Estimating Labor Supply Functions

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February 1972
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Controversy over the adoption of various proposals for a negative income tax has focused on the effect of these programs on the work effort of potential program participants. On the one hand, current welfare recipients will presumably face a lower marginal tax on earned income and perhaps a change in level of income subsidy. On the other hand, the current "working poor" will face an increased marginal tax on earned income and a hitherto nonexistent income subsidy. Though it is expected that the former group will increase its labor supply,¹ it is widely agreed that the latter group is likely to reduce its labor supply. With regard to this second group the crucial question, of course, is the likely quantitative magnitude of any labor supply reduction, since it is this that will determine both the tax revenue costs of financing the program and the effects of decreased factor supplies on the prices of the goods and services produced by industries employing low income workers. Precisely the same issues are involved in the somewhat older question of the effect of income taxation on work effort, although no clear prediction on the direction, much less the size, of the labor supply effects is possible in this case.²

¹Very little empirical literature on the labor supply behavior of this group exists, but some recent efforts are reported by Hausman [8], Kowlatt [17], and Saks [18]. Though we recognize the importance of the behavior of current welfare recipients in response to changes in the welfare system, we will follow the lead of others and ignore this important group in the sequel.

²See Musgrave [14] and Kosters [12].
Both discussions derive their frame of reference from an application of the classical theory of consumer choice to the demand for leisure or non-market time. Their importance for practical matters has resulted in a very substantial volume of empirical research using this basic framework in recent years. Despite minor variations, however, the basic econometric specification has changed little from that proposed in the pioneering papers of Mincer [13] and Kosters [11]. This state of affairs stands in rather sharp contrast to work where the classical theory of consumer choice is applied to the demand for market goods. In this latter area a variety of functional forms and estimation procedures have been proposed with an eye toward improving the efficiency of estimation of important parameters. In somewhat the same spirit, the purpose of this paper is to present an alternative scheme for the choice of parameterization of labor supply functions that offers some distinct advantages for purposes of estimation. In the first section of the paper we briefly review the classical theory as applied to the supply of labor and the implications of that theory for the effects on work effort of a negative income tax plan. The second section contains a discussion of the choice of parameters in labor supply analysis for purposes of empirical implementation, while the last section contains an empirical test of the proposed procedure using microeconomic data on male heads of families from the 1967 Survey of Economic Opportunity.

3/ For a comprehensive survey of this literature see Goldberger [5].
I. The Theory of Labor Supply

The force of the classical theory of labor supply resides in the assumption that the individual consumer behaves as if he maximizes a well-behaved preference function subject to a constraint on total available resources. The individual's basic resource is time \( T \), which may be used in work, or for a variety of non-market activities. The consumer spends his earnings on market goods, which may be aggregated into a "composite commodity" \( X \), with price \( P \), so long as the prices of the goods within this bundle do not change relative to each other. Precisely the same aggregation over all non-market uses of time can be performed so long as it is assumed that time may be freely substituted among its uses. This composite commodity is commonly called leisure \( L \), with price \( W \), the price of time in each of its uses. Assuming that the consumer-worker has \( Y \) dollars of unearned income, we may summarize his budget constraint as

\[
(1) \quad PX = W(T-L) + Y = WL + Y
\]

where \( H = T-L \) is the hours allocated to work. This constraint is sometimes written as

\[
PX + WL = WT + Y = F
\]

to emphasize that the consumer's "full income" \([3]\), \( F = WT + Y \), may be thought of as \( WL \) dollars spent on leisure and \( PX \) dollars spent on market goods. Faced with exogenous values of \( F, W, \) and \( Y \), the consumer
has chosen the values of $L$ and $X$ that satisfy (1) and maximize his preference function, $U = U(L, X)$, only if the marginal utility of leisure equals the marginal utility of income times the price of leisure, 
\[ \frac{\partial U}{\partial L} = \lambda W, \] and the marginal utility of consumption goods equal the marginal utility of income times the price of consumption goods, 
\[ \frac{\partial U}{\partial X} = \lambda P. \] The budget constraint, along with these latter two conditions, may be thought of as three equations in the three unknowns $L$, $X$, and $\lambda$. For a given set of values of $P$, $W$, and $Y$, therefore, they may be solved for the former as functions of the latter. The first of these is

\[(2) \quad L = L(W, P, Y),\]

the demand function for leisure. Since $T-L = H$ we have $-dH = dL$, so that the partial derivatives of the labor supply function

\[(3) \quad H = H(W, P, Y)\]

will be equal and opposite in sign to those of (2).

