Unemployment as Disequilibrium in a Model of Aggregate Labor Supply

by

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*I am indebted for financial support to the Guggenheim Foundation. This paper was written during my stay at the Centre for Labour Economics, London School of Economics, and I am indebted to seminar participants there, at Tel-Aviv University, CORE--University of Louvain, Nuffield College, Oxford, Princeton University, and UCLA for helpful comments. John Ham is owed a special debt for his assistance throughout.
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Even though the memory of the Great Depression of the 1930’s still lingers in the background of professional discussions of unemployment, by now it has nearly faded away. The most influential modern work on the theory of unemployment has proceeded in two directions. In one case unemployment has been treated as a form of choice about the length of time to search for a job.1/ In the other case unemployment has been explained as the result of the inter-temporal substitution of leisure for work. Lucas and Banzing [26], for example, interpret unemployment as the difference between the offer to sell labor at the current wage and the offer that would be forthcoming at the normal wage. What both of these approaches have in common is the assumption that unemployment time is a result of choices by individual workers in the face of parametric wage rates, so that the hours at work of these individuals are consistent with their conventionally defined labor supply functions.

At the same time there has also been a rebirth of interest in old fashioned views of unemployment. One strand of this work spells out the reasons why optimal employment contracts may involve the implicit joint choice of the wage rate and hours at work.2/ Another strand examines the implications of

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1/See Mortenson [29] and the references therein.

2/See Azariadis, [6], Bally [7], Feldstein [13], and D. Gordon [14].
institutionally inflexible wage and price mechanisms for the nature of economic equilibria. What both of these approaches have in common is the explicit recognition of unemployment as disequilibrium in the sense that workers' actual hours at work are not consistent with the predictions of their conventionally defined labor supply functions. In the first case this happens because hours of work are not chosen in the face of a parametric wage, but in either case the worker behaves as if facing a constraint on the hours that may be sold in the labor market.

The purpose of this paper is to explore the empirical implications for the consumer-worker's behavior of treating unemployment as a constraint on labor supply rather than the result of it. The main importance of this issue is for establishing an appropriate model to untangle constrained from unconstrained behavior and to test for the existence of the former. Also of importance, however, are the normative insights for unemployment compensation that such constraints imply. The plan of the paper is therefore as follows: Section I contains a discussion of the general implications for household commodity demands and the labor supply of other family members that result from unemployment. These implications are so simply obtained, in contrast with the usual analysis of "straight" rationing, that they may also be of interest for this reason as well as for the empirical strategies for testing that they suggest. Section II contains a discussion of the derivation of a simple set of aggregate labor supply and commodity demand functions that are based on the utility maximization paradigm but that explicitly recognize the presence of both constrained

\[3/\text{A useful exposition and synthesis of this "modern" Keynesian approach has recently been written by Malinvaud [25], which also contains the appropriate references.}\]
and unconstrained microeconomic units. These equations are then fitted to aggregate data over the past four decades. The final section contains a discussion of the limitations of the results and the additional work that may be useful.
I. Labor Supply and Unemployment

The power of the application of the classical utility maximization framework in a model of family labor supply derives from the assumption that the family's preferences among non-market time (leisure) and commodities may be represented by the conventional quasi-concave utility function

\[ u = u(l_1, \ldots, l_m, x_{m+1}, \ldots, x_{m+n}). \]

Here \( l_i \) is the family's consumption of the non-market time of the \( i^{th} \) member of the \( m \) family members and \( x_i \) is the family's consumption of the \( i^{th} \) of the \( n \) commodities. Assuming that the family behaves as if maximizing (1) subject to a budget constraint that requires money income to equal expenditure,

\[ y + \sum_{i=1}^{m} (T-l_i)w_i = \sum_{i=m+1}^{m+n} p_ix_i, \]

where \( y \) is unearned income, \( h_i = T-l_i \) and \( w_i \) are the hours of market work and wage of the \( i^{th} \) family member, and \( p_i \) is the price of the \( i^{th} \) commodity leads to the \( m \) demand functions for the family's non-market time

\[ l_i = l_i(w, p, y) \quad i = 1, \ldots, m \]

and the \( n \) demand functions for commodities

\[ x_i = x_i(w, p, y) \quad i = m+1, \ldots, m+n. \]

Since the total amount of time available to each family member, \( T \), is fixed, the \( m \) labor supply functions for the \( m \) family members,

\[ l_i = T-l_i = l_i(w, p, y), \]
have derivatives that are equal but opposite in sign to those of the demand functions (3a).

The actual, maximum attainable, or indirect utility level achieved by the family at the price vector \( p \), wage vector \( w \) and unearned income level \( y \) is

\[
(4) \quad v = u(x_1(w, p, y), \ldots, x_m(w, p, y), x_{m+1}(w, p, y), \ldots, x_{m+n}(w, p, y)) = v(w, p, y).
\]

For what follows it is useful to solve equation (4) for unearned income to obtain

\[
(5) \quad y = E(w, p, v).
\]

Just as the function \( v(w, p, y) \) gives the maximum utility level obtainable with the wage vector \( w \), the price vector \( p \), and an unearned income of \( y \), the inverse function \( E(w, p, v) \) gives the minimum unearned income necessary to obtain the utility level \( v \) at these wages and prices. Alternatively, since \( y = \sum p_i x_i - \sum w_i h_i \) and \( \sum p_i x_i \) is family expenditure on commodities while \( \sum w_i h_i \) is family earnings, \( E(w, p, v) \) is the minimum excess of expenditure over earnings required to reach the utility level \( v \).\(^{1/2}\) For this reason it may be useful to call \( E(w, p, v) \) the family's excess-expenditure function.

