Time Series Representations of Economic Variables and
Alternative Models of the Labor Market

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Introduction

The inability of the empirical models of wage inflation built in the 1960's to predict the simultaneous high inflation and high unemployment of the 1970's led to their virtual demise and to a subsequent rebirth of interest in the theoretical foundations of these models. Both the empirical failure and the state of theoretical work leave the impression with many economists that virtually any theory is likely to be consistent with the "facts" of the aggregate labor market, and that there simply are not enough facts to discriminate among leading candidates.

At the same time, having learned the hard way from the poor performance of many models in the 1970's, econometric practice has changed so as to emphasize the importance of the dynamic structure of most time series data. In this new view parsimonious descriptions of the data are the autoregressive and moving average (ARMA) characteristics of the various time series that represent the data history of particular markets. Since most of the cyclical characteristics of movements in labor market variables seen to be satisfactorily represented by relatively low order ARMA models, these representations are then taken to be the

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1 See particularly Granger[9], Sims[26], Sargent[23], Hendry[10], and Wallis[23].
"facts".\(^2\)

If the ARMA representations of labor market variables are an adequate description of the data, then it seems that a useful theory is one that likewise delivers a linear ARMA representation of the data. Tests of the theory then involve straightforward comparisons of the observed and predicted ARMA representations of the data.\(^3\)

In this paper we employ this research strategy by first summarizing the time series "facts" about the aggregate labor market with which a useful theory must be consistent. Our empirical strategy is to first set out the unrestricted reduced forms from a vector autoregression that contains nominal wages, consumer prices, nominal interest rates, and unemployment. From there we are able to test and catalogue the "exclusion

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\(^2\) It is worth observing that these are representations for the stochastic parts of the various time series only. Deterministic parts of the time series are typically represented by non-stochastic trends that are removed from the data before or during the analysis.

\(^3\) In using this methodology it is important to recognize that theories about the deterministic movement of the variables are not being tested. In effect, what is being offered are tests of explanations for what once was called the Trade Cycle, and they leave open the question of what determines the long run average levels of labor market variables. As a result, much of this research differs sharply in style from the continuing Keynesian tradition of analysing the determinants of long run slack in the labor market. Keynes himself wrote in his Trade Cycle chapter of the General Theory that "Since we claim to have shown in the preceding chapters what determines the volume of employment at any time, it follows, if we are right, that our theory must be capable of explaining the phenomenon of the Trade Cycle...", but concludes that "to develop this thesis would occupy a book rather than a chapter, and would require a close examination of the facts." [11], p.313.
restrictions" that are consistent with the quarterly U.S. time series data. We then compare the facts against the predictions of several elegant and straightforward models due to Lucas[15], Fischer[6], and Taylor[27] and others that satisfy our methodological criterion for a useful theory. These are also models of considerable practical significance, since the continuing debate over the effectiveness of monetary policy in stabilizing aggregate employment and output has been conducted around them.

Much to our surprise, the facts are not only sufficient to discriminate among these models, they are also sufficient to demonstrate serious problems with at least the simplest specifications of all of them.

I. The Time Series Data

Table I(a) provides one elementary description of the basic U.S. quarterly time series on the logarithm of the nominal wage (\( \tilde{\pi} \)), the logarithm of the consumer price index (\( P \)), the logarithm of the unemployment rate (\( U \)), and the 90 day Treasury Bill rate (\( R \)). In this study we have used average hourly straight time earnings in manufacturing as an index of aggregate wages. Precise data definitions and sources are contained in the Appendix. For each of these time series we present in Table I(a) the fourth order univariate autoregressions (AR4) obtained by least squares fit over the period indicated. In all cases here, and in subsequent tables, we have included seasonal dummy variables and
Table I (a)

Univariate AR 4 Representations

<table>
<thead>
<tr>
<th>Regressors</th>
<th>W</th>
<th>F</th>
<th>U</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>.97</td>
<td>1.54</td>
<td>1.49</td>
<td>1.46</td>
</tr>
<tr>
<td></td>
<td>(.11)</td>
<td>(.11)</td>
<td>(.10)</td>
<td>(.10)</td>
</tr>
<tr>
<td>AR(2)</td>
<td>-.02</td>
<td>-.63</td>
<td>-.90</td>
<td>-.90</td>
</tr>
<tr>
<td></td>
<td>(.15)</td>
<td>(.21)</td>
<td>(.18)</td>
<td>(.18)</td>
</tr>
<tr>
<td>AR(3)</td>
<td>.09</td>
<td>.29</td>
<td>.51</td>
<td>.63</td>
</tr>
<tr>
<td></td>
<td>(.15)</td>
<td>(.20)</td>
<td>(.18)</td>
<td>(.19)</td>
</tr>
<tr>
<td>AR(4)</td>
<td>-.13</td>
<td>-.26</td>
<td>-.21</td>
<td>-.33</td>
</tr>
<tr>
<td></td>
<td>(.10)</td>
<td>(.11)</td>
<td>(.10)</td>
<td>(.12)</td>
</tr>
</tbody>
</table>

| Standard error | .0046   | .0034   | .0688   | .0054   |
| BF (significance) | 2.79 (.514) | 7.22 (.125) | 7.58 (.109) | 5.55 (.234) |
| KS c/ | .07 .11 .12 .08 |

Notes:

a/ All regressions cover 1956I - 1980I, and include linear and quadratic trends and quarterly dummy variables.

b/ Box Pierce statistic. The statistic is defined as \( \sum_{i=1}^{k} \hat{\rho}_i^2 \), where \( n \) is the number of observations, \( \hat{\rho}_i \) is the \( i \) th estimated residual correlation, and \( k = \frac{p}{2} \) in this and subsequent Tables. The statistic has an asymptotic \( X^2 \) distribution with \( k-p \) degrees of freedom, where \( p \) is the number of AR and MA coefficients estimated in the regression. The number in parentheses is the marginal significance of the test statistic.

c/ Kolmogorov Smirnov statistic for estimated residual periodogram. The 5% critical value is .13.
linear and quadratic trend terms.

Even the simple data analysis in Table I(a) is revealing because it suggests that these four time series have quite different properties. On the one hand, the nominal wage rate may apparently be represented as an extremely low order process, perhaps an AR1. The consumer price index, on the other hand, apparently does not have such a low order representation, and significant coefficients appear at three of the four lags present. Higher order autoregressive terms are also important in the representations of unemployment and interest rates. The similarity of the univariate representations of prices, unemployment and interest rates, and the difference between these three and the representation of wages, are remarkable.

As an alternative to these pure AR representations, we give selected low order autoregressive moving average (ARMA) representations of each of the four time series in Table I(b). The similarities and differences of the four series are even more apparent here. The ARMA(1,2) representation of nominal wages in the first column of the Table has small and insignificant moving average coefficients at one and two lags. Likewise, the estimated MA coefficients in the ARMA(2,2) representation of the consumer price index are both insignificant. On the other hand, the first

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4 While we only present data for short (90 day) interest rates, the qualitative properties of time series of longer term bond rates (1 year and 3 year Treasury bonds) are very similar.

5 In Table I(b) we adopt the sign convention that a positive MA(1) coefficient indicates that the lagged white noise error enters positively into the current composite residual.}
Table I (b)

Univariate ARMA Representations

<table>
<thead>
<tr>
<th>Dependent Variable (^a)</th>
<th>W</th>
<th>P</th>
<th>U</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>W. (Estimated standard errors in parentheses)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR1</td>
<td>.89</td>
<td>1.79</td>
<td>.92</td>
<td>.92</td>
</tr>
<tr>
<td>( .03)</td>
<td>( .07)</td>
<td>( .16)</td>
<td>( .18)</td>
<td></td>
</tr>
<tr>
<td>AR2</td>
<td>-</td>
<td>-.83</td>
<td>-.21</td>
<td>-.04</td>
</tr>
<tr>
<td>(       )</td>
<td>( .07)</td>
<td>( .16)</td>
<td>( .18)</td>
<td></td>
</tr>
<tr>
<td>MA1</td>
<td>.05</td>
<td>-.25</td>
<td>.39</td>
<td>.55</td>
</tr>
<tr>
<td>( .11)</td>
<td>( .15)</td>
<td>( .14)</td>
<td>( .14)</td>
<td></td>
</tr>
<tr>
<td>MA2</td>
<td>.05</td>
<td>-.10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( .11)</td>
<td>( .14)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard error</td>
<td>.0046</td>
<td>.0035</td>
<td>.0696</td>
<td>.0055</td>
</tr>
<tr>
<td>BP (significance)</td>
<td>1.04</td>
<td>9.14</td>
<td>10.39</td>
<td>7.02</td>
</tr>
<tr>
<td>( .244)</td>
<td>( .58)</td>
<td>( .069)</td>
<td>( .220)</td>
<td></td>
</tr>
<tr>
<td>K.S.</td>
<td>.06</td>
<td>.14</td>
<td>.09</td>
<td>.09</td>
</tr>
</tbody>
</table>

\(^a\) All regressions cover 1956I - 1980I and include linear and quadratic trends and quarterly dummy variables. Estimates were obtained by numerically minimizing the conditional sum of squared errors of the regression, setting pre-sample errors to zero.
order moving average coefficients estimated for both unemployment and interest rates are sizeable and statistically significant. Furthermore, while the the addition of moving average errors to the representation of prices does not alter the estimated AR part of the time series in any appreciable way, the same is not true for either unemployment or interest rates. In fact, while we do not present them here, ARMA(1,1) representations of unemployment and interest rates appear to be as good descriptions of the two time series as ARMA(2,1) representations.

