The Optimal Timing of Subsidies: Triggers for Training Programs

Daniel S. Hamermesh*

*Assistant professor of economics, Princeton University. Robert Goldfarb, Wallace Oates, Albert Rees, Neil Soss and members of seminars at Princeton and Rutgers provided helpful comments. The material in this paper was prepared under Grant No. 91-34-72-51 from the Manpower Administration, U.S. Department of Labor, under the authority of title I of the Manpower Development and Training Act of 1962, as amended. Researchers undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment. Therefore, points of view or opinions stated in this paper do not necessarily represent the official position or policy of the Department of Labor.
The funding for an increasing number of social programs is being linked to variations in the level of aggregate economic activity, and pending legislation proposes to tie the rate of funding of training programs in a similar way.⁴ No framework has yet been constructed within which to analyze the economic effects of varying expenditures on these subsidy programs, but it is evident that the timing of a subsidy affects its eventual success in achieving its goals. In this study we present a general theoretical model of cyclical variations in the benefits and costs of a subsidy. We then analyze a particular subsidy, manpower training expenditures, to discover how variations in it might affect its results. Such diverse subsidies as those for housing construction, school dropout prevention programs and urban renewal will have benefits and costs which vary cyclically, and the extent of their variation should be amenable to similar empirical analysis.

Although several economic and social goals could be served by a flexible subsidy for training, we concentrate on increasing the overall economic efficiency of these programs. The effects of other goals on our results for training programs are discussed in the context of a simulation model presented elsewhere.⁵ The model constructed here seeks to discover the variation in

---

¹/ The Emergency Employment Act of 1971 triggers expenditures for job creation in the public sector when the unemployment rate exceeds 4.5 percent. Triggers of various magnitudes are included in the Manpower Revenue Sharing bill, 92nd Congress, H.R. 6181, and the Esch-Steiger bill, 92nd Congress, H.R. 11688. A trigger was also contained in the Manpower Act of 1969, the amended version of which was vetoed by President Nixon in 1970.

²/ Economic stabilization has been cited by some as a major purpose of the trigger mechanism. By report, The Optimal Timing of Training Subsidies, to the Office of Research and Development, Manpower Administration, 1972, forthcoming, presents a discussion of the macroeconomic effects of different trigger mechanisms. The rankings of the various mechanisms by this criterion is similar to that by the efficiency criterion discussed in this paper.
the social benefits and costs produced by differential subsidies during the business cycle. Galbraith, in [17, pp. 3-4], was aware of the effect of variations in labor market activity on the desirability of a public employment program. Haveman and Krutilla [7] were the first to recognize explicitly the importance of this variation for project evaluation, but there has been no analysis of how it might be used to change the timing of subsidies to improve their overall efficiency. Our theoretical model provides some general results designed to guide changes in the timing of subsidies.

We use a simulation approach to analyze alternative trigger mechanisms to learn which produces the greatest increase in the average efficiency of training programs. In evaluating the trigger proposals we ignore a number of economic factors which may change their relative desirability. Especially important may be the displacement problem and its potential negative effects on the true benefit-cost ratios, effects which may vary systematically over the cycle. (See [4] for a discussion of the role of this difficulty in the construction of a trigger mechanism.)

I. A Benefit-Cost Model of the Timing of Subsidies

The true benefits of some subsidized activity will often vary over the business cycle. The value of a subsidized unit which comes onto the market when there is excess demand for this product will exceed the value of a similar subsidized unit appearing when there is excess supply. In the case of subsidies for training, a trainee who completes training when the market for his skill is tight will, other things equal, receive higher
lifetime earnings because he is less likely to suffer an initial period of unemployment than one who finishes when the market is loose.

The true costs of a subsidized activity also vary cyclically, for the shadow prices of inputs vary with the degree of excess demand in the markets for them. Certainly, the opportunity cost of a trainee's time is greater when he has a higher probability of finding employment in a job that requires no training. The social costs of an urban renewal program are lower when there is greater unemployment among construction workers.

