INCOME DISTRIBUTION AND THE AGGREGATION OF PRIVATE DEMANDS FOR LOCAL PUBLIC EDUCATION

by

Byron W. Brown

and

Daniel H. Saks*

*The authors are Associate Professors at Michigan State University. Mr. Saks is also currently Acting Director of the Industrial Relations Section and Visiting Associate Professor of Economics at Princeton University. They wish to thank their colleagues in the Economics Department at Michigan State University, especially James Ramsey, Robert Rasche, and Daniel Suits for helpful advice. They also wish to thank Joseph Wisenbaker for his help in calculating the parameters of the school district income distributions. This paper has benefitted from comments received during presentations at the National Bureau of Economic Research-Educational Testing Service joint conference on the "Economics of Education" and to the Econometric Research Program seminar at Princeton. A perceptive referee saved us from a logical inconsistency in our presentation of the theory in an earlier version of the paper.
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Introduction

Although one could certainly envisage other institutional arrangements, schooling in this country is locally provided and partially paid for as one of the services that go along with the right to residential use of a parcel of land. As government in recent years has attempted to equalize the quality of schooling among income groups, it has (with perhaps the single exception of cross-district bussing) not tried to meddle with these basic local institutional arrangements for provision of schooling. Instead, it has used transfer schemes to offset the school-finance consequences of inter-district variations in the base of the property tax. In order to understand the distributional and allocative consequences of such transfer schemes, one has to understand how communities respond to changes in income and the price of schooling.

The literature on this issue often uses words like "fiscal capacity" and "burden"; yet in the economist's argot, the real question is how does a family's and community's demand for local schooling vary with changes in income (Engel curves) and the price of schooling (demand curves). Many previous researchers have in fact asked the question in this way, but they have often ignored the consequences of our particular institutional arrangements for providing local public education. They get around these community-choice problems by invoking a median voter theorem which tells them that under certain conditions they can treat communities as individuals. This is an extremely convenient assumption which, in the cases we examine, seems also to be inappropriate. Unfortunately, except for some very special cases, the
proper use of a median voter rule is difficult and we have only taken a first step here toward solving the problems arising when the conventional assumption does not apply. If we are right, however, then the previous estimates of demand for local schooling were incorrectly estimated and policy implications may be invalid. In what follows, we first develop our critique of the way the median voter theorem has been applied in estimating the demand for local public schooling. Our critique suggests an alternative theoretical model and empirical framework leading to estimates of Engel curves for local schooling that differ substantially from earlier results. Our results suggest that the proportion of income people desire to be spent on schooling is not a monotonic function of their income. Although inconsistent with the assumptions of the previous literature on this topic, the result is much more consistent with the literature on Engel curves for many other commodities.

Voting Models Where the Median Income Voter is Decisive

People can adjust their expenditures on local public schooling in either of two ways: they can leave communities with the wrong levels ("exit" in Hirschman's (1970) taxonomy) or they can exercise political influence to change the level of expenditures in the community where they already reside ("voice"). Under the first behavioral assumption (the Tiebout (1956) model), people would search over all relevant communities until they found the bundle of housing services which included the right level of schooling, type of neighborhood, journey to work and shopping, and mix of other local public services. Of course, people may not find or be allowed to buy the exact bundle they want.\(^\frac{1}{2}\) Hence the necessity for a second behavioral postulate that school boards search over possible school programs till they find the program which satisfies the largest number of voters in a community.
A complete structural model would include these two behavioral relations explicitly. Here, we ignore the exit versus voice distinction because we do not care whether an observed school budget gets that way because the majority voters move to a suitable community or because they modify the community in which they already reside. We do expect both mechanisms to be working simultaneously. What we cannot then ignore is the necessity for a community-choice model or voting scheme to resolve intra-community differences in desired levels of educational expenditure.\(^2\) A voting scheme is simply a rule which tells how individual family demands for school expenditures are aggregated to determine the community's decision. The assumption, typically, is that the expenditure level chosen is the one to command a simple majority of the votes.\(^3\)

How do families vote? If one is willing to assume that each family in a community receives a determinate portion of the total school expenditures (benefit shares which need not sum to one, cf. Denzau and Mackay (1976)), then one can talk about a family's demand for school expenditures as a function of family income, the prices of local schooling and all other goods, and tastes. One can, following Bergstrom and Goodman (1973), derive such a demand curve from the theory of the utility-maximizing consumer.\(^4\) The family chooses a preferred tax rate for local schools and a level of expenditures for all the other goods in its utility function.

One twist here is that the wealth of the community is the base to which the tax rate is applied. The same tax rate buys a lot more schooling for a given family in a wealthy community than it would have purchased in a poor community so long as school services (benefit shares) are not distributed
exactly in proportion to the parent's tax share. In short, the average wealth of the community acts as a price variable for schooling. Higher community wealth may be thought of as a matching grant from inside the community similar to an external matching grant from state and federal levels. And it will have an income and substitution component.

The important issue is not what individual families demand, but how those individual demands get aggregated to yield a community decision. Our voting model is that of Black (1948). Individual preferences are assumed to be single-peaked as they will be in the standard consumer-choice model where there is always a unique optimal basket of goods and services for any given income and set of prices. The community choice is found by distributing the individual choices according to the percentage of income (i.e. the preferred proportional tax rate) that the individual wishes to spend on, say, schooling and determining the median individual's choice. According to Black's model, the community choice will be the tax rate or proportion of income which will win in a simple majority vote over any alternative tax-rate proposal. This equilibrium rate, which is unique, is the one which has half the voters wanting more, half less.