On the basis of Tables I(a) and I(b) we can identify a number of preliminary facts about the data in our analysis. First, nominal wages are well represented as an AR1 process. Second, the price level is a higher order AR with complex roots capable of generating business-cycle like responses to innovations. Third, unemployment and interest rates are remarkably similar time series, with each series apparently admitting a parsimonious ARMA(1,1) representation. Finally, the stochastic parts of each of these time series have largest roots that are not too far from, but always less than, unity. Although it is slightly misleading to say so, rough lower order approximations to these series could accordingly be obtained by first differencing. In the case of nominal wages, first differencing would lead to a (roughly) random series. With prices, first differencing would

5 We are aware of the difficulty of testing the null hypothesis of a largest root equal to unity. Fuller[7] shows that under the null, in a regression that includes constant and trend, the test statistic is biased toward rejection of the null in favour of the alternative of stationarity.
lead to a first order autoregression with a coefficient of (roughly) .7. And, in the case of unemployment and interest rates, first differencing would lead to a first order moving average process with a coefficient of (roughly) .50.

In Table II we report the four variable vector autoregression fitted by least squares over the sample period indicated, again including four lags in each variable. Starting with the first column of the Table, there is no indication of effects of lagged unemployment or interest rates on nominal wages, but there is some indication that lagged prices affect nominal wages. A more formal test of each of these hypotheses is contained in Table IV. Here we record the F-ratios to test whether nominal wages are Granger-caused by prices, unemployment, and interest rates, under various maintained hypotheses. In all cases, we find that the test that prices Granger-cause wages is short of statistical significance at the 5 percent level, but not by a great deal. There is no evidence from this table, however, that either unemployment or interest rates Granger-cause wages.

The second column of Table II indicates statistically significant coefficients of interest rates and wages at one quarter and three quarter lags, respectively, in the regression for prices. The causality tests reported in Table IV provide strong evidence that nominal interest rates cause prices: this

7 We use the (admittedly imprecise) phrase "X Granger causes Y" when lagged values of X improve the prediction of Y, maintaining the effects of lagged values of Y as predictors. See Granger[9]. We implement this definition by the usual F-ratio for a joint test of the effects of lagged X's on Y.
### Table II

Wages, Prices, Unemployment and Interest Rates: Vector AR Representation

<table>
<thead>
<tr>
<th>Regressors</th>
<th>W_t</th>
<th>P_t</th>
<th>U_t</th>
<th>R_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>W_{t-1}</td>
<td>0.85 (0.11)</td>
<td>0.09 (0.07)</td>
<td>3.21 (1.55)</td>
<td>-0.21 (0.13)</td>
</tr>
<tr>
<td>W_{t-2}</td>
<td>-0.05 (0.15)</td>
<td>-0.11 (0.09)</td>
<td>-0.84 (2.04)</td>
<td>0.08 (0.16)</td>
</tr>
<tr>
<td>W_{t-3}</td>
<td>0.13 (0.15)</td>
<td>0.19 (0.09)</td>
<td>-0.46 (2.03)</td>
<td>0.17 (0.16)</td>
</tr>
<tr>
<td>W_{t-4}</td>
<td>-0.14 (0.11)</td>
<td>-0.02 (0.07)</td>
<td>0.25 (1.48)</td>
<td>0.12 (0.12)</td>
</tr>
<tr>
<td>P_{t-1}</td>
<td>0.60 (0.18)</td>
<td>1.23 (0.11)</td>
<td>2.84 (2.47)</td>
<td>0.04 (0.20)</td>
</tr>
<tr>
<td>P_{t-2}</td>
<td>0.43 (0.29)</td>
<td>-0.52 (0.18)</td>
<td>-1.64 (3.94)</td>
<td>0.28 (0.32)</td>
</tr>
<tr>
<td>P_{t-3}</td>
<td>0.12 (0.29)</td>
<td>0.36 (0.18)</td>
<td>1.49 (3.95)</td>
<td>-0.27 (0.32)</td>
</tr>
<tr>
<td>P_{t-4}</td>
<td>-0.15 (0.19)</td>
<td>-0.28 (0.11)</td>
<td>-1.05 (2.56)</td>
<td>-0.19 (0.21)</td>
</tr>
<tr>
<td>U_{t-1}</td>
<td>-0.003 (0.01)</td>
<td>-0.007 (0.01)</td>
<td>1.16 (0.12)</td>
<td>-0.033 (0.01)</td>
</tr>
<tr>
<td>U_{t-2}</td>
<td>0.007 (0.01)</td>
<td>0.011 (0.01)</td>
<td>-0.64 (0.17)</td>
<td>0.029 (0.01)</td>
</tr>
<tr>
<td>U_{t-3}</td>
<td>-0.011 (0.01)</td>
<td>-0.003 (0.01)</td>
<td>0.33 (0.17)</td>
<td>-0.018 (0.01)</td>
</tr>
<tr>
<td>U_{t-4}</td>
<td>0.008 (0.01)</td>
<td>0.001 (0.01)</td>
<td>0.08 (0.11)</td>
<td>0.010 (0.01)</td>
</tr>
<tr>
<td>R_{t-1}</td>
<td>-0.04 (0.11)</td>
<td>0.18 (0.07)</td>
<td>-3.10 (1.50)</td>
<td>1.13 (0.12)</td>
</tr>
<tr>
<td>R_{t-2}</td>
<td>-0.02 (0.16)</td>
<td>0.04 (0.10)</td>
<td>1.53 (2.17)</td>
<td>-0.74 (0.18)</td>
</tr>
<tr>
<td>R_{t-3}</td>
<td>-0.17 (0.16)</td>
<td>-0.01 (0.10)</td>
<td>0.02 (2.22)</td>
<td>0.41 (0.18)</td>
</tr>
<tr>
<td>R_{t-4}</td>
<td>0.00 (0.13)</td>
<td>0.07 (0.08)</td>
<td>4.63 (1.75)</td>
<td>-0.07 (0.14)</td>
</tr>
</tbody>
</table>

Standard error: 0.0065, 0.0028, 0.0612, 0.0050

B.P. (significance): 1.80 (0.772), 4.92 (0.296), 9.58 (0.048), 8.25 (0.083)

K.S.: 0.06, 0.07, 0.08, 0.09

\(^5/\) See Notes to Table I(a).
Table III

Correlation Matrix of Innovations in Vector AR.

<table>
<thead>
<tr>
<th>Innovations in $a/$:</th>
<th>W</th>
<th>P</th>
<th>U</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Innovations in:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>-.05</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>.03</td>
<td>.05</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>-.17</td>
<td>.11</td>
<td>-.32</td>
<td>1.00</td>
</tr>
</tbody>
</table>