Now consider the effect of a simple negative income tax on work effort. Such a program effectively changes the budget constraint to

\[(4) \quad P(X - G + (1-t)[WH + Y]),\]

where $G$ is a flat grant called the "guarantee level" and $0 < t < 1.0$ is the tax rate on income. Since $WH + Y = I$ is money income before the program, and $G + (1-t)I$ is money income after the program, the subsidy to the consumer unit is $\text{SUB} = G + (1-t)I - I = G - tI$. Negative subsidies (positive taxes) would result if $G - tI < 0$, so it is assumed
that the budget constraint (4) is imposed only for \( I < G/t \) and the budget constraint (1) holds for \( I > G/t \). The parameter \( G/t \) of the program is called the "break-even" income level because persons with incomes higher than this do not participate in the program. It is interesting that increases in \( G \), holding \( t \) constant, increase the break-even level of income and thus the coverage of the program. Likewise, decreases in \( t \), holding \( G \) constant, increase the break-even level of income and thus the coverage of the program. A crucial problem for public policy, therefore, is to find satisfactory values for \( G \) and \( t \) that do not imply inordinately high break-even levels. Notice that (4) is equivalent to (1), except that \( Y \) is replaced by \( G + (1-t)Y \) and \( W \) is replaced by \( (1-t)W \). It follows that the optimum level for \( H \) for the consumer who is under the program is obtained by making these substitutions in (3). The change in hours worked is then

\[
\Delta H = H(1-t)W, \ P, \ G + (1-t)Y - H(W, P, Y). \]

Even assuming that \( \partial H/\partial Y < 0 \), so that leisure is a normal good, without further analysis it is not obvious what sign we should expect this expression to take.

Of course, the classical theory is not so empty as this. The basic result of the application of this theory to the supply of hours is the decomposition of \( \partial H/\partial W \) into a substitution (S) and income effect [(\( \partial H/\partial Y \)H),

\[
(5) \ \partial H/\partial W = S + (\partial H/\partial Y)H,
\]

with the important result that utility maximization requires \( S > 0 \).
Differentiating (3) totally and substituting (5) then gives

\[(6) \quad dH = SdW + \partial H/\partial Y[HdW + dY],\]

where we have set \(dP = 0\) here and in the sequel.\(^4\) Suppose that we may treat \(S\) and \(\partial H/\partial Y\) as constants over the range of variation in \(Y\) and \(W\) that we wish to examine. Under these conditions we may treat the effect of the program as producing the changes \(dW = -tW\) and \(dY = G - tY\). Substituting, we have

\[(6a) \quad dH = -tWS + \partial H/\partial Y[G - t(WH + Y)].\]

This says that the change in hours worked resulting from the negative income tax program may be found by multiplying the negative of the tax rate times the wage rate times the substitution effect and adding the product of the income effect and the initial subsidy (= \(G - tI\)).

The first part of this effect must be negative and the second part will also, so long as leisure is not an inferior good. This is the conventional result that a negative income tax program must reduce the hours of work of rational consumer units previously not covered by such a program. Of course, the crucial determinants of the magnitude of \(\Delta H\) are the size of \(S\), the substitution effect, and \(\partial H/\partial Y\), the income derivative. We turn next to procedures for estimating these quantities.