It is at this point that a crucial assumption has always been introduced in empirical work on demand for schooling: an individual family's preferred tax rate is assumed to be a monotonic function of income, ceteris paribus. Another way to put it is that when families are ordered by income, the rankings are identical with a ranking of the same families by preferred tax rate. To explain why this is so useful, it will help to add two less important assumptions which could be released later. First, assume that each family has the same preference function so that families in similar circumstances would like
similar amounts of local schooling (i.e., everyone has the same demand curve). Second, assume that the residential part of the property tax is distributed in direct proportion to permanent income so that the school millage rates can be talked about as if they were proportional income tax rates.\(^6\) 

Given these assumptions, the tax rate preferred by the median income voter is the only rate to get a bare majority when paired against any other proposed rate.\(^7\) Figure 1 illustrates why. The upper panel shows two alternative proportional tax rates as \(\bar{t}\) and \(t'\). It also shows a family's desired proportion of income to be spent on schooling given the mean income of the community (a price variable). We call this form of the income demand curve a fractional Engel curve\(^8\) since it is the Engel curve for desired educational expenditures divided by income. It is drawn as a monotonic (downward) function of income. The distribution of income in the community is shown in the lower panel. \(\bar{t}\) is the tax rate preferred by the voter with the median income \((Y_{\text{med}})\). When voters in this community are confronted with an alternative of \(\bar{t}\) or any other rate such as \(t'\), \(\bar{t}\) will get half the votes (in this case all families with incomes less than \(Y_{\text{med}}\)) plus some part of the votes of families with incomes in the range \(Y_{\text{med}}\) to \(Y'\). \(\bar{t}\) will clearly win over any arbitrary \(t'\) above or below \(\bar{t}\). 

In the case of the downward sloping fractional Engel curve (implicitly favored in the literature), an increase in median community income (given the mean income) will lead to a lower equilibrium tax rate. Variations in median community income will then, under these assumptions, permit us to identify the family fractional Engel curve exactly since all observed points of intersection between median income and tax rate will be on the true fractional Engel curve.
Figure 1.—Desired tax rates as a function of income and community income distribution. $\bar{t}$ is the chosen tax rate.
Varying the mean income (given the median income) will enable us to isolate the price effect for individual families. For example, a higher mean income (by bringing in more total revenue for a given millage rate) would shift the fractional Engel function upwards. Thus the monotonic fractional Engel curve has the effect of letting us treat communities as if they were individuals by making the median (decisive) voter the voter with the median income.

This simple standard model has two empirical consequences. First, it is impossible to disentangle price and income effects unless median and mean income have some independent variation across communities. Second, and most important, no aspects of the income distribution other than its mean and median are relevant for explaining community choice in the model. We could change the form of the distribution drastically and in so doing greatly alter the character of the community, yet if the mean and median are unaffected so will expenditures be unaffected. That no parameters of the income distribution are relevant to decision-making other than the mean and the median is, on the face of it, a strange thing to assume no matter how convenient. At a minimum, it is the sort of restriction which ought to be tested before being imposed.

If we check some of the literature on Engel curves for goods other than schooling [see, for example, Ramsey (1974)], we see that the expenditure patterns are often well approximated by logistic or sigmoid-shaped curves. These curves often suggest fractional Engel curves which are not monotonic and in fact have multiple extrema. This means the demand for schooling would need to be quite unlike that for most other commodities if the corresponding fractional Engel curve were to have the convenient monotonicity always
implicitly imposed for empirical work. Clearly a more general model is called for so that the assumption can be tested.

**Toward a More General Model**

How does the relation between median community income and underlying family demands (the fractional Engel curve) change when the fractional Engel curve is not monotonic? Figure 2 is an example of how to proceed on this question in the case we are considering in this paper. Suppose that instead of being monotonic, the fractional Engel curve for the families in a community were either U-shaped or inverted U-shaped. The former case\(^1\) (shown in Figure 2) is one which has been hypothesized by Wilson and Banfield (1964) who used the notion of "public-regardingness" to explain voting patterns. The notion is that the rich and the poor are disproportionately likely to favor public expenditures: the poor because the ratio of benefit shares to tax contribution is likely to be large and the rich because they feel a "respon-
sibility" to the community.

The equilibrium tax rate for the fractional Engel curve in Figure 2 is \(\overline{t}\). This rate is chosen so that one half of the families will desire a higher rate and one half a lower rate. Note that those families desiring the higher rate will be from different parts of the income distribution.

In general, it will be impossible to predict the vote of a particular family, since it will depend on the actual alternative tax rate presented. There is an alternative which will permit us to identify the coalition for and against \(\overline{t}\) precisely, however. \(\overline{t}\) versus a tax rate marginally less than \(\overline{t}\) will yield a winning coalition for \(\overline{t}\) made up of voters with incomes less than or equal to \(Y_0\) and greater than or equal to \(Y_1\). This shaded
Figure 2. -- When the fractional Engel curve is U-shaped, the equilibrium tax rate $\xi$ is favored by a coalition of voters from different parts of the income distribution.
area of the income distribution will make up the base majority necessary for victory. On the other hand, when $t$ is paired against a marginally greater rate, families with incomes from $Y_0$ through $Y_1$ will pick $t$, the equilibrium rate. With appropriate modifications, the same procedure could be applied to the case of an inverted U-shaped fractional Engel curve.

Now that we know how to analyze these coalitions when fractional Engel curves are U-shaped or inverted U-shaped, we can ask how equilibrium tax rates (as a proportion of income) change in response to changes in the community income distribution, particularly changes which are mean and median preserving.

