PLASMA INTERACTION WITH ELECTRON-EMITTING SURFACES

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Electron emission from surfaces occurs in many plasma systems. Several types including secondary, thermionic and photon-induced emissions are intense under certain conditions. Understanding the effects of emission on the “sheaths” that govern plasma-surface interaction is important. This dissertation predicts some emitting sheath phenomena that were not reported in past studies. For example, most previous theoretical models assumed that an emitting sheath potential is always negative and that ions always accelerate into the wall. We show when the emission is intense that the sheath potential can become positive, fundamentally changing how the plasma and wall interact. In this “inverse sheath” state, ions are repelled, suggesting for instance that (a) no presheath exists in the plasma interior, (b) emitting walls could be used in applications to stop sputtering. Another topic considered is the “transit” of emitted electrons across the plasma to other surfaces, which is possible in low collisionality plasma systems. When transit occurs, the flux balance is a complex global problem where the sheaths at opposite surfaces are coupled through their exchange of emitted electrons. We also show that secondary emission can trigger a variety of sheath instability phenomena that change the state of the plasma-wall system or cause oscillations preventing steady state. Lastly, we analyze a mechanism where emitted electrons return to the same surface and knock out secondaries, which return and knock out more secondaries, etc., feedback amplifying the emission intensity. The four phenomena will be analyzed theoretically and verified with particle-in-cell simulations: (a) inverse sheath, (b) sheath coupling via transiting electrons, (c) sheath instabilities, (d) returning electron amplification. Consequences of these processes on the sheath potentials, wall heating, loss rate of charge, and cross field transport (near-wall conductivity) are discussed throughout.
Possible implications are suggested for fusion machines, plasma propulsion engines, probes, dusty plasmas, RF discharges, and surfaces in space.
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1. INTRODUCTION

1.1 Motivation

Practically all laboratory plasmas contact solid surfaces [1]. The plasma-surface interaction (PSI) is always important. In tokamaks for example [2], bombardment by plasma ions heats the device “walls” and sputters away wall atoms. Sputtering erodes the walls and contaminates the plasma with detrimental impurities. Bombardment by plasma electrons further heats the walls, increasing the risk of melting. In lower temperature plasma devices, the walls might not be severely damaged by the plasma, but the PSI is still always important. The properties of a plasma depend on balances between heating and ionization versus losses of energy and charges at the boundaries. The potentials of surfaces relative to the plasma and relative to other surfaces are important quantities in laboratory and astrophysical plasma systems. In space, the ambient plasma sources such as the solar wind charge up spacecraft, the moon, comets, and dust [3,4].

In many if not most plasma-surface interaction situations, electrons are emitted from the surface. Several mechanisms of emission are significant under different conditions. When energetic electrons (tens of eV or more) strike a solid, they can knock out one or more electrons [5]. Electron impact secondary electron emission (SEE) is therefore intense in hot plasma devices including tokamaks [6], magnetic mirrors [7] and Hall thrusters [8]. Lower energy electrons, even a few eV or less, have been shown to “backscatter” efficiently off solids [9]. So some degree of emission will still be present at low plasma temperature. Ion impacts in the keV energy range can eject electrons [10]. This is important in capacitively coupled plasmas [11] and plasma processing applications [12] where electrical biasing leads to a strong negative sheath...
potential that accelerates the ions. Solids exposed to photons can emit electrons through the photoelectric effect [13]. Photoemission from sunlit objects in space is intense [3,4]. Solids emit electrons thermionically when heated to high enough temperature [14]. Plasma-facing surfaces can be heated to thermionic emission levels by the plasma itself like in a tokamak [15]. In some applications including emissive probes [16] and thermionic converters [17], the surfaces are heated on purpose to emit thermionic electrons.

Overall, we see that the topic “Plasma Interaction with Electron-Emitting Surfaces” is relevant to a broad variety of plasma systems. Understanding the effects of emission on plasma-surface interaction is essential from both fundamental and applied perspectives.

Figure 1.1: Common mechanisms of electron emission include electron-impact SEE, ion-impact SEE, photoemission and thermionic emission.

1.2 Conventional Predictions of Electron Emission Effects on PSI

In the plasma physics field, it has been known since Langmuir’s work [18] that plasma-surface interaction revolves around thin “sheaths” that form at the interface. In plasmas, the electron thermal velocities far exceed the ion thermal velocities due to the mass difference. So
when a plasma contacts a non-emitting wall, the wall charges negatively and acquires a negative potential relative to the plasma. The potential profile $\phi(x)$ takes the form of the classical Debye sheath sketched in Figure 1.2. Ions are accelerated into the wall and most plasma electrons are reflected away. The floating sheath potential $\Phi_f$ that maintains zero current is about $-4T_e$ in terms of the electron temperature $T_e$, though it may be a bit more or less depending on other factors [1].

When electrons are emitted, it is intuitive that more plasma electrons must reach the wall to balance the ion flux. So $\Phi_f$ becomes less negative. Many authors have constructed rigorous models of emitting sheaths which include the charge density from emitted electrons in addition to the plasma electrons and ions [19,20,21,22,23,24]. These authors model a planar plasma contacting a wall where the “secondary emission coefficient” $\gamma$ is a variable parameter. They show as $\gamma$ crosses a critical value $\gamma_{cr}$ slightly below unity, a “virtual cathode” or “dip” forms in $\phi(x)$ near the wall. In this “space-charge limited” (SCL) regime, the overall sheath potential $\Phi_f$ is still negative (about $-1T_e$) but $\phi(x)$ is nonmonotonic, see Figure 1.2. It is assumed that if $\gamma$ increases any further, even beyond unity, the “extra” electrons will reflect in the virtual cathode and return to the wall, not significantly changing $\Phi_f$. 
Figure 1.2: Qualitative drawings of the potential distribution relative to the wall in the classical, SCL and inverse sheath regimes. The floating sheath potential $\Phi_f$, defined as $\varphi(wall) - \varphi(plasma)$, is negative in the classical and SCL regimes, but positive in the inverse regime.

Because sheaths in laboratory plasmas are in most cases very thin and difficult to measure directly, there are only a few experimental studies of sheaths at emitting surfaces, see Section 4 of Robertson’s recent review [25]. Particle-in-cell (PIC) simulations have been used as “experiments” to study emitting sheaths with more precision [21,26]. Past simulation studies have corroborated the basic predictions of past sheath theories, finding that the sheath potential becomes less negative with increasing emission intensity until the sheath becomes SCL [21].

Sheath theories are important for setting boundary conditions in analytical models and fluid simulation codes of plasmas including tokamak scrape-off layers [1]. When emission is considered, the sheath potential and energy transmission factor are adjusted in terms of $\gamma$ in accordance with conventional emitting sheath theories. The increase of plasma electron flux to the walls with $\gamma$ causes an enhancement of the wall heating and enhanced cooling of the plasma [1]. This is undesirable in most applications. However, a desirable reduction of sputtering is also expected because ion impact energies are reduced [15]. The critical emission coefficient $\gamma_{cr}$
at which the sheath becomes SCL is considered the maximum possible $\gamma$ for practical purposes. In the literature, it is assumed that if $\gamma > \gamma_{cr}$, the sheath potential and energy transmission are the same as when $\gamma = \gamma_{cr}$ because the extra electrons are returned to the wall and presumably play no role [1,15,19,27]. Another conventional prediction is that the loss rate of plasma charge to wall recombination is unaffected by emitted electrons. This is because the flux of ions is assumed to follow from the Bohm criterion at the sheath edge, and the Bohm velocity varies negligibly with $\gamma$ [19].

1.3 Thesis structure

In this dissertation, we show with theory and simulations that the effects of electron emission on PSI can be much different from what are normally predicted. Possible applications will be discussed for a variety of plasma systems including tokamak divertors, plasma propulsion engines, dusty plasmas, heated cathodes, RF discharges and space objects. Results presented in this document also appear in recent published papers, see References [28,29,30,31,32].

In Chapter 2, we will show that when the emitted flux is strong, the sheath potential does not need to remain negative as in SCL sheath theory but can become positive. In this “inverse sheath” regime [28,29] sketched in Figure 1.2, electrons are unconfined and ions are confined, opposite to the classical and SCL sheath regimes. Therefore the energy flow through the sheath and the loss rate of charge to the wall change drastically. We will explain that the inverse sheath was not captured in past studies because ions were always assumed to fall to the wall in theoretical models and were forced to fall to the wall by an artificial “source sheath” in simulation models.
In Chapter 3, we analyze an important process for low collisionality plasma systems where the electron mean free path exceeds the plasma size. In this regime, the perspective used in most theory and simulation models where a plasma contacts one emitting wall is no longer valid because the emitted electrons can transit between opposite surfaces, changing the flux balance. In the past, transit was only discussed in the context of planar symmetric plasmas where the two transiting electron beams self-cancel [33,34]. Interesting new phenomena arise in systems with asymmetries from different wall materials or geometries, because the beams no longer cancel [30]. The sheaths become coupled to each other through their unequal exchanges of electrons.

In Chapter 4, we show that SEE can cause “sheath instabilities” that abruptly change the sheath potential or make it oscillate in time. Instabilities attributed to SEE were observed in past simulation studies [35,36,37] but the precise causes were not known. Here, a general theory is derived [31] showing conditions where perturbations of the surface charge amplify, driving a sheath instability. One of the conditions is shown to explain three different types of instability phenomena observed in plasma simulations [32].

Throughout this dissertation, the above effects are demonstrated with a simulation code developed by Dmytro Sydorenko [38] for his PhD thesis in collaboration with Andrei Smolyakov and this writer’s advisor Igor Kaganovich. The 1D3V electrostatic direct implicit particle-in-cell (EDIPIC) code simulates a planar plasma bounded by walls with SEE. EDIPIC was used to model Hall discharges in several papers from 2006 to 2009 including Refs. [34,35,37,39]. This code, with modest enhancements, was very useful for studying PSI with emission more fundamentally for this dissertation work. In EDIPIC, the simulated plasma is produced by volume ionization and is bounded by two walls, so the sheath physics is more
realistic than in simulations where the plasma was injected across an artificial boundary [21] towards one wall.

The plasma domain in EDIPIC contains a background $E$ field parallel to the walls and a $B$ field normal to the walls, for modeling Hall thrusters. This captures an important emission effect relevant to $E\times B$ plasma systems known as near-wall conductivity (NWC). NWC is cross field transport caused by electrons emitted from the wall moving across $B$ in the $E\times B$ drift [40]. We will show that NWC is influenced by the inverse sheath, transiting electron, and sheath instability phenomena in Chapters 2 through 4. The final chapter of results, “Self-Amplification of Emitted Electrons that Return to Surfaces” in Chapter 5, shows that even secondaries that return to the wall without ever reaching the plasma (in the inverse and SCL states) contribute NWC. NWC from returning electrons was not previously considered in calculations counting only secondaries that enter the plasma [41,42,43,44]. We show that returning electrons can actually dominate the cross field transport if they knock out more secondaries and “self-amplify” to extreme values.
2. A POSITIVE EMITTING SHEATH POTENTIAL – INVERSE SHEATH

2.1 The Strong Emission Problem - Overview

It is intuitive that when electrons are emitted from a plasma-facing wall, more plasma electrons must reach the wall to satisfy the zero current condition.

\[ \Gamma_{e,\text{net}} = \Gamma_p - \Gamma_{\text{emit}} = \Gamma_{\text{ion}} \]  

The plasma electron flux \( \Gamma_p(\Phi) \) is a function of sheath potential \( \Phi \) and the ion flux is governed by the Bohm criterion [45]. So the floating potential \( \Phi_f \) is usually calculable in terms of known plasma properties and \( \Gamma_{\text{emit}} \). But an interesting and important fundamental question is: what happens when a wall emits more electrons than it can collect from the plasma? If \( \Gamma_{\text{emit}} > \Gamma_p \), the net electron flux is negative and Eq. (2.1) cannot be satisfied because \( \Gamma_{\text{ion}} \) cannot be negative. Such a paradox arises when the secondary electron emission (SEE) coefficient of the plasma electrons \( \gamma \) exceeds unity. When \( \gamma > 1 \), \( \Gamma_{\text{emit}} = \gamma \Gamma_p > \Gamma_p \) for any \( \Phi \). So there appears to be no allowable \( \Phi_f \). Most plasma-facing materials have \( \gamma > 1 \) beyond a certain \( T_e \) (tens of eV or more) [46]. SEE coefficients exceeding unity are predicted to occur under certain conditions at surfaces in tokamak scrape-off layers [6,47], plasma thruster channels [48], dusty plasmas [49] and hot astrophysical plasmas [3].

It is also possible to have \( \Gamma_{\text{emit}} > \Gamma_p \) at surfaces emitting a thermionic or photoelectron current that exceeds the plasma electron saturation current \( \Gamma_{p0} \), the maximum possible \( \Gamma_p \). Examples include heated cathodes [50], emissive probes [16], and sunlit objects in space [3]. In this chapter, we will treat the “strong emission problem” in terms of a SEE coefficient \( \gamma \). The results apply
without loss of generality to other emission types. For example, if a surface emits a strong thermionic flux \( \Gamma_{\text{emit}} \), the equivalent \( \gamma \) is given by \( \Gamma_{\text{emit}} / \Gamma_{p0} \).

Most works predict that when \( \gamma > 1 \), a “space-charge limited” (SCL) sheath forms [19,21,26,23,24] as sketched in Figure 2.1. In theory, a potential “dip” called a virtual cathode can return some emission to the wall make the net current zero. Here we will show the “inverse sheath [28]” in Figure 2.1 is another solution. The inverse regime has some important features that differ from the familiar classical and SCL regimes. (a) The plasma potential is negative relative to the wall (or equivalently, the sheath potential is positive, \( \Phi_f > 0 \)). (b) Ions are confined, so no loss of charge from the plasma occurs when the ions are cold. (c) No ion-accelerating presheath structure needs to exist in the plasma interior. (d) The electron energy flux to the wall is the maximum possible thermal value.

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**Figure 2.1:** Sketch of \( \phi(x) \) relative to the wall in the classical, SCL, and inverse sheath regimes. The “transition” between the classical and SCL sheaths, where the electric field is zero at the wall, is also sketched.
In Sec. 2.2, we discuss the physical origin of the different sheath structures in Figure 2.1. In Sec. 2.3, we explain why the inverse sheath was not captured in past theoretical studies of emission. In Sec. 2.4, a mathematical model proving the inverse sheath solution when $\gamma > 1$ is presented. Estimates of the amplitude (voltage) and spatial width of the inverse sheath are derived. In Sec. 2.5, implications of the inverse sheath effect on plasma-surface interaction are elaborated. In Sec. 2.6, we attempt to determine whether an inverse sheath or SCL sheath forms at strongly emitting surfaces. Past empirical studies are reviewed and a new simulation is conducted. In Sec. 2.7, we present an example illustrating how strongly inverse sheaths can change the properties of a plasma. Two Hall discharge simulations, one in the classical regime and one in the inverse regime, are compared.

### 2.2 Sheath Structure Variation with Emission Intensity

In this section, we will analyze why each sheath structure in Figure 2.1 can exist, and under what conditions. Revisiting the origins of the classical and SCL sheaths will help us explain why the inverse sheath was not captured in past theories. This discussion is kept conceptual to focus on the physical meaning that is often not explained in mathematical sheath treatments. Note throughout this section, the sheath potential “$\Phi$” is defined as $\varphi(\text{wall}) - \varphi(\text{plasma})$, so it is independent of where the reference potential is.

Consider an unmagnetized planar plasma with Maxwellian electrons and cold ions in contact with a non-emitting floating wall. It can safely be assumed that in equilibrium, the wall must be negatively charged, and ions will be attracted to the wall. Because the distant plasma must be shielded from the negative wall, the ion density must fall off more slowly than the electron
density as the wall is approached, so that the net space charge near the wall is positive. From these assumptions, the Bohm criterion for the ion velocity into the “sheath” is derivable [45,51]. The Bohm criterion necessitates a presheath structure to accelerate the ions. The presheath solution essentially fixes $\Gamma_{\text{ion}}$ at the sheath edge, independent of $\Phi$. In equilibrium, $\Phi_f$ must be sufficiently negative to maintain $\Gamma_f(\Phi_f) = \Gamma_{\text{ion}}$. The exact $\Phi_f$ can be calculated, and then the full sheath structure can be derived if desired. Qualitative charge density distributions for a non-emitting classical Debye sheath are plotted in Figure 2.2(a).

Figure 2.2: Qualitative plots of the electron and ion density distributions near the wall for different $\gamma$. For each charge distribution, the sheath structure in Figure 2.1 that corresponds to it is indicated in text. The sign of the wall charge is marked in the squares. In (b-d), there are two oppositely charged layers in the sheath. The combined charge of the two layers is positive in (b), zero in (c), and negative in (d).
With electron emission, a few changes occur. For $\gamma > 0$, the floating potential amplitude $|\Phi_f|$ is reduced from the zero current condition $\Gamma_p(\Phi_f) = \Gamma_{\text{ion}}/(1-\gamma)$. Also, emitted electrons add negative charge to the sheath, see Figure 2.2(b). Their contribution creates an electron density peak near the wall because emitted electrons start with small initial velocities before being accelerated away to higher velocity by the sheath. For small $\gamma$, the negative charge layer in the sheath is smaller in magnitude than the positive layer, so the wall charge still must be negative and the potential must increase outward from the wall. Hence the sheath structure for small $\gamma$ is still a classical sheath.

As $\gamma$ approaches unity, the emission flux increases sharply, $\Gamma_{\text{emit}} = \gamma \Gamma_{\text{ion}}/(1-\gamma)$. For some critical $\gamma_{cr}$ below unity, the electron charge in the sheath will equal the ion charge, see Figure 2.2(c). In this situation, the wall charge must be zero for the distant plasma to be shielded. The resulting $\phi(x)$ is the “transition” sheath in Figure 2.1, where the electric field vanishes at the wall. For any further increase of $\gamma$, the total charge in the sheath must be negative, Figure 2.2(d), so the wall charge must be positive. The resulting $\phi(x)$ is nonmonotonic for $\gamma_{cr} < \gamma < 1$, taking the shape of the SCL sheath in Figure 2.1. Although the wall charge is positive in the SCL regime, the combined charge of the wall and the negative space charge layer is negative. The positive charge layer near the sheath edge still shields the plasma, meaning the same Bohm criterion [45,51] must be met at the edge, and an ion-accelerating presheath exists.

The SCL sheath remains a physically possible sheath when $\gamma > 1$. In the SCL regime, there is an influx of emitted electrons that reflect in the potential dip and return to the wall $\Gamma_{\text{ref}}$. Zero current can be maintained by a SCL sheath if,

$$\Gamma_{e,\text{net}} = \Gamma_p(1-\gamma) + \Gamma_{\text{ref}} = \Gamma_{\text{ion}}$$

(2.2)
However note that when $\gamma > 1$, Eq. (2.2) is solvable with $\Gamma_{\text{ion}} = 0$ and $\Gamma_{\text{ref}} = \Gamma_p(\gamma-1)$. This means zero current can be maintained without ions reaching the wall at all! One could propose that a fundamentally different type of solution should exist, an “inverse sheath,” where the overall sheath potential is positive, as sketched in Figure 2.1. In this state, the ions are confined, the wall draws the full thermal electron flux from the plasma ($\Gamma_p = \Gamma_{p0}$) and the inverse sheath returns the “extra” emitted electrons to the wall $\Gamma_{\text{ref}} = \Gamma_{p0}(\gamma-1)$. A formal derivation of the inverse sheath solution is presented in Section 2.4. We will find that the charge density profiles appear as sketched in Figure 2.2(e).

