal productivity of labor. Therefore, the percentage change in money wages would be related to both the percentage changes in product prices and the marginal physical productivity of labor. This model of wage adjustments may be said to be a continuous equilibrium model; it is particularly relevant when the time period of adjustment is shorter than that defined by our data. One should also note that in the continuous equilibrium model wage changes would depend on variables such as price changes as a result of market perfection. 1/ Indeed, the above argument implies that the existence of a relationship between wage adjustments and unemployment depends upon market imperfection.

1/ As an example, escalation clauses in labor contracts may serve the purpose of perfecting the market.
The approach taken in formulating the wage equation in this paper is a mixture of the discrete and the continuous models of wage adjustments.

That is, percentage changes in money wages, $\dot{w}$, are related to the rate of unemployment, $U$, and to percentage changes in both product prices and the marginal productivity of labor, $\dot{p}$ and $MPL$.\(^{1/}\) Percentage changes in the value of the marginal product were decomposed into $\dot{p}$ and $MPL$ in order to allow for differences in their respective reaction parameters. Somewhat more formally, the wage equation considered was

\[(4.9) \quad \dot{w} = f(U, MPL, \dot{p}, D) + e\]

where $e$ is a disturbance term and $D$ is a variable which measures changes in the distribution of output between one-digit industries. The variable $D$ was considered in order to account for changes in the rate of vacancies, $V$ and also to account for changes in average hourly earnings that may result from shifts in the distribution of employment from low-wage to high-wage industries (Routh (1959)). It should be noted that unless $V$ is held constant or otherwise accounted for, the relationship between $U$ and the rate of excess demand will not be monotonic; thus, the relationship between $\dot{w}$ and $U$ will not be stable (see Lipsey (1960)).

Changes in the distribution of output at time $t$ were measured in terms of the formula

\[(4.10) \quad D_t = \sum_{j=1}^{12} (S_{jt} - S_{jt-1})^2,\]

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\(^{1/}\) Observations on the marginal productivity of labor were derived by differentiating the C.B.S. production function with respect to manhours and then using equation 3.1 of Section 3 to estimate the unknown parameters.
where $S_{jt}$ is the percentage of total output produced by one-digit industries which is produced at time $t$ by the $j^{th}$ one-digit industry. It is clear that $D_t$ will be zero if the economy grows, or remains stagnant, in such a way that each sector continues to produce the same percentage of output. The time path of $D_t$ is outlined in Chart 1. A point to note is that distributional changes in output have been especially mild in the period since 1962.

We turn now to consider a technical difficulty encountered in translating equation (4.9) into an operational regression model. Specifically, when using quarterly data, the dependent variable in the wage equation is often formulated in terms of overlapping four-quarter percentage changes in money wages. Two assumptions underlie such a formulation. First, wages are assumed to be adjusted once a year, and second, one-fourth of all such adjustments are assumed to take place in each quarter. However, it is shown in the Appendix that these assumptions are not sufficient to eliminate an aggregation bias, and further, that the assumptions necessary to minimize such a bias are strong enough to enable us to use a much less restrictive model. Specifically, it is shown that the wage variable should be formulated as a one-quarter percentage change regardless of how frequently wage agreements are negotiated. It is further shown that the formulation of the independent variables should correspond to the hypothesis concerning the frequency with which wages are adjusted. For instance, if wages are adjusted once every $j$ quarters the relevant percentage change in prices is the change over the preceding $j$ quarters.

1/ See, for instance, Dicks-Mireaux and Dow (1959), Klein and Ball (1959), Perry (1966), and Klein and Evans (1967).
The TSLS results corresponding to (4.9) are given by equation (3.5) in section 3 above. The variables $\dot{w}$, $\dot{p}$, and MPL are formulated in terms of one-quarter percentage changes (see Appendix). Lags were considered in each of the regressors but, with the exception of the unemployment variable, were not significant. It is noted that each of the parameter estimates given in (3.5) has the expected sign and is statistically significant at the 5% level. Further, the value of the Durbin-Watson $d$ statistic suggests that the residuals are not autocorrelated. Therefore, the results given in (3.5) are taken to support the hypothesis that wage adjustments are determined by a process which can be considered as a mixture of the discrete and continuous models of wage adjustments.\footnote{The steady state Phillips curve implicit in (3.5) is obtained by setting $D = \Delta U = 0$ and $\ddot{p} = \dot{w} - APL$ with MPL and APL set at their 1948-65 averages. With both variables on an annual basis in percentage points, the result is $\dot{w} = 9.8 - 1.2U$. Thus at 4% unemployment, steady-state $\dot{w}$ = 5% per annum and $\ddot{p} = 2.2\%$ per annum.}

The formulation of $\dot{p}$ and MPL in (3.5) is implicitly based upon the assumption that the wage variable is considered for adjustment every quarter. Because compensation per manhour is largely determined by average hourly earnings, this assumption seems reasonable (see Appendix). However, the alternative assumption usually considered -- that wages are adjusted once a year -- was tested by expanding the basic model (4.9) to include the four-quarter percentage change in prices, $\dot{p}_4$, and the four-quarter percentage change in the marginal product of labor, MPL$_4$.

