Large Eddy Simulation of Vertical Axis Wind Turbines

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Abstract

Due to several design advantages and operational characteristics, particularly in offshore farms, vertical axis wind turbines (VAWTs) are being reconsidered as a complementary technology to horizontal axial turbines (HAWTs). However, considerable gaps remain in our understanding of VAWT performance since they have been significantly less studied than HAWTs. This thesis examines the performance of isolated VAWTs based on different design parameters and evaluates their characteristics in large wind farms. An actuator line model (ALM) is implemented in an atmospheric boundary layer large eddy simulation (LES) code, with offline coupling to a high-resolution blade-scale unsteady Reynolds-averaged Navier-Stokes (URANS) model. The LES captures the turbine-to-farm scale dynamics, while the URANS captures the blade-to-turbine scale flow. The simulation results are found to be in good agreement with existing experimental datasets. Subsequently, a parametric study of the flow over an isolated VAWT is carried out by varying solidities, height-to-diameter aspect ratios, and tip speed ratios. The analyses of the wake area and power deficits yield an improved understanding of the evolution of VAWT wakes, which in turn enables a more informed selection of turbine designs for wind farms. One of the most important advantages of VAWTs compared to HAWTs is their potential synergistic interactions that increase their performance when placed in close proximity. Field experiments have confirmed that unlike HAWTs, VAWTs can enhance and increase the total power production when placed near each other. Based on these experiments and using ALM-LES, we also present and test new approaches for VAWT farm configuration. We first design clusters with three turbines then configure farms consisting of clusters of VAWTs rather than individual turbines. The results
confirm that by using a cluster design, the average power density of wind farms can be increased by as much as 60% relative to regular arrays. Finally, the thesis conducts an investigation of the influence of farm length (parallel to the wind) to assess the fetch needed for equilibrium to be reached, as well as the origin of the kinetic energy extracted by the turbines.

*Keywords: Large Eddy Simulation, Actuator Line Model, Vertical Axis Wind Turbine, Wakes, Wind Energy, Vertical Axis Wind Turbine Clusters*
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Chapter One

1 Introduction

Shifting to renewable energy production modes and improving the efficiency of energy extraction from clean sources are paramount grand challenges for scientists, engineers, industry managers, and policy makers. The adverse climatological impacts of fossil fuels are intensifying the need for this shift, and the various policies that set objectives for decreasing CO₂ emissions, and that are being implemented all over the world especially in the US, Europe and China, are setting the timetable for the wider adoption of renewables in energy production. Denmark, for example, has defined one of the most ambitious targets of 50% renewable energy by the year 2030, increasing to 100% by 2050. In March 2007, the European Union adopted a target of 20% renewable energy by the year 2020 (Lund and Mathiesen 2009).

These plans invariably foresee a mix of renewable sources implemented and used in tandem to provide a reliable supply of energy, with wind power often playing a key role in the projected production capacity. Wind is an abundant renewable source of energy; it can provide widely distributed generation; and its operational emissions of greenhouse gases are nearly zero. The United States’ Department of Energy has developed a scenario whereby wind energy would provide 20% of U.S. electricity needs by 2030 (Breu, Guggenbichler, and Wollmann 2008). In order to reach these goals, wind power capacity must increase from the 66 GW installed by the end of 2014 to more than 300 GW in 2030 (Figure 1-1). This scenario requires the installation of more than 16 GW each year after...
2018 (Breu et al. 2008). As the cumulative and new annual installed capacities increase, wind turbines as well as wind farm designs need to be improved to reduce the required land use. Moreover, various configurations and models need to be available to suit the diverse locations and wind climatologies of the different farm sites.

![Cumulative and annual wind installations graph](image)

Figure 1-1 Annual and cumulative wind installations until 2030 according to one DOE scenario (Breu et al. 2008)

There are two main configurations of wind turbine designs: horizontal axis wind turbines (HAWTs) and vertical axis wind turbines (VAWTs) (Figure 1-2). Currently, the majority of research and development in wind energy is directed towards HAWTs, which remain about 20% more efficient than VAWTs when a single turbine in a steady wind is considered, a highly idealized configuration. However, HAWTs require specific wind and terrain conditions in order to be in their optimal working conditions. In comparison, VAWTs present several important advantages including: (1) their blades can increase in size and be manufactured less expensively since they can be built from several sections
and attached to the main hub at several points; (2) since the axis of rotation is vertical, VAWTs can operate with the same efficiency with winds from any direction (omni-directional) and are thus more suitable for locations with variable winds such as over complex terrain or near shores where the land and sea breeze cycle shifts the wind direction rapidly at sunrise and sunset; (3) their optimal performance is at a lower tip speed ratio than HAWTs and consequently they have lower environmental impacts on ecological systems (e.g. birds) and result in lower noise pollution; and (4) the vertical rotation axes allow the electric generators to be placed at the bottom of the tower, making both construction and maintenance of such turbines easier and less expensive. Since the installation and foundation costs are a large fraction of the total cost of aeolic power plants, VAWTs may offer an appealing economic advantage, particularly for offshore farms.

Figure 1-2 Schematic view of HAWT and VAWT (picture courtesy of the Sandia National Lab)
There are two major types of VAWTs, the Savonius and the Darrieus turbines. In the case of the Savonius type, power is generated using a drag device, while for Darrieus type, the power is generated using the lift force over airfoils. Due to their different characteristics, specifically the rotation speeds that in Darrieus models are higher, the Darrieus type VAWTs are more efficient (Sutherland, Berg, and Ashwill 2012). Sandia National Lab is leading research and development of VAWT technology at the turbine level to make it more feasible for large commercial applications in the future [3, 4]; however, several research questions remain open particularly related to VAWT farm designs and performance, and these questions frame the goals of this thesis. HAWT wakes and farm configurations have been studied very extensively both experimentally and numerically [5, 6]. Porté-Agel et al. [7, 8] conducted LES simulations of HAWTs using three different models, an actuator-disk model without rotation (ADM-NR), an actuator-disk model with rotation (ADM-R), and an actuator-line model (ALM), which is the technique we apply here for VAWTs. In terms of comparison with experimental data, they found that ADM-NR tends to overestimate the average velocity in the near-wake region, but ADM-R and ALM provide accurate predictions of various turbulence statistics in HAWT wakes. Shamsoddin and Porté-Agel (Shamsoddin and Porté-Agel 2014) recently conducted a study of the accuracy of ALM-LES models in capturing the wake of a VAWT, but they focused primarily on the influence of the subgrid-scale model of the LES. Their results generally displayed good agreement with experimental data.

In a wind farm, the turbine spacing is also an important consideration. Turbines should be far enough apart to allow wind speeds to recover after deceleration by the upwind generator (by lateral or vertical momentum entrainment), and to reduce the fatigue load
generated by turbulence from the upstream turbines to increase the turbine lifetime (Chamorro and Porté-Agel 2009). The large majority of existing farms are using horizontal axis wind turbines, HAWTs. The behavior of HAWTs in large wind farms, and the required spacing between them have been extensively studied [4, 5]. Calaf et al., [6, 7], investigated fully developed HAWT wind turbine array boundary layer (WTABL) and quantified the vertical transport of momentum and kinetic energy across the boundary layer. They have shown that in large wind farms the kinetic energy regeneration is mainly from downward vertical fluxes across the plane delineating the top of the farm, unlike farms with a limited number of wind turbine rows where the streamwise advection of kinetic energy dominates. The concept of WTABL is valid for wind farms whose length is an order of magnitude larger than the height of atmospheric boundary layer (ABL) since the influence of such a farm would extend all the way to the top of the ABL. Meyers and Meneveau (Meyers and Meneveau 2010) used an actuator disk model and large eddy simulations (LES) to simulate large HAWTs wind farm and understand their interaction with the ABL. They have shown that a staggered wind farm can extract 5% more power than an aligned configuration. In a follow-up study in 2011 (Meyers and Meneveau 2011), the same authors investigated the optimization of turbine spacing in fully developed wind farms; they showed that varying the ratio of land cost to turbine cost in the financial optimization analysis (minimizing power per unit cost) results in different optimal spacing. Meyers and Meneveau’s results indicate that the optimal turbine spacing is higher than what is currently being used in HAWT wind farms. Recently, it has also been shown that highest mean wind farm power is highly dependent on the alignment angle (of turbine rows relative to the wind) and contrary to previous
results, the optimal angle is significantly smaller than the one in a perfectly staggered farm (Stevens, Gayme, and Meneveau 2014). In wind farm sites with a dominant wind direction, these findings can be implemented to improve wind farm designs.

These papers and the large majority of research and development efforts focused on wind farms consisting of HAWTs (Leonardo P. Chamorro and Porté-Agel 2010; Lu and Porté-Agel 2011; Meyers and Meneveau 2012; Porte-agel, Fernando, Yu-ting 2011). However, recent investigations by Dabiri et al. (Dabiri 2011a) have suggested the possibility of an order of magnitude increase in power densities for wind farms when vertical axis wind turbines (VAWTs) are used. VAWTs have vertical blades that rotate about a vertical axis, and the flow in a VAWT farm is thus distinctively different from a conventional HAWT farm. This increase in power density can be achieved by configuring VAWT farms with a closer spacing to better exploit the flow patterns created by upstream turbines. In their studies, they performed experiments on various counter-rotating configurations of 9-m high VAWTs and demonstrated that, unlike the typical performance reduction of HAWTs with small spacing, there is an increase in VAWTs performance when adjacent turbines are arranged to interact synergistically. However, high experimental costs and time requirements prevent the extension of these field investigations to large farm scales or the assessment of a large number of configurations. The previous findings thus only pertain to a limited number of turbines where the mean kinetic energy (MKE) is primarily replenished by streamwise advection and cross-stream turbulent transport, rather than by vertical transport as in large farms.

In the light of the above, a combination of LES and actuator line model has been developed in this thesis to fully investigate the dynamic interactions of vertical axis wind
turbine in atmospheric boundary layer, from single turbine scale to wind farm scale, under conditions that mimic the real atmosphere as closely as possible.

Chapter 2 introduces the ALM-LES numerical model and, after validation against three sets of laboratory experimental results, investigates different design parameters on performance of single VAWT. The aim of this chapter is to answer following questions: 1) How does the size ratio and solidity of VAWT affect the wake recovery downstream of a single turbine? 2) Which turbine design is most suitable to use in large wind farms?

Chapter 3 investigates interactions of multiple VAWTs in wind farms and introduces the VAWT triangle clusters as a basis for creating large compact wind farms. The LES-ALM model is validated against field experimental data. Following the ideas presented by Dabiri et al (Araya et al. 2014; Dabiri 2011b; Whittlesey, Liska, and Dabiri 2010), the aim of the third section of this thesis is to assess the feasibility of increasing power density in large VAWT farm using synergistic clustering of turbines.

In Chapter 4, we assess the role of farm fetch by investigating large “infinite” wind farms and finite fetch farms. We compare wind power recovery mechanisms and power outputs to answer the following research questions: 1) What transport mechanisms dominate the regeneration of mean kinetic energy in VAWT wind farms, and how do they vary with wind farm scale? 2) How large should VAWT wind farms be to display the dynamics of fully developed wind-turbine array boundary layers?

A short appendix is also included to illustrate the important effects of ABL stratification on the performance of large VAWT wind farms. Chapter 5 summarizes the entire thesis, draws the general conclusions, and suggests future research directions.
Chapter Two

2 Simulation and Wake Analysis of a Single Vertical Axis Wind Turbine

2.1 Introduction

As detailed in the introduction, the large majority of research efforts have focused on horizontal axis wind turbines. HAWT wakes and farm configurations have been studied very extensively both experimentally and numerically (Chamorro and Porté-Agel 2010; Lu and Porté-Agel 2011), and these studies could be exploited in our efforts to understand VAWTs. For simulating the wind farm-ABL interactions for example, Porté-Agel tested various approaches that do not resolve flow over individual blades to enable the simulations to cover a whole farm. They conducted large eddy simulation (LES) of HAWTs using three different turbine models, an actuator-disk model without rotation (ADM-NR), an actuator-disk model with rotation (ADM-R), and an actuator-line model (ALM). In terms of comparison with experimental data, they found that ADM-NR tends to overestimate the average velocity in the near-wake region, but ADM-R and ALM provide accurate predictions of various turbulence statistics in HAWT wakes. Shamsoddin and Porté-Agel (Shamsoddin and Porté-Agel 2014) recently conducted a study of the accuracy of ALM-LES models in capturing the wake of a VAWT, but they focused primarily on the influence of the subgrid-scale model of the LES. Their results generally displayed good agreement with experimental data.
In light of these and other HAWT studies, in this chapter we detail the implementation and evaluation of an LES-ALM approach for simulating VAWT wakes and farms. We then investigate the flow recovery in the wake of a single VAWT and how it is influenced by the turbine design, with the long-term aim of using this knowledge to better configure VAWT farms. The questions we seek to answer are: 1) How does the size ratio and solidity of VAWT affect the wake recovery downstream of a single turbine? 2) Which turbine design is most suitable to use in large wind farms?

In Section 2.2 the numerical techniques (LES, ALM, and URANS) are presented; in Section 2.3 the combined model is validated against laboratory data. Section 2.4 includes the analyses of the wakes and how they are influenced by the turbine design. Section 2.5 concludes with a summary and discussion.

2.2 Numerical Models

Investigating full-sized VAWTs experimentally in the field, or reduced-sized models in a wind tunnel, is a straightforward and accurate way to investigate their performance, but the cost of this approach quickly becomes prohibitive when various models and configurations are to be tested. Numerical flow simulations provide a very economical alternative to investigate different design parameters and operational conditions. LES is emerging as a tool of choice for many environmental and engineering problems [15–17] since it provides robust and detailed results at a manageable computational cost. Many of the simulations of HAWT wind farms surveyed in the introduction use LES, and it is currently the most widely adopted tool for simulations of ABL flows since it can directly resolve the impact of important ABL features, including as static stability and the far-
wake of bluff bodies such as wind turbines or buildings. However, LES is inherently more computationally expensive than Reynolds-averaged Navier Stokes (RANS) simulations, which remain sufficient for many practical applications.

In this thesis, we combine LES with unsteady RANS (URANS) to simulate the fluid dynamics of wind turbines. The forces exerted on the flow by the blades are represented in the LES by an actuator line model (ALM) that will be detailed later in this section, and the LES therefore captures the turbine-to-ABL scale eddies. The ALM however requires knowledge of sub-blade scale dynamics to translate the flow field around the blade, provided by the LES, into a force interaction between the blade and the flow. These sub-blade scales involve the complex unsteady dynamics of flow separation and vortex shedding around each blade, but only their aggregate average effect is needed by the ALM since they occur at scales below the LES grid scale and thus only their averaged effect is sensed by the resolved scales of the LES. As such, URANS can be used to simulate these blade-to-turbine scale dynamics, and provide the ALM with the needed force coefficients to parameterize the force exerted by the blades in the LES. The coupling is offline in the sense that the force coefficients for any desired turbine design at a given Reynolds number ($Re$) are provided at each azimuth angle by an independent, uncoupled URANS simulation. These coefficients are then ingested into the ALM-LES for simulating the wakes and farm-scale flow.