### 2.3 Comparing the Inverse Sheath Theory to Past Theories

The possibility of an emissive plasma sheath being positive ($\Phi_f > 0$) was not reported in the past to our knowledge. Throughout the literature it has widely been predicted that the sheath potential is negative (SCL) under strong emission. A closer look reveals that the sheath potential being negative was an assumption of the past theoretical sheath models, not a derived result.

Hobbs and Wesson presented the pioneering theoretical treatment of sheaths with SEE [19]. They solved Poisson’s equation in a planar sheath assuming Boltzmann plasma electrons, cold ions and cold emitted electrons. They showed that the electric field at the floating wall drops to zero when $\gamma$ reaches a critical value $\gamma_{cr}$ below unity (SCL transition sheath in Figure 2.1). Other researchers have since considered the influence of the kinetic correction to the electron velocity distribution in the sheath [21,24], nonzero ion temperature [21], nonzero emitted electron temperature [24], current-carrying surfaces [20], the presence of incident electron beams [22], and supermarginal Mach numbers [23] on emissive sheath structures.
In all of the aforementioned models, it is assumed that ions enter the sheath with a (Bohm) flow velocity, and the charge densities $n_e(x)$, $n_{ion}(x)$ are expressed in terms of a $\varphi(x)$ that is assumed below the sheath edge potential everywhere in the sheath. The only possible sheath under these assumptions when $\gamma > 1$ is the SCL sheath. The SCL sheath is rarely derived in papers because Poisson’s equation becomes very complicated when $\varphi(x)$ has a dip. Usually the transition sheath (Figure 2.1) with $\gamma = \gamma_{cr}$ is modeled and it is stated in the papers that for $\gamma > \gamma_{cr}$, the sheath develops the dip. Hobbs and Wesson [19] wrote “For $\gamma > \gamma_{cr}$ no monotonic solution for $\varphi(x)$ exists and a potential well forms such that a fraction of the emitted electrons are returned to the wall in order to maintain the effective $\gamma$ equal to $\gamma_{cr}$. Under these conditions the emission current is space-charge limited.”

So we see that the conventional prediction of the sheath potential always being negative is based on an assumption of ions falling to the wall. The key concept to report in this chapter is simple: the ion flow assumption can break down when $\gamma > 1$. As we saw earlier, ion flow is only necessary for current balance when $\gamma < 1$ because the effective electron flux $\Gamma_p(1-\gamma)$ is positive, requiring $\Gamma_{ion}$ to be nonzero. Sheath theories will often attribute ion flow to the Bohm criterion too. But recall that the Bohm criterion is a requirement for forming a positive charge layer to shield a negative charge [45]. When $\gamma > 1$, because the wall must be positively charged to pull back some of the emission, the wall can be shielded by a single layer of negative charge in Fig. 2(e). The double layer of Fig. 2(d) is unnecessary.

Overall, we have shown that both fundamental requirements of plasma interaction with a floating surface, (a) zero current at the surface, and (b) shielding of the plasma interior, are met by the monotonic inverse sheath solution when $\gamma > 1$. 
2.4 Structure of an Inverse Sheath

2.4.1 Overview

It might be unsurprising that a positively charged wall can be shielded by a negatively charged sheath, as sketched in Figure 2.2(e). But proving that the inverse sheath can exist still requires showing that the \( \phi(x) \), \( N_e(x) \) and \( N_{\text{ion}}(x) \) sketched in the figures can form self-consistently.

Suppose \( \phi(x) \) is flat in the neutral plasma interior and starts increasing monotonically from the sheath edge to the wall. Assuming the plasma ions are cold and have no flow velocity, the ions cannot climb to a higher potential. Therefore, the ion density is zero in the sheath, as in Figure 2.2(e). It follows that the charge density will be negative everywhere in the sheath. So from Poisson’s equation, \( \phi(x) \) monotonically increases from the edge to the wall. If the positively charged wall balances the negative charge in the sheath, the plasma is shielded and \( \phi(x) \) can indeed be flat in the plasma.

We conclude that the inverse sheath’s charge density profiles can exist self-consistently with the potential profile. To more closely investigate properties of the inverse sheath such as its size and amplitude, we present an analytical model.

2.4.2 Mathematical Model

Consider an unmagnetized planar plasma contacting a floating wall with a given \( \gamma > 1 \). Let \( \Phi_1 \) denote the positive potential difference between the wall and sheath edge, see Figure 2.3. Let
N denote the neutral plasma density at the edge. Assuming the ions are cold, the ion density $N_{ion}$ drops abruptly from N to zero at the edge, so it can be neglected in the sheath. The electron density $N_e$ in the inverse sheath consists of three distinct components, one corresponding to each flux component in Figure 2.3.

![Figure 2.3: Parameters and notations used for the analytical inverse sheath model. The sheath is assumed collisionless.](image)

Let us assume that the hot plasma electrons approaching the wall have a half-Maxwellian distribution of temperature $T_p$, starting with density $N_p^{SE}$ at the edge. The plasma electrons accelerate through the inverse sheath towards the wall, producing a density in terms of $\varphi$ given by,

$$N_p(\varphi) = N_p^{SE} \exp\left(\frac{e\varphi}{T_p}\right) \text{erfc}\left(\sqrt{\frac{e\varphi}{T_p}}\right)$$  \hspace{1cm} (2.3)
Suppose the secondaries are emitted with a half-Maxwellian distribution of temperature $T_{\text{emit}}$, starting with a density $N_{\text{wall}}$ at the wall. The density of secondaries traveling away from the wall under the retarding force is expressible by a Boltzmann factor,

$$N_{\text{emit}}(\varphi) = N_{\text{emit}}^{\text{wall}} \exp\left(\frac{e(\varphi - \Phi_{-1})}{T_{\text{emit}}}\right)$$  \hspace{1cm} (2.4)

The portion of secondaries emitted with kinetic energy normal to the wall exceeding $e\Phi_{-1}$ will escape the inverse sheath. The rest will reflect and return to the wall. The charge density at each point from reflected secondaries moving towards the wall is,

$$N_{\text{ref}}(\varphi) = N_{\text{emit}}^{\text{wall}} \exp\left(\frac{e(\varphi - \Phi_{-1})}{T_{\text{emit}}}\right) \text{erf}\left(\sqrt{\frac{e\varphi}{T_{\text{emit}}}}\right)$$  \hspace{1cm} (2.5)

Eqs. (2.3)-(2.5) are formally derived by using the Vlasov equation to solve for the velocity distribution functions in the sheath in terms of $\varphi$, accounting for the cutoff velocities, and then integrating the distributions to get the densities. We omit the details because similar expressions are ubiquitous in sheath theories where half-Maxwellians are accelerated, decelerated and reflected. For instance, the emitted and reflected secondaries in Eqs. (2.4) and (2.5) are respectively analogous to incident and reflected plasma electrons in a classical sheath (c.f. Ref. [20]).

So far, $N_P^{SE}$ and $N_{\text{emit}}^{\text{wall}}$ in (2.3)-(2.5) are unspecified quantities which should be expressed in terms of the known $N$. The condition that the plasma is neutral at the sheath edge must account for the secondaries that escape the inverse sheath and enter the plasma,
\[ N_P^{SE} + N_{\text{emit}}^{SE} = N \]  

(2.6)

The secondary electron density at the edge \( N_{\text{emit}}^{SE} \) is expressible in terms of \( N_{\text{wall}}^{SE} \) using (2.4) with \( \phi \) set to zero.

\[ N_{\text{emit}}^{SE} = N_{\text{wall}}^{SE} \exp \left( \frac{-e\Phi_{-1}}{T_{\text{emit}}} \right) \]  

(2.7)

\( N_{\text{wall}}^{SE} \) and \( N_P^{SE} \) can be linked through \( \gamma \). The plasma electron influx is the full thermal flux of the half-Maxwellian source, \( \Gamma_p = \Gamma_{p0} = N_P^{SE} \left( 2T_p / m_e \pi \right)^{1/2} \). The emitted flux from the wall is \( \Gamma_{\text{emit}} = N_{\text{emit}}^{wall} \left( 2T_{\text{emit}} / m_e \pi \right)^{1/2} \). Then because \( \Gamma_{\text{emit}} = \gamma \Gamma_{p0} \), we have,

\[ N_{\text{emit}}^{wall} \sqrt{T_{\text{emit}}} = \gamma N_P^{SE} \sqrt{T_p} \]  

(2.8)

Now to determine \( \Phi_{-1} \), we use the zero current condition, \( \Gamma_p - \Gamma_{\text{emit}} + \Gamma_{\text{ref}} = 0 \). With \( \Gamma_p = \Gamma_{\text{emit}} / \gamma \) it follows,

\[ \Gamma_{\text{ref}} = \frac{\gamma - 1}{\gamma} \Gamma_{\text{emit}} \]  

(2.9)

In terms of \( \Gamma_{\text{emit}} \), the flux of the half-Maxwellian secondaries that escape the inverse sheath barrier is \( \Gamma_{\text{emit}} \exp(-e\Phi_{-1}/T_{\text{emit}}) \). So \( \Gamma_{\text{ref}} \) is just the complement,

\[ \Gamma_{\text{ref}} = \Gamma_{\text{emit}} \left[ 1 - \exp \left( \frac{-e\Phi_{-1}}{T_{\text{emit}}} \right) \right] \]  

(2.10)

Equating (2.9) with (2.10) yields a simple expression for the inverse sheath amplitude \( \Phi_{-1} \).
\[ e^{\Phi_1} = T_{\text{emit}} \ln \gamma \]  \hspace{1cm} (2.11)

Now plugging (2.11) into (2.7) and solving Eqs. (2.6)-(2.8) for \( N_p^{SE} \) and \( N_{\text{emit}}^{wall} \) gives,

\[
N_p^{SE} = \frac{N}{1 + \sqrt{T_p/T_{\text{emit}}} \gamma} \hspace{1cm} (2.12)
\]

\[
N_{\text{emit}}^{wall} = \frac{\gamma N \sqrt{T_p/T_{\text{emit}}}}{1 + \sqrt{T_p/T_{\text{emit}}} \gamma} \hspace{1cm} (2.13)
\]

Summing Eqs. (2.3)-(2.5) where \( \Phi_1, N_p^{SE} \) and \( N_{\text{emit}}^{wall} \) are defined in (2.11)-(2.13) gives the total electron density in the inverse sheath \( N_e = N_p + N_{\text{emit}} + N_{\text{ref}} \) in terms of \( \varphi \).

### 2.4.3 Discussion and Application of the Model

The exact \( \varphi(x) \) and corresponding \( N_e(x) \) for a given \( \{N, T_p, T_{\text{emit}}, \gamma\} \) can be calculated by solving Poisson’s equation numerically using the expression for \( N_e(\varphi) \) derived above. We will show that the most important properties of the inverse sheath can be described in simple terms analytically.

One property of the inverse sheath is that its amplitude is very small compared to classical and SCL sheaths. Classical and SCL sheaths always have amplitudes \( \geq \sim T_p \). On the other hand, the inverse sheath amplitude \( T_{\text{emit}} \ln(\gamma) \) is determined by \( T_{\text{emit}} \) no matter how large \( T_p \) is. Although \( \gamma \) itself varies with \( T_p \) if the emission type is SEE, the function \( \gamma(T_p) \) has a maximum less than 2 for most materials [46]. So in general for SEE, \( e^{\Phi_1} < T_{\text{emit}} \).
To investigate how the electron density behaves in the sheath, we insert \( e\varphi = e\Phi_{\text{1}} = T_{\text{emit}} \ln(\gamma) \) into the formula for \( N_e(\varphi) \) to give an expression for the total electron density at the wall interface \( N_e^{\text{wall}} \). We write \( N_e^{\text{wall}} \) in terms of the dimensionless \( T_R \equiv T_p/T_{\text{emit}} \) because only the temperature ratio appears in the expression.

\[
N_e^{\text{wall}} = N \frac{\frac{1}{\sqrt{T_R}} \text{erfc} \left( \sqrt{\ln \gamma} \frac{T_R}{T_R} \right) + \sqrt{T_R} \left[ 1 + \text{erf} \left( \sqrt{\ln \gamma} \right) \right]}{1 + \sqrt{T_R}}
\] (2.14)

In Figure 2.4, we plot \( N_e^{\text{wall}} \) versus \( T_R \) for various \( \gamma \) values. We see that \( N_e^{\text{wall}} > N \) when \( T_R > 1 \). The range \( T_R < 1 \) is not of practical interest because \( T_{\text{emit}} \) for various emission types is only a few eV or less. For plasmas hot enough to induce \( \gamma > 1 \) from a typical material, \( T_p \) is from tens to hundreds of eV [46]. While thermionic or photoemission can induce an inverse sheath in a colder plasma, \( T_p \) will still substantially exceed \( T_{\text{emit}} \) in practical conditions.

Figure 2.4: Variation of \( N_e^{\text{wall}} \) with \( \gamma \) and \( T_R \equiv T_p/T_{\text{emit}} \). \( N_e^{\text{wall}} \) is normalized to the sheath edge plasma density \( N \). The range \( T_R < 1 \) is unrealistic, so we conclude that \( N_e^{\text{wall}} > N \).
We conclude that the electron density in an inverse sheath increases towards the wall. (While this technically does prove that $N_e$ monotonically increases towards the wall, monotonicity can be shown by evaluating $dN_e(\phi)/d\phi$ analytically, confirming it is positive for $T_R > 1$ and using $dN_e/dx = dN_e/d\phi \times d\phi/dx$. We omit this calculation for brevity.)

In Figure 2.4, $N_{e,wall}^\gamma$ increases with $\gamma$ and with $T_R$. In the limit $T_R >> 1$, (2.14) reduces to,

$$N_{e,wall}^\gamma = N \gamma \left[1 + \text{erf} \left( \sqrt{\ln \gamma} \right) \right]$$  \hspace{1cm} (2.15)

Because usually $\gamma < 2$ for SEE [46], and because $N_{e,wall}^\gamma < 2N\gamma$ via (2.15), this puts an upper bound on $N_e$ in an inverse sheath of about four times the interior plasma density.

A useful approximation of the inverse sheath structure can now be derived. The preceding analysis shows that the electron density in an inverse sheath always exceeds $N$, but not by more than a factor of a few. Therefore, given that $N_{ion}$ is zero, it is reasonable to approximate the charge in the inverse sheath as a uniform density $-N$. Poisson’s equation in the sheath is then approximately,

$$\frac{d^2 \varphi(x)}{dx^2} = \frac{eN}{\varepsilon_0}$$  \hspace{1cm} (2.16)

Setting the origin $x = 0$, $\varphi = 0$ at the sheath edge, assuming zero electric field at the edge, and integrating (2.16) twice gives a parabolic $\varphi(x)$,

$$\varphi(x) = \frac{eN}{2\varepsilon_0} x^2$$  \hspace{1cm} (2.17)
Applying the boundary condition $\varphi(x_{\text{wall}}) = \Phi_1$ at the wall, we can determine the location of the wall relative to the sheath edge from (2.17). This gives a simple estimate of the spatial size of an inverse sheath $\Delta x_{\text{inv}}$,

$$\Delta x_{\text{inv}} \approx \sqrt{\frac{2e_0 T_{\text{emit}} \ln \gamma}{e^2 N}}$$  \hspace{1cm} (2.18)

Eq. (2.18) is a robust estimate for the size of SEE-driven inverse sheaths. Even if $N_e(x)$ were to increase by the maximum possible factor of $\sim 4$ from the sheath edge to the wall, (2.18) would be accurate within a factor of two. For thermionic emitting surfaces, the equivalent $\gamma$ can be much larger (e.g. up to 52 in Ref. [50]), so that $N_{e \text{wall}}^w >> 4N$. An improved estimate for $\Delta x_{\text{inv}}$ is obtainable by using $\frac{1}{2}N_{e \text{wall}}^w$ instead of $N$ in (2.18), where $N_{e \text{wall}}^w$ is calculated from (2.14).

Another important property of the inverse sheath is that its spatial size is very small. To see this quantitatively, we compare $\Delta x_{\text{inv}}$ to a common estimate of a non-emitting classical Debye sheath size $\Delta x_D \approx 10\lambda_D$, (from p. 76 of Ref. [1]), where $\lambda_D = (\varepsilon_0 T_p/e^2 N)^{1/2}$ is the Debye length. Dividing $\Delta x_{\text{inv}}$ by $10\lambda_D$ gives,

$$\frac{\Delta x_{\text{inv}}}{\Delta x_D} \approx \sqrt{\frac{T_{\text{emit}} \ln \gamma}{50T_p}}$$  \hspace{1cm} (2.19)

Because $T_p >> T_{\text{emit}}$ and because of the $(50)^{1/2}$ factor, it follows $\Delta x_{\text{inv}} << \Delta x_D$. We conclude that the inverse sheath arising when $\gamma > 1$ is far smaller than the classical sheath that would arise if the plasma (same $N$ and $T_p$) were facing a non-emitting material. The inverse sheath is also far smaller than the SCL sheath that could arise in theory for the same $\gamma > 1$. (The structure of the ion-rich part of a SCL sheath is similar to that of a non-emitting sheath, so it has a similar size.)
As a final comment, we can test the accuracy of the equations by checking limits. As \( \gamma \to 1 \) from above, \( N_e^{\text{wall}} \to N \) in (2.14), \( \Phi_1 \to 0 \) in (2.11), and \( \Delta x_{\text{inv}} \to 0 \) in (2.18), as should be expected because if \( \gamma = 1 \) exactly, no sheath is needed between the plasma and the wall. For \( \gamma < 1 \), the inverse sheath solution should break down. Indeed with \( \gamma < 1 \), \( N_e^{\text{wall}} \) and \( \Delta x_{\text{inv}} \) are undefined, and \( \Phi_1 < 0 \) in (2.11), contradicting the premise that the wall potential exceeds the sheath edge potential.

### 2.4.4 Effect of Ion Temperature on the Inverse Sheath

We used \( T_{\text{ion}} = 0 \) in the model for simplicity. For nonzero \( T_{\text{ion}} \), the inverse sheath solution still exists when \( \gamma > 1 \). Ions will enter the sheath and some will even reach the wall, but the key concept remains; ions entering the sheath to not need to have a flow velocity. When there is no flow velocity at the inverse sheath edge, \( N_{\text{ion}}(\varphi) \) decreases with increasing \( \varphi \) as the wall is approached. So because the electron density increases with increasing \( \varphi \) (shown in Sec. 2.4.3), the charge layer between the edge and the wall is automatically negative to shield the positively charged wall. Hence the argument of Sec. 2.4.1 that the inverse sheath solution exists is also valid for \( T_{\text{ion}} > 0 \).

The quantitative model can be extended to account for thermal ions. Thermal ions will enter and reflect from an inverse sheath in the same way that thermal plasma electrons behave in a classical sheath. When \( T_{\text{ion}} \approx T_{\text{emit}} \), the influence of ions is negligible, and the cold ion approximation is valid. For larger \( T_{\text{ion}} \), the ions produce a significant charge density in the inverse sheath, which will cause the sheath spatial size to increase.
Nonzero $T_{\text{ion}}$ will also produce a nonzero $\Gamma_{\text{ion}}$. If the ions approaching the wall are half-Maxwellian at the sheath edge, the flux in terms of $\Phi_{-1}$ is,

$$
\Gamma_{\text{ion}} = \frac{N}{2m_{\text{ion}} \pi} \sqrt{\frac{T_{\text{ion}}}{2m_{\text{ion}} \pi}} \exp \left( \frac{-e\Phi_{-1}}{T_{\text{ion}}} \right) \frac{1}{1 + \text{erf} \left( \frac{e\Phi_{-1}}{T_{\text{ion}}} \right)}
$$

(2.20)

When deriving (2.20), it was assumed that the total ion density at the sheath edge is $N$. So the denominator gives the fraction of ions approaching the wall, accounting for wall losses and the return of ions reflected in the sheath. Now if $\Gamma_{\text{ion}}$ is included in the zero current condition (2.9) it can be shown that the (transcendental) solution for $\Phi_{-1}$ becomes,

$$
e\Phi_{-1} = T_{\text{emit}} \ln \gamma + \ln \left( 1 + \text{erf} \left( \frac{e\Phi_{-1}}{T_{\text{ion}}} \right) \right)
$$

(2.21)

In (2.21), the first term $T_{\text{emit}} \ln(\gamma)$ is the same as the $T_{\text{ion}} = 0$ solution. The second term is a positive valued term appearing due to the nonzero $T_{\text{ion}}$. Because of the smallness of $m_e/m_{\text{ion}} (< 10^3)$, the argument of the logarithm is very close to unity for most realistic values of $T_{\text{ion}}/T_{\text{emit}}$ and $T_{\text{ion}}/T_p$. We conclude that hot ions make the sheath potential larger, but not by much. The $\Phi_{-1}$ is dominated by $T_{\text{emit}}$ and $\gamma$. 
2.5 Implications of the Inverse Sheath Effect

The sheath physics for strongly emitting surfaces is relevant to a diverse variety of systems including those mentioned in Section 2.1. The inverse sheath has important implications on PSI because it differs in critical ways from the SCL sheaths that are usually assumed to form.

