WORKING PAPER # 266
INDUSTRIAL RELATIONS SECTION
PRINCETON UNIVERSITY
JUNE, 1990

Testing the Stationary Search Model

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March 1989, revised January 1990
for the Workshop "Applied Econometrics", Munich¹

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¹I thank the Zentralarchiv für Empirische Sozialforschung, Köln, for providing the analyzed data set, Angus Deaton, Richard Quandt and in particular David Card for many helpful suggestions and advice, and the participants of the Princeton Labor Lunch for fruitful discussions. I am responsible for all remaining errors.

Abstract

This paper uses tight parametric assumptions to model individual unemployment histories in a structural form. Starting from a simple search model it is shown that individual spells are distributed exponentially with different hazard rates for different individuals. In contrast to the usual reduced form approach the choice of the reservation wage is directly incorporated into the analysis. In effect, a log-linear simultaneous equations system is derived that explains both reservation wages and elapsed spell durations of currently unemployed individuals in terms of exogenous variables. Moreover, it is demonstrated that apart from constant terms an identical structural form should be valid for accepted wages and completed spell durations for individuals who are re-employed at the time of the survey.

The empirical part of the paper uses data from the Federal Republic of Germany to estimate the parameters of this structural form both for a subsample of unemployed and a subsample of re-employed individuals. Evidence on the performance of structural search models was previously provided only for currently unemployed individuals by the studies by Kiefer/Neumann (1979), Lancaster (1985), and Jones (1988). This study is therefore the first to provide evidence for re-employed individuals. Furthermore it is the first of its kind to investigate German data. In this paper the main emphasis is given to the robustness of the estimation results across subsamples, not to the formulation of formal tests for the validity of cross-subsample restrictions. The resulting evidence is mainly favorable, and the paper concludes with an extensive discussion of merits and remaining deficiencies of the proposed model.

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I. Introduction

This work uses very tight parametric assumptions to model individual unemployment histories and confronts the resulting structural form with data from the Federal Republic of Germany. It thereby provides favorable evidence for the usefulness of models that deviate from the familiar reduced form approach in an attempt to give more economic structure to the analysis of individual unemployment spell durations. It follows the work of Kiefer/Neumann (1979), Lancaster (1985), and, more recently, Jones (1988). The model allows derivation of the same log-linear simultaneous equation representation for elapsed durations and reservation wages of currently unemployed individuals and for completed durations and accepted wages of reemployed individuals.

Moreover, apart from constant terms the coefficients estimated for both groups of individuals should be identical. Jones (1988) provides estimates for a sample of unemployed individuals, but at the present time there is no evidence on the behavior of the model when data on both re-employed and unemployed individuals from the same environment are used. Here the model is estimated both for subsamples of unemployed and reemployed individuals. At the present stage emphasis is given to the qualitative robustness of the estimation results across these subsamples, not to the formulation of formal tests.

The paper is organized as follows. Section 2 introduces the model and derives the estimable log-linear specification first for currently unemployed individuals and then for reemployed individuals and discusses the issue of identification. Section 3 describes the data set and the eliminations and manipulations necessary to derive the variables used in the analysis. Section 4 discusses the appropriate econometric techniques, presents the estimation results and discusses their implications for the

validity of the proposed model. Finally Section 5 gives a closing discussion, stressing the limits of the analysis and possible improvements.

II. The Model

Suppose the environment an unemployed individual is facing is characterized by stationarity in the sense that he searches for a new job by drawing from a known offer distribution $F(w)$ according to a constant arrival rate, where wages w completely characterize jobs and the searcher is facing an infinite horizon. In particular, unemployment benefits are paid indefinitely, a new job will last forever, and there is no uncertainty regarding economic variables. Moreover, think of only two states being possible, unemployment and employment, and ignore the possibility of leaving from unemployment into nonparticipation. It is well known² that in this situation the individual's optimal policy is to reject all offers below a certain wage r , the reservation wage, and to accept all offers exceeding r . Over time this reservation wage is constant since in each instant of time the unemployed worker faces the same optimization problem. This has stochastic implications for the duration of his unemployment spell: The probability of leaving unemployment in a short interval $(t, t+\delta t)$, conditional on still being unemployed at t , is also constant.

More formally, let the probability that a job offer arrives in the interval $(t, t+\delta t)$ be $\mu \cdot \delta t$, with μ being a constant, and consider the probability that a given offer exceeds the reservation wage r , $1-F(r)$. Then the probability of receiving and accepting a job offer in that interval is

$$(1) \quad \pi \cdot \delta t = \mu (1-F(r)) \cdot \delta t.$$

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²see e.g. Lippman/McCall (1976)

The constant $\pi = \mu (1-F(r))$ is the hazard rate, which completely describes the stochastic properties of the underlying random variable spell duration. In a homogeneous population, specification of the form $F(\cdot)$ would specify the form of π and thus would already suffice to yield an estimable model. In this paper I follow the example of Lancaster (1985) and Jones (1988) and postulate that arriving job offers follow a Pareto distribution. This distribution has a positive probability density only for values exceeding a certain cutoff value w^0 and its pdf. is given by

$$(2) \quad f(w) = \phi/w \cdot (w^0/w)^{\phi}, \quad \phi > 0, w > w^0.^3$$

The probability of drawing a wage offer exceeding r is therefore $1-F(r) = (w^0/r)^{\phi}$, which in turn means that the hazard function can be written as

$$(3) \quad \pi(r) = \mu (w^0/r)^{\phi}.$$

A constant hazard means that the duration of completed spells t is distributed exponentially with pdf $g(t) = \pi \exp(-\pi t)$, which in the light of the given specification for F is

$$(4) \quad g(t|r) = \mu (w^0/r)^{\phi} \cdot \exp(-\mu t (w^0/r)^{\phi}).$$

Individual or match-specific heterogeneity is a necessary condition to justify the existence of a wage offer distribution as opposed to a single equilibrium wage. If one wanted to consider a parametric reduced form model that captures individual differences in a proportional hazard, the hazard π would directly be taken as a function of exogenous variables. Here it is decomposed into two

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³This pdf. shows also a constant elasticity of the hazard with respect to the chosen reservation wage and the coefficient of variation is easily calculated as $((1/\phi) / (1-2/\phi)^{1/2})$ for $\phi > 2$.

parts in order to enable us to focus on the relationship between reservation wages and durations directly.