It is difficult to envision holding the mean and median constant and changing the other characteristics of the income distribution. We do have a device to help one's intuition about the effect of changing the distribution in this way. Apply a transformation to the income distribution and to the fractional Engel curve as well so that the income distribution after the transformation is symmetric. Or simply assume, for sake of argument, that the income distribution is symmetric. The transformed distribution will have its median equal to its mean.\textsuperscript{11/}

Figure 3 shows the analysis of this case for a symmetric income distribution. For the income distribution with the smaller variance, $t_a$ is the equilibrium rate. For the distribution with larger variance, $t_b$ is the rate to command a simple majority. An increase in the variance, because it shifts more of the distribution to the tails, increases the tax rate even with median income constant.\textsuperscript{12/}

But a U-shaped fractional Engel curve has another important and disturbing consequence (already apparent as point C in Figure 2) -- the tax
rate chosen at a given community median income lies above the fractional Engel curve, not on it. This destroys the neat one-to-one correspondence between individual and group behavior. In the case shown, estimates of the proportion of income spent by income class will be systematically biased upward ($t_b > t_a > t_o$ in Figure 3) and if the empirical estimates are always derived from an assumed monotonic (e.g., log-log) equation, the bias will not be apparent.

The reader should note that if the fractional Engel curve is an inverted U, the bias will be negative and we will underestimate the individual demand from observing variations in group demand.

In any event, this possibility of coalitions of voters from non-contiguous portions of the income distribution suggests a simple yet conservative test of the simple model. If we could control for symmetry in the income distribution and then examine how tax rates vary, we would expect the tax rate to be a function of mean and median income. If tax rates also depend on the spread of the income distribution, that would be evidence that monotonicity of the fractional Engel curve is an incorrect assumption. Furthermore, depending on whether an increase in the spread of the distribution increases or decreases the tax rate, we might deduce whether the fractional Engel curve is inverted U-shaped or is as shown in Figure 2. But to estimate properly the impact of income distribution on school expenditures, we need to make some assumptions about the functional forms of this bias and of the fractional Engel curve.
Figure 3.—When the fractional Engel curve is U-shaped, the majority in favor of a tax rate is a coalition of voters from the tails of the income distribution. Increasing the variance of income will increase the tax rate given the mean income.
Towards a Simple Functional Form

So far, we have suggested that associating community-school expenditures per family with the preferences of the median-income family in that community may be a biased procedure. The problem depends upon whether the ranking of voters by preferred tax rates is different from the ranking by incomes. Thus in Figure 2 some voters from the upper tail of the income distribution substitute their votes for others in the middle of distribution in forming a decisive coalition. We call this, for lack of a better name, the substitute voter effect and we think it is a key to simple estimation of a more correct fractional family Engel curve.

Consider Figure 3 again. Because voters from the upper tail of the income distribution come in and vote with the poor, we would be mistaken if we were to estimate $t_b$ as the desired proportion of income that people with median income ($Y_{med}$) would like to spend. $t_o$ is the "true" proportion of income those with $Y_{med}$ would like to spend on schooling. Would it be possible to identify this difference and thus predict $t_o$ when in fact we observe only $t_b$? As the distribution of income gets narrow, the potential number of people from the tails of the income distribution who might join a coalition becomes smaller. As the variance of the income distribution becomes smaller, the bias becomes smaller. When, in the limit, the income distribution finally collapses to a spike, there are just no substitute voters left in the community to join the coalition. As the variance of income in the community vanishes so does the bias at the median income.

The bias due to the substitute voter effect will also vary over another dimension. Given the variance of the income distribution and assuming a U or inverted U-shaped fractional Engel curve, the location of the median income will help determine the opportunity for finding members on
the other end of the income distribution who share preferences about school
millage rates. If the income distribution is far to the right or left, the
distribution will generally fall within a monotonic portion of the fractional
Engel curve and the decisive voter will again be the voter with the median
income. The bias would be smaller for very high and very low median incomes.
Indeed, proper estimation seems to require explicit use of information on the
degree to which communities have more of their families within a monotonic
portion of the typical family’s fractional Engel curve.

The bias (B) due to the substitute voter effect under the assumption
that the fractional Engel curve is U-shaped or inverted U-shaped is:

\[ B = g(Y_{med}, G) \]  \hspace{1cm} (1)

where G is some measure of the spread of the income distribution (e.g.,
the Gini coefficient). Our theory says that the bias will be non-negative
(non-positive) if the fractional Engel curve is U-shaped (inverted U-
shaped). Furthermore, the bias should approach zero as G, the spread of
the income distribution, goes to zero, and should become smaller the further
median income lies from some central value.

Using the Gini coefficient to measure the dispersion of income, a
polynomial in median income will approximate the bias in estimating the
fractional Engel curve:

\[ B = [\beta - \alpha(Y_{med} - \gamma)^2]G \]  \hspace{1cm} (1.a)

Using the example of the U-shaped fractional Engel curve, if \( \beta \),
\( \alpha \), and \( \gamma \) are all greater than zero, the bias is greatest at \( Y_{med} = \gamma \) and
diminishes for increases or decreases in \( Y_{med} \) until it becomes zero at
\( Y_{med} = \gamma \pm \sqrt{\beta/\alpha} \). \( \frac{13}{1} \)
Having picked a form for the bias, we must now consider the form of the fractional Engel curve. Following a convention in the literature, we will estimate the Engel curve (expenditures per family) rather than the fractional Engel curve (proportion of income expended per family). A quadratic functional form for the Engel curve has, besides the virtue of simplicity, the property of being able to describe a wide range of outcomes over a particular range of data, including monotonic and U or inverted U-shaped fractional Engel curves. Thus:

$$E^*(Y) = a + bY + cy^2,$$

(2)

where the asterisk implies the true family Engel curve. The observed curve would contain, in addition, the bias formula presented above multiplied through by Y to convert to expenditures rather than fractions of income:

$$E(Y) = a + bY + cy^2 + [\beta - a(Y - \gamma)^2]GY.$$

(3)

An alternative functional form with a theoretically more restricted structure will be discussed below. However, at this point we do not have any strong a priori reasons for doing anything better than seeing how best to approximate the data points.