2.2.1 The Large Eddy Simulation Setup

In the LES code used here, the filtered Navier-Stokes equations are explicitly solved for the large scales using a pseudo-spectral approach in the horizontal directions and second-order finite differencing in the vertical direction. The effect of the subgrid scale (SGS)
eddie on the resolved ones is modeled using a scale-dependent Lagrangian dynamic subgrid scale model (Bou-Zeid, Meneveau, and Parlange 2005), and the second order Adam-Bashforth scheme is used for time integration. The effect of VAWTs on the ABL flow are represented as horizontal forces imposed by the blades at each LES time-step in each grid cell where a blade element is present. These forces are added to the filtered Navier Stokes equations, written in rotational form but here without Coriolis or buoyancy forces, to yield:

\[
\frac{\partial \tilde{u}_i}{\partial t} = 0 ,
\]

\[
\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \left( \frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i} \right) = -\frac{1}{\rho} \frac{\partial \tilde{p}^*}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + F_i + F_i' ,
\]

where \( \tilde{u}_i \) is the resolved velocity with the tilde denoting a filtered quantity; \( \tilde{p}^* \) is a modified pressure that includes the resolved and subgrid scale turbulent kinetic energies; \( F_i \) is the mean pressure gradient forcing the flow; \( \tau_{ij} \) is the deviatoric subgrid scale stress tensor; and \( F_i' \) represents the aerodynamic forces of the turbine blades on the air.

The horizontal boundary conditions are numerically periodic, but non-periodic flows can be simulated using an inlet sponge region as detailed later. At the top vertical boundary, zero vertical velocity and zero shear stress are imposed. The bottom boundary has zero vertical velocity, while the surface shear stress is imposed using an equilibrium log-law wall model with a wall roughness of \( z_0 = 10^{-6} z_i \), where \( z_i \) is the depth of computational domain, which is used to normalize all length scales in the code. The same LES code with the same numerical approaches and SGS model used here has been widely validated.
[19, 21] and also used to simulate HAWT wakes and farms (Calaf et al. 2010b, 2011). As such, we restrict our validation here to the implementation of the ALM for VAWTs. The ALM computes $F_i'$ and imposes it at the grid locations that contain blade elements.

### 2.2.2 The Actuator Line Model

In the ALM, the VAWT blades are represented as vertical lines and their airfoil characteristics are used for computing their forces on the air (Hezaveh and Bou-Zeid 2013). These actuator lines are integrated in the LES, and their locations in the computational grid are tracked in time. At each time step, the LES local horizontal wind velocity averaged over the past 10-minutes is used for computing the local tip speed ratio and relative wind speed $V_{rel}$. Due to the blockage effect of the VAWT, the flow is diverted sideways; therefore, a blade will not see a flow that is strictly along the streamwise direction. The direction of the local velocity must hence be considered when computing $V_{rel}$ (Figure 2-1-a) and the resulting local angle of attack, $\alpha$, which are given by:

\[
\alpha = \cos^{-1} \left( \frac{\sin}{\left( \sin + \cos^2 \right)^{1/2}} \right), \quad (2-3)
\]

\[
V_{rel}^2 = \left[ \sin^2 + \cos^2 \right], \quad (2-4)
\]

\[
\beta = \theta + \arctan \left( \frac{\tilde{v}}{\tilde{u}} \right), \quad \theta = \sqrt{\tilde{u}^2 + \tilde{v}^2}, \quad (2-5)
\]
\[ \theta = \frac{R}{J} \quad \text{,} \quad (2-6) \]

where \( \theta \) is the azimuthal angle (Figure 2-1-b); \( \lambda \) is the local tip speed ratio; \( R \) is VAWT radius; \( \tilde{u} \) and \( \tilde{v} \) are respectively the streamwise and spanwise local wind speeds, and \( \omega \) is the angular velocity of the blades.

Figure 2-1 a) Angle of attack and relative velocity (vectors are in bold). \( \mathbf{u} = \tilde{u}\mathbf{i} + \tilde{v}\mathbf{j} \) is the incoming flow horizontal velocity vector. b) Schematic 2-D cross section (top view) of the VAWT blade path, the forces on the blades, and representative LES grid cells. The relative scales of the blade chord length to the LES grid cell vary in the different simulations, but the chord length is comparable to the cell size (might be larger or smaller).

Knowing \( V_{rel} \), the normal and tangential blade forces on the flow can be computed as a function of \( \theta \) following:

\[ dF_N(\theta) = \frac{1}{2} \rho \alpha c V_{rel}^2 C_N \, d\zeta \quad \text{,} \quad (2-7) \]
\[
\begin{align*}
 dF_T(\theta) &= \frac{1}{2} \rho \omega c V_{rel}^2 C_T dz , \\
\end{align*}
\]

where \( c \) is the blade chord length and \( dz \) is the vertical length of the blade occupying each LES grid cell. This requires knowledge of the normal and tangential force coefficients \( C_N \) and \( C_T \), which can be directly expressed in terms of the lift \( C_L \) and drag \( C_D \) coefficients:

\[
C_N = |C_L| \cos \alpha + |C_D| \sin \alpha , \quad (2-9)
\]

\[
C_T = |C_L| \sin \alpha - |C_D| \cos \alpha . \quad (2-10)
\]

\( C_L \) and \( C_D \) are thus the main “input” for the LES. They describe the interaction of the blades with the surrounding flow and they vary with blade type and with \( Re \). It is these coefficients that the URANS (or potentially direct experimental measurements) need to provide to the LES.

From \( dF_N \) and \( dF_T \), the upwind \(-\pi/2 < \theta < \pi/2\) (quadrants I and IV) Cartesian force components (aligned with the LES grid) needed for solving the LES momentum budget equation can be computed as:

\[
\begin{align*}
 dF_x &= (dF_N(\theta) \cos \alpha + dF_T(\theta) \sin \alpha) , \\
 dF_y &= (dF_N(\theta) \sin \theta - dF_T(\theta) \cos \theta) . \\
\end{align*}
\]

Similarly, for the downwind half-cycle of the rotor \( \pi/2 < \theta < 3\pi/2 \) (quadrants II and III) these forces are:

\[
\begin{align*}
 dF_x &= (-dF_N(\theta) \cos \theta + dF_T(\theta) \sin \theta) , \\
\end{align*}
\]

\[14\]
\[ dF_y = (-dF_N(\theta) \sin \theta - dF_T(\theta) \cos \theta) . \quad (2-14) \]

In order to increase the accuracy of the ALM when the blade is passing between two or more grid cells, and to make it less sensitive to the grid size (recall that the chord length can be smaller or larger than the cell size), a distributed force method is implemented. First, instead of imposing the force at the center of the airfoil chord, it is distributed equally (though this can be modified if better force distribution information is available) over 5 points spanning the chord length. The position of each of these 5 points is tracked at each time-step and the forces are applied to the corresponding grid cell. In addition, since the force imposed by a blade on the flow is not “local” in the sense that the pressure gradients resulting from the presence of the blade span a finite flow region, it is common in simulations with the ALM and ADM approaches to further distribute the force over multiple grid cells adjacent to the blade [7, 25]. We also adopt this force distribution here, and the forces computed for each of the 5 points spanning a chord length are further distributed over the surrounding cells. The distribution weights are 2/10 for the cell confining the point, and 1/10 for the 8 surrounding cells. This approach reduces sharp discontinuities in the flow and is useful for simulations using pseudo-spectral discretizations (like the ones in this chapter). When applicable, the central mast is modeled as a cylinder with a drag coefficient of 1.0.

2.2.3 Obtaining the Lift and Drag Coefficients

Since the main aim of this chapter is to study the flow recovery in wake of a VAWT, the wake needs to be tracked for a long distance downstream. A future goal of the project in general is to simulate VAWT farm configurations. Both of these aims require a domain size on the order of 50\(\times R\) or even larger. Spanning such a large domain precludes the
possibility of resolving the sub-blade scale dynamics explicitly, but as discussed above, these dynamics are important for the larger scales. In the ALM-LES they are strictly represented by the lift and drag coefficient and hence accurate estimate of these coefficient is essential. Here, we will mainly rely on a well-tested and robust URANS to obtain such coefficients, but one could also potentially obtain them from wind tunnel tests, or from direct numerical simulation or blade-resolving LES. Our study however documents the feasibility of conducting joint uncoupled simulations for the blade-to-turbine and turbine-to-farm scale flow dynamics, with good results, as long as robust simulation approaches are used for each range of scales. Laboratory measured airfoil data for lift and drag forces, such as those provided by the Sandia National Lab, can be used in the LES-ALM model. These data should provide $C_L$ and $C_D$ as a function of the attack angle, Reynolds number, and blade type. However, during each cycle of rotation of a VAWT, the blades are constantly encountering wakes shed from other blades and this complex interaction will not be captured by wind tunnel tests which are typically done with a single fixed blade. This will result in errors, particularly for low tip-speed ratios and high angles of attack. Furthermore, when the angle of attack is changing rapidly in time, a flow phenomenon called dynamic stall will be encountered. It refers to the delay in the separation of the flow along the blade, which will now occur at an angle of attack higher than the one for static stall conditions. The errors in $C_L$ and $C_D$ that would result from using data from static wind tunnel tests can be reduced by implementing a dynamic stall correction; two approaches have been investigated here for this correction.
2.2.3.1 Static Wind Tunnel Data with Dynamic Stall Correction

In the first approach, the modified Boeing-Vertol dynamic stall model (Paraschivoiu and Delclauxt 1982; Paraschivoiu and Desy 1986; Rahimp, Allet, and Paraschivoiu 1995) has been used with the ALM to correct the aerodynamic force coefficients obtained from static tests in Sandia National Lab anywhere this was needed in this thesis. In this model, the lift-curve slope observed for small angles of attack remains the same, but the stall angle increases. A modified angle of attack \( \alpha_m \) (different for lift \( m_l \) and drag \( m_d \) coefficients) is computed as a function of the relative velocity \( V_{rel} \) and of the instantaneous rate of change of the angle of attack \( \dot{\alpha} = d\alpha / dt \):

\[
\alpha_{ml} = \alpha - \gamma_l \kappa \left( \frac{c\dot{\alpha}}{2V_{rel}} \right)^{\frac{1}{2}} \frac{\dot{\alpha}}{\alpha}, \tag{2-15}
\]

\[
\alpha_{md} = \alpha - \gamma_d \kappa \left( \frac{c\dot{\alpha}}{2V_{rel}} \right)^{\frac{1}{2}} \frac{\dot{\alpha}}{\alpha}, \tag{2-16}
\]

where \( \gamma_l, \gamma_d, \kappa \) are dimensionless empirical constants, which for low Mach numbers are given by the following expressions:

\[
i = 1.4 \quad 6 \quad 0.06 \quad \frac{d}{c} \quad \text{and} \quad \frac{d}{c} = 1 \quad 2.5 \quad 0.06 \quad \frac{d}{c}, \tag{2-17}
\]

\[
\kappa = 0.75 + 0.25 \frac{\dot{\alpha}}{\alpha}, \tag{2-18}
\]
where \( d \) is the airfoil thickness. The lift and drag coefficients at the modified angles of attack \( C_L(m_l) \) and \( C_D(m_d) \) are then evaluated from the experimental data, and then a further correction is applied to obtain the lift and drag coefficients to be used in the ALM:

\[
C_{Lm} = \frac{\alpha_0}{m_l} C_L(m_l),
\]

(2-19)

\[
C_{Dm} = C_D(m_d),
\]

(2-20)

where \( \alpha_0 \) is the angle of zero-lift needed for curved blades; in this study it is equal to zero since only straight blades are investigated. Figure 2-2 shows the lift coefficient, \( C_L \), versus angle of attack, \( \alpha \), for a NACA 0018 blade. The green line shows statically measured \( C_L \) values and the red line shows corrected lift coefficients considering the dynamic stall effects. The black crosses are previously computed CFD results (Paraschivoiu and Desy 1986).
2.2.3.2 High Resolution URANS Simulations of VAWTs

In the second approach, we simulate the dynamics of the flow at the blade-to-turbine scales using the full URANS equations, and a Menter’s (Menter 1994) two-equation SST (shear stress transport) $k - \omega$ turbulence model for closure, which is known to produce realistic results in separated flows. The URANS equations are discretized by a numerical scheme of second-order accuracy in space and time, which has been extensively validated over the course of the last twenty-five years (Martinelli and Jameson 2012). The URANS solver used in this work has been recently detailed and validated for VAWTs against highly resolved PIV measurements (Buchner et al. 2015). The simulations conducted for the current study are two dimensional with a total of 198,000 grid cells; the grid is stretched and the first grid point is at 1 viscous unit from the blade surface.

The URANS model computes the entire flow field as a function of time, allowing us to model the forces of VAWT blades on the ABL without any other assumption: dynamic stall, as well as the influence of the wake of a blade on the subsequent ones are automatically included in the evaluation of the aerodynamic forces. An example of the tangential force coefficients versus azimuthal angle from URANS are shown in Figure 2-3, along with experimental data from McLaren (McLaren 2011) (the experiments are detailed in section 3.2). In the upwind section of the rotor (quadrants I and IV), the URANS is tracking the reported experimental value for $C_T$ quite well. In the downwind section, there are differences between the experimental and numerical results (for both the URANS and the numerical model used by McLaren, although the URANS captures the average $C_T$ in these quadrants better). However, as shown in the validation section, this discrepancy between the URANS and experimental $C_T$ has a minor influence
on the ability of the ALM-LES-URANS to capture the downwind wake accurately (the main forces on the flow are in fact exerted in the upwind sector where $C_T$ is captured well). This approach captures dynamic stall, as well as the influence of the wake of a blade on the subsequent one. In this case we have $dF_x$ and $dF_y$ in equations 10-13 which are shown in Figure 2-4.

![Figure 2-3 Tangential force coefficients versus $\theta$ obtained from URANS, compared to experimental and numerical results reported by McLaren (McLaren 2011).](image)
2.3 Code Validation

In this section, we use three experimental benchmarks for VAWTs to validate the ALM-LES-URANS model. The experiments have different boundary and inflow conditions, as well as different VAWT setups and flow patterns that will be presented along with the validation in the following subsections. However, more detailed and comprehensive experiments certainly remain needed for a better understanding of the flow patterns around VAWTs and for advanced LES validation (Bou-Zeid 2014).