It is assumed that the individuals' wage offer distributions given the reservation wage differ systematically according to measured personal characteristics, as captured in a vector of variables X , a conformable parameter vector β and a mean-zero error term u whose distribution does not depend on the observables X , and that reservation wages themselves vary systematically according to the same variables X and a conformable parameter vector α with mean-zero error term ϵ_2 with a given variance σ_2^2 . In effect one can write

$$(5) \quad \mu \cdot (w^o)^{\phi} = \exp(X'\beta + u)$$

$$(6) \quad r = \exp(X'\alpha + \epsilon_2)$$

It has been one of the main lessons of the literature on parametric models of unemployment duration that the proper likelihood function for a sample of unemployment spells has to take into account whether the observations are on elapsed or completed spells, and how the individuals have been selected into the sample. This means in particular, that in a situation where we have two types of observations, say elapsed and completed spells, the derived estimates for both subsamples should be mutually compatible. The approach taken by this paper is to derive the estimable structure following from the model proposed above for two types of observations in the available data set, unemployed and reemployed individuals, and to compare the estimation results across these subsamples.

Consider first the situation where we observe elapsed durations and reported reservation wages. If the behavior of the inflow into unemployment is stable enough to assume that a spell is equally likely to start at any time and its elapsed duration at survey time

is drawn from the same distribution regardless of the beginning of the spell⁴, we can interpret elapsed duration as the backwards recurrence time of the process⁵. Thus, if one samples randomly from the population of currently unemployed individuals, one can express the distribution of elapsed spell durations s at sampling time in terms of the distribution of completed spells and in the specific case of the exponential distribution elapsed durations are distributed exponentially with hazard $\pi = \exp(X'\beta + u) \cdot r^{-\phi}$ for a given reservation wage r , which is the same distribution as the distribution of completed spells. It is a well known result that in this case the expectation of the logarithm of elapsed spell duration $\log s$ is

$$(7) \quad E(\log s | \pi) = \Gamma'(1)/\Gamma(1) - \log \pi,$$

and therefore⁶

$$(8) \quad E(\log s | r, X) = -.5772 + \phi \cdot \log r - X'\beta - E(u | r, X)$$

Adding a mean-zero error term ϵ_1 capturing the unsystematic deviation of an observed deviation from its expectation gives the complete equation for $\log s$. Since $\log s$ follows an extreme-value distribution, the variance of ϵ_1 is given by $\sigma_1^2 = 1.6449$. In particular ϵ_1 is uncorrelated with ϵ_2 . Together with the postulated reservation wage equation

$$(6a) \quad \log r = X'\alpha + \epsilon_2$$

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⁴see Salant(1977), p.40

⁵see Cox(1962), p.27

⁶the value of the digamma function $\Gamma'(s)/\Gamma(s)$ at $s=1$ is equal to $-C$, where C is Euler's constant = 0.5772

we have a system of two recursive equations, with possible simultaneity introduced by correlation of the error terms.

The reservation wage equation is immediately identified and identification of the duration equation is ensured by exclusion restrictions if the following argument holds. Any variables in X influencing the wage offer distribution also have an influence on the reservation wage chosen by a rational decision maker, but not necessarily vice versa. If we succeed in finding at least one variable influencing the individuals' reservation wage choice while leaving the offer distribution untouched, the first equation is identified. Natural candidates are variables relating to the cost of search like "# of dependent children" or unemployment support and the empirical part of this paper discusses several different choices.

Consider now the situation where we randomly select observations on completed spells and accepted wages in the new job. This situation will arise whenever one selects randomly out of the population of those leaving their spell of unemployment at sampling time, if the inflow into unemployment satisfies the stability properties mentioned above. Here we don't have to rely on the cancellation of interruption and length bias due to the assumption of an underlying exponential distribution as for elapsed durations, but on the other side we can't observe the individual reservation wages directly. Instead we observe the realization of a random variable accepted wage w^* , whose distribution is the distribution of the offered wage w conditional on this offer being higher than the reservation wage r and whose pdf is

$$(9) \quad f(w^*) = f(w|w>r) = \frac{f(w)}{1 - F(r)}.$$

Note in particular that the probability of observing a high accepted wage is time-independent because of the constancy of the

reservation wage: longer spells make it more likely for many offers to arrive, but given that no previous offer exceeded the reservation wage, any new arriving offer is independent of these previous draws. From the discussion above, the completed spell duration t is distributed exponentially with pdf $g(t)=\pi \cdot \exp(-\pi t)$. Following the previous argument we get

$$(10) \quad E(\log t|\pi) = -0.5772 - \log \pi$$

and thus

$$(11) \quad E(\log t|r,X) = -0.5772 + \phi \log r - X'\beta - E(u|r,X)$$

It can be shown that

$$(12) \quad E(\log w^*|r) = 1/\phi + \log r$$

and therefore

$$(13) \quad E(\log w^*|X) = 1/\phi + X'\alpha.$$

Adding the a mean zero error term τ with variance σ_τ^2 , multiplying this equation by ϕ and subtracting it from the expected duration equation yields the following equation for the expected unemployment duration:

$$(14) \quad E(\log t|w^*,X) = -1.5772 + \phi \cdot \log w^* - X'\beta - E(u|w^*,X)$$

In effect we find a log-linear simultaneous equation model with identical coefficients for completed spells and accepted wages in the case of reemployed individuals and for elapsed spells and reservation wages in the case of unemployed individuals apart from constants and error terms. The formal derivation of these

relationships and of the exact cross-equation restrictions is contained in the appendix.

III. Data and Sampling

The analyzed data were provided by the Zentralarchiv für Empirische Sozialforschung, Universität Köln. The survey was initiated by the Bundesminister für Arbeit und Sozialordnung, Federal Republic of Germany, and covers 2873 individuals. The actual interviews were conducted orally in February 1978 following prespecified questionnaires. 1637 of the respondents were chosen out of the population of those registered as unemployed as of November 1977 and 1236 individuals were chosen out of the population of those leaving the official unemployment register in a two week period in September/October 1977. All respondents were German citizens living in one of 25 regions in the Federal Republic of Germany, which were selected as representative by the Bundesanstalt für Arbeit.

The data set provides extensive information on the individuals' demographic characteristics and their history in the labor market, in particular on their education, their previous position, and if reemployed at survey date their current position. All searching individuals were asked what minimum net monthly earnings they would require in order to induce them to take an offered position. In what follows their answer to this question is interpreted as their reservation wage. All analysis of "unemployed individuals" in this paper refers to those individuals that are registered as unemployed and actually searching for a job in order to avoid the problems arising from the apparent misstatement of unemployment in administrative data.