This is the test suggested in the Appendix for choosing between these hypotheses. The results of that regression are

$$\begin{align*}
(4.11) \quad \dot{w} &= .0097 + .002(\ddot{U}) - .0059(U-U_{-1}) + .042(U-U_{-1}) + .078\text{MPL} + .448 \dot{p} \\
&\quad (2.7) \quad (2.7) \quad (.93) \quad (2.7) \quad (1.63) \quad (2.5) \\
&\quad + .058 \dot{p}_4 - .0091 \text{MPL}_4 \\
&\quad (1.2) \quad (.6)
\end{align*}$$

$\hat{\sigma}_u = .00355, d = 2.1, 19483-654, \bar{w} = .0113, R^2 = .6858.$
It is noted that $\dot{p}$ and MPL are jointly significant at the 5% level, 
$F = 3.48 > F_{2.62}.95 = 3.15$, while $\dot{p}_4$ and MPL$_4$ are not, $F = 1.014 < 3.15$. Therefore, the four-quarter percentage change model was rejected.

The estimated residuals corresponding to (3.5) are outlined in Chart 2. It should be noted that there is no evidence of a negative trend in the residuals after 1962. Dummy variables accounting for the establishment of the wage-price guideposts and the Wage Stabilization Board were considered but did not contribute significantly to the analysis. Thus it appears that governmental attempts at wage stabilization have not been successful in the postwar period in affecting the aggregate rate of wage increase in the non-farm business sector.

The result for the wage-price guidepost is in sharp contrast to that described by Simler and Tella (1968) concerning the negative trend in the projected residuals after 19621 from Perry’s (1964) equation for the manufacturing sector. One reason for this is that Perry's equation did not have a variable, such as D, which accounts for the rate of vacancies and shifts in employment. Concerning this point, the reader should recall that distributional changes in output as measured by D were especially mild in the period after the first quarter of 1962. It is also likely that guideposts were more significant in a highly unionized sector such as manufacturing, while being less significant in the aggregate.

---

1/ The guidepost dummy variable took on the value of 1 for the quarters 19621-19654 and was zero otherwise. The other dummy variable reflecting the operation of the Wage Stabilization Board took the value 1 during the Korean War and zero elsewhere.
Two other modifications of the wage equation were considered. First, because of various institutional considerations concerning the wage adjustment process, equation (4.9) was broadened to include the ratio, and the change in the ratio of corporate profits to output in the nonfarm private business sector. In the second modification considered, equation (4.9) was broadened to include the square of the unemployment variable. This was done in order to account for the possibility that the conditional function relating $\dot{w}$ to $U$ is concave upward as suggested by Phillips (1958) and Lipsey (1960). However, both modifications were rejected because the added variables were not statistically significant at the 5% level. \textsuperscript{1/} Thus the results of this section do not support the contention that wage adjustments are determined primarily by institutional forces (see Kaldon (1959) or Hines (1964)).

Supply of Labor

The cyclical behavior of the aggregate labor force is best explained by participation functions as developed by Tella (1964, 1965) and modified by Black and Russell (1966). These functions relate the labor force participation rates, defined as the ratio of labor force LF to population $P$, of various age-sex subsets of the working population to measures of

\textsuperscript{1/} The reader should note that the arguments given by Phillips and Lipsey for the shape of the curve relating $\dot{w}$ to $U$ probably do not apply to the bulk of U.S. postwar experience. First, the Phillips argument that wages are downwardly rigid defines the shape of the curve relating $\dot{w}$ to $U$ in the fourth quadrant. Since, in the postwar period in the U.S., $\dot{w} > 0$ for all but one quarter, 1949Q4, the observable relationship between $\dot{w}$ and $U$ lies in the first quadrant. Second, the argument given by Lipsey concerning nonlinearities between the rate of excess demand and that of unemployment, has empirical relevance only to the extent that the rate of unemployment hovers at and below the frictional rate. Thus, the bulk of U.S. postwar experience lies between the ranges considered by Phillips and Lipsey.
employment opportunities. Tell's work (1965) has shown clearly that there are two major age-sex groups with differing cyclical behavior (apart from trend): the primary labor force of males aged 25-54, characterized by high average participation rates and little response to cyclical changes; and the secondary labor force of women aged 14 and over plus males aged 14 to 24 and 55 and over, characterized by lower average participation and higher responsiveness to cyclical changes.

The average participation rate of one of these groups can be regarded as the probability that a member of the group population will be a member of the group labor force. Three hypotheses are made concerning this probability.