In general, the state of a plasma depends on the balance between the generation of ion-electron pairs and their losses [1]. In the classical and SCL regimes, the loss rate of charged particles to the boundaries is a value (independent of $\gamma$) determined by the Bohm velocity and plasma density at the sheath edge. In the inverse regime, the loss rate of ions and electrons to the boundaries is zero for low $T_{\text{ion}}$, meaning the wall is no longer a plasma sink. No neutrals will recycle from the wall in this state. There will be some charge loss if $T_{\text{ion}} \geq ~\Phi_1$ because some ions will have enough thermal energy to escape. But the ion loss rate is always smaller compared to classical and SCL regimes where the ions are accelerated out of the plasma.

Another important consequence of the inverse sheath regime is that the impact energies of any ions hitting the wall are as small as possible. Reduced sputtering could be beneficial in many systems, especially fusion machines. It has long been proposed that deliberate use of emitting wall materials could benefit future tokamaks [15,52]. The argument was that emission reduces the amplitude of classical sheaths, thereby reducing the ion acceleration into the walls. The phenomenon of space charge limitation was thought to limit the maximum benefit of the emission, as $|\Phi_1|$ was assumed to never fall below the SCL limit [15,52]. However, in light of the inverse sheath result, it should be possible to reduce ion impact energies down to their thermal energies. If the thermal energies fall below the sputtering threshold, sputtering would vanish!
When attempting to exploit the inverse sheath for sputtering mitigation, a possible drawback is that the electron energy flux to the wall and the corresponding plasma energy loss are larger. It has always been known that emission enhances the energy flux because the extra plasma electrons which must reach the wall to compensate the emission have larger temperature than the emitted electrons that enter the plasma ($T_p > T_{\text{emit}}$). Authors have stated that the SCL regime is “considered the maximum plasma interaction of ambient plasmas with the surrounding boundary [23]” because $\Gamma_p$ is assumed to never exceed its value at space charge saturation. But in the inverse sheath regime, the energy flux is even larger because all plasma electrons are unconfined.

The difference between $\Phi_f > 0$ and $\Phi_f < 0$ is significant in any application where the surface potentials are an important quantity. For example, when emissive probes are used to measure space potential in plasmas with the floating point method, it is assumed that the sheath is SCL. The space potential is taken to be about $1T_p$ above the measured floating potential of the probe [16]. But if the probe were in the inverse regime, the space potential is below the measured floating probe potential, by a small margin $\sim T_{\text{emit}}$.

2.6 Which Sheath Structure Forms in Reality when Emission is Strong?

2.6.1 SCL or Inverse? - Theoretical Considerations

This dissertation does not intend to claim that SCL sheath theories are “wrong”. Both the SCL sheath and inverse sheath are legitimate theoretical solutions to the strong emission problem. Because the two regimes have drastically different properties as discussed in the previous section,
it is important to determine which sheath will appear in practice. A variety of theoretical arguments could be made favoring either sheath.

For instance, the inverse sheath configuration consisting of a single negative charge layer shielding the positively charged wall (Figure 2.2(e)) is simpler than the SCL configuration consisting of a negative charge layer, a positive charge layer further inward, and an ion-accelerating presheath, Figure 2.2(d). The inverse sheath is also a lower potential energy configuration, making it more stable from an electrodynamics viewpoint. In addition, it may seem more natural for ions to be repelled from a positively charged wall than drawn to it.

On the other hand, it could be argued that a SCL sheath should exist at most surfaces with $\gamma > 1$ as long as $\gamma$ was below unity at some time in the past. For example, a wall contacting a hot laboratory plasma that started from a colder initial state would transition past $\gamma = 1$ as the temperature rises. An analogy for thermionic emission is when an emissive probe is inserted into a plasma before it is heated to emit. In these cases, a SCL sheath with a presheath and a potential dip must already exist before $\gamma$ crossed unity, so the sheath might remain SCL after the transition. But this argument does not guarantee that a SCL sheath would persist indefinitely. Experiments have shown that virtual cathodes are spontaneously destroyed by accumulation of slow ions produced from i-n collisions or charge exchange with slow neutrals [50].

2.6.2 Past Empirical Studies

Because theoretical arguments alone cannot determine which sheath will form under strong emission, empirical studies would be valuable. Unfortunately, probing the space potential in
sheaths is difficult due to their small width. So there are few direct probe measurements of $\varphi(x)$ in emitting sheaths in the literature.

Measurements of space potential showing a virtual cathode dip structure near a surface with SEE were reported recently [53] by Li et al. But the surface was forced below the plasma potential by an electrical bias, not floating as we wish to study here. Intrator et al. measured space potential near a floating thermionic emitting cathode [50]. In Fig. 6 of the paper, it was found when the emission was strongest, the cathode floated more positive than the background plasma, as in an inverse sheath regime, but the $\varphi(x)$ was also nonmonotonic with a dip resembling a SCL sheath. So the result cannot be classified exclusively as one of the planar sheaths in Figure 2.1. Of course, in experiments the sheath physics is often more complex than 1D models can explain. The potential distribution in Ref. [50] was shown to be influenced by 3D nonuniformities in the cathode region and by the presence of an ion beam injected from the plasma source region.

If $\varphi(x)$ cannot be probed, the overall sheath amplitude is still measurable by measuring the impact energy of ions. Schwager et al. [52] used an energy analyzer to measure the energies of ions that passed through a small hole in a thermionically emitting floating plasma-facing surface. The ion energies should indicate the (negative) sheath potential that accelerates them towards the cathode. The authors expected that as the emission intensity was increased, the ion energies would reduce and level off at a minimum value of about $1.7T_e$ when the SCL limit is crossed. Instead, the ion energies approached zero! This result was a major discrepancy with SCL theory. We believe it is consistent with inverse sheath formation reducing ion impact energies down to their small thermal energies.
Another way one can empirically study plasma-surface interaction with emission is by simulation. Particle simulation allows maintaining a simple plasma in one spatial dimension, measuring $\phi(x)$ exactly, and tracking emitted electrons, so that basic sheath physics can be analyzed closely.

In most particle simulation studies of PSI, a plasma is produced at a “source” boundary in front of the “collector” (wall) [21,26,54,55]. Ions and electrons are injected into the plasma domain at the same rate to maintain global neutrality. Because ions and electrons have different thermal velocities, this injection mechanism creates a potential drop called a “source sheath”. The source sheath is not caused by plasma interaction with the collector, which could be arbitrarily far from the source. But the source sheath accelerates ions towards the collector, so that the plasma source facing the collector has drifting ions. This type of simulation setup can artificially distort the physics of the $\gamma > 1$ problem because it “forces” ions to flow to the wall, similar to the ion flow assumption in the SCL sheath theories discussed in Section 2.3.

Schwager presented the seminal simulation-based study of sheaths with electron emission [21]. In Fig. 9 of the paper, a SCL collector sheath, i.e. a $\phi(x)$ with a “dip”, was observed in a simulation with $\gamma = 1.5$. But the same $\phi(x)$ near the source boundary showed an ion-accelerating source sheath of amplitude $\sim$30 times larger than the dip, so the ions were clearly forced to the wall. More recently Zhang et al. simulated PWI with strong SEE to investigate interesting sheath oscillation effects [56]. In the simulation the ions were modeled as a spatially uniform background density flowing to the wall at a fixed velocity set to the Bohm velocity.

Overall, it is not yet known what $\phi(x)$ looks like at strongly emitting floating surfaces where ions are not forced towards the surface. So here we will simulate a full bounded plasma system where the charged particles, and their temperatures, are sustained naturally within the plasma.
itself, and no ion beams are produced. That way ions will flow to the walls if and only if they “need to”.

2.6.3 A New Empirical Study

We will simulate a planar plasma bounded by two walls, see Figure 2.5(a). A background $E$ field parallel to the walls and a $B$ field normal to the walls is included in the model as a natural heating mechanism for plasma electrons, like in a Hall discharge. This $E \times B$ feature is not expected to reduce the generality of the “$\gamma > 1$ problem” results because the $E \times B$ field does not alter particle velocities normal to the walls, and hence does not distort the sheath physics. An electrostatic direct implicit particle-in-cell (EDIPIC) code for this configuration was produced by D. Sydorenko [38]. EDIPIC has been used for modeling PPPL Hall thruster (HT) plasmas [34,35,37,39]. The results were successfully applied to explain experimental measurements discussed in a recent review paper by Raitses et al., see Ref. [8].

A theoretical analysis of the plasma properties and the wall fluxes as a function of the controllable simulation parameters is given in Ref. [34] for applications to HT’s. However, the theory assumes negative sheaths always exist at the walls. When the $E \times B$ drift velocity is sufficiently high, the electrons incident on the walls eject more than one secondary on average ($\gamma > 1$). A detailed theoretical explanation of why this happens is given in the next section. For now we just want to analyze the sheath physics for a simulation with $\gamma > 1$.

Figure 2.5 shows the profiles of $\phi(x)$, $N_e(x)$, $N_{\text{ion}}(x)$ and $V_{x,\text{ion}}(x)$ in the plasma domain. The plasma width was set to only 1mm so that the plasma and sheath regions could be resolved and
the data could be time-averaged over a long enough interval to remove fluctuations in a reasonable computation time. There are 229 grid points spaced uniformly 4.4μm apart.

Figure 2.5: (a) Schematic of the simulation model with the main discharge parameters for the current run listed. (b) The electrostatic potential relative to the right wall. (c) Charge density profiles. (d) Ion velocity normal to the walls. \( V_{x,\text{ion}}(x) \) is the mean velocity of the ions in the two cells neighboring each grid point, normalized to \( c_s = 2053 \text{m/s} \), the ion sound speed calculated from the electron distribution over \( v_x \) in the simulation. Note the grid spacing 4.4μm is more than 10 times smaller than the sheath size, so the sheaths are well resolved. “S.E.” marks the right sheath edge.
Usually for a plasma between two walls [1], \( N_e(x) \) and \( N_{\text{ion}}(x) \) decrease by a factor of about two from the plasma center to the sheath edges because of the presheaths. There is a substantial ion flow velocity throughout the plasma domain; \( V_{x,\text{ion}}(x) \) increases from zero at the plasma center, to \( \sim c_s \) (ion sound speed) at the sheath edges. The potential \( \varphi(x) \) has a local maximum at the center and is positive relative to the walls everywhere between the two sheath edges. The presheath properties are indeed observed for simulations with \( \gamma < 1 \) using the same simulation model [34]. These presheath properties should remain present with secondary emission according to conventional papers that assume the floating potential is negative for all \( \gamma \).

But the profiles in Figure 2.5 sharply differ from conventional PSI. There is clearly no ion-accelerating presheath structure in the system because \( N_e(x) \) and \( N_{\text{ion}}(x) \) are flat between the two sheath edges and the ion mean velocity \( V_{x,\text{ion}}(x) \) is negligible everywhere compared to \( c_s \). The sheath regions consist not of a double charge layer but instead a single negative charge layer (c.f. Figure 2.2). From the sheath edges moving towards the wall, \( N_e(x) \) and \( \varphi(x) \) monotonically increase, and \( N_{\text{ion}}(x) \) monotonically decreases. The potential \( \varphi(x) \) is negative and flat between the sheaths. Overall, the properties of the simulation profiles match the characteristics of the inverse sheath regime predicted theoretically in Section 2.4.

The inverse sheath’s spatial width is 68 \( \mu \text{m} \). This value is within a factor of two of the estimate \( (2\varepsilon_0\Phi_1/eN)^{1/2} = 37 \ \mu\text{m} \) from (2.18) using \( \Phi_{-1} = 2.2\text{V} \) and \( N = 1.8 \times 10^{17} \ \text{m}^{-3} \). The derivation of (2.18) assumed the ion density was zero in the sheath. So the underestimate is likely attributable to the presence of ions in the sheath. With \( T_{\text{ion}} = 0.5\text{eV} \), ions in the simulation can penetrate a significant distance, though very few can reach the wall as \( \exp(-2.2/0.5) \approx 0.01 \).

The author has conducted studies of the sheath structures in this simulation model as the parameters (\( E, B \), and other conditions) are varied over a wide range. It turns out whenever \( \gamma > 1 \),
inverse sheaths form at the walls. While it should also be theoretically possible for a SCL sheath to form, a SCL sheath has not yet been observed in steady state. So it seems the inverse sheath is the more natural solution.

Interestingly, a nonmonotonic $\varphi(x)$ that looks like the SCL sheath does appear in some simulations, as shown previously in Fig. 3 of Ref. [37] by Sydorenko et al. However, it was later shown that the nonmonotonic $\varphi(x)$ appears because a classical sheath with a presheath exists initially, and then the “weakly confined electron” instability causes the wall charge to change from negative to positive before the heavy ions have a chance to respond, (see Fig. 5 of Ref. [31] and the discussion therein). The nonmonotonic $\varphi(x)$ is not a true SCL sheath because the corresponding charge density profiles cannot exist in steady state. In this simulation model, only inverse sheaths have been observed in steady state when $\gamma > 1$.

2.7 Inverse Sheath Effects on a Hall Discharge

Up to now in this chapter, we focused on the properties of the inverse sheath. Because the properties of any laboratory plasma are coupled to the sheaths, a change in the sheath potential from negative valued to positive will have important feedback effects on the plasma. A practical example is illustrated in this section.

It has been deduced that $\gamma > 1$ in Hall thruster channel walls at high voltage based on measurements of the plasma temperature and known SEE yield of the materials [48]. It has always been assumed that the sheaths are SCL [8], though direct measurement of $\varphi(x)$ was not possible. Because the EDIPIC simulation code models a Hall discharge and inverse sheaths appear when $\gamma > 1$, inverse sheaths could be occurring in the experiments too. Here we will
show how sharply the interior plasma properties differ in simulations when the sheath potential is negative compared to when it is positive.

Recall that EDIPIC simulates a planar E×B xenon plasma bounded by floating walls made of BNC, see Figure 2.6. The motion of electrons in the x-direction is governed by the plasma’s self-generated field $E_x(x)$. The motion in the y-z plane parallel to the walls includes E×B drift from the background fields. Electrons also suffer collisions. There are collisions with neutral atoms depending on the gas density $n_a$. Coulomb collisions are implemented with a Langevin model, but can usually be neglected as they only weakly affect the plasma [57]. Turbulent collisions of frequency $\nu_{turb}$ effectively simulate anomalous conductivity by scattering the y-z component of the velocity vector [58]. Each scatter leads to an energy gain parallel to the walls on average of $<\Delta W_\parallel> = m_e V_D^2$.

Past simulations modeling the PPPL HT found that in the low collisionality regime anticipated in experiments, the emitted electrons form beams that cross the plasma and strike the other wall [34]. When the sheath potentials are negative, most plasma electrons are trapped. The particle flux at each wall consists of collision-ejected electrons (CEE’s) scattered into the loss cone by impacts with neutrals, “beam” electrons from the other wall, and ions. $\Gamma_{ion}$ is given by the Bohm criterion in terms of the electron temperature normal to the walls, $T_x$ [34]; $\Gamma_{ion} \approx (n/2)(T_x/m_i)^{1/2}$. In quasisteady state, the zero current condition applies. By symmetry, the two beams are equal and opposite. So at each wall, the incident beam and outgoing SEE are equal. The floating condition becomes $\Gamma_{e,net} = (\Gamma_{CE} + \Gamma_b) - \Gamma_b = \Gamma_{CE} = \Gamma_{ion}$. Also, the SEE produced by the influx must yield the outgoing beam $\Gamma_b$. That is, $\gamma_{CE} \Gamma_{CE} + \gamma_b \Gamma_b = \Gamma_b$. We obtain,
Above, $\gamma_b$ and $\gamma_{CE}$ are the partial SEE coefficients. (e.g., $\gamma_{CE} \equiv$ ratio of secondary flux produced by CEE’s to $\Gamma_{CE}$). The coefficient $\gamma_{net}$ is the net emission induced by both electron populations, equivalent to the “$\gamma$” used earlier in this chapter. It has been found [34] that a classical (non-SCL) negative sheath potential forms even if $\gamma_{CE}$ is well above unity because as long as $\gamma_b < 1$, $\gamma_{net} < 1$ also via (2.24). Past simulations typically used $B_x = 100G$ and $E_z = 50$-200V/cm. For $E_z = 200V/cm$, it was found that $\gamma_b$ approaches unity (~0.92-0.95). This is possible because the drift energy gained by secondaries crossing the plasma can range up to $2m_eV_D^2 = 45eV$, comparable to the $\gamma(\epsilon) = 1$ threshold for B.N.C., where $\gamma(\epsilon) \approx 0.17\epsilon^{1/2}$ ($\epsilon$ in eV) [59].

The plasma electron temperature in EDIPIC simulations is usually governed by the $E\times B$ drift energy and the turbulent collision frequency, $T_e \sim v_{turb}(E_z/B_x)^2$. We run a Simulation A with $E_z = 200V/cm$, $B_x = 100G$, $n_a = 10^{12} \text{ cm}^{-3}$, $n_0 = 10^{11} \text{ cm}^{-3}$, $v_{turb} = 1.4\times10^6 \text{s}^{-1}$ and $H = 2.5cm$. Simulation A exhibits the familiar behavior discussed in previous papers. We compare it to Simulation B with all conditions equal except $E_z = 250V/cm$ and $v_{turb} = 2.8\times10^6 \text{s}^{-1}$. One may expect the larger $E_z$ and $v_{turb}$ to produce a hotter plasma with a larger sheath potential in Sim. B.

But instead, the physics of Sim. B fundamentally changes. Figure 2.6(b) shows the electrostatic potential function $\phi(x)$ in both runs, relative to the right wall. Sim. A exhibits a nearly symmetric potential well of amplitude $\Phi \approx 21V$ with well-defined classical sheaths near
the walls. In Sim. B, \( \varphi(x) \) has no apparent sheath structure at all. Fluctuations of a few Volts dominate. (The fluctuations are due to two-stream instability caused by the SEE beams [60]. If the \( \varphi(x) \) data is time-averaged over a long interval, only the sheath and presheath structures remain. The data for Figure 2.5 was time-averaged over a sufficiently long interval to eliminate the fluctuations but the data for Figure 2.6 below was not).

Figure 2.6 (a) Simulation model. (b) \( \varphi(x) \). (c) \( \varphi(x) \) near the left wall. Electron and ion densities near the left wall in Sim. A (d) and Sim. B (e).