We define the logarithm of the benefit-cost ratio per unit subsidized as:

\[
R(t) = B_1 + B_2 \sin \frac{\pi(t+\lambda)}{L} - C_1 - C_2 \sin \frac{\pi t}{L},
\]

where we have decomposed both benefits and costs into two components, one constant and one that varies cyclically. In general, we are assuming the business cycle peaks at \( t = \frac{L}{2} \) and has a trough at \( t = \frac{3L}{2} \). We assume further that the cycle has a duration of \( 2L \) periods, and that there is a lag of \( \lambda \) periods between the time the subsidy begins and the time the benefits are reaped. This fixed lag abstracts from the possibility that the duration of the subsidy can be varied. Implicitly, we assume that the production process of the subsidized activity is characterized by fixed proportions between time and the other factors of production. Finally, we assume that at any point of time \( R(t) \) is independent of the number of units subsidized. For small programs this abstraction is not a serious problem.
Consider a planner who has complete freedom to vary the number of units, \( v(t) \), who are being subsidized at any time \( t \). We assume that the cost per unit subsidized is greater when the number of units subsidized is unusually high; overhead costs, that are based on the peak \( v(t) \), should be charged against those times when \( v(t) \) is large. We assume, therefore, that the cost per unit is also an increasing function of \( [v(t) - \bar{v}] \), where \( \bar{v} \) is the mean number of units subsidized per period. The planner thus wishes to maximize:

\[
Q = \sum_{t=1}^{2L} v(t) [r(t) - a(v(t) - \bar{v})],
\]

where \( a \) is some constant. We assume that the planner operates subject to a constraint on the total number of units subsidized over the entire cycle:

\[
Z = \sum_{t=1}^{2L} v(t).
\]

While one could assume a total cost rather than a quantity constraint, the former is quite intractable mathematically and not clearly any more realistic.

The planner's problem is the maximization of:

\[
Q = \sum_{t=1}^{2L} v(t) [r(t) - a(v(t) - \bar{v})] - \lambda \left[ Z - \sum_{t=1}^{2L} v(t) \right],
\]

where \( \lambda \) is a Lagrangean multiplier applied to constraint (3). After performing the required differentiation, one sees that the maximizing sequence \( v(t) \) is:

\[
v(t) = \frac{1}{2a} \left[ B_2 \sin \frac{\pi(t+1)}{L} - C_2 \sin \frac{\pi t}{L} \right] + \bar{v} = \frac{r(t)}{2a} + \bar{v},
\]

\( t = 1, \ldots, 2L \).
The optimal number of units that receive the subsidy in any period is a simple linear function of \( r(t) \), the cyclical component of the logarithm of the benefit-cost ratio. The slope of that function depends only on the inverse of \( \alpha \), a measure of the extent to which deviations above the average number of units subsidized increase overhead costs.

If we maximize (5) with respect to \( t \), we find that point \( t^\star \) when the optimal number of subsidized units is greatest:

\[
\frac{3v(t)}{dt} = \frac{\pi^2}{2L^2} \left[ C_2 \cos \frac{\pi t}{L} - B_2 \cos \frac{\pi (t + 1)}{L} \right] = 0.
\]

Solving (6) for \( t \), we have:

\[
t^\star = \frac{L}{\pi} \arctan \left( \tan \frac{\pi}{2} - \frac{B_2}{C_2} \right),
\]

where \( k = \frac{C_2}{B_2} \). In Table 1 we list \( \frac{t^\star}{L} \) as a function of \( k \), the relative magnitude of the cyclically varying components of costs and benefits, and \( \frac{N}{L} \), the time interval between incurring costs and receiving benefits relative to the length of the cycle. If, as is likely in the case of training subsidies, \( k \) is substantially greater than unity, our results suggest that the best time for extra subsidies is fairly insensitive to variations in the duration of training.

Under what conditions does a policy of flexible timing produce the greatest gain over one that holds subsidies constant? Substituting (5) into (2) and rearranging, we have:

\[
Q^\star = 2[B_1 - C_1] + \frac{1}{4\alpha} \sum_{t=1}^{2L} r(t)^2.
\]
### Table 1

$\frac{n^*}{L}$ - Best Period for Extra Spending

<table>
<thead>
<tr>
<th>$\frac{n}{L}$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td>.4000</td>
<td>1.9500</td>
<td>1.5912</td>
<td>1.5242</td>
<td>1.5109</td>
<td>1.5010</td>
</tr>
<tr>
<td>.2</td>
<td>.3000</td>
<td>1.9000</td>
<td>1.655</td>
<td>1.5444</td>
<td>1.5204</td>
<td>1.5019</td>
</tr>
<tr>
<td>.333</td>
<td>.1666</td>
<td>1.8333</td>
<td>1.6666</td>
<td>1.5606</td>
<td>1.5289</td>
<td>1.5028</td>
</tr>
<tr>
<td>.5</td>
<td>.0000</td>
<td>1.7500</td>
<td>1.6476</td>
<td>1.5628</td>
<td>1.5318</td>
<td>1.5032</td>
</tr>
<tr>
<td>.667</td>
<td>1.8333</td>
<td>1.6666</td>
<td>1.6061</td>
<td>1.5497</td>
<td>1.5262</td>
<td>1.5028</td>
</tr>
<tr>
<td>1</td>
<td>1.5000</td>
<td>1.5000</td>
<td>1.5000</td>
<td>1.5000</td>
<td>1.5000</td>
<td>1.5000</td>
</tr>
</tbody>
</table>
Using several trigonometric identities to evaluate the second term in (8), we derive:

\[ Q^* = 2(B_1 - C_1) + \frac{L}{4a} \left\{ \frac{B_2^2}{2} + C_2^2 - 2B_2C_2 \cos \frac{2\pi}{L} \right\}. \]

The larger the degree of cyclicality in both benefits and costs, and the closer the length of the production period is to one-half the business cycle, the greater the potential advantage of a perfectly flexible subsidy.

The discussion thus far has dealt only with a planner who is completely free to vary the number of units subsidized. In the context of our model in (2) this strategy is best, but it may be that there are costs of adjustment that we have ignored. We thus consider two other strategies: (1) A fixed rule, which subsidizes some fixed number of units for \( SL \) periods during the cycle and some lesser fixed number for the remaining \((2 - \delta)L\) periods \((0 < \delta < 2)\); (2) A proportional rule, which subsidizes some fixed number of units for some fraction of the cycle and varying greater amounts in other periods. The terminology is like that used to describe formula flexibility in countercyclical policy (see [12] and [14]). The major difference is that in our case there is no feedback from an endogenous variable to the magnitude of the proportional rule. 3/

3/ Sar Levitan, in a speech quoted in Bureau of National Affairs, Manpower Information Service (January 15, 1972), p. 198, argues for a proportional mechanism which triggers an additional ten percent of spending for each additional .2 percent rise in unemployment above 4.5 percent. Title II of the Esch-Steiger bill, 92nd Congress, H.R. 11688, includes a provision for an additional fifteen percent spending for each additional .5 percent rise in unemployment above 4.5 percent. The Administration manpower reforms provide only for a fixed-rule trigger mechanism.
Given constraint (3) and the parameter $\beta$, the planner's problem is to pick:

$$v(t) = \begin{cases} 
\bar{v} + (2-\beta)x, & \text{if trigger is on} \\
\bar{v} - \beta x, & \text{if trigger is off}
\end{cases}$$

where $x$ is the magnitude of the fixed rule and is subject to choice by the planner. Substituting (9) into (2), we have:

$$Q_f = \sum_{t \in S} \left[ \bar{v} + (2-\beta)x \right] \left[ R(t) - a(2-\beta)x \right] + \sum_{t \in S} (\bar{v}-\beta x) \left[ R(t) + a \beta x \right],$$

where $S$ is the set of periods when the rule is on. (It is a function only of $\beta$.) If we maximize $Q_f$ with respect to $x$, we find:

$$x^* = \frac{\sum_{t \in S} r(t)}{2aL \overline{R}(2-\beta)}.$$

For any $\beta$ the value of $Q_f$ for the optimal $x^*$ can be shown to be:

$$Q_f^* = Z[C_1 - C_1] + \frac{\sum_{t \in S} r(t)}{2aL \beta(2-\beta)}.$$

If we rewrite (12) in continuous time, we can easily show that $\beta^* = 1$.

$$\frac{d}{dt} Q_f = \left[ t^* + \frac{\beta}{2} \right] \left[ r(t) \right]^2 \frac{1}{\beta(2-\beta)} \frac{2}{aL} + Z[C_1 - C_1].$$

Differentiating with respect to $\beta$, setting the derivative equal to zero and cancelling terms:

$$L(2-\beta) r(t^* + \frac{L}{2}) - (2-2\beta) \int_{t^*}^{t^* + \frac{\beta L}{2}} r(t) dt = 0$$

continued on p.8
In the class of fixed rules that which provides the extra subsidy half of the time is optimal, given our assumptions about overhead costs and about the time paths of cyclical variation in benefits and costs.