The first hypothesis is usually called the "discouraged-worker effect". This suggests that, mainly for secondary workers, the smaller is the probability of finding work, the smaller will be the probability that any given person will be in the labor force. This effect might better be called the "opportunity" effect, because as Mincer (1966) pointed out, the cyclical response of secondary workers may be more a result of voluntary choice of the timing of partial labor force participation than a result of involuntary withdrawal from the labor force. Some recognition lag is likely before the labor force responds to changes in the opportunity to find a job. The probability of finding "jobs in general" is measured by the lagged overall employment/labor force ratio, which is the conditional probability of being employed, given that one is in the labor force. The overall employment rate is preferred to Tell's use of the group employment rate because it is felt that there is substantial substitutability among age-sex groups in filling jobs.
The second hypothesis may be called the "specific opportunity" effect. When jobs most frequently held by members of a specific group become less available, the probability that members of the group will be in the labor force is lower. The share of manufacturing employment in total employment is used to measure this effect, since gains in manufacturing relative to non-manufacturing increase opportunities for primary workers and reduce them for secondary workers.

The third hypothesis, the "additional-worker effect," suggests that when primary workers' incomes are reduced, additional secondary workers come into the labor force to bolster family income. This effect is measured by the ratio of disposable labor income to household wealth, which includes not only earned income, but also transfer payments less taxes.

A statistical controversy of some significance for the estimation of participation functions was raised when Mincer (1966) pointed out that non-cyclical or autonomous factors might produce a positive correlation between labor force LF and employment ET independently of the cyclical factors discussed above. For example a reduction in the average number of children per family would increase both female employment and labor force. Tell's (1964, 1965) regressions of the form \( \frac{LF}{P} = a + b \frac{ET}{P} \) would thus give an upward-biased estimate of cyclical factors in employment and labor force because of the non-cyclical correlation between ET and LF. As Black and Russell (1968) have shown, this bias is eliminated when the regression is formulated as \( \frac{LF}{P} = a + b \frac{ET}{LF} \), because the common autonomous factors cancel out of the ET/LF ratio. This solution to the problem at first glance appears to introduce a downward bias, however, because the effect of a disturbance term is transmitted to the endogenous
labor force variable, which appears both in the numerator of the participation rate and in the denominator of the employment rate. But in the estimated equations the employment rate is lagged one period, thus eliminating this source of bias. Note that autocorrelated disturbances would re-introduce the bias, but the equations estimated have been transformed by the use of Durbin's method to eliminate the autocorrelation.

The equations (3.3) and (3.4) listed in the table in section 3 for the primary and secondary labor force uphold the hypotheses cited above. The general opportunity effect as measured by the overall employment rate raises the participation rate of secondary workers by a third of a point for a one-point rise in the overall employment rate. This effect was negative but insignificant for primary workers. A one-point rise in the share of manufacturing employment raises the primary participation rate by .38 points and lowers the secondary rate by .28 points, although these changes are not directly comparable because the secondary population is much larger than the primary population. At 1965 levels this would reduce the total labor force about 160 thousand, or a little over 0.1%. Most significant is the finding of a substantial income effect which reduces secondary labor force participation and represents the additional worker effect.\footnote{Ray Fair suggested these estimates might be biased upward, since the disturbance terms might be correlated with EM/EP if EM/EP is an increasing function of $E_1/E_2$. As a test, EM/EP was lagged one period, resulting in only minor changes in coefficients.}

Both equations explain about 50% of the variance about the mean of the untransformed dependent variable. Given the substantial sampling errors in the data, this degree of fit is reasonably good and accords with the best results of other workers. The standard error of estimate of the secondary equation is 0.8% of the mean of the untransformed dependent variable, or about a third of a percentage point of participation.
The standard error of estimate of the primary equation is only 0.3% of
the mean of the dependent variable, or a fourth of a percentage point,
the better performance resulting from less intrinsic variation in the
primary labor force rate.

Equations (3.3) and (3.4) were used to generate potential employ-
ment $\bar{E}$ at a 4% unemployment rate as follows. EM/EP was regressed on
ET/LF and time, and the resulting equation substituted into (3.3) and
(3.4) in place of EM/EP. The ratio of disposable labor income to
household wealth was set at its postwar mean value. $(ET-AF)/(LF-AF)$
was set at 96% to obtain high employment trend participation rates.
Together with population data, these trend rates yield total potential
labor force and employment corresponding to a 4% unemployment rate.

5. Method of Simulation

Having estimated our model we turn now to investigate a few of
its properties. We do this by simulating the model under various con-
ditions. Unfortunately, classical multiplier analysis could not be
undertaken because we were unable to obtain closed reduced-form
equations containing additive disturbances.

The solution to the equations of the model were obtained as follows.
First, equations (3.3) and (3.4) were aggregated into a supply equation
with dependent variable LF. Then, using historical values of the pre-
determined variables, and also for $Q$, $p$, and initially, $w$, equations
(3.1), (3.2), and the aggregate supply equation were solved recursively
for the corresponding endogenous variables. Because the derived supply
equation is a nonlinear difference equation with respect to unemployment,
this equation was solved by an iterative procedure which converged to a

\footnote{The disturbance terms in equations (3.1-3.4) were set equal to zero
in the simulations.}
solution vector of values of unemployment. In this procedure, the initial value of unemployment in 19493 was held constant at its historical value for a formal presentation of such procedures, see Klein and Evans (1967, pp. 39-39).