\(^4\)Since we deal with cross-sectional data below, the assumption that \(dP = 0\) is equivalent to the assumption that each worker faces similar prices for market goods.
II. Estimating Labor Supply Parameters

The principle unsettled issue in the classical labor supply analysis is the manner of its empirical implementation. In practice this involves a choice of which functions to treat as constant parameters for purposes of estimation. Most studies are based on an approach suggested by Kosters [11], which starts from a linear approximation to the labor supply function (3):

\[
H = \alpha_0 + \alpha_1 W + \alpha_2 Y.
\]

The \( \alpha_i \) are assumed constant for purposes of estimation and observations on different sets of \( H, W, \) and \( Y \) are used to estimate these parameters. This scheme is equivalent to requiring that \( \partial H/\partial W = \alpha_1 \) and \( \partial H/\partial Y = \alpha_2 \) be constants. Of course, these partial derivatives are not constants in the theory, and hence this particular choice of parameterization is only one of many possible empirical strategies. One obvious disadvantage with this approach is that it implies that the substitution effect, \( S \), varies systematically from observation to observation in the sample. In particular, from (5) we have for the \( i^{th} \) observation that

\[
S_i = \alpha_1 - \alpha_2 W_i.
\]

Since \( \alpha_1 \) and \( \alpha_2 (\lt 0) \) are constant, this implies a numerically larger substitution effect for persons who work longer hours. Moreover, since the slope of the labor supply function, \( \alpha_1 \), often turns out to be negative,\(^5\)

\(^5\)That is, the labor supply function turns out to be backward bending, empirically.
it is generally possible to choose a non-negative value for \( H_1 \) such that \( S_1 < 0 \). This, of course, violates the crucial empirical prediction of the classical theory, and is not a reassuring sign for the latter's use in practical matters of affair.\(^6\)

It is also possible to specify a linear approximation to (3) that will be similar to (7), but which uses full income, \( F = W_T + Y \), as an independent variable:\(^7\)

\[
(8) \quad H = \beta_0 + \beta_1 W + \beta_2 F.
\]

Since substitution of the identity \( Y = F - WT \) into (7) allows it to be rewritten as

\[
(7a) \quad H = \alpha_0 + (\alpha_1 - \alpha_2 T)W + \alpha_2 F,
\]

it follows that (7) and (8) are identical when we set \( \beta_1 = \alpha_1 - \alpha_2 T \) and \( \beta_2 = \alpha_2 \). Moreover, so long as \( T \) is not allowed to vary in the sample, least squares estimates of the coefficients of (8) will be identical to these implied linear transformations of the coefficients of (7). The substitution effect for the \( i^{th} \) observation from (8), using (5) again, is

\[
(5b) \quad S_i = \beta_1 + \beta_2 T - \beta_2 H_i.
\]

\(^6\) The average substitution effect in the sample using this approach is, from (5a), \( \bar{S} = \bar{\alpha}_1 + \bar{\alpha}_2 \bar{H} \), and is often reported, in what is obviously imprecise language, as "the" substitution parameter.

\(^7\) Mincer [13] apparently had this in mind.
which is identical to (5a) after the substitutions for \( \beta_1 = \alpha_1 - \alpha_2 T \) and \( \beta_2 = \delta_2 \). It follows that this approach to parameterization is formally identical to the preceding one.\(^8\)

An alternative scheme that we have proposed is to treat \( S \) and \( \partial H/\partial Y \equiv B \) as constants for the purpose of estimation.\(^9\) To do this we start directly with the total differential of the labor supply function, (6), and replace the unobservable infinitesimal changes \( dH \), \( dW \), and \( dY \) by the observable finite changes \( \Delta H \), \( \Delta W \), and \( \Delta Y \) to get

\[
(9) \quad \Delta H = S \Delta W + B[H^*(\Delta W) + \Delta Y],
\]

where \( H^* \) is the appropriate point of income compensation for a wage change of size \( \Delta W \). In a time-series or panel-data context it would be natural to treat the operator \( \Lambda \) as implying first differences.\(^{10}\)

In cross-sectional data a natural procedure is to treat these differences as deviations from the sample mean, and this is the procedure we follow.