The unusual behavior in Sim. B occurs because the SEE beams themselves induce more than one secondary on average. The flux components and partial SEE coefficients are listed in Table 2.1. In Sim. A, a classical sheath appears because \( \gamma_b < 1 \) and thus \( \gamma_{\text{net}} < 1 \). Eqs. (2.22)-(2.24) apply. In Sim. B, the E×B drift energy is \(~50\%\) larger, making \( \gamma_b \) exceed unity! Eq. (4) suggests
a classical sheath cannot exist because $\gamma_{\text{net}}$ would exceed unity and the ion flux could not be balanced. This means either a SCL or inverse sheath must form to reflect some secondaries to the walls.

<table>
<thead>
<tr>
<th>Sim.</th>
<th>$\gamma_{\text{CE}}$</th>
<th>$\gamma_b$</th>
<th>$\gamma_{\text{net}}$</th>
<th>$\Gamma_{\text{CE}}$</th>
<th>$\Gamma_b$</th>
<th>$\Gamma_{\text{ref}}$</th>
<th>$\Gamma_{\text{ion}}$</th>
<th>$\langle W_x \rangle$</th>
<th>$\langle W_{\parallel} \rangle$</th>
<th>$\langle V_z \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.75</td>
<td>0.94</td>
<td>0.96</td>
<td>3.21</td>
<td>78.7</td>
<td>N/A</td>
<td>2.51</td>
<td>5</td>
<td>89</td>
<td>-6.5</td>
</tr>
<tr>
<td>B</td>
<td>1.28</td>
<td>1.22</td>
<td>1</td>
<td>18.2</td>
<td>1104</td>
<td>248</td>
<td>0.63</td>
<td>2.5</td>
<td>46</td>
<td>-50</td>
</tr>
</tbody>
</table>

Table 2.1: Key parameters in both runs, after quasisteady state was reached. Fluxes are at the left wall in units of $10^7 \text{ cm}^2\text{ns}^{-1}$. Average electron energies $\langle W_x \rangle$ and $\langle W_{\parallel} \rangle$ are in eV. Axial electron velocity $\langle V_z \rangle$ is in km/s.

Closer study of Sim. B confirms that the system is in the inverse sheath regime. Ions are repelled away from the wall and the net space charge near the interface is negative, see Figure 2.6(e). Some secondaries are reflected to the wall, producing a flux $\Gamma_{\text{ref}}$. This is why the net emission coefficient $\gamma_{\text{net}}$ does not exceed unity even though both $\gamma_b$ and $\gamma_{\text{CE}}$ exceed unity in Table 2.1. In fact, $\gamma_{\text{net}}$ is almost exactly unity. To see why, first consider the ion flux. In Sim. A, ions are accelerated in the sheath to the wall, forming a substantial flux $\Gamma_{\text{ion}}$. The sheath limits $\Gamma_{\text{CE}}$ to maintain (2.22) roughly. In Sim. B, because there is no ion acceleration, the Bohm criterion does not apply. $\Gamma_{\text{ion}}$ is merely 3% of $\Gamma_{\text{CE}}$. ($\Gamma_{\text{ion}}$ is nonzero because with $T_{\text{ion}} \approx 1\text{eV}$ in the simulation, some ions have sufficient thermal energy to overcome the barrier $\Phi_{-1}$ and reach the wall.) With very small $\Gamma_{\text{ion}}$, the net electron flux $\Gamma_{\text{e,net}}$ must be near zero in Sim. B if the current is to be balanced. Hence $\gamma_{\text{net}} = 0.9994 \approx 1$.  

As predicted earlier, the sheath potential changing from negative to positive should have feedback effects on the interior plasma properties. Negative sheath potentials “insulate” the walls from the plasma by reflecting most incoming electrons. Figure 2.7 shows the EV$_x$DF (the EVDF in the direction normal to the walls, integrated over V$_y$ and V$_z$) in both simulations. In Sim. A with classical sheaths, bulk plasma electrons in the interior of the plasma volume with $\frac{1}{2}m_eV_x^2 < e\Phi$ are trapped and oscillate in the potential well. They cannot hit the wall unless they have large V$_{\parallel}$ and get scattered into the loss cone (W$_x > e\Phi$) by a collision. Because collisionality is low, replenishment of the loss cone is weak and there is a sharp cutoff in the Gaussian bulk EV$_x$DF at $V_x = \pm V_{\text{cutoff}} \equiv (2e\Phi/m_e)^{1/2}$. Secondaries form small humps beyond the cutoff velocity. Overall, in Sim. A, the walls are protected from most of the hot electrons in the system.

Figure 2.7: EV$_x$DF for electrons in the “middle” of the system (0.8cm < x < 1.7cm) in both runs. V$_x$ is in units of V$_{\text{cutoff}} \approx 2.7 \times 10^8$ cm/s (the cutoff velocity for Sim. A with $\Phi \approx 21$V). For Sim. A, secondaries and bulk plasma electrons are plotted separately. In Sim. B, all electrons are secondaries. The “humps” in the EV$_x$DF are responsible for the two-stream fluctuations observed in Figure 2.6 [60].

Because the sheaths are positive in Sim. B, the interior plasma EVDF is different. Electrons travel freely into the walls and are replaced by secondaries. So all electrons are actually
“secondaries” recently emitted from a wall. The EVₓDF takes the form of two oppositely directed SEE beams, similar in shape to the smaller SEE beams in Sim. A. EDIPIC diagnostics record the average kinetic energy of all electrons in the plasma in the directions normal and parallel to the walls. In Sim. A, \( \langle W_x \rangle = 5\text{eV} \) and \( \langle W_\parallel \rangle = 89\text{eV} \). Sim. B has \( \langle W_x \rangle = 2.5\text{eV} \) and \( \langle W_\parallel \rangle = 46\text{eV} \). So the plasma in Sim. B that was expected to be hotter due to larger \( E \times B \) drift energy is actually colder because of the positive sheaths! The reason is that the electrons are untrapped; most electrons will reach a wall before suffering any collisions that lead to increases of \( W_x \) or \( W_\parallel \).

Another important feature of Sim. B is that the energy flux to the walls is enormous. With positive sheaths and \( \gamma_{\text{net}} \approx 1 \), all electrons in the system can be thought of as traveling back and forth from wall to wall repeatedly. The secondaries, though emitted cold, will gain drift energy and displace along \( E_z \) before impacting the other wall, enhancing energy flux and cross field transport (NWC). Compared to Sim. A, the energy flux is found to be 20 times larger in Sim. B and the near-wall conductivity transport \( \sim \langle V_z \rangle \) is 8 times larger (see Table 2.1).

We have found with \( B_x \) fixed at 100G, the inverse sheath appears in simulations with electric field \( E_z \) exceeding 200V/cm. The transition occurs because even the “coldest” electrons in the system have drift energies parallel to the wall oscillating from 0 to \( 2m_eV_D^2 \). Therefore, when \( V_D \) reaches a critical value, the average emission induced by secondary electron beams \( \gamma_b \) exceeds unity. When this happens, a classical sheath cannot maintain zero current anymore and the inverse sheath appears. This result may have a connection to an important effect attributed to SEE in HT experiments. For wall materials with substantial SEE yield such as B.N.C., SEE leads to saturation of the temperature \( T_e \) and the maximum thrust field \( E_z \) at high voltages [8]. In experiments, the discharge voltage is set. \( E_z \) and \( T_e \) are axially nonuniform, determined self-
consistently with the axial transport and the balance between heating and losses. In EDIPIC, the fields are fixed and uniform, so the self-consistency is not captured. But the simulations suggest that as the voltage is increased in a HT, the E×B drift energy of electrons will reach a critical value in which the insulating negative sheaths begin to collapse. Further increases in $E_z$ and $T_e$ would be suppressed by the enhanced transport and energy loss that occur in the positive inverse sheath regime.

2.8 Summary and Context

In this chapter, a type of sheath structure that can theoretically appear at plasma-facing surfaces emitting strong secondary, thermionic or photoelectron fluxes was introduced. We explained that the past theoretical models which predicted negative sheath potentials (“SCL” sheaths) to form when $\gamma > 1$ contained an explicit assumption that ions must fall to the wall. But we showed that when $\gamma > 1$, the zero current condition and the plasma shielding requirement can be maintained without ion flow to the wall. Relaxing the ion flow assumption at the sheath edge leads to the new “inverse sheath” solution.

In the inverse sheath regime, the potential $\phi(x)$ monotonically increases from the sheath edge to a positively charged wall. The sheath’s charge distribution is not a double layer, but a single layer of negative charge. Ions are repelled from the wall and the ion velocity is zero everywhere in the plasma. An analytical inverse sheath solution was derived for a plasma-wall system where the plasma electrons and emitted electrons are Maxwellian with temperatures $T_p$ and $T_{emit}$. The inverse sheath amplitude $e\Phi_1 = T_{emit}ln\gamma$ and its spatial width, estimated as $\Delta x_{inv} = (2\varepsilon_0 T_{emit} ln\gamma/e^2N)^{1/2}$ are much smaller than a classical Debye sheath or SCL sheath.
The inverse sheath effect drastically alters how a plasma interacts with a wall. Most importantly, with ions being confined, the sputtering and charged particle loss to the wall are reduced or eliminated. No presheath potential gradient exists inside the plasma to accelerate the ions. The distributions of potential, ion density and electron density in the sheath and the plasma are much different in the inverse regime compared to the classical and SCL regimes.

There is significant empirical evidence to claim that the inverse sheath is a realistic phenomenon. An experiment by Schwager et al. reported ion impact energies nearing zero at a thermionically emitting surface, contradicting their prediction based on SCL sheath theory that the ions must be accelerated to a larger energy [52]. This writer believes their result is consistent with inverse sheath formation. More empirical evidence of inverse sheaths was presented in this chapter, where a high voltage Hall discharge with SEE coefficients exceeding unity at the walls was simulated. It was found that inverse sheaths formed; ions were confined in the plasma and there was zero ion flow throughout the domain. Although past particle simulation studies including Schwager’s [21] reported formation of a SCL sheath at an emitting wall, the models contained unrealistic “source sheaths” that accelerated the ions towards the wall.
3. **TRANSIT OF EMITTED ELECTRONS BETWEEN SURFACES**

3.1 Motivation

Recall that most theoretical models [19,21,23,24] and particle simulation studies [21,26,56] of sheaths with emission treat a plasma source contacting one floating wall. In one-wall models, the flux balance is simple. An increased emitted flux is compensated by an increased influx of plasma electrons up until the sheath reaches the SCL/inverse thresholds, as discussed in Chapter 2. The sheath potential and energy transmission are therefore always calculable in principle if the plasma properties and the one wall’s emitted flux are known.

However, because plasmas are usually surrounded by surfaces, it is important to consider whether the emitted electrons transit from surface to surface and change the flux balance. Probes have detected secondary electrons accelerated by (negative) sheaths propagating deep into a plasma as a directed beam [61]. Naturally by energy conservation, secondaries have enough energy to overcome a sheath of amplitude equal to or smaller than the one they came from.

The fate of the secondaries depends on their collisional mean free path relative to the distance between surfaces. In the high collisionality limit, secondaries thermalize with the interior plasma electrons before reaching surfaces, making each surface behave like a one-wall system. In the low collisionality limit, secondaries should propagate to surfaces without colliding, thereby contributing another flux source in addition to the plasma electrons. Transit is relevant not only for secondaries but any emission type. For example, direct flight of electrons from cathode to anode occurs in Knudsen thermionic converters[17].
Observations of secondaries transiting to surfaces can be found in the recent literature. SEE from the lunar surface was detected reaching a spacecraft in orbit [62]. At low operating pressure, energetic secondaries from plasma immersion ion implantation targets generate x-rays upon impacting the surrounding chamber walls [12]. Hall thruster (HT) simulations show secondaries crossing from each channel wall to the other [34,63] (also in Section 2.7). In simulations of a low pressure hollow cathode discharge, some trajectories of secondaries from the cylinder appear to reach the cylinder again, see Fig. 2(f) of Ref [64]. In most real systems, it may be difficult to directly prove that secondaries reach surfaces especially if they have low impact energy or if the surfaces are dielectrics. But in light of the energy conservation argument and the diverse examples above, transit is likely a common phenomenon.

The possibility of transit has been acknowledged in some past sheath models. A rigorous model of a planar plasma bounded by walls with equal electron-induced SEE coefficients accounting for transit was treated by Ahedo and Parra [33]. In that configuration, the two transiting SEE “beams” cancel exactly due to the symmetry. This is critical because the emission has no influence on the wall potentials no matter how intense it is!

But if a system is asymmetric, interesting new complexities arise because the transiting beams will be unequal. For example, if one wall has a larger SEE yield (Figure 3.1(a)) or larger surface area (Figure 3.1(b)) it will emit more electrons and thereby transmit more electrons to the other wall than it receives. Also, even if two opposite walls emit the same flux of electrons, the transiting beams will be unequal if some secondaries from one wall cannot reach the other wall due to a potential difference from biasing (Figure 3.1(c)), or a magnetic mirror force (Figure 3.1(d)). In this chapter, we will analyze effects of transit on PSI in asymmetric conditions. First in Section 3.2, we discuss some general concepts. Then we simulate a practical example in
Section 3.3 and apply the general concepts to analyze the results. Some applications of transit are pointed out in Section 3.4.

![Diagram of asymmetries](image)

Figure 3.1: Examples of asymmetries that cause surfaces to exchange emitted electrons at unequal rates. Arrows show representative trajectories through the interior region.

### 3.2 Flux Balance with Transit – General Phenomena

In this section, we discuss some effects of transiting electrons in qualitative terms before verifying the concepts in a simulation later. Consider a weakly collisional plasma between two planar surfaces that face each other. Suppose the walls emit fixed thermionic fluxes $\Gamma^L_{\text{emit}}$ and $\Gamma^R_{\text{emit}}$. The left and right walls have negative sheath potentials defined by positive amplitudes $\Phi_L$ and $\Phi_R$. Assume the plasma is uniform, meaning the temperature $T_p$ of plasma electrons approaching each wall is the same and the plasma electron flux $\Gamma_p(\Phi)$ at each wall is the same function of $\Phi$. 
One conclusion that can be drawn whether the system is symmetric or asymmetric is that transiting electrons make both walls float more negatively relative to the plasma than they would if the emission thermalized. When the emission thermalizes, only plasma electrons are collected. Transit causes both walls to collect more electrons \textit{for a given sheath potential}. Both sheath potentials therefore must be more negative to balance the Bohm ion flux.

Suppose the thermionic fluxes are unequal, $\Gamma_{\text{emit}}^R > \Gamma_{\text{emit}}^L$. It is clear that if the wall potentials were equal ($\Phi_L = \Phi_R$), the plasma electron flux to each wall would be the same, but the \textit{net} flux of electrons at the left wall is larger because the transiting beams are unequal. Both walls cannot be floating if they have different electron fluxes because the ion fluxes, governed by the Bohm criterion, are the same at each wall. So the only possibility is for $\Phi_L$ to exceed $\Phi_R$. The potential difference $\Delta\Phi = \Phi_L - \Phi_R$ (a) causes fewer plasma electrons to hit the left wall and (b) causes fewer electrons from the right wall to reach the left wall (they reflect off the left sheath and return to the right wall). These two factors both serve to reduce the imbalance of the net electron fluxes. So for some positive $\Delta\Phi$, both walls will float.

How large is $\Delta\Phi$? In the low collisionality limit, all emitted electrons from both walls are recaptured at one of the walls. We call this concept the “transit principle”. For given $\Delta\Phi$, the emitted electron temperature $T_{\text{emit}}$ determines how many electrons from the right wall will reach the left wall and how many return to the right wall. We surmise that $\Delta\Phi$ cannot exceed a few $T_{\text{emit}}$ or else all emitted electrons from both walls will hit the right wall, causing the net electron flux at the right wall to be larger than the left wall. This is in sharp contrast to how sheaths behave in systems where the emission thermalizes. In the thermalization case, each wall behaves like a one-wall system [19] and has $\Phi$ ranging from $\sim 4T_p$ without emission to $\sim 1T_p$ when $\Gamma_{\text{emit}}$ is large (SCL/inverse sheath threshold). Two opposite walls with $\Gamma_{\text{emit}}^R > \Gamma_{\text{emit}}^L$ can therefore have
ΔΦ up to a few T_p. Because T_{emit} << T_p, we see that transiting electrons cause two walls to float at much closer potentials.

The general results discussed above can be applied to a variety of systems. We discussed a case of two planar surfaces with \( \Gamma_{emitt}^R > \Gamma_{emitt}^L \). Suppose instead that we have an annular plasma between two circular surfaces. If both walls emit the same flux \( \Gamma_{emit}^{inner} = \Gamma_{emit}^{outer} \) (same number of electrons per unit area) but the surfaces have different areas, there will be a net flow of electrons from the larger outer surface to the smaller inner surface. This requires that \( \Phi_{inner} > \Phi_{outer} \) for the walls to float. By comparison, when the emission thermalizes, the inner and outer walls behave like a one-wall system where the sheath potential is a function only of T_p and \( \gamma \), independent of surface areas. We see that an intricate coupling between the sheaths of both walls occurs for systems with any kind of asymmetry that causes an imbalance in the transiting beams. As another example, if the walls are planar and emit the same flux \( \Gamma_{emit}^{inner} = \Gamma_{emit}^{outer} \) but there is a nonuniform B field that mirror reflects some electrons from the right wall (Figure 3.1(d)), then \( \Phi_R > \Phi_L \). In an arbitrary system with any type of asymmetry, it can be determined which surface floats more negatively by determining which surface would have a larger net electron flux if ΔΦ were zero.

### 3.3 Flux Balance with Transit – Simulated Example

We will simulate a planar plasma between floating walls using EDIPIC as before. We set the plasma width H = 2.5 cm, xenon neutral density \( n_n = 10^{12} \text{ cm}^{-3} \), plasma density \( n_p = 5 \times 10^{10} \text{ cm}^{-3} \), field values \( E = 100\text{V/cm} \), \( B = 100\text{G} \), and turbulent collision frequency \( v_{turb} = 2.8 \times 10^6 \text{ s}^{-1} \).
These parameters are similar to those in past papers where classical sheaths formed at the walls [34,35,39]. But in all past papers the left and right SEE yields were both set to model boron nitride ceramics with $\gamma_{BNC}(\varepsilon) \approx 0.17\varepsilon^{1/2}$ (in eV). The transiting beams self-canceled in the flux balance and both sheath potentials were equal. Here we introduce asymmetry by setting the SEE yield function of the left wall to $\beta\gamma_{BNC}$, with $\beta$ an adjustable factor.

Figure 3.2: (a) The simulated system. The various particle flux components to the walls that exist when $\Phi_L > \Phi_R$ are sketched. (b) Potential energy of an electron relative to the extremum in the plasma interior. Note the plotted function is “$-\varphi(x)$” so the sheath potentials shown are negative classical sheaths. Sheath amplitudes $\{\Phi_L, \Phi_R\}$ are defined to be positive for convenience.

In this simulation, the anisotropic plasma has approximate electron temperatures parallel $T_{//} = 51$eV and normal $T_x = 7$eV to the walls. Initially, $\beta = 1$. Since the system is symmetric at first, the potential difference between the plasma interior and each wall is equal, $\Phi_R \equiv -\varphi(x=H) = \Phi_L \equiv -\varphi(x=0) = \Phi_{symm} = 19$V, see Figure 3.2(b). The sheaths are classical and electrons with energy normal to the walls $w_x = \frac{1}{2}m_e v_x^2 - e\varphi(x)$ below $e\Phi_{symm}$ are trapped regardless of their parallel
energy $w_\parallel$. The plasma electron flux to each wall ($\Gamma_p$) comes from initially trapped electrons with total energy $w = w_\parallel + w_x$ exceeding $e\Phi_{symm}$ that get scattered into the loss cone ($w_x > e\Phi_{symm}$). (Note $\Gamma_p$ is equivalent to the $\Gamma_{CE}$ used in Section 2.7). SEE from the other wall produces “beam flux” ($\Gamma_B$). More details on the physics behind the plasma properties, $T_\parallel$, $T_x$, $\Phi_{symm}$ and their dependence on control parameters are in Ref. [34].

