NON-LINEARITIES IN THE LABOR MARKET
ADJUSTMENT PROCESS

by

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It is a now rather widely accepted view that the nature of the inter-
relationships between various labor market variables depends on the degree of
tightness of the labor market. Decisions concerning labor mobility, for
example, depend upon whether there are or are not vacancies in the more
desirable sectors. Formal wage theory, however, has heretofore rarely taken
the problems associated with less-than-full employment into account. The
purpose of this paper, therefore, is to present some rather simple models of
labor mobility, relative wage determination, and their interaction, with par-
ticular emphasis on the differences in behavior between greater-than-full, full,
and less-than-full employment situations. In order to isolate these problems,
it is assumed throughout that all individuals in a particular labor market — a
region, industry, or occupation — are homogeneous with respect to ability and
that the aggregate supply of labor does not change over the time interval under
consideration. Sections I and II derive equations for intermarket supply adjust-
ment under two alternative assumptions concerning the nature of labor turnover,
and Section III explores some of the implications of different degrees of tight-
ness of the aggregate labor market for the behavior of relative wage levels over
time by means of a two-sector, linear model. In Section IV the approach is
generalized (somewhat cautiously) to the case of N markets.

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ing many of the ideas which are developed in the paper.
I. Behavioral Foundations

Consider an individual who is located in the \( j \)th of two labor markets in the economy. Let \( Q_j(t) \) be the probability that he will be employed in that market in time period \( t \), where \( 0 \leq Q_j(t) \leq 1 \), \( \phi_j \) be the probability he will become unemployed in period \( t \) given that he is employed in period \( t-1 \), and \( \psi_j \) be the probability he will become employed in period \( t \) given that he is unemployed in period \( t-1 \). [Thus, the conditional probabilities of remaining employed and remaining unemployed are \( 1 - \phi_j \) and \( 1 - \psi_j \) respectively.]

Assuming that \( \phi_j \) and \( \psi_j \) remain constant over time, the change from \( t-1 \) to \( t \) in the probability that the individual will be employed is given by

\[
Q_j(t) - Q_j(t-1) = -\phi_j Q_j(t-1) + \psi_j (1-Q_j(t-1))
\]

The solution of this difference equation is

\[
Q_j(t) = Q_j^* \left[ \frac{\psi_j}{\psi_j - \phi_j} \right] (1-\phi_j)^t + \frac{\psi_j}{\psi_j - \phi_j} Q_j(0) (1-\phi_j)^t
\]

where \( Q_j^* = \frac{\psi_j}{\phi_j + \psi_j} \). \( Q_j(t) \) goes asymptotically to \( Q_j^* \) if \( -1 < (1-\phi_j - \psi_j) < 1 \), and since both \( \phi_j \) and \( \psi_j \) cannot exceed unity in absolute value by definition, the stability conditions are not satisfied only when \( \phi_j \) and \( \psi_j \) are either both zero (the individual always maintains his employment status of last period) or both one (the individual changes his employment status in each successive period).

The present value of the individual's expected income stream at the beginning of period \( t \) is

\[
P_j(l) = \sum_{t=1}^{T+1} Y_j(t)(1+r)^{-(t-1)}
\]
where $Y_j(t)$ is his expected income in period $t$, $r$ is some positive discount rate, and $T$ is the number of years he expects to remain in the labor force.

Expected income is defined as

$$Y_j(t) = W_j Q_j(t) H,$$  

where $W_j$ is the money wage rate in the $j^{th}$ market and $H$ is the normal number of hours of work per period. Substituting (1.2) for $Q_j(t)$ in (1.4) and the result for $Y_j(t)$ in (1.3) gives

$$P_j(l) = W_j H \left\{ Q_j \sum_{t=1}^{T+1} \left[ (1+r)^{-(t-1)} \right. \right. \left. \left. - \left( \frac{1-\phi_j}{1+r} \right) \frac{\psi_j}{(1-\phi_j)} \right] + Q_j(0) \sum_{t=1}^{T+1} \left( \frac{1-\phi_j}{1+r} \right) \frac{\psi_j}{(1-\phi_j)} \right\},$$  

which, as $T$ gets large, becomes

$$P_j(l) = W_j H \left[ \frac{1+r}{(x+\phi_j^*+\psi_j^*)^r} + Q_j(0)(1-\phi_j) \right].$$

The same individual also evaluates the present value of the income stream associated with the other, the $k^{th}$ labor market. Since the individual was located in market $j$ in the initial period, $Q_k(0) = 0$, and

$$P_k(l) = W_k H \left[ \frac{(1+r)^2 \psi_k}{(r+\phi_k^*+\psi_k^*)^r} \right].$$

The individual desires to move from the $j^{th}$ to the $k^{th}$ market when

$$P_k(l) > P_j(l) + M_{j,k},$$

where $M_{j,k}$ is the out-of-pocket cost of moving from $j$ to $k$. By substituting (1.6) and (1.7), this condition becomes

$$W_k \left[ \frac{(1+r)^2 \psi_k}{(r+\phi_k^*+\psi_k^*)^r} \right] > W_j \left[ \frac{(1+r)^2 \psi_j}{(r+\phi_j^*+\psi_j^*)^r} \right] + Q_j(0)(1-\phi_j) \psi_j.$$

It is useful to define a new variable, $\Delta$, as the Propensity to Move coefficient, where
\[ \Delta = \frac{P_k(l)}{P_j(l) + M_{j,k}} , \]

which, upon substitution of (1.6) and (1.7), is seen to be

\[ \Delta = \left( \frac{W_k}{W_j} \right) \frac{1 + \frac{1}{r} \frac{\psi_k + \psi_j}{r}}{\frac{1}{r} \frac{\psi_j + \psi_j}{r} + \frac{1}{r} \frac{\psi_j + \psi_j}{r}} \left( 1 - \frac{M_{j,k}}{P_j + M_{j,k}} \right) , \]

so long as \( P_j > 0 \).