Here I concentrate on two subsamples of individuals. The first subsample contains 944 individuals that left the unemployment register at sampling time (September/October 1977) and were still

employed at survey time (February 1978). The second subsample contains 573 individuals that were selected into the sample as unemployed in November 1977 and were still found unemployed at the time of the survey (February 1978). For the first group we have information on completed spells, for the latter group on elapsed spells. One can view these sample definitions as an attempt to recreate the information available at sampling time.

Several caveats have to be kept in mind, however. The derivation of the estimable structure for both subsamples rests on the assumptions on the stability of the inflow described above. Furthermore the sample definitions used necessarily discard potentially useful information. For example, some of those individuals included in the sample as unemployed in November 1977 have found a job by the time of the survey in February 1978. In this paper I make no attempt to analyze these individuals. The underlying assumption of exponential spells implies that there is no sample selection bias caused by the omission of this group in the analysis.

Similarly, no use is made of information on the length of individuals' employment spell durations following the transition into employment. Potentially the durations of employment and unemployment spells are linked, and this information may be important. Finally, monetary variables, in particular previous wages, have not been adjusted for inflation. Note, however, that due to underlying assumption of the constancy of reservation wages, the reservation wages at the time of the survey and those at the time of the interviews have necessarily to be the same.

The variables used in the estimations reported in the next section are the logarithm of the duration of the last (elapsed or completed) unemployment spell in months (LOGDURAT), the logarithm of actual net monthly earnings in the current position or minimum

net monthly earnings in the envisaged position (LOGWAGE), the logarithm of the net monthly earnings in the previous position (OLDWAGE), the number of children below 14 years of age (CHILDREN), years of schooling constructed according to educational institutions visited and degrees completed (SCHOOLING), and a number of dummy variables (yes=1): whether the individual completed education to a specific occupation⁷ (OCC), experienced multiple spells of unemployment within the last five years (MULT), has had a tenure in the previous position from 1 to 3 years (TEN1), 3 - 10 years (TEN2), or over 10 years (TEN3), whether he/she worked in a position not demanding qualified work (UNQUAL), whether the employer initiated the termination of the previous employment spell (FIRED), and whether the individual's experience in the labor market accumulates to 3 to 10 years (EXP1), 10 to 20 years (EXP2), or over 20 years (EXP3). Finally, I constructed dummies indicating whether the respondent is a man (MALE), rents as compared to being a landlord (RENT), or is married (MARRIED).

As a variable capturing the pecuniary cost of search I use a dummy variable FULLUB equal to unity for all persons qualifying for the highest type of unemployment support at the start of their spell. In the Federal Republic of Germany unemployment benefits can only be collected when the individual registers as unemployed and basically take two different forms. If the unemployed individual is eligible for unemployment support (Arbeitslosengeld) he receives roughly 68% of average net earnings obtained before the start of the unemployment spell⁸ for a time period which depends on the time he contributed to the insurance. If the unemployed individual contributed to the unemployment insurance fund within the year preceding unemployment and does not qualify for Arbeitslosengeld

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⁷such an education often comes in form of an apprenticeship, but can definitely take other forms

⁸this amount varies with the number of dependent children from 63% to 70%, and there exists an upper limit

or already has exhausted benefits, he is eligible for unemployment help (Arbeitslosenhilfe), which amounts to 58% of average net earnings in the period preceding unemployment.

Individuals not qualifying for these types of unemployment benefits may receive welfare (Sozialhilfe), which is administered by a different government agency (Sozialamt). In addition to the described benefits the Arbeitsamt uses other kinds of financial benefits to support unemployed workers, like Winterhilfe, Überbrückungsgeld or the payment of training costs.

A remarkable pattern emerges in the analyzed data set. Only roughly 10% of the individuals finally contained in the analyzed data set did not receive the highest type of support, Arbeitslosengeld, when their spell commenced and only roughly another 10% had to switch to the lower type of support, Arbeitslosenhilfe. Only very few persons started out with Arbeitslosenhilfe, and all the other types of support (welfare, health insurance payments, additional payments by the employment agency) hardly played any role. Because of this extensive coverage one can expect high multicollinearity in a regression involving the amount of support paid and OLDWAGE, and this consideration lead to the choice of the simple measure described.

Not all individuals included in the analyzed data set could actually be used in the investigation. I decided to eliminate all those individuals who had not been employed at some point in their lives since for them no information on previous earnings - presumably our main indicator for the mean of the appropriate offer distribution - was available. Furthermore all handicapped individuals were excluded, since their treatment in the labor market is heavily influenced by federal laws (Ausgleichsabgabe). Finally, I excluded all persons who own a farm as source of

additional earnings or who had been self-employed in their previous job, since their search behavior is hard to interpret.

A third group of excluded individuals are the nonparticipants, a fourth group are those re-employed individuals who returned to their previous employer after their unemployment spell, and a fifth group are those unemployed individuals not searching but waiting to start in a particular job. Finally the exclusion of a sixth group was dictated by the choice of the wage variables: since I decided to use net monthly earnings I had to exclude all those individuals exclusively looking for parttime jobs, and I could not use those unemployed individuals who had not a clear idea about their reservation wage⁹. Moreover, all individuals with missing values for any of the relevant variables were excluded.

As a result of these exclusions 374 individuals were left in the sample of completed spells, and 378 individuals were left in the sample of elapsed spells. For a list of variables and some descriptive statistics see Table 1.

IV. Empirical Results

The presentation of the empirical results concentrates on two issues. Firstly, the system of equations is estimated for the two different subsamples. The main emphasis is given to the question of the robustness of the estimation results. Secondly, the validity of the underlying model is discussed in the light of these

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⁹942 individuals in the original data set stated to have a clear idea about the minimum net earnings envisaged, but asked for this amount 32 of them answered "don't know" and 14 didn't answer. Of the 453 individuals saying that they didn't have a very clear idea about minimum earnings, 337 actually stated an amount. I decided to leave all individuals in the data set, which stated an amount independently of their answer to the first question.

estimates. At this point, no formal tests for the equality of coefficients across subsamples were devised.

The choice of the estimation procedure¹⁰ depends heavily on the assumptions imposed on the covariance structure of the system of equations¹¹. By construction the error term τ in the (accepted) wage equation for reemployed individuals is uncorrelated with all observable exogenous variables X , and similarly the error term ϵ_1 in the duration equation is uncorrelated with X and with the reservation wage r or the accepted wage w^* . By assumption any correlation between the observables X and the error terms u (in the re-employment hazard) and ϵ_2 (in the reservation wage equation) was excluded when relationships (5) and (6) were imposed¹². Furthermore, given our assumptions, only ϵ_2 and u could possibly have a nonzero covariance. It is this correlation which requires some further thought.