2.3.1 Water Channel Experiments of Bachant and Coworkers

Bachant (Bachant and Wosnik 2013) performed water channel experiments for a small VAWT the characteristics of which are given in Table 2-1 (Sheldahl and Klimas 1981). The water channel facility is 36 m long, 3.7 m wide, and its height is 2.4 m. The turbine
is towed in the tank at a speed \( (U_\infty) \) of 1 m/s. Acoustic Doppler velocimetry (ADV) was used for measuring the velocity at a downstream distance of 1 diameter from the center of the turbine. The diameter of the center mast of the turbine was 9.5 cm. The experiments were conducted at two different tip speed ratios of 1.4 and 1.9, resulting in \( Re = cU_\infty/\nu \) (where \( \nu \) is the kinematic viscosity of water) of \( 2 \times 10^5 \) and \( 3 \times 10^5 \), with the latter corresponding to the highest power output. We have simulated both tip speed ratios, but we only show \( \lambda = 1.9 \) here since the validation for \( \lambda = 1.4 \) gave comparable results.

Table 2-1 Numerical and experimental configuration (Bachant and Wosnik 2013)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades</td>
<td>3</td>
</tr>
<tr>
<td>Rotor diameter, ( m )</td>
<td>1</td>
</tr>
<tr>
<td>Blade vertical length, ( m )</td>
<td>1</td>
</tr>
<tr>
<td>Blade chord length, ( m )</td>
<td>0.14</td>
</tr>
<tr>
<td>Airfoil section</td>
<td>NACA 0020</td>
</tr>
<tr>
<td>Solidity ( Nc/\pi D )</td>
<td>0.13</td>
</tr>
<tr>
<td>Tip speed ratio ( \lambda )</td>
<td>1.9</td>
</tr>
<tr>
<td>( Nx \times Ny \times Nz ) (fine LES grid cells)</td>
<td>288\times144\times60</td>
</tr>
<tr>
<td>( Nx \times Ny \times Nz ) (intermediate LES grid cells)</td>
<td>144\times72\times60</td>
</tr>
<tr>
<td>( Nx \times Ny \times Nz ) (coarse LES grid cells)</td>
<td>72\times36\times60</td>
</tr>
<tr>
<td>Domain size ( Lx \times Ly \times Lz ) (m)</td>
<td>7.2\times3.7\times2.4</td>
</tr>
</tbody>
</table>

In the numerical simulation setup, the VAWT is fixed and the inflow is set to a uniform 1 m/s far upstream of the turbine. The cross-section of the simulation domain matches that of the water channel. However, since the velocity profiles are only available at a distance of one diameter downstream of the turbine, for the sake of computational efficiency, the length of the simulated domain in the streamwise direction is set to 7.2 m (the full length of the actual channel does not need to be matched). The force coefficients for the LES in this subsection are obtained from tabulated Sandia National lab wind tunnel data, with a
subsequent dynamic stall correction following the model in the appendix, the main aim here being to assess the influence of grid resolution on the ALM-LES results.

Figure 2-5 shows comparisons between the simulated streamwise velocity and experimental results at a tip speed ratio of 1.9. Both the velocity deficit magnitude and the wake shape (skewed with higher velocities on one side) are in good agreement with the experimental results for all grid resolutions, indicating that the mean velocity is insensitive to resolution.

![Figure 2-5 Comparison of streamwise velocity cross-section for \( \lambda = 1.9 \) obtained using 3 grid resolutions to experimental measurements.](image)

Figure 2-6 depicts the streamwise velocity standard deviation (or root mean square velocity) cross-section. While for all cases the ALM-LES predicted the peak locations and the trends correctly, the two higher resolution cases are capable of capturing the magnitudes much better than the low resolution case. This is partially due to the fact that the higher resolutions resolve more turbulent scales in the LES; it is clear in the figure that the velocity variance increases with resolution. Therefore, as expected, higher order
statistics are more sensitive to grid resolution than the means, and higher resolutions result in better quantitative agreement with experimental results.

![Comparison of streamwise root-mean-square velocity cross-section for $\lambda=1.9$ obtained using 3 grid resolutions to the experimental measurements.](image)

**Figure 2-6** Comparison of streamwise root-mean-square velocity cross-section for $\lambda=1.9$ obtained using 3 grid resolutions to the experimental measurements.

### 2.3.2 Wind Tunnel Measurements of McLaren

In 2011, McLaren performed numerical and experimental simulations of a relatively large, high solidity VAWT under low tip speed ratios (McLaren 2011). The wind tunnel facility was 19.5 m long, 15.4 m wide and 7.6 m high, with an inflow of 10 m/s with 10% turbulence intensity. Table 2-2 provides the details of the experimental configuration and concomitant simulation. The computational cross section is slightly larger than the experimental one, but the ratio of the turbine area to the wind tunnel cross section is about 7% (5.8% in the simulations), and therefore this discrepancy should not have a significant impact on the results (particularly that here we only compare simulated wakes to each other since no experimental measurements of the wake velocity were made).
The goal of this sub-section is to assess the discrepancy between LES results obtained with different sources for the force coefficients. The experimentally-measured normal and tangential force coefficients at each tip speed ratio were reported by McLaren. We therefore extracted them and imposed them directly in the LES (run #1). We also obtained $C_L$ and $C_D$ for the NACA 0015 at the same $Re$ of 50000 from the steady tests of the Sandia National lab, and imposed them directly in the LES without correction (run #2), as well as with the dynamic stall correction model detailed in the appendix (run #3). Finally, we simulated the same turbine using URANS, computed the force coefficient from the URANS independently from the measurements, and used these force coefficients in the ALM-LES to simulate the wake (run #4). A direct comparison of the tangential force coefficients was depicted in Figure 2-3. Using these four different sources for the force coefficients, we simulated the wake profiles behind the turbine using the ALM-LES. To match the turbulent experimental inflow, a turbulent LES inflow was created in a separate simulation using a similar domain without a turbine, producing a logarithmic inflow profile with a magnitude of 10 m/s at the mid height of the turbine. To
impose such an inflow in the horizontally periodic LES, a buffer zone over several $y$-$z$ planes is added upstream of the turbine to smoothly transition from the outflow at the end of the periodic computational domain, to the desired logarithmic inflow.

In Figure 2-7, we compare the downstream wakes of the turbine for the highest experimental tip speed ratio of 1.8. The reference wake here (the one considered the most accurate) is the one obtained with the direct force coefficients reported by McLaren. As illustrated in the figure, the ALM-LES with dynamic stall correction is able to reproduce the LES wake generated using the measured force coefficients very well. However, when the coefficients were not corrected for the dynamic stall, the errors in the wake velocity were clearly higher. Figure 2-9 also demonstrates that the ALM-LES-URANS coupled approach is able to match the wake profiles that use the measured coefficients very reasonably, giving better results than the simulations with the coefficient corrected for dynamic stall. This is not surprising since the URANS captures the detailed unsteady flow around the blades and no ad hoc or empirical dynamic stall correction is needed; the force coefficients the URANS provides to the LES capture all the complexities of the flow including wakes from other blades and unsteady dynamics. However, while this validation test allows us to confirm that the wake generated with URANS force coefficients is similar to the one produced using measured force coefficients, the accuracy of ALM-LES-URANS simulated wakes remains untested. Therefore, in the next subsection we will compare the model results directly to measurements of mean and root-mean-square velocities in the wake.
Figure 2-7 ALM-LES wake comparison when the force coefficients are directly measured (McLaren force coefficients), obtained from a static test without dynamic stall correction, or with dynamic stall correction, and simulated by URANS. Note that none of the profiles depict an experimentally measured wake directly.

2.3.3 Water Channel Measurements of Brochier et al

Brochier et al (Brochier et al. 1986) conducted gravity driven water channel measurements of the flow around a VAWT, the characteristics of which are listed in Table 2-3, using Laser-Doppler Velocimetry (LDV). The blades are composed of NACA 0018 airfoils (Sheldahl and Klimas 1981), and the central mast has a diameter of 1 cm.
The water channel is vertical and its dimensions are $1.5 \times 0.2 \times 0.2$ m; the gravity driven flow speed is 15 cm/s. The wake measurements were done for two tip speed ratios of 2.14 and 3.85, where only the former corresponds to an operating regime with dynamic stall.

**Table 2-3 Numerical and experimental configuration** (Brochier et al. 1986)

<table>
<thead>
<tr>
<th>Number of blades</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor diameter, m</td>
<td>0.12</td>
</tr>
<tr>
<td>Blade vertical length, m</td>
<td>0.20</td>
</tr>
<tr>
<td>Blade chord length, m</td>
<td>0.02</td>
</tr>
<tr>
<td>Airfoil section</td>
<td>NACA 0018</td>
</tr>
<tr>
<td>Solidity $Nc/\pi D$</td>
<td>0.1</td>
</tr>
<tr>
<td>Tip speed ratios $\lambda$</td>
<td>2.14</td>
</tr>
<tr>
<td>$Nx \times Ny \times Nz$ (LES grid cells)</td>
<td>300×30×80</td>
</tr>
<tr>
<td>Domain size $Lx \times Ly \times Lz$ (m)</td>
<td>$2 \times 0.2 \times 0.2$</td>
</tr>
</tbody>
</table>

For the simulations, and again due to the periodicity of the numerical boundary conditions in the LES code, two buffer zones in the spanwise and streamwise directions were added for this test. In the streamwise direction, a buffer zone over several $y$-$z$ planes was added upstream of the turbine to smoothly transition from the outflow at the end of the domain to the imposed inflow we want to simulate, which consists of a uniform 0.15 m/s velocity. For the spanwise direction, a buffer zone was added to smoothly force the spanwise velocity to zero at the impermeable side walls (only needed at one spanwise edge of the domain due to periodicity Figure 2-8).

Using these setups, the VAWT was simulated at the tip speed ratio of 2.14 since it is the one that will operate under dynamic stall and is hence more challenging to capture. The force coefficients for this test were obtained from URANS simulations and also from the
Sandia National Lab static wind tunnel tests for the NACA 0018 blade, with a dynamic stall correction applied as in the appendix.

The wakes at different distances upstream and downstream of the turbine ($X/R = -1.67, 1.67, 3.33, 5$ and $8.4$) were recorded and compared to the water channel measurements. It is clear that not only can the ALM-LES-URANS predict comparable and realistic wake velocities (Figure 2-9), but also at a low tip speed ratio, where dynamic stall effects are far more consequential, it can capture turbulence intensity patterns and magnitudes quite well even in the near wake regime (Figure 2-10). The ALM-LES with dynamic stall model also reproduces the mean wake pattern and profiles with good agreement with experimental results. Both the ALM-LES-URANS and the ALM-LES with a dynamic stall model predict a turbulence intensity lower than what is reported in the experiment; however, the former is predicting the pattern and magnitude better. The underestimation of the turbulence intensity directly behind the turbine ($X/R = 1.67$) is due to the fact that in that region a large fraction of the turbulent kinetic energy is at the subgrid scales (~blade chord), which cannot be captured in LES.
Figure 2-9 Cross stream profile of the normalized mean stream-wise velocity at different downstream locations for $\lambda = 2.14$, ALM-LES with URANS model and ALM LES with dynamic stall correction (DS-model).

Figure 2-10 Cross section of the turbulence intensity at different downstream locations for $\lambda = 2.14$, ALM-LES with URANS model and ALM LES with dynamic stall correction (DS-model).
2.4 Wake Analysis

With the code validated, we turn our attention to its application to provide detailed information on the influence of solidity ($S$), tip speed ratio ($\lambda$), and aspect ratio ($D/L$) on VAWT wake characteristics. Details of a suite of 15 simulations we conducted are provided in Table 2-4. This study only considers one turbine to better isolate a single wake, but future applications of the code are aimed at wind farms. Three different $S$ values of 0.08, 0.15 and 0.25, along with three $D/L$ ratios of 5, 1.25 and 0.2 were chosen, resulting in 9 base cases. Varying the tip speed ratio for the lowest $S$ cases then results in 6 supplementary cases (in italics in Table 2-4). All 15 cases have the same frontal area of 2000 m$^2$ and the height above ground of the bottom of the blades in all cases is 20 m (relative to a domain height of 300 m). The simulations are designated as follows: in each simulation label, the first letter denotes the aspect ratio and varies between S (Slender $D/L = 0.2$), I (Intermediate $D/L = 1.25$), and F (Flat $D/L = 5$), while the second letter denotes the solidity and varies between H (High $S = 0.25$), M (Medium $S = 0.15$) and L (Low $S = 0.08$). Following the letters, we add digits denoting the tip speed ratio to the case labels.
The LES domain is 1152×1152×300 m in the x, y and z directions. This domain is divided into 288×288×336 grid cells of uniform dimensions (4 m × 4 m × 1 m). Using URANS and NACA 0018 blades, each of these turbines was simulated at different tip speed ratios to provide \( C_p \) vs. \( \lambda \) curves (Figure 2-11) and the force coefficients needed by the ALM-LES. The figure illustrates that increasing the solidity shifts the power curves to the left as expected: the peak \( C_p \) occurs at a lower \( \lambda \). Moreover, by increasing the solidity, the peak \( C_p \) drops from 55% for the lowest \( S \) cases to 40% for the highest. Based on these generated power curves, a tip speed ratio is selected for each solidity to yield the same \( C_p \) of 40% in the 9 base cases. Since this \( C_p \) could occur at two distinct tip speed ratios, the lower one has been selected. While we also have simulated each turbine at its peak \( C_p \), we report here the results for all turbines at the same \( C_p \) in order to make the wake
comparison more consistent and realistic; the same $C_P$ ensures that the power extracted by the turbines from the flow have the same value relative to the upstream power. All turbines have a central mast with a diameter equal to 10% of the turbine’s diameter. It should be mentioned that for the flat turbine cases with high and medium solidities (F-H and F-M), we have not considered the effect of tip vortices, which can have significant impact on the performances of these turbines given their relatively high ratio of $c/L$.

![Figure 2-11 $C_P$ vs. $\lambda$ from URANS.](image)

In order to mimic the behavior of these turbines in the real ABL, a realistic turbulent inflow was generated and applied to the simulations as an inflow boundary condition by running a separate periodic simulation over a flat wall, and regularly saving the velocity components over $y$-$z$ planes to provide to the VAWT simulations. After a start-up period of 200,000 time-steps, flow statistics were collected over the following 200,000 time-steps, corresponding to 3 to 12 boundary-layer-scale eddy turnover time (estimated as the depth of the domain over the friction velocity), or about 500 vortex shedding cycles from a cylinder with the same diameter as the rotor. Figure 2-12 and 2-13 show $x$-$y$ cross

33
sections of time-averaged normalized streamwise and cross-stream velocity, respectively, for the 9 base cases. The velocities at each height are normalized by the upstream $U_\infty$ at the corresponding height. Qualitatively, it is clear that both the solidity and the aspect ratio have a significant impact on the wake shape and on the amplitude of the velocity deficit. Furthermore, the cross-stream velocities have different trends for different S and D/L (Figure 2-13).