We do not, of course, expect the Engel curve to be the same in all school districts. There are some systematic variations for which we can try to control. The average per family property tax base (residential and non-residential) is, as mentioned above, a price variable, since, the larger the base, the more schooling a dollar of one's own school expenditures will buy. If schooling behaves normally, we expect an upward shift in the Engel curve due to a rise in the base. Adjustments will also have to be made in
expenditures if schooling is more costly in a particular community (e.g., if teachers of similar quality are paid less, ceteris paribus, that also lowers the price of schooling). There will also be shifts in desired expenditures due to differences in the bundle of school characteristics provided by a district. Two variables reflecting such characteristics are average achievement score for students in each school district and the standard deviation of that score. These variables are intended to reflect two performance characteristics of the schools, namely, the extent to which student achievement levels are raised on the average and the way in which high achieving students are treated relative to low achievers. Other things the same, a district with a higher spread of student scores reflects less egalitarian treatment of individuals in opposite tails of the distribution. Some families, preferring schools with more or less egalitarian aims, will vote accordingly.¹⁴/

Preferences or demands for schooling should vary with the demographic characteristics of the community. Communities having a larger fraction of families with school-aged children should want to spend more on schooling per family in the district. Similarly, the more children in a community attending non-public schools, the lower the average family's desire to support high local public school expenditures. One expects the average number of school-aged children per family with such children to have a non-linear impact on expenditures, rising with small numbers of children and then falling as more children make other demands on scarce family resources. Preferences might also vary among cultural groups so that, for example, racial composition of the school district might affect expenditures. Finally, prices as well as preferences may vary with the degree of urbanization of a school district.¹⁵/
Data Used in Engel Curve Estimation

Our dependent variable is local revenue raised per family in each school district. Total current revenues for education are these local revenues plus state and federal aid and any special fees collected. While some other studies of school finance have tried to explain total current revenues (or expenditures), our model suggests that determination of the amount to be paid locally is a better focus of the analysis.

Our explanatory variables fall into three groups: (a) characteristics of the income distribution and the level of wealth; (b) factors reflecting differences in tastes for education; (c) factors measuring characteristics of the entire bundle of community services, including education, offered within a community. As we shall see, it is hard to say in some cases whether a particular variable measures tastes or community characteristics or both.

The data came from two sources: the 1970 Census Fourth Count (Population) School District Tapes prepared by the National Center for Educational Statistics; and the Michigan School Assessment Program, 1970-71, reported by the Michigan Department of Education. We obtained matched data for 500 Michigan school districts from both of these sources.

Since the problem of parameterizing the 500 school-district-income distributions absorbed quite a bit of our time, it is worth mentioning the problems we encountered. Economists generally expect income distributions to be skewed to the right and so the obvious approach was to follow Metcalf (1972) in assuming that school-district-income distributions are log-normal or displaced log-normal. We could then have estimated the variance of distributions of the logarithm of income for each district and used that
parameter in our equation as a measure of the spread of the distribution. When we tried to use some of the estimation methods for this distribution given in Metcalf, it soon became clear that not very many of our distributions could be closely approximated by a log-normal curve. In fact in more than half of our 500 cases, the original distributions were either almost symmetric before transformation or skewed to the left rather than to the right.\textsuperscript{16}

We finally settled on the three-parameter approximation suggested by Kakwani and Poddar (1976).\textsuperscript{17} These parameters yield a direct measure of skewness and permit easy calculation of the Gini coefficient. The Kakwani and Poddar technique involves estimating the Lorenz Curve for an income distribution. The trick is to transform the usual Lorenz curve coordinates and then fit a Beta distribution to the transformed curve. This has provided a very good fit over a wide range of cases using a small number of parameters. We used data from the Census on the distribution of family income to estimate a Lorenz curve for each school district. This was then used to compute our measure of dispersion, the Gini coefficient, and measure of skewness, the ratio of alpha to beta in Kakwani and Poddar's parameterization. For a symmetric distribution $\alpha/\beta = 1$. For distributions skewed to the right (left), we have $\alpha/\beta < 1 (> 1)$. Our Lorenz curves using the Kakwani and Poddar approximation gave a superb fit of the data. The coefficients of determination for the school-district-Lorenz curves all exceeded .98.\textsuperscript{18}

Other income-wealth data for each district include median family income estimated from the Census income distributions, the state equalized valuation of property per family, and the state school aid per family. The state equalized property valuation per family (SEV) serves as the
price variable for families in any income class since it determines the
tax rate which any family would have to pay to achieve a given level of
local revenue per family. At the time these data were generated, state
aid was distributed to Michigan's school districts according to a "founda-
tion plan." Under such a plan, each district receives a basic grant per
pupil which is then reduced depending on the district's state equalized
valuation per pupil. Richer districts (in terms of SEV) receive smaller
grants per pupil. This will attenuate the impact of SEV on our estimates.

Data on demographic characteristics of the community (family and
racial composition) were taken from the Census data. Achievement test
data were taken from the Michigan Assessment data and are more fully
described in Brown and Saks (1975). The cost of schooling relative to
other goods should affect the level of school expenditures. Teachers'
salaries, an often-used measure of these costs, are not satisfactory for
this purpose because teacher quality varies among districts. Our pro-
cedure was to adjust average teacher salaries for quality differences due
to variations in years of experience and the proportion of teachers with
master's degrees. This was done by regressing observed average teacher
salary in a school district on the average years teacher experience and
proportion of teachers with master's degrees (both in quadratic form).
This equation was used to find the difference between the observed and
predicted salary level for each district, given its teacher-quality
composition. The result, residual average teacher salary, is a measure
of the differences in costs between districts.

Finally, our equation includes the total number of families in
each district (to capture scale effects) and a set of dummy variables for
each of five community types: (1) metropolitan core, (2) city, (3) suburb, (4) town, and (5) rural.²⁰/

Table I gives a summary of the variable names and their units of measurement, means and standard deviations.