It is well known, and easy to establish, that the derivative of this excess-expenditure function with respect to the \( i \)\(^{th} \) wage gives the negative of the \( i \)\(^{th} \) compensated (or utility-constant) labor supply function, while the derivative of this excess-expenditure function with respect to the \( i \)\(^{th} \) commodity price

\(^{1/2}\) For an accessible discussion of the expenditure function see Diamond and McFadden [12]; a much earlier discussion is McKenzie [27], and a very detailed discussion is McFadden [26].
gives the \( i \)th compensated (or utility-constant) commodity demand function. Thus 
\[
E_i(w, p, v) = \delta E(w, p, v)/\delta w_i = -h_i(w, p, v) = e_i(w, p, v) - T
\]
for \( i = 1, \ldots, m \) and 
\[
E_i(w, p, v) = \delta E(w, p, v)/\delta p_i = x_i(w, p, v)
\]
for \( i = m + 1, \ldots, m + n \). The compensated (Slutsky) effects of wage or price changes on
the demands for non-market time and commodities, \( S_{ij} \), are then conveniently
arrayed in the negative semi-definite matrix of second derivatives of the
excess-expenditure function, so that \( [S_{ij}] = [E_{ij}] \). As in the conventional
case, two commodities and/or non-market time are said to be substitutes if
\( S_{ij} > 0 \) and complements if \( S_{ij} < 0 \).

A. Constraints on Labor Supply

With the stage set in this way it is a straightforward matter to examine
the effect of a spell of unemployment by the first family member on the labor
supply functions of other family members or on the family's commodity demand
functions. In this context the first worker suffers a spell of unemployment
during a period of length \( T \) if, for a given set of wages, prices and level
of unearned income, \( h_1(w, p, v) > \bar{h}_1 \), that is, if the worker's desired level
of hours of market work is greater than some fixed constraint level \( \bar{h}_1 \).

Of course, if this constraint is effective, as is assumed to be the case,
actual hours of work will equal the constraint level, \( \bar{h}_1 \). In this framework
a worker who suffers a spell of unemployment of length \( h_1 - \bar{h}_1 \) is, for one
reason or another, unable to sell all the hours on the labor market that he
desires to sell at his current market wage rate. As a consequence the family
is forced to consume more of the non-market time (leisure) of the first family
member than is desired.\(^5\)

\(^5\)Rea [34] has suggested a different approach to this problem where unemployment
time is taken to reduce a worker's fixed time endowment, \( T \). This seems a concept-
ually inappropriate procedure since it confuses technological differences with
variable constraints.
Put in this way, unemployment is a constraint on labor supply similar in form to the "straight" rationing that has been analyzed by Tobin and Houthakker [38]. Since the constraint on the hours of work of the first worker, \( \bar{h}_1 \), is binding we may as well follow their procedure and build this into the set of constraints the family faces. In particular, because of the presence of the constrained worker the family now has one less choice to make and maximizes

\[
(6) \quad u = u(T - \bar{h}_1, \bar{h}_2, \ldots, \bar{h}_m, x_1, \ldots, x_{m+n})
\]

subject to the budget constraint

\[
(7) \quad y + \nu_1 \bar{h}_1 + \sum_{i=2}^{m_1} \nu_i h_i = \sum_{i=m+1}^{m+n} P_i x_i.
\]

This leads to the new set of \( m-1 \) demand functions for the family's non-market time

\[
(8a) \quad \hat{\lambda}_1 = \hat{\lambda}_1(\bar{h}_1, \nu', p, y + \nu_1 \bar{h}_1)
\]

and the new set of \( n \) commodity demand functions

\[
(8b) \quad \hat{x}_1 = \hat{x}_1(\bar{h}_1, \nu', p, y + \nu_1 \bar{h}_1),
\]

where the vector of wage rates \( \nu' = (\nu_2, \ldots, \nu_m) \) does not include \( \nu_1 \). Of course, if there were only one worker in the family (or if all workers were constrained), these commodity demands would depend, apart from the level of the hours constraint, only on total money income, \( y + \nu_1 \bar{h}_1 \). This is apparently the basis for Clower's [10] remark to the effect that Keynesian consumption functions, which do not distinguish between the effects of a consumer's wage and unearned income, are consistent with the usual model of consumer choice.
if the consumer-worker faces a constraint on labor supply and is, in this
sense, off his labor supply function. The existence of such constraints
also suggests the possibility that direct measures of the extent of these
constraints, such as the unemployment rate, might be suitable candidates
for inclusion as explanatory variables in commodity demand functions.

The actual, maximum attainable, or indirect utility level achieved
by the constrained family is obtained by substituting (8) into (6) to get

\[ v^* = u(T - \bar{h}_1, \bar{h}_1, w', p, y + v_1 \bar{h}_1), \ldots, \bar{h}_1(\bar{h}_1, w', p, y + v_1 \bar{h}_1) \]

\[ = v^*(\bar{h}_1, w', p, y + v_1 \bar{h}_1). \]

It is useful to solve equation (9) for the sum of unearned income and con-
strained earnings to obtain the conventional expenditure function

\[ y + v_1 \bar{h}_1 = R(\bar{h}_1, w', p, v^*). \]

The constrained excess-expenditure function is then

\[ y = R(\bar{h}_1, w', p, v^*) - v_1 \bar{h}_1. \]

As before, the negative of the derivatives of this function with respect to
wages, \(-R_i(\bar{h}_1, w', p, v^*)\) for \(i = 2, \ldots, m\), and the derivatives with respect
to prices, \(R_i(\bar{h}_1, w', p, v^*)\) for \(i = m+1, \ldots, m+n\) are the compensated (or
utility-constant) labor supply and commodity demand functions of the constrained
family. Likewise, \([R_i, S_i] = [S_i, \hat{S}_i] \) is the matrix of Slutsky effects derived
from the remaining \(m+1\) non-market time and \(n\) commodity demand functions of
the constrained family.

In order to examine the effect of changes in the length of the spell of
unemployment on the labor supply of the unconstrained family members, or to
compare the response of the family to wage and price changes in the presence
and absence of the unemployment of one or more family members, it is necessary
to choose a constraint level \( \vec{h}_1 \) at which the comparison is to be made. A
natural and convenient point to choose is the value of \( \vec{h}_1 \) that would have
been chosen in the absence of unemployment. Although they have considerable
intuitive appeal, the results obtained from this procedure can be very mis-
leading and require careful interpretation. The appropriate caution has been
indicated by Pollak [32] in his comments on Tobin and Routhakker's [38]
analysis, and the reader is referred there for further discussion.