Figure 2-12 Time-averaged normalized streamwise velocity, $U/U_\infty$, averaged in $z$ over the length of the blades.
Figure 2-13 Time-averaged cross-stream velocity, $V/U_\infty$, averaged in $z$ over the length of the blades.

Figure 2-14 depicts $x$-$z$ cross sections of the flow. The middle solidity cases (S-M-1.6, I-M-1.6 and F-M-1.6) are blocking the incoming flow more significantly compared to the lowest and highest solidity cases. When compared to lowest solidity cases, it is expected that the medium solidity cases will generate greater forces at the blade scale, and therefore result in stronger blockage. However, the trends in the results are not expected when medium and high solidity cases are compared since the medium solidity should not result in higher blockage. A possible explanation (that will in fact be confirmed in section 2.4.3) for the observed trends is that the tip speed ratio seems to have a higher effect than solidity on flow obstruction: since the highest solidity cases have a lower $\lambda$ than the medium ones (to obtain the same $C_p$), the resulting flow blockage is lower with high solidity.
Figure 2-14 Time-averaged normalized streamwise velocity in an $x$-$z$ plane passing through the middle (hub) of the turbines.

However, these depictions of the wake only yield qualitative insight. A more quantitative characterization of the wakes is obtained by considering the normalized averaged velocity deficits in the wake, which are calculated in the following subsection. At varying $x$-locations, the deficit is averaged in time and over a $y$-$z$ rectangle that is aligned in the $x$ direction with the turbine cross-section projected area, i.e. a rectangle directly behind the turbine and of equal area to the turbine plane.

2.4.1 Solidity

The solidity $S$ is the first parameter that was varied. In Figure 2-15, the velocity deficits are shown with a fixed $D/L$ in each panel. For the turbines with $D/L$ ratios of 0.2 and 1.25 (Figure 2-15-a and b), the middle solidity cases (S-M-1.6 and I-M-1.6) are producing a larger deficit near the turbine (the deficit exceeding 1 even implies that there is flow reversal (negative $U$) inside and/or behind the rotor). However, the recovery for these
cases is the fastest and their deficit drops to 20% faster than the other cases. For these two lower $D/L$ ratios, the mean kinetic energy (MKE) recovery from the sides (transport of MKE in the cross-stream direction) is important due to the aspect ratio and the results suggest that designs that produce the highest deficit directly behind the turbine create the largest shear in the cross-stream direction and the fastest recovery (due probably to stronger turbulence generation). On the other hand, for a flat $D/L$ ratio of 5, the medium solidity turbine still produces the highest deficit directly behind the turbine, but its recovery is no longer the fastest. In these cases, a decreasing solidity results in longer recovery distances. This is due to the fact that, for this flat aspect ratio, MKE recovery is mainly through vertical transport. Results not shown here indicate that the turbulence intensity near the turbine decreases with lower solidity for flat turbines, explaining why the regeneration of mean kinetic energy is taking longer to recover in this case (recall that MKE recovers primarily through transport by turbulence). Overall, the trends of recovery are not monotonic with changing solidity, which may be related to the need to change the tip speed ratio to maintain the same $C_p$. Medium solidities consistently produce the strongest wake directly behind the turbine, however, and this is consistent with the observation in the slices of the previous section where we attributed this behavior of the medium solidity models to the combination of $S$ and $\lambda$ yielding the highest streamwise force on the flow.
2.4.2 D/L Ratio

Wake deficits for cases with the same $S$, but different $D/L$ ratios, are compared in Figure 2-16 a-c. The influence of D/L is not consistent across all solidities as can be noted in subplots a-c. Figure 2-16-a depicts cases S-H-1.3 and I-H-1.3 and F-H-1.3 with $S = 0.25$ (highest solidity) and the same $\lambda$. The deficit near the turbine for the flat rotor (F-H-1.3) is less than for the other 2 cases, but its recovery is the fastest. Figure 2-16-b shows the wakes for the middle solidity cases, for which it is clear that increasing D/L results in slower recovery to 20% deficit. The low solidity cases in Figure 2-16-c also indicate that a higher $D/L$ leads to a slower recovery (although the flat and intermediate $D/L$ are rather similar). Therefore, the overall trend that is consistent with the previous subsection is that slender turbines have the shorter wakes (although at the highest solidity the recovery of the intermediate aspect ratio turbine is slightly faster).
In Figure 2-16 (d-f), the weighted normalized areas of the wakes, $A_w$, are plotted. $A_w$ is defined as the area in a $y$-$z$ wake plane where the deficit of the velocity exceeds 20%, weighted by that deficit, and then normalized by the turbine cross-sectional area $A$. We express it here at any downstream distance $x$, in terms of a conditional sum (where the condition $\Gamma$ is $\left(1 - \frac{U}{U_{\infty}}\right) > 0.2$) over discrete $y$-$z$ grid points as follows:

$$A_w(x) = \frac{1}{A} \sum_{j=1}^{Ny} \sum_{k=1}^{Nz} \left(\frac{U(x,j,k)}{U_{\infty}}\right)^2 dy \, dz .$$

Equation (2-21)

$A_w$ can be understood as a measure of the size of the wake; the smaller it is the better since that would allow placing downwind rotors at a small cross-stream offset relative to upwind ones. Figure 2-16 (d-f) displays the wake areas downstream of the turbines. The trends for this parameter are consistent: it is clear that by increasing the $D/L$ ratio, the wake areas are decreasing (except for a limited $x$ range when $S = 0.15$). That is, flatter turbines create smaller wakes.
2.4.3 Tip Speed Ratio

In order to isolate the role of the tip speed ratio in wake recovery, we performed six supplementary simulations for the 3 low $S$ turbines at 2 additional $\lambda$ values. This resulted in a set of nine simulations at three $\lambda$ values of 2.1, 2.8 and 4.1, for each $D/L$ ratio. The velocity deficit results are depicted in Figure 2-17 (a-c). Increasing $\lambda$ consistently results in stronger wake deficit near the turbine (confirming our previous explanation for why the middle $S$ base cases, which had a higher $\lambda$, yielded stronger deficits). However, due to
the increase in generated turbulence (analyzed but not shown here) at these higher $\lambda$ values, the wake recovery distance is consistently decreased. We note that for the low $S$ cases, the change in $\lambda$ must lead to a concomitant change in $C_P$, (see Table 2-4) and both of these parameters are playing an important role in these figures. $\lambda$ increases the generated turbulence, while $C_P$ controls the amount of energy extracted from the wind. When these values vary simultaneously, different patterns of wake recovery could emerge. However, here we consistently find that the highest $\lambda$ values that generated the largest deficit immediately behind the turbine recover the fastest.

While all of the figures above depict the velocity deficit, it is important to note that one could also define the deficit based on the power (and, in fact, this might be more pertinent for farm design) and the wakes will have different behaviors, sizes and recovery distances when this alternative definition is adopted. In order to illustrate these differences, the analyses based on velocity deficit in Figure 2-17 (a-c) are reproduced based on the power deficit and the results are depicted in Figure 2-17 (d-f). From these figures, it is clear that the recovery distance based on power deficit is larger compared to velocity deficit recovery, although the trends of the recovery distance as $\lambda$ varies remain consistent.
Figure 2-17 Comparison (a-c) between averaged velocity deficits and (d-f) between averaged power deficits, at different tip speed ratios, $\lambda$. The $x$-axis is in meters and should be compared to the characteristic scale of the VAWT, $A^{1/2} \approx 45$ m.

2.5 Conclusions

An ALM-LES model for simulation of vertical axis wind turbines was developed, validated, and applied to the analysis of turbine wakes in this chapter. The ALM-LES can capture turbine-to-farm scale flow structures, and hence it was also combined with a URANS model that captures blade-to-turbine scale flow dynamics. The URANS is first applied to obtain lift and drag coefficients at a given $Re$ and for a given blade design, at
all azimuth angles, and then these coefficients are passed on to the ALM-LES model. Although these coefficients can also be obtained from tabulated steady state tests (e.g. from Sandia National Lab data) and then empirically corrected for dynamic stall effects, or obtained from experimental measurements directly, the ALM-LES-URANS coupled framework has the advantage of allowing faster and wider exploration of the parameter space that influences turbine performance for arbitrary airfoil and blade configurations. Different evaluation tests were conducted, comparing ALM-LES-URANS outputs to available experimental results. The model was able to capture the mean wake deficit and structure remarkably well, and was also successful in reproducing the more challenging root-mean-square streamwise velocity patterns in the wake.

A parametric study of the performance of a single VAWT was carried out by first simulating nine base cases with different diameter to length $D/L$ ratios and solidities $S$. Then, for 3 combinations of $D/L$ and $S$, six supplementary simulations at different tip speed ratios $\lambda$ were conducted. Increasing $D/L$ at high $S$ reduced the wake recovery length, but at medium and low solidities increased it. Slender rotors are therefore preferable unless $S$ is very high. Moreover, at a fixed $D/L$ ratio, the medium $S$ cases had the highest wake deficit near the turbine due to their higher $\lambda$. Increasing the tip speed ratio consistently decreased the wake recovery distance since it produced stronger wakes behind the rotors; this holds regardless of whether the recovery assessment was based on velocity or power deficit.

In summary, the rotor design parameters considered have been found to alter the wake shapes and intensities in interconnected ways, and isolating the influence of one parameter was not always straightforward. Particularly when $D/L$ or $S$ are changed, one
can fix either $C_p$ or $\lambda$, but not both; this results in simultaneous changes in another key parameter for turbine wakes. These results however provide guidance for selecting the optimal VAWT model for a large wind farm configuration. In particular, we can conclude that:

1. Increasing the tip speed ratio (up to the optimal value that results in the largest $C_p$) consistently reduce the recovery distances.

2. For medium to slender turbines ($D/L \leq 1.25$, more likely to be employed than the flatter ones with $D/L = 5$), a medium optimal $S$ exists that minimizes the wake recovery distance. $D/L$ is probably the first parameter that would be set in designing farms from considerations related to array configuration or turbine manufacturing. Once it is fixed, the corresponding optimal $S$ can be inferred from the ALM-LES-URANS.

3. Strong deficits directly behind the turbines do not necessarily imply longer wakes; in fact, a stronger initial deficit most likely will dissipate faster due to higher shear and turbulence generation at the edge of the wake.

4. While qualitatively the trends in the recovery of the velocity and power deficits are similar, recovery to 80% of upstream power invariably takes significantly longer than recovery to 80% of upstream velocity, and the power recovery might be the more pertinent criteria for wind farm design.
Chapter Three

3 Interactions of Multiple VAWTs

3.1 Introduction

Despite the concerted effort to improve energy efficiency and decouple economic growth from energy consumption, the U.S. Energy Information Administration projects that global total energy consumption will grow by about 45% between 2015 and 2040 (U.S. Energy Information Agency 2013). Mitigating the large increase in greenhouse gasses (GHG) emissions that such growth would entail necessitates exploring alternative low-GHG energy sources, particularly that the majority of the current fossil-based energy resources are finite and non-renewable, and have other adverse side effects on the environment. Wind energy is expected to be one of the primary sources of clean, renewable energy that would allow the transition away from fossil-based energy to accelerate; in the US for example, wind is projected to contribute around 20% of electric energy by the year 2030 (Marquis et al. 2011). As a result, large wind farms are being deployed, and the continued spread and expansion of these farms pose a challenge since the required land area will increase. A major goal of current research is thus to increase the power density of wind farms, i.e. how much energy is produced per unit land area used.

Considerable work has been done on optimizing the layouts of HAWT wind farms to maximize power density as detailed in the introduction of this thesis, but the parallel work for VAWTs remain lacking. VAWTs have vertical blades that rotate about a
vertical axis, and the flow in a VAWT farm is thus distinctively different from a conventional HAWT farm. The importance of understanding air flow in a VAWT farm was highlighted by the recent investigations by Dabiri et al. (Dabiri 2011a) who have suggested, based on field experimental tests, the possibility of an order of magnitude increase in power densities for wind farms when vertical axis wind turbines (VAWTs) are used. This increase in power density can be achieved by configuring VAWT farms with a closer spacing to synergistically exploit the flow patterns created by upstream turbines. However, high experimental costs and time requirements prevent the extension of these field investigations to large farm scales or the assessment of a large number of configurations. The previous findings thus only pertain to a limited number of turbines where the TKE is primarily replenished by streamwise advection and cross-stream turbulent transport, rather than by vertical transport as in large farms. The aim of this chapter is, therefore, to bridge this research gap and assess the feasibility of increasing power density in large VAWT farm using synergistic clustering of turbines. Following on the in the previous chapter, we here simulate the interactions of multiple VAWTs in small clusters, and subsequently use these clusters to design large VAWT wind farms.

3.2 Numerical Model

Same LES-ALM model is used for the simulations in this section with minor changes in the model used for computing the lift and drag coefficients. In order to find $C_L$ and $C_D$ in this section, actual dynamic curves for lift and drag forces determined experimentally were implemented into the numerical code. Claessens in 2006 has conducted several numerical and wind tunnel experiments at various Reynolds numbers and tabulated several lift and drag curves for different airfoil blade types (Claessens 2006). Figure 3-1
depicts the static values of the coefficients (no dynamic stall) and the measured values that included the dynamic stall effect (directly taken from wind tunnel measurements). Based on the sign of instantaneous rate of change of the angle of attack, different paths emerge in these curve in Figure 3-1; $C_{Lm+}$ versus $C_{Lm-}$ and $C_{Dm+}$ versus $C_{Dm-}$. These results are for the DU-06-W-200 blade type, which is redesigned from NACA 0018 airfoil and is 2 % thicker, and cambered rather than symmetric (Claessens 2006). This blade type and the experimental results reported in Figure 3-1 are used throughout this study.

Figure 3-1 Static Lift and Drag Coefficient curve for NACA 0018 blade (Sheldahl and Klimas 1981) and Dynamic $C_L$ and $C_D$ for DU 06-W-200 measured by Claessens (Claessens 2006). The + subscript is for $\dot{\alpha} > 0$ and the – is for $\dot{\alpha} < 0$.

3.3 Results and Discussions

3.3.1 Validation against field measurements of counter-rotating VAWTs

As mentioned before, Dabiri et al. (Dabiri 2011a) have performed several field measurements with different configurations of 9-m tall VAWTs using the same blade
type and rotor configuration we adopt here. They have shown that, by using counter rotating VAWTs and special configurations, the VAWTs can exploit the flow deflection from upwind adjacent rotors and there is a potential of an order of magnitude increase in power density. To complement the previous validation of the ALM-LES code performed against laboratory experiments (Hezaveh et al. 2016), and to ensure that the simulations accurately represent the flow in between multiple turbines and therefore in turbine clusters, we will compare our LES results to the field measured data described in Dabiri et al. (Dabiri 2011a) for two adjacent counter-rotating turbines. This is the first validation of our model against data from real-sized VAWT field measurements, and in fact, to the best of our knowledge, the first validation of any VAWT ALM-LES against field data.