Figure 3.3: Plot of the fluxes and $\Phi$ at each wall vs. $\beta$. The beam flux $\Gamma_B$ is separated into true ($\Gamma_{B,t}$) and non-true ($\Gamma_{B,n}$) parts. The tables give the net SEE coefficient $\gamma_{net}$ and partial coefficient of each flux component for several $\beta$. A hyphen means no coefficient exists because the flux component is $\approx 0$.

There is another flux $\Gamma_{wc}$ from “weakly confined electrons”. Field fluctuations from plasma waves nudge electrons with $w_x$ slightly below $e\Phi_{symm}$ into the loss cone. The fluctuations also cause some beam electrons to become trapped. In most situations in quasisteady state, the rate of
electrons entering and leaving the loss cone “diffusively” are practically equal; for flux balance one can equivalently assume no beam electrons get trapped this way and $\Gamma_{wc}$ does not exist. Then since the collisional mean free path is large enough that secondaries rarely suffer collisions, one can assume the emitted beams transit fully and cancel in the flux balance. These assumptions are justified by the flux data in Figure 3.3 because when $\beta = 1$, $\Gamma_P = \Gamma_{ion}$ at each wall.

Now $\beta$ is varied quasistatically from 1 to 0 in the simulation. The plasma properties and EVDF are unaffected, so the variation of fluxes and wall potentials is due only to the wall material asymmetry. In Figure 3.2(b), $\varphi(x)$ for three $\beta$ values is plotted. We see that reducing $\beta$ causes $\Phi_L$ to increase. Now some emission from the right wall $\Gamma_{emit}^R$ is unable to overcome the left sheath. Hence in Figure 3.3 as $\Phi_L$ increases, $\Gamma_P^L$ decreases; the secondaries from the right wall that cannot overcome the left sheath reflect back through the plasma to the right wall, producing a “reflected beam” $\Gamma_{B,ref}^R$. Similarly, as $\Phi_L$ increases, $\Gamma_P^R$ decreases. Now plasma electrons that approach the left wall with $e\Phi_R < w_x < e\Phi_L$ reflect off the left sheath and then hit the right wall ($\Gamma_{P,ref}^R$).

Treating PSI with transiting secondaries appears to be a very complicated problem. There are many different flux components in Figure 3.3 and each component varies with the SEE asymmetry parameter $\beta$. Also, each component induces secondaries at a different average rate (see the “partial SEE coefficient” tables), so the total emitted flux from each wall is not known a priori. The destination of each secondary, its impact energy and SEE induced depends on its emission energy, the potential difference $\Delta \Phi \equiv \Phi_L - \Phi_R$, and $\beta$. Overall, calculating each flux component self-consistently would be formidably complicated. However some simple relations between the components can be derived from the “transit principle” mentioned in the previous
section: in the low collisionality limit, all secondaries are recaptured at a wall. So emission produces no net electron flow into the plasma globally.

The condition for global charge balance is thus that the total flux to all surfaces from plasma electrons (determined by $\Phi_L$, $\Phi_R$) must add up to the total ion flux. The emitted and incident beams add up to zero by the transit principle. In the simulated slab, the ion flux $\Gamma_{ion}$ at each wall is equal and independent of emission by Bohm’s criterion. Because $\Phi_L \geq \Phi_R$ when $\beta \leq 1$ (to be proven later) we have,

$$\Gamma^R_P(\Phi_R) + \Gamma^R_{P,ref}(\Phi_L,\Phi_R) + \Gamma^L_P(\Phi_L) = 2\Gamma_{ion}$$ \hspace{1cm} (3.1)

Note the plasma electrons approaching the left wall with $w_x > e\Phi_R$ will ultimately hit either the left wall (if $w_x > e\Phi_L$ ) or right wall (if $e\Phi_L > w_x > e\Phi_R$ ), producing the same total flux $\Gamma^R_P$. That is, $\Gamma^R_P = \Gamma^R_{P,ref} + \Gamma^L_P$. It follows,

$$\Gamma^R_P(\Phi_R) = \Gamma_{ion}$$ \hspace{1cm} (3.2)

Eq. (3.2) shows the right wall must float at fixed potential $\Phi_R = \Phi_0$, where $\Phi_0$ denotes the potential a wall floats at if there is no SEE! This is why $\Phi_R$ remains almost constant for all $\beta < 1$ in Figure 3.3. It is quite surprising that even though the flux components and coefficients vary with $\beta$ in Figure 3.3, $\Phi_R$ does not. Only $\Phi_L$ varies with $\beta$ because $\Delta\Phi$ is governed by the transiting beams. Since nearly all secondaries reach a wall, the net electron flux to each wall is expressible as,
\[
\Gamma_{e,\text{net}}^L = \Gamma_p^L + \Gamma_b^L - \Gamma_b^R
\]  \hspace{1cm} (3.3)

\[
\Gamma_{e,\text{net}}^R = \Gamma_p^R + \Gamma_{p,\text{ref}}^R + \Gamma_b^R - \Gamma_b^L
\]  \hspace{1cm} (3.4)

Since the walls float, \( \Gamma_{e,\text{net}}^R = \Gamma_{\text{ion}} = \Gamma_{e,\text{net}}^L \). Equating (3.3) with (3.4), using \( \Gamma_p^R = \Gamma_{p,\text{ref}}^R + \Gamma_p^L \), gives a floating condition,

\[
\Gamma_{p,\text{ref}}^R (\Phi_L, \Phi_0) = J_{\text{trans}} (\Phi_L, \Phi_0)
\]  \hspace{1cm} (3.5)

where \( J_{\text{trans}} \equiv \Gamma_b^L - \Gamma_b^R \) is the net “transit current” density exchanged by the walls. If the plasma EVDF is known, \( \Phi_0 \) from (3.2) is calculable and then \( \Gamma_{p,\text{ref}}^R \) is known as a function of \( \Phi_L \). But solving for \( \Phi_L \) in (3.5) exactly is unfeasible because calculating \( J_{\text{trans}} \) vs. \( \Phi_L \) requires calculating the energy distribution of beams self-consistently with \( \Phi_L \).

For the gist of how \( \Phi_L \) is determined, first suppose \( \Delta \Phi = 0 \). Then \( \Gamma_{p,\text{ref}}^R = 0 \), and both beams transit fully to the other wall; \( \Gamma_b^L = \Gamma_{\text{emit}}^R, \Gamma_b^R = \Gamma_{\text{emit}}^L \). The outflux \( \Gamma_{\text{emit}} \) comes from SEE produced by impacting plasma and beam electrons:

\[
\Gamma_b^L = \Gamma_{\text{emit}}^R = \gamma_p^{R} \Gamma_p^R + \gamma_b^{R} \Gamma_b^R
\]

\[
\Gamma_b^R = \Gamma_{\text{emit}}^L = \gamma_p^{L} \Gamma_p^L + \gamma_b^{L} \Gamma_b^L
\]  \hspace{1cm} (3.6)

The SEE coefficient of plasma electrons \( \gamma_p \) depends on plasma temperature and surface material. Generally the SEE coefficient of beam electrons \( \gamma_b \) is less than \( \gamma_p \), but not negligible. Some non-true (backscattered) secondaries will have high enough energies to induce SEE. \( E \times B \) drift energy gained parallel to the walls in transit can also raise \( \gamma_b \) to higher values, as discussed in Section 2.7. For \( \Phi_L = \Phi_R = \Phi_0, \Gamma_p^L = \Gamma_p^R = \Gamma_{\text{ion}} \). Plugging this into (3.6), we can solve for \( \Gamma_b^L \).
and $\Gamma_B^R$; taking their difference gives the transit current that would flow if the potential difference were zero $J^\Delta\Phi=0_{\text{trans}}$,

$$J^\Delta\Phi=0_{\text{trans}} = \Gamma_{\text{ion}} \frac{\gamma_P^R (1-\gamma_B^L) - \gamma_P^L (1-\gamma_B^R)}{1-\gamma_B^R \gamma_B^L} \quad (3.7)$$

If $\beta < 1$, then $\gamma_B^R > \gamma_P^L$ and $\gamma_B^B > \gamma_P^L$. It follows $J^\Delta\Phi=0_{\text{trans}} > 0$. So to maintain $(3.5)$, the wall potentials cannot be equal because $\Gamma_{P,\text{ref}}^R$ would be zero. Since the wall with smaller $\Phi$ must float at $\Phi_0$ for global charge balance, $\Phi_L$ must increase above $\Phi_0$. This increases $\Gamma_{P,\text{ref}}^R$ and decreases $J_{\text{trans}}$ below $J^\Delta\Phi=0_{\text{trans}}$ as some emission from the right wall is sent back to the right wall ($\Gamma_{B,\text{ref}}^R$) contributing no transit current. Since $J_{\text{trans}} \to 0$ for large $\Phi_L$, a solution to $(3.5)$ with $\Phi_L > \Phi_0$ exists by the intermediate value theorem. The smaller $\beta$ is, the larger $J^\Delta\Phi=0_{\text{trans}}$ is, and the further $\Phi_L$ must exceed $\Phi_0$, as in Figure 3.3.

The result that $\Phi_L > \Phi_0$, i.e. that a wall floats more negatively than a non-emitting wall, violates familiar PSI principles. For PSI in a slab without transit, there are no beam influxes. Each wall independently satisfies the floating condition for one-wall models; $\Gamma_P(\Phi) = \Gamma_{\text{ion}}(1-\gamma_P)$ [19,21,23]. If both walls emit, then both have $\Gamma_P > \Gamma_{\text{ion}}$ and $\Phi < \Phi_0$. For a Maxwellian EVDF and xenon ions, $\Phi$ drops from $e\Phi_0 \approx 5T_e$ for $\gamma_P = 0$ to $e\Phi \approx T_e$ for $\gamma_P \geq 1$ if the sheath is SCL. With transit, unless the beams themselves induce $\gamma_b > 1$, neither sheath becomes SCL or inverse (Figure 3.2(b)) even if $\gamma_P^R$ and $\gamma_P^L$ exceed unity (as in this simulation, Figure 3.3). The net emission $\gamma_{\text{net}} < 1$ at both walls. Also since the total plasma electron flux to all walls ($2\Gamma_{\text{ion}}$ via $(3.1)$) is independent of the emission, the energy flux does not increase with emission yield, in contrast to one-wall PSI.
Figure 3.4: Simulation from Figure 3.3 repeated with the same total SEE yield function $\gamma_{BNC}$, except all secondaries are “true secondaries”. Notice here that $\Delta \Phi$ is smaller for a given $\beta$.

The presence of backscattered secondaries plays an important role in PSI with transit. Notice in Figure 3.3 as $\Phi_L$ increases, the true part of the left wall beam flux $\Gamma_{B,L}^L$ decreases faster than the non-true part $\Gamma_{B,L}^N$ because non-true secondaries have a broader range of emission energies. If all SEE was “cold” true SEE, then $\Delta \Phi$ could never exceed a few $T_{\text{emit}}$ or else $\Gamma_B^L$ would be zero, giving the two sides of (3.5) opposite signs. In Figure 3.4, the simulation is rerun, but now the non-true part of the SEE yield is replaced with true SEE. In this run as $\beta$ is varied from 0 to 1.75, $|\Delta \Phi|$ never exceeds 5V (compare to $\Delta \Phi = 18V$ in Figure 3.3 when $\beta = 0$). Figure 3.4 also shows an interesting transition occurs when $\beta$ crosses 1. Because $J_{\text{trans}}^{\Delta \Phi=0}$ changes sign, $\Phi_L$ becomes roughly fixed at $\Phi_0$; then further increasing the emission yield of the left wall only changes the right sheath!

We can also use the simulation model to study transit between mutually biased walls (c.f. Figure 3.1(c)). We model the same plasma as earlier, with $\beta = 1$, but vary the potential difference between the walls, $\Delta \Phi$. Now $\Delta \Phi$ is the known parameter and the current through the walls is the unknown. Because the transit principle still applies, we can determine $\Phi_L$ and $\Phi_R$
using the same charge balance constraint. The wall with less negative potential must have $\Phi = \Phi_0$.

So the other wall has $\Phi = \Phi_0 + |\Delta \Phi|$.

Although the wall materials are symmetric, the sheath asymmetry from biasing drives transit current which influences the current-voltage trace of the walls. In Figure 3.5, we plot the net electron current $J_{net}^R = \Gamma_{e,net}^R - \Gamma_{ion}$ vs. $\Delta \Phi$. The current from just the plasma $J_p^R = \Gamma_p^R + \Gamma_{p,ref}^R - \Gamma_{ion}$ is also plotted. The function $J_p^R(\Delta \Phi)$ resembles a double probe trace, saturating at $\pm \Gamma_{ion}$ for large bias. The difference between $J_p^R$ and $J_{net}^R$ is $J_{trans}$. $J_{net}^R$ has a large slope near the origin because $J_{trans}$ changes sharply, as a few volt bias stops most true secondaries from one wall from reaching the other. $|J_{net}^R|$ actually exceeds $\Gamma_{ion}$ for a range of $\Delta \Phi$, but still approaches $|\Gamma_{ion}|$ for large bias, making the I-V trace nonmonotonic. Note if the wall materials were asymmetric, the trace would be more irregular than Figure 3.5. $J_{trans}(\Delta \Phi)$ and $J_{net}(\Delta \Phi)$ are not odd functions anymore, and surprisingly there is a nonzero current for zero bias ($J_{trans}^{\Delta \Phi=0} \neq 0$), as expected from (3.7).

Figure 3.5: Right wall net current of electrons in units of $\Gamma_{ion}$. The wall material SEE yields are both equal ($\beta = 1$). The asymmetry considered here is just the bias $\Delta \Phi$. 
3.4 Applications

We will briefly discuss some applications where transit in asymmetric conditions may have an important effect on the sheath physics. A recent review of dust grain charging [49] reports evidence that secondaries from grains in dusty plasmas are captured by nearby grains when the grain concentration is high, naturally making grains charge more negatively than they otherwise would. Since it is known that the SEE yield of grains varies sharply with size [49], we predict transit currents between small grains and large grains driven by surface area and SEE yield asymmetries should affect the potential differences of interacting grains.

Transit is expected to occur in HT’s [8,34,63]. There is experimental evidence of asymmetric wall materials influencing radial potential profiles [65]. Other asymmetries that can affect transit current in HT’s are annular geometry and the 1/r magnetic field variation that mirror reflects part of the emission from the outer wall[63], c.f. Figure 3.1(b,d). So based on the results here, it is clear that accurate computation of the sheath potentials in a HT must account self-consistently for the asymmetric transiting beams in the flux balance.

Secondaries and photoelectrons from spacecraft are predicted to be recaptured by its other surfaces in certain situations [3,4]. Differential charging asymmetries arise from sunlight exposure on part of the craft, different component materials, sizes or shapes, etc. For any differentially charged surfaces exchanging electrons, transit will reduce the differential charging. In other words, transit favors reduction of the potential difference because the surface with more negative potential can donate all of its emitted electrons to the other surface whereas some electrons moving in the other direction will be blocked by the potential difference.
Ion-induced SEE is important for RF discharges. Recent work shows that asymmetric electrode materials can drive substantial dc bias across geometrically symmetric capacitively coupled plasmas due to the unequal SEE fluxes [11]. Ref. [11] studied a collisional regime where the SEE is roughly a constant outflux at each electrode. In low collisionality RF discharges, secondaries propagate across the plasma [66]. They can impact the other electrode or reflect off the sheath, eventually reaching either electrode depending how the sheath potentials oscillate in time. Thus transit can make the net flux from SEE at RF discharge electrodes exhibit a complex time dependence that was absent in the collisional system.

3.5 Summary and Context

The interaction of plasmas with electron-emitting surfaces is much different in weakly collisional systems compared to collisional systems. In collisional systems, emission is just a local correction to the flux balance at each wall because the emitted electrons thermalize in the plasma. We showed in weakly collisional systems that applying a “one-wall” treatment [19] to each wall would lead to very inaccurate results. The flux balance becomes a complex global problem where the sheaths are coupled to each other through the electrons that transit from surface to surface. The potential difference of two interacting surfaces is influenced by the emission from both walls as well as the energy distribution of electrons from both walls. The transiting beams only cancel in systems with special asymmetries [33]. We showed that a variety of asymmetric surface conditions from different materials, areas and biasing can lead to net flows of electrons between two walls, called “transit currents”.
Calculation of the sheath potentials in systems with transit must account for the transit currents. Self-consistent calculation is quite complicated. But some general conclusions are always applicable. Firstly, transit makes the sheath potentials at all interacting surfaces more negative. Secondly, transit reduces the potential difference between interacting floating surfaces. These results were verified and illustrated by a Hall discharge simulation where the SEE yields of the two walls were varied. The concepts discussed in this chapter are expected to be relevant to other plasma systems where the electron mean free path exceeds the system size.
4. SHEATH INSTABILITIES CAUSED BY SECONDARY EMISSION

4.1 Introduction

Although plasmas can exhibit numerous types of instabilities, plasma-surface interaction itself is usually assumed to be stable. If a classical sheath potential at a floating surface is made less negative due to a perturbation, more plasma electrons will reach the wall, increasing the negative wall charge and restoring the sheath potential to its initial value. The sheath potential should change substantially only if the plasma properties (such as $T_e$) change.

It is not obvious whether the above stability argument holds in the presence of secondary emission, because the emitted electron flux also changes under a sheath potential perturbation. But in emitting sheath theories, all quantities including the sheath potential, charge distributions, and the emission coefficient $\gamma$ are time-independent $[19,20,23,26]$ and implicitly assumed stable. Nevertheless, “instability” effects attributed to secondary emission (SEE) have been reported in many plasma simulations $[35,36,37,56]$ and experiments$[67,68]$. The instabilities can have important consequences. Changes of the sheath potential modulate critical parameters such as the energy flux to the walls. Because the plasma properties in any device are coupled to the PSI, sheath instabilities can have feedback effects on the plasma. For example, sheath instabilities were reported to change the energy of the whole plasma $[37]$, enhance cross-B transport $[35]$ and launch plasma waves $[67]$.

The precise causes of SEE sheath instabilities and under what conditions they occur have not yet been explained theoretically in the literature. In some simulations, instabilities were known to be connected to SEE because the changes of the sheath potential were accompanied by
simultaneous changes of the emitted electron flux [36,37]. In some experiments, the instabilities were attributed to SEE because their presence depends on wall material [68]. But this does not explain why instabilities occur. One well understood case of instability arises when an energetic electron beam is incident on a surface. The emitted current induced by the beam depends on the beam impact energy, which varies with sheath potential in a way that gives the current-voltage (I-V) trace an unstable branch (“negative differential conductivity”) [67]. However, this mechanism cannot explain all SEE sheath instabilities. Instabilities reported in some simulations and experiments do not contain a high energy beam incident on the surface [37,68]. And if a beam always made a sheath unstable, it could be argued that every sheath is unstable because the plasma electrons hitting a surface are like a superposition of monoenergetic beams.

In this chapter, we aim to classify the causes of and conditions for SEE sheath instabilities in a complete theoretical framework. In Section 4.2, we express the electron current perturbation caused by a sheath potential perturbation in terms of the energy distribution of all electrons approaching the wall. From this we show that the stability or instability of the sheath is determined by a competition between two terms. In Section 4.3, we compare the two terms for various plasma properties to classify conditions that lead to instability. In Section 4.4, the theory is shown to explain three different instability phenomena observed in simulations.

### 4.2 Mechanism of Sheath Instability – General Theory

Although the term “sheath” by definition refers to the region of space charge in front of a wall, the essential force that attracts or repels incoming charged particles is provided by the charge on the wall. It is perturbations of the wall charge that govern whether the sheath is stable
or not. Let us assume for now that the sheath is in the classical regime. Let $\sigma_e$ denote the negative surface charge density and $\Phi$ the sheath amplitude. This way for convenience we have $\sigma_e$ and $\Phi$ as positive quantities throughout.