Requiring that the constraint \( \vec{h}_1 \) be imposed at the point that would
have otherwise been chosen implies that \( \vec{h}_1 = -E_1(w, p, v) \) and that \( v = v^* \).
Consequently, the relationship

(11) \( R_i(-E_1(w, p, v), w', p, v) \equiv E_i(w, p, v) \)

holds as an identity for \( i = 2, \ldots, m+n \). Differentiating this identity with
respect to \( w_1 \) implies that \( -(\partial R_i / \partial h_1) E_{i1} = E_{i1} \), which establishes

(12) \( \partial R_i / \partial h_1 = E_{i1} / (-E_{i1}) \quad (i = 2, \ldots, m+n) \)

as a simple formula for the effect of an increase in employment on the compen-
sated demands for non-market time or commodities. Moreover, it is a straight-
forward matter to establish that the effects of a change in employment on the
uncompensated demands are identical to this.\(^6\)

\(^6\) Note that \( -k_i(\vec{h}_1, w', p, y = w_1 \vec{h}_1) \equiv R_i(\vec{h}_1, w', p, v(\vec{h}_1, w', p, y = w_1 \vec{h}_1)) = T \)
from the substitution of (9) into the compensated (or utility-constant) demand
function. Differentiating this identity with respect to \( \vec{h}_1 \) gives \( -\partial k_i / \partial h_1 = \partial R_i / \partial h_1 + (\partial R_i / \partial v^*)[3v^*/3h_1 + w_1 3v^*/3y] = \partial R_i / \partial h_1 \), and similarly for the
commodity demands.
Since $P_i = -h_i$ (for $i = 2, \ldots, m$) it follows from (12) that

\[(12a) \quad \frac{\delta h_i}{\delta h_1} = \frac{S_{i1}}{S_{11}} \quad (\text{for } i = 2, \ldots, m).\]

Since $S_{11} < 0$, according to (12a) a decrease in the employment of the first family member tends to increase (decrease) the labor supply of family members whose non-market time is substitutable (complementary) with that of the first family member. This result has considerable intuitive appeal. For example, suppose that in the absence of unemployment the non-market time of the husband and wife in a family are substitutes. In this case an exogenous spell of unemployment for the husband would be expected to lead the wife through a pure substitution effect alone to seek more employment in the labor market. This is the so-called "added worker" effect of unemployment that has been studied empirically by labor-force analysts since the 1930's.\footnote{Bowen and Fingeran [9] find considerable evidence for this effect, but Cohen, Lerman, and Rea [11], using what may be better data, do not.} Of course, whether the non-market time of the husband and the non-market time of the wife are substitutes or complements is primarily an empirical issue.

It is also evident from (12) that

\[(12b) \quad \frac{\delta x_i}{\delta h_1} = \frac{S_{i1}}{S_{11}} \quad (i = m + 1, \ldots, m + n),\]

so that a decrease in the employment of the first family member tends to increase (decrease) the demand for commodities that are complementary (substitutable) with the non-market time of the first family member. This result also has considerable intuitive appeal. For example, it is often suggested that the demand for commodities such as books, cinema, and home maintenance materials
are considerably less sensitive to the vagaries of the business cycle because these goods are complementary with the non-market time of consumer-workers and there is consequently a substitution of expenditure toward these commodities in periods of substantial unemployment.  

Finally, differentiating (11) with respect to the \( j^{th} \) wage or price (\( P_{j} \)) implies that \(- (\partial R_{i}/\partial P_{j}) E_{ij} + R_{ij} = E_{ij} \), which, after the substitution of (12) establishes

\[
(13) \quad S_{ij} = S_{ij} - (S_{ij})(S_{ij})/S_{ii} \quad (i,j\#1)
\]

as a simple formula relating the (Slutsky) substitution effects when the first worker is unemployed to the (Slutsky) substitution effects prevailing in the absence of unemployment. Although (13) implies a whole set of changes in the matrix of substitution effects in the family's demand functions as a result of unemployment, the most obvious impact is on the own substitution effects \( S_{ii} (i\#1) \). Since \( S_{ii} = S_{ii} \) and \( S_{ii} < 0 \), these own substitution effects must generally become smaller in absolute value as a result of the unemployment of the first family member. Similar relationships may also be established for the uncompensated wage and price derivatives.  

\(^{8/}\) For some direct evidence that these commodities are complementary with leisure see Owen [31].

\(^{2/}\) The results above do not assume that \( \bar{h} > 0 \), and the case \( \bar{h} = 0 \) could also be taken as the case where a family member chooses not to work. This problem has been analyzed recently by Kneser [21] to determine the effect of the presence of a working wife on the labor supply function of a working husband. Compare his equation (50) with equation (13) above, but note also the involved way in which the former is derived.
B. The Welfare Implications of Unemployment

The family constrained by the presence of an unemployed worker will generally suffer a loss of welfare compared to an identical family that does not face this constraint. One natural way to examine the quantitative nature of this loss is to explore the lump-sum compensation that would restore the unemployed worker's family to the welfare level of the fully employed family.\textsuperscript{10}

Since $E(w, p, v)$ is the unearned income required by the family to reach the welfare level $v$ in the absence of unemployment, while $R(h_1, w', p, v) - w_1 h_1$ is the unearned income required to reach the same welfare level in the presence of unemployment, the quantity

$$C(h_1, w, p, v) = R(h_1, w', p, v) - w_1 h_1 - E(w, p, v)$$

is the lump-sum compensation that would make a family with an unemployed worker as well off as a fully employed family. The amount of unemployment compensation depends, of course, on the amount of employment, wages, and prices. In general, the unemployment compensation will differ from the earnings loss due to unemployment, $w_1(h_1 - h_1)$, because of the additional non-market time available to the family with an unemployed worker.\textsuperscript{11} The family compensated by the amount $w_1(h_1 - h_1)$ would be over-compensated so long as non-market time was desirable because such a family would have the same command over consumption goods as the fully employed family and a greater command of non-market time.