Not only does $\beta^* = 1$ produce the largest $Q^*_f$; when $\beta^* = 1$, one can also show that $x^*$ is largest. $^5$ The largest trigger, measured by the value of $x$, is also the optimal fixed-rule trigger. The optimizing sequence $v(t)$ is thus:

$$v(t) = \bar{v} + \frac{1}{2aL} \int_{t_0}^{t_{2L}} r(t) = \bar{v} + \frac{r}{2a},$$

where $\bar{v}$ is simply the average of the positive values of $r(t)$. Setting $\beta = 1$ and rewriting (12):

$$(12') Q^*_{f} = 2[ L - C_1 + \frac{L - 2}{2a}].$$

Subtracting (12') from (9) and rearranging we have:

$$Q^* - Q^*_{f} = \frac{2L}{4a} \sum_{t=1}^{2L} \left\{ r(t)^2 - 2L^{-2} \right\} > 0,$$

Now, by the symmetry of $r(t)$ around $t^*$, the first term in (11) is zero if and only if $\beta = 1$ (since $\beta \neq 0, \beta \neq 2$). The second term is also zero if $\beta = 1$, so that $Q^*_f$ is maximized at $\beta = 1$ for $0 < \beta < 2$.

$^5$ Writing (11) in continuous form, differentiating with respect to $\beta$ and setting the result equal to zero, we have:

$$\beta(2 - \beta) \frac{L}{2} r(t_{t - \frac{\beta L}{2}}) = (2 - \beta) \int_{t^*}^{t_{2L}} r(t) dt = 0.$$

For the same reasons as in footnote 4, this equality holds for $0 < \beta < 2$ only if $\beta = 1$. 
with the absolute magnitude of the loss from moving to a fixed rule
from complete flexibility depending on the degree of cyclical variation
in benefits and costs. Using (2) one can easily show that if \( v(t) \) is
held constant at \( \bar{v} \) for the entire cycle, the value of \( Q \) is:

\[
Q_c^* = Z \left[ B_1 - C_1 \right].
\]

The gain from a fixed rule over inflexibility is thus \( \frac{Ls^2}{2a} \), a function of
the degree to which overhead costs must be incurred and the variation in
\( R(t) \).

The optimal proportional rule sets \( \beta = 2 \), i.e., allows complete
flexibility of the sort we discussed above. If \( \beta \) is fixed at any value
0 < \( \beta \) < 2, we can show that the optimal sequence of \( v(t) \) is:

\[
v(t) = \frac{\bar{v} + \frac{R(t)}{2a}}{t \in S} + \frac{\sum_{t \in S} r(t)}{2aL(2-\beta)}
\]

Furthermore, the advantage of a proportional over a fixed rule increases
with increased \( \beta \).

One implication of our results for policy is that it is hardly
ever optimal to base a variable subsidy solely on the rate of aggregate
activity. The best time to subsidize coincides with a cyclical peak or
trough only if either costs or benefits do not vary cyclically. Further-
more, we have shown, under some fairly restrictive assumptions about
the time path of the benefit-cost ratio of the subsidy, that the optimal
fixed rule is one that provides extra expenditures for exactly half the
cycle. Finally, a proportional rule is more flexible than a fixed rule and can thus be expected to produce a higher average benefit-cost ratio.

II. A Simulation Model for the Evaluation of Trigger Mechanisms

We wish to construct a model that embodies the considerations of Section I and that provides a series on the benefit-cost ratio of adding an additional trainee at any time $t$. We do not consider such costs as trainee allowances or such benefits as decreased unemployment compensation payments; these are merely transfers and do not represent resources consumed or new resources (of human capital) produced. If we assume the unit of time is a month, the discounted social benefits of starting training at time $t$ are:

$$B(t) = [1 - \bar{U}] \left[ W_a - W_b \right] \left[ 1 - e^{-r(N + N')} \right] \frac{1}{r},$$

where

- $r$ = social discount rate;
- $\bar{U}$ = average unemployment rate over the time period;
- $W_b$ = average monthly wage before training;
- $W_a$ = average monthly wage after training;
- $N$ = duration of training

and $N'$ = expected duration of unemployment facing a trainee who finishes at time $t + N$.

Implicit in this model is the assumption that it takes $N'$ months for the new trainee who started training at time $t$ to reach the front of the queue of unemployed workers and find a job. This lag will vary inversely with the level of aggregate activity. We assume that once
the trainee finds work (at time $t+N$) his future unemployment experience is that of the average worker, and thus his training is in use all but a fraction $\bar{U}$ of the time. This assumption may be incorrect, but as long as the probability of unemployment varies slowly relative to $N$, the discounting factor will diminish the bias to our results.

We assume that all the social costs of training are incurred during the $N$-month training period, so that the discounted costs of starting a trainee at time $t$ are:

$$(14) \quad C(t) = \int_0^N \left[ \frac{K}{N} + \mu k(t+\tau) \right] e^{-\tau} \, d\tau$$

where $K$ = out-of-pocket resource costs per trainee;

$k(\cdot)$ = a variable ranging between zero and one, which indicates the shadow price of labor at a point in time,

and the other variables are already defined.