Using the solution results of these three equations along with the input required to obtain that solution, equation (3.5) was then solved iteratively for compensation per manhour, w. In solving equation (3.5), the initial value of w in 19494 was set equal to its historical value. Equations (3.1), (3.2), and the aggregate supply equation were then resolved in the same manner as before with the exception that the solution values, rather than the historical values of w were used as part of the input. This sequence was continued until the solution vectors of the endogenous variables became invariant to the fourth significant figure.

6. Simulation of the Macro-Model of the Labor Market

Basic Simulation

In the five equation model described above, wages depend on unemployment, employment depends on wages, and the circle is closed by relating total labor force (and hence unemployment) to employment opportunities and disposable income. Given prices and GNP, this simultaneous model determines wages, employment, disposable labor income, unemployment, and labor force. A stringent test of the model's explanatory power is given by computing the levels of wages, employment, unemployment, labor income, and labor force predicted by the entire model over the postwar period. Prices, GNP and variables such as population and the trend level of employment are taken to be equal to their actual
values. In this test, the predicted values of all other endogenous variables are used whenever lagged values of endogenous variables appear in the model. The results of this basic simulation for the unemployment rate and other major variables are presented in Charts III-VII and in Table I.

Table I

Measures of Fit of Basic Simulation: 1950I-1965II

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\frac{1}{1 - r^2}$</th>
<th>$\text{RMSE} \frac{1}{1 - r^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compensation per manhour, NFBS</td>
<td>.999</td>
<td>$0.018$</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>.816</td>
<td>0.53%</td>
</tr>
<tr>
<td>Total employment</td>
<td>.982</td>
<td>471 thousand</td>
</tr>
<tr>
<td>Hours per week, NFBS</td>
<td>.953</td>
<td>0.13 hours</td>
</tr>
<tr>
<td>Disposable labor income</td>
<td>.999</td>
<td>$1.83$ billion</td>
</tr>
</tbody>
</table>

It can be seen that the model reproduces labor market behavior quite accurately. Turning points in unemployment are all predicted correctly, and the root mean square error (RMSE) is about a half a percentage point. The wage rate is tracked very closely, with RMSE of less than two cents. This is significant because the rate of change method of predicting wages could have resulted in errors which increase in size through time. Turning points in total economy employment and the nonfarm business sector workweek are also predicted correctly. The

\[
\frac{1}{1 - r^2} \text{ is the square of the ordinary correlation coefficient between a variable } x \text{ and its predicted value } \hat{x}. \text{ RMSE is the root mean square error of prediction, or } \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{x}_i)^2}. \]
root mean square error is 471 thousand for employment, relative to a mean value of 67.5 million, while the RMSE for the workweek is 0.13 hours per week. Disposable labor income is predicted with a root mean square error of only $1.8 billion, relative to a mean of $280 billion.

There are some significant errors. Wages are over-predicted in 1955-56. Employment is under-predicted and unemployment over-predicted in 1956-57 and 1960 because of the well-known but puzzling low productivity gains in those peak years. In 1963-64 wages are under-predicted, in contrast to the over-predictions found by Perry (1966) and Simler and Teis (1968). As a result, employment is over-predicted and unemployment is under-predicted in this period.

To our knowledge, no other model of the labor sector has been subjected to this strenuous test over the entire period of estimation, using predicted values for lagged endogenous variables. Three other models have been simulated over a shorter period, which is included in the period of our simulation. The Federal Reserve-MIT model has been simulated from 1961 to 1964 by Teis and Tinsley (1968). The Wharton Econometric Forecasting Unit model has been simulated by Evans and Klein (1967) for 1963 and 1964 only. The Brookings-SSRC model has been simulated by Fromm and Taubman (1968) from 1960 to 1962. It should be emphasized that the Wharton, Brookings, and FRB-MIT models simulate the behavior of the entire economy, and thus can have errors in prediction of GNP and prices, which are taken as exogenous to our model. Both the Wharton and FRB-MIT simulations are within 1% of actual real GNP, however. The Wharton model is within 1/2% on prices, while the FRB-MIT model takes prices as exogenous. The Brookings model has errors of the order of 2% in GNP and 4% in prices, however, so any
comparison of its labor market results with our model would be meaningless. The results of simulations of the Wharton and FRB-MIT models are compared with the simulation of our model in Table II. As the figures show, the labor sector of the Wharton model is clearly inferior to the other two models. The only cases in which the FRB-MIT model out-performs our model in predicting unemployment are 1963 and 1964, while our model is better in 1962. On the other hand, our model is closer on wages in 1964. It should be noted that while the FRB-MIT simulation begins in 1961, ours begins in 1950.

Table II

Percentage Errors in Different Models

<table>
<thead>
<tr>
<th>Unemployment Rate Predictions</th>
<th>Wage Rate Predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black-Kelejian Model</td>
<td>FRB-MIT Model</td>
</tr>
<tr>
<td>1961 1%</td>
<td>-1%</td>
</tr>
<tr>
<td>1962 1%</td>
<td>+8%</td>
</tr>
<tr>
<td>1963 -5%</td>
<td>+1%</td>
</tr>
<tr>
<td>1964 -14%</td>
<td>+8%</td>
</tr>
</tbody>
</table>

Sources: See footnote 14.