\textsuperscript{10}This is not the only way to do so, however. Other possible forms of compensation might include, for example, the provision of commodity subsidies to families with unemployed workers, or higher wage rates for the hours that less than fully employed workers do work. The latter would be a natural approach to the derivation of compensating wage differentials.

\textsuperscript{11}This point, and some related analysis involving the use of unemployment time in the search for employment, are discussed by Gordon [15].
A natural procedure to gain some insight regarding the unemployment compensation \(C(\bar{h}_1, w, p, v)\) is to approximate it by a second-order Taylor series around the fully employed equilibrium. At this point \(R(\bar{E}_1, p, v, w, p, v) + \frac{\partial R}{\partial \bar{h}_1} (w, p, v) \equiv E(w, p, v)\) so that differentiation with respect to \(\bar{w}_1\) establishes that \(\frac{\partial R}{\partial \bar{w}_1} - \bar{w}_1 = 0\) and \(\frac{\partial^2 R}{\partial \bar{h}_1^2} = 1/E_{11} = -1/S_{11}\). Consequently

\[
(14a) \quad C(\bar{h}_1, w, p, v) \approx \left(\frac{\partial R}{\partial \bar{h}_1} - \bar{w}_1\right) \frac{\partial R}{\partial \bar{h}_1} + \frac{1}{2} \left(\frac{\partial^2 R}{\partial \bar{h}_1^2}\right) \left(\frac{\partial \bar{h}_1}{\partial \bar{w}_1}\right)^2
= 1/2 \left(\frac{\partial \bar{h}_1}{\partial \bar{w}_1}\right)^2 / \left(-S_{11}\right),
\]

which is a conventional "triangle measure" of welfare loss.\(^{12}\) Since \(-\frac{\partial \bar{h}_1}{\partial \bar{w}_1} = h_1 - \bar{h}_1\) is unemployed hours, \((14a)\) may be expressed in elasticity terms as \(1/2 \frac{D_1}{e_{11}}(h_1 - \bar{h}_1)\). D\(_1\)/e\(_{11}\), where \(e_{11} = -S_{11}(w_1/h_1)\) is the utility-compensated elasticity of labor supply and \(D_1 = (h_1 - \bar{h}_1)/h_1\) is the proportional duration of unemployment.

It follows that unemployment compensation as a fraction of lost earnings is merely

\[
(15) \quad C^* = \frac{C}{w_1(h_1 - \bar{h}_1)} = \frac{1}{2} D_1 / e_{11}.
\]

A result similar to \((14a)\) may be established by a similar method when two family members are unemployed and constrained to work \(\bar{h}_1\) and \(\bar{h}_2\) hours respectively. In this case it is a straightforward matter to establish the matrix equality \(\left[R_{ij}\right] = -\left[E_{ij}\right]^{-1} = \left[S_{ij}\right]^{-1}\), where \(R_{ij} = \frac{\partial^2 R}{\partial \bar{h}_i \partial \bar{h}_j}\) evaluated at the point where \(\bar{h}_i = h_i\) (\(i = 1, 2\)) and \(\left[S_{ij}\right]^{-1}\) is the inverse of the appropriately conformable sub-matrix of the Slutsky matrix. It then follows that

\[
(14b) \quad C(\bar{h}_1, \bar{h}_2, w, p, v) = \frac{1}{2} \left(\frac{\partial \bar{h}_1}{\partial \bar{w}_1}\right)^2 \left(\frac{\partial \bar{h}_2}{\partial \bar{w}_2}\right)^2 / \left(-S_{11}\right) s_{11}\).
\]

\(^{12}\) See Harberger [18].
where the vector $\overline{d\nu'} = [d\overline{h}_1 \ d\overline{h}_2]$. Equation (14a) is obviously a special case of (14b). Putting the Slutsky matrix into elasticity form as $\eta = [\eta_{i,j}] = h^{-1} \beta_{i,j} w$, where $h$ and $w$ are diagonal matrices with $h_i (i=1,2)$ and $w_i (i=1,2)$ on the diagonals, then establishes that

(15a) $C^* (\overline{h}_1, \overline{h}_2, \nu, p, v) = C(\overline{h}_1, \overline{h}_2, \nu, p, v)/[w_1(h_1-\overline{h}_1) + w_2(h_2-\overline{h}_2)]$

$\quad\quad\quad = 1/2 [w_1(h_1-\overline{h}_1) + w_2(h_2-\overline{h}_2)]^{-1} dh', \nu [\eta_{i,j}]^{-1} h^{-1} dh' = 1/2 \theta' \eta^{-1} D$,

where $\theta'$ is a (row) vector with elements $\theta_i = v_i (h_1-\overline{h}_1)/[w_1(h_1-\overline{h}_1) + v_2(h_2-\overline{h}_2)]$ that are the $i^{th}$ family member's share in lost earnings and $D$ is a (column) vector with elements $D_i = (h_i-\overline{h})/h_i$ that are the proportional unemployment durations. The approximation (15a) establishes that the family's unemployment compensation as a fraction of the family's lost earnings depends on the (proportional) duration of the unemployment of the family members and on the (inverse of the) matrix of Slutsky substitution elasticities. Equation (15) is obviously a special case of (15a).

The function $C^*$ is basically a criterion for establishing the appropriate value of what is known in the practical literature of unemployment compensation as the "earnings replacement ratio." It is obvious that $0 < C^* < 1$, but it may also be interesting to examine explicit computed values of this quantity under various assumptions about the incidence and duration of unemployment among the family members. Table 1 contains such calculations on the basis of Ashenfelter and Heckman's $[14]$ estimates of $\eta$ for husband's and wives.13

The elements of $\eta$ are $\eta_{11} = .0600$, $\eta_{15} = .0415$, $\eta_{21} = .207$, and $\eta_{22} = 1.157$. These imply the inverse elements $\eta_{11} = 19.02$, $\eta_{12} = -.682$, $\eta_{21} = -3.40$, and $\eta_{22} = .986$. The male/female wage ratio used in the calculations is $w_1/w_2 = 1.73$. 