$a/\$: Residuals obtained from estimated equations reported in Table II. The approximate standard error of each correlation is .10.
<table>
<thead>
<tr>
<th>Line No.</th>
<th>Test for</th>
<th>Causality of:</th>
<th>By:</th>
<th>Maintained Lagged Regressors</th>
<th>Test Statistic</th>
<th>Marginal Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>W</td>
<td>P</td>
<td>-</td>
<td>2.09</td>
<td>.090</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>W</td>
<td>U</td>
<td>-</td>
<td>.10</td>
<td>.922</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>R</td>
<td>-</td>
<td>-</td>
<td>1.26</td>
<td>.268</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>P, U, R</td>
<td>-</td>
<td>-</td>
<td>1.41</td>
<td>.180</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>P</td>
<td>W</td>
<td>-</td>
<td>4.62</td>
<td>.002</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>P</td>
<td>U</td>
<td>-</td>
<td>3.19</td>
<td>.017</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>P</td>
<td>R</td>
<td>-</td>
<td>7.65</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>P, U, R</td>
<td>U, R</td>
<td>-</td>
<td>4.41</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>W</td>
<td>U, R</td>
<td>-</td>
<td>3.54</td>
<td>.011</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>W</td>
<td>W, R</td>
<td>-</td>
<td>.90</td>
<td>.469</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>R</td>
<td>W, U</td>
<td>-</td>
<td>4.35</td>
<td>.003</td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>R</td>
<td>R</td>
<td>-</td>
<td>3.72</td>
<td>.008</td>
<td></td>
</tr>
<tr>
<td>13.</td>
<td>R</td>
<td>W</td>
<td>-</td>
<td>6.52</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>14.</td>
<td>U</td>
<td>W</td>
<td>-</td>
<td>3.06</td>
<td>.021</td>
<td></td>
</tr>
<tr>
<td>15.</td>
<td>U</td>
<td>P</td>
<td>-</td>
<td>3.89</td>
<td>.006</td>
<td></td>
</tr>
<tr>
<td>16.</td>
<td>U</td>
<td>R</td>
<td>-</td>
<td>5.70</td>
<td>.001</td>
<td></td>
</tr>
<tr>
<td>17.</td>
<td>W, P</td>
<td>W, P</td>
<td>-</td>
<td>2.71</td>
<td>.011</td>
<td></td>
</tr>
<tr>
<td>18.</td>
<td>W, P, R</td>
<td>R</td>
<td>-</td>
<td>2.90</td>
<td>.002</td>
<td></td>
</tr>
<tr>
<td>19.</td>
<td>W</td>
<td>P, R</td>
<td>-</td>
<td>1.69</td>
<td>.161</td>
<td></td>
</tr>
<tr>
<td>20.</td>
<td>P</td>
<td>W, R</td>
<td>-</td>
<td>1.12</td>
<td>.354</td>
<td></td>
</tr>
<tr>
<td>22.</td>
<td>W, P</td>
<td>R</td>
<td>-</td>
<td>1.40</td>
<td>.211</td>
<td></td>
</tr>
<tr>
<td>23.</td>
<td>R</td>
<td>W</td>
<td>-</td>
<td>1.25</td>
<td>.296</td>
<td></td>
</tr>
<tr>
<td>24.</td>
<td>R</td>
<td>P</td>
<td>-</td>
<td>.58</td>
<td>.688</td>
<td></td>
</tr>
<tr>
<td>25.</td>
<td>R</td>
<td>U</td>
<td>-</td>
<td>2.67</td>
<td>.038</td>
<td></td>
</tr>
<tr>
<td>26.</td>
<td>R</td>
<td>W, P</td>
<td>-</td>
<td>1.47</td>
<td>.182</td>
<td></td>
</tr>
<tr>
<td>27.</td>
<td>R</td>
<td>W, P, U</td>
<td>-</td>
<td>2.24</td>
<td>.018</td>
<td></td>
</tr>
<tr>
<td>28.</td>
<td>W</td>
<td>U</td>
<td>-</td>
<td>1.39</td>
<td>.245</td>
<td></td>
</tr>
<tr>
<td>29.</td>
<td>W</td>
<td>P</td>
<td>-</td>
<td>.93</td>
<td>.451</td>
<td></td>
</tr>
<tr>
<td>30.</td>
<td>W</td>
<td>P, U</td>
<td>-</td>
<td>2.80</td>
<td>.032</td>
<td></td>
</tr>
<tr>
<td>31.</td>
<td>W</td>
<td>P, U</td>
<td>-</td>
<td>2.33</td>
<td>.063</td>
<td></td>
</tr>
<tr>
<td>32.</td>
<td>W</td>
<td>P, U</td>
<td>-</td>
<td>3.42</td>
<td>.013</td>
<td></td>
</tr>
</tbody>
</table>

\*All regressions cover 1956 I - 1980 I, and include linear and quadratic trends and quarterly dummy variables.

\*The causality test statistic is an F-ratio for the null hypothesis that the coefficients of four lagged values of each of the variables in this column are jointly equal to zero. Four lagged values of the dependent variable are included in both the restricted and unrestricted regressions.

\*In computing the F-ratio, four lagged values of each of the variables in this column appear in both the restricted and unrestricted regressions.

\*Probability of obtaining an F-ratio at least as large as the test statistic under the null hypothesis. A marginal significance level smaller than .05 indicates rejection of the null hypothesis at the 5% significance level.
fact is robust to the inclusion of lagged wages and/or unemployment in the price equation. A similar conclusion emerges when we test for the significance of lagged wages in determining prices. However, the evidence that unemployment causes prices is weak. When interest rates and wages are excluded, the statistic for the test of causality from unemployment to prices is significant at conventional levels. However, maintaining the presence of lagged interest rates and wage rates, lagged unemployment terms aid little to the precision of the forecast for prices.

The third column of Table II provides estimates of the coefficients of lagged prices, wages, and interest rates in the regression for unemployment. Again, the impact of lagged interest rate terms is immediately apparent. There are also relatively large, though imprecisely estimated, effects of lagged wages and prices on current unemployment rates. The test results in Table IV confirm Granger causality from interest rates to unemployment, both with and without lagged wages and prices included in the unemployment regression. The causality tests for wages and prices are more ambiguous. When interest rates are excluded, wages and prices together and separately appear to cause unemployment. However, with interest rates included, the opposite conclusion holds. Finally, it is interesting to observe that there is no strong tendency for lagged wages and prices to enter the unemployment regression with equal and opposite sign.\(^3\) There is

\(^3\) Excluding interest rates, the F-statistic for the null hypothesis that wages and prices have equal and opposite
however some indication that the sum of the coefficients of lagged wages and prices is zero, confirming the long run homogeneity of unemployment with respect to nominal wages and prices.

The fourth column of Table II gives the estimated regression equation for interest rates. Inspection of the coefficient estimates suggests that lagged unemployment terms have a significant role in the time series representation of interest rates. Indeed, the tests in Table IV indicate that the null hypothesis of no causality from unemployment to interest rates is easily rejected, both in the presence and in the absence of lagged wage and price regressors. By the same token, the test statistic for joint causality from wages, prices, and unemployment to interest rates is highly significant. While taken individually, neither wages nor prices cause interest rates, in combination with unemployment these two series add precision to the forecast of interest rates.\(^9\)

In many applications it is appropriate to consider employment rather than unemployment in the analysis of the aggregate labor market. Although employment and unemployment are not precise mirror images, it is well known that they move in

\(^9\) Further investigation reveals that causality from prices to interest rates is stronger, the longer the term of the interest rate being considered. For 90 day Treasury Bills, the F-statistic is 0.53. For one year Treasury Bonds, the statistic is 1.30, and for three year Treasury Bonds, the statistic is 2.15.
nearly equal, but opposite directions over time. While we do not report the results, our finding is that the substitution of employment for unemployment does not substantially alter any of the properties of the vector autoregression recorded in Table II, or most of the conclusions from the causality tests in Table IV. Similarly, the Treasury Bill rate can be replaced by a longer term interest rate without any important qualitative differences.

In Table III we provide estimates of the correlations among the innovations (residuals) from the regressions reported in Table II. Given information on lagged values of nominal wages, prices, unemployment, and interest rates, these are the contemporaneous correlations among the unpredicted "surprises" in each time series. Surprisingly, none of the correlations between innovations in wages, prices, or unemployment is very large or statistically significant. On the other hand, innovations in all three series are correlated with innovations in interest rates. The strongest correlation exists between innovations in unemployment and interest rates: the unpredicted parts of these two series move in opposite directions, just as the levels of both series move in opposite directions over the business cycle. As our previous analysis has shown, interest rates and unemployment are closely linked, and this linkage apparently extends to the surprises in each series.

---

10 The only difference in the conclusions from the causality tests is that wages and prices (together and separately) Granger-cause employment, controlling for the effects of lagged interest rates.
To this point, we have presented our results in terms of nominal wages and prices. Somewhat different insights are gained by considering real rather than nominal wages. Tables V(a) and V(b) give a brief summary of the data analysis, recast in terms of real wages, prices, unemployment, and interest rates. Table V(a) presents the estimated univariate AR4, AR2 and ARMA(1,2) representations of real wages. Not surprisingly, the representation of real wages is somewhere between the very low order AR for nominal wages and the higher order AR for prices. Neither of the estimated MA coefficients in the ARMA representation of real wages are significant at conventional levels, however, and the first difference of real wages is not far from a white noise process.\[1\] Table V(b) presents some selected causality tests for real wages. Neither prices nor unemployment cause real wages, alone or in the presence of lagged interest rates. However, the evidence for causality from interest rates to real wages is stronger. As with prices and unemployment, the forecast of real wages can be significantly improved by taking account of past movements in nominal interest rates.