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\(^8\) Since the value taken on by \( T \) is arbitrary, it may be assigned the value \( H \). In this case the average substitution effect is just \( \beta_1 \). Of course, if \( T \) is assigned a value other than \( H \), then \( \beta_1 \) will not equal \( S \) and will be either higher or lower according to whether the value assigned to \( T \) is larger or smaller than \( H \). Hall [7] uses an equation like (6), for example, and assigns \( T = 2,000 \) hours per year.

\(^9\) See Ashenfelter and Heckman [1].

\(^{10}\) In the case of a negative income tax experiment \( \Delta W = -tW \) and \( H^*(\Delta W) + \Delta Y \) may be taken as the (initial) income subsidy, which gives a very natural interpretation to (9).
The point of compensation, \( H^* \) in (9), is uniquely determined as the point of equilibrium hours in the case of infinitesimal changes. Determining this point for finite changes in the independent variables is an old, but important problem in economics. To see the difficulty, differentiate the budget constraint (1) totally to get

\[
(10) \quad P(dX) - W(dH) = P(dX) + W(dL) = H(dW) + dY.
\]

In (10) we have separated the effects of a small change in \( W \) and \( Y \) into the change in the value of expenditures on the left hand side, which equals the change in the value of consumption \( [P(dX)] \) plus the change in the value of leisure \( [W(dL)] \); and the change in money income on the right hand side, which equals the change in the value of work effort \( [H(dW)] \) plus the change in non-labor income \( (dY) \).\(^{11}\) We should like to obtain the empirical analogue of the change in money income, which is on the right hand side of (10), but for finite changes. To proceed, we may write the first difference about the mean of the budget constraint (1) as

\[
(11) \quad \bar{F}(\Delta X) = H(\Delta W) + W(\Delta H) + (\Delta H)(\Delta W) + \Delta Y.
\]

This differs from (10), of course, because of the second order effect \( (\Delta H)(\Delta W) \) which does not vanish on taking finite differences. Let us first separate the change in the value of expenditures from the change

\(^{11}\) This corresponds to putting the changes in the dependent variables of the analysis, \( dX \) and \( dH \), on the left, and the changes in the independent variable of the analysis, \( dW \) and \( dY \), on the right.
in money income in (11) by evaluating expenditures at the mean wage level. This gives

$$\bar{P}(\Delta X) - \bar{W}(\Delta H) = \bar{P}(\Delta X) + \bar{W}(\Delta L) = \overline{H}(\Delta W) + (\Delta H)(\Delta W) + \Delta Y = (\overline{H} + \Delta H)\Delta W + \Delta Y.$$  

The deviation of money income from the mean is $(\overline{H} + \Delta H)\Delta W + \Delta Y$. This suggests using $\overline{H} + \Delta H$, which is actual equilibrium hours, as the point of compensation. On the other hand, if we separate the change in the value of expenditures from the change in money income in (11) by evaluating expenditures at the new $(\bar{W} + \Delta W)$ wage level, we have

$$\bar{P}(\Delta X) - \bar{W}(\Delta H) - (\Delta W)(\Delta H) = \bar{P}(\Delta X) - (\overline{H} + \Delta W)\Delta H = \bar{P}(\Delta X) + (\overline{W} + \Delta W)\Delta L = \overline{H}(\Delta W) + \Delta Y.$$  

The deviation of money income from the mean is now $\overline{H}(\Delta W) + \Delta Y$, and this suggests using $\overline{H}$ as the point of compensation. The disparity between the two measures of money income change occurs only because of the finite nature of the wage change.\textsuperscript{12/}

Choosing either measure imparts a certain asymmetry to the analysis. Using the first measure, for example, if we initially raise the wage by some amount from the mean and use the final equilibrium $H$ as the point of compensation, and then reduce the wage by the same amount using the original value of $H$ as the point of compensation, we will not leave the consumer at the value of $H$ from which he started. The same problem would result from using the second measure. A straightforward way to avoid this asymmetry is to use a simple average of these two possible values.