Table 1

Family Earnings Replacement Ratios for Unemployment Compensation
Calculated from Equations (15) and (15a)

<table>
<thead>
<tr>
<th>Proportion of the Year Unemployed by the Male</th>
<th>0</th>
<th>.05</th>
<th>.1</th>
<th>.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion of the Year Unemployed by the Female</td>
<td>0</td>
<td>.02</td>
<td>.27</td>
<td>.69</td>
</tr>
<tr>
<td>.05</td>
<td>.42</td>
<td>.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.1</td>
<td>.04</td>
<td>.19</td>
<td>.54</td>
<td>.95</td>
</tr>
<tr>
<td>.15</td>
<td>.06</td>
<td>.15</td>
<td>.44</td>
<td>.81</td>
</tr>
</tbody>
</table>

\[1/\text{The approximation (15a) implies } c^* > 1, \text{ which is clearly inadmissible.}\]
As would be expected, $C^b$ increases with the duration of unemployment because of the declining value of the (enforced) consumption of additional non-market time. The majority of completed unemployment spells in the U.S. in the typical post-war year undoubtedly satisfy the constraint $D \leq .15$, but Table 1 suggests two interesting conclusions. First, the computed earnings replacement ratio varies dramatically with the duration of unemployment and the group of workers unemployed, whereas in most actual government unemployment programs this is not the case. Second, the calculations in Table 1 undoubtedly cannot be taken very seriously, both because of the known imprecision in the estimate of $n$ and because the approximation (15a) is clearly unsatisfactory for even the calculations in Table 1, as is demonstrated by the first two entries in its fourth column.$^{14}$

II. Empirical Testing

In order to put the notion of unemployment as a constraint on labor supply to empirical use it is necessary to examine two issues of empirical strategy. The first is the question of how and in what ways the empirical analysis will remain faithful to the utility maximization hypothesis. This is basically a question of how one is to choose a functional form for the commodity demand and labor supply functions of the employed and unemployed consumer-workers. A second problem of empirical strategy concerns how the conjunction of constrained and unconstrained workers is to be related to aggregate behavior.

A. Choice of Functional Form

One scheme for the choice of functional form for the commodity demand

$^{14}$Further discussion of these issues is contained in Ashenfelter [3].
and labor supply functions that is faithful to the utility maximization hypothesis involves specifying a functional form for the constrained indirect utility function (9) and generating the unemployed consumer worker's commodity demand functions by the use of Roy's identity. With the assumption that the preferences of employed and unemployed consumer-worker's are identical the maximum of (9) over \( \bar{h} \) then gives (14), the indirect utility function of the employed consumer-worker, and Roy's identity may then be used with (14) to generate the latter's commodity demand and labor supply functions. The primary difficulty with this approach is finding a tractable functional form for the constrained indirect utility function (9) that can both be solved for (14) and will generate estimatable demand and supply functions. Until this problem has been resolved it is necessary to consider alternative approaches.

A second approach to the choice of functional form would be to use the restrictions on the partial derivatives of the commodity demand and labor supply functions derived from the utility maximization hypothesis as restrictions on the finite approximations to the differentials of the employed consumer-worker's commodity demand and labor supply functions (3).\(^{15}\) The relations between the partial derivatives of the employed and unemployed consumer-worker's commodity demand and labor supply functions in (12) and (13) could then be exploited directly. The primary difficulty with this approach is that the relationships (12) and (13) are valid only at the point where \( h = \bar{h} \), which, of course, will be far from the observed situation. Even so, this procedure is no doubt worth

\(^{15}\)The use of these restrictions as exact information has been considered extensively by Barten \([8]\) and Theil \([37]\) and in the context of interrelated commodity demands and labor supply by Abbott and Ashenfelter \([1]\); their use as prior information only is considered by Kiefer \([20]\).
further exploration.

A third scheme, and the one followed here, is to assume a specific functional form that is common to both employed and unemployed workers for the direct utility function and then to solve the appropriate marginal conditions for the constrained and unconstrained commodity demand and labor supply functions (8) and (3). Since Abbott and Ashenfelter [1] conclude that even the simple augmented Stone-Geary utility function

\[(16)\quad u = B_1 \ln(\gamma_1 - h) + \sum_{i=2}^{m+n} B_i \ln(x_i - y_i),\]

where \( EB_1 = 1 \), is not entirely unsatisfactory for the purpose of examining aggregate labor supply it seems useful to start with this case.\(^{16/}\)

For the fully employed consumer-worker the maximization of (16) subject to (2) leads to the labor earnings and commodity expenditure functions

\[(17a)\quad w^e = \gamma_1 w - B_1[y + \gamma_1 w - E_{11} p_1],\]

\[(17b)\quad p_1 x^e_1 = \gamma_1 p_1 + B_1[y + \gamma_1 w - E_{11} p_1] \quad (i = 2, \ldots, m+n),\]

where \( w^e \) and \( p_1 x^e_1 \) represent earnings and expenditure for the employed worker. The same worker who is unemployed and may work no longer than \( \bar{h} \) hours has labor earnings of

\[(18a)\quad w^c = \bar{w}^e = w^e - w^e D,\]

and commodity expenditures of

\(^{16/}\) As it turns out, Malinvaud's [25] expository analysis uses a special case of equation (16) to describe the preferences of consumer-workers.
\( (18b) \quad p_{i1}^c = \gamma_{i1} p_{i1} + B_i^* \left[ y + h \nu \gamma_{i1} p_{i1} \right] \\
= \gamma_{i1} p_{i1} + B_i^* \left[ y + \gamma_{i1} w - \gamma_{i1} p_{i1} \right] \\
= \gamma_{i1} p_{i1} + B_i^* \left[ y + \gamma_{i1} w - \gamma_{i1} p_{i1} \right] - B_i^* \left[ y + \gamma_{i1} w - \gamma_{i1} p_{i1} \right] \\
= \gamma_{i1} p_{i1} + B_i^* \left[ y + \gamma_{i1} w - \gamma_{i1} p_{i1} \right] - B_i^* \left[ y_{i1} w - B_1 (y + \gamma_{i1} w - \gamma_{i1} p_{i1}) \right] \)

where, as before, \( D = (h^n - n)/n^e \) is the proportional duration of unemployment
and \( B_i^* = B_i^{t+1} \prod_{t=2}^{n} B_j = B_i^1 (1 - B_i) \) (for \( i = 2, \ldots, m+n \)). It is worth observing
that the reference period for the maximization of utility underlying equations
(18) may be longer than, say, a single week so that \( D \) is not generally equal
only to one or zero. In fact, we will apply these equations to annual data,
and it is thus necessary to recognize explicitly that in general we may assume
only that \( 0 < D \leq 1 \) for a worker who has been unemployed. \(^{17}\)