The macro-model emerges from this test as an imperfect but usable tool for the explanation of labor market behavior in the postwar period. Its largest errors occur because we do not yet fully understand the movement of productivity at all times. Most of the time, however, it performs well enough, both absolutely, and relative to alternative models. With this evidence in hand, it was decided that the model was reliable enough to be used to predict the response of the labor market to exogenous shocks.
Armed Forces Simulation

The number of men in the armed forces enters the model in two ways—as an exogenous element of demand for labor, and as a portion of total "job opportunities" to which the total supply of labor to civilian and military occupations responds. Thus a simulated increase in the armed forces, while interesting in itself for policy purposes, demonstrates the functioning of the model as an inter-related system.

The simulation takes the form of a gradual increase in the armed forces beginning in early 1961 to a peak 500 thousand above the actual armed forces in 1962, followed by a decrease back to normal in 1963. This pattern is super-imposed on the actual armed forces figures from 1961 to 1963. It is assumed that compensatory fiscal and monetary policies keep GNP and prices unchanged.

The results are shown in Table III. Concentrating on the peak period, 1962IV, it can be seen from columns (1) to (4) that when the armed forces increase 500 thousand, the model suggests that the total labor force expands by 105 thousand, or 21% of the increase. Reduced civilian employment accounts for 127 thousand, or 25.4% of the increase, while the remaining 268 thousand, or 53.6% come from the ranks of the unemployed. This is not to suggest that the composition of the men taken in by the armed services will be in these proportions, but that the economy will respond in such a way as to bring about this result regardless of where the men actually come from.

The causation runs as follows: the unemployment rate falls a third of a percentage point, pushing up wages by about one cent an hour. In response to higher wage costs, employers economize on labor and reduce total manhours slightly. The reduced availability of men, except at even higher wages, leads to a slightly longer workweek for the smaller number working at civilian jobs.
<table>
<thead>
<tr>
<th>Quarter</th>
<th>△ Civil. Enpl.</th>
<th>△ Armed Forces</th>
<th>△ Total Unempl.</th>
<th>△ Total Labor Force (1)+(2)+(3)</th>
<th>△ Unemployment Rate %</th>
<th>△ Comp. per MH NFBS</th>
<th>△ Hours per M-Y NFBS</th>
<th>% Change in Comp. per Manhour</th>
<th>△ in Disposable Labor Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960IV</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6.18</td>
<td>0</td>
<td>0</td>
<td>+0.74</td>
<td>+0.74</td>
</tr>
<tr>
<td>1961I</td>
<td>-37</td>
<td>+100</td>
<td>-65</td>
<td>-2</td>
<td>6.82</td>
<td>0.001</td>
<td>+1.2</td>
<td>+0.44</td>
<td>+0.49</td>
</tr>
<tr>
<td>1961II</td>
<td>-58</td>
<td>+200</td>
<td>-118</td>
<td>+24</td>
<td>7.01</td>
<td>0.002</td>
<td>+1.7</td>
<td>+0.98</td>
<td>+1.03</td>
</tr>
<tr>
<td>1961III</td>
<td>-79</td>
<td>+300</td>
<td>-175</td>
<td>+46</td>
<td>6.91</td>
<td>0.004</td>
<td>+2.1</td>
<td>+0.51</td>
<td>+0.57</td>
</tr>
<tr>
<td>1961IV</td>
<td>-102</td>
<td>+400</td>
<td>-229</td>
<td>+69</td>
<td>6.49</td>
<td>0.006</td>
<td>+2.6</td>
<td>+1.01</td>
<td>+1.08</td>
</tr>
<tr>
<td>1962I</td>
<td>-126</td>
<td>+500</td>
<td>-284</td>
<td>+90</td>
<td>5.90</td>
<td>0.008</td>
<td>+3.0</td>
<td>+0.95</td>
<td>+1.03</td>
</tr>
<tr>
<td>1962II</td>
<td>-115</td>
<td>+500</td>
<td>-272</td>
<td>+113</td>
<td>5.88</td>
<td>0.009</td>
<td>+2.2</td>
<td>+0.78</td>
<td>+0.82</td>
</tr>
<tr>
<td>1962III</td>
<td>-120</td>
<td>+500</td>
<td>-274</td>
<td>+106</td>
<td>5.23</td>
<td>0.010</td>
<td>+2.0</td>
<td>+0.96</td>
<td>+1.01</td>
</tr>
<tr>
<td>1962IV</td>
<td>-127</td>
<td>+500</td>
<td>-268</td>
<td>+105</td>
<td>5.51</td>
<td>0.011</td>
<td>+1.9</td>
<td>+0.70</td>
<td>+0.74</td>
</tr>
<tr>
<td>1963I</td>
<td>-96</td>
<td>+400</td>
<td>-201</td>
<td>+103</td>
<td>5.42</td>
<td>0.011</td>
<td>+0.7</td>
<td>+0.84</td>
<td>+0.84</td>
</tr>
<tr>
<td>1963II</td>
<td>-82</td>
<td>+300</td>
<td>-145</td>
<td>+73</td>
<td>5.70</td>
<td>0.011</td>
<td>+0.2</td>
<td>+0.85</td>
<td>+0.84</td>
</tr>
<tr>
<td>1963III</td>
<td>-68</td>
<td>+200</td>
<td>-84</td>
<td>+48</td>
<td>5.36</td>
<td>0.011</td>
<td>-0.3</td>
<td>+0.99</td>
<td>+0.97</td>
</tr>
<tr>
<td>1963IV</td>
<td>-52</td>
<td>+100</td>
<td>-26</td>
<td>+22</td>
<td>4.92</td>
<td>0.010</td>
<td>-0.8</td>
<td>+1.13</td>
<td>+1.11</td>
</tr>
<tr>
<td>1964I</td>
<td>-35</td>
<td>0</td>
<td>+32</td>
<td>-3</td>
<td>4.56</td>
<td>0.010</td>
<td>-1.2</td>
<td>+1.04</td>
<td>+1.01</td>
</tr>
<tr>
<td>1964II</td>
<td>-54</td>
<td>0</td>
<td>+25</td>
<td>-29</td>
<td>4.41</td>
<td>0.010</td>
<td>-0.4</td>
<td>+1.06</td>
<td>+1.06</td>
</tr>
<tr>
<td>1964III</td>
<td>-56</td>
<td>0</td>
<td>+29</td>
<td>-27</td>
<td>4.61</td>
<td>0.010</td>
<td>-0.3</td>
<td>+1.00</td>
<td>+0.99</td>
</tr>
<tr>
<td>1964IV</td>
<td>-55</td>
<td>0</td>
<td>+27</td>
<td>-28</td>
<td>4.45</td>
<td>0.010</td>
<td>-0.3</td>
<td>+1.02</td>
<td>+1.01</td>
</tr>
<tr>
<td>1965I</td>
<td>-55</td>
<td>0</td>
<td>+28</td>
<td>-27</td>
<td>3.94</td>
<td>0.010</td>
<td>-0.3</td>
<td>+1.52</td>
<td>+1.52</td>
</tr>
<tr>
<td>1965II</td>
<td>-55</td>
<td>0</td>
<td>+27</td>
<td>-28</td>
<td>4.04</td>
<td>0.010</td>
<td>-0.3</td>
<td>+1.00</td>
<td>+0.99</td>
</tr>
</tbody>
</table>