The experimental setup details are presented in Table 3-1 and the schematic configuration is shown in Figure 3-2. The VAWTs were $1.6D$ ($D$ is their rotor diameter) apart, and the velocity profiles were measured at 16 points with streamwise coordinates (relative to the line joining the centers of the turbines) at $x = -15D$, $-1.5D$, $2D$, $8D$ and elevations above ground of $z = 3m$, $5m$, $7m$ and $9m$. All of the velocities are normalized using a 10 m meteorological tower in the vicinity of the experiments (Araya et al. 2014).

In order to match inflow conditions such as turbulence intensity and mean upstream velocity profile in LES to the observed field data, a precursor periodic simulation was run to generate the inflow. The surface roughness and friction velocity of this precursor simulation were calibrated to yield the experimentally-observed log-law velocity profile. Then, $y-z$ slices of instantaneous velocity and pressure were saved every time step and fed to the simulation with the turbines as upwind inflow.
Table 3-1 Experimental configuration (Dabiri 2011a)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of turbines</td>
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</tr>
<tr>
<td>Number of blades per turbine N</td>
<td>3</td>
</tr>
<tr>
<td>Rotor diameter $D$, $m$</td>
<td>1.2</td>
</tr>
<tr>
<td>Blade vertical height, $m$</td>
<td>6.1</td>
</tr>
<tr>
<td>Blade chord length $c$, $m$</td>
<td>0.11</td>
</tr>
<tr>
<td>Airfoil section type</td>
<td>DU 06-W-200</td>
</tr>
<tr>
<td>Solidity $Nc/\pi D$</td>
<td>0.275</td>
</tr>
<tr>
<td>Tip speed ratio $\lambda$</td>
<td>2.18</td>
</tr>
</tbody>
</table>

Figure 3-2 Schematic of (a) the two turbines in the 3-D flow domain and (b) Top view of the computational domain
The results are shown in Figure 3-3, and it is clear that the ALM-LES is capable of closely reproducing the wake generated by the interactions of the two counter rotating VAWTs (the blades move towards the back when facing the other turbine, such that the flow acceleration in between the two rotors is maximized). We underline the observation we made here that it was essential to provide the simulation with the correct inflow (left panel of Figure 3-3, from the precursor simulation) for the profiles near and behind the turbines (right 3 panels of Figure 3-3) to be reproduced accurately. These results confirm that the ALM-LES model produces realistic wakes even where turbines are interacting, and, therefore, investigating large wind farms and VAWT clusters can proceed with confidence.

Figure 3-3 ALM-LES incoming velocity and wakes versus field measurements data
3.3.2 VAWTs Clusters

Clustering VAWTs in small arrangements was shown to have several advantageous (Dabiri 2011a) implications for power generation. The global performance of the turbines is enhanced since the downstream turbines can benefit from the flow deflection effect and the resulting higher velocity induced by upstream turbines. However, depending on the wind direction, compact clustering might also have negative effects when one turbine is mainly in the wake/shadow of an upstream one. For example, if two turbines are clustered together, the range of wind directions for which one of the turbines is affected by being in the shadow (partially or fully) of the other is $2\beta$, where $\beta = \tan^{-1}\left(\frac{2D}{L}\right)$ (Figure 3-4, left panel), $L$ being the turbine spacing (center to center) in a cluster. When the wind is on the other hand approximately perpendicular to the center-to-center axis, the higher induced velocity in between the two turbines is not being exploited in such cases.

![Diagram showing wind directions](image)

**Figure 3-4** Different wind directions in which VAWTs are in the wake of an upstream turbine for 2, 3 and 4 turbines ($\gamma \approx \beta$)

By only introducing the third turbine, the number of wind directions where 2 turbines can directly shadow each other is increased to $6\beta$ (Figure 3-4, middle panel). However, the
third turbine can benefit from the higher velocity induced in between the two upstream ones or the two downstream rotors can benefit from the transverse flow deflection of the upstream turbine (depending on wind direction). This has the potential to result in a power production of these 3 turbines that is greater than the power from three distant non-interacting ones (this improvement will also depend on \( L/D \) as we will show). By increasing the number of turbines in the cluster beyond 3, the flow-related benefits will drop and the number of wind directions where the turbines shadow each other will increase to \( n (n-1) \beta \), where \( n \) is the number of turbines in the cluster (e.g., Figure 3-4, right panel). In Figure 3-5, the variation of \( \beta \) with \( L/D \) and \( n \) for various clusters is shown. By increasing the \( L/D \) of a cluster, the \( \beta \) value is reduced. On the other hand, increasing \( n \) results in higher \( \beta \). For \( n > 3 \), the \( \beta/2\pi \) value can become larger than 1, which indicates there is no wind direction where the turbines are not casting shadows on each other. However, one can note that for \( L/D > 5 \), the differences between \( n = 2 \) and \( n = 3 \) are minor. Moreover, the clustering with higher \( n \) has the important benefit of using a smaller land area. Therefore, the most efficient design for a cluster, when there is no dominant wind direction of the site, seems to be a triangle, with \( n = 3 \) since it has a limited \( \beta \) while at the same time allowing for compact clustering and synergistic interaction between the turbines. Going to \( n = 4 \) almost doubles the shadowing angle \( \beta \), with no increase in the wind direction range for which synergistic interactions occur. Therefore, for the rest of this chapter we will focus on clusters with \( n = 3 \).
Figure 3-5 Variation of $\beta$ with the $L/D$ ratio and the number of turbines in a cluster, $n$

In order to investigate the proposed triangle cluster, several parameters are defined in Table 3-2; these remain constant for the analyses in this subsection (except the grid and domain sizes as detailed later). In addition to these fixed parameters, the wake deficit increases with $D$ and decreases with the distance between the turbines $L$, thus, $L/D$ is an important dimensionless number to consider; it also controls the shadow angle $\beta$ as illustrated in Figure 3-5. In order to investigate the optimal distance, various $L/D$ ratios ranging from 2 to 8 were simulated. However, before conducting these simulations the computational setup needed verification; therefore, for one fixed $L/D = 6$, analysis of the sensitivity to the domain size was performed (such that the domain size and number of grid points increase proportionally, and thus the grid resolution is unchanged). Two parameters were investigated for sensitivity to the domain size: the average cluster power
coefficient $C_P$ and the wake velocity deficit values at 15D and 20D downstream of the cluster. The wake velocity deficit is averaged in time and over a $y$-$z$ rectangle that is aligned in the $x$ direction with the turbine cross-section projected area. As can be seen from Figure 3-6, changes in domain length $L_x$ have little impact on the average $C_P$; changes in domain width $L_y$, on the other hand, can be significant when $L_y$ becomes very small (the crossflow area blocked by the turbines become large) and prevents correct sideways deflection of the streamlines. The figure suggest that a minimal $L_y \geq 40 \, D$ should be used, since increasing the transverse domain size to $L_y = 54 \, D$ results in insignificant changes in the average $C_P$. However, the velocity deficit values are also changing with $L_x$ (due to downstream boundary condition effects). Based on the sensitivity analysis results in Figure 3-6, a $L_x > 40 \, D$ is required. Therefore, a domain size of $L_x = 60 \, D$ by $L_y = 40 \, D$ is adopted for the single cluster simulations. All these simulations were conducted using laminar logarithmic profile inflow. To assess the influence of the turbulence levels in the inflow, a simulation was conducted using inflow planes from a precursor periodic run. As can be seen in Figure 3-6, using a turbulent inflow has reduced the deficit values at 15D and 20D downstream of clusters significantly, which can be explained by the fact that the increase in turbulence intensity increases momentum entrainment into the wake and speeds up its recovery.
Table 3-2 Triangle clusters experiments

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of turbine $n$</td>
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</tr>
<tr>
<td>Number of blades per turbine $N$</td>
<td>3</td>
</tr>
<tr>
<td>Rotor diameter $R$, m</td>
<td>1.2</td>
</tr>
<tr>
<td>Blade vertical length, m</td>
<td>6.1</td>
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<td>Airfoil section type</td>
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<tr>
<td>Solidity $Nc/2R$</td>
<td>0.275</td>
</tr>
<tr>
<td>Tip speed ratio $\lambda$</td>
<td>2.18</td>
</tr>
<tr>
<td>Power coefficient $C_p$ for isolated turbine</td>
<td>0.36</td>
</tr>
<tr>
<td>Grid size $N_x \times N_y \times N_z$, nodes</td>
<td>88×192×192</td>
</tr>
<tr>
<td>Domain size $L_x \times L_y \times L_z$, m</td>
<td>72×48×25</td>
</tr>
</tbody>
</table>

Figure 3-6 Domain size sensitivity analysis. The adopted size is 60D×40D,

\[
C_p = \frac{\text{Turbine Power}}{\left(\frac{1}{2} \rho A (u_{\infty})^3\right)}
\]
With the domain size set, simulations with triangle clusters with different $L/D$ ratios were conducted, with the parameters shown in Table 3-2 fixed. Based on simulations for different cases at wind direction of 60 degrees (see Figure 3-6), it is obvious that by increasing $L/D$, the performance of first/upwind turbine is increased due to the decrease in upstream blockage effect from turbines 2 and 3. Due to the rotation direction of the 1$^{st}$ turbine (shown in Figure 3-6) and deviation of flow, 3$^{rd}$ turbine has a higher $C_P$ compared to the 2$^{nd}$ turbine. On the other hand, the performances of 2$^{nd}$ and 3$^{rd}$ turbines are decreased at higher $L/D$ since they are less able to utilize the higher induced velocity from the flow deflection by the upstream rotor. The average $C_P$ (related to the upstream wind speed $U_\infty$) therefore peaks at an intermediate $L/D$. The three cases with $L/D$ of 3, 4 and 5, which had the highest average $C_P$, were hence selected for further analyses.

![Figure 3-7 Triangle cluster $C_P$ for each turbine, and averaged for the cluster, as a function of $L/D$.](image)

56
These analyses consisted of simulations where all parameters remain the same for a given 
$L/D$, but with different incoming wind orientation. We aim to investigate omni-
directionality of the proposed VAWT clusters, as well as to find the most efficient 
VAWT spacing averaged over all wind directions. Figure 3-8-a shows the average $C_P$
versus incoming wind direction. The case with $L/D$ equals to 5 has the highest $C_P$
averaged over all turbines for all wind directions. This is confirmed in Figure 3-8-b
where the $C_P$ averaged over all wind directions and all turbines is first normalized by the 
$C_P$ of a single isolated turbine, and then plotted for different $L/D$. $L/D = 5$ performs better
because the angles where VAWTs are casting shadows on downstream turbines ($\beta$) are
reduced by increasing the distance between them, as well as because wake recovery is
improved when there is not such shadowing. Finally, a key observation from Figure 3-8-b
is that the average $C_p$ is about 8% higher than for a single isolated turbine when $L/D = 5$;
this confirms that the synergistic interaction between closely-space turbines can indeed
result in a higher overall power generation when used adequately.

![Figure 3-8](image)

Figure 3-8 a) Triangle cluster average $C_P$ versus wind direction. b) Triangle cluster average $C_P$, also
averaged over all wind directions and normalized by single isolated turbine $C_P$ (angled brackets
denote averaging).
With the cluster design selected, we now turn our attention to the design of farms based on clusters. An important parameter in designing and optimizing wind farms is the distance required for wind velocity and power recovery downstream of turbines (Hezaveh et al. 2016); this applies for layouts consisting of single turbines as well as for clusters (unless there is a dominant known wind direction). The wind velocity deficit 

\[
(1 - \frac{U(x, y, z)}{U_\infty(z)})
\]

was averaged over the y-z planes encompassing the whole cluster (projected wind-normal area), and over equivalent planes at varying x distances from the hub (Figure 3-9). We also investigated various L/D to confirm our choice of L/D = 5 made based on the power output of an isolated cluster. By increasing the distance between the turbines in each triangle cluster, the distance needed for the wind velocity to recover to 75% of upstream velocity \(U_\infty\) is decreased significantly. It is clear from the figure that the recovery distance is reduced from 25D for \(L/D = 3\) to 15D for \(L/D = 5\).

This is an important criterion for designing a wind farm, which further solidifies the option of \(L/D = 5\) as the optimal. In a wind farm, it is important that downstream turbines are placed at distances were the available wind has recovered to at least 75% of its upstream undisturbed velocity so the power generation capacity of these turbines is not underutilized. Furthermore, as shown in the figure, by increasing the distance between the turbines in the clusters, the effect of incoming wind direction on the recovery distance is reduced. The \(L/D = 3\) recovery is sensitive to the change in incoming wind angle; the recovery distance to 75% occurs anywhere between 18D and 28D as the wind angle changes. The recovery for \(L/D = 5\) on the other hand is much less sensitive to wind direction and thus yields omnidirectional farms. Figure 3-9 also indicates that when
designing farms based on clusters with $L/D = 5$, the required separation between clusters for $\approx 75\%$ recovery ranges between 15 and 18 $D$.

Figure 3-9 Comparison between averaged velocity deficit at various $L/D$, and different wind direction

3.3.3 Wind Farms

Now we tackle the main question of the chapter: can synergistic interactions between VAWTs increase wind farm power density. Practically, we need to investigate whether farms of synergistic clusters perform better (produce more power per unit land and/or per unit invested cost) than two prototypical layout of wind farms, aligned and staggered regular arrays. Based on the size of the selected turbine and the results obtained in the previous section, 11 farm configurations were simulated. The turbines are the same as the ones detailed in Table 3-2. The simulations in this section are all periodic (an infinite farm is simulated), with $N_x \times N_y \times N_z = 320 \times 160 \times 336$ (grid nodes) and $L_x \times L_y \times L_z = 96 \times 48 \times 32$ (m).