In the following section it will be argued that the actual movement into or out of each of the markets depends upon \( \Delta \), but it is first necessary to relate the \( \phi \)'s and \( \psi \)'s to conceptually observable variables. In a "structureless" labor market, \( ^4 \) such as the markets for agricultural and casual labor, the probability of being employed in period \( t \) is independent of the individual's employment status in period \( t-1 \), so \( \phi_j = 1 - \psi_j \). In this case (1.9) reduces to

\[ \Delta = \left( \frac{W_k}{W_j} \right) \frac{\psi_k}{\psi_j} \left( 1 - \frac{M_{j,k}}{P_j + M_{j,k}} \right) , \]

where \( \psi_k \) and \( \psi_j \) are the probabilities that the individual can obtain employment in a given period if he locates in \( k \) and \( j \) respectively at the beginning of the period. The values of these parameters will depend upon the state of labor demand relative to supply in each particular market. Define \( X_j = D_j/S_j \) (the ratio of labor demand to labor supply in the \( j^{th} \) market) as the rate of excess demand for labor in the \( j^{th} \) market. \( \psi_j \) may be considered a function of \( X_j \), say

\[ \psi_j = h(X_j) , \]

where \( h' > 0 \). The shape of this relation is shown in Figure 1. For rates of excess demand considerably less than unity, there is a one-to-one
correspondence between \( \psi_j \) and \( X_j \) for unemployment is so considerable that all available job vacancies are quickly filled. However, as \( X_j \) gets very close to unity, some vacancies will remain unfilled because of frictional problems, and at \( X_j = 1 \) the probability an individual will find employment in a particular period is close to but less than one. As \( X_j \) gets very large, the chance of not obtaining employment in a particular period will become very small, and \( \psi_j \) goes to unity asymptotically. Substituting (1.10) for \( \psi_j \) and \( \psi_k \), the Propensity to Move Coefficient for structureless labor markets becomes simply

\[
\Delta = \left( \frac{W_k}{W_j} \right) \left( \frac{h(X_j)}{h(X_k)} \right) \left( 1 - \frac{M_{j,k}}{P_j + M_{j,k}} \right). 
\]

The majority of labor markets, however, are not featured by such easy entry. In the typical situation an individual may retain his position with a particular firm from period to period if he chooses, so, ignoring layoffs, \( \phi_j = 0 \). An individual who is not employed in a market in period \( t-1 \) has little or no chance of obtaining employment in that market unless some firms have vacancies, i.e., unless \( X_j > 1 \). In its most extreme form this means that

\[
\psi_j = \begin{cases} 
1, & \text{if } X_j > 1 \\
0, & \text{if } X_j \leq 1 
\end{cases}
\]

which is, of course, quite different from the turnover assumption represented by (1.10). This leads to four separate cases of the Propensity to Move Coefficient, and these, which are discerned from inspection of (1.9), are presented in Table 1. When \( X_k < 0 \), there is no incentive to move from \( j \) to \( k \), for expected income in \( k \) is zero. The expression \( 1 - \frac{M_{j,k}}{P_j + M_{j,k}} \) is
<table>
<thead>
<tr>
<th>$X_j$</th>
<th>$X_k$</th>
<th>$&gt;1$</th>
<th>$&lt;1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&gt;1$</td>
<td>$\frac{W_k}{W_j} \left( 1 - \frac{M_{j,k}}{F_j + M_{j,k}} \right)$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\leq 1$</td>
<td>$\frac{W_k}{W_j} \left( 1 - \frac{M_{j,k}}{F_j + M_{j,k}} \right)$, $Q_j(0) = 1$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\frac{W_k(1+r)}{r M_{j,k}}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1

important because it determines the relation between $\Delta$ and the wage differential. When both $X_j$ and $X_k$ exceed unity, the expression becomes

$$1 - \frac{M_{j,k}}{W_k H \left( \frac{1+r}{r} \right) + M_{j,k}}$$

and it is closer to unity the larger is $W_k$ and the smaller are $M_{j,k}$ and $r$.

II. A Linear Model of Inter-market Supply Adjustment

The analysis of individual preferences and the two alternative specifications of labor turnover will now be used to construct a linear model of supply adjustment between the two markets. Since we are examining the phenomenon of inter-market adjustment and are not concerned with choices by new entrants to the labor force, we shall assume that there are $S_j$ individuals in the economy who work in either market $j$ or market $k$ ($S_j = S_j + S_k$). Now $\Delta$ is an index of the desirability of moving from $j$ to $k$ relative to remaining in $j$ for an individual who is initially located in $j$. It is plausible to suppose,
therefore, that the fraction at some point in time who will decide to move
depends positively on $\Delta$ for $\Delta > 1$. Similarly, for individuals who are
located in $k$ there is a similar index which will determine how many indi-
viduals will choose to flow to $j$. Letting the index for individuals in $j$ be
given by $\Delta_j$ and the equivalent for individuals in $k$ by $\Delta_k$, we can thus
say that

\[
\hat{\delta}_k = \begin{cases} 
  g(\Delta_j), & \Delta_j > 1 \\
  -g(\Delta_k), & \Delta_k > 1 \\
  0, & \text{both } \Delta_j, \Delta_k \leq 1,
\end{cases}
\]

(2.1)

where $g' > 0$. Given that $M_{j,k}$ and $M_{k,j}$ are both positive, there are some
combinations of wage rates and/or rates of excess demand in which
$P_{j}(l) \neq P_{k}(l)$ but movement from the less desirable market is not profitable.
Having made this point, however, we shall now assume that out-of-pocket
movement costs are negligible, so it is approximately true that $\Delta_k = \Delta_j^{-1}$.