3. Aggregating Employed and Unemployed Workers

It is natural to apply equations (17) and (18) to aggregate time-series data
since it is in this context that variations in unemployment are generally taken
to unambiguously indicate variability in constraints on consumer-worker's labor
supply. It is first necessary to decide, however, how variations in unemployment

\(^{17}\) There are a number of issues related to equations (17) and (18) that are
often ignored, but that deserve some attention. First, equations (17) and (18)
are consistent with a simple intertemporal additive model of utility maximization
of the type suggested by Lluch [23] and lead to savings (\( S \)) functions of the form
\( S^c = u(y + \gamma_{i1} w - \gamma_{i1} p_{i1}) \) and \( S^c = u^*(y + \nu w - \gamma_{i1} p_{i1}) \), respectively, where \( u \) and \( u^* \)
are the marginal propensities to save from unearned income. Thus savings are not
ignored, but it must be understood that proper estimation requires that \( y \) in
(18) be interpreted as unearned income minus savings. This is guaranteed in
practice by requiring that \( y = \gamma_{i1} p_{i1} - w h \) as an "adding up" constraint in (18).
Second, as Abbott and Ashenfelter \( ^{11} \) observed, both (17) and (18) aggregate
consistently even if wage rates differ across individuals so long as individuals
have the same preferences. However, Muller [60] has also established that
there are more general preference structures that satisfy this condition than (16),
which suggests some interesting possibilities for future work. Third, equations
(17) and (18) treat hours of work as a single aggregate and ignore the presence
of several family members. This choice might be justified on the grounds that as
the male/female wage ratio has changed very little over the last twenty-five years
\( h \) is to be thought of as a Hicksian composite good, but, in fact, it actually
reflects data availability.
over time are to be related to the presence of the two regimes (17) and (18). In most of the literature using the disequilibrium concept for aggregate markets it is assumed that at a particular time period the entire market is either at an equilibrium or not, and the econometric problem is then essentially one of switching regressions. 18/ Information on the nature and extent of rationing is used only to establish the nature of the prevailing regime.

Although this approach may be satisfactory in the study of commodity markets it does not seem very well suited to the analysis of unemployment in the labor market. For the unique characteristic of unemployment in market economies is its uneven incidence, with the result that even in periods of considerable measured unemployment a large fraction of all workers do not appear to suffer whatever from any constraints on their ability to sell hours on the labor market. The result is that the aggregate behavior of hours at work in any period appears to be a mixture of some consumer-workers who are governed by the unconstrained regime (17) and some consumer-workers who are governed by the constrained regime (18). Moreover, the unemployment rate is a statistic that is meant, at least in principle, to be related to the fraction of consumer-workers who fall into the constrained regime. This suggests that it may be useful to use the measured unemployment rate as exogenous information relevant to the actual mixing of equations (17) and (18) in the aggregate data.

In order to proceed it is worth observing that in a steady-state the measured unemployment rate \( u \) is simply the product of the mean proportional duration of a completed spell of unemployment and the probability \( s \) of suffering a spell of unemployment, so that

18/ This problem was introduced by Richard Quandt [33] and its application to the labor market is studied by Rosen and Quandt [35].
(19) \( u = D \cdot s^{19} \)

Since \( s \) is essentially a measure of the fraction of consumer-workers whose behavior is described by (18) while \((1-s)\) is the fraction of consumer-workers whose behavior is described by (17), aggregate earnings and commodity expenditures may be written as

\[
(20a) \quad wh = s(wh^c) + (1-s)(wh^e)
\]

\[
(20b) \quad p_i x_i = s(p_i x_i^c) + (1-s)(p_i x_i^e) \quad (i = 2, \ldots, m+n)
\]

Substituting (17) and (18) into (20) and using (19) then gives

\[
(21a) \quad wh = y_1w - B_1[y+y_1w-Sy_jP_j] - [y_1w-B_1(y+y_1w-Sy_jP_j)]u
\]

\[
(21b) \quad p_i x_i = y_1p_i + B_1[y+y_1w-Sy_jP_j] - B_1^*[y_1w-B_1(y+y_1w-Sy_jP_j)]u \quad (i = 2, \ldots, m+n).
\]

Although (21) contains additional nonlinearity in the parameters over (17) because of the relationship between the parameters \( B_1 \) and \( B_1^* = B_1/(1-B) \), it does not depend on \( D \) and \( s \) separately, for which there do not exist separate data, but only on their conventionally measured product. This is a remarkable simplification and makes the empirical task considerably more tractable.

Of course, the unemployment proportion \( u \) that appears in (21) is not conceptually identical to the unemployment proportion \( u^* \) that is measured directly. First, there are technical differences. The measured unemployment rate counts as unemployed only those workers who have no employment whatever in a survey week, so that unless all hours of work adjustments are in the dimension of weeks at work \( u^* \) will differ in principle from \( u \). Of course, there is also the problem that observed unemployment rates generally do not represent steady

\[19\text{See Kaitz [19] and Salant [36] for useful discussions of the algebra of stocks and flows in the measurement of unemployment.}\]
states so that (19) and hence (21) will be only approximately accurate. Second, there is the problem that some workers may not report themselves as unemployed in response to the conventional survey question because in response to the constraints they face they do not report themselves as "active" labor force participants. In both of these cases the measured unemployment rate \( u^* \) is less than \( u \). Finally, some of those workers who report themselves as unemployed may not actually face any constraint on their hours of employment at all and may have simply classed some part of their chosen hours of non-market time as unemployment. One way this might happen is if those workers unemployed during a part of the year were able to find additional hours during a different part of the year to fully compensate for their unemployment spell, but there are surely many other ways this might happen as well. In these cases \( u^* \) will be greater than \( u \). All of these arguments suggest that it is desirable to test whether the observed measure of unemployment \( u^* \) behaves in (21) as \( u \) would be expected to behave. A simple way to do this is to write

\[
(22) \quad u = au^*
\]

and substitute into (21) prior to estimation. If in the estimates \( a = 0 \), then the conventionally measured unemployment proportion does not correspond to a measure of the constraints workers face, whereas \( a = 1 \) suggests that \( u^* \) is a good measure of those constraints. Of course, (22) is a very simple relationship, so it also seemed useful to replace it with

\[
(23) \quad u = a_0u^* + a_1(u^*)^2.
\]