---

\[1\] This is consistent with Altonji and Ashenfelter's[1] conclusion from seasonally adjusted aggregate quarterly data, and also with VaCurdy's[13] analysis of individual longitudinal data. Tests of the hypothesis that the first difference of real wages is serially uncorrelated (apart from deterministic components) yield a Box-Pierce statistic with a marginal significance level of .07, and a Kolmogorov-Smirnov statistic of .13, which is just significant at the 5 percent level. VaCurdy's analysis of microeconomic data suggests that aggregation biases are not the source of this phenomenon. He finds that an ARMA(1,2) representation of the annual real wage process, with an AR coefficient very close to unity, gives a good fit to the data.
<table>
<thead>
<tr>
<th>Regressors</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR1</td>
<td>.99</td>
<td>1.10</td>
<td>.85</td>
</tr>
<tr>
<td></td>
<td>(.11)</td>
<td>(.11)</td>
<td>(.09)</td>
</tr>
<tr>
<td>AR2</td>
<td>.02</td>
<td>-.22</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(.15)</td>
<td>(.12)</td>
<td></td>
</tr>
<tr>
<td>AR3</td>
<td>.04</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(.15)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR4</td>
<td>-.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.11)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NA1</td>
<td></td>
<td></td>
<td>.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(.13)</td>
</tr>
<tr>
<td>NA2</td>
<td></td>
<td></td>
<td>.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(.11)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standard error</th>
<th>.0059</th>
<th>.0063</th>
<th>.0060</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.F. (significance)</td>
<td>8.68</td>
<td>12.50</td>
<td>12.41</td>
</tr>
<tr>
<td>(standard errors in parentheses)</td>
<td>(.070)</td>
<td>(.052)</td>
<td>(.047)</td>
</tr>
<tr>
<td>K.S.</td>
<td>.12</td>
<td>.15</td>
<td>.10</td>
</tr>
</tbody>
</table>

\footnote{See Notes to Table I(a)}
Table Vb

Selected Causality Tests for Real Wages $^a/$

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Test for Causality of Real Wages By:</th>
<th>Maintained Lagged Regressors:</th>
<th>Test Statistic</th>
<th>Marginal Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>P</td>
<td>-</td>
<td>1.26</td>
<td>.213</td>
</tr>
<tr>
<td>2.</td>
<td>U</td>
<td>-</td>
<td>.92</td>
<td>.156</td>
</tr>
<tr>
<td>3.</td>
<td>R</td>
<td>-</td>
<td>4.15</td>
<td>.004</td>
</tr>
<tr>
<td>4.</td>
<td>P, U</td>
<td>-</td>
<td>1.66</td>
<td>.122</td>
</tr>
<tr>
<td>5.</td>
<td>P</td>
<td>R</td>
<td>1.78</td>
<td>.141</td>
</tr>
<tr>
<td>6.</td>
<td>U</td>
<td>R</td>
<td>.95</td>
<td>.440</td>
</tr>
<tr>
<td>7.</td>
<td>R</td>
<td>P</td>
<td>5.12</td>
<td>.001</td>
</tr>
</tbody>
</table>

Notes: $^a/$ See notes to Table IV.
To conclude our empirical analysis, we investigate the appropriate representation of ex post real interest rates in Table VI. The ex post interest rate is defined as the difference between the nominal interest rate and the realized percentage increase in prices over the relevant holding period. By construction, ex post interest rates differ from their anticipated or ex ante counterparts by the amount of unexpected price increases over the holding period. Under the assumption of rational expectations, the deviation of ex post and ex ante real interest rates is therefore serially uncorrelated and orthogonal to information available at the start of the holding period.\textsuperscript{12} It follows that tests of the lagged effects of \( \Delta, p, U, \) and \( R \) on the ex post interest rate are interpretable as tests of the effects of these variables on its ex ante counterpart, under the assumption of rational expectations.\textsuperscript{13}

\textsuperscript{12} This point has been exploited in previous empirical work by Fama\textsuperscript{[5]} and Vishkin\textsuperscript{[19]}.

\textsuperscript{13} Let \( E_t \) denote expectations conditional on information at \( t \). The ex ante real interest rate is \( r^*_{t} = R_t - (E_t | R_{t+1}, P_t) \) and the ex post real interest rate is \( r^p_{t} = R_t - (P_{t+1} - P_t) \). The deviation of ex ante and ex post rates is \( r^*_{t} - r^p_{t} = E_t | P_{t+1} - P_t \). Since

\[
E_t - 1 E_t = E_t - 1 R_t - E_t - 1 | P_{t+1} + E_t - 1 P_t
\]

the reduced forms for \( r^* \) and \( r^p \) (in terms of variables dated \( t-1 \) and earlier) are identical. Furthermore, the innovation in \( r^* \) is an exact linear combination of innovations in \( R_t, P_t \) and the variables used to predict \( P_t \). Finally, the reduced forms for \( r^* \) and \( r^p \) include only those variables needed to predict \( R_t \) and \( P_t \). For our purposes, these include lagged values of \( p, U, v, \) and \( R \).

The tests we report are convenient because of the ease of their computation and interpretation. They may not be the most
The theoretically useful hypothesis that ex ante real interest rates are constant was supported in early empirical work by Fama[5]. Under the assumption of rational expectations, this implies that ex post real interest rates are composed of a constant term and a serially uncorrelated error. However, as has been reported by Mishkin[19], we find that this hypothesis may be easily rejected in a sample that includes the post-1972 period. In a regression for ex post real interest rates that uses the 90 day Treasury Bill rate over the sample period 1956-1980, the marginal significance level of a pair of linear and quadratic trend terms is less than .1 percent. Furthermore, the hypothesis that the non-deterministic component of real interest rates is serially uncorrelated is easily rejected.\textsuperscript{14}

Prediction tests for ex post real interest rates are reported in Table VI. Taken one variable at a time, lagged wages and prices have a statistically significant impact on the forecast error variance of ex post interest rates. Lagged unemployment terms are not quite significant at the 5 percent level, while lagged nominal interest rates add very little to the regression for ex post real rates. Similar conclusions emerge when lagged values of all other variables are maintained while computing the

\textbf{---------------}

powerful since they ignore the structure of price forecasts implicit in the data under the rational expectations hypothesis.

\textsuperscript{14} The Box-Pierce and Kolmogorov-Smirnov statistics for this hypothesis are both significant at the 1 percent level. For the sample period 1956-1972, linear and quadratic trend terms are also jointly significant in a regression for ex post real interest rates. However, there is weaker evidence that the non-deterministic component of ex post real rates is serially correlated over this period.
Table VI

Selected Prediction Tests for Ex Post Real Interest Rates

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Test for Prediction of Real Interest Rates By:</th>
<th>Maintained Lagged Regressors</th>
<th>Test Statistic</th>
<th>Marginal Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>P</td>
<td>-</td>
<td>6.38</td>
<td>.000</td>
</tr>
<tr>
<td>2.</td>
<td>W</td>
<td>-</td>
<td>3.63</td>
<td>.009</td>
</tr>
<tr>
<td>3.</td>
<td>U</td>
<td>-</td>
<td>2.42</td>
<td>.054</td>
</tr>
<tr>
<td>4.</td>
<td>R</td>
<td>-</td>
<td>1.58</td>
<td>.187</td>
</tr>
<tr>
<td>5.</td>
<td>P,W,U,R</td>
<td>-</td>
<td>3.32</td>
<td>.000</td>
</tr>
<tr>
<td>6.</td>
<td>P</td>
<td>W,R,U</td>
<td>5.50</td>
<td>.000</td>
</tr>
<tr>
<td>7.</td>
<td>W</td>
<td>P,R,U</td>
<td>2.74</td>
<td>.025</td>
</tr>
<tr>
<td>8.</td>
<td>U</td>
<td>P,W,R</td>
<td>1.54</td>
<td>.200</td>
</tr>
<tr>
<td>9.</td>
<td>R</td>
<td>P,W,U</td>
<td>0.85</td>
<td>.498</td>
</tr>
<tr>
<td>10.</td>
<td>U</td>
<td>W,P</td>
<td>1.96</td>
<td>.109</td>
</tr>
<tr>
<td>11.</td>
<td>R</td>
<td>W,P</td>
<td>1.22</td>
<td>.309</td>
</tr>
<tr>
<td>12.</td>
<td>U,R</td>
<td>W,P</td>
<td>1.40</td>
<td>.211</td>
</tr>
</tbody>
</table>

\*\ See Notes to Table IV. In computing the tests in this Table, lagged values of the dependent variable are not included in the regressions.
tests. However, maintaining the other variables, the marginal significance level of the test for the lagged effects of unemployment is greatly increased. The last three rows of the Table report prediction tests for unemployment and nominal interest rates, individually and jointly, maintaining wages and prices in the representation of ex post real rates. Under the assumption of rational expectations, these results indicate that the ex ante real interest rate is a function of lagged nominal wages and lagged prices, but not of lagged unemployment or lagged nominal interest rates.\textsuperscript{15}

It is worth indicating that the conclusions from our data analysis of wages, prices, interest rates and unemployment are largely unaffected by considering only the quarterly U.S. time series data to the end of 1972. For the shorter sample, the estimated univariate representations are very similar to those reported in Tables I(a) and I(b). Furthermore, most of the conclusions of the causality tests in Tables IV and V(b) are invariant to the choice of the shorter or longer sample period. The only qualitative change from Table IV is that there is much weaker evidence of causality from interest rates to prices and unemployment in the pre-1973 sample. The test results in Table V(b) are also representative of those for the shorter period.

\textsuperscript{15} These results are not inconsistent with a recent and interesting analysis of real interest rates by Litterman and Weiss[12]. They test and accept the hypothesis that the reduced form for ex post real interest rates is consistent with a univariate representation for ex ante real interest rates. Their tests exploit the implied cross-equation restrictions on the observable vector autoregression implied by the hypothesis.
although again the evidence for causality from interest rates to real wages is weaker in the pre-1973 data.

On the other hand, the causality tests for ex post real interest rates yield somewhat different results in the two sample periods. In particular, the evidence against the hypothesis that ex ante (and ex post) real interest rates are composed of deterministic components and a serially uncorrelated error is weaker in the earlier data. None of the prediction tests reported in Table VI are significant at the 5 percent level when the analysis is restricted to pre-1973 data. However, the strongest evidence continues to be that ex post real interest rates are predicted by lagged wages and prices only. With the possible exception of the behavior of real interest rates, we find that the conclusions from our analysis of 1955-1930 data are fair representations of the data in the early part of the sample.