For structureless labor markets the Propensity to Move Coefficient for
individuals in $j$ was seen by (1.11) to be

\[
\Delta_j = \frac{w_k}{w_j} \frac{h(x_k)}{h(x_j)}.
\]

(2.2)

The combinations of $\frac{w_k}{w_j}$ and $\frac{x_k}{x_j}$ for which $\Delta_j = 1$ are shown in Figure 2
below. When the values of both $x_j$ and $x_k$ are such that the relation between
the chance of being employed and the rate of excess demand is linear (see
Figure 1), the combinations of $\frac{w_k}{w_j}$ and $\frac{x_k}{x_j}$ for which $\Delta_j = \Delta_k = 1$ is
represented by a rectangular hyperbola (Curve A). When, however, both
$x_j$ and $x_k$ are well in excess of unity, both $h(x_j)$ and $h(x_k)$ are close to
unity, and the line representing indifference between markets becomes very steep (Curve B). Of course, all combinations of \( \frac{W_k}{W_j} \) and \( \frac{X_k}{X_j} \) to the right of the boundary imply \( \hat{S}_k > 0 \); those to the left \( \hat{S}_k < 0 \).

Substitution of (2.2) into (2.1) gives the supply adjustment function for market \( k \),

\[
\dot{S}_k = \begin{cases} 
-g \left( \frac{W_j h(X_j)}{W_k h(X_k)} \right), & \Delta_j < 1 \\
g \left( \frac{W_k h(X_k)}{W_j h(X_j)} \right), & \Delta_k > 1 
\end{cases}
\]

and, since \( S_\Sigma \) is fixed by hypothesis, \( \dot{S}_j = -\dot{S}_k \). Later we will require solutions of systems of equations for which the supply adjustment function is an important part, it is necessary to take a linear approximation of (2.3), i.e.,

\[
\dot{S}_k = aw + bv
\]

where \( w = W_k - W_j \) is the absolute wage differential and \( v = V_k - V_j \) \( \equiv (D_k - S_k) - (D_j - S_j) \) is the excess demand differential, the difference between the levels of excess demand for labor in \( k \) and \( j \). \( a \) and \( b \) are considered to be positive constants, but the clear implication of Figure 2 is that the size of \( b \) depends on the level of aggregate excess demand for labor,

\[
V_\Sigma = D_\Sigma - S_\Sigma \equiv V_j + V_k.
\]

As \( V_\Sigma \) gets very large, \( b \) should diminish to zero.

Because of the rather subtle nature of the labor turnover process, supply adjustment for structured markets cannot be represented by a single equation. It is necessary to distinguish between the nine cases for which \( V_j \) and \( V_k \) are each positive, zero, or negative, and we shall identify each case by a \( C \) with subscripts from Table 2. By inserting the Propensity to Move Coefficients from Table 1 into (2.1) and following a procedure similar to the derivation of
(2.4), we get equations for \( \hat{S}_k \) for each of the nine cases, and these are shown in Table 3 below. For the cases for which both levels of excess demand are

\[
\begin{array}{c|c|c|c}
V_k & + & 0 & - \\
+ & \alpha w & \alpha w, w<0 & \beta v \\
0 & 0, w>0 & 0 & 0 \\
\end{array}
\]

Table 3

negative, \((C_{12}, C_{23}, C_{32}, \text{ and } C_{33})\), there is no incentive for individuals to move out of or into either market, so \( \hat{S}_j = \hat{S}_k = 0 \). For \( C_{31} \), there are vacancies in \( k \) and unemployed workers in \( j \). As long as this situation lasts, \( \Delta_j \) is very large (infinitely large if \( M_{j,k} = 0 \)), and \( \hat{S}_k \) may be considered a function of the difference between the number of vacancies in \( k \) and \( j \).

Further, to the extent that \( \Delta_j \) is very large, it is quite likely that \( \beta \) will be
very large due to the great advantage of moving to $k$. The explanation for $C_{13}$, of course, is analogous to that for $C_{31}$. At $C_{11}$ both $V_j$ and $V_k$ are positive, so the incentive to move depends only on the wage differential, there being no chance of suffering unemployment after a move. Finally, at $C_{21}$ individuals will be attracted to $k$ only if $W_k > W_j$, and the rate at which individuals respond to a positive wage differential will be the same as in $C_{11}$. If $W_k < W_j$, there is no incentive for individuals to move to market $k$. A similar argument explains the equation for $C_{21}$.

III. Wage, Demand, and Supply Adjustment in a Two-Sector Model

In the static treatment of the equilibrium of these two markets, both $V_j$ and $V_k$ are set equal to zero, and, since each individual prefers more to less income, $W_j$ and $W_k$ must be identical. Given $S_{Z^*}$, the value of $W_j$ and $W_k$ is then determined by the positions and shapes of the two labor demand functions. Given a shift to the right of the demand for labor function in, say, market $k$, the value of $W_j$ and $W_k$ will increase, but it is still true that $V_k = V_j = 0$. If, however, the process of adjustment is considered explicitly, there is a question of whether the variables will ever reach equilibrium.