The sheath is I-V stable [69] at an equilibrium surface potential relative to the plasma $\Phi_{eq}$ if $d\Gamma_{e,net}/d\Phi < 0$ at $\Phi = \Phi_{eq}$, (or equivalently if $d\Gamma_{e,net}/d\sigma_e < 0$ at $\sigma_e = \sigma_{e,eq}$). Any perturbation of $\Phi$ will perturb $\Gamma_{e,net}$ in a way that serves to restore $\Phi$ to $\Phi_{eq}$. On the other hand, the sheath cannot be a static structure if $d\Gamma_{e,net}/d\Phi > 0$ because perturbations of the surface charge would amplify. For a floating surface, $\Phi_{eq}$ is the value of $\Phi$ satisfying the zero current condition. For conducting surfaces, nonzero currents are allowed in some configurations. For instance, if a plasma-facing surface is inductively coupled to a conductor biased at (negative) potential $-V$, then $\Phi_{eq} = V$. Hence any value of $\Phi_{eq}$ is possible depending on the system. Overall, the condition for the sheath potential to be stable at $\Phi = \Phi_{eq}$ is separate from the condition used to determine $\Phi_{eq}$. One must compute $\Gamma_{e,net}$, the incident minus emitted electron fluxes, as a function of all $\Phi$ and differentiate at the relevant $\Phi_{eq}$. For a given interior electron energy distribution function (EEDF) over kinetic energies parallel and normal to the wall $f(w_\parallel, w_\perp)$, along with $\gamma(\varepsilon)$, the SEE yield as a function of kinetic impact energy $\varepsilon$,

\[
\Gamma_{e,net} = \int_0^\infty \int_{-\infty}^\infty f(w_\parallel, w_\perp)[1 - \gamma(w_\parallel + w_\perp - e\Phi)]dw_\perp dw_\parallel.
\]

(4.1)

The form of (4.1) indicates that electrons in the plasma interior with $w_\perp \geq e\Phi$ will reach the wall and have impact energy $\varepsilon = w_\parallel + w_\perp - e\Phi$. Differentiating with respect to $\Phi$ yields two terms,
The stability condition can be expressed in a more lucid form. First, let us write the electron flux as a summation to treat cases where there may be multiple electron components in the source distribution (e.g. a plasma with a beam). Let $\Gamma_{S,\text{in}}(\Phi)$ denote the total influx from component “S”. The differential electron flux terms in (4.2) can be rewritten in terms of the influxes alone to express the stability condition as,

$$
\frac{d\Gamma_{e,\text{net}}}{d\Phi} = \sum_S \left[ \frac{d\Gamma_{S,\text{in}}}{d\Phi} \left(1 - \gamma_{\|,S}(\Phi)\right) + \Gamma_{S,\text{in}} \left\langle \frac{d\gamma}{d\varepsilon} \right\rangle_S \right] < 0.
$$

Equation (4.3) gives general insight into the physics of stability. The first “collection” term is due to the change in number of electrons that reach the surface as $\Phi$ varies, $d\Gamma_{S,\text{in}}/d\Phi$. Because marginally collected electrons with $w_\perp \approx e\Phi$ in the plasma interior will strike the wall with zero normal kinetic energy, their average SEE yield $\gamma_{\|,S}(\Phi)$ is due only to their $w_\|$, as seen in the first integral of (4.2). Because it is always true that $d\Gamma_{S,\text{in}}/d\Phi \leq 0$, the collection term is stabilizing (provided $\gamma_{\|,S} < 1$). This is why sheaths without SEE are I-V stable and can exist statically.

The second “energy” term of (4.3) is the change in emission caused by the change in impact energy of the incident electrons as $\Phi$ varies. Quantitatively as seen in the second integral of (4.2), the energy term is just the influx $\Gamma_{S,\text{in}}$ times the average of $d\gamma(\varepsilon)/d\varepsilon$ over the influx. The energy term is usually destabilizing because $d\gamma(\varepsilon)/d\varepsilon > 0$ for materials in the energy range of the electrons in most systems. Thus overall, we see the collection term must outweigh the energy term for a static sheath to exist with $d\Gamma_{e,\text{net}}/d\Phi < 0$. 
4.3 Types of Instabilities – Specific Cases

4.3.1 Instability due to Fast Electrons or Beams

We can apply the competition between terms in (4.3) to a well understood example shown in Griskey and Stenzel’s experiment [67]. A ~200eV electron beam was projected towards an electrode immersed in a cold $T_e \sim 3$eV background plasma. The I-V trace of this electrode had a negative (unstable) differential resistance in the voltage range where all beam electrons were collected and no plasma electrons were collected. For instance at -100V, the collection term of (4.3) is clearly zero for both the beam and plasma electron components, so the beam’s energy term drives instability. It was shown that spontaneous oscillations would arise if the electrode were inductively coupled to a grid biased to a potential in the unstable branch of the I-V trace. On the other hand, the differential conductivity was positive for electrode voltages where plasma or beam electrons were partially collected (e.g. near -200V or near -1V) because the respective collection term in (4.3) outweighed the energy term(s).

There were three floating potentials satisfying zero current. Two were in the voltage branches where beam and plasma electrons were partially collected, both stable. The third was near -100V in the unstable I-V branch, meaning it would not be observable on an unbiased floating surface. The concepts in this example are important in various systems where fast electrons are present among the colder plasma electrons. For instance, a qualitatively similar I-V trace can be measured for probes in plasmas produced by energetic electrons emitted from hot biased filaments [70]. Triple valued floating potentials, with the middle one being unstable, is also present for spacecraft exposed to ambient energetic electron beams [3,4].
4.3.2 Instability in Hot Maxwellian Plasmas

For higher temperature plasma applications, the SEE induced by the thermal plasma electrons becomes important. To examine stability in such systems, we first consider a simple example; a 1-D Maxwellian source EEDF in front of a wall (with no parallel energy \( w_\parallel \)).

\[
f(w_\perp) = n_0 \left( \frac{m_e}{2\pi T_e} \right)^{1/2} \exp\left( -\frac{w_\perp}{T_e} \right)
\]

(4.4)

To illustrate the basic physics, we use an approximate function for the SEE yield. If we let \( \gamma(\varepsilon) = \varepsilon / \varepsilon_1 \), where \( \varepsilon_1 \) is the energy at which \( \gamma(\varepsilon) \) crosses unity, \( \Gamma_{\text{max,net}}(\Phi) \) can be calculated analytically,

\[
\Gamma_{\text{max,net}} = n_0 \left( \frac{m_e}{2\pi T_e} \right)^{1/2} T_e \left( 1 - \frac{T_e}{\varepsilon_1} \right) \exp\left( -e\Phi / T_e \right)
\]

(4.5)

Rewriting \( \Gamma_{\text{max,net}} \) in terms of the influx \( \Gamma_{\text{max,in}} \), the positive part of (4.5), gives

\[
\Gamma_{\text{max,net}} = \Gamma_{\text{max,in}} \left( 1 - \frac{T_e}{\varepsilon_1} \right)
\]

(4.6)

By analogy to the general stability equation (4.3), the differential flux can be expressed in terms of \( \Gamma_{\text{max,in}} \),

\[
\frac{d\Gamma_{\text{max,net}}}{d\Phi} = -\frac{e\Gamma_{\text{max,in}}}{T_e} + \frac{e\Gamma_{\text{max,in}}}{\varepsilon_1}
\]

(4.7)
If \( T_e < \varepsilon_1 \), then \( d\Gamma_{\text{Max,net}}/d\Phi < 0 \) for all \( \Phi \), meaning the plasma electrons produce a positive differential resistance. Hence sheaths are I-V stable in systems with low temperature plasmas, as might be expected. But as \( T_e \) increases, the EEDF over \( w_\perp \) spreads out, thereby weakening the collection term. Beyond a critical temperature \( (T_e > \varepsilon_1) \), we see that \( d\Gamma_{\text{Max,net}}/d\Phi > 0 \) in (4.7). This raises doubts about whether a static sheath can exist in high temperature plasmas.

It is necessary to note from (4.6) that when \( T_e > \varepsilon_1 \), the emission coefficient of the Maxwellian electrons \( \gamma_{\text{Max}} = T_e/\varepsilon_1 \) exceeds unity for all \( \Phi \). So for a floating surface, a classical Debye sheath with \( \sigma_e > 0 \) could not exist due to the zero current condition. Zero current can instead be maintained with a SCL or inverse sheath with a positively charged wall as discussed in Chapter 2. A new flux component \( \Gamma_{\text{ret}} \) appears in addition to the Maxwellian plasma electrons \( \Gamma_{\text{max}} \), so that \( \gamma_{\text{net}} \leq 1 \). Analyzing stability when \( \sigma_e < 0 \) is more complex because surface charge perturbations will also affect \( \Gamma_{\text{ret}} \) and equation (4.3) must include this species. The influence of returning electrons is stabilizing \( (d\Gamma_{\text{ret}}/d\Phi < 0) \) because if the wall charge becomes more positive, \( \Gamma_{\text{ret}} \) increases and more electrons return to the wall. The returning electron influence makes SCL and inverse sheaths stable, so no sheath instabilities occur at positively charged floating surfaces even if \( \gamma > 1 \).

However, if we consider a biased plasma-facing surface with \( \gamma > 1 \), there is still a branch of negative surface potentials with a negative surface charge \( (\sigma_e > 0) \). The I-V trace is unstable in this branch when \( T_e > \varepsilon_1 \). This is important for conducting surface applications where zero current is not required. For hot plasmas contacting conductors in various circuit arrangements, the same types of spontaneous oscillations demonstrated in the Griskey and Stenzel experiment [67] with a beam are possible and the implications discussed in that paper carry over directly.
In reality, SEE yields \[5, 71\] are nonlinear, nonmonotonic functions of energy and have an angular dependence. But this does not change the conclusion. For arbitrary \(\gamma(e_{\parallel}, e_{\perp})\), the flux of a source EEDF \(f(w_{\parallel}, w_{\perp}) = f_{\parallel}(w_{\parallel})f_{\perp}(w_{\perp})\) with arbitrary \(f_{\parallel}(w_{\parallel})\) that is Maxwellian over \(w_{\perp}\) is,

\[
\Gamma_{e,\text{net}} \propto \int \int f_{\parallel}(w_{\parallel})e^{-\frac{(w^*_{\perp} + e\Phi)}{T_{\parallel}}} [1 - \gamma(w_{\parallel}, w^*_{\perp})] dw^*_{\perp} dw_{\parallel}
\]  

(4.8)

To derive (4.8), we started with (4.1), inserted \(f_{\perp}(w_{\perp})\) from (4.4), and then performed the change of variable \(w^*_{\perp} = w_{\perp} - e\Phi\). Differentiating (4.8) we obtain,

\[
\frac{d\Gamma_{e,\text{net}}}{d\Phi} = -\frac{e}{T_{\perp}} \Gamma_{e,\text{net}}
\]  

(4.9)

Thus if \(\Gamma_{e,\text{net}} < 0\), or equivalently if the SEE coefficient exceeds unity, it follows that \(d\Gamma_{e,\text{net}}/d\Phi > 0\). This is critical because it is well known that SEE coefficients for commonly used plasma-facing components can exceed unity at plasma temperatures in the tens of eV range and above [46]. Eq. (4.9) proves that the components have an I-V characteristic with a negative differential resistance in the voltage range where the surface is negatively charged. Sheath instability phenomena are possible.

### 4.3.3 Influence of Electron Energies Parallel to the Wall on Instability

Equation (4.9) holds independently of the structure of the parallel component of the EEDF, \(f_{\parallel}(w_{\parallel})\), as long as the full EEDF is separable, i.e. \(f(w_{\parallel}, w_{\perp}) = f_{\parallel}(w_{\parallel})f_{\perp}(w_{\perp})\). Parallel energies are often ignored in sheath theories because they are not altered by the sheath and do not affect the
number of electrons that reach the wall. But parallel energies enhance the SEE yield and weaken the collection term relative to the energy term via the $\gamma_{\parallel,S}$ parameter in (4.3). This is relevant to all real plasmas, which have three velocity dimensions. For the case of a 3D isotropic Maxwellian, the critical $T_e$ required for instability is less than the 1-D Maxwellian example.

Parallel energies can also alter the I-V trace for surfaces in plasmas with beams. For example, a “simple beam” with a small spread in energy, normally incident on a surface in a plasma, will contribute a positive electron flux and positive differential resistance in part of the branch of surface voltages where the beam is partially collected. Also, provided the beam flux approaching the wall exceeds $\Gamma_{\text{ion}}$, the surface has a floating potential in this branch. But if the beam is incident at an oblique angle such that $\gamma_{\parallel,\text{beam}} > 1$ in (4.3), it follows that the collection term becomes destabilizing. In addition, $\Gamma_{\text{beam,net}} \leq 0$ for all $\Phi$ and no floating potential near the beam’s energy exists.

The energy term can also change its usual sign if the beam is incident at oblique angle. Materials that obey the universal SEE yield curve shape[71] exhibit a local maximum of $\gamma(\epsilon)$ typically somewhere between 0.2-1 keV. So for most materials, there is an energy range in which $d\gamma(\epsilon)/d\epsilon < 0$ and $\gamma(\epsilon) > 1$. For beams with an energy component parallel to the wall in this range, the energy term is stabilizing.

### 4.3.4 Instability in Hot Weakly Collisional Plasmas – The WCE Instability

A high collisionality is necessary to maintain a Maxwellian EEDF. In applications where collisionality is low, the bulk plasma is non-Maxwellian and secondaries transit to other surfaces [33,34,63] as discussed in Chapter 3. The source EEDF $f(w_{\parallel},w_{\perp})$ in front of each surface is
much different from the Maxwellian case of Sec. 4.3.2, so the weakly collisional case must be analyzed separately. We will find that sheath instability can still occur at high temperature, but the mechanism is somewhat different.

Consider a symmetric weakly collisional planar plasma bounded by floating walls with SEE. Such a plasma is produced in the EDIPIC E×B discharge model in Figure 4.1 at low neutral pressure. Assuming the E×B drift energy is not too large, classical sheaths form at each wall. The sheaths maintain $\Gamma_{e,\text{net}} = \Gamma_{\text{ion}}$. A potential well $\varphi(x)$ of some equilibrium amplitude $\Phi_{\text{eq}}$ exists. Figure 4.1(b) shows a sample of electrons in energy space in a typical simulation. Electrons with total energy normal to the walls $w_x = \frac{1}{2}m_e v_x^2 - e\varphi(x) < e\Phi_{\text{eq}}$ are trapped and oscillate in the potential well. The average energy parallel to the walls $<w_\parallel> = <w_z+w_y>$ far exceeds $<w_x>$ and $e\Phi_{\text{eq}}$ because energy is gained from the electric field $E_z$ while all electrons with $w_x > e\Phi_{\text{eq}}$ quickly escape to the walls. The loss cone is depleted of hot (red) electrons because the collisionality is insufficient to replenish the loss cone. Secondaries (blue dots) are in the loss region because they are emitted from a wall (the “top” of the potential well) and have $w_x > e\Phi_{\text{eq}}$ automatically. They are accelerated through the plasma by the sheath and reach the other wall. Most secondaries are emitted cold with small initial velocities. In transit across the plasma, they undergo drift motion with $w_\parallel$ bounded between 0 and $2m_e V_D^2 = 45\text{eV}$ as seen in Figure 4.1(b). A small portion of the SEE consists of electrons that backscattered off the wall. These may have energies above $45\text{eV}$.
To treat the weakly collisional case with the generalized stability equation (4.3), we must consider positive and negative perturbations of $\Phi$ separately because of the EEDF discontinuity at $w_x = e\Phi_{eq}$. For negative perturbations, the fact that the electron density in the bulk is orders of magnitude larger than in the loss region causes the $d\Gamma_{in}/d\Phi$ factor in (4.3) to be much larger than the energy term. Therefore, the sheath is unstable when the collection term itself is destabilizing, i.e. $(1 - \gamma_{//}) < 0$. This occurs when the effective electron temperature component parallel to the wall $T_{//}$ is large enough that the SEE yield of the marginally trapped “weakly confined electrons” (WCE’s) $\gamma_{WC}$ exceeds unity. Formally, $\gamma_{WC}$ is just $\gamma_{//}$ in (4.3) when $\Phi$ is perturbed negatively. Instability should not occur for positive perturbations of $\Phi$ because marginally lost electrons are mostly cold beam electrons. Positive perturbations of $\Phi$ are canceled by a decreased collection of beam electrons from the other wall with $\gamma_{//,beam} < 1$. 

Figure 4.1 (a) Model of the acceleration region of a HT. (b) Phase plot of bulk (red) and secondary (blue) electrons at $t = 1000$ns in the simulation detailed in the next section.
4.4 Consequence of Instabilities – Simulated Examples

To further understand the effects of sheath instabilities on PSI, one must study not just why they occur, but also what happens when they do occur. A system with unstable sheaths could either undergo a transition to a stable state or oscillate perpetually. The exact behavior will depend on specifics of the plasma device including the EEDF and type of surface. As an illustrative example, we will analyze the time evolution of a weakly collisional plasma slab simulated with EDIPIC.

We recall from earlier that for the E×B discharge simulations in the classical sheath regime, the particle flux at each wall consists of a “collision-ejected electron” (CEE) flux $\Gamma_{CE}$ and a “beam flux” due to transiting SEE from the opposite wall. In equilibrium, it was shown that $\Gamma_{CE} = \Gamma_{ion}$ because the beams cancel in the flux balance. The equilibrium potential $\Phi_{eq}$ limits $\Gamma_{CE}$ to maintain this balance. The beam flux $\Gamma_b$ was shown to be,

$$\Gamma_b = \left( \frac{\gamma_{CE}}{1 - \gamma_b} \right) \Gamma_{CE}. \quad (4.10)$$

The net emission $\gamma_{net}$ is the ratio of the total emitted flux to the incident electron flux at a wall, $(\gamma_{CE} \Gamma_{CE} + \gamma_b \Gamma_b)/(\Gamma_{CE} + \Gamma_b)$. Using (4.10), we obtain,

$$\gamma_{net} = \frac{\gamma_{CE}}{1 + \gamma_{CE} - \gamma_b}. \quad (4.11)$$

There is an additional flux component $\Gamma_{WC}$ formed by weakly confined electrons (WCE) with $w_x$ slightly below $e\Phi$ that are nudged into the loss cone by fluctuations. We mentioned earlier that WCE’s basically trade places with some beam electrons and thereby cancel in the flux
balance in steady state. However, Section 4.3.4 predicts that WCE’s can drive sheath instability if $\gamma_{WC} > 1$. Using the simulation’s particle tracking diagnostics to keep track of the average emission induced by the WCE’s hitting the walls $\gamma_{WC}$ as the plasma evolves is therefore expected to provide a metric of the sheath stability.

We now show that three seemingly different instability phenomena observed in EDIPIC HT simulations are all in fact caused by $\gamma_{WC} > 1$! We present a run for illustration. System parameters are in the ranges used to model contemporary experiments [34]. $E_z = 200 \text{ V/cm}$, $B_x = 0.01 \text{T}$, $n_a = 10^{12} \text{ cm}^{-3}$, $n_0 = 1.1 \times 10^{11} \text{ cm}^{-3}$, $\nu_{turb} = 4.2 \times 10^6 \text{ s}^{-1}$ and $H = 2.5 \text{cm}$. The initial plasma state ($t = 0$) is a uniform Maxwellian EVDF with $T_e = 10 \text{eV}$ in a cold ion background. The sheaths and a depleted loss cone form quickly over ~100ns, so the theory developed here applies to the subsequent evolution of the plasma. The phase plot in Figure 4.1 was taken at $t = 1000\text{ns}$ of this simulation. We used EDIPIC diagnostics to record temporal data of the fluxes and partial SEE coefficients by component (CEE, WCE, beam). Data is plotted in Figure 4.2.