Included in $K$ are the administrative, selection, instructional and placement costs of conducting the training program; these measure resources which are consumed during the provision of training.

Measuring the opportunity cost of the trainee's time is the least secure aspect of the benefit-cost model in (13) and (14). In a world of perfect competition and homogeneous labor in a single labor market the shadow price of that labor would either equal the wage (if the economy were at full employment) or would equal the value of an additional hour of enforced leisure (if employment were less than full). In a complex economy characterized by many types of labor and by markets which are occupationally and geographically separated the simple polar distinction
is incorrect. To account for this complexity we assume that the shadow
price of labor moves between zero and $V_b$ and is inversely related to
the aggregate unemployment rate. This assumption is admittedly arbitrary,
yet it does capture the phenomenon of segmented markets while at the
same time reflecting the theoretical observation that the shadow price
of labor varies over the cycle.

Both the benefits and costs are discounted back to period $t$. The
appropriate rate of discount has been a subject of extensive
discussion (cf. [1] and the references it contains). To simplify we
assume an annual rate of six percent; the experiments undertaken in
Section IV were also performed with other values of $r$, and the results
were quite insensitive to reasonable variations in this parameter.

Although some of the parameters in our benefit-cost model will
not qualitatively affect the results of our evaluation, we nonetheless
attempt to find estimates of them in order to derive benefit-cost
estimates which are quantitatively reasonable. The parameters $W_a$
and $V_b$ are outside the integral in (13) and thus affect only the level of
$B(t)$, not its variation over time. Varying these parameters is equivalent
to multiplying the estimated benefit-cost series by a constant and thus
leaves unaffected the relative rankings of the time periods in our
study. The parameter $K$ also functions essentially this way, although
it cannot be taken outside the integral in (14).

The first two columns in Table 2 present estimates of trainees' monthly wages after and before entry into the training program. In
all cases the after-training wage is that of the average trainee who
<table>
<thead>
<tr>
<th>Study</th>
<th>$u_a$ ($/ month)</th>
<th>$u_b$ ($/ month)</th>
<th>K ($)</th>
<th>N (months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borus¹</td>
<td>384</td>
<td>347</td>
<td>218</td>
<td></td>
</tr>
<tr>
<td>Cain-Stromsdorfer²</td>
<td>232</td>
<td>152</td>
<td>373</td>
<td></td>
</tr>
<tr>
<td>Hansen et al³</td>
<td>170</td>
<td>145</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hardin-Borus⁴</td>
<td>146</td>
<td>125</td>
<td>677</td>
<td>4</td>
</tr>
<tr>
<td>Levitan-Nangun⁵</td>
<td>304</td>
<td>252</td>
<td>420/855</td>
<td>6-8</td>
</tr>
<tr>
<td>Main⁶</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Page⁷</td>
<td>293</td>
<td>237</td>
<td>567</td>
<td></td>
</tr>
<tr>
<td>Sevall⁸</td>
<td>201</td>
<td>164</td>
<td>1223/1693</td>
<td></td>
</tr>
<tr>
<td>Somers-Stromsdorfer⁹</td>
<td>424</td>
<td>354</td>
<td>1051</td>
<td></td>
</tr>
</tbody>
</table>

¹Wages computed from [2, pp. 380-81]; K computed from [2, p. 413].
²Wages computed as weighted averages for completers and nonapplicants, from [3, p. 317]; K estimated as weighted average of direct training costs, [3, p. 313].
³Wages computed using means of income and the training dummy variable in [5, p. 413].
⁴Wages computed from average gain and base wage in [6, pp. 63, 214]; K includes administrative and instructional costs per enrollee in [6, pp. 124, 130].
⁵Average wages of completers, pre- and post-training 1962-66, [8, p. 51]; first estimate of K is the average for OJT, second is for institutional training, average for 1962-66, [8, p. 75]; N is the range of average durations of training, fiscal years 1963-67, [8, p. 79].
⁶N is average duration for completers, [10, p. 94].
⁷Wages are for control group before training and trainees adjusted for controls' experience after training, [13, pp. 262-63]; K is tuition cost per enrollee, [13, p. 261].
⁸Wages are for trainees, non-trainees, [15, p. 103]; first estimate of K is weighted average for OJT, second is for institutional training, [15, pp. 96-97].
⁹Wage before training is the control group's average, after training wage adds the excess indicated by the regression, [16, pp. 152-53]; K is weighted average per enrollee of sponsor costs, [16, p. 88].
completed the program and who was employed at the time the follow-up study was made. The estimates vary among the studies listed because of differences in the types of programs analyzed, in the time when they were undertaken and in their location. For purposes of our study we assume $U_b = \$250$ and $U_a = \$300$; these estimates fall roughly in the middle of those presented in the table. The variation in $K$ among those studies listed in Table 1 is much greater than in the other parameters, again implying for our purposes that less precision will be possible in the calculation of discounted costs. Here too we use a value approximately at the median of those presented and let $K = \$750$.