It is necessary to make assumptions concerning the determination of wage rates and demand levels. Initially we assume that the change in the money wage rate with respect to time in each market is a homogeneous function of the level of excess demand in that market, that is

$$\dot{W}_j = \sigma V_j.$$  \hspace{1cm} (3.1)

This implies that the absolute wage differential changes at a rate

$$\dot{W} = W_k - \dot{W}_j = \sigma v.$$  \hspace{1cm} (3.2)
The initial assumption concerning demand changes is simply that the change in the level of demand in each market with respect to time is a negative multiple of the change in the money wage rate, that is

\[ \dot{D}_j = -\rho \dot{W}_j, \]

so, by substitution of (3.1) into (3.3), we get

\[ \dot{D}_j = -\rho \sigma V_j. \]

The change in the level of aggregate demand for labor is \( \dot{D}_\Sigma = \dot{D}_j + \dot{D}_k = -\rho \sigma V_\Sigma \), so, since \( S_\Sigma \) is fixed, the change in the aggregate level of excess demand for labor is

\[ \dot{V}_\Sigma = -\rho \sigma V_\Sigma. \]

The solution of this equation is

\[ V_\Sigma = V_\Sigma^0 e^{-\rho \sigma t}, \]

where \( V_\Sigma^0 \) is the level of \( V_\Sigma \) in some initial time period. Thus, \( V_\Sigma \to 0 \) as \( t \to \infty \).

The time derivative of the excess demand differential is

\[ \dot{V} = (\dot{D}_k - \dot{D}_j) - (\dot{S}_k - \dot{S}_j) = -\rho \sigma V - 2\hat{S}_k. \]

For a pair of structureless markets this becomes, upon substitution of (2.4) for \( \dot{S}_k \) in (3.7),

\[ \dot{V} = -(\rho \sigma + 2b)V - 2aw. \]

This equation and (3.2) form a system of two linear differential equations in two variables, and their roots are \( m = -\lambda + \mu \), where \( \lambda = \frac{\partial \sigma}{\partial \Sigma} + b \) and \( \mu = (\lambda^2 - 2a\sigma)^{1/2} \). The system is stable if \( \rho, \sigma, \) and \( a \) are positive (since \( b > 0 \) by hypothesis), and the form of the solution depends on whether the roots
are: (i) real and unequal, $\lambda^2 > 2a\sigma$; (ii) real and equal, $\lambda^2 = 2a\sigma$; or (iii) complex, $\lambda^2 < 2a\sigma$. The solutions equations for each case are:

(i) $v = A_{11} e^{(-\lambda+\mu)t} + A_{12} e^{(-\lambda-\mu)t}$  
$w = A_{21} e^{(-\lambda+\mu)t} + A_{22} e^{(-\lambda-\mu)t}$

(3.9) (ii) $v = e^{-\lambda t} [A_{11} + A_{12} t]$  
$w = e^{-\lambda t} [A_{21} + A_{22} t]$  

(iii) $v = e^{-\lambda t} [A_{11} \cos \theta t + A_{12} \sin \theta t]$  
$w = e^{-\lambda t} [A_{21} \cos \theta t + A_{22} \sin \theta t]$ ,

where $\theta = (2a\sigma - \lambda^2)$ and the $A$s are arbitrary constants depending upon the initial values of $v$ and $w$. For cases (i) and (ii), both $v$ and $w$ approach zero either without changing signs or with a single change of sign apiece depending upon the values of the constants. For (iii), both $v$ and $w$ approach zero in an oscillatory manner. The existence of cobwebs (and I know of no examples of these in labor market behavior) requires that $2\sigma(a - \rho b) - \frac{\rho^2 \sigma^2}{4} - b^2 > 0$.

Thus, the greater are $\rho$ and $b$ and the smaller is $a$ the less likely that $v$ and $w$ will oscillate. (The effect of $\sigma$ on $\mu$ is ambiguous.)

An interesting question concerns the quantitative effect of the size of the parameters on the speed of adjustment of $v$ and $w$. For cases (ii) and (iii) the speed of adjustment is determined by $\lambda$ — the greater is $\lambda$ the more quickly the variables go to zero. Hence, the greater are $\rho$, $\sigma$, and $b$, the greater the speed of adjustment. For case (i), the speed of adjustment is determined by the size of the larger root, $-\lambda + \mu$, and this is the smaller (implying faster adjustment) the greater are $\rho$, $b$, and $a$. (The effect of $\sigma$ in this case depends on the relative values of the other parameters.) In any event, for structureless markets both $v$ and $w$ go to zero in the long run.
The analysis of the dynamic behavior of \( v \) and \( w \) for structured markets is not so straightforward, for the specification of the supply adjustment equation depends on the signs of \( V_j \) and \( V_k \). By substituting the expressions in Table 3 for \( \dot{S}_k \) into (3.7), we get the equations for \( \dot{v} \) in each of the nine cases, and these are shown in Table 4 below.

\[
\begin{array}{c|c|c}
\ & + & - \\
\hline
+ & -\rho \sigma v - 2\sigma w & -\rho \sigma v - 2\sigma w, w < 0 \\
& & 0 , w \geq 0 \\
V_j & -\rho \sigma v - 2\sigma w, w > 0 & -\rho \sigma v \\
& & 0 , w \leq 0 \\
\hline
- & -(\rho \sigma + 2\beta)v & -\rho \sigma v \\
\hline
\end{array}
\]

Table 4

The second equation of the system is given by (3.2) in all nine cases, for the wage determination process has been assumed to take the same form in situations of positive and negative excess demand.

In the four cases for which both \( V_j \) and \( V_k \) are not positive (\( C_{22}, C_{23}, C_{32}, \) and \( C_{33} \)), the system is of the recursive form

\[
\dot{v} = -\rho \sigma v \\
\dot{w} = \sigma v .
\]

Hence,

\[
(3.10) \quad v = v_0 e^{-\rho \sigma t}
\]
and

\[ w = w^0 + \frac{\nu^0}{\rho} \left(1 - e^{-\rho \sigma t}\right), \]

where \( w^0 \) and \( \nu^0 \) are the initial values of the wage and excess demand differentials. The system never moves from this set of cases to any of the other five, for neither \( \nu_j \) nor \( \nu_k \) ever becomes positive. Since \( \nu_{\Sigma} = \nu_j + \nu_k \) and \( \nu = \nu_k - \nu_j \), \( \nu_k = \frac{1}{2} (\nu_{\Sigma} + \nu) \) and \( \nu_j = \frac{1}{2} (\nu_{\Sigma} - \nu) \). Thus, by (3.6) and (3.11),

\[ V_j = V_j^0 e^{-\rho \sigma t} \quad \text{and} \quad V_k = V_k^0 e^{-\rho \sigma t}, \]

so both \( \nu_j \) and \( \nu_k \) never exceed zero. Now \( \nu \rightarrow (w^0 + \nu^0/\rho) \) and \( t \rightarrow \infty \), and in the limit \( w - w^0 \rightarrow 0 \) as \( \nu^0 \rightarrow 0 \).