At this stage it is a straightforward matter to summarize the time series facts at our disposal. First, nominal wages are well represented by an AR1 and there is no evidence that they are Granger caused by unemployment or interest rates, while there is only weak evidence that they are caused by consumer prices. Second, consumer prices are better represented by a higher order AR process and there is strong evidence that this series is Granger caused by wages and interest rates. Third, unemployment and interest rates are also better represented as higher order AR processes, or alternatively, as low order mixed autoregressive moving average processes. Fourth, unemployment and interest rates Granger cause each other. Controlling for interest rates, lagged
Wages and prices do not seem to add to the precision of the forecast of unemployment, although they do have a significant impact on the forecast of interest rates. Fifth, innovations in wages, prices, and unemployment are essentially uncorrelated, while innovations in all three series are correlated with innovations in interest rates. Sixth, the real wage is reasonably described by a low order autoregressive process, or even as a random walk. There is no evidence of causality from prices or unemployment to real wages, but fairly strong evidence of causality from interest rates to real wages. Seventh, ex post real interest rates are predicted by wages and prices but not by nominal interest rates or unemployment. Finally, all of the series we have investigated are characterized by a high degree of serial persistence. In fact, several series, most notably prices and real wages, exhibit largest roots quite close to unity. Innovations introduced into any of these series tend to persist for relatively long periods of time.\(^{16}\)

**Alternative Models of the Labor Market**

We next turn to the time series implications of several alternative models of the aggregate labor market. Prior to doing this it is worth observing that there are a number of anomalies in the time series representations set out in Tables I - VI that

\(^{16}\) Although we do not give the moving average representations associated with the vector autoregression in Table II, we have found that they tend to exhibit cycles of 16 to 20 quarters, and only fairly weak dampening.
are going to pose problems for the explanatory power of most simple models of the labor market. First of all, as indicated earlier, nominal wages and prices do not enter the unemployment regression with equal and opposite coefficients at each lag. This characteristic is shared by the (unreported) employment equation in a vector autoregression of wages, prices, employment, and interest rates.\textsuperscript{17} One natural interpretation of such an equation is as a reduced form of a dynamic labor demand schedule where nominal wages and prices are allowed separate coefficients.\textsuperscript{13} Under this interpretation, however, and abstracting from interest rate effects, the real wage should be driving employment and nominal wages and prices should enter the equation with equal but opposite coefficients. On the other hand, once current and expected future real interest rates are admitted into the labor demand equation, the coefficients of wages and prices are freed. Apparently, any simplified model that ignores the role of interest rates or otherwise fails to distinguish nominal wages and prices in the determination of employment and unemployment will be easily rejected by the data.

\textsuperscript{17} The F-statistic for the hypothesis of equal and opposite coefficients has a marginal significance level of .33, maintaining lagged nominal interest rates.

\textsuperscript{13} In fact, this is precisely a generalization of the results reported by Veftolli\textsuperscript{20} and the interpretation offered by Sargent \textsuperscript{24} of those results. Geary and Kennan \textsuperscript{3} have observed that Sargent's interpretation has some obvious problems, since the price level should in this case be a measure of the producer price index, rather than the consumer price index. However, we have found that entering both the wholesale and consumer price indexes into the employment equation leads us to reject any causal role for wholesale prices, while retaining the consumer price index.
Secondly, aggregative models of the labor market typically highlight the role of monetary forces in the explanation of unemployment, wages, and prices. As a consequence, perhaps for simplicity, many such models assume either that the real wage is deterministic, or that it is a serially uncorrelated random variable. It is clear from our analysis in Table V, and from previous work by Veefoï[20], Sargent[24], and Altonji and Ashenfelter[1] that this characterization of the real wage process is simply inadequate. Since these "monetary" models of the business cycle are not intended to address the determination of real as opposed to nominal wage rates, they certainly cannot be faulted for this shortcoming. Nevertheless, this empirical feature of the time series process for real wages may be a clue that suggests that a more appropriate interaction of real and monetary forces will ultimately have to be addressed in a satisfactory model of the labor market and the business cycle.

A. The Intertemporal Substitution Theory of Unemployment

In this section we catalogue the implications of Lucas and Rapping's [14] intertemporal substitution theory of unemployment for the time series representations of wages, prices, unemployment and interest rates. The basic hypothesis of this theory is that differences in the prices of consumption and leisure between the present and future periods induce workers to alter their supply of labor in the current period. The deviation of labor supply from its trend level is interpreted as a measure of individual unemployment. Cyclical movements in labor supply
and unemployment are therefore attributed to (potentially misperceived) changes in real interest rates and real wage rates. In the spirit of the approach outlined in the introduction, we ask how this linkage between unemployment and wages, prices and interest rates can be tested against the data as summarized by the unrestricted vector autoregressions presented in the previous section.

The current labor supply decision of a worker can be written as a function of the discounted prices of consumption and leisure in each of the periods in his planning horizon. Adopting a log-linear approximation to this function, we assume that

\[
(1) \quad u_{it} = \sum_{j=0}^{\infty} b_j E_{it}(w_{it+j} - p_{t+j}) + \sum_{j=0}^{\infty} a_j E_{it}(r_{t+j} - p_{t+j+1} + p_{t+j}) + v_{it},
\]

where \( u_{it} \) is the log of current (measured) individual unemployment, \( w_{it+j} \) is the log of the nominal wage rate earned by individual \( i \) in period \( t+j \), \( p_{t+j} \) is the log of the price level in period \( t+j \), \( r_{t+j} \) is the nominal interest rate in period \( t+j \), \( b_j \) and \( a_j \) are constants, \( E_{it} \) is the expectations operator conditional on information available to \( i \) at period \( t \), and \( v_{it} \) is an error term in individual unemployment.

For simplicity we neglect the impact of current wealth, although in principle it is a legitimate argument of the labor supply function.

Equation (1) is easily derived by writing the unemployment of \( i \) in period \( t \) as a function of discounted wages and prices in each of the periods in his planning horizon and then using the homogeneity of the demand function to divide through by the
\((R_{t+j} - p_{t+j} + \eta_{t+j} p_{t+j})\) is just the real interest rate between periods \(t+j\) and \(t+j+1\). According to the intuitive argument given by Lucas and Rapping, the effect of current real wages on unemployment is negative \((\beta_j < 0)\), while the effect of expected future real wage rates on current unemployment is positive. The sign pattern of the \(\alpha_j\) is unrestricted, although if current leisure is a substitute for leisure and consumption in every future period, then the \(\alpha_j\) are positive and non-increasing in absolute value. It is useful as well as traditional, and perhaps empirically harmless in the analysis of business cycle movements in unemployment, to simplify equation (1) by assuming that the elasticity of labor supply with respect to permanent increases in real wage rates is 0. In that case, \(\Sigma_j \beta_j = 0\), and the first summation in (1) can be replaced by

\[(2) \beta_j (\bar{w}_{it} - \bar{p}_t) - (\bar{w}_{it} - \bar{p}_t)\]

where

\[\bar{w}_{it} - \bar{p}_t = \langle 1/\beta_j \rangle \Sigma_j \beta_j E_{it}(\bar{w}_{it+j} - \bar{p}_{t+j})\]

has the interpretation of a long run average expected future real wage rate. In previous studies by Sargent[22] and Altonji and Ashenfelter[11], and in Lucas and Rapping's original empirical work, the effects of current and expected future real interest rates on current unemployment were neglected, and tests of the intertemporal substitution hypothesis involved regressions of current unemployment on expressions like (2).

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current price level. For example, see Sargent[25], pp. 366-370.
To explore the empirical implications of the hypothesis described by (1) we need to specify the formation of individual expectations and the nature of the error term in the unemployment function. Throughout, we assume that aggregate nominal wages, prices, and interest rates follow a vector autoregression. Furthermore, we assume that the individual wage rate $w_{it}$ differs from the aggregate by a fixed effect $w_i$ and a serially uncorrelated error $z_{it}$:

$$w_{it} = w_i + \omega_t + z_{it}.$$  

Finally, we assume that the individuals' expectations are formed rationally, and that individuals' information sets include $w_{it}$, $w_i$, $p_t$, and $R_t$, and past values of the relevant aggregate variables.

**A Simplified Basis Case.**

As a basis case, consider a model in which the expected real interest rate is deterministic and the aggregate real wage can be adequately forecast from its own lagged values and lagged values of the nominal interest rate.\(^{21}\) For the period from the early 1950's to the early 1970's, empirical work by Fama[5] and Vishkin[19] suggests that anticipated real interest rates did not depart significantly from a constant.\(^{22}\) Likewise, at least in

\(^{21}\) This is formally expressed in the requirement that none of the variables in our system except interest rates Granger cause real wages.