Since \( u/u^* = a_0 + a_1u^* \), it follows that \( a_0 \) reflects the fraction of measured unemployment that represents true constraints on hours of work at low levels of \( u^* \) and that \( a_1 > 0 \) indicates that this fraction increases with \( u^* \).
C. Empirical Results

Tables 2 and 3 contain the maximum likelihood estimates of the parameters of equations (21a) and (21b) and the related statistics under various specifications of the error terms added to these equations. The period of fit is 1930-1967 and the basic data are the corrected version of the data used by Abbott and Ashenfelter [1], who describe their construction in detail. As often occurs with such data the fitted residuals of equations (21) initially indicated considerable serial correlation. As a result the disturbances appended to equations (21) were assumed to follow a first-order autoregressive with the same parameter \( \rho \) in each equation. Since the fitted parameter \( \rho \) was often near unity, the results in Table 2 were also obtained under this assumption, which amounts to fitting these equations in first-differences. Unfortunately, as can be seen from Table 2, the method used to deal with serial correlation affects somewhat the estimates of the parameters \( a \) and \( a_i \) in (22) and (23), although it had little effect on the \( B_i \) and \( \gamma_i \) parameters in (21). As a result the estimates of the \( a_i \) parameters are reported in Table 2 for every specification fitted, but only the three sets of estimates of \( B_i \) and \( \gamma_i \) that correspond to the cases where \( \rho \) is taken to be a free parameter are reported in Table 3. It is straightforward to verify that equations (21) satisfy the budget adding up condition exactly, and this implies that one equation may be dropped

20/ These computations were made using an extremely efficient scheme devised by John Ham [16], to whom I am indebted for assistance.

21/ The errors in the original data series were confined to four of the seven commodity price series, and the corrected data are available from the author in the form of a processed appendix. Corrected versions of the original computations are available in Abbott and Ashenfelter [2]. The unemployment data are from Lebergott [22].
## Table 2

Estimated Parameters and Summary Statistics for Testing for the Presence of Unemployment in Aggregate Commodity Demand and Labor Supply Functions

<table>
<thead>
<tr>
<th>Row Number</th>
<th>Twice the Logarithmic Likelihood $\mathcal{L}^/$</th>
<th>Estimates (and Estimated Standard Errors) of:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>72.8</td>
<td>$\hat{a}$</td>
<td>$\hat{a}_0$</td>
</tr>
<tr>
<td>2</td>
<td>81.6</td>
<td>0</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>79.1</td>
<td>.477 (.187)</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>85.6</td>
<td>.358 (.176)</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>83.8</td>
<td>-- .265 (.204)</td>
<td>2.23 (1.03)</td>
</tr>
<tr>
<td>6</td>
<td>86.9</td>
<td>-- .233 (.210)</td>
<td>1.14 (1.03)</td>
</tr>
</tbody>
</table>

$\mathcal{L}^/$: The tabulated critical values for the chi-square statistic with one and two degrees of freedom at the .05 level are 3.84 and 5.99, respectively.
<table>
<thead>
<tr>
<th>Commodity Group</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>$R^2$</th>
<th>Durbin-Watson Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durables</td>
<td>0.027</td>
<td>0.219</td>
<td>-0.004</td>
<td>0.239</td>
<td>-0.023</td>
</tr>
<tr>
<td></td>
<td>(0.384)</td>
<td>(0.027)</td>
<td>(0.389)</td>
<td>(0.029)</td>
<td>(0.400)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.029)</td>
</tr>
<tr>
<td>Food</td>
<td>0.893</td>
<td>0.167</td>
<td>0.905</td>
<td>0.174</td>
<td>0.878</td>
</tr>
<tr>
<td></td>
<td>(0.117)</td>
<td>(0.117)</td>
<td>(0.124)</td>
<td>(0.019)</td>
<td>(0.130)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.019)</td>
</tr>
<tr>
<td>Clothing</td>
<td>0.557</td>
<td>0.064</td>
<td>0.553</td>
<td>0.069</td>
<td>0.543</td>
</tr>
<tr>
<td></td>
<td>(0.093)</td>
<td>(0.008)</td>
<td>(0.095)</td>
<td>(0.008)</td>
<td>(0.098)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.009)</td>
</tr>
<tr>
<td>Other Nondurables</td>
<td>0.997</td>
<td>0.108</td>
<td>1.022</td>
<td>0.113</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>(0.148)</td>
<td>(0.006)</td>
<td>(0.140)</td>
<td>(0.007)</td>
<td>(0.151)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.007)</td>
</tr>
<tr>
<td>Housing</td>
<td>1.09</td>
<td>0.159</td>
<td>1.135</td>
<td>0.165</td>
<td>1.120</td>
</tr>
<tr>
<td></td>
<td>(0.190)</td>
<td>(0.015)</td>
<td>(0.190)</td>
<td>(0.016)</td>
<td>(0.186)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.017)</td>
</tr>
<tr>
<td>Transport</td>
<td>0.884</td>
<td>0.020</td>
<td>0.881</td>
<td>0.021</td>
<td>0.881</td>
</tr>
<tr>
<td></td>
<td>(0.197)</td>
<td>(0.004)</td>
<td>(0.197)</td>
<td>(0.004)</td>
<td>(0.204)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.004)</td>
</tr>
<tr>
<td>Other Services</td>
<td>0.594</td>
<td>0.143</td>
<td>0.607</td>
<td>0.150</td>
<td>0.588</td>
</tr>
<tr>
<td></td>
<td>(0.133)</td>
<td>(0.016)</td>
<td>(0.133)</td>
<td>(0.017)</td>
<td>(0.134)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.018)</td>
</tr>
<tr>
<td>Labor Supply</td>
<td>2356.</td>
<td>0.120</td>
<td>2263.</td>
<td>0.069</td>
<td>2230.</td>
</tr>
<tr>
<td></td>
<td>(79.8)</td>
<td>(0.030)</td>
<td>(83.6)</td>
<td>(0.037)</td>
<td>(96.7)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.039)</td>
</tr>
</tbody>
</table>