NFBS = Nonfarm Business Sector

△ = Simulation of Armed Forces Increase Minus Basic Simulation
The more rapid rate of increase of wages lasts until the armed forces build-up peaks out in 1962. The wage effect thus becomes larger throughout this period. As manhours are cut back further in response to wages, the need for a longer workweek is lessened, so the workweek falls back from its peak expansion in 1962.

As the armed forces are cut back in 1963, wages do not fall back immediately to their original level. As a result, both employment and the workweek are slightly lower than normal. Unemployment is higher, and the labor force is smaller. The net effect on the unemployment rate is an increase of .03 percentage points.

Labor Force Reduction

Having seen the effects of an increase in demand, we turn now to a simulation of reduced supply, again postulating no change in GNP or prices. It is assumed that the participation rates of primary and secondary workers each shift down by one percent of their existing levels beginning in 1961 and that this altered behavior remains in effect through 1965. This equip-proportionate downward shift in participation would result in a ceteris paribus reduction in the labor force of about 780 thousand people at 1965 levels, but as can be seen from Table IV, the reduction averages only 650 thousand in the 1961-65 period. This is because the lower unemployment rates draw about 130 thousand additional secondary workers into the labor force as an offset.

Initially the major impact of the reduction in labor force is borne by reduced unemployment, but then tighter labor markets drive wages up from one cent to eventually four cents an hour above the basic solution rate. This increase in labor cost tends to shift the burden of the reduction in labor force to reduced employment, so that by the middle of 1965 employ-
<table>
<thead>
<tr>
<th>Quarter</th>
<th>Δ Civil. Empl.</th>
<th>Δ Total Unempl.</th>
<th>Δ Total Labor Force (1)+(2)</th>
<th>Unemployment Rate %</th>
<th>Δ Comp. per MH NFBS</th>
<th>Δ Hours per M-Y NFBS</th>
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NFBS = Nonfarm Business Sector  
Δ = Simulation of Labor Force Reduction minus Basic Simulation
ment is down almost as much as unemployment. At first the workweek expands 0.18 hours on a weekly basis, but then it is slowly cut back as manhours are reduced steadily under the impact of higher wages. The relative flatness of the Phillips curve implied by our model is shown by the fact that while the unemployment rate drops by about a half a point, wage rates rise only about four cents an hour above the basic solution by the middle of 1965.\footnote{Actually, this comparison exaggerates the flatness a bit, since the simulation assumes that productivity rises to permit output to remain constant with reduced labor input.} The wage rate winds up 1.3% above its basic solution level, but .36% or about a fourth of that increase comes in the first quarter, when the effect of the change in unemployment and the lower level of unemployment are both acting on the wage equation.
Appendix