Six of the cases were prototypical wind farms with staggered or regular array configurations and with separation distances of 5D, 10D or 20D for each configuration;
these configurations result in 128, 32 and 8 turbines in the computation domain, respectively. Four additional experiments were conducted with wind angles of 0 or 60 degrees, using staggered clusters with $L/D = 5$ for each cluster, and with inter-cluster distances of 20D or 10D for each wind angle, corresponding to 24 and 96 turbines in the domain, respectively. It should be mentioned that since the LES code uses periodic boundary conditions in both $x$ and $y$ directions, these simulations correspond to infinite wind farms with infinite number of turbines. Therefore, the number of turbines in the computational domain will not influence the results when normalized per turbine. Finally, one experiment is conducted using 20D spacing and aligned clusters with a 60 degree wind angle. To visualize the differences in the flow patterns in these design, the average streamwise velocities for a few selected configurations are shown in Figure 3-10. The lower velocity in the staggered configurations reflect the higher power extraction resulting from a larger number of turbines, and therefore metrics that allow consistent comparison of these configurations are needed.
The most direct consistent metric is the average wind farm $C_P$, which is shown for different configurations in Figure 3-11. The average wind farm $C_P$ is computed using the average wind velocity in the whole wind farm domain containing the VAWTs as reference velocity (i.e. over a volume spanning a full $x$-$y$ plane and the $z$ domain from the bottom to the top of the blades). As anticipated the staggered cases have higher $C_P$ compared to the aligned cases, for both the clustered and regular designs. What is more interesting and relevant is that the clustered designs consistently produce higher power than the prototypical design for any spacing. As indicated in Table 3-2, the $C_P$ for an isolated turbine is 0.36 and the cluster-staggered designs with an inter-cluster spacing of 20D are surpassing this value over the whole wind farm for both wind angles. This is due to the gain in average $C_p$ that clusters allow, and the large inter-cluster spacing in this case that minimizes the penalty of being in the wake of the upstream cluster. By reducing
the distance between clusters to 10D, the average $C_p$ drops, but remains significantly higher than for the corresponding regular wind farms. Another interesting aspect of the results is that the staggered configurations, even at small separation distances, consistently perform better than the aligned ones. The streamwise separation in the staggered 10D case for example is the same as the separation in the 20D aligned case, and yet the staggered 10D layout yields a higher $C_p$. This would indicate that the two configurations have an influence on the performance that is not limited to the recovery distance, but seems to also affect the downward transport of mean kinetic energy, which would be higher for the staggered cases.

![Figure 3-11 Average wind farm $C_p$ for various configurations.](image-url)
The results in Figure 3-11, however, exclude an important difference between these periodic simulations. Due to higher drag forces exerted on the ABL in cases with higher densities (5D spacing) or cases with more efficient farm layouts, the required pressure gradient imposed in the simulations to yield a steady state mean flow will also be higher. In the LES, at each time step, the drag exerted on the ABL by the whole wind farm and the ground surface is computed and the needed mean streamwise pressure gradient is imposed. This gradient reaches a steady state eventually, when the mean flow equilibrates. Due to this feature, the different cases have unique pressure gradients over the wind farm. This pressure gradient is an important parameter in such periodic simulations that are mimicking very large farms. In large farms, the wind speed (streamwise-averaged over the footprint of a row) is constant over most of the rows, and the main source of energy is this pressure drop rather than the upwind velocity (the latter being the main parameter for small farms). Thus, the kinetic energy that can be extracted in large farms scales with the pressure drop that scales with the total surface drag, while the advective velocity for the energy (to obtain power) is the constant wind speed. That implies that for large farms the total pressure drop along the farm (and thus the available energy) will be limited since these large farms influence pressure fields in the atmosphere significantly. Therefore, in order to be able to compare the various configurations without this pressure drop discrepancy, the following $C_P^*$, normalized per unit pressure gradient, is introduced:

$$C_P^* = \frac{\text{Average Wind Farm Power}}{\frac{1}{2} \rho A \left( u^* \right)^2 \left< u_T \right>}, \quad u^{**} = \sqrt{\frac{\Delta P_{(Loss)} L_x}{L_z}}, \quad (3-1)$$
where $\sqrt{u^{+\ast}}$ is the root square of total surface drag (on ground + turbines), which is related to the total pressure drop $P_{Loss}$, $\langle u_F \rangle$ is mean streamwise velocity over the wind farm (average of the domain containing the blades as before), and $A$ is the rotor area. The comparison of this new performance metric for the various configurations is presented in Figure 3-12. Since $u^{+\ast}$ is about 10 percent of horizontal velocity, $C_P^*$ is 100 times $C_P$. Even after normalizing the total power generated in these layouts by the exerted pressure gradient for each case, the cluster cases maintain the highest $C_P^*$. This means that these cases were able to extract more energy from the applied pressure gradient in the field compared to regular wind farms. The relative differences in the performance of the farms is expected to be closer to the differences depicted in Figure 3-11 for smaller farms, and closer to the ones in Figure 3-12 for larger farms.
A comparison of the power density per unit land area used for the various configurations was also performed. It confirmed that clustered designs increased the power density, validating the hypothesis of the chapter. However, the results have the caveat that the power density will be invariably higher for very close spacing, even when the turbines in the farm are not being used efficiently (low $C_p$). Therefore, power density itself cannot be used a metric for optimizing farm layout. In order to have a more realistic and practical metric, the total capital cost per unit power generation is computed. Since the power generation for each farm is proportional to the sum of the $C_p$ values of all the individual turbines, $C_{p,\text{total}}$, we use this sum for normalization instead of the actual power. The capital costs consist mainly of the cost of the land and the turbines:

**Figure 3-12 Average wind farm $C^*$, normalized per unit pressure gradient applied, for all cases.**
\[
\frac{\text{Cost}_{\text{total}}}{C_{P,\text{total}}} = \left( \frac{\text{Cost}_{\text{Land}} \times A + \text{Cost}_{\text{Turbine}}}{C_{P,\text{total}}} \right) \times \frac{A \times \text{Cost}_{\text{Turbine}}}{C_{P,\text{total}}} 
\]

(3-2)

Where \(\Gamma_A\) is the wind turbine density per unit area, \(A\) is the total land area. \(\text{Cost}_{\text{Land}}\) is the cost of land per unit area and \(\text{Cost}_{\text{Turbine}}\) is the cost of a single turbine. Using different land cost to turbine cost ratios, and the cost for a typical individual turbine (\(\approx \$10,000\) US dollars) (Dabiri 2011a) in Equation 9, the total cost over total \(C_P\) ratios were computed and plotted in Figure 3-13. Using this comparison metric also indicates that the triangle cluster staggered layout has the lowest capital cost per projected unit power generated, and is therefore the best design among the ones we investigate here.

![Figure 3-13 Total capital cost per unit power generated for the various cases](image)

Similar analysis has been made using total \(C_P^*\) and the results confirmed that wind farms with cluster designs are still the most optimal. Again we reiterate that the comparison
with $C_p$ is more relevant for small farms while if one uses $C_p^*$ the results are more representative of large farms.

### 3.4 Conclusions

This chapter presents a novel concept for optimizing the layout of large VAWT wind farms. It takes advantage of synergistic interactions between closely spaced turbines that were previously shown to yield higher power for a limited number of turbines. Using an ALM-LES model, the modeled wake generated by two-counter rotating turbines is first successfully validated against results from field experiments. To take advantage of the high wind velocity created by the flow deflection of VAWTs when placed in close vicinity, we proposed a triangular cluster design consisting of 3 VAWTs, which can form the basis for larger wind farms. The triangular design is the one that best exploits flow acceleration, with a minimal increase in wake shadowing.

The influence of inter-turbine spacing relative to their diameter $L/D$ was then investigated to optimize a single cluster in terms of the total generated power, the omni-directionality of its performance, and the needed downstream wake recovery distance. Changing the turbine spacing, the cases with $L/D$ ratio of 3, 4 and 5 were shown to generate the highest cluster averaged power. Further tests were then performed with these three spacings only, and the case with $L/D$ of 5 emerged as the one with the highest cluster-averaged power over all wind directions: the generated power averaged over all wind direction for this case is about 10% higher than the power generated by three isolated turbines. Furthermore, $L/D = 5$ results in the lowest variation of the generated power with wind direction, and the downstream wake recovery distance is shorter (since the cluster is more
“porous”). Therefore, this cluster design confirmed the potential to use synergistic VAWT interactions to increase power production, and would generate a higher power density (power generated per unit land used) due to the proximity of the rotors. It was hence adopted for configuring large VAWT wind farms.

Farms that use this advanced cluster design and a sufficient distance for wake recovery between clusters were then compared to prototypical aligned and staggered designs for infinitely large wind farms, with different turbine horizontal spacings of 5D, 10D and 20D. For the very large wind farms simulated, the results show that the average wind farm $C_P$ for the case with staggered triangle clusters was much higher than for the wind farms with regular configurations, reaching almost double the value of the prototypical designs for 20D turbine or cluster spacing. Using these average $C_P$ results and a simple capital cost function for the whole wind farm and varying the land to turbine cost ratio, we also showed that the wind farm design with staggered triangle clusters is the optimal design (amongst the ones considered here) in terms of cost per unit power produced. This normalized cost is almost reduced in half compared to prototypical configurations when in areas with relatively high land costs.

These results strongly indicate that VAWT farms can and should be configured using different approaches than the ones used for horizontal axis rotors (although the potential benefits of HAWT clustering could also be investigated). A significant increase in power and decrease in capital costs can be achieved using the ability of VAWTs to positively boost the power production of nearby turbines if properly configured. It should also be mentioned that one of the criteria in optimizing the clusters and farms in this chapter was omni-directionality. We sought to propose configurations the performance of which is not
strongly dependent on wind direction since this is also a major advantage of individual VAWTs. However, if this criterion is relaxed, e.g. in places where there is a dominant wind direction, the optimal cluster designs can be very different and can use the synergistic interaction between clusters as well, with potentially much higher power densities.
Chapter 4

4 Investigation of Mean Kinetic Energy Replenishment Mechanisms in Wind farms: The Effect of Farm Fetch

4.1 Introduction

As reviewed by McKay (McKay 2008), the current power density of wind farms, defined as the power per unit land used, is about 1 W/m$^2$. This might seem very low, but since the land between individual turbines can be used for other purposes (e.g. most current farms lease small areas for the tower base in agricultural lands), it has not been a significant detriment for wind energy expansion. However, to reach the goal of 20% electricity provided by wind power by the year 2030, the cumulative installed capacity should reach 300 GW (Gigawatt) as outlined by the DOE (Breu et al. 2008). This entails a significant expansion of wind farms, and hence raises the appeal and advantages of higher land power densities.

To reach such higher power density values, innovative farm designs are needed. As shown in the previous chapter, using a novel design of VAWT clusters instead of individual VAWTs in large wind farms increases the power output and $C_P$ values for whole wind farm per unit land used, thus increasing the power density. However, in the previous chapter, an infinite wind farm was effectively simulated due to the periodic boundary conditions. In the limits of such a very large fully developed wind farm boundary layer, where the frontal power that was advected into the farm has been...
exhausted, only the downward plane-form energy from above the wind farm is available to replenish the mean kinetic energy of the air inside the farm, as discussed by Dabiri et al (Dabiri 2011a). This downward transfer of MKE is thus the upper limit on the power extraction by VAWT arrays in very large farms, unlike a single standing turbine or a small farm where the advected power controls the potential generation. Dabiri et al further suggested that as long as the wind velocity above the wind farm array remains higher than 75% of undisturbed velocity, the power extraction enabled by this upper limit will be one order of magnitude greater than the actual extraction of current HAWT wind farms. However, due to insufficient measurements, they were not able to completely confirm this hypothesis.

Calaf et al, (Calaf, Meneveau, and Meyers 2010c) addressed these same challenges for HAWT wind farms; they found that farms with a horizontal length larger than 10-20 km (an order of magnitude larger that ABL height), will reach this infinite length wind farm regime we simulated in the previous chapter. They also simulated various HAWT configurations and investigated the effect and magnitude of different kinetic energy transport mechanisms in such fully developed boundary layer regime. They showed that vertical fluxes of kinetic energy have the same order of magnitude as the power extracted by the wind turbines (which is expected). This implies that the proximity of turbines in such infinite farms is limited by the downward replenishment of TKE. Due to the ability of VAWTs to be placed in closer proximity that was demonstrated in the previous chapter, therefore increasing the power density, questions arise as to the limits on such proximity that might arise from the constraint of MKE replenishment and whether even closer arrangements can be used in farms with limited fetch (farm fetch is the length of
the farm in the along-wind direction). Using our ALM-LES model for VAWT wind farms, in this chapter we perform simulations on finite and infinite length wind farms and investigate the mechanisms responsible for kinetic energy replenishment in wind farms.

The simulations in chapter 3 were conducted using periodic boundary condition, and to compensate for the losses in the pressure gradient term due to the turbines, an additional pressure gradient was imposed to force the flow and compensate for total losses. It was suggested that the relative differences in the performance of the farms are expected to be closer to the differences depicted in Figure 3-11 for $C_P$ for smaller farms, and in Figure 3-12, $C_P^*$, for larger farms. In this chapter, using precursor simulations to generate realistic turbulent inflows, we investigate that hypothesis and compare power recovery mechanisms and power output between large “infinite” wind farms and finite ones to answer the following research questions:

1- What transport mechanisms dominate the regeneration of mean kinetic energy in VAWT wind farms, and how do they vary with wind farm scale?

2- How large should VAWT wind farms be to display the dynamics of fully developed wind-turbine array boundary layers?

### 4.2 Numerical Model

To investigate VAWTs in ABL, the LES code with the actuator line model (ALM-LES) presented and validated in previous chapter is used. The numerical model with wind-tunnel measured dynamic $C_L$ and $C_D$ force coefficients is used. For the finite wind farm simulation, a turbulent inflow was created using a separate precursor periodic run (without turbines) and inflow planes were saved every 5 timesteps. These inflow planes
are provided as inflow boundary conditions for finite wind farms simulations, and the values are interpolated within each 5 timesteps intervals.

The budget equation for mean kinetic energy involves multiple terms reflecting the various mechanisms for generation, dissipation and transport (Stull 1988). If we consider the budget equation for the x-component of the MKE alone $0.5U^2$, and assuming the farm is infinite in the cross stream direction and a statistically steady-state, two terms will dominate the vertical transport of MKE in and out of wind farms through the horizontal plane at the top of the farm. The first one is the vertical advection of MKE by mean vertical wind, $0.5\langle W \rangle \langle U \rangle^2$ (its spatial average gives rise to the so-called dispersive fluxes), and the second one is through the interaction of mean flow and turbulence which is $\langle u'w' \rangle \langle U \rangle$. Using LES results, we will compute the vertical and horizontal profiles of these terms, and will compare their magnitude with the initial advective influx of MKE through the frontal area, $0.5\langle U \rangle^3$ and with the power extracted by the wind turbines. The notations used in the terms above and the rest of this chapter are as follows: an instantaneous turbulent parameter $q$ is Reynolds decomposed into its mean $Q$ (estimated as the time average) and the perturbation $q'$. Spatial averages are denoted by angled brackets $\langle \rangle$, and they are performed either over some horizontal plane $x$-$y$, or only in the cross stream direction $y$ (as indicated in the relevant sections).