Consider first the case where the covariance between ϵ_2 and u is zero. Here both the wage equation and the duration equation can be estimated consistently by performing Least Squares (LS) for each equation separately. LS estimates of the coefficients of the wage equation are reported in Table 2, LS estimates of the duration equation are presented in Table 3.

If there exists a nonzero correlation between ϵ_2 and u the LS estimation of the duration equation turns out to be problematic. One scenario that argues for a positive correlation between the two error terms is the case where both potential employers and

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¹⁰given that both equations are identified by exclusion restrictions as discussed above

¹¹see the appendix for details on the error terms involved

¹²correlation of e.g. u and X is definitely a potentially very interesting problem, but its discussion is beyond the scope of this paper.

searching individuals observe an individual characteristic, unobserved by the econometrician, whose presence moves the respective wage offer distribution towards higher offers (positive u) and therefore influences the choice of the reservation wage in an upward direction (positive ϵ_2). A simple example could be the tendency to leave a good impression in interviews. In this case the reservation wage r , and thus the accepted wage w^* , will be negatively correlated with the error term in the duration equation and we can expect the estimate of ϕ to be downward biased.

In order to investigate the presence of this problem and to assess the size of a potential bias the duration equation was estimated by Two Stage Least Squares (2SLS). The results are reported for all samples in Table 4. Note that the LS estimates of the wage equation presented in Table 2 are identical to estimates derived by 2SLS, since the wage equation is already a reduced form. On a different note one might argue that a system estimator possibly improves the efficiency of the estimates. This consideration is not pursued here, but one should keep in mind that the special structure of the considered system of equations will only allow the estimation results of the wage equation to be altered from the second to the third stage of a 3SLS Procedure, and the coefficients of the duration equation and their standard errors stay identically the same as reported in Table 4. Moreover, combining the subsamples of re-employed and unemployed persons into one sample introduces a specific type of heteroskedasticity whose exact properties are worked out in the appendix and which should be accounted for in a system estimator.

Finally, the choice of exclusion restrictions made in order to enable identification of the duration equation has to be discussed. I always excluded the indicator for living in a rented apartment, RENT, and the number of children, CHILDREN, from the duration equation, in a first set of estimations alone, and in a second set

together with the indicator of the extent of unemployment support, FULLUB¹³. The wage equations were estimated with and without the measure of the previous wage OLDWAGE included as a regressor (specifications (1) and (2)). The latter specification should allow better comparison with studies that are based on data sets that do not provide information on previous wages.

Turning first to the estimated wage equations presented in Table 2, one can observe that both specifications seem to explain the formation of wages quite well. In general, specifications for reservation wages (elapsed(1) and elapsed(2)) show R^2 statistics a bit higher than for accepted wages(completed(1) and completed(2)). This fact is totally in accordance with the idea that accepted wages are influenced by an additional source of randomness. For both samples inclusion of OLDWAGE increases the explanatory power of the regression dramatically, and, in general, the magnitudes of all other regressor increase by a sizeable factor upon exclusion of OLDWAGE.

As to the group of demographic characteristics we see that age influences the reservation wage choice in the nonlinear fashion typically observed in earnings functions with the effect of AGE significantly positive and that of AGESQ significantly negative and of smaller order. Being male has a strong and significant positive effect, but it is somewhat puzzling to find married individuals with significantly lower wages, in particular since married individuals should be aware of the tax advantages they face and both reservation and accepted wages are expressed in net monthly earnings. One should keep in mind, however, that the labor supply decisions of the partner are not incorporated in the analysis. Reservation wages significantly increase with the number of

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¹³An earlier version of the paper discussed alternative choices for the cost of search measure, and the use of MARRIED as an instrument.

children, whereas one might rather expect individuals to lower reservation wages since the presence of dependent children should make it more urgent to find a new job. Accepted wages, on the other hand, do not show this pattern.

Looking at education, years of schooling, SCHOOLING, and the indicator for completed training in a specific occupation, OCC, both have significantly positive effects on the formation of reservation wages, but OCC is consistently insignificant, and in some cases even negatively related to wages of re-employed individuals. Moving to the individuals' history in the labor market, having had multiple unemployment spells, indicated by MULT, tends to decrease reservation wages, but not significantly. The effect on accepted wages is also insignificant, but here the sign tends to be positive. This erratic behavior of the coefficient of MULT is not too surprising and will be discussed below.

The indicator for unskilled work in the previous job, UNQUAL, turns out to be an excellent predictor for wages. Both accepted and reservation wages are significantly lower for those workers who previously held an unskilled position. Reservation wages also decrease significantly with the dummy capturing the reason for the last job termination, FIRED, but for accepted wages the effect of this variable is insignificant and even mainly positive. It seems that employers do not weigh the reason for termination as a signal, although individuals seem to expect that. It would be interesting to compare this finding with data from another time period when aggregate labor market conditions were different.

The time spent in the workforce should increase the available wage offers and thus reservation and accepted wages. Indeed the estimated coefficients increase with one minor exception from EXP1 to EXP3. However, for the first two dummies they tend to be

insignificant and to start off negatively¹⁴ for the specifications including OLDWAGE. Experience measures are inevitably correlated with age, and these somewhat disappointing results might be due to problems of multicollinearity. If tenure in the old position predominantly captures the amount by which previous earnings were induced by firm specific human capital we would expect the coefficients of TEN1 to TEN3 to be negative and decreasing. In fact, the behavior of the estimates is not totally unproblematic. In the majority of cases the coefficients are insignificant and the sign of the changes is contrary to the argument provided.

Moving finally to the cost of search variables the effect of living in an apartment, indicated by RENT, is significantly positive, which could be explained by credit constraints imposed on homeowners. The coefficient of the indicator of reception of unemployment support to the full extent, FULLUB, is insignificant and interestingly changes signs whenever one compares the same specification across sample definitions. One could argue, however, that the bad statistical performance of FULLUB is unsurprising given the very high fraction of individuals in the data receiving the highest level of UI benefits.

One of the main implications of the theoretical model is the equality of coefficients across sub-samples of re-employed and unemployed individuals. As a general observation, several point estimates are quite different in magnitude and some show even different signs. It turns out, however, that these coefficients tend to be estimated relatively imprecisely. Overall the estimated coefficients seem to reveal the same causal relationships for both accepted and reservation wages.