However, unlike the case of the structureless markets, there is no automatic tendency for \( w \) to go to zero — even though, given the strictly neoclassical assumption concerning wage movements, both \( \nu_j \) and \( \nu_k \) (and hence \( \nu_{\Sigma} \)) go to zero with time.

When \( \nu_k > 0 \) and \( \nu_j < 0 \) (C31) the system becomes

\[ \dot{\nu} = -((\rho \sigma + 2\beta) \nu \quad \dot{w} = \sigma \nu, \]

and, as long as \( \nu_k \) and \( \nu_j \) remain positive and negative respectively, we get

\[ \nu = \nu^0 e^{-(\rho \sigma + 2\beta)t} \quad \text{and} \]

\[ w = w^0 + \frac{\sigma \nu^0}{\rho \sigma + 2\beta} \left(1 - e^{-(\rho \sigma + 2\beta)t}\right). \]

Now

\[ \nu_j = \frac{1}{2} (V_{\Sigma}^0 e^{-\rho \sigma t} - \nu^0 e^{-(\rho \sigma + 2\beta)t}). \]

and
\[ V_k = \frac{1}{2} (V_\Sigma^0 e^{-\rho \sigma t} + V^0 e^{-(\rho \sigma + 2\beta)t}) \]

Since \( -\rho \sigma \) is greater than \( -(\rho \sigma + 2\beta) \), \( \nu \) goes to zero faster than \( V_\Sigma \). For \( V_\Sigma^0 = 0, \ V_j \) rises to and \( V_k \) falls to zero at the same time, so the system moves from \( C_{31} \) to \( C_{22} \) in the limit, and the change in the wage differential is \( \sigma V^0 / (\rho \sigma + 2\beta) \). The implication of the mobility theory of the last section, however, is that \( \beta \) should be quite large, so the wage change should be negligible. If \( V_\Sigma^0 < 0, \ V_k \) will fall to zero before \( V_j \) rises to zero, and the system will move to \( C_{32} \). Once in \( C_{32} \), the variables will follow (3.10) and (3.11), and the wage differential will increase. If \( V_\Sigma^0 > 0, \ V_j \) will rise to zero before \( V_k \) falls to zero, and the system will move to \( C_{21} \). The analysis for \( C_{13} \) is analogous to that for \( C_{31} \).

For the case in which \( V_k > 0 \) and \( V_j = 0 \) \( (C_{21}) \) the form of the equations depends upon the sign of \( w \). If \( w > 0 \), the system has the same equations as \( C_{11} \), but if \( w < 0 \) the system has the same equations as the zone for which both levels of excess demand are zero. In the latter case \( \nu \) and \( w \) adjust according to (3.10) and (3.11) until \( w \) falls to an increment less than zero, which will occur only if \( V^0 < \rho w_0 \), and the system takes on the equations of \( C_{11} \).

The final case is that of both \( V_j \) and \( V_k \) positive \( (C_{11}) \). The equations of the system are

\[
\dot{\nu} = -\rho \sigma \nu - 2\alpha w \\
\dot{w} = \sigma \nu ,
\]

and the time paths of the variables are derived in the same way that (3.9) was derived. As long as \( V_j \) and \( V_k \) remain positive, \( \nu \) and \( w \) approach zero, but the condition that \( V_j, V_k > 0 \) requires that \( -V_\Sigma < \nu < V_\Sigma \). The solutions for \( V_\Sigma \) and \( \nu \) are given by
\[ V_\Sigma = V_\Sigma^0 e^{-2\lambda t} \]

and

\[
v = \begin{cases} 
A_{11} e^{(-\lambda+\mu)t} + A_{12} e^{(-\lambda-\mu)t}, & \lambda^2 > 2\alpha \sigma \\
e^{-\lambda t}[A_{11} + A_{12} t], & \lambda^2 = 2\alpha \sigma \\
e^{-\lambda t}[A_{11} \cos \theta t + A_{12} \sin \theta t], & \lambda^2 < 2\alpha \sigma 
\end{cases}
\]

where \( \lambda = \frac{\sigma \rho}{2} \) and \( \mu = (\lambda^2 - 2\alpha \sigma) \). The speed at which \( V_\Sigma \) goes to zero is governed by \(-2\lambda\), which is more negative (implying faster adjustment) than \(-\lambda\) and \(-\lambda+\mu\). \( V_\Sigma^0 > 0 \) by hypothesis, so unless both \( v^0 \) and \( w^0 \) are zero, eventually \( |v| > V_\Sigma^0 \). Thus, except in a trivial case, the system will not stay in \( C_{11} \) for the full adjustment period.