\(^{22}\) Our own results do not reject the hypothesis of a deterministic anticipated real interest rate over this period.
systems including only real wages and employment or unemployment, the evidence of previous studies is that real wages are reasonably forecast from their own lagged values (Sargent[24], Altonji and Ashenfelter[1]). Our own results in Table V suggest that in fact forecasts of the aggregate real wage can be significantly improved by taking into account the effects of lagged nominal interest rates. However, maintaining interest rates, unemployment and prices aid very little to the precision of the forecasts. Consequently, this basis case may be a reasonable framework for initial empirical testing, at least over this particular period.

In this simplified framework, the intertemporal substitution hypothesis can be represented by the set of equations consisting of

\[ u_{it} = b_3(\omega_{it} - \rho_{it}) - (\omega_{it} - \rho_{it}) + v_{it}, \]

equation (3), and the autoregressive processes generating real wages, interest rates, and prices. By assumption, the deviation of current individual real wages from aggregate real wages is transitory (normalized for fixed effects). An individual's forecast of own normalized real wage rate in any future period coincides with his forecast of the aggregate real wage rate in that period. If the reduced form for aggregate real wages includes lags of real wages and interest rates, then

\[ \omega_{it}^* - \rho_{it}^* = \sum_{j=0}^{k} c_j \Delta_t \epsilon_t \left( r_{it-J} - j \right) + \sum_{j=0}^{k} i_j \Delta_t r_{t-j} \]

where \( r_{it} = \omega_{it} - \rho_{it} \), and \( \{ c_j \} \) and \( \{ i_j \} \) are sets of coefficients.
Individuals have three pieces of information with which to update their expectation of the current aggregate real wage: their own wage rate \( w_{it} \), the aggregate price level \( p_t \), and the current nominal interest rate \( R_t \). Assuming that this information is used to form linear forecasts,

\[
E_{it}(w_t) = E_{it-1}(r_{it}) + \varphi_1(w_{it-1}(w_t)) \\
+ \varphi_2(p_t - E_{it-1}(p_t)) \\
+ \varphi_3(R_t - E_{it-1}(R_t))
\]

for a set of coefficients \( \varphi_1, \varphi_2, \) and \( \varphi_3 \), where \( E_{it-1} \) is the expectations operator conditional on information dated \( t-1 \) and earlier.\(^{23}\) If the variance of the individual shock to wages is zero, then individuals have complete information on current aggregate real wages and \( \varphi_1 = \varphi_2 = 1 \), while \( \varphi_3 = 0 \). Otherwise, \( 0 < \varphi_1 < 1 \). Upon substitution, individual unemployment in the current period is given by:

\[
\begin{align*}
\left( z_{it} - b_1 \right) & = b_2 (1-c_0) + b_3 (1-c_0) \varepsilon_t - b_4 (c_0 \delta_3 + d) x_t \\
+ b_5 (1-c_0) E_{it-1} r_{it-1} - b_6 (c_0 \delta_4 + d) E_{it-1} R_{it-1} \\
- b_7 (c_0 \delta_2 + d) E_{it-1} R_{it-1} - b_8 (c_0 \delta_3 + d) E_{it-1} R_{it-1} \\
+ \varepsilon_{it}
\end{align*}
\]

or, after aggregation,

\[
\begin{align*}
\sum_{i} (z_{it} - b_1) & = b_2 (1-c_0) + b_3 (1-c_0) \varepsilon_t - b_4 (c_0 \delta_3 + d) x_t \\
+ b_5 (1-c_0) E_{it-1} r_{it-1} - b_6 (c_0 \delta_4 + d) E_{it-1} R_{it-1} \\
- b_7 (c_0 \delta_2 + d) E_{it-1} R_{it-1} - b_8 (c_0 \delta_3 + d) E_{it-1} R_{it-1} \\
+ \delta_{it}
\end{align*}
\]

\(^{23}\) Assuming the joint normality of \( z_{it} \) and the innovations in aggregate nominal wages, interest rates and prices, the coefficients \( \varphi_1, \varphi_2, \) and \( \varphi_3 \) are easily derived from the expression for the conditional mean of the innovation in aggregate wages, given the other three (see for example chrymes[4], pp. 15-13.). More generally, \( \delta \) can be interpreted as a linear least squares projection operator, and the \( \varphi_i \) as population regression coefficients that minimize the mean square forecast error of the innovation in aggregate real wages, given aggregate prices, interest rates, and individual wages (see Sargent[25], Chapter X).
(4a) \[ u_t = \beta_0 (1 - \phi_1 \phi_2) z_t - \beta_1 (\phi_2 + (1 + \phi_2) \phi_2) \eta_t - \beta_2 (\phi_2 \phi_2 + \phi_2) \xi_t \]
\[ + \beta_3 (1 - \phi_1) \bar{F}_t - \beta_4 \phi_1 \bar{F}_t - \cdots - \beta_5 \phi_{k-1} \bar{F}_t - k + 1 \]
\[ - \beta_6 \phi_1 \bar{R}_t - \beta_7 \phi_2 \bar{R}_t - \cdots - \beta_8 \phi_{k-1} \bar{R}_t - k + 1 \]
\[ + \nu_t, \]

where \( z_t \) is the aggregate innovation in wages, \( \eta_t \) is the innovation in prices, \( \xi_t \) is the innovation in nominal interest rates, and \( \nu_t \) is the average of the \( \nu_{it} \). Individual unemployment is a distributed lag on aggregate real wages and interest rates, plus the sum of the error term \( \nu_{it} \) and linear combinations of the current innovations in individual wages, interest rates, and prices. Aggregate unemployment is a function of \( k \) lags of real wages and nominal interest rates, plus an error term composed of combinations of the aggregate innovations in wages, interest rates, and prices.

The implications of the basis case version of the intertemporal substitution hypothesis for the behavior of real wages, prices and unemployment are summarized in Table T-1. Depending on the error term \( \nu_{it} \) and the real wage forecasting equation, the implications are more or less easily tested against the unrestricted vector autoregressions and causality tests in the previous section. For example, if \( \nu_{it} \) is serially uncorrelated, then we can conclude from (4) that prices do not cause unemployment, maintaining \( k \) lags of real wages and interest rates in the unemployment regression, since \( \eta_t, \xi_t, z_t \) and \( \nu_{it} \) are all uncorrelated with lagged prices.\(^{24}\) In fact, this

\(^{24}\) These implications are very similar to the set of implications tested and rejected in early empirical work by Sargent[22] and Nelson[21].
### Table T-1

**Implications of the Intertemporal Substitution**

**Hypothesis: Basis Case**

<table>
<thead>
<tr>
<th>Assumptions</th>
<th>Implications</th>
</tr>
</thead>
</table>
| (1) errors in individual unemployment serially uncorrelated | (a) real wage forecast by univariate AR | (i) prices and interest rates fail to cause unemployment, maintaining real wages.  
(ii) lagged unemployment does not improve the forecast of unemployment, maintaining lagged real wages. |
| | (b) real wage caused by interest rates | (i) prices fail to cause unemployment, maintaining real wages and interest rates.  
(ii) lagged unemployment does not improve the forecast of unemployment, maintaining lagged real wages and interest rates. |
| (2) errors in individual unemployment AR(h) | (a) real wage caused by interest rates | (i) unemployment is a distributed lag on real wages, interest rates, prices, and h lags of unemployment. |
implication is consistent with the aggregate time series evidence. More generally, if \( v_{it} \) is serially uncorrelated, then the only variables in the reduced form for unemployment are those that enter the forecasting equation for real wages. Thus the causality from interest rates to unemployment noted in Table IV is consistent with the theory and the fact that interest rates help to predict real wages. On the other hand, lagged unemployment terms are included in the unemployment equation only to the extent that unemployment Granger causes real wages. Since causality from unemployment to real wages is easily rejected in Table IV, the implication of serially uncorrelated labor supply errors is that lagged unemployment terms should not improve the forecast error of the unemployment regression, maintaining lagged real wages and nominal interest rates. As indicated in Table II, however, three out of four lagged unemployment terms are highly significant in the unrestricted reduced form for unemployment.

On the other hand, if the \( v_{it} \) are serially correlated, the implications of the theory are less transparent. If, for example, \( v_{it} \) follows an \( n^{th} \) order autoregression of the form:

\[
4(L)v_{it}=3_{it},
\]

for \( 3_{it} \) serially uncorrelated, and if the parameters of the lag operator \( 4(L) \) are common to all individuals, then (4) and (4a) can be pre-multiplied by \( 4(L) \) to give current unemployment as a function of lagged unemployment, lagged real wages, lagged

\[\text{------------------------}\]

\( ^{25} \) The marginal significance level of the test statistic is .13.
interest rates, lagged prices, and a serially uncorrelated error. While the theory continues to generate cross equation restrictions between the unemployment equation and the reduced form equations for real wages and interest rates, as Table T-1 indicates there are no simple exclusion restrictions on the system that we can verify against the data as summarized in Section I.

While it is clear that the most restricted versions of the basis case are rejected by the data, the simple addition of serial correlation in the labor supply errors makes it more difficult to test the theory. We conclude that further research effort will be required to assess the empirical support for even this basis case version of the intertemporal substitution hypothesis so long as serial correlation in the labor supply errors is taken to be a reasonable hypothesis for the explanation of persistence in the aggregate unemployment rate.\textsuperscript{25} Since the goal of most business cycle theories is to explain this persistence without resort to ad hoc assumptions about microeconomic behavior, however, we would not count this as a particularly successful feature of the simplified intertemporal substitution model.