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¹⁴recall that the reference group has an experience below 3 years

The performance of the LS and 2SLS estimations of the duration equation turns out to be quite poor relative to that of the wage equation. Turning first to the LS estimates presented in Table 3 (specification (3)) a few observations suffice. The R^2 statistics of all regressions are very low, although for most variables the behavior of the estimated coefficients is stable across samples. In general, the coefficients that vary most across samples are those that were also problematic in the wage equations discussed above.

In the discussion of the results derived by 2SLS reported in Table 4 (specifications (4) and (5)) the main emphasis should be given to the behavior of the estimated coefficient of the wage variable, LOGWAGE. Recall that this coefficient should be positive and big enough to represent a reasonable variation in the wage offer distribution. If there exists a positive correlation between the error terms affecting wage offers and reservation wages, u and ϵ_2 , we would expect OLS estimates of this coefficient to be downward biased. Indeed this is apparently true.

The OLS estimate of the wage coefficient in the elapsed duration equation (elapsed(3) in Table 3) is small and positive, whereas the OLS estimate of the wage coefficient in the completed duration equation (completed(3) in Table 3) is actually negative. By comparison, The 2SLS estimates of these coefficients reported in Table 4 are uniformly positive and significant. They are of an order of magnitude sufficient to imply coefficients of variation of the wage offer distribution as small as 0.4 (elapsed(5)).

This pattern confirms the earlier finding of Jones (1988) based on a sample of unemployed individuals. Moreover, the results are very similar for both re-employed and unemployed respondents. There are sizable differences in the magnitude of point estimates, however. Estimates for completed spells imply larger coefficients of the

wage offer distribution of up to 1.2 (completed(4)). These results seem to reflect the additional source of randomness incorporated in accepted wages in contrast to reservation wages.

As it was true that age significantly influences wages in a nonlinear fashion, so does it influence the duration of unemployment with a positive coefficient for AGE and a negative coefficient of smaller order for AGESQ. Again the imprecision of the estimated experience coefficients suggests the presence of multicollinearity with age. Being male considerably shortens the unemployment spell, but much more so for completed durations, a result that mirrors the different behavior of the same variable in the wage equations.

The effect of being married is inconclusive, but it tends to decrease the length of elapsed spells, and to significantly increase that of completed spells. The level of the previous wage, OLDWAGE, significantly decreases spell duration, and the higher magnitudes of LOGWAGE and OLDWAGE for the unemployed individuals (elapsed(4) and elapsed(5)) tend again to outweigh each other. The negative coefficients of the variables capturing differences in education, SCHOOLING and OCC, are quite weak both for elapsed and completed spells and the positive coefficient of the indicator FIRED is far more important for elapsed spells, thereby also mirroring the results of the wage equations.

Different arguments can be offered to explain the big and significant negative effect of MULT, the indicator for multiple spells of unemployment in the previous 5 years. In general, it will be hard to distinguish between individuals who have been fired (frequently) in the past and individuals who changed firms (frequently) to enhance their career on the basis of the number of unemployment spells alone. Given the extensive coverage of the unemployment insurance system discussed above, there is an

incentive to register as unemployed even for a relatively short spell. If multiple unemployment spells are a stigma, the individual might lower reservation wages considerably. In this case, however, one might have found a bigger negative effect of MULT in the estimates of the wage equation. An alternative explanation is that the experience of previous unemployment spells provides individuals with an advantage in searching.

Finally, having previously held an unskilled job (UNQUAL) turns out to be a good predictor of spell lengths, but more so when one considers elapsed spells than completed spells. The behavior of the tenure variables, TEN1 to TEN3, shows more or less a pattern that reflects the discussion given for the wage equations.

Regarding the R^2 statistics the estimations based on observations of elapsed spells seem to have more explanatory power, but although completed spells in principle provide more information, the necessity to use accepted wages instead of reservation wages might explain this behavior. For all specifications the overidentifying restrictions were tested by Basmann's F - Test and the null hypothesis that the instruments are mutually consistent could never be rejected¹⁵. In the estimations it does not seem to matter whether the indicator of receiving unemployment benefits to the full extent, FULLUB, is actually used as an instrument or not. This seems to indicate that in an unemployment insurance system with extensive coverage the cost of search and consequently search behavior are mainly influenced by variables unrelated to unemployment support.

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¹⁵For comparison, the respective 5% critical values given degrees of freedom n_1 and $n_2=400$ are: $F_{n_1,400} = F_{1,400} = 3.86$ and for $F_{n_1,400} = F_{2,400} = 3.02$

V. Concluding Remarks

This paper uses tight parametric assumptions to model individual unemployment histories in a structural form. Starting from a simple search model it is shown that individual spells are distributed exponentially with different hazard rates for heterogeneous individuals. In contrast to the usual reduced form approach the choice of the reservation wage is directly incorporated into the analysis. In effect, a log-linear simultaneous equations system is derived that explains both reservation wages and elapsed spell durations of currently unemployed individuals in terms of exogenous variables. Moreover, it is demonstrated that apart from constant terms an identical structural form should be valid for accepted wages and completed spell durations for individuals who are re-employed at the time of the survey.

The empirical part of the paper uses data from the Federal Republic of Germany to estimate the parameters of this structural form both for a subsample of unemployed and a subsample of re-employed individuals. In this paper the main emphasis is given to the robustness of the estimation results across subsamples, not to the formulation of formal tests of the validity of cross-subsample restrictions. Although the results for both subsamples were remarkably similar and the model's empirical implications were not violated in an obvious way, some important differences across subsamples could be observed.

Mainly the results are reassuring, however. As a minimum this work confirms the finding that the choice of reservation wages indeed does matter for the length of individual unemployment experience and that there is a relationship between the duration of a spell and the chosen reservation wage that one cannot control for in a reduced form approach. The stable behavior of the results is

especially remarkable if one realizes not all information available in the data set has been used in the analysis.

Furthermore, this paper shows that in principle a formal test of the proposed model is possible if one exploits the implications given by the model for different subsamples of observations in the data set. It will require proper account of the specific type of heteroskedasticity involved that is worked out in the appendix. The development of such a formal test should be on the agenda for future work.

Despite the favorable evidence for the validity of the model one cannot overlook a few negative facts. The excellent performance of the estimated wage equations is not too surprising, given the well-documented success of empirical earnings equations. And this relationship was merely imposed, not derived out of an optimizing model. Moreover, irrespective of the estimation technique used, the duration equations perform relatively badly, even for cross-sectional data. In addition, one should note that the use of reported reservation wages for unemployed individuals as the counterpart of accepted wages for employed persons certainly requires some faith in the accuracy of the given answers.