The degree to which \( w \) adjusts depends upon the initial conditions \( (v^0, w^0, \text{ and } V_\Sigma^0) \) and upon the relative size of the parameters \( (\rho, \sigma, \text{ and } \alpha) \). There is a large variety of behavior possible, but delineation of the combinations of initial values and parameters which yield particular types of behavior would be quite tedious. Instead, two of the more typical patterns of behavior are illustrated in Figures 3 and 4 below. In the situation depicted in Figure 3, \( v^0 = 0 \) and \( w^0 > 0 \). This induces individuals to transfer to \( k \), so \( v \) falls — at a diminishing rate because \( w \) begins to fall and demand decreases in \( k \) at a slower rate than in \( j \). At period \( t_1 \), \( V_k \) has reached zero, so there is no more supply adjustment, and \( v \) increases slowly due only to declines in demand in \( j \) caused by wage increases. This goes on until \( V_j \) reaches zero. If \( V_\Sigma^0 \) were greater, say \( V_\Sigma^0' \), there would be more wage adjustment, for supply would stop adjusting later than \( t_1 \). This is illustrated by the dotted line.

The case in which wage changes are more responsive to excess demand is represented in Figure 4. The initial conditions are the same as in the
previous example, so the initial wage differential bids individuals from \( j \) to \( k \). At \( t_1 \), however, \( w \) reaches zero, and, after that, there is a reverse movement of supply to \( j \) until at \( t_2 \) \( w \) becomes zero again. At \( t_3 \) \( v = V_\Sigma \), and there are no jobs left in \( j \). The system is now in \( C_{12} \) with \( w < 0 \). After an interval, \( w \) rises to zero — if \( v(t_3) > -\rho w(t_3) \), and the system then moves back to \( C_{11} \).

We thus see that there is a tendency for \( |w| \) to diminish, but it is by no means assured that \( w \) will reach zero in the long run.

IV. Generalization to \( N \) Sectors

The approach to the construction of labor market adjustment models of the preceding sections may now be summarized in terms of a more general — but less precise — system, which permits us to study a wider range of behavior than did the linear model of the last section. It is useful at this point to revert back to the use of relative rather than absolute differentials, for the latter are better for comparison of the desirability of different markets. In particular we are concerned with the relative rate of excess demand, \( x_1 \), and the relative wage rate, \( \gamma_1 \), in the typical market, where

\[
x_1 = \frac{X_i}{X_\Sigma} = \frac{D_i/S_i}{D_\Sigma/S_\Sigma}
\]

and

\[
\gamma_1 = \frac{W_i}{W_\Sigma}.
\]

(\( X_\Sigma \) and \( W_\Sigma \) are the averages of the \( X_i \)'s and \( W_i \)'s respectively.) Differentiating the definition of \( x_1 \) logarithmically with respect to time yields

\[
x_1' = (\delta_1 - \delta_\Sigma) - (\delta_1' - \delta_\Sigma'),
\]

(4.1)
where the prime denotes a logarithm (so \( x_i' = x_i / x_i \), etc.), and we thus require equations to explain relative demand and supply changes in the typical market.

The relative proportionate change in the level of supply in the \( i \)th market may be written as

\[
(4.2) \quad (\hat{S}_i - \hat{S}_\Sigma) = f(x_i, y_i; X_\Sigma) + s_i,
\]

where \( s_i \) is the difference between the endogenous rate of growth of supply in the \( i \)th market (that increase in supply which is not due to movement into or out of the market from or to other markets) and the rate of growth of the aggregate labor force. Normally we would expect \( f \) to be homogeneous in the sense that \( f(0, 0; X_\Sigma) = 0 \), but it must also be true that the average proportionate change of supply across market, \( \Sigma \frac{\hat{S}_i - \hat{S}_\Sigma}{\hat{S}_\Sigma} \), equals the average rate of growth of supply from endogenous sources, \( \Sigma s_i \frac{1}{\hat{S}_\Sigma} \). Hence, \( f \) must be such that

\[
\Sigma f(x_i, y_i; X_\Sigma) \frac{S_i}{\hat{S}_i} = 0
\]

for all combinations of \( x_i, y_i \), and \( \frac{S_i}{\hat{S}_i} \). This creates potential aggregation problems, but we shall assume that these can be represented by a constant in \( f \) and will henceforth be ignored. The signs and magnitudes of the derivatives \( f_x \) and \( f_y \) depend upon \( X_\Sigma \) and whether the markets are structureless or structured. For the case of a set of structureless markets, \( f_y \) (which is related to a in the previous linear model) is positive over the whole range of \( x_i \) regardless of the value of \( X_\Sigma \). \( f_x \), however, depends upon the value of \( X_\Sigma \). We shall distinguish between three degrees of tightness of the aggregate labor market:

(i) \( X_\Sigma \) (high), the situation in which \( X_\Sigma \) is considerably greater than unity;
(ii) $X_\Sigma$ (med), $X_\Sigma$ in the neighborhood of unity; and
(iii) $X_\Sigma$ (low), $X_\Sigma$ well below unity.\textsuperscript{10}

From the analysis of Section II we would expect that $f_x$ (which is related to $b$) will be positive in the zones $X_\Sigma$ (low) and $X_\Sigma$ (med) but will diminish to zero as $X_\Sigma$ moves toward $X_\Sigma$ (high). The representation of relative supply adjustment among a set of structured markets by a single equation is somewhat heroic, for the implication of Section II is that the change in supply for the typical market depends critically on the distribution of the $x_i$'s and $y_i$'s as well as upon $x_i$, $y_i$, and $X_\Sigma$. However, by assuming that the $x_i$'s are distributed around unity fairly "normally" (or, to put it another way, not too strangely), some qualitative insights into the adjustment process may be gleaned.\textsuperscript{11} The influence of $x_i$ on $x_i'$, holding $w_i$ constant, is shown for the three hypothetical values of $X_\Sigma$ in Figure 5 below. At $X_\Sigma$ (low) very few—if any—markets, have rates of excess demand in excess of unity, and $f_x$ is very great for markets with $X_i > 1$ (or $x_j > \frac{1}{X_\Sigma}$), but $f_x$ is negligible for markets with $X_i < 1$. This is because the outflow of individuals from the average market with $X_i < 1$ is much less than the inflow to the average market. Of course, if there are no markets with $X_i > 1$, there is no supply adjustment, and $f_x = 0$ over the whole range of $x_i$. If, say, only two of a great many markets have vacancies, these will be filled quickly from the other markets without measurably increasing the relative rates of excess demand in the larger group. When $X_\Sigma$ is in the moderate range ($X_\Sigma$ (med)), there are vacancies in about half the markets and surpluses in the others, so the relation between $x_i'$ and $x_i$ is more-or-less symmetric around $x_i = 1$ (and is related to $\beta$ of Section II). Finally, when $X_\Sigma$ is very large ($X_\Sigma$ (high)), there will be very few markets with $X_i < 1$. In those few markets $f_x$ is quite
positive, but there will be little effect on markets with \( X_i > 1 \) \((x_i > \frac{1}{X_S})\). If there are no markets with surpluses, \( f_x \) is zero over the entire range of \( x_i \).
The sign and magnitude of \( f_y \) also depend upon the tightness of the aggregate labor market. Unless \( X_S \) is very large, wage differentials are a minor consideration to individuals contemplating movement, and this implies that \( f_y > 0 \) only when \( X_S \) is large.