The General Case

Evidence on both realized and anticipated real interest rates

\textsuperscript{25} This is an issue that could usefully be studied with longitudinal microeconomic data. This research has just been started in recent years. See, for example, \text{McCurdy[17]}.
suggest that their levels have not been constant throughout the
decade after 1970. The impact of expected real interest rates
on current unemployment is therefore a potentially significant
factor in tests of the intertemporal substitution hypothesis.
Next, we derive from equation (1) the representation of current
unemployment assuming that real wages are forecast as part of a
vector autoregressive system, and without the assumption that
expected real interest rates are constant. Not surprisingly,
there are relatively few conclusions that can be drawn from the
simple data analysis of the first section for this more general
specification of the intertemporal substitution hypothesis.

Suppose that aggregate nominal wages, prices and nominal
interest rates are generated by a vector autoregressive system
that includes at most k lags of each variable and k lags of the
unemployment rate in each equation. Individual unemployment is
generated by (1), taking as given the processes for \( w_t \), \( p_t \), and
\( R_t \). For simplicity, we assume that the individual specific
errors \( v_{it} \) are serially uncorrelated. This allows us to postulate
a solution for wages, prices, interest rates, and unemployment of
the form:

\[
y_t = A y_{t-1} + e_t
\]

where

\[
y_t = (w_t, w_{t-1}, \ldots, w_{t-k+1}, p_t, \ldots, p_{t-k+1}, R_t, \ldots, R_{t-k+1}, u_t, \ldots, u_{t-k+1}),
\]

and

\[
27 \text{ See for example Mishkin[19].}
\]
\[ e_t^* = (e_{1t}, \ldots, e_{2t}, \ldots, e_{3t}, \ldots, e_{4t}, \ldots, \ldots) , \]

is a vector of serially uncorrelated innovations, and \( A \) is a \( 4 \times 4 \) matrix of coefficients. The \( 4k \) coefficients in the row of \( A \) corresponding to \( u_t \) are treated as undetermined. Since

\[ E_{lt}(w_{lt+j}) = E_{lt}(w_{lt+j}) \]

for \( j > 0 \), (1) can be expressed as:

\[ u_{lt} = b_0 (w_{lt} - p_t) + \Theta E_{lt}(y_t) + v_{lt} \]

for a vector of coefficients \( \Theta = (\Theta_1, \ldots, \Theta_{4k}) \) whose elements are functions of \( \{ b_j \}, \{ a_j \} \), and the elements of \( A \). Taking expectations conditional on information available at time \( t-1 \), and noting that \( E_{t-1}(u_{lt}) = E_{t-1}(u_t) \), we obtain:

\[ E_{t-1}(u_t) = b_0 (E_{t-1}(w_t - p_t)) + \Theta E_{t-1}(y_t) \]

Since \( E_{t-1}(u_t) \), \( E_{t-1}(w_t) \), \( E_{t-1}(p_t) \), and \( E_{t-1}(y_t) \) are all functions of \( \psi_{t-1} \), for (5) to hold identically in \( y_{t-1} \), \( A \) must satisfy a set of \( 4k \) restrictions. Given the coefficients of the autoregressive system generating \( w_t \), \( p_t \), and \( R_t \), the coefficients of the unemployment equation are determined by (5).

While the force of the cross equation restrictions imposed by the general model of intertemporal substitution is not transparent, several conclusions are possible with respect to the vector autoregressive system of wages, prices, interest rates and unemployment. First, and fundamentally, Granger causality of unemployment by any variable not directly useful in predicting
either real wages or real interest rates is ruled out.\footnote{This conclusion is false if the $v_{it}$ are serially correlated.} Alternatively, assuming that the $v_{it}$ are serially uncorrelated, lagged unemployment terms should enter the regression for unemployment only to the extent that they help to predict real wages or real interest rates. As we have seen in Tables V(b) and VI, there is no evidence that unemployment helps predict either variable. We therefore conclude that the intertemporal substitution hypothesis by itself is not capable of describing the aggregate time series data on wages, prices, interest rates and unemployment. On the other hand, by augmenting the model with serial correlation in individual labor supply it may be possible to set out a version of the intertemporal substitution model that is consistent with the data we have presented in the first section of this paper. A definitive assessment of this possibility requires an alternative research strategy that concentrates on the cross equation (rather than the exclusion) restrictions of the model.

3. Long Term Wage Contracts and Unemployment

For the purposes of discussing the role of monetary policy in the determination of aggregate employment and unemployment, models with sticky wages or prices appear to be the leading alternative contenders to the intertemporal substitution...
framework.\textsuperscript{29} Models that incorporate what are interpreted to be long term contracts have a plausibility based on the observation of the apparent existence of such contracts, and they have been advanced by Fischer\textsuperscript{[6]} and Taylor\textsuperscript{[27]} among others. The set-up due to Taylor is a remarkably clear example of the kind of testable model that delivers the concise ARMA representations that may be so easily contrasted with the "facts" in Tables I - VI. Although far from identical, Taylor's set-up has many of the same implications for the data as does Fischer's, and so we restrict attention here to the examination of the former.

Taylor assumes that all workers are employed in $N$-period fixed nominal wage contracts and concentrates on how these wages may be set in the face of a known money supply feedback rule and rational expectations. For the fraction of workers in the labor force whose wages are set in period $t$, the (logarithm of the) wage is $x_t$ for the next $N$ periods. Assuming a uniform distribution of contract expirations, the average nominal observed wage $w_t$ is a simple moving average of current and past $x_t$'s:

\begin{equation}
   w_t = D(L)x_t,
\end{equation}

where the $N$-1 order lag polynomial has all coefficients equal to $1/N$. Taylor assumes that the nominal wage established in $t$ depends on nominal wages established in the previous ($N$-1) periods, on expectations of nominal wages to be established in

\textsuperscript{29} See for instance, Lucas' discussion of these two competing views in [16].
the next \((N-1)\) periods, and on the expected state of aggregate demand over the life of the contract. Let \(B(L)\) be a lag polynomial of order \((N-1)\) with \(B(1)=1/2\), let \(e_t\) represent a measure of aggregate demand in period \(t\), and let \(\varepsilon_t = L^{-1} e_t\), and \(\varepsilon_t = L^{-1}(x_t)\). Taylor assumes that \(x_t\) is established according to

\[
(3) \quad x_t = B(L)x_t + B(L^{-1})\varepsilon_t + hD(L^{-1})\varepsilon_t + z_t,
\]

where \(h>0\) gives the response of the negotiated wage in \(t\) to average expected aggregate demand over the life of the contract, and \(z_t\) is a serially uncorrelated error.\(^3\) Note that the weights applied to negotiated wages \(k\) periods in the past and expected negotiated wages \(k\) periods in the future are equal. The requirement that the coefficients of \(B(L)\) sum to 1/2 implies that the symmetric polynomial \(B(L)+B(L^{-1})\) has coefficients that sum to unity.

The model is closed by adding a quantity theoretic aggregate demand equation relating \(e_t\) to \(w_t\). A simple formulation is

\[
(9) \quad \varepsilon_t = \gamma w_t + v_t,
\]

where \(v_t\) is a white noise error and \(\gamma<0\) reflects the fact that in the absence of full accommodation by the monetary authority, higher average nominal wages reduce the level of aggregate demand. Substituting (9) into (3), taking expectations at \(t-1\), and noting that \(x_{t-k} = \varepsilon_{t-k}\) for \(k>0\), we obtain a difference equation in \(\varepsilon_t\):

\[
------------
\]

\(^3\) \(B(L^{-1})\) and \(D(L^{-1})\) are polynomials in the lead operator \(L^{-1}\).
(10) \[ R_t = B(L)R_t + B(L^{-1})R_t + \gamma hD(L)D(L^{-1})R_t. \]

Now \( D(L)D(L^{-1}) \) is a symmetric polynomial and can be written as

(11) \[ D(L)D(L^{-1}) = 1/N + C(L) + C(L^{-1}), \]

where \( C(L) \) is a one sided polynomial of order \( N-1 \) with a zero constant term. Substituting (11) into (10) and re-arranging, we have:

(12) \[ \gamma = \left[ B(L)+\gamma hC(L) \right] R_t + \left[ B(L^{-1})+\gamma hC(L^{-1}) \right] R_t \]

\[ = \beta^*(L)x_t, \]

where \( \beta^*(L) \) is a polynomial with leads and lags to order \( N-1 \).

Since \( \beta^*(L) \) is constructed to be symmetric, there exists \( \theta \neq 0 \) and a one sided polynomial \( A(L) \) of order \( N-1 \) with a unit constant term and all roots less than or equal to 1 in modulus such that

\[ \beta^*(L) = \theta A(L)A(L^{-1}). \]

Substituting into (12) and dividing by \( A(L^{-1}) \) gives a solution for \( R_t \) in terms of past values alone:

\[ A(L)R_t = \gamma. \]

From equation (3) it is evident that \( x_t \) and \( R_t \) differ by \( z_t \). Since \( x_{t-k} = R_{t-k} \) for \( k > 0 \),

(13) \[ A(L)x_t = A(L)R_t + x_t - R_t \]

\[ = z_t, \]

which gives the reduced form solution for the contract wage at
time t.