\textsuperscript{12/}This is precisely the problem Hicks [9] addressed in regard to the measurement of consumer surplus for finite price changes. The first procedure corresponds to his "equivalent variation" while the second corresponds to his "compensating variation."
Hence we define $H^* = (H + \hat{H})/2$.\footnote{This is the procedure suggested by Barten [2] and Theil [19] in a time-series context.}

The procedure we have suggested is not, however, without its drawbacks. In particular, given the way we have defined $H^*$, any disturbance term added to (9) will necessarily be correlated with the right hand "real income" variable in (9) because of the presence of the budget constraint (1). Intuitively, this implies that the ordinary least squares estimator of the coefficient of $[H^*\Delta W + \Delta Y]$ would be biased (even asymptotically) upward because part of the "credit" for the definitional effect of increased labor supply on money income would be given to the effect of money income on labor supply.\footnote{Or, as Hincer remarked in his well known paper [13, p. 69], "Instead of serving as a determinant of labor force behavior, it [money income] already reflects such decisions."}

Our procedure for resolving this problem is the use of an instrumental variables estimator where the actual values of $[H^*\Delta W + \Delta Y]$ are replaced in a least squares regression by their predicted values from a regression that includes a function of the wage rate and other presumably predetermined variables. This approach to estimation is very similar to the approach used in some cross-sectional consumption function studies, where actual income is replaced by predicted income on the basis of demographic and other characteristics and this new measure is taken as more correctly representative of "permanent income." Note that $\Delta Y$ is also taken as endogenous in response to the argument of Greenberg.
and Kosters [6] that those persons working longer than normal hours have a taste for net assets, implying that any disturbance term in (9) would also be positively correlated with $\delta Y$.

III. Empirical Results

In principle, the classical theory and the procedures for estimation outlined above may be applied to any data on labor supply behavior. In practice, the choice of the appropriate dependent variable for empirical implementation is more difficult. One important difficulty is the choice of a time horizon over which the decision-making process described in Section I will be appropriate. Although it seems plausible to assume that the theory is appropriate to the decision about lifetime working hours, for example, it may be inappropriate to the decision about hours worked in a given year because of systematic variation in wages and family conditions over the life cycle.\(^1\)

Here we estimate equation (9) for the annual hours of married men in their prime working years, ages 25 to 64, on the assumption that current hours of work may well be an appropriate proxy for life-cycle hours of work for this group. In essence, if a male is almost always in the labor force in this age range, as we observe, it is reasonable to expect him to work the same number of hours in each year. For married women, on the other hand, labor force attachment

\(^1\)What we have in mind, of course, is a model in which the consumer faces a series of known (or expected) wage rates over his lifetime and determines an optimal set of annual hours so as to maximize a preference function that has inter-temporal elements. Changes in a wage rate in one period would then have effects on labor supply in that and all other periods as well. In the empirical results that we report we include age and age squared as regressors, which, under certain conditions, will be a satisfactory solution to this problem.
is not as complete, and the hours worked in a given year may consequently be a poor proxy for the life-cycle hours of this group. We choose men with non-zero hours in the sample year on the assumption that men who did not work during the sample year were at corner solutions and do not have labor supply functions defined for them, even though this may not be the case over a longer decision-making period. We also choose men whose wives did not work during the sample year and we assume that the wives of these men were at corner solutions, so that it is legitimate to maintain that variations in the wage rates that these wives would receive if they were in the market are of no consequence. Both of these assumptions are adopted for their tractability and not their realism.

Table 1 contains instrumental variables estimates of equation (9) from the national probability sample component of the 1967 Survey of Economic Opportunity.¹⁶/ Unlike most samples of this type, two separate methods exist for estimating annual hours for a worker in the sample. On the one hand, a respondent's estimate of annual weeks worked may be multiplied by his estimate of hours worked last week to obtain annual hours worked. Alternatively, since the 1967 SEO provides a particularly good estimate of the respondent's normal hourly wage, this may be divided into the respondent's estimate of annual earnings to obtain annual hours worked.¹⁷/ Both procedures have advantages and disadvantages, but preliminary calculations did not suggest much

¹⁶/ For a discussion of the instrumental variables estimator in a nonlinear equation like (9), see Kelejian [10].