The rate of growth in the \( i^{th} \) market less the rate of growth of demand in the aggregate market may be considered a function of the rate of growth of the relative wage rate in the \( i^{th} \) market and the relative wage itself plus a demand shift parameter. Symbolically, this is

\[
(4.3) \quad (\dot{D}^i - \dot{D}^*_i) = g(\dot{y}_1^i, y_1^i) + C_i.
\]

Normally we would expect that \( g_y < 0 \), reflecting short run demand adjustment, and \( g_y < 0 \), reflecting long run demand adjustment. Further, \( g(0, 1) = 0 \) subject to qualifications concerning aggregation effects. The differential demand shift parameter reflects changes in technology, product demand, and a host of other factors exogenous to the labor market, and its average across all markets is zero by definition. Upon substituting (4.2) and (4.3) into (4.1), we see that

\[
(4.4) \quad \dot{x}_i^i = (G_i - s_i) + g(\dot{y}_1^i, y_1^i) - f(x_1, y_1, X_S).
\]

We now require an equation to explain changes in relative wage level, and it is instructive to examine the effects of three different hypotheses. The first two hypotheses may be represented by

\[
(4.5) \quad \dot{y}_1^i = h(x_1, X_S).
\]
The first, the strict neoclassical hypothesis, is the strong variant of the Hansen-Phillips-Lipsey theory which was employed in the last section, and it
maintains that $h' > 0$ over the whole range of $x_1$ regardless of the value of $X_\Sigma$. The second wage determination hypothesis is based on the assumption of downward wage flexibility, i.e., money wages increase when $X_1 > 0$ but remain constant when $X_1 < 0$. In this model the sign and magnitude of $h'$ depend upon $x_1$, $X_\Sigma$, and the distribution of the $x_1$'s. Following an argument similar to the derivation of $f_x$ for a set of structured labor markets, we get three different relations between $y_1$ and $x_1$ for different values of $X_\Sigma$, and these are displayed in Figure 6. Of major importance is the fact that for low values of $X_\Sigma$ $f_y$ is close to zero for most (or all) values of $x_1$. A third wage determination hypothesis is the strict spillover model in which the relative wage structure is essentially fixed over long periods of time. Thus, instead of (4.7), we get simply that

$$y_1 = y_1^O,$$

where $y_1^O$ is the relative wage rate in some initial time period.

We can now put the assumptions concerning supply, demand, and wage adjustment together to form a system with which to analyze the labor market adjustment process. Substituting (4.5) for $\dot{y}_1$ in (4.4) gives

$$\dot{x}_1 = (G_i - s_1) + g(h(x_1; X_\Sigma), y_1) - f(x_1, y_1; X_\Sigma),$$

which represents the growth of the relative rate of excess demand under the first two wage determination hypotheses. (4.7) and (4.5), then, compose a system of two differential equations in two variables, $x_1$ and $y_1$, and their solution may be written as

$$x_1 = x_1^* + F(t; x_1^O, y_1^O, X_\Sigma)$$

and

$$y_1 = y_1^* + G(t; x_1^O, y_1^O, X_\Sigma).$$
The system is said to be globally stable if both \( x_1 \) and \( y_1 \) go to their steady values, \( x_1^* \) and \( y_1^* \), as \( t \) gets large for any initial values of \( x_1 \) and \( y_1 \).

This requires that both

(i) \[ \frac{\partial x_1'}{\partial x_1} < 0 \]

and

(ii) \[ \frac{\partial x_1'}{\partial y_1} \frac{\partial y_1'}{\partial x_1} < 0 \]

over the whole range of \( x_1 \) and \( y_1 \). In terms of the derivatives of \( f, g, \) and \( h, \) these conditions become

(i) \[ g_y h' - f_x < 0 \]

and

(ii) \[ (g_y - f_y)h' < 0 \]

Thus, the system is stable if \( h' > 0, g_y < 0 \) or \( f_x > 0 \), and \( g_y < 0 \) and \( f_y > 0 \) for all \( x_1 \) and \( y_1 \).

For low values of \( X_\Sigma \), it is clear that the system is stable only if wage determination is strict neoclassical, for \( h' > 0 \). For structureless labor markets, \( f_y > 0 \) for all \( x_1 \) and \( X_\Sigma \), \( f_x > 0 \) for all \( x_1 \) when \( X_\Sigma \) is not very large, and \( g_y < 0 \) always, so the system is stable regardless of the value of \( X_\Sigma \). The stationary values of \( x_1 \) and \( y_1 \) are obtained by setting (4.5) and (4.7) equal to zero, differentiating totally, and solving for \( dx_1^* \) and \( dy_1^* \).