The coefficients of A(L) depend on the coefficients of B(L) and the parameters \( \gamma \) and \( h \). To obtain an expression for the aggregate observed wage, pre-multiply (13) by \( D(L) \) to obtain

\[
A(L)w_t = D(L)z_t.
\]

The time series representation of aggregate demand follows by substituting (14) into (9).

The empirical implications of Taylor's model are summarized in Table I-2. First, as (14) indicates, the nominal aggregate wage has a conoiso univariate ARMA representation, with the order of both the AR and MA parts equal to the length of the underlying contracts.\(^{31}\) Second, by assumption, aggregate demand (or unemployment) has the same basic stochastic structure as nominal wages and prices. Third, the AR coefficients of the nominal wage process are closely linked to the product of the structural parameters \( \gamma \) and \( h \). If neither \( \gamma \) nor \( h \) is zero, then all roots of \( A(L) \) are less than one in modulus. On the other hand, \( \gamma h = 0 \) implies and is implied by a unit root in the polynomial \( A(L) \).\(^{32}\) If workers fail to consider the aggregate consequences of their wage demands, or if the monetary authority fully accommodates their wage demands, then the nominal wage process will be non-

\[^{31}\text{Note that the MA error } D(L)z_t \text{ is non-invertible since } D(L) \text{ has all roots on the unit circle.}

\[^{32}\text{Recall } B(L) = QA(L)A(L^{-1}). \text{ Note that } B'(1) \text{ is the sum of the coefficients of the polynomial } B. \text{ It is easy to show } C(1) = 1/2(1-1/N), \text{ and since } B(1) = 1/2, \ B'(1) = \gamma h. \text{ If } A(L) \text{ has a unit root then } A(1) = 0 \text{ so } B'(1) = QA(1)A(1) = 0, \text{ which implies } \gamma h = 0.\]
Table T-2

Implications of Taylor's
Overlapping Contracts Model

<table>
<thead>
<tr>
<th>Assumptions</th>
<th>Implications</th>
</tr>
</thead>
</table>
| 1. N period contracts; errors in wage setting and aggregate demand equations serially uncorrelated. | (i) aggregate nominal wages follow ARMA(N,N). MA part is an unweighted moving average \((1+L+L^2+...L^N)\).  
   (ii) unemployment follows ARMA(N,N) with same AR part as nominal wages.  
   (iii) unit root in AR part of nominal wages if and only if full accommodation or no dampening effect of unemployment on wage demands.  
   (iv) unemployment fails to cause current wage settlements. |
| 2. N period contracts; errors in wage setting and aggregate demand equations MA(K). | (i) aggregate nominal wages follow ARMA(N,N+K).  
   (ii) unemployment follows ARMA(N,N+K) with same AR part as nominal wages.  
   (iii) unit root in AR part of nominal wages if and only if full accommodation or no dampening effect of unemployment on wage demands.  
   (iv) unemployment fails to cause current wage settlements. |
stationary.

A forth implication of Taylor's model is that aggregate demand fails to Granger-cause $x_t$, the level of nominal wages established in currently negotiated contracts. This is a simple consequence of the rational expectations of wage setters: information on past levels of all variables is incorporated into the current decision, and the innovation in $x_t$ is therefore orthogonal to $e_{t-k}$ for $k>0$. However, since $w_t$ is an average of past $x_t$, and since $x_t$ is in general correlated with $e_t$, aggregate wages may be Granger caused by aggregate demand.

At this juncture it is worthwhile pointing out the robustness of most of these implications to the specification of the model. First, as noted by Taylor, the shocks $v_t$ and $z_t$ may be serially correlated. This has the effect of adding a term in the forecasts of $v_t$ and $z_t$, $\hat{v}_t$ and $\hat{z}_t$ respectively, to the right hand side of (12). The solution of the model can then be written as:

\[
(13a) \quad A(L)x_t = A^{-1}A(L^{-1})^{-1} \{ YD(L^{-1})\hat{v}_t + \hat{z}_t \} \\
+ z_t - 2_t,
\]

which differs from (13) by the addition of a possibly serially correlated error. Clearly none of the basic properties of the simpler model are lost.

A second possible modification is to replace the relative wage setting rule (3) by a purely forward looking real wage setting
The average expected price level over the life of the contract is \( D(L^{-1})\hat{\pi}_t \). Suppose that currently negotiated wages are set according to

\[
(3a) \quad x_t = D(L)\hat{\pi}_t + hD(L^{-1})\hat{\pi}_t + z_t.
\]

This implies that current wages are set to achieve an expected real wage target, modified to the extent that anticipated aggregate demand deviates from trend. Substituting for the definitions of average wages and excess demand, (3a) leads to an expression exactly analogous to (12). The switch from relative to real wage setting leaves the model essentially unchanged.

It will now be obvious that this model is going to have a hard time explaining the "facts". First, as Table I(b) indicates, we find no evidence of moving average errors in even an AR1 representation of nominal wages. The presence of such moving average errors is also implied by Fischer's overlapping contracts set-up, and seems to be a fairly broad implication of the presence of overlapping wage contracts of the type that it is usually suggested do exist. Our empirical results imply either that these contracts are not very prevalent, or that they take a different form than is usually suggested. Likewise, since the "facts" suggest that an AR1 adequately describes the quarterly wage data, if overlapping contracts of the type suggested by

\[33\] This suggestion is pursued in Buiter and Jewitt[1].

\[34\] See Barro[2] for a discussion of how optimal overlapping contracts would be constructed and how they contrast with the type of contracts suggested by Taylor and Fischer.
Taylor are actually prevalent, then they must be very short (that is, of two quarters duration or less).

Perhaps a more fundamental difficulty still is our finding that the stochastic structure of the nominal wage process differs strongly from the stochastic structure of both prices and unemployment. Taylor and Fischer set out models in which the real wage is constant and then derive the implication that unemployment has the same basic ARMA structure as wages. A more reasonable empirical statement is that unemployment has the same basic ARMA structure as prices, and that neither of these series have much in common with nominal wages. It does not seem possible to attribute the business-cycle responses to innovations that we observe in the time series representations of unemployment, employment, and prices to the underlying structure of the nominal wage determination process.

**Concluding Remarks**

In this paper we have tried to emphasize the usefulness of setting out the unrestricted final forms of the time series data that form the endogenous variables in models of the aggregate labor market before proceeding to the elaborate fitting to the data of models that incorporate strong prior information on these final forms. A decade ago this would have been akin to urging that the unrestricted reduced forms of aggregate models of the

35 Although it is not quite true that unemployment and prices have the same ARMA representations—see Table 1(b).
labor market be fitted to the data and scrutinized before strong prior information in the form of structural exclusion restrictions was imposed.\footnote{In fact, two decades ago the validity of imposing this kind of prior information on the data was strongly questioned by Liu\cite{13}.} The factual turn of events during the 1970's has finally turned attention to truly dynamic models, and this has likewise turned attention to the dynamic final form representations of time series variables.

Our approach is to first catalogue the "facts", which we take to be the univariate and vector autoregressive and moving average representations of the time series data on unemployment, nominal wages, consumer prices and nominal interest rates, together with the maximal set of exclusion restrictions with which these data are consistent. The next step is to catalogue the implications of the various models that deliver linear ARMA representations of the time series data, and then to compare these implications against the "facts". From the point of view of this research strategy, the regressions reported in Tables I-VI provide the data that challenge any proposed model that purports to explain the time series behavior of the aggregate labor market.

As we have seen, this challenge is a formidable one. Neither parsimonious models based on intertemporal substitution in labor supply with imperfect information, nor explicit models of wage and price stickiness seem to be consistent with the data. This suggests that there is a large agenda for further research.

\footnote{In fact, two decades ago the validity of imposing this kind of prior information on the data was strongly questioned by Liu\cite{13}.}
First, it may be useful to more carefully explore the temporal stability of the results in Tables I-VI. One important message of existing models is that the reduced form representations of various time series variables may not be invariant to changes in public policy. Further data analysis would afford the opportunity to both explore the data and test this proposition. Second, it is important to catalogue and compare in more detail the time series implications of the theoretical models examined here, simple modifications of these models, and other models that exist in the literature or have been suggested.

It is remarkable that the debate over the role of the effectiveness of demand management policies has thus far been carried out in the context of models for which very little in the way of empirical support exists. It is perhaps not very surprising that neither academics nor public officials have thus far listened very carefully to it.
References


Appendix: Data Definitions and Sources

Definitions and Sources of the variables in Tables I - IV are as follows:

\[ W = \] average hourly earnings in manufacturing, excluding overtime and unadjusted for industry composition. Source: Citibase.

\[ P = \] consumer price index, wage earners and clerical workers (CPI-W), 1967 = 100. Source: Citibase.


\[ R = \] yield on U.S. Government 3-month Treasury Bills (rate on new issues). Source: Citibase.