¹⁷/ Hall [7] seems to have been the first writer to suggest this procedure.
difference in the results using the two procedures and we finally 
adopted the latter. As the empirical counterpart to unearned income 
we pooled rent, dividend, and interest receipts with royalty income, 
private transfer receipts, and alimony payments. Workers receiving 
payments from work-determined public transfer programs, including 
social security, public welfare, and unemployment insurance payments, 
were excluded from the sample since this income does not correspond 
to the unearned income concept in the classical theory, and is more 
likely to be the result than the cause of labor supply behavior. 
The sample constituted in this way contained 3,203 male workers. 
The results in line 1 of Table 1 also contain a dummy 
variable to allow for any differences in hours between whites and 
non-whites not accounted for by wages and incomes. As can be seen 
from the table, the results imply that at given wage and income levels 
non-white workers averaged about three hundred hours per year less than 
white workers. We do not speculate on the reasons for this difference 
here. In lines 2 and 3 we add a set of dummy variables to the equation 
to account for differences in workers' city sizes and geographical 
location, on the assumption that these variables will be good proxies 
for differences in the price (? of consumption goods. As can be seen 
from the table, most of these variables' coefficients are not estimated 
with much precision. In line 4 we also add a quadratic in the age of 
the worker to allow for any systematic life-cycle changes in annual 
hours that we have ignored. As can be seen from the table, these latter
Table 1
Instrumental Variables Estimates of Equation (9) for 3,203 Male Workers, Spouse Present but Not Working
(with estimated asymptotic standard errors in parentheses)

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Estimates of:</th>
<th>Estimated Coefficients of:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S</td>
<td>B</td>
</tr>
<tr>
<td>1</td>
<td>68.2</td>
<td>-.065</td>
</tr>
<tr>
<td>2</td>
<td>68.9</td>
<td>-.066</td>
</tr>
<tr>
<td>3</td>
<td>66.9</td>
<td>-.066</td>
</tr>
<tr>
<td>4</td>
<td>67.3</td>
<td>-.070</td>
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</tr>
</tbody>
</table>

Notes: The instrumental variables used are polynomials in the wage (to three powers), race, the region and SMSA-size dummy variables, age and age squared, plus: a dummy variable for the presence of a child less than age six, a set of occupational dummy variables, and polynomials in the worker's years of schooling (to three powers). For a core complete discussion of the instrumental variables estimator in a nonlinear equation such as (9) see Kelejian [10]. The dummy variable for race takes on the value of unity if the respondent is non-white and zero otherwise. The SMSA-size dummy variables are in reference to the category "not in SMSA" and the numbers denote the size class, where SMSA <250 means SMSA of size 0 to 250,000 and so on. The regional dummy variables are in reference to the category South.
<table>
<thead>
<tr>
<th>Line No.</th>
<th>Estimates of:</th>
<th>Estimated Coefficients of:</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>B</td>
<td>SMSA&lt;250</td>
</tr>
<tr>
<td>1</td>
<td>66.6</td>
<td>-.064</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>2.0</td>
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<td>64.7</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>64.7</td>
</tr>
</tbody>
</table>

Notes: See notes to Table 1
variables have highly significant coefficients and imply a steady growth in annual hours until around age 44, and a steady decline thereafter. Finally, the significance of the dummy variable for race in all of these equations suggested the possibility that the basic labor supply parameters might differ for one reason or another as between whites and non-whites. We consequently re-estimated the equations in lines 1-4 of Table 1 for white workers only, and these results are contained in Table 2. As can be seen from this latter table, most of the estimated coefficients change very little as a result of restricting the sample in this way. Hence, we concentrate on the results in line 4 of Table 1.