An Estimate of the Simple General Model

Equation 3 augmented by a stochastic term and a linear combination of the control variables discussed above was estimated from the data on the 500 Michigan school districts. Because we were dealing with grouped data from the individual school districts, we used weighted least squares in which the weights were the square root of the number of families in the district.²¹/

These estimates for the Engel curve are presented in Table II. The set of terms involving the Gini coefficient (i.e., the bias) is significantly different from zero at the .05 level. We take this as evidence in favor of rejecting the hypothesis that the fractional Engel curve is monotonic. Figure 4 contains a graph of the Engel curve and the fractional Engel curve for the estimates of Table II assuming that all variables except income and the Gini coefficient are at their means. Two sets of curves are presented: one assuming the Gini coefficient is equal to .33 (the average value for Michigan school districts in 1970); and the other assuming that the Gini coefficient is zero, representing "unbiased" estimates of the curves since equation 1.a goes to zero when the Gini coefficient is set to zero. The curves are plotted for the range over which median community incomes are observed, about $4,000-24,000.
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<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
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<tr>
<td>Local revenue/family ($1000)</td>
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<td>Median family income ($10,000)</td>
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<td>State aid/family ($1000)</td>
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<td>Mean achievement score (fourth graders)</td>
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<td>Standard deviation of achievement scores</td>
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<td>Number of families in district (10,000)</td>
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<td>Fraction of families with children</td>
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<td>Average children/family in families with children</td>
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<td>Fraction of students in private schools</td>
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**Sample Information**

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*Weighted in proportion to the square root of the number of families in the school district.
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</tr>
<tr>
<td>School-age children/family with children</td>
<td>-.471** (.216)</td>
</tr>
<tr>
<td>(School-age children/family with children)$^2$</td>
<td>.115** (.0584)</td>
</tr>
<tr>
<td>Residual average teacher salary</td>
<td>-.0406 (.0381)</td>
</tr>
<tr>
<td>Total number of families</td>
<td>-.000214** (.0000734)</td>
</tr>
</tbody>
</table>
TABLE II.---Continued.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Regression Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index of income distribution skewness ($\alpha/\beta$)</td>
<td>.0204 (.0872)</td>
</tr>
<tr>
<td>Dummy Variables:</td>
<td></td>
</tr>
<tr>
<td>Metropolitan core</td>
<td>.116** (.0174)</td>
</tr>
<tr>
<td>City</td>
<td>.0327** (.0137)</td>
</tr>
<tr>
<td>Town</td>
<td>-.000234 (.00913)</td>
</tr>
<tr>
<td>Urban fringe</td>
<td>.0294** (.0113)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.83</td>
</tr>
</tbody>
</table>

*Weighted in proportion to the square root of the number of families in the school district.

**Significant at the 5% level.
Figure 4.--Estimated demand for local public education as a function of income for different values of the Gini coefficient.
The "unbiased" fractional Engel curve estimated in this way is U-shaped with a minimum at a family income of about $10,300. In contrast, the biased curve \( G = .33 \) falls monotonically, a result consistent with some previous estimates which have ignored the substitute voter effect.22/ Our theoretical discussion of the bias leads us to predict that where the bias is negative, the fractional Engel curve will be shaped like an inverted U. Note that in Figure 4 the bias is positive at low incomes but at higher incomes becomes negative. This result could occur if the fractional Engel curve turned down at some still higher income.

We were puzzled by this result at first since the fractional Engel curve is clearly U-shaped implying the bias must be positive. Yet it is not reasonable to suppose that the fraction of income spent on any good, including education, will rise without limit. The negative bias seems to be telling us that the fraction of income desired to be spent on education must eventually fall. This sensible (and, indeed, required) result is consistent with many other Engel curve studies and for a wide range of commodities [see Ramsey (1974)]. These studies have suggested that the Engel curves for many goods are sigmoid shaped and well-approximated by logistic curves. In such a case, the proportion of income spent on a good may, as income increases, fall, rise, and then fall again. Furthermore, if this form is correct, then the coalitions of the voting model are more complex than postulated because every extremum of the fractional Engel curve creates a new possibility for a coalition. The negative bias in our estimates at higher incomes probably comes from the Gini coefficient giving us information on the education demands of families with incomes outside the observed range of median community incomes. We discuss in the next
section of the paper some attempts we made to estimate what happens in this "unobserved" range of income. Before turning to those estimates, we must discuss the other variables in our estimated equation.

There were two important price variables that should affect local educational expenditures. One is an external price subsidy from the state (state aid per family) which, in this case, is supposed to be an inverse function of wealth. The other is the average property tax base per family which can be thought of as an internal price subsidy. As these subsidies increase, individual family-school expenditures increase significantly.

The question of cultural background and preferences for higher school expenditures was examined only with respect to Blacks and Whites. The coefficient on fraction of students who are White implies that Blacks significantly favor higher school expenditures per family. All-White districts would spend $72 less per family per year than would similar all-Black districts. This is a difference of over 15% for the average district.23/

The characteristics of the performance of the school system seem to affect how much people wish to spend on schooling. Families will spend more for each point their school system's mean achievement test score increases. They also seem willing to pay more for schools which have a greater dispersion (standard deviation) of student performance scores. In general, people are willing to pay more for elitist schools [in the sense defined by Brown and Saks (1975)].

The effects of the demographic variables are more ambiguous than expected. The fraction of families with school children has no effect on school expenditure levels. This strange finding is likely to stem from the fact that schooling is bundled with the capital expenditure on housing. Families planning to have children want to live in neighborhoods
which will have the right kind of schools when they need them and those whose children have completed school know that the market value of their house will reflect the value of the school system. They would assume that buyers might want the same kind of school system that they wanted when they had children in school. There is, however, a distinction to be made between transitory changes in the fraction of families which are without school children and districts where there are permanently fewer such families. In the latter case, if expenditures levels do not adjust, then a change in quality is implied since per pupil expenditure must reflect the varied demographic patterns.