Further extensions of this analysis model should concentrate on the treatment of unobserved heterogeneity and should allow for time-dependent reservation wages as in Lancaster(1985). A related line of work might attempt to understand the different magnitudes of the classical wage determinants like years of schooling in the different samples, possibly by allowing for different information structures for job searchers and potential employers.

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Table 1: List of variables used, frequencies and means

Spell		completed	elapsed	
OCC	yes	303	265	occupation learned
	no	71	113	
MULT	yes	74	120	multiple unemployment spells
	no	300	258	
TEN1	yes	134	93	tenure last job 1-3 years
	no	240	285	
TEN2	yes	156	120	tenure last job 3-10 years
	no	218	258	
TEN3	yes	24	43	tenure last job over 10 years
	no	350	335	
UNQUAL	yes	186	228	last job involved unqualified work
	no	188	150	
FIRED	yes	185	237	last employment spell terminated by employer
	no	189	141	
EXP1	yes	149	115	time spent in the workforce 3-10 years
	no	225	263	
EXP2	yes	107	107	time spent in the workforce 10-20 years
	no	267	271	
EXP3	yes	86	131	time spent in the workforce over 20 years
	no	288	247	
MALE	yes	198	203	sex
	no	176	175	
RENT	yes	264	264	renting an apartment
	no	110	114	
MARRIED	yes	223	204	marital status
	no	151	174	
FULLUB	yes	343	345	eligible for Arbeitslosengeld at start of spell
	no	31	33	

Table 1 (continued)

Spell		completed	elapsed	
CHILDREN	0	238	236	
	1	90	89	
	2	36	33	
	3	6	13	
	4	3	4	
	5	0	1	
	6	1	2	
SCHOOLING	8	6	8	
	9	39	67	
	9.5	0	1	
	10	49	61	
	11	163	171	
	11.5	1	0	
	12	40	25	
	13	30	24	
	14	26	7	
	15	4	5	
	16	6	3	
	17	2	0	
	18	3	0	
	19	3	5	
	20	0	1	
	21	2	0	
LOG DURAT	mean	1.56	1.66	log duration of unemployment
	s.d.	0.94	1.02	
LOG WAGE	mean	7.00	7.06	log net monthly earnings
	s.d.	0.46	0.36	
OLDWAGE	mean	6.94	6.99	log previous net monthly earnings
	s.d.	0.57	0.42	
AGE	mean	31.34	35.55	age
	s.d.	10.79	12.99	

Table 2: OLS estimations of the wage equation*

	elapsed(1)	elapsed(2)	completed(1)	completed(2)
CONSTANT	3.1752 (0.2573)	5.7023 (0.2044)	4.3244 (0.2978)	5.9455 (0.2560)
AGE	0.0186 (0.0094)	0.0302 (0.0113)	0.0018 (0.0133)	0.0238 (0.0144)
AGESQ	-0.0002 (0.0001)	-0.0004 (0.0001)	-0.0001 (0.0002)	-0.0004 (0.0002)
MALE	0.1573 (0.0277)	0.2857 (0.0314)	0.3225 (0.0380)	0.4635 (0.0379)
MARRIED	-0.0441 (0.0259)	-0.0202 (0.0313)	-0.0683 (0.0360)	-0.0424 (0.0395)
SCHOOLING	0.0199 (0.0083)	0.0432 (0.0098)	0.0272 (0.0097)	0.0400 (0.0105)
OCC	0.0787 (0.0303)	0.0659 (0.0366)	0.0070 (0.0468)	-0.0002 (0.0515)
MULT	-0.0192 (0.0290)	-0.0097 (0.0352)	0.0313 (0.0454)	-0.0054 (0.0498)
TEN1	0.0368 (0.0324)	0.0582 (0.0392)	-0.0366 (0.0496)	-0.0487 (0.0546)
TEN2	0.0381 (0.0344)	0.0670 (0.0416)	-0.0294 (0.0537)	-0.1049 (0.0583)
TEN3	-0.0183 (0.0464)	0.0076 (0.0562)	-0.0324 (0.0827)	-0.0379 (0.0910)
OLDWAGE	0.4670 (0.0359)		0.3241 (0.0372)	
UNQUAL	-0.0735 (0.0280)	-0.1350 (0.0334)	-0.1084 (0.0361)	-0.1527 (0.0393)
FIRED	-0.0411 (0.0247)	-0.0413 (0.0300)	0.0076 (0.0341)	0.0138 (0.0376)
EXP1	-0.0961 (0.0559)	0.0686 (0.0659)	0.0527 (0.0658)	0.0980 (0.0722)
EXP2	-0.0784 (0.0752)	0.1177 (0.0892)	0.0382 (0.0842)	0.1107 (0.0922)
EXP3	-0.0485 (0.0878)	0.2048 (0.1038)	0.2157 (0.1039)	0.2914 (0.1139)
RENT	0.0505 (0.0260)	0.0675 (0.0314)	0.0833 (0.0356)	0.0961 (0.0391)
CHILDREN	0.0305 (0.0126)	0.0447 (0.0152)	-0.0275 (0.0203)	-0.0307 (0.0224)
FULLUB	0.0155 (0.0426)	0.0469 (0.0515)	-0.0089 (0.0593)	-0.0118 (0.0653)
RSQUARE	0.6446	0.4768	0.5797	0.4895
* observations	378	378	374	374
Standard Errors in Parentheses				

Table 3: OLS estimations of duration equation*

	elapsed(3)	completed(3)
CONSTANT	0.8066 (1.3102)	0.6272 (1.0951)
LOG WAGE	0.2148 (0.2263)	-0.3296 (0.1565)
AGE	0.1099 (0.0407)	0.0747 (0.0388)
AGESQ	-0.0009 (0.0005)	-0.0008 (0.0005)
MALE	0.0577 (0.1243)	-0.1436 (0.1234)
MARRIED	-0.1883 (0.1080)	0.1322 (0.1059)
SCHOOLING	-0.0037 (0.0362)	-0.0034 (0.0291)
OCC	-0.0977 (0.1306)	-0.1280 (0.1390)
MULT	-0.3782 (0.1257)	-0.0017 (0.1348)
TEN1	0.1512 (0.1396)	0.1821 (0.1467)
TEN2	0.0902 (0.1475)	0.2891 (0.1594)
TEN3	-0.1024 (0.1994)	0.2045 (0.2443)
OLDWAGE	-0.4078 (0.1887)	0.2167 (0.1218)
UNQUAL	0.0941 (0.1215)	-0.0437 (0.1073)
FIRE	0.0923 (0.1066)	0.0804 (0.1014)
EXP1	0.0043 (0.2431)	0.3856 (0.1937)
EXP2	-0.1788 (0.3260)	0.0665 (0.2485)
EXP3	-0.5517 (0.3804)	0.0674 (0.3078)
RSQUARE	0.1516	0.1196
observations	378	374

*Standard errors in parentheses.