![Figure 4.2: Evolution of key parameters in the simulation including the sheath amplitude $\Phi$, the flux components and the SEE coefficients. Note $\gamma_{CE} \approx 2$ (out of range above). Important times and time intervals in the simulation are labeled from 1 to 7 at the top.](image)
Relaxation sheath oscillations (RSO’s) are quasiperiodic instabilities appearing in interval 3 of Figure 4.2. Typical oscillatory behavior of the total electron flux, average electron energy and potential in RSO’s was first reported in Ref. [37], but the cause of instability was unknown. Here, we see in interval 3 that the instabilities occur when $\gamma_{WC}$ reaches unity. The plot of $\gamma_{WC}$ appears noisy as the WCE flux in steady state is intrinsically fluctuation-driven, but it has been verified over dozens of simulations with RSO’s that $\gamma_{WC} = 1$ is the critical point of instability. The RSO process can now be explained as follows. When $\gamma_{WC}$ reaches unity, the sheaths become unstable; $\Phi$ rapidly drops while $\Gamma_{WC}$ jumps (see the magnified box near $t = 1780\text{ns}$). This causes a corresponding jump in the emitted flux. Because $\gamma_b < 1$, when the larger emitted fluxes transit to the opposite walls and $\Gamma_b$ increases, there is a net absorption of electrons and $\Phi$ rises from its minimum $\Phi_{\text{min}}$ back to its initial value, $\Phi_{\text{eq}}$. The sheath potential increase traps some cold secondaries emitted during the instability into the WCE region of the EEDF ($e\Phi_{\text{min}} < w_x < e\Phi_{\text{eq}}$). This is why $\gamma_{WC} < 1$ after $\Phi$ restores and the plasma enters a stable interval. Gradually, the WCE’s regain energy until $\gamma_{WC}$ reaches 1 again. The process repeats periodically.

Figure 4.3: Closer view of the “beam instability” (point 2 of Figure 4.2).
Another type of instability observed in HT simulations occurs at point 2 in Figure 4.2 (shown more closely above in Figure 4.3). The plasma evolves smoothly from its initial state until at t = 430ns, Φ abruptly drops by ~half and the total electron flux becomes ~10 times larger afterward. In contrast to RSO’s, Φ does not return to its initial value and the plasma permanently changes its state. In Ref. [35], this type of instability was thought to be caused by the sinusoidal modulation of the phase of beam drift energy in its flight time \( \tau_{\text{flight}} \) between the walls, since the beam energy changes after the instability (see \( \gamma_b \) in Figure 4.3). Because the emission velocity is assumed small, \( \tau_{\text{flight}} \) is roughly the same for all secondaries (\( \tau_{\text{flight}} \sim 1/\nu_{x,\text{avg}} \sim \Phi^{-1/2} \)). Thus the beam is coherent and its impact energy becomes, where \( \omega_c = eB_x/m_e \),

\[
    w_b \approx m_e V_D^2 [1 - \cos(\omega_c \cdot \tau_{\text{flight}}(\Phi))].
\]  

A decrease of Φ increases \( \tau_{\text{flight}} \), changing the beam’s phase of \( \mathbf{E} \times \mathbf{B} \) energy upon impact. Ref. [35] argues if \( d\gamma_b/d\Phi < 0 \), the SEE outflux increases so that \( \Delta \sigma_e < 0 \) and instability occurs, similar the case of Ref. [67]. But the derivation overlooked the effect of WCE’s on the system when Φ decreases. The number of WCE’s that reach the walls during a potential drop far exceeds the initial beam fluxes (compare the \( \Gamma_{\text{WC}} \) peak in Figure 4.3 to \( \Gamma_b \) before the instability). So the WCE’s influence on stability dominates when \( \gamma_{\text{WC}} < 1 \). Also, the beam phase theory implies Φ would be unstable in both directions, but potential jumps are never observed in simulations. Only drops occur, as predicted in the WCE theory.

Focusing on the WCE’s reveals that “beam instabilities” always occur as \( \gamma_{\text{WC}} \) crosses unity, as in Figure 4.3. So we see the beam phase changes as a result of a WCE instability. To see why \( \gamma_b \) changes, consider the equilibrium condition (4.10). From (4.12), \( w_b \) can range from 0 to \( 2m_e V_D^2 \). In runs with \( E = 200\text{V/cm} \) and \( B = 0.01\text{T} \), \( 2m_e V_D^2 = 45\text{eV} \). For B.N.C. where \( \gamma(e,0) \approx \ldots \)
$\gamma(\varepsilon) \approx 0.17\varepsilon^{1/2}$ (\varepsilon in eV), $\gamma_b$ could vary in principle from zero to unity. But surprisingly, all simulations with instabilities tended to have $\gamma_b$ near unity. The fact that $\gamma_b$ jumps from 0.55 to 0.96 after instability in Figure 4.3 cannot be a coincidence. Explaining the origin of this behavior is critical because $\Gamma_b$ becomes very large compared to $\Gamma_{CE}$ via (1) when $\gamma_b \to 1$. The E\times B drift motion of secondaries increases axial transport (NWC) and adds to the energy loss in HT’s [8]. When $\Gamma_b$ jumps by a factor of \sim 15 after the transition in Figure 4.3, the total power loss and axial conductivity increase dramatically (~10 times). So this effect has implications on HT efficiency.

Similarly to RSO’s, when $\gamma_{WC}$ reaches unity in Figure 4.3, $\Phi$ drops, causing a jump in $\Gamma_{WC}$ (at both walls). When the intense SEE crosses the plasma, $\Gamma_b$ jumps and $\Phi$ is no longer decreasing. At this point, $\Gamma_{WC}$ is small again, $\Gamma_{CE}$ is weakly changed, but $\Gamma_b$ is still very large. For any $\Gamma_b$, $\Gamma_{CE}$, and $\gamma_{CE}$, there is a $\gamma_b$ such that equilibrium condition (4.10) holds. So after instability, the beams recharge the walls only to the extent needed for a self-consistent equilibrium to establish between the CEE flux, beam flux and emitted flux. Since there is ample freedom in $\gamma_b$ via drift rotation in (4.12), the system is able to remain in a state with larger $\Gamma_b$ simply by restoring to a $\Phi$ in which $\gamma_b$ becomes closer to 1. In general after the first instability of a simulation, as in Figure 4.2, the new $\Phi$ is lower than before the instability. However, once $\gamma_b$ is already near unity, $\Phi$ must restore close to its initial value. This explains why further instabilities after point 2 in Figure 4.2 became quasiperiodic RSO’s.
A newly discovered regime appears in this run when $\gamma_{WC}$ crosses unity at point 6 in Figure 4.2, shown more closely in Figure 4.4. Recall in the previous two instability cases we discussed, $\gamma_{WC}$ reaches unity from below, instability occurs and the system restores to a stable state with $\gamma_{WC} < 1$. But in interval 7, $\gamma_{WC}$ reaches well above unity, so the plasma is perpetually unstable and a new type of oscillation occurs. In Figure 4.4, starting at $t = 3692$ ns, $\Phi$ drops slightly, causing $\Gamma_{WC}$ to increase. $\Gamma_{b}$ then increases after a delay $\tau_{\text{flight}} \approx 10$ ns when the secondaries emitted during the drop transit the plasma. Because $\gamma_{b} < 1$, the excess beam flux recharges the walls to the initial potential. The fundamental difference between this new regime and RSO’s is that here, $\gamma_{WC}$ still exceeds unity even when $\Phi$ restores, so instability quickly reoccurs. This is a true oscillation unlike the periodic instabilities in RSO’s. The characteristic frequency is $\sim 10$ times higher because there is no stable time interval in this regime.
4.5 Summary and Context

Sheaths are usually assumed to be stable structures. In this chapter a theory was presented showing various conditions under which sheaths can become unstable due to secondary emission. For example, we showed that whenever a plasma’s electron temperature exceeds a critical value, the I-V trace of a surface contacting the plasma has a negative differential resistance branch which can cause perturbations of the wall charge to amplify. When the plasma EEDF is Maxwellian, the critical temperature is just the temperature in which the SEE coefficient $\gamma$ exceeds unity for the surface material, a well-defined condition known to be possible at common plasma-facing materials [46]. Other examples were also discussed.

As a practical example, we studied by simulation the time evolution of a sheath instability likely to occur under experimental conditions in Hall thrusters. The “weakly confined electron” instability [32] causes a runaway loss of electrons from the surface and corresponding collapse of the sheath amplitude. The WCE instability can manifest itself in several different ways, (a) as abrupt one-time transitions, (b) as periodic instabilities with quasisteady behavior in between, or (c) as high frequency oscillations. The instabilities caused major changes to the sheath potential, energy flux and near wall conductivity.

We were able to verify that SEE caused the observed sheath instabilities because the secondary electrons could be tracked and the sheath structure could be measured closely in time. This cannot be done in experiments. In plasma devices, unexplained transitions and oscillations are often assumed to be driven by instability processes in the plasma interior. But light of the effects observed in this chapter and elsewhere [35,36,37,56,67,68], SEE-driven sheath instabilities might sometimes be responsible.
5. SELF-AMPLIFICATION OF EMITTED ELECTRONS THAT RETURN TO SURFACES

5.1 Motivation

We have seen that when electrons are emitted from a plasma-facing wall, there are important consequences on the sheath potential and energy transmission factors. In addition, some magnetized plasma systems contain an applied electric field $E$ perpendicular to $B$ [72, 73], so that emission from the boundaries also causes “near wall conductivity” (NWC) [40]. NWC is transport caused by emitted electrons moving across $B$ in the $E \times B$ drift. It is especially important in Hall thrusters (HT’s) [8] and electron ring accelerators [74].

In the literature, the enhanced energy flux and NWC caused by emission are generally predicted to have certain limits. The flux of plasma electrons to a wall has a maximum possible value (the “thermal flux”) in terms of plasma density and temperature. The flow of emitted electrons entering the plasma cannot exceed the thermal flux either (zero current consideration). If the emitted flux from the wall exceeds the thermal flux, we know the sheath must change from the classical Debye to the space-charge limited (SCL) or inverse sheath states, see Figure 5.1(a). A potential barrier returns the “surplus” electrons to the wall. Overall, when an emission barrier exists, the flows of electrons from the plasma into the wall (responsible for energy flux), and from the wall into the plasma (responsible for NWC), are maximum. Therefore, it is normally assumed that the energy flux [19, 27] and NWC [8, 41] cannot get larger even if the emission intensity gets larger. Any additional emitted electrons are assumed to return promptly to the wall without playing a role.
In this chapter we show that returning electrons do play a major role under certain conditions in E×B discharges. Because the returning electrons do not return instantly, they have time to accelerate in the background E field, enhancing NWC, and return to the walls with extra energy, enhancing the energy flux. These enhancements increase further if returning electrons induce secondaries, some of which return to induce more secondaries, etc., feedback amplifying. In this chapter, the physics of returning electrons and their “self-amplification” are explored by theory and simulation.

Figure 5.1: Qualitative sketch of the physics under consideration. (a) Possible potential distributions near plasma-facing surfaces that emit electrons. The SCL and inverse sheaths have potential barriers that return some electrons to the wall. (b) Averaged flows (the trajectory of the centroid) of plasma electrons, escaped electrons and returned electrons in an E×B system with an emission barrier. Incoming plasma electrons have zero average motion across B because their E×B drifts are phase mixed. Emitted electrons start to drift quasi-coherently across B. Some escape the barrier; their average motion across B oscillates but damps with increasing x due to phase mixing [40]. The other emitted electrons return to the wall displaced along E_z. Having more energy than they started with; they may induce secondary emission.
5.2 Theoretical Analysis

Consider a plasma contacting a floating wall, as diagrammed in Figure 5.1. Let $E_z$ and $B_x$ represent the magnitudes of uniform crossed background fields. Plasma electrons striking the wall produce an influx $\Gamma_p$. Let $\gamma_p$ denote the average number of secondaries induced per plasma electron. When $\gamma_p > 1$, the sheath must be SCL or inverse. In the SCL regime, the “virtual cathode” $\Phi_{vc}$ is a potential barrier to secondaries. In the inverse regime, the inverse sheath $\Phi_{-1}$ is a barrier. Amplification would have similar consequences in both regimes. We will treat the inverse case here.

The zero current condition used in Chapter 2 for the inverse sheath derivation must be modified when the returning electrons have a nonzero secondary emission coefficient $\gamma_{ret}$. The floating condition at the wall is now

$$\Gamma_p (1-\gamma_p) + \Gamma_{ret} (1-\gamma_{ret}) = \Gamma_{ion} = 0. \quad (5.1)$$

We recall that the ion flux is zero in the inverse regime because plasma ions are trapped and do not produce a significant wall flux. $\Gamma_p$ is the thermal flux of electrons from the plasma, which is known in terms of plasma density and temperature. The wall material and $T_p$ determine $\gamma_p$ [46]. So $\Gamma_{ret}$ is calculable in terms of known plasma and wall properties if $\gamma_{ret}$ can be calculated. Rewriting Eq. (5.1),

$$\Gamma_{ret} = \frac{\Gamma_p (\gamma_p - 1)}{1-\gamma_{ret}}. \quad (5.2)$$
That is, the returning flux is self-amplified by a factor $(1-\gamma_{\text{ret}})^{-1}$ and becomes large if $\gamma_{\text{ret}}$ approaches unity. This is important because any energy flux and transport produced by returning secondaries amplifies by the same factor.

Amplification also changes the sheath potential as follows. The total emitted flux from the wall is

$$\Gamma_{\text{emit}} = \gamma_p \Gamma_p + \gamma_{\text{ret}} \Gamma_{\text{ret}}. \quad (5.3)$$

Secondaries emitted with initial energies less than $q_e \Phi_{-1}$ will return to the wall (we omit the “-1” subscript from now on in this chapter). If the initial energy distribution of secondaries is an isotropic Maxwellian with temperature $T_{\text{emit}}$, then

$$\Gamma_{\text{ret}} = \left[1 - \exp\left(-\frac{q_e \Phi}{T_{\text{emit}}}\right)\right] \Gamma_{\text{emit}}. \quad (5.4)$$

Combining Eqs. (5.2)-(5.4) yields an expression for $\Phi$,

$$\Phi = \frac{T_{\text{emit}}}{q_e} \ln\left(\frac{\gamma_p - \gamma_{\text{ret}}}{1 - \gamma_{\text{ret}}}\right). \quad (5.5)$$

Recalling that $\gamma_p > 1$, and assuming $\gamma_{\text{ret}} < 1$, one concludes that $\Phi$ increases with $\gamma_{\text{ret}}$. Naturally the barrier amplitude increases because when the secondary emission self-amplifies, a larger fraction of it must be returned for the wall to float. We should note that secondary electron amplification is also possible at surfaces emitting thermionic or photoelectron currents. For example, emissive probes in $E\times B$ discharges collect returning thermionic electrons which could gain enough drift energy to induce secondaries, amplifying the same way. This would increase
the probe heating and the probe’s cross field current perturbation. Also, any emitting conductor biased above plasma potential collects returning electrons [75] that could amplify. In all cases, the degree of amplification is governed by $\gamma_{\text{ret}}$ which we now seek to calculate.

$\gamma_{\text{ret}}$ depends on the impact energies of the returning electrons and the wall’s secondary emission yield. The parameter $\varepsilon_{\text{max}} \equiv 2m_e(E_z/B_x)^2$ is the maximum energy returning electrons could gain from E×B drift motion in the direction parallel to the wall before impact. We will show that two fundamentally distinct amplification types may lead to a high $\gamma_{\text{ret}}$. One occurs at small $\varepsilon_{\text{max}}$ (less than ~1eV) and the other at large $\varepsilon_{\text{max}}$ (tens of eV).

In systems where $\varepsilon_{\text{max}} < \sim 1 \text{eV}$ (including systems with no E×B field), the drift motion is insignificant. Returning electron impact energies equal their emission energies $\sim T_{\text{emit}}$ (a few eV). Such low energy electrons cannot knock out “true secondaries” from solids [5], but modern experiments show that backscattering probabilities can be high [9]. So returning electrons might backscatter efficiently, making $\gamma_{\text{ret}}$ approach unity. Strong experimental evidence of this phenomenon exists in Ref. [75] where the authors deduced that secondaries returning to a positively biased plate bounced off many times before getting absorbed. This did not affect the sheath potential because it was fixed by the bias. But the same “backscatter amplification” could increase amplitudes of potential barriers at floating surfaces in various other systems. The energy flux and NWC would not increase in this regime because backscattering deposits no energy, and on average causes no transport when $\varepsilon_{\text{max}}$ is small.

In systems where $\varepsilon_{\text{max}}$ well exceeds a few eV, returning electrons could gain enough energy to eject true secondaries efficiently. In this regime, the initial parallel energy $\sim T_{\text{emit}}$ is small compared to $\varepsilon_{\text{max}}$, so the parallel motion resembles an electron starting from rest in an E×B field. The impact energy $\varepsilon_{\text{ret}}$ depends on the time it takes to return to the wall $\tau_{\text{ret}}$. 
\[ \varepsilon_{\text{ret}}(\tau_{\text{ret}}) = \frac{\varepsilon_{\max}}{2} \left[ 1 - \cos \left( \frac{B_A q_e}{m_e} \tau_{\text{ret}} \right) \right]. \]  
(5.6)

The \( \tau_{\text{ret}} \) for each secondary depends on its initial velocity \( v_{x,\text{emit}} \) normal to the wall and the potential profile \( \varphi(x) \). Calculating \( \tau_{\text{ret}} \) exactly would require the exact solution to Poisson’s equation for \( \varphi(x) \). For analytical estimation, we approximate that the inverse sheath’s potential gradient is uniform, given by the average gradient \( \Phi/\Delta x_{\text{inv}} \). Its spatial width \( \Delta x_{\text{inv}} \) is estimated in Chapter 2 to be \( (2\varepsilon_0 \Phi/q_e N)^{1/2} \), where \( N \) is the interior plasma density. Each secondary thus faces deceleration \( a = q_e \Phi/m_e \Delta x_{\text{inv}} \). The total return time is then \( \tau_{\text{ret}} = 2v_{x,\text{emit}} a = m_e v_{x,\text{emit}} (8\varepsilon_0 / \Phi q_e^3 N)^{1/2} \).

Now \( \gamma_{\text{ret}} \) is calculable in terms of the measured secondary emission yield function \( \gamma(\varepsilon) \) for the wall material. For returning electrons with initial velocity \( v_{x,\text{emit}} \), their induced emission is \( \gamma(\varepsilon(\tau_{\text{ret}}(v_{x,\text{emit}}))) \), where \( \varepsilon(\tau_{\text{ret}}) \) is (5.6), and \( \tau_{\text{ret}}(v_{x,\text{emit}}) \) was derived above.

The average number of secondaries induced by all returning electrons \( \gamma_{\text{ret}} \) is the average of \( \gamma(\varepsilon(\tau_{\text{ret}}(v_{x,\text{emit}}))) \) over the Maxwellian \( v_{x,\text{emit}} \) distribution,

\[ \gamma_{\text{ret}} = A^{-1} \int_0^{2q_e \Phi} \varepsilon_{\max} \left[ 1 - \cos \left( \frac{v_{x,\text{emit}}^2}{2T_{\text{emit}}} \right) \right] \gamma \left( \frac{B_A^2 \varepsilon_0}{q_e \Phi N} \right) dv_{x,\text{emit}}. \]  
(5.7)

The upper integration limit \( (2q_e \Phi/m_e)^{1/2} \) is the cutoff velocity separating returning secondaries from the secondaries that enter the plasma. The constant \( A \) is the normalization, given by the same integral without the “\( \gamma[] \)”.