The higher the fraction of children attending private or parochial schools, the lower will be school expenditures. This was the expected result.

The effect of family size on expenditures is peculiar. We had expected the quadratic equation to be concave from below and instead the function is U-shaped. Two-child families spend the least per family and expenditures increase with children after two. One-child families are similar in expenditure pattern to three-child families. 24

The degree of urbanization of the school district affects the level of local school expenditures. The most urban school districts spent the most per family and town and rural districts spent the least with the spread being some 25% of the expenditures of the average district. As with other variables, with a reduced form we cannot be sure whether this reflects higher costs or higher preferences. The explicit teacher-cost variable seemed to have no impact. We tried segmenting the sample by community type, but we were unable to reject the hypotheses that the sets
of estimated coefficients were equal for all community types. Community size does seem to affect expenditure levels. Larger communities tend to spend less per family. Bergstrom and Goodman (1973) have suggested that this reflects the higher transactions costs of making public decisions about publicly provided goods in larger communities. It may, then, be an example of the underproduction of semi-public goods in large groups.

Estimates of Logistic Engel Curves for Local Schooling

Two important results emerged from our attempt to estimate a polynomial form for the Engel curve. The first was that the fractional Engel curve was U-shaped over the observed range of income. Second, the bias due to the substitute voter effect became negative at higher incomes. Together, these results suggest a fractional Engel curve which falls, rises and then falls again. We noted earlier that such a curve would be consistent with an Engel curve which is sigmoidal and, in the spirit of this observation, we proceeded to estimate some logistic equations.\(^{25/}\)

The function we use for the Engel curve is

\[
E(Y) = \frac{Z}{1 + \exp(-z_0 + z_1 Y + z_2 z_3 Y)).
\]

(4)

\(Z\), the upper asymptote, is a function of some of the price and taste variables discussed above.\(^{26/}\)

\[
Z = \sum_{i=1}^{5} Y_i X_i, \text{ where}
\]

\(X_1 = \text{state equalized valuation per family.}\)

\(X_2 = \text{state aid per family.}\)
\( X_3 \) = proportion of white students.
\( X_4 \) = average achievement score.
\( X_5 \) = standard deviation of achievement scores.

This specification assumes changes in these variables have larger absolute impacts on the desires of families with larger incomes. The units of measurement for the X's are described in Table I above.

The terms in the Gini coefficient (G) represent the substitute voter bias and when set equal to zero would give us an estimate of the underlying family Engel curve. While the bias in this specification is not forced to be positive at low incomes and negative at high incomes, such a result is not excluded. Furthermore, the logistic is not inconsistent with a monotonic fractional Engel curve and so gives another test of that hypothesis.

We estimated equation (14) by weighted non-linear least squares for three sub-samples of our data: \( 21/38 \) metropolitan core and other city districts, \( 114 \) suburban districts, and \( 348 \) town and rural districts. Table III gives the results.

For the cities, the small sample size relative to the number of parameters to be estimated and the relatively high correlations among some of the variables leads to high standard errors for most of the estimates. In the other equations, all the estimated values are significantly different from zero at the 5 percent level or better. The coefficients on the control variables in the numerator of the logistic all have the same signs as their comparable estimates in Table II.

The estimates of the parameters in the exponent which determine the position of the logistic as well as its rate of increase are of similar magnitude across community types. The estimated bias due to the substitute voter effect is significant.
TABLE III.—Estimated Parameters of Logistic Form of Engel Curves (Standard Errors in Parentheses).

<table>
<thead>
<tr>
<th>Variable (Coefficient)</th>
<th>Cities &amp; Metro. Core</th>
<th>Suburbs</th>
<th>Town &amp; Rural</th>
</tr>
</thead>
<tbody>
<tr>
<td>State equalized valuation per family ($\gamma_1$)</td>
<td>1.83 (1.05)</td>
<td>.378** (.064)</td>
<td>.546** (.049)</td>
</tr>
<tr>
<td>Aid per family ($\gamma_2$)</td>
<td>2.62 (3.12)</td>
<td>2.92** (.58)</td>
<td>2.06** (.19)</td>
</tr>
<tr>
<td>Proportion white students ($\gamma_3$)</td>
<td>-.280 (.263)</td>
<td>-.0963** (.029)</td>
<td>-.121** (.015)</td>
</tr>
<tr>
<td>Average achievement ($\gamma_4$)</td>
<td>.0166 (.0159)</td>
<td>.0416** (.0097)</td>
<td>.0339** (.0031)</td>
</tr>
<tr>
<td>S.D. of achievement ($\gamma_5$)</td>
<td>.0239 (.0263)</td>
<td>.0141** (.0047)</td>
<td>.0132** (.0018)</td>
</tr>
<tr>
<td>Income ($\beta_1$)</td>
<td>1.22** (.29)</td>
<td>1.24** (.12)</td>
<td>1.44** (.071)</td>
</tr>
<tr>
<td>Gini coefficient ($\beta_2$)</td>
<td>2.87 (1.76)</td>
<td>4.09** (.77)</td>
<td>2.47** (.231)</td>
</tr>
<tr>
<td>(Income) * (Gini) ($\beta_3$)</td>
<td>-2.71** (.91)</td>
<td>-2.88** (.38)</td>
<td>-3.03** (.232)</td>
</tr>
<tr>
<td>($\beta_0$)</td>
<td>-5.56** (.70)</td>
<td>-4.64** (.14)</td>
<td>-4.64** (.091)</td>
</tr>
</tbody>
</table>

| Number of school districts | 38 | 114 | 348 |
| Value of Z at variable means | 31.26 | 9.71 | 10.35 |
| $R^2$                        | .77 | .87 | .81 |

**Significant at the 5 percent level.
A more intuitive notion of these estimates can be gained from Figure 5 which shows the Engel curves and fractional Engel curves for the suburban school-district sample. Once again, two estimates are shown for different values of the Gini coefficient. For $G = .33$, the Engel curve rises much less rapidly than for $G = 0$. This shows up as widely divergent estimates for the two fractional Engel curves. If we have approximated the voter bias correctly, then earlier estimates of the Engel curve have seriously underestimated desired expenditures of higher income families.