Table 2 also presents estimates of the average duration of the training programs, $H$, in several studies. Since this parameter is crucial to our model because it contributes to variations in the simulated benefit-cost ratio over the cycle, we assume $H = 4$ or 8. These values bracket those listed in the Table, and both are used to test how sensitive the relative rankings of the various trigger mechanisms are to varying assumptions about the duration of training.

The variable $k(t)$, the measure of the shadow price of labor at each point in the sample period, was computed under several assumptions about the rate at which it declines as unemployment rises from its minimum to its maximum. We present results only for a full-employment unemployment rate of four percent, and we assume:

$$k_j(t) = \left[1 - U_r(t)\right]^{\frac{U(t) - U_r(t)}{U(t)_{MAX} - U(t)_H}}^j, \quad j = 0, 1 \text{ or } 3,$$
where \( u_h \) is the expected unemployment rate of a person with the demographic characteristics of the average trainee. The assumption implicit when we let \( j = 1 \) or \( 3 \) is analogous to the transformation from excess demand in individual markets to the aggregate unemployment rate that appears in the literature on the Phillips curve (cf., e.g., [10, p.14]).

The remaining variable is \( n_t^i \), the length of the queue of unemployed workers facing the newly-trained worker whose training began at time \( t \). We postulate that upon completing the program the trainee enters the end of the queue, moves forward if people ahead of him become employed, but moves back if a previously employed worker loses his job. The new trainee thus has priority over later new entrants to the labor force and over later trainees, but he yields priority to newly unemployed experienced workers. We thus assume implicitly that the new trainee is also a new entrant to the labor force who lacks the specific training which would enable him to compete with the experienced unemployed for job vacancies in times of high unemployment. He is assumed to find employment at those times only because of the natural attrition of currently employed workers resulting from deaths and retirements. These assumptions are contained in our calculation of \( n_t^i \) from:

---

6/ In (15) we envision a random process involving the probability of obtaining employment, \( 1 - u_h(t) \), facing a potential trainee. Assuming \( j = 0 \) implies that his probability of employment is always the same as that of the average individual with his demographic characteristics; assuming \( j = 1 \) or \( j = 3 \) implies that it drops very rapidly as \( u_h \) increases and approaches zero when \( u_h \) nears its peak.
where \( L(t) \) is the civilian labor force at time \( t \), \( E \) is civilian employment and \( U \) is the aggregate unemployment rate.\(^7\) The figure \( .0017 \) is used to represent the monthly rate of deaths and retirements of current employees.\(^8\) The worker entering the labor market at \( t+N \) is assumed to avoid waiting in a queue if the fraction of unemployed workers is below \( .04 \) (our assumption about full employment). If it is above this amount, he enters a queue which is larger as the fraction unemployed is larger; he moves toward the front of the queue as employment increases and as individuals retire, and he reaches the front at that time \( t+N+N' \) at which \( A(t, t+N+N') \) becomes zero. The lag \( N' \) can thus vary from zero months upward.

These assumptions about the parameters and variables in (13) and (14) are used to compute six series of simulated benefit-cost ratios for the period July 1948 through December 1970. (The first six months

\(^7\) With one exception published data on the unemployment rate (Employment and Earnings, February 1972, p. 182) are used. The published seasonally adjusted civilian unemployment rate for October 1949 is 7.9 percent, but the figures for September and November are only 6.6 and 6.4 respectively. Moreover, the unusually large deviation is caused mainly by a large shift from the employed to the unemployed, a shift which was reversed in the following month. Since both a steel and a coal strike fell during October 1949, the best guess is that the enumerators mistakenly counted as unemployed a large fraction of these strikers. For purposes of our calculations we therefore set the value of the unemployment rate for this month at 6.5 percent, the average for September and November.