Table 4: 2SLS estimations of duration equation*

	elapsed(4)	elapsed(5)	completed(4)	completed(5)
CONSTANT	-8.9572 (5.8207)	-9.0163 (5.9008)	-10.6943 (6.5973)	-11.5488 (6.6843)
LOG WAGE	3.3073 (1.7930)	3.4534 (1.8197)	2.2843 (1.4999)	2.3655 (1.5165)
AGE	0.0503 (0.0607)	0.0389 (0.0619)	0.0746 (0.0519)	0.0845 (0.0527)
AGESQ	-0.0002 (0.0007)	-0.0001 (0.0007)	-0.0006 (0.0007)	-0.0007 (0.0007)
MALE	-0.4319 (0.3196)	-0.4884 (0.3257)	-0.9854 (0.5059)	-1.0003 (0.5114)
MARRIED	-0.0953 (0.1433)	-0.0758 (0.1457)	0.3302 (0.1807)	0.3448 (0.1828)
SCHOOLING	-0.0652 (0.0568)	-0.0714 (0.0577)	-0.0792 (0.0579)	-0.0823 (0.0586)
OCC	-0.3058 (0.2003)	-0.2970 (0.2031)	-0.1398 (0.1857)	-0.1288 (0.1878)
MULT	-0.3119 (0.1595)	-0.3072 (0.1617)	-0.0727 (0.1845)	-0.0839 (0.1865)
TEN1	0.0152 (0.1888)	0.0426 (0.1921)	0.2642 (0.2015)	0.2349 (0.2043)
TEN2	-0.0145 (0.1914)	0.0261 (0.1955)	0.3651 (0.2172)	0.3504 (0.2197)
TEN3	-0.0096 (0.2514)	0.0315 (0.2560)	0.3133 (0.3322)	0.2845 (0.3361)
OLDWAGE	-1.8845 (0.8771)	-1.9378 (0.8898)	-0.6418 (0.5142)	-0.6671 (0.5199)
UNQUAL	0.3033 (0.1918)	0.2998 (0.1945)	0.2306 (0.2118)	0.2658 (0.2149)
FIRE	0.1931 (0.1435)	0.2134 (0.1459)	0.0689 (0.1356)	0.0634 (0.1371)
EXP1	0.3096 (0.3469)	0.3131 (0.3517)	0.2207 (0.2751)	0.1523 (0.2807)
EXP2	0.0698 (0.4263)	0.1053 (0.4326)	-0.0596 (0.3396)	-0.1283 (0.3453)
EXP3	-0.3738 (0.4798)	-0.3626 (0.4865)	-0.4738 (0.5134)	-0.5799 (0.5222)
FULLUB		-0.3895 (0.2300)		0.4231 (0.2358)
RSQUARE	0.1114	0.1153	0.0704	0.0770
BASMAN'S F**	1.5758 (2,378)	0.1890 (1,378)	1.7589 (2,374)	0.2089 (1,374)
observations	378	378	374	374

* Standard Errors in Parentheses

** Overidentification test, degrees of freedom in Parentheses

Appendix

In this appendix the model is derived for both elapsed and completed unemployment durations and the covariance structure of the involved error terms is explained. Starting again with the unemployed individuals the complete model is

$$(A1) \quad \mu \cdot (w^0)^\phi = \exp(X' \beta + u)$$

$$(A2) \quad r = \exp(X' \alpha + \epsilon_2)$$

In the situation where we observe elapsed durations and reported reservation wages in an environment satisfying the described stationarity requirements elapsed durations are distributed exponentially with hazard $\pi = \exp(X' \beta + u) \cdot r^{-\phi}$ for a given reservation wage r . This leads us to a moment generating function for log elapsed duration $\log s$, given by

$$(A3) \quad M(q) = E(s^q) = \int_0^\infty u^q \cdot \pi e^{-\pi u} du = \frac{\pi \cdot \Gamma(q+1)}{\pi^{q+1}} = \frac{\Gamma(q+1)}{\pi^q}$$

From the cumulant generating function $K(q) = \log \Gamma(q+1) - q \cdot \log \pi$ we can easily infer the expectation of $\log s$ given π by taking the derivative of K at $q=0$ to get

$$(A4) \quad E(\log s | \pi) = \Gamma'(1)/\Gamma(1) - \log \pi,$$

and since the value of the digamma function $\Gamma'(s)/\Gamma(s)$ at $s=1$ is equal to $-C$, where C is Euler's constant = 0.5772, we have

$$(A5) \quad E(\log s | r, X) = -.5772 + \phi \cdot \log r - X' \beta - E(u | r, X)$$

Adding a mean-zero error term ϵ_1 capturing the unsystematic deviation of an observed deviation from its expectation gives the complete equation for $\log s$. The variance of this error term σ_1^2 is given by the second derivative of the cumulant generating function at $q=0$, i.e. the trigamma function evaluated at 1, which is 1.6449. In particular ϵ_1 is uncorrelated with ϵ_2 . Together with the postulated reservation wage equation

$$(A2a) \quad \log r = X' \alpha + \epsilon_2$$

we have a system of two recursive equations, with possible simultaneity introduced by correlation of the error terms.