We note that the upper asymptote for the Engel curve for cities is a hefty $31,000 when the other variables are evaluated at their means. Unless we are prepared to believe that the super rich want their children tutored by highly paid Ph.D.'s, such a result strains credulity. In the suburban school districts, the maximum amount which would be spent is about $9,700 per family, still a rather high figure.

As the reader may suspect, the bias is sensitive to the specification of the functional form we assume for it. In the case of the logistic estimates of the suburban sub-sample, the "true" maximum proportion of income desired to be spent is about 16 percent at an income of $50,000. If the bias in the estimated equation is set equal to $G(\beta_2 + \beta_3 \ln Y)$ instead of $G(\beta_2 + \beta_3 Y)$, the difference between the biased and "true" estimates is reduced but still is large. With the former specification, the maximum proportion spent on education is about 9.5 percent when income is about $90,000. (In dollar terms, however, these upper asymptotes are rather close.) Obviously, when results about the upper (unobserved) portion of the curves are sensitive to specification, we cannot place any confidence in the exact
Figure 4.—Logistic Engel curves and fractional Engel curves for the sample of suburban school districts for different values of the Gini coefficient.
curve beyond our observed range. We do, however, believe this indicates
that the form of the Engel curve for schooling does not differ from the
form found for other commodities.

Conclusion

Earlier studies of the determination of local public school expendi-
tures have conveniently assumed that the fractional Engel curve for typical
families is monotonic in income. As a consequence, they have assumed that
the voter with the median income is decisive and that community decisions
therefore truly represent the preferences of the median-income voter. We
have presented evidence that this assumption is untenable and that it leads
to significant biases in the estimation of demand curves for local public
schooling. Indeed, it is hard to imagine two more vastly different esti-
mates of the same curve than one observes in the lower panel of Figure 5.
While we have reason to believe that our estimate of the "unbiased" frac-
tional Engel curve is deficient in the range above, say, $25,000, we do
believe that we have discovered the correct approximate shape for the curve.
While this shape is not consistent with the earlier empirical work in
public expenditure analysis, it is completely consistent with the empirical
work on Engel curves for other commodities. If we are right, then the
basic assumption of the previous empirical literature on demand for local
public schooling is seriously deficient and the heterogeneity of a com-

munity is an important determinant of demand for schooling and of the
response of local communities to intergovernmental subsidies. Unfortunately,
our own estimates do not correctly represent the upper part of the Engel
curve where data are sparse and we do not feel we can solve that problem
without more information or a more ingenious way of using the information we already possess.

There are some policy implications in even our tentative results. First, it may be wrong to ignore community income distribution in devising equalizing inter-district school-aid plans. Second, communities which are homogeneous in income will not necessarily be the best [in the McGuire (1974) sense] jurisdictions for providing schooling. People with different incomes may well desire to spend similar proportions of their income on schooling. Third, court-favored district equalizing State aid plans which make expenditures depend only on local tax rates do not go far toward breaking the relation between income and expenditures.
FOOTNOTES

1/ Only a small set of feasible bundles is likely to exist even in large metropolitan areas. Part of this limitation of alternatives may be due to problems of finding enough people with similarly "peculiar" tastes and part is due to social and political constraints such as racial discrimination and zoning. There is also a dynamic component to this problem because elements of the bundles may change continuously and uncontrollably (e.g. neighborhoods change, the housing stock filters up or down, job locations move) and there are costs to residential relocation.

2/ If the Tiebout model works perfectly, then everyone is always perfectly happy with their school system or they leave the community. Political voting is trivial because all decisions are unanimous.

3/ The word "voting" in these models is not to be taken literally. These are not models of how political influence is exercised but rather of the consequence of a process of influence where the majority in the community usually find the observed school budget to be acceptable. We do not care whether they vote, demonstrate, or threaten to blow up the school to achieve that result (except, of course, in our own school district). It may be true, however, that such models have predictions about votes on school referenda as well as about school budgets. See Deacon and Shapiro (1975).

4/ We do not linger on this problem because we do not wish to detract from our main argument about aggregation of family demands. However, the interesting problem of how properly to incorporate the bundling of public
services has yet to be addressed. If, for example, the utility function contains housing services, local school expenditures, and some composite of all other commodities, the constraints have to be written so as to reflect the fact that more housing increases school expenditures unless there is a compensatingly lower tax rate.

\[2/\] Their desired rate corresponds to a point on their demand curve generated as usual by utility maximization subject to their budget constraint. Assuming that the school tax is proportional to income and that it is a small share of income, then the rate will be approximately the slope of the ray from the origin to the optimal point on the budget constraint when the vertical axis is school expenditures and the horizontal axis is all other goods and services.

\[6/\] The issue of the income incidence of the property tax is unsettled (cf. Aaron (1975)). Treating it as proportional simply makes our analysis easier and the exposition simpler. Our data force us to talk in terms of the income distribution rather than the wealth distribution.

\[7/\] We are ignoring the logrolling problems raised by Tullock (1959).

\[8/\] Frankly, we do not know what to call this curve. It is really a special form of an Engel curve where expenditures on the particular commodity, schooling (expressed as a proportion of income), are a function of income. If \( E \) is the fractional Engel curve for school expenditures and \( D \) is the standard demand curve for school expenditure then \( E(Y) = D(Y,P)/Y \) where \( Y \) is income and \( P \) is price.

\[2/\] This assumes that it is educational expenditures, and not physical quantities of education, that enter the consumer's utility function, and that these expenditures are not a Giffen good.
This case was noted by Bergstrom and Goodman (1973), but they do not pursue it. They do take up the possibility that not all voters in an income class have the same tastes and therefore do not all vote the same way. They offer a theorem about income distribution which may solve that problem. It is interesting that the fractional Engel curve they display in their argument about heterogeneous tastes is U-shaped. No evidence for this is offered and other possibilities are not mentioned.