\(^8\) The estimated rate of deaths and retirements of males in all occupations is exactly two percent per year. See Bureau of Labor Statistics, Bulletin 1606, p. 64.
of 1948 are excluded because they are to be used in constructing the trigger mechanisms, and the data for 1971 were needed to compute the values of $N'_t$ for the last few months of 1970.) The computations were carried out using $U_H$ both for hypothetical JOBS and hypothetical institutional trainees. Since further results varied only minutely across these, we concentrate on the JOBS results.\footnote{JOBS (Job Opportunities in the Business Sector) provides subsidies to employers who contract to hire disadvantaged workers at what are mainly entry-level positions. Institutional programs teach trainees specific skills at centers devoted solely to this purpose and divorced from the usual work environment.} \footnote{Wide variations in the simulated benefit-cost ratios, $R'(t)$, are caused by the variations over time in $k_j(t)$ and $N'_t$; this estimated lag reached fourteen months several times during the 1958 and 1961 recessions. The highest values of the $R'(t)$ series occur just after business-cycle troughs, and their lowest values are observed shortly before cyclical peaks.} All the trigger proposals that we construct involve increasing government spending on training during those times when unemployment is relatively high. These are likely to be the only mechanisms which are a politically acceptable response to the demands for aid to the unemployed. We should stress that despite their short-run appeal increased training subsidies, especially if concentrated among disadvantaged workers, can produce detrimental long-run political effects as a result of the greater displacement occurring during high-unemployment periods. We ignore this.

III. The Specification of Trigger Mechanisms
problem here, even though its importance for the construction of trigger proposals should not be underestimated.

For each of the mechanisms constructed we examine its success first for a fixed and then for a proportional rule. We define each of the triggers in terms of a variable $V(t)$ which takes the value one if the trigger is on and zero if not. The rule triggers on in that month when the change in $V$, $\Delta V(t)$, equals one, and it triggers off when $\Delta V(t) = -1$. Each rule is constructed for $U^* = .045$, the fraction of aggregate unemployment that triggers the rule. The simulations for the proportional rules link increased spending to each additional rise in $U$ of $.005$ above $.045$.\footnote{A 4.5 percent unemployment rate, used as the trigger point to turn extra spending on and off, appears in all three pieces of proposed legislation that have contained a trigger mechanism.} We describe below the six rules for which results are presented; others were examined and found to be uniformly dominated by at least one of these.

Rule I: \[ \begin{align*}
\Delta V(t) &= 1 \quad \text{if } U(t-1) \geq .045 \text{ and } V(t-1) = 0 \\
\Delta V(t) &= -1 \quad \text{if } U(t-1) < .045 \text{ and } V(t-1) = 1 \\
\Delta V(t) &= 0 \quad \text{otherwise.}
\end{align*} \]

This very simple rule triggers extra training expenditures when unemployment is above the triggering rate and reduces them when it again lies below the same rate. Because it is linked only to one month's value of the unemployment rate, it may produce frequent triggering of the extra expenditure and a relatively short average duration between times when the trigger is off.
This rule, like the five that follow, makes the value of $V$ contingent upon the value of $U$ in the previous month. This assumption enables us to account for the lag between the time the Current Population Survey is taken and the release of the unemployment statistics. $^{11/}$

Rule II: $\Delta V(t) = 1$ if $U(t-1), U(t-2), U(t-3) \geq .045$ and $V(t-1) = 0$

$\Delta V(t) = -1$ if $U(t-1), U(t-2), U(t-3) < .045$ and $V(t-1) = 1$

$\Delta V(t) = 0$ otherwise.

This trigger mechanism is similar to Rule I except that it allows for a more rigid criterion for deciding when unemployment is serious enough to warrant increased training expenditures. It describes exactly the provisions for a trigger mechanism embodied in pieces of legislation proposed recently.

Rule III: $\Delta V(t) = 1$ if $U(t-1) - U(t-4), U(t-2) - U(t-5), U(t-3) - U(t-6) \geq .005$ and $V(t-1) = 0$

$\Delta V(t) = -1$ if $U(t-1) - U(t-4), U(t-2) - U(t-5), U(t-3) - U(t-6) < -.005$ and $V(t-1) = 1$

$\Delta V(t) = 0$ otherwise.

This rule links extra expenditures to the change in the fraction unemployed, and it tries to avoid very short durations of the trigger-on period by requiring unemployment to be rising for several consecutive months. If the cycle in the unemployment rate had a smooth sinusoidal path, this rule could trigger expenditures on and off at the inflection points of that path.