*These statistics refer only to the model based on the results in row 6 of Table 2, treat each equation as if it were separate, and are based on the quasi-differences in the left-hand variables.*
arbitrarily for purposes of estimation. Equation (21a) was therefore deleted, but it should be understood that the estimates in Tables 2 and 3 are invariant to this choice.

The results in these tables suggest several conclusions. First, incorporating measured unemployment into the explanation of aggregate commodity demands does significantly increase the value of the maximized likelihood. In the simplest case, using specification (22), this amounts to a test of whether \( a \) is significantly different from zero. Both the normal statistic associated with this coefficient or a comparison of twice the (logarithmic) likelihood in rows (3) or (4) against rows (1) or (2) imply the rejection of this hypothesis at conventional test levels. Using the specification in (23) requires a comparison of likelihood values and also rejects the joint hypothesis \( a_o = a_1 = 0 \), but only at the .1 level in row (6). This suggests that the constraints on hours of work faced by consumer-workers may be of both theoretical and empirical interest.

Second, although this implies that both \( a \) and \( a_o \) are not zero, the estimates of these parameters are closer to zero than to unity. Taking the estimate of \( a \) from row (4) of Table 2, for example, suggests that only about 36 percent of those hours measured as unemployed actually represent constraints on consumer-workers' choices. Despite the imprecision in the estimates the results in rows (5) and (6) of Table 2 clearly imply that \( u/u^* \) is an increasing function of \( u^* \). Thus, the results in row (6) imply that in periods of low measured unemployment perhaps thirty percent of measured unemployment represents constraints on choice, while in periods of high unemployment, as in the Depression of the Thirties when measured unemployment reached twenty-five percent, perhaps one-half of measured unemployment represents constraints on choice. As should also be clear from Table 2 these results are not entirely independent of the way in which
the autocorrelation in the disturbances of equations (21) is handled. Moreover, measurement error in $u^*$ would presumably bias downward the estimates of $\alpha$ and $a_0$, although this would also imply that measured unemployment is a poor indicator of the actual extent of constraints in the labor market.

Third, as can be seen from Table 3 the estimates of the parameters in the labor earnings function (21a) change systematically when unemployment is introduced. In particular, both $\gamma_1$ and the marginal propensity to consume leisure $(B_1)$ are decreased, which implies an unconstrained labor supply function that is generally less responsive to economic forces. On the other hand, equations (21) clearly imply that in the aggregate all income and price elasticities will change systematically as the state of the labor market changes. For example, in the aggregate the marginal propensity to consume the $i^{th}$ commodity is

$$\frac{\partial p_i x_i}{\partial y} = \frac{B_i (1-B_1) + B_1 u_0}{1-B_1},$$

and equals $B_i$ when $u = 0$, but approaches $B_1/(1-B_1)$ as $u$ approaches unity. Likewise, the marginal propensity to consume non-market time is

$$\frac{\partial (w h)}{\partial y} = B_1 - B_1 u,$$

and equals $B_1$ when $u = 0$, but approaches zero as $u$ approaches unity. Nevertheless, the estimates of the relationship between $u$ and $u^*$ contained in Table 2 imply that the sample period estimates of (24) and (25) and the comparable expressions for the price and wage rate elasticities differ very little apart from the years of the 1930's.

III. Conclusion

Treating the presence of unemployment as a constraint on choice rather than the result of it has a long theoretical tradition. The results in this paper
suggest that this approach may also be of empirical interest in the study of labor supply and commodity demand behavior. Apart from providing a simple treatment of this behavior that is consistent in principle with both the Keynesian and classical approaches, there is clear empirical evidence that actual aggregate demand behavior represents a combination of both the former and the latter. The estimates here of critical parameters must nevertheless be treated with some caution, both because of the weakness of the data and the simplicity of the preference and aggregation structures that have been imposed on the data. There consequently remains a large agenda for further research.

First, it should be clear that most of the normative issues involved in the discussion of unemployment have been left unanswered. In particular, since the causes of the constraints on choice that unemployment is taken to represent have not been specified, there is no remedy for the removal of these constraints implied by the analysis. Moreover, to the extent that movements in unemployment represent temporary layoffs that result from optimal employment contracts it is possible that inter-industry mobility results in compensating wage differentials or unemployment benefit schemes that are also optimal. In either case the estimation of labor supply and commodity demand functions requires a resort to the methods used here, but the normative implications for government policies may be vastly different. Further work on these issues would clearly be very desirable.

Second, a variety of empirical issues remain open. For one thing, it would be very useful to expand the generality of both the functional form and the approach to choice of functional form that have been adopted. The additive utility used here removes many problems of estimation, but it also removes all but the income effects of unemployment from the constrained worker's demand functions, and this should be remedied. Likewise, even the simple model of
equations (21) has implications for savings behavior that might usefully be explored.

Finally, the presence of constraints on labor supply can never be entirely ignored in the empirical estimation of labor supply functions. If nothing else, this issue must be treated in order to define the appropriate measurement of labor supply. It is time for this issue to be faced directly and explicitly in studies of both cross-sectional and experimental data.22/

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22/ For example, in the earliest cross-sectional studies, such as Mincer's [28], labor supply is defined to include the sum of employment and unemployment, which implies that unemployment results from a constraint on choice and should be counted as part of the offer to supply labor. In later studies, such as Ashenfelter and Heckman's [5], labor supply is defined to include only employment, which implies the contrary. The results of a recent effort to study this issue empirically are reported by John Ham [17].
REFERENCES