The estimates of $S$, the substitution effect, and $B$, the income derivative, are remarkably stable in these two tables at around 67.0 and -.070, respectively; and both coefficients would clearly be judged significantly different from zero at conventional test levels. This gives a fairly small substitution elasticity (evaluated at the means) of around .12. Differentiating the budget constraint (1) partially with respect to $Y$ gives $P(\partial X/\partial Y) - W(\partial H/\partial Y) = 1$, which implies that if consumption goods are to be normal (i.e., if $\partial X/\partial Y > 0$), then $W(\partial H/\partial Y)$ must not be less than minus one. The quantity $\frac{\partial}{\partial y}(\partial H/\partial Y)$ is not a constant in this model of course, but it clearly satisfies this constraint within the sample. Evaluated at the mean wage ($3.86$) we have $\bar{W}(\partial H/\partial Y) = \bar{W} = -.27$, which implies that the average worker allocates .27 of each additional dollar of non-labor income to the purchase of non-market time and the other .73 to the purchase of consumption goods. On the other hand, the comparable estimate for $W(\partial H/\partial Y)$ at a wage of $2$ is only -.14, which
implies that only .14 of each additional dollar of unearned income is allocated to the purchase of non-market time with the remaining .86 allocated to consumption goods. The slope of the labor supply function implied by equation (9), \( (A/3Y)_1 = S + H_1(3H/3Y) \), is also not a constant in this model and depends on the point of compensation \( H_1 \). Except for levels of annual hours below 800 this quantity is always negative in our results, which implies that we observe a backward bending labor supply function over most of the range of variation in the sample. The sum of the substitution elasticity and \( W(3H/3Y) \) is equal to the wage elasticity of the uncompensated supply function.\(^{18}\)

Evaluated at the mean wage and hours (2,272) this is \( .12 - .27 = -.15 \) and seems broadly consistent with Rosen's [16] estimates of -.07 to -.30 from inter-industrial data, Finegan's [4] estimates of -.25 to -.35 from inter-occupational data, Winstone's [20] estimates of -.07 to -.10 from inter-country data, and Owen's [15] estimates of -.11 to -.24 from U.S. time-series data.

Another way to evaluate the magnitudes of our estimated labor supply parameters is to see what they imply about the effect on annual hours worked of an illustrative negative income tax plan. Consider a plan with a guarantee level of \( G = 2,400 \) and a tax rate of \( 1/2 \) and assume the case of a worker earning one 1967 dollar per hour and working 2,000 hours per year, so that he obtains an initial subsidy of \( $1400 \) per year. We may then make the appropriate substitutions into (6a) to obtain

\[ \frac{(3H/3Y)}{W} \text{ for } W/H \text{ we have } \frac{(3H/3Y)}{W} = \frac{(3H/3Y)}{W}S + W(3H/3Y). \]
$\Delta H = -131.5$ hours, or about a 6 1/2 percent decline in annual hours.\footnote{Actually, using (6a) in this way ignores the fact that the appropriate finite approximation to (6a) uses $(H + H_0)/2$ as the point of compensation. The difference between the decline in hours from (6a) and the solution to its finite approximation in the form of (9) is only four hours, however, which seems justification enough for using the simpler procedure of evaluating (6a) directly.}

The same worker confronting a program with a similar guarantee level but a tax rate of 2/3 would have a $\Delta H$ of only -119, because his subsidy would be so much smaller. Other combinations can easily be worked out in this framework and will obviously give differing results depending on the initial wage and income level of the worker and the parameters of the plan.

IV. Conclusion

In this paper we have offered an alternative scheme for the choice of parameters in labor supply functions that offers some distinct advantages, but disadvantages also, for purposes of estimation. Our empirical results provide some evidence that the procedure we suggest for estimating labor supply parameters may be a fruitful one. The basic results are consistent with the predictions of the classical theory of labor supply and the crucial parameters seem to be estimated with tolerable precision. Of course, far more additional evidence is necessary before any firm conclusions can be drawn concerning the usefulness of our estimated parameters or, indeed, the usefulness of the classical theory for practical matters of affair.
REFERENCES