This is a common practice when, for example, the distribution of income is assumed to be log-normal or displaced log-normal. Indeed, if we had reason to believe either of these distributions were a good approximation of our data, some of our difficult empirical problems would have been easily and perhaps better solved.

There are other ways the variance of the income distribution could be increased than the way shown in Figure 3. In fact some of these ways, if they affect only the ends of the tails, or only the very center, might leave the tax rate unchanged. Hence even if we observed cases where communities had the same tax rate, the same mean and median income, and different variance, we could not be sure the fractional Engel curve was monotonic.

This specification of the bias for estimating the fractional Engel curve has the disadvantage of becoming negative for some extreme values of \( Y_{med} \), but we used it initially in the hope that it would have the right properties at least over the range of our data. This "disadvantage" of the specification forces us, as the reader will see below, to realize that the fractional Engel curve has more than one turning point. We would probably
not have discovered this if we had forced a specification more in conformity with our simple theory.

14/ Brown and Saks (1975) have argued for the importance of the standard deviation of achievement as one output of schools. It is possible that voter dissatisfaction over the nature of school outputs would manifest itself through the selection of school board members or other administrators, rather than by changing the level of financing. Furthermore, we have no prior information on whether most people prefer more or less egalitarian schools.

15/ In the absence of any information, we are ignoring the nasty problem of unsystematic variation in preferences for individuals who are similar in all characteristics which we observe.

16/ The sample has a high proportion of small rural school districts and these were the ones with income distribution at variance with most economists' preconceived notions of what "the" income distribution looks like (i.e., the long tails were on the left).

17/ One other abortive attempt is worth reporting. Rather than trying to capture all the relevant information in the income distribution in two or three parameters, why not simply include as variables the proportion of families in each income category? Since there are 15 categories in the U.S. Census classification, 1\frac{1}{2} variables are necessary. This did not seem cumbersome considering we had observations on 500 school districts. In fact this approach was suggested by Blinder (1975) in another context, but he despaired using it because the data requirements were excessive.
We, on the other hand, had plenty of degrees of freedom. The upshot was that there is such a high correlation between the income variables that the standard errors of all coefficients became very large. Even combining several of the categories at the lower range of the income distribution did not help significantly. Thus our falling back on approximating the observed distribution.

Estimation of Lorenz curves requires information on the total income accruing to families in each income class as well as the number of families. Regretably the Census does not provide the total or mean income for each class in each tract so this had to be estimated or guessed at. This is a particularly troublesome problem for the large and open-ended classes (25,000-49,999 and above $50,000). Happily or not, we found that our estimates of skewness and the Gini coefficient were quite insensitive to variations in the assumed class means, probably because most families fall in narrowly defined classes where the error in guessing the mean cannot be very large.

The assumed mean incomes by class for the estimates presented below are: (a) the class interval midpoint for income classes up to and including $14,999; (b) $15,500 for the $15,000-24,999 class, $31,200 for the $25,000-49,999 class, and $67,000 for the above $50,000 class. These were chosen from more aggregate data.

In 1973, Michigan changed to a matching grant or variation of the "power equalization" plan. See Coons, Clune and Sugarman (1970) or Barro (1974). Under a matching grant state aid becomes a component of
price for families trying to decide how much to spend. Feldstein (1975) recently analyzed such a plan for Massachusetts school districts.

20/ The definitions of the community type variables are:

Metropolitan core—a city of 50,000 or more population.
City—a community of 10,000 to 50,000.
Town—a community of 2,500 to 10,000
Rural Community—a community of less than 2,500.
Suburb—a community of any population size that has as its economic focal point a metropolitan core or a city.

Each community type variable was given the value 1.0 for a district of corresponding type, 0.0 otherwise. In the regressions reported below, the variable for rural community was omitted.

21/ Since sampling theory tells us that the error term will be a function of the size of the population tested (heteroskedasticity), ordinary least squares would be inappropriate [see, e.g., Kmenta (1971), pp. 322-336].

22/ Piele and Hall (1973) in their review of the political science (survey) literature find support for the U-shaped form of the fractional Engel curve.

23/ This finding is consistent with data on voting behavior of Blacks and Whites with respect to school millage elections. Cf. Piele and Hall (1973), p. 106.

24/ Ex post, we can devise an explanation for this or any other result. One might call it the "permanent child hypothesis." Families with only one school-age child are much more likely than other family sizes to be families whose long-term size is different from current size either because they are planning to have more children or because their older
children have graduated from school. Thus communities with unusually small numbers of children may be in greatest disequilibrium. Such periods would, of course, provide windfalls to school administrators for a period of time.

\[25/\] We made some unsuccessful attempts to augment the specification of Table II by using a higher degree polynomial in income to see if the fractional Engel curve could be made to turn down. These higher degree terms in income never succeeded in making the Engel curve "flatten out." The polynomial form simply did not contain enough restrictions on the possible outcomes.

\[26/\] We did not include all the variables that were in the polynomial form for two reasons. First, not all seemed in retrospect to be empirically important, and second, the great expense of non-linear estimation with a large number of parameters led us to economize. Comparing the logistic and polynomial results within the range of the data leads us to conclude we did not give up very much in accuracy.

\[27/\] The non-linear estimation program used here was provided by Metzler, Elfring and McEwen of the Upjohn Corporation. We derived the starting values for the parameter estimates from information provided in Table II.
REFERENCES