### 4.3 Simulations Setup

In order to investigate the mean kinetic energy transport mechanism in wind farms, a suite of simulations has been conducted. For finite wind farms, to be able to reach fully
developed wind farm boundary layer conditions, a long domain in streamwise directions was used. One factor that constrains the size of the domain is the size of the wind turbine; the grid resolution should be fine enough to capture each turbine, while also allowing for a large domain size with a manageable number of grid nodes. For these simulations, 9.1 m tall wind turbines, with 1.2 m diameter, were selected. The practical domain size we used is 198×48×32 m, spanned by 660×160×336 grids in x, y and z directions. The grid resolution is thus $dx \times dy \times dz = 0.3 \times 0.3 \times 0.0955$ m. The VAWT clusters studies in the previous chapter, with a spacing of $5D$, are used as the wind farm basic elements. Four wind farm layouts with aligned and staggered configurations and cluster spacings of 10D and 20D are simulated. The details for a single wind turbine are shown in Table 4-1, while the layouts and wind farm simulations setups are shown in Table 4-2. For one of the cases, the 20D-Staggered wind farm, a longer domain with double the number of turbines was also simulated to investigate the influence of fetch more thoroughly (last column in Table 4-2).

<table>
<thead>
<tr>
<th>Table 4-1 Wind turbine experimental configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades per turbine $N$</td>
</tr>
<tr>
<td>Rotor diameter $D$, m</td>
</tr>
<tr>
<td>Blade vertical height, m</td>
</tr>
<tr>
<td>Blade chord length $c$, m</td>
</tr>
<tr>
<td>Airfoil section type</td>
</tr>
<tr>
<td>Solidity $Nc/\pi D$</td>
</tr>
<tr>
<td>Tip speed ratio $\lambda$</td>
</tr>
</tbody>
</table>
For infinite wind farms, the setups are almost identical to the ones described in chapter 3 where periodic boundary conditions are used. The major difference is that since we aim to compare the results with finite wind farms, the pressure gradient is not augmented to compensate for the drag from turbines. Therefore, only the 20D spacing infinite cases, with aligned and staggered layouts are simulated (in these 10D cases the flow cannot be sustained with an augmented pressure gradient). The simulations setups for infinite wind farms are shown in Table 4-3.

### Table 4-3 Infinite wind farms layout configuration with period domains

<table>
<thead>
<tr>
<th>Simulations name</th>
<th>20-S-P and 20-A-P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clusters horizontal spacing ($S_X \times S_Y$)</td>
<td>20D×20D</td>
</tr>
<tr>
<td>Wind farm layout</td>
<td>Staggered and aligned</td>
</tr>
<tr>
<td>Number of turbines</td>
<td>24</td>
</tr>
<tr>
<td>Number of rows</td>
<td>4 (infinite)</td>
</tr>
<tr>
<td>Number of clusters/row</td>
<td>2</td>
</tr>
<tr>
<td>Grid size $N_x \times N_y \times N_z$, nodes</td>
<td>320×160×336</td>
</tr>
<tr>
<td>Domain size $L_x \times L_y \times L_z$, m</td>
<td>96×48×32</td>
</tr>
</tbody>
</table>
The simulations are performed for 200000 timesteps until the power production of each wind turbine are converged to a steady value. The time averaged 3D flow field variables are outputted at the end of simulations.

4.4 Results and Discussion

4.4.1 ALM-LES wind farm results

Figure 4-1 shows the streamwise velocity in the finite cluster wind farms with 10D and 20D spacings, and with aligned and staggered configurations. It is clear from this figure that the turbines in the 10D spacing cases, both aligned and staggered, are decreasing the velocity within the wind farm more than the 20D spacing cases. The 20-A case has higher wind velocity in between each line of turbine clusters, and the velocity is reduced significantly in the wakes of the clusters. These line of higher velocity speeds are only visible till the sixth row in the 10-A case, after that distance, the wakes generated by the turbine clusters merge. For both the 10D and 20D staggered cases, except within the first row, such higher velocity regions cannot be found due to the layout. The important role played by the initial advective influx of MKE through the frontal area, $0.5\langle U \rangle^3$, is clear in the 10-A simulations. It enables the high velocity regions in between rows to persist till the 6 row and to serve as a source of MKE that can be transported laterally. However, this initial influx seems to be exhausted by the 6th row, after which the downward entrainment of MKE from above (which will be discussed later) become important and limits further reduction in the flow velocity. For the 20D spacing cases, we conducted a longer domain length to be able to confirm the same results (20-S-L); its flow field is depicted in Figure 4-2. Like the 10-S case, after half the fetch of the wind farm in the
longer domain, the reduction of flow speed is significant but begins to plateau as MKE is replenished from downward entrainment into the wind farm.

Figure 4-1 Streamwise velocity in finite-fetch wind farms with inflow for 10D and 20D horizontal spacings, mean wind from left to right: a)10D Aligned, b)10D Staggered, c)20D Aligned, d)20D Staggered
In Figure 4-3, the spatial averages of the vertical profiles of the interaction of mean flow and turbulence (a), the spatial average of vertical profiles of downward advection of MKE (b) and the sum of these two fluxes (c) are shown (colored lines for finite wind farms and black line for infinite one). We only show the staggered cases since each aligned case had very similar profiles to the corresponding staggered case. Note that the spatial average of the vertical advection term encompasses the advection by the spatially-averaged velocity (which is negligible since the spatially averaged vertical velocity $\langle W \rangle \approx 0$) and the so-called dispersive fluxes that arise from the spatial covariance of the time-averaged quantities (Poggi and Katul 2008; Raupach and Shaw n.d.) For the 10-S, the vertical advection of MKE is always negative for finite wind farms, which means that due to the reduction of streamwise wind velocity in the wind farm, the vertical velocity is always upwards and is thus advecting MKE upwards and out of the wind farm domain (this is like a farm-scale Betz limit effect). This is also the case for 20-S simulation. The vertical profile of the interaction term between turbulence and the mean stream ($\langle u'w' \rangle \langle U \rangle$) is shown in Figure 4-3 (a). That transport is mostly positive (downward), replenishing the turbine layer’s MKE from above but also transporting MKE out of the layer at the bottom. The net transport is positive since the fluxes at the top are higher than at the bottom. For the 10-S case, there is an upward flux of energy from the bottom of turbine blades (at 3 m), as well as a downward flux from the top of wind farm (at 9.1 m), both replenishing the turbine layer with MKE. For 20D, the fluxes are smaller since the power extracted is less than for the 10D cases (due to the smaller number of turbine per unit land area). For the periodic, infinite simulations, the net transport is always downwards; this will discussed in detail later in this chapter.
Figure 4-2 Streamwise velocity in the longer wind farm with inflow with 20D horizontal spacings, case 20-S-L, mean wind from left to right.

The sum of these two transport terms is shown in (c), which depicts a net downward flux over the whole wind farm from the top of wind farm domain (at 9.1 m), and for 10D cases also an upward flux of energy from bottom of wind farm domain. The net flux is positive in all cases, indicating a net transport of MKE into the turbine layer. The vertical derivatives of these fluxes are shown in (d); they reflect the net transport at any layer (influx – efflux) and illustrate the positive rate supply of energy all over the turbine layer.

In Figure 4-3 the black line is vertical profiles averaged in x and y directions of staggered infinite wind farm with 20D horizontal spacing. Since the simulations are periodic in both x and y directions, the wind farms have an infinite number of turbines. For both layouts, (aligned case is similar to staggered case) there is downward flux of kinetic energy from interactions of mean flow and turbulence from top of wind farms; however, in contrast to finite wind farms with finite length, there is no upward flux of kinetic energy from vertical advection of MKE. Therefore, all of the power transported downward is available for extraction by the wind turbines. In these wind farms, there is no frontal advection of MKE available, and all the power transported to the wind farm domain is thorough downward fluxes of kinetic energy.
Figure 4-3 Vertical profiles of fluxes of MKE averaged over whole wind farms domains. Turbine layer is shown with cyan color.

In order to investigate these MKE fluxes at the top of turbine layer in more detail, their averages in the y-direction, at increasing streamwise distances from the upwind edge of the wind farm, are shown in Figure 4-4 (for now we focus only on the solid lines). As shown, over the length of the wind farm, there is a downward flux of energy from the
interactions of turbulence and the mean flow, and an upward advective transport of MKE from the top of the finite wind farms. However, the net effect (sum) of these two terms shows net upward fluxes over the first few rows of clusters, and the switch to a net source of MKE only occurs after the sum changes sign. That is, for the first few rows, not only are the turbines extracting MKE from the flow, but there is also a net loss of MKE by transport. For the 10D spacing cases, the switch to net replenishment occurs about half way through the streamwise fetch of the wind farm, but for the 20D spacing cases with same domain length, this transition point is not obvious. Focusing now on the 20-S-L case (dotted green line), we see that after the midpoint of the wind farm this flux also becomes positive, implying that MKE is being transported down into the turbine layer. This will help us understand the relationship between cluster spacings and the needed length for the wind farm to reach the limits of fully developed wind farm array boundary layers, where the dominant MKE transport mechanism is thorough downward interactions of mean and turbulent flow. The 20-S and the 20-S-L results match quite well near the end of the short farm of the 20-S case, but it seems this fetch remains relatively short and the condition at the end are very far from the fully developed (infinite) farm conditions. The longer farm on the other hand starts to display variations in the flux that seem self-similar at various rows, implying an approach to the fully developed regime.

Infinite wind farms fluxes are shown in Figure 4-4 with black lines (the fluxes are plotted from X/D of 170 to 240, but this is just for visibility, their location is irrelevant since these farms are periodic). We can now check whether at the downstream end of finite length wind farms, the fluxes are reaching the limits of infinite wind farms. As shown in
In this figure, the vertical fluxes of the interaction of mean flow and turbulence are comparable with the last rows of clusters of finite wind farms. Furthermore, since the vertical advective term decreases significantly over the last rows in finite wind farms, the magnitude and pattern of the sum of these two terms become very similar to the case with an infinite wind farm.

**Figure 4-4** Fluxes of kinetic energy averaged over the cross-stream y-direction. The short wind farms starts and ends are shown by cyan color and long wind farm with magenta color.
The available power and cumulative extracted power are now compared. The available power is computed as the sum of frontal advected power plus the turbulent and dispersive transported power from top and bottom of the turbine layer:

\[ P_{\text{Available}} = P_{\text{Frontal}} + P_{\text{Top}} - P_{\text{Bottom}} \]  

(4-1)

\[ P_{\text{Frontal}} = \frac{1}{2} \rho L_y H \langle U \rangle^3 \]  

(4-2)

\[ P_{\text{Top}} = \int_{x=0}^{X} L_y \left( \left\langle u' w' \right\rangle \langle U \rangle + \frac{1}{2} \left\langle W^2 \right\rangle \langle U \rangle \right) \bigg|_{z=9.1 \text{m}} \, dx \]  

(4-3)

\[ P_{\text{Bottom}} = \int_{x=0}^{X} L_y \left( \left\langle u' w' \right\rangle \langle U \rangle + \frac{1}{2} \left\langle W^2 \right\rangle \langle U \rangle \right) \bigg|_{z=3 \text{m}} \, dx \]  

(4-4)

where \( L_y \) is the width of the wind farm; \( H \) is the rotor blade length; \( \rho \) is the air density; \( x=0 \) is the start of the farm and \( X \) is an arbitrary location in the farm; the spatial averaged are done over the \( y-z \) plane defining the upwind inflow boundary of the wind farm in 4-2, and over \( x-y \) planes defining the top and bottom boundaries of the farm in 4-3 and 4-4.

The available power is shown in Figure 4-5 with lines without markers and the cumulative extracted power is shown by lines with markers. The value of the available power at \( x = 0 \) is simple \( P_{\text{Frontal}} \), and change after \( x=0 \) are due to \( P_{\text{Top}} \) and \( P_{\text{Bottom}} \). For the 10D spacing cases, up to the sixth row of turbines, the available power decreases and then plateaus after that, which means no net energy is entering or leaving the turbine layer. The subsequent increase marks the start of the regeneration. In addition, the slope of the cumulative extracted power is larger in the first few rows of these 10D cases, which means they are producing more energy than subsequent rows due to high energy.
provided by the frontal plane advection. However, after the midpoint of the farm, this slope becomes smaller and constant (i.e. rows extract the same amount of power) till the end of the wind farm. The net transport of energy into the turbine layer remains negative, so the energy extracted from these subsequent rows is still from the $P_{\text{Frontal}}$, which now is being redistributed inside the farm layer from region of high velocity to regions of lower velocity. Furthermore, it is obvious that the staggered design is extracting more energy compared to aligned case.

For 20D spacing cases, the total available power is decreasing less rapidly than for the than 10D cases, which can be attributed to the fact that the lower turbine density induces a weaker flow deceleration and therefore a lower uplift positive $W$, which minimizes the loss of MKE through the dispersive fluxes as shown in Figure 4-4. The extracted power by the 20-S case is higher than the 20-A case and their difference is higher in comparison with the difference between the 10-S and 10-A cases. For all of the cases, the sudden increase in available power after last row of clusters is due to sudden and large amount of energy transported downward. Since there are no more wind turbine drag in this region, the wind is accelerating in $x$ direction and creating a downward velocity, which brings significant amounts of MKE from aloft.

Figure 4-5 and Figure 4-6 also show the available and extracted power for infinite wind farms. Since there is no frontal power available at infinite farms, the only source of power is from downward plan-form power form top of wind farm, which has a smaller magnitude compared to frontal power. Therefore, the amount of extracted power is also less than finite wind farms.
In addition, in Figure 4-5, the available and extracted power for longer finite wind farm with 20D spacing is shown. Over the length of first half of wind farm, which is the same as shorter wind farms, the available power is following the same pattern as the shorter 20D staggered wind farms. However after the half length of the wind farm, the available power is increased due positive downward flux of kinetic energy from the top and the slope is the same as the infinite wind farms.

Figure 4-5 Available power (lines with no markers) and extracted power (lines with markers) for all of the cases. Markers are at location of each row of clusters.
Figure 4-7 shows the average $C_p$ per row of wind turbines in the finite wind farm. Although based on the previous figure the 10D staggered cases were producing more energy, this figure indicates that the efficiency of the 20D staggered case in extracting energy from the available frontal power is higher. The $C_p$ of the second row is higher in 20D staggered case which is due to higher velocity available at the second row due to the flow constriction between the two upstream clusters. The longer 20D staggered wind farm is following the same trend as the shorter domain, and after the sixth row its $C_p$ has becomes constant.
4.5 Conclusion

In this chapter, using the ALM-LES model developed and tested in the previous chapter for vertical axis wind turbines, the transport mechanisms into the wind farm domain are investigated. For finite-fetch wind farms, there is a net downward turbulent flux of mean kinetic energy from the top of the wind farm, but its magnitude is smaller than the net upward advective efflux resulting from the upward velocity that is generated by the slowing down of the flow in the farm. Shorter farms thus lose energy over almost half of their fetch. A longer finite farm with 20D staggered configuration was simulated, and the
result do show that at larger fetch there is a significant net replenishment of MKE into farm. At such large fetches, flows in finite farms tend to become similar to those in infinite wind farms.

The results for finite wind farms show that the bulk of the extracted power is actually provided through the initial streamwise advective influx, despite the fact that beyond the half-length of the wind farm the available power is regenerated by the downward flux of MKE from the top of turbine layer, which prevents significant slowdown of the flow. The extracted power per row after the first few rows becomes constant, and the power regeneration per row also becomes constant.