In the situation where we observe completed spells and accepted wages in the new job we observe the realization of a random variable accepted wage w^* , whose distribution is the distribution of the offered wage w conditional on this offer being higher than the reservation wage r and whose pdf is

$$(A6) \quad f(w^*) = f(w | w > r) = \frac{f(w)}{1 - F(r)}.$$

The completed spell duration t is distributed exponentially with pdf $g(t) = \pi \cdot \exp(-\pi t)$. Following the previous argument we get

$$(A7) \quad E(\log t|\pi) = -0.5772 - \log \pi$$

and thus

$$(A8) \quad E(\log t|r, X) = -0.5772 + \phi \log r - X'B - E(u|r, X)$$

The moment generating function for $\log w^*$ is

$$(A9) \quad M(q) = E(w^q) = \int_r^\infty w^q \frac{f(w)}{1 - F(r)} dw$$

and by substituting the assumed Pareto distribution (for $q < \phi$)

$$\begin{aligned} (A10) \quad M(q) &= \int_r^\infty w^q \frac{\frac{\phi}{w} \left[\frac{w^\phi}{r^\phi} \right]}{\left[\frac{w^\phi}{r^\phi} \right]} dw = \int_r^\infty w^q \frac{\phi}{w} \left[\frac{r}{w} \right]^\phi dw \\ &= \phi r^\phi \int_r^\infty w^{q-1-\phi} dw = \phi r^\phi \left[\frac{1}{q-\phi} w^{q-\phi} \right]_r^\infty \\ &= \phi r^\phi \cdot \frac{1}{\phi-q} r^{q-\phi} = \frac{\phi}{\phi-q} r^q \end{aligned}$$

From this we derive the cumulant generating function

$K(q) = \log \phi - \log(\phi-q) + q \cdot \log r$, with derivative

$K'(q) = (1/\phi-q) + \log r$, so that

$$(A11) \quad E(\log w^*|r) = 1/\phi + \log r$$

and therefore

$$(A12) \quad E(\log w^*|X) = 1/\phi + X'\alpha.$$

Adding the a mean zero error term τ with variance σ_τ^2 , multiplying this equation by ϕ and subtracting it from the expected duration equation yields the following equation for the expected unemployment duration:

$$(A13) \quad E(\log t|w^*, X) = -1.5772 + \phi \cdot \log w^* - X'B - E(u|w^*, X)$$

The formal derivation of the exact cross-equation restrictions for the resulting log-linear simultaneous equation model is contained in what follows.

Consider again

$$(A5a) \quad \log s = -.5772 + \phi \cdot \log r - X'\beta - u + \epsilon_1$$

and

$$(A2a) \quad \log r = X'\alpha + \epsilon_2,$$

where

$$\begin{aligned} \epsilon_1 &\sim (0, 1.6449) \\ \epsilon_2 &\sim (0, \sigma_2^2) \\ u &\sim (0, \sigma_u^2) \\ \text{cov}(\epsilon_1, u) &= 0, \text{cov}(\epsilon_1, \epsilon_2) = 0, \text{and } \text{cov}(\epsilon_2, u) \neq 0 \\ (\epsilon_1 - u) &\sim (0, 1.6449 + \sigma_u^2) \text{ and } \text{cov}((\epsilon_1 - u), \epsilon_2) = -\text{cov}(\epsilon_2, u) \end{aligned}$$

For completed durations we have

$$(A8a) \quad \log t = -.5772 + \phi \cdot \log r - X'\beta - u + \epsilon_1$$

and

$$(A2a) \quad \log r = X'\alpha + \epsilon_2,$$

and

$$(A12a) \quad \log w^* = 1/\phi + X'\alpha + \epsilon_2 + \tau$$

Since the variance of τ is equal to the variance of $\log w^*$, we have from the second derivative of the cumulant generating function $K''(q) = 1/(\phi - q)^2$, evaluated at $q=0$: $K''(0) = 1/\phi^2 = \sigma_\tau^2$.

Multiplying (a12a) by ϕ yields

$$(A12a') \quad \phi \cdot \log w^* = 1 + \phi \cdot X'\alpha + \phi \cdot \epsilon_2 + \phi \cdot \tau$$

Subtract (a12a') from (a8a) to get

$$\begin{aligned} (A13) \quad \log t &= -1.5772 + \phi \cdot \log r - \phi \cdot X'\alpha - \phi \cdot \epsilon_2 - X'\beta \\ &\quad + \phi \cdot \log w^* - u + \epsilon_1 - \phi \cdot \tau \\ &= -1.5772 + \phi \cdot \log w^* - X'\beta - u + \epsilon_1 - \phi \cdot \tau \end{aligned}$$

where

$$\begin{aligned} \epsilon_1 &\sim (0, 1.6449) \\ \epsilon_2 &\sim (0, \sigma_2^2) \\ u &\sim (0, \sigma_u^2) \\ \tau &\sim (0, 1/\phi^2) \text{ and thus } \phi \cdot \tau \sim (0, 1) \\ \text{cov}(\epsilon_1, u) &= 0, \text{cov}(\epsilon_1, \epsilon_2) = 0, \text{cov}(\epsilon_1, \tau) = 0, \\ \text{cov}(u, \tau) &= 0, \text{cov}(\epsilon_2, \tau) = 0 \text{ and } \text{cov}(\epsilon_2, u) \neq 0 \end{aligned}$$

and thus

$$\begin{aligned} (\epsilon_1 - u - \phi \cdot \tau) &\sim (0, 2.6449 + \sigma_u^2), \quad (\epsilon_2 + \tau) \sim (0, \sigma_2^2 + 1/\phi^2) \\ \text{and } \text{cov}((\epsilon_1 - u - \phi \cdot \tau), (\epsilon_2 + \tau)) &= -\text{cov}(\epsilon_2, u) - 1/\phi \end{aligned}$$

Now define the dummy EMP (=1 for reemployed individuals), and let $T = s$ and $W = r$ for unemployed individuals and $T = t$ and $W = w^*$ for re-employed individuals and write the system for all individuals together:

$$(A14) \quad \log T = -.5772 - EMP + \phi \cdot \log W - X'B - u + \epsilon_1 - \phi \cdot \tau \cdot EMP$$

$$(A15) \quad \log W = \quad 1/\phi \cdot EMP \quad + X'\alpha \quad + \epsilon_2 \quad + \tau \cdot EMP$$

where the variance/covariance matrix is

$$\begin{bmatrix} \sigma_u^2 + 1.6449 + EMP & -\text{cov}(\epsilon_2, u) - 1/\phi \cdot EMP \\ -\text{cov}(\epsilon_2, u) - 1/\phi \cdot EMP & \sigma_2^2 + 1/\phi^2 \cdot EMP \end{bmatrix} \quad @ \quad I$$

In effect, we have a very special type of heteroskedasticity, which we can parameterize in the fashion documented by Goldfeldt/Quandt(1972), Chapter 3, or Amemiya(1977), where the squared residuals of a first step estimation (here 2SLS) are regressed on explanatory variables (here a constant and EMP). The model thus yields restrictions on the variances and covariances, and on the structural parameters, both across equations, and within. One proper technical way to test the model will be to estimate it unrestrictedly with slope dummies, estimate (possibly iteratively) the parameters explaining the heteroskedasticity and finally test simultaneously for the slope dummies all being zero and the other described restrictions by performing a nonlinear Wald-Test.