Assume that wage agreements are, on the average, adjusted once every \( j \) quarters so that over any consecutive \( j \)-quarter period all wage agreements are considered for adjustment. In the analysis below it is assumed that the quarterly time periods are numbered sequentially and indexed by the variable \( t \). Consider now the relation

\[
(A.1) \quad w(t) = \alpha_1 w_1(t) + \ldots + \alpha_j w_j(t), \quad t = 1, \ldots
\]

where \( w(t) \) is the average wage level at time \( t \); \( \alpha_i, \ i = 1, \ldots, j \), is the percentage of total wage agreements which are adjusted in time period \( i \), \( j + i, 2j + i \), etc.; \( w_i(t) \) is the corresponding average wage level at time \( t \) for the group of workers whose wage agreements are adjusted in time periods \( i, j + i, 2j + i \), etc. The assumption that all wages are adjusted over a \( j \)-quarter period implies

\[
(A.2) \quad \sum_{i=1}^{j} \alpha_i = 1
\]

The problem is to derive the wage equation corresponding to a \( j \)-quarter percentage change in \( w(t) \) given the wage equations corresponding to the quarterly percentage changes in the component wage series. The procedure is to express the \( j \)-quarter percentage change in \( w(t) \) as a function of the quarterly percentage changes in its component series and then aggregate the component wage equations.

The absolute change in \( w(t) \) over a \( j \)-quarter period can be expressed as the sum of the corresponding quarterly changes:

\[
(A.3) \quad w(t) - w(t-j) = (w(t) - w(t-1)) + (w(t-1) - w(t-2)) + \ldots (w(t-j+1) - w(t-j)).
\]
For convenience we now define the function \([t]\), where \([t]\) is a function of \(t\) such that \([t] = i\) when \(t = Kj + i, K = 0, 1, \ldots\). That is, if a calendar year had \(j\) quarters, the value of \([t]\) would be the calendar quarter corresponding to \(t\).

The quarterly change in \(w(t)\) is proportional to the change in \(w_{[t]}(t)\), which is the wage level of those workers whose wages were adjusted in that quarter. In particular, from (A.1) we have

\[
(A.4) \quad w(t) - w(t-1) = \alpha_{[t]} \Delta w_{[t]}(t),
\]

where \(\Delta w_{[t]}(t) = w_{[t]}(t) - w_{[t]}(t-1)\).

Substituting (A.4) into (A.3) we have

\[
(A.5) \quad w(t) - v(t-j) = \sum_{i=0}^{j-1} \alpha_{[t-i]} \Delta w_{[t-i]}(t-1).
\]

Equation (A.5) simply demonstrates that the \(j\) quarter absolute change in \(w(t)\) is a weighted average of the quarterly changes in the component wage series. Dividing both sides of (A.5) by \(w(t-j)\) and defining

\[
\dot{w}_{[t]}(t) = \frac{\Delta w_{[t]}(t)}{w_{[t]}(t-1)}
\]

we have

\[
(A.6) \quad \frac{w(t) - v(t-j)}{w(t-j)} = \sum_{i=0}^{j-1} B_{[t-i]}(t-i) \dot{w}_{[t-i]}(t-i),
\]

where \(B_{[t-i]}(t-i) = \alpha_{[t-i]} w_{[t-i]}(t-i-1)\). We see, therefore, that the \(j\)-quarter percentage change in \(w(t)\) can be expressed as a weighted sum of the quarterly percentage changes in its component wage series. In order to show that the weights sum to unity we note

\[
(A.7) \quad w_{[t-i]}(t-i-1) = w_{[t-i]}(t-j)
\]
because the average wage series of a quarterly group of workers would only be subject to change in those quarters that the corresponding wage agreements are negotiated. We have therefore, from (A.7) and (A.1)

$$\sum_{i=0}^{j-1} B_{[t-i]}(t-i) = \frac{1}{\alpha_{w(t-j)}} \sum_{i=0}^{j-1} \alpha_{w(t-i)} = 1.$$  

Therefore, with obvious notation, if the wage equations corresponding to the component wage series are of the form $1/

$$\dot{w}_{[t]}(t) = b_0 + b_1 X_{1t} + \ldots + b_m X_{mt} + \epsilon_{[t]}(t),$$

the wage equation corresponding to the j-quarter percentage change in $w(t)$ is, from (A.6) and (A.8), of the form

$$\frac{\dot{w}(t)}{w(t-j)} = b_0 + b_1 \overline{X}^{w}_{1t} + \ldots + b_m \overline{X}^{w}_{mt} + \overline{\epsilon}^{w}(t),$$

where $\overline{X}^{w}_{Lt} = \sum_{i=0}^{j-1} X_{Lt-i} B_{[t-i]}(t-i)$, $L = 1, \ldots, m$, and $\overline{\epsilon}^{w}(t) = \sum_{i=0}^{j-1} \epsilon_{[t-i]}(t-i)$. 