Because equation (5.7) contains \( \Phi \), it is coupled to Eq. (5.5) which contains \( \gamma_{\text{ret}} \). Solving the equations numerically gives \( \gamma_{\text{ret}} \) and \( \Phi \) in terms of known system properties.
In Figure 5.2, $\gamma_{\text{ret}}$ and $\Phi$ are plotted over a range of plasma properties. We considered $E_z$ values from 67 to 282 V/cm with $B_x = 0.01$T. The $\varepsilon_{\text{max}}$ values are indicated in Figure 5.2. Typical values 2eV for $T_{\text{emit}}$ [5] and 1.5 for $\gamma_p$ were chosen. To compute $\gamma_{\text{ret}}$, the function $\gamma_{\text{BNC}}(\varepsilon) = 0.17\varepsilon^{1/2}$ is used in (5.7) serving as a good fit to a common plasma-facing material boron nitride ceramics (BNC) [59]. The sheath potential for this system would be $\Phi_0 \equiv T_{\text{emit}}\ln(\gamma_p)/q_e = 0.81$V by (5.5) if $\gamma_{\text{ret}} = 0$. To illustrate the extent of amplification, the plotted potential is normalized to $\Phi_0$.

Figure 5.2: Variation of amplification with $\varepsilon_{\text{max}}$ and N. We plotted $\Phi^* \equiv \Phi/\Phi_0$, so that the dotted line $\Phi^* = 1$ represents the inverse sheath potential without amplification.

Figure 5.2 shows the plasma density N has a strong influence on $\Phi$. This is surprising because in most sheath models with SEE [19,23], even the inverse sheath model of Chapter 2, the sheath potential is independent of N (the fluxes of ions, plasma electrons and e-induced secondaries are all proportional to N). The unexpected behavior predicted here is due to the influence of N on $\gamma_{\text{ret}}$. When N is large, return times are too short for returning electrons to gain
energy accelerating in $E_z$, so $\gamma_{ret} \approx 0$. As $N$ drops, the sheath’s width $\Delta x_{inv}$ increases. This reduces the potential gradient $\Phi/\Delta x_{inv}$ responsible for returning the electrons, thereby increasing return times, making $\gamma_{ret}$ increase. Eventually $\gamma_{ret}$ reaches a maximum and decreases when many returning electrons have time to complete more than a half gyration in the $E\times B$ drift, reducing their energies. For further decreases of $N$, $\gamma_{ret}$ exhibits a damped oscillation and converges to a limit where returned electrons with different $v_{x,emit}$ are well phase mixed.

$\gamma_{ret}$ increases with $\varepsilon_{\text{max}}$ in Figure 5.2 as expected because returning electrons gain more drift energy. Interestingly, since true secondary yield functions can well exceed unity at 10’s of eV energies, one might expect that $\gamma_{ret}$ should exceed unity at high $\varepsilon_{\text{max}}$. This would cause a runaway generation of secondaries similar to a multipactor [76]. Instead, as $\gamma_{ret}$ nears unity, $\Phi$ increases rapidly via (5.5). The increase of $\Phi$ helps limit return times, preventing $\gamma_{ret}$ from getting larger in (5.7). Overall, a different regime of behavior occurs at high $\varepsilon_{\text{max}}$; the sheath potential blows up at low $N$ rather than converge (see the $\varepsilon_{\text{max}} = 90\text{eV}$ curve in Figure 5.2).

### 5.3 Possible Implications

The possibility of returning electrons driving cross-$B$ transport was not previously considered. In $E\times B$ systems including HT’s [41], Penning-type systems [77], magnetrons [78], and electron ring accelerators [74], transport is known to be dominated by anomalous mechanisms [8,36,72,73] including fluctuations and NWC. Past theoretical models of NWC [40,79] calculated transport by secondaries entering a plasma. Because their outflow cannot exceed the maximum $\Gamma_p$, the NWC’s contribution to an $E\times B$ discharge current is assumed to have a
corresponding maximum, see the review of Ref. [8] and references therein. So if returning electrons contribute NWC, as our theoretical analysis suggests, the total NWC will be larger than previously believed possible. Because $\Gamma_{\text{ret}}$ can well exceed $\Gamma_p$ at high $\gamma_{\text{ret}}$ via Eq. (5.2), transport from returning secondaries could be the dominant transport mechanism under such conditions.

5.4 Simulation Study

We now use EDIPIC to simulate an E×B discharge with a high $\epsilon_{\text{max}}$ to investigate returning electron effects. In the early EDIPIC simulations of Ref. [34] with modest $\epsilon_{\text{max}}$, classical sheaths formed at the walls. All NWC was due to escaped secondaries entering the plasma (and transiting to the other wall). Experiments show as the HT voltage is increased, enhanced transport and energy loss arise, limiting the achievable thrust field $E_z$ [8]. This problem is attributed to secondary emission. Some theories predict the sheaths are SCL [8] at saturation. Inverse sheaths are also possible in light of Chapter 2. In either case, returning secondaries exist. But previous theoretical works [41,43,80,81] assumed the NWC saturates beyond the SCL threshold (i.e. returning electrons cause no transport).

Here, we run simulations of a Hall discharge with $E_z = 325$ V/cm and $B_x = 115$ G, giving $\epsilon_{\text{max}} = 91$eV. These field magnitudes were measured in the PPPL HT with BNC walls, see Fig. 9 of Ref. [8]. We set the neutral xenon density to $10^{18}$ m$^{-3}$ and use a turbulent collision frequency $0.7 \times 10^6$ s$^{-1}$. The plasma width is 20mm. Four simulations are run at plasma densities N from $10^{14}$ to $10^{17}$ m$^{-3}$. Results are presented in Table 5.1 and Figure 5.3.
Table 5.1: Simulation data for different plasma densities $N$. $R_{\text{trans}}$ and $R_{\text{heat}}$ describe approximately the enhancement of cross-field conductivity and electron heat flux due to returning electrons, respectively.

Table 5.1 shows a strong variation of $\Phi$ with $N$ even though all other system properties are fixed. This observation is consistent with returning electron amplification, as theorized earlier. To measure amplification, we tracked the average number of secondaries induced by returning electrons $\gamma_{\text{ret}}$ versus plasma electrons (coming in from beyond the sheath edge) $\gamma_p$. Table 5.1 indicates that $\gamma_{\text{ret}}$ approaches unity as $N$ drops, driving $\Phi$ to much higher values. Although $\gamma_p$ also increases at lower $N$, this is not the cause of the stronger sheaths but is an effect of plasma electrons accelerating through them.

Figure 5.3 shows the cross-B current density near the left wall. Because the current in a discharge increases proportionally to $N$ if all else equal, the plotted currents are each normalized $J_z^*(x) = J_z(x)/N$ to compare transport “efficiency” at different $N$. The NWC transport dominates over transport from neutral and turbulent collisions in all runs, so the currents in Figure 5.3 are essentially all NWC. For $N = 10^{17}$ m$^{-3}$, almost all transport is to the right of the sheath edge, from escaped secondaries. This NWC oscillates in space and damps with distance due to phase
mixing, as expected from NWC models [40], c.f. Figure 5.1(b). As N drops, the emitted flux increases sharply because $\gamma_{ret}$ increases. But the transport to the right of the sheath edge varies weakly in magnitude because when an emission barrier is present, the flow of secondaries entering the plasma cannot get larger. This is why NWC is predicted to saturate in conventional theories.

However, the transport efficiency inside the sheath rises drastically as N drops. Quantitatively, the ratio of integrated cross-B current inside the sheath to outside (from the sheath edge to the midplane $x = 10$ mm) increases from $|R_{trans}| = 0.03$ to 30 as N drops from $10^{17}$ to $10^{14}$ m$^{-3}$. Even though some transport inside the sheath in each run is due to the secondaries that escape, they cannot be responsible for the increased transport because their NWC is zero at the wall [40]. Returning electron NWC generally has its maximum at the wall, so the transport efficiency growth at lower N in Figure 5.3, which is largest at the wall, is attributable to returning electron amplification. Amplification enhances the wall’s heating in a similar way. The ratio $R_{heat}$ of the returning electron energy flux to the plasma electron energy flux is much larger when $\gamma_{ret}$ is larger, see Table 5.1.

Figure 5.3: The cross-B current density versus distance from the left wall. The two plots show the same data with different scales for clarity.
5.5 Summary and Context

In this chapter, we showed that the emitted electrons that return to surfaces in E×B plasma systems can constitute a significant mechanism of wall heating and near wall conductivity that was not previously analyzed. In past works, it was assumed that the energy flux comes only from plasma electrons hitting the wall, and that the NWC comes only from emitted electrons that enter the plasma. But the returning electrons contribute to both because they have time to undergo E×B drift motion before impacting the wall, leading to a displacement across B and enhanced impact energy.

When the E×B energy gained is large, returning electrons can eject secondaries, amplifying the emission. When amplification is sufficiently intense, the returning electron flux exceeds the flux of plasma electrons. In this regime, returning electrons can have a dominant influence on the sheath amplitude, energy flux to the wall, and cross field transport. This result was confirmed in an E×B discharge simulation with realistic values of E and B.

If the E×B drift energy gained is small or zero, all returning electrons impact with low energy. In this regime, high backscattering probabilities are possible according to Refs. [9,75]. So even in systems without E×B drift, “backscatter amplification” may make the potential barriers stronger than are normally predicted. In most theoretical calculations of potential barrier amplitudes, it is implicitly assumed that \( \gamma_{ret} = 0 \). Similarly, in applied particle simulations of emitting sheath phenomena, such as the charging of thermionically emitting dust grains [82], the returning electrons are set to absorb with 100% probability.
6. CONCLUSIONS

6.1 Review of this Dissertation and its Applicability

It has long been known or predicted in many plasma applications that electron emission from the bounding surfaces plays an important role. Plasma physicists often apply theoretical emitting sheath models to estimate the effects of emission on the surface potential, energy transmission, sputtering, cross field transport and charged particle losses in the applications. In this dissertation, we showed that the physics of plasma-surface interaction with emission is sometimes much different from what is predicted based on conventional sheath models. Several fundamental phenomena were studied using mathematical theory and particle-in-cell simulations.

In Chapter 2, we showed that the usually negative sheath potential can become positive when the emitted electron flux is strong. The “inverse sheath” transition is reached when the secondary electron emission coefficient reaches unity. SEE coefficients exceed unity when $T_e$ exceeds a threshold (typically 10’s of eV) depending on the wall material. Inverse sheaths could therefore arise in hot plasma systems such as tokamak divertors and Hall thruster channels under operating conditions that yield sufficiently high $T_e$. Inverse sheaths can exist at surfaces with other types of emission when the emitted flux exceeds the plasma electron saturation current. Thermionic emission from emissive probes and photoemission from sunlit objects in space often exceed this threshold.

In Chapter 3, we showed that the emitting sheath potentials at opposite plasma-facing surfaces become coupled in an intricate way when the plasma is weakly collisional. Because the electron mean free path exceeds the distance between surfaces, the emitted electrons do not
collisionally thermalize in the plasma but instead “transit” across it unperturbed, eventually hitting a surface again. Systems where transiting electrons could significantly affect the flux balance include conventional Hall thrusters, dusty plasmas with high grain concentration, and low pressure RF discharges. Surfaces of spacecraft also capture electrons from its other surfaces under certain conditions.

In Chapter 4, we showed that secondary electron emission can cause the sheath potential to undergo a rapid transition or oscillate in time. The sheath instabilities, triggered when the differential conductivity of the sheath is negative, are possible at surfaces facing a plasma that contains beams or has a high $T_e$. Negative differential conductivity branches via beams arise at spacecraft surfaces due to ambient plasma beams and in filament discharges due to the energetic filament electrons. Instabilities can occur spontaneously at conducting surfaces facing a hot Maxwellian plasma (e.g. a collisional scrape-off layer) when the conductor is connected to certain circuits. Instabilities can occur at floating surfaces if the plasma is hot and weakly collisional, as in a Hall thruster discharge.

In Chapter 5, we showed that the emitted electrons that return to strongly emitting surfaces can knock out more electrons, thereby amplifying the emission intensity. Amplification arises at emitting surfaces in E×B plasma systems if the returning electrons gain enough energy parallel to the wall in the drift motion to cause secondary emission upon impact. Magnetized plasma systems including Hall thrusters, magnetrons and electron ring accelerators have applied E×B fields. The amplification is expected to be strongest in HT’s which have the highest E×B drift energy. However, since backscattering probabilities can be high at low energy, a related “amplification” by repeated backscatters is possible at any strongly emitting surface, even if no
$E \times B$ field is present. This includes strong *thermionic* or *photoelectron* emitters in the laboratory and in space.

In the previous chapters, the phenomena of inverse sheath, transiting electrons, sheath instabilities and self-amplification were explained in detail. It was shown that in general, the phenomena have major effects on the surface potentials, plasma charge loss rate, ion impact sputtering, near-wall conductivity and wall heating in plasma systems. Implications on specific plasma systems were predicted in the chapters. This writer hopes that the results motivate further studies. Some suggested avenues of research are discussed in the next section.

### 6.2 Suggested Future Work

The simulations in this dissertation provide strong evidence that inverse sheaths, transiting electrons, sheath instabilities and self-amplification are realistic phenomena. Future experimental confirmation of these phenomena would be particularly valuable to the plasma physics community.

In light of Chapter 2, it is an important open question whether an inverse or SCL sheath forms at strongly emitting floating surfaces. One way to measure the sheath potential is to measure $\phi(x)$ with an emissive probe using the inflection point technique in the limit of zero emission [16]. Although sheaths are too thin to be resolved this way in many applications, the method works when the plasma density is low. In very low density plasma sources produced in Ref. [50] and Ref. [53], emitting sheaths were large enough to be probed, but the $\phi(x)$ profiles were influenced by plasma source acceleration and negative surface biasing, respectively. A
future experiment should measure $\varphi(x)$ at a floating surface in an apparatus where the plasma source is non-drifting; this represents the most common situation. Another method to measure a (negative) sheath potential is to measure energies of ions that pass through an aperture in the surface. Existing experimental data by Schwager et al. show ion energies approaching zero as the emission intensity crosses a threshold [52]. This result is consistent with a positive sheath formation, so a reproduction and further study of this type of experiment is encouraged. Another method used to study sheaths is laser-induced fluorescence of ions (LIF) [25]. Because an inverse sheath leads to a different ion velocity distribution in the sheath than a SCL sheath, LIF might be utilized to infer the sheath type. Note that the LIF and ion energy analyzer techniques can be used for sheaths that are too thin for probes to resolve. If inverse sheath existence can be convincingly demonstrated in any experiments with any technique, then an important application should be explored. Divertor tiles deliberately heated to emit a strong thermionic flux in a tokamak can test the prediction of Chapter 2 that the ion impact damage will reduce or disappear.

Demonstrating the effects of transiting electrons in an experimental system would be more difficult than in simulations because particles cannot be tagged. The flux contributions from emitted electrons, incident beam electrons and bulk plasma electrons are indistinguishable in measurements of the total electron current. In some applications such as plasma immersion ion implantation [12] and RF discharges [66], energetic sheath-accelerated secondaries cause emission of light or x-rays and thereby become detectable. In most systems, transiting electrons will not be energetic enough to detect in this manner. Nevertheless, two distinguishable features of transiting electrons are (a) their characteristic transit time between the surfaces and (b) their sensitivity to the potential difference of the surfaces. Effects of transiting electrons could therefore be studied fundamentally using an apparatus with two emitting electrodes that face
each other within a larger weakly collisional plasma chamber. The presence of transiting electrons will be inferable from the temporal response of the electrode currents to changes of the bias that are faster than the transit time. A strong change of the electrode currents as their potential difference switches sign is also indicative of transiting electrons, as discussed in Chapter 3.

Secondary emission sheath instabilities have already been demonstrated in the experiment by Griskey and Stenzel [67]. Spontaneous oscillations arose when a plasma-facing electrode was linked through a parallel L-C circuit to a grid biased to a potential in the unstable branch of the I-V trace caused by the presence of an electron beam. As shown theoretically in Chapter 4, surfaces facing plasmas where the secondary emission coefficient exceeds unity also contain an unstable branch. A simple suggestion for a future experiment is to verify this prediction by connecting a probe to a similar circuit and inserting it into a sufficiently hot plasma in a tokamak or Hall thruster. Oscillations of the probe potential governed by the circuit inductance, capacitance, and the plasma properties, are expected.

Self-amplification of emitted electrons is measurable in principle if the electron density in the sheath is measurable. Chapter 5 emphasized how the energy flux, NWC and inverse sheath amplitude increase due to amplification. Another effect of amplification is an increased electron density in the sheath. The electron density $n_e$ in the sheath is predictable in terms of the total emission intensity and the known plasma density. One can conclude that amplification is happening if $n_e$ in the sheath is too large to explain otherwise. This reasoning was used in Ref. [75] to show that secondaries returning to a positively biased plate backscattered repeatedly. The $n_e$ in the sheath was inferred from measured velocities of ions accelerated out of the sheath after ionization from electrons accelerated through the sheath. The ionization occurs because of the
large positive bias, so this method would not work at most floating surfaces where the electron acceleration will be too weak to cause ionization. However, we predicted that amplification is most significant in E×B discharges with high drift energy and low plasma density. Low density plasmas have a large sheath within which $n_e$ could be measured by a collecting Langmuir probe.

Future theoretical and simulation research should also be conducted on PSI with emission. This dissertation invoked planar geometry in the theoretical and simulation treatments to analyze the basics of the inverse sheath, transiting electron, sheath instability and self-amplification phenomena. The same phenomena are also expected to be present in 2D and 3D systems under the relevant conditions. However, complexities of multidimensional systems will lead to interesting additional effects important to consider for experimental plasma applications.

For example, plasma-facing surfaces will often have areas that are strongly emitting next to areas that are not. This could be due to a gradient of the plasma temperature along the direction parallel to the wall causing a corresponding gradient in the SEE coefficient, such as in the radial direction of a tokamak SOL or the axial direction of a Hall thruster. It could also be due to a juncture between two different materials with different secondary, thermionic or photoemission properties (e.g. as in Ref. [24]). In such systems, we expect the sheath to be classical at parts of the surface and inverse at others. Recalling from Chapter 2 that the interior plasma structure (presheath) is completely different in the classical and inverse regimes, it follows that large gradients of plasma density in the plasma interior along the direction parallel to the wall must be set up above where the sheath transition occurs. This may have important consequences which are testable by setting up a 2D simulation.

Transiting electrons will also have interesting behaviors in multidimensional systems because the transit currents flowing between surfaces are influenced by asymmetries in their size, shape
and orientation. (In the planar 1D simulations of Chapter 3, the only testable asymmetries were from different wall materials and electrical biases.) Sheath instabilities too will exhibit behaviors in multidimensional systems that do not appear in 1D. The 2D \((r,\theta)\) Hall thruster simulations by Taccogna et al. [36] show radial SEE sheath instabilities, perhaps related to the WCE instability in Chapter 4, driving fluctuations in the azimuthal direction. The azimuthal fluctuations cause increases of axial transport that cannot be studied by EDIPIC’s planar (radial direction only) simulations. Estimating the influence of returning electron amplification on a real Hall discharge also requires at least a 2D treatment. Recall that \(E_z\) was set to a fixed value in the 1D simulations. In practice, \(E_z\) is determined self-consistently by the applied discharge voltage and the cross field transport (and other factors). Increases of transport triggered by self-amplification in the radial sheaths will have feedback effects on \(E_z\), so a \((r,z)\) simulation is needed to capture the coupling.

Overall, there are numerous opportunities for experimental, theoretical and simulation research on plasma interaction with electron-emitting surfaces in which the results of this dissertation can be tested, extended and applied.
BIBLIOGRAPHY