Since the only available source of power for infinite wind farms is from the plan-form influx from above the wind farm, the available and extracted powers are smaller compared to finite wind farms. However, in last rows of finite wind farms, the slope of extracted power is comparable with infinite wind farms. This confirms that in large wind farms, after the large amount of available frontal power is dissipated and extracted by the initial front rows, the rest of energy is provided by the power regenerated from the top of wind farm domain.
Chapter Five

5 Conclusions and Future Research Directions

5.1 Conclusions

The general aim of this dissertation research is to better understand and model VAWTs in the ABL to use them as complementary means for extracting clean energy from wind. Specifically, this thesis investigated the effect of VAWT from a single turbine scale to wind farm scale and introduced smart designs for wind farms.

After introducing the research questions in first chapter, in the second chapter, a novel Actuator Line Model implemented in a Large Eddy Simulation code is presented, validated, and applied to the analysis of turbine wakes. This model is combined with a URANS model that captures blade-to-turbine scale flow dynamics to obtain lift and drag coefficients at any given $Re$ and for any blade design. These coefficients are then passed on to the ALM-LES model. The resulting ALM-LES-URANS coupled framework has the advantage of allowing faster and wider exploration of the parameter space that influences turbine performance for arbitrary airfoil and blade configurations. After complete validation of model performance, a parametric study of a single VAWT was carried out by first simulating nine base cases with different diameter to length $D/L$ ratios and solidities $S$. Then, for 3 combinations of $D/L$ and $S$, six supplementary simulations at different tip speed ratios $\lambda$ were conducted. Increasing $D/L$ at high $S$ reduced the wake recovery length, but at medium and low solidities it increased it. Slender rotors are therefore preferable unless $S$ is very high. Moreover, at a fixed $D/L$ ratio, the medium $S$
cases had the highest wake deficit near the turbine due to their higher \( \lambda \). Increasing the tip speed ratio consistently decreased the wake recovery distance since it produced stronger wakes behind the rotors; this holds regardless of whether the recovery assessment was based on velocity or power deficit. In particular, we can conclude that increasing the tip speed ratio (up to the optimal value that results in the largest \( C_p \)) consistently reduces the recovery distances and for medium to slender turbines (\( D/L \leq 1.25 \), more likely to be employed than the flatter ones with \( D/L = 5 \)), a medium optimal \( S \) exists that minimizes the wake recovery distance. Strong deficits directly behind the turbines do not necessarily imply longer wakes; in fact, a stronger initial deficit most likely will dissipate faster due to higher shear and turbulence generation at the edge of the wake.

In chapter 3, a novel concept for optimizing the layout of large VAWT wind farms is presented. It takes advantage of synergistic interactions between closely spaced turbines that were previously shown to yield higher power for a limited number of turbines. Using the ALM-LES model, the modeled wake generated by two-counter rotating turbines is first successfully validated against results from field experiments. To take advantage of the high wind velocity created by the flow deflection of VAWTs when placed in close vicinity, we proposed a triangular cluster design consisting of 3 VAWTs, which can form the basis for larger wind farms. The triangular design is the one that best exploits flow acceleration, with a minimal increase in wake shadowing.

The influence of inter-turbine spacing relative to their diameter \( L/D \) was then investigated to optimize a single cluster in terms of the total generated power, the omni-directionality of its performance, and the needed downstream wake recovery distance. Changing the turbine spacing, the cases with \( L/D \) of 5 was shown to generate the highest cluster
averaged power and the generated power averaged over all wind direction for this case is about 10% higher than the power generated by three isolated turbines. Furthermore, $L/D = 5$ results in the lowest variation of the generated power with wind direction, and the downstream wake recovery distance is shorter (since the cluster is more “porous”). Therefore, this cluster design confirmed the potential to use synergistic VAWT interactions to increase power production, and would generate a higher power density (power generated per unit land used) due to the proximity of the rotors. It was hence adopted for configuring large VAWT wind farms.

Farms that use this advanced cluster design and a sufficient distance for wake recovery between clusters were then compared to prototypical aligned and staggered designs for infinitely large wind farms, with different turbine horizontal spacings of 5D, 10D and 20D. For the very large wind farms simulated, the results show that the average wind farm $C_P$ for the case with staggered triangle clusters was much higher than for the wind farms with regular configurations, reaching almost double the value of the prototypical designs for 20D turbine or cluster spacing. Using these average $C_P$ results and a simple capital cost function for the whole wind farm and varying the land to turbine cost ratio, we also showed that the wind farm design with staggered triangle clusters is the optimal design (amongst the ones considered here) in terms of cost per unit power produced. This normalized cost is almost reduced in half compared to prototypical configurations when in areas with relatively high land costs.

These results strongly indicate that VAWT farms can and should be configured using different approaches than the ones used for horizontal axis rotors (although the potential benefits of HAWT clustering could also be investigated). A significant increase in power
and decrease in capital costs can be achieved using the ability of VAWTs to positively boost the power production of nearby turbines if properly configured. It should also be mentioned that one of the criteria in optimizing the clusters and farms in this paper was omni-directionality. We sought to propose configurations the performance of which is not strongly dependent on wind direction since this is also a major advantage of individual VAWTs. However, if this criterion is relaxed, e.g. in places where there is a dominant wind direction, the optimal cluster designs can be very different and can use the synergistic interaction between clusters as well, with potentially much higher power densities.

In chapter 4, using the ALM-LES model developed and tested in the previous chapter for vertical axis wind turbines, the transport mechanisms into the wind farm domain are investigated. For finite-fetch wind farms, there is a net downward turbulent flux of mean kinetic energy from the top of the wind farm, but its magnitude is smaller than the net upward advective efflux resulting from the upward velocity that is generated by the slowing down of the flow in the farm. Shorter farms thus lose energy over almost half of their fetch. A longer finite farm with 20D staggered configuration was simulated, and the result do show that at larger fetch there is a significant net replenishment of MKE into farm. At such large fetches, flows in finite farms tend to become similar to those in infinite wind farms.

The results for finite wind farms show that the bulk of the extracted power is actually provided through the initial streamwise advective influx, despite the fact that beyond the half-length of the wind farm the available power is regenerated by the downward flux of MKE from the top of turbine layer, which prevents significant slowdown of the flow. The
extracted power, as well as the power regeneration, per row after the first few rows become constant.

Since the only available source of power for infinite wind farms is from the plan-form influx from above the wind farm, the available and extracted powers are smaller compared to finite wind farms. However, in last rows of finite wind farms, the slope of extracted power is comparable with infinite wind farms. This confirms that in large wind farms, after the large amount of available frontal power is dissipated and extracted by the initial front rows, the rest of energy is provided by the power regenerated from the top of wind farm domain.

5.2 Future Research Guidelines

Based on the results obtained in this dissertation, a number of future research directions can be identified and proposed. Thermal stability has a significant effect on ABL flow and will thus influence farm performance. During day time, when the sun is heating the ground and the air is colder than the ground, positive buoyancy will increase the turbulent kinetic energy production and transport of momentum and heat. However, during the night and when the air is hotter than the ground, the opposite will happen and negative buoyancy will dissipate the turbulent kinetic energy and will reduce the transport of momentum and heat. Brief results of these effects are presented as an appendix in this thesis, but it is important to conduct a thorough study of the effect of stability on the performance of VAWT wind farms, and especially the novel cluster design layouts presented here.
Alternative cluster designs can also be investigated, particularly in regions with a relatively constant and known wind direction. In addition, other VAWT designs will have their own optimized configurations. Using the methods and models developed in this thesis, future analyses can investigate a broad range of conditions not covered here. For example, it would be useful to apply the simulations to offshore farms that offer a complementary solution for global energy demand, and as previously mentioned VAWT are inherently more suitable for offshore wind farms than HAWT.
Appendix A

A. Impacts of atmospheric boundary layer stability on VAWT wind farms

A.1 Introduction

As shown in previous chapters, wakes from upstream turbines impact the performance of downstream turbines. In infinite wind farms, the wake recovery mechanism is due to entrainment of MKE from the interaction of turbulent and mean flows. So the more turbulent the flow, the faster the recovery. Static stability in the ABL, influenced by thermal gradients, has a significant effect on flow and turbulence. During day-time, when the sun is heating up the ground and the air is colder than the ground, positive buoyancy will increase the turbulent kinetic energy production and transport of momentum and heat. However, during the night and when the air is hotter than the ground, the opposite will happen and negative buoyancy will dissipate the turbulent kinetic energy and will reduce transport of momentum and heat. It has been shown that diurnal cycle and different ABL stabilities have a significant effect on the performance of large HAWT wind farms (Abkar, Sharifi, and Porté-Agel 2016; Calaf et al. 2011). These studies showed that under convective conditions, the wake recovery is faster compared to the nighttime is due to the higher turbulence level of the incoming wind that leads to a higher turbulent entrainment flux into the wake region. However in the stable regime, the relatively low turbulence intensity results in a relatively slow rate of entrainment of momentum into the wakes, and a slow wake recovery. The averaged power deficit in the
wind farm will thus increase as thermal stability increases. Several wind tunnel experiments have been conducted on HAWTs to investigate the effect of the convective boundary layer on wind turbines wakes and performance (Leonardo P Chamorro and Porté-A gel 2010; Zhang, Markfort, and Porté-A gel 2013). These experiments are limited by the weak thermal stratifications and low Reynold numbers. As discussed in the first chapter, the majority of research studies are toward HAWTs and ABL stability effect has not been investigated in VAWT wind farms. In this appendix, we will use ALM-LES model to give a brief overview of the impacts of neutral and stable boundary layers on the performance of VAWT wind farms, especially the novel cluster designs of wind farms presented in chapter 3. The numerical methods are as before, but atmospheric stability effects and the buoyancy force are represented using the Boussinesq approximation. The final governing equations for LES and potential temperature equations in the ABL are:

\[ \frac{\partial \tilde{u}_i}{\partial x_i} = 0 \]  \hspace{1cm} (A-1)

\[ \frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \left( \frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i} \right) = -\frac{1}{\rho} \frac{\partial \tilde{p}^*}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_i} + g \frac{\partial \tilde{\theta}}{\partial x_j} + F_i + F_i' \]  \hspace{1cm} (A-2)

\[ \frac{\partial \tilde{\theta}}{\partial t} + \tilde{u}_j \frac{\partial \tilde{\theta}}{\partial x_j} = -\frac{\partial \pi_j}{\partial x_j} \]  \hspace{1cm} (A-3)

where \( \tilde{u}_i \) is the resolved velocity with the tilde denoting a filtered quantity; \( \tilde{p}^* \) is a modified pressure that includes the resolved and subgrid scale turbulent kinetic energies; \( F_i \) is the mean pressure gradient forcing the flow; \( \tau_{ij} \) is the deviatoric subgrid scale stress tensor; and \( F_i' \) represents the aerodynamic forces of the turbine blades on the air; \( \theta \)
is potential temperature; \( q \) is the deviation of \( \theta \) from its horizontal average, and \( \pi \) is the SGS heat flux vector.

For the stable boundary layer simulations with a constant negative surface heat flux, a sponge layer with a Rayleigh damping method is used as the upper boundary condition to damp all the vertical perturbations. For these simulations, the domain height is \( L_z = 544 \) m with an inversion base height of \( z_i = 512 \) m, at which the sponge layer effect starts. All wind farms simulation setups are presented in Table A-2. Using the results from the second chapter, we are using the intermediate aspect ratio wind turbine with medium solidity (I-M in Figure A-1). This VAWT shows the highest deficit at the turbine position and the wake recovery happens at a shorter distance compared to the other cases.

![Figure A-1 Power recovery downstream of different VAWTs](image_url)
Table A-1 Wind turbine experimental configuration

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades per turbine $N$</td>
<td>3</td>
</tr>
<tr>
<td>Rotor diameter $D$, m</td>
<td>50</td>
</tr>
<tr>
<td>Blade vertical height, m</td>
<td>40</td>
</tr>
<tr>
<td>Blade chord length $c$, m</td>
<td>7.8</td>
</tr>
<tr>
<td>Airfoil section type</td>
<td>NACA 0018</td>
</tr>
<tr>
<td>Solidity $Nc/\pi D$</td>
<td>0.15</td>
</tr>
<tr>
<td>Tip speed ratio $\lambda$</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table A-2 Finite wind farms layout configuration

<table>
<thead>
<tr>
<th>Simulation name</th>
<th>N-S, N-A</th>
<th>-0.015-S, -0.015-A</th>
<th>-0.03-S, -0.03-A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface heat flux $\nu'$ mK/s</td>
<td>0</td>
<td>-0.015</td>
<td>-0.03</td>
</tr>
<tr>
<td>Number of turbines</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Cluster horizontal spacing $(S_X \times S_Y)$</td>
<td>20D×20D</td>
<td>20D×20D</td>
<td>20D×20D</td>
</tr>
<tr>
<td>Number of rows</td>
<td>12</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>Number of clusters/row</td>
<td>2 (infinite)</td>
<td>2 (infinite)</td>
<td>2 (infinite)</td>
</tr>
<tr>
<td>Grid size $N_x \times N_y \times N_z$, nodes (coarse)</td>
<td>50×50×64</td>
<td>50×50×68</td>
<td>50×50×68</td>
</tr>
<tr>
<td>Grid size $N_x \times N_y \times N_z$, nodes (fine)</td>
<td>200×200×256</td>
<td>200×200×272</td>
<td>200×200×272</td>
</tr>
<tr>
<td>Domain size $L_x \times L_y \times L_z$, m</td>
<td>2000×2000×512</td>
<td>2000×2000×544</td>
<td>2000×2000×544</td>
</tr>
</tbody>
</table>

As can be seen in Table A-2, 6 cases of aligned and staggered wind farms with cluster horizontal spacing of 20D, in both $x$ and $y$ directions, are simulated. Each case has been simulated for 40 hours with coarse resolution and 2 hours of fine resolution to confirm the statistical convergence. The boundary conditions are periodic in $x$ and $y$ direction.

A.2 Results and Discussion

In this section, results of fine resolution simulations are presented. Figure A-2 shows horizontally averaged vertical velocity profiles for all six cases. As the stability increases,
the velocity is also increasing for all cases reflecting the lower surface and wind farm drag imposed on the ABL. Due to the higher drag created by staggered configurations, the velocity is lower for these cases.

Figure A-2 Vertical profiles of streamwise velocity normalized by $u^*_{TL}$ computed at top of wind farm layer
Reference


Hezaveh, Seyed Hossein and Elie Bou-Zeid. 2013. “Large Eddy Simulations of Vertical Axis Wind Turbines to Optimize Farm Design.” in APS DFD.


Meyers, Johan and Charles Meneveau. 2010. “Large Eddy Simulations of Large Wind-Turbine