Because the weights defined in (A.6) are generally unknown functions of time, the regressors in (A.10) would be approximated by the simple averages $\overline{X}_{Lt} = (1/j) \sum_{i=0}^{j-1} X_{Lt-i}$, $L = 1, \ldots, m$. It is clear that a specification error is avoided only if $B_{[t]}(t) = 1/j$, for all $t$. For this reason, apparently, Dicks-Mireaux and Dow (1959), Perry (1966), and others, in dealing with the special case of $j = 4$, have assumed that wage adjustments

---

$1/$ Notice the restrictiveness of this assumption. Not only are the j wage equations assumed to have the same parameters but also the same series as regressors. For instance, if $X_{t}$ represents profits, then a formulation such as (A.9) implies that all quarterly groups of workers adhere to the same profit series.
are uniformly distributed throughout the year. However, it is clear from
(A.6) that \(a_t = 1/j\) does not imply \(b_t(t) = 1/j\). Therefore, unless
further assumptions are considered, a specification error is committed
when using the simple averages as regressors in models such as (A.10).
However, if, in fact, wage adjustments are uniformly distributed through-
out the \(j\)-quarter period, this error may not be serious. The reason for
this is that one would expect the ratios in (A.6) of the component wage
series to the aggregate to be close to unity unless the labor markets
corresponding to the component wage series were highly demarcated.
Therefore, assuming that such demarcations are not very sharp, we may, as
a first approximation take \(b_t(t) = 1/j\).

The question now arises, however, as to why a \(j\)-quarter
percentage change in \(w(t)\) is considered in the first place. That is,
if \(a_t = 1/j\) and the ratios of the component wage series to the aggreg-
ate are approximately unity, then it can be shown from (A.4) that

\[
(A.11) \quad \frac{w(t) - w(t-1)}{w(t-1)} \approx \frac{1}{j} \dot{w}_t(t).
\]

Therefore if the wage equation corresponding to \(\dot{w}_t(t)\) is given by
(A.9), the wage equation corresponding to the one-quarter percentage
change in \(w(t)\) is

\[
(A.12) \quad \frac{w(t) - w(t-1)}{w(t-1)} = \frac{b_0}{j} + \frac{b_{1t}}{j} X_{1t} + \ldots + \frac{b_{mt}}{j} X_{mt} + \frac{\varepsilon_t(t)}{j}.
\]

Therefore, if \(j\) is a given constant (implicitly assumed in the formul-
ation of (A.10), the only problem associated with the estimation of (A.12)
is a trivial scaling problem. That is, if \(\hat{b}_p\) is the estimate of \(b_p\) then
\(j\hat{b}_p\) is the estimate of \(b_{pj}\).
In comparing the formulation of (A.12) with that given by (A.10) a number of considerations should be noted. First, the formulation of the dependent variable in (A.12) (in contrast to that in (A.10)) does not depend upon a knowledge of j. Therefore, various formulations of the independent variables conditional upon particular values of j can be considered and compared. For example, assume that \( X_1 \) is (A.9) and (A.12) represents the percentage change in consumer prices since the time of the last wage adjustment. Then, if the hypothesis is that \( j = 4 \) or \( j = 1 \), both formulations of the price variable may be considered as regressors in (A.12) and compared in terms of the significance of their respective regression coefficients. Because the dependent variable in (A.10) is a function of j, that formulation of the wage equation precludes such experimentation.

It should also be noted that artificial autocorrelation is not induced by the formulation of the wage estimation as given by (A.12). On the other hand, since the disturbance term in (A.10) is a moving average of quarterly disturbance terms, that stochastic element will, in general, be autocorrelated. The consequences of such autocorrelation concerning the efficiency of the parameter estimates as derived from models (A.10) and (A.12) concerns the fact that the number of observations lost due to the formulation of the dependent variable is a minimum when \( j = 1 \). Therefore, the parameter estimates derived from (A.10) would be based on fewer observations and, hence, less efficient.

\[ \frac{1}{p(t) \text{ be consumer prices at time } t. \text{ Then } (p(t) - p(t-1))/p(t-1) \text{ and } (p(t) - p(t-4))/p(t-4) \text{ correspond respectively to } j = 1 \text{ and } j = 4. \]
A final point to consider in comparing (A.10) and (A.12) is that the wage variable used in most wage studies in the U.S. is average hourly earnings. For practical purposes, constant weighted averages of wage rates corresponding to most major sectors of the U.S. economy are not available. The significance of this is that the basic assumption of discrete component wage series (see (A.1) which lead to the construction of (A.10) is no longer tenable. For instance, even if wage rates were fixed, average hourly earnings could vary because of interindustry shifts, occupational shifts, changes in the percentage of hours worked at for premium pay, changes in the productivity of workers employed on a piece-rate basis, and number of other considerations. Therefore, when the wage variable used is average hourly earnings, (A.12) should be considered with \( j = 1 \). In brief, it is difficult to conceive of a situation for which model (A.10) is preferable to (A.12).
References


 Nerlove, Marc, "Notes on the Production and Denied Demand Relations Included in Macro-Econometric Models," International Economic Review, 8(2) (June 1967) A.


-----, "Labor Force Sensitivity by Age, Sex," Industrial Relations, IV (February 1965).

References Continued