From this we see that \( x_1^* = 0 \) and \( y_1^* \) depends upon \( (G_i - s_i) \), where

\[ \frac{\partial y_1^*}{\partial (G_i - s_i)} = -\frac{1}{[g_y - f_y]} > 0 \]

Of course, the dependence of \( G_i \) and \( s_i \) is the less the greater are the absolute values of \( g_y \) and \( f_y \). For structured markets, \( f_Y > 0 \) only for very large
so the system is not stable in this range unless $g_y < 0$. Given that the stability conditions are met, however, the steady state values of the variables will be qualitatively similar to those in structureless markets.

When the relative wage structure is frozen, either because the strict spillover hypothesis is correct or because the downward wage rigidity assumption is correct and $X_\Sigma$ is not large, the system is not stable by definition, but it is interesting to investigate the behavior of $x_1$ under the two specifications of labor turnover and for various $X_\Sigma$'s. \eqref{eq:4.7} now becomes

$$x_1'(t) = (G_1 - s_1) + g(0, y_1^0) - f(x_1, y_1^0; X_\Sigma),$$

and $x_1$ is stable (in the sense that $x_1$ goes to some steady state value) if and only if $\frac{dx_1'}{dx_1} = -f_x < 0$ over the whole range of $x_1$. If this is satisfied, $x_1^*$ will depend upon $(G_1 - s_1)$ and $y_1^0$, where

$$\frac{\partial x_1^*}{\partial (G_1 - s_1)} = \frac{1}{f_x},$$

and

$$\frac{\partial x_1^*}{\partial y_1^0} = \frac{g_y - f_y}{f_x}.$$  

For a set of structureless labor markets $f_x > 0$ unless $X_\Sigma$ is very large, and $f_y$ is always negative. Hence, $x_1$ should depend positively upon $(G_1 - s_1)$ and negatively upon $y_1^0$. For a set of structured markets, on the other hand, $f_x$ is positive (and large) in the moderate range of $X_\Sigma$, and both these derivatives are small. When $X_\Sigma$ is very small, $f_x$ is zero for most markets and $f_y$ is also zero. In this range, then, there is little which can be said about the adjustment of the $x_1$'s. If wage determination is strict spillover and $X_\Sigma$ is very large, there is a tendency for $x_1$ to get very large in markets with low wage rates and very small in markets with high wage rates. Undoubtedly, though, the spillover mechanisms would break down in such a situation.
V. A Concluding Remark

A principal theme of the preceding analysis has been that the adjustment of variables in structured labor markets is "efficient" (in the sense that the net advantages of different markets necessarily tend to equalize) only when the economy experiences sustained over-full employment. Since most labor markets in a modern economy conform with the structured case, it is not surprising that few labor economists feel that the price system is the primary allocator of the aggregate labor supply. The only times in recent history in which we would expect the price system to have fulfilled its allocative function with respect to the labor sector are the few periods of extensive mobilization (but during these times the price system has been controlled). On the other hand, for those occupations which are generally characterized by a state of high excess demand (like the market for scientists in recent times) neoclassical propositions concerning equalization of advantages should be consistent with observed behavior.
Footnotes:


5. This is derived from assumptions similar to those employed in Section II of Richard G. Lipsey, "The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1862-1957: A Further Analysis," Economica, N.S., 27 (February 1960), pp. 1-30.

6. Unless, of course, both $\Delta_k$ and $\Delta_j$ are zero.

7. If a constant, say $\gamma$, is added to (3.1), $V_t \sim -\frac{\gamma}{\sigma}$ as $t \to \infty$. We shall continue to assume that the wage adjustment function is homogeneous, but the analysis could be modified to cover the more general assumption.
8. If (as is likely the case) \( \gamma > 0 \), \( v \) and \( w \) still go to zero even though the steady state value of \( V_\Sigma \) is negative.

9. The case of downward wage flexibility is handled in the next section.

10. In terms of the aggregate U.S. unemployment rate we would expect that \( X_\Sigma \) (high) conforms roughly with \( u < 3\% \), \( X_\Sigma \) (med) with \( 3\% < u < 5\% \), and \( X_\Sigma \) (low) with \( u > 5\% \).

11. A forthcoming task of the Princeton labor market project, for which this essay has been written, is to modify the equations to apply to regional, occupational, and industrial movement. Having estimated the parameters (as well as possible with existing data) of these relations and other aspects of the system, the effects over time of various exogenous disturbances (such as labor market policies) may then be traced by simulation methods.

12. Since \( \Sigma (D_i - D_i') \frac{D_i}{D_\Sigma} \) and \( \Sigma G_i \frac{D_i}{D_\Sigma} \) are each zero by definition, \( \Sigma g(y_i', y_i) \frac{D_i}{D_\Sigma} \) must also be zero.

13. Of course, this applies to the case in which there is a non-zero floor to wage increases. For evidence that this floor is positive see W. P. Albrecht, Jr., "The Relationship Between Wage Changes and Unemployment in Metropolitan and Industrial Labor Markets," Yale Economic Essays, 6 (Fall 1966).

14. This is the extreme form of the approaches to wage determination in which "emulative factors" or "custom" cause individuals to derive utility not from income but relative income. See J. R. Hicks, "Economic Foundations of Wage Policy," Economic Journal, 65 (September 1955), pp. 398-401.
15. The results of Section III were based on the assumption that $g_y = 0$.

16. In this situation supply will adjust to any job opportunities very quickly, but that is all. This adjustment has been called the "job vacancy thesis" and has been considered as an alternative theory to the "wage-pull" or neoclassical supply adjustment theory (see Ulman, op. cit.). However, the two approaches to supply adjustment are simply the predicted behavior of a single theory in two different degrees of tightness of the labor sector.
Figures:

Figure 1
Figure 5