$^{11/}$ The Current Population Survey is taken during the week containing the twelfth of the month. Unemployment rates for each month of 1971 were announced between the second and the eighth of the following month.
Rule IV: \( \Delta V(t) = 1 \) if \( U(t-1) \geq .045, U(t-1) - U(t-4) \geq .005 \) and \( V(t-1) = 0; \)
\( \Delta V(t) = -1 \) if \( U(t-1) - U(t-4) \leq -.005 \) and \( V(t-1) = 1; \)
\( \Delta V(t) = 0 \) otherwise.

The uniqueness of this rule lies in its use of both absolute levels and changes in the fraction unemployed. It triggers on when unemployment is high and rising and triggers off when unemployment has been falling for several months. If the path of the unemployment rate were smooth and sinusoidal, Rule IV would trigger on at a point some distance before the peak unemployment rate and trigger off a shorter distance after the peak.

Rules V and VI:
\( \Delta V(t) = 1 \) if \( U(t-1), U(t-2), U(t-3) \geq [.045 - DU1], U(t-1) - U(t-7) \geq .005 \) and \( V(t-1) = 0; \)
\( \Delta V(t) = -1 \) if \( U(t-1), U(t-2), U(t-3) < [.045 + DU2] \) and \( V(t-1) = 1; \)
\( \Delta V(t) = 0 \) otherwise.

Rule V sets \( DU1 = 0 \) and \( DU2 = .005 \), while Rule VI sets both equal to .0025. Both rules trigger on when unemployment is high and rising rapidly and trigger off when it is even higher. Rule VI triggers on earlier in a recession than any other rule, but it triggers off earlier in the upswing that all others except Rule V. The provision that \( U(t-1) - U(t-7) \geq .005 \) is necessary to ensure that these rules do not trigger on when unemployment is between \( .045 - DU1 \) and \( .045 + DU2 \) and falling slowly.

Although the results in Table 1 suggest centering the trigger on a point after the cyclical trough, our theoretical model was based implicitly on the halfway point during the production process of the subsidized activity.
Here we are talking of the **starting point** of the training activity, so that centering the trigger on a point before the trough may well be reasonable.

IV. The Success of Alternative Triggers

It is difficult to establish simple criteria by which to judge the triggers embodied in Rules I-VI. Ideally one would wish to use all the past values of the unemployment rate to derive some relationship between it and the series $R(t)$ and employ the result to predict when triggered increases in expenditures would be most successful in future years.\textsuperscript{12} Since time does not allow us to wait to test those predictions, we evaluate each rule instead by considering its relative merits if each had been in effect from July 1948 through December 1970. Assuming that the process is stationary, the length of our sample period should ensure that our relative ranking of the six rules will correspond to their success in the future.

To evaluate the merits of each fixed rule, we calculate:

\[
(15) \quad \text{RWD}_m = \frac{1}{270} \left\{ \sum_{t \in S} R(t)(1 + x(1-\frac{N_m}{270})) + \sum_{t \in S} R'(t)(1-\frac{N_m}{270}) \right\},
\]

where $m$ is an index of rules, $m = 1, \ldots, 6,$

$N_m$ is the number of months the trigger is on for Rule $m$;

$x$ is some arbitrary proportionality constant indicating the size of the trigger;

and 270 is the number of months in the sample.

\textsuperscript{12} Nerlove [11] presents a detailed discussion of a technique to accomplish this sort of prediction.
\( RWTD_m \) is analogous to \( Q_f \) in equation (10) in that both embody the constraint that the total number of units subsidized be constant no matter what time path of subsidies is chosen. Manipulating (15), we have:

\[
(16) \quad RWTD_m - \bar{\mu} = \frac{x}{270} \left\{ \sum_{t \in S} R(t) \left[ 1 - \frac{N}{270} \right] - \frac{N}{270} \sum_{t \in S} R(t) \right\},
\]

where \( \bar{\mu} \) is the unweighted average benefit-cost ratio over the entire sample. We set the arbitrary constant \( x = 1 \), and evaluate each fixed rule by comparing the calculated value of (16) under different assumptions on \( k \) and \( N \). The estimate indicates the gain in program efficiency produced by the fixed rule compared to an unvarying subsidy.

We also present estimates of \( RON \), the average of \( R(t) \) for \( t \in S \). (16) is the appropriate criterion for evaluation if there is a constraint on costs or number of units subsidized. \( RON \) may, however, be more useful if the planner feels no such constraint but instead wishes to maximize the efficiency of the program when the extra spending is offered. \( NPLUS \) is the number of times in the sample the rules switches on; \( NDUR \) is the average length in months of the intervals when the trigger is on; and \( NFRAC \) is the fraction of the sample period for which \( t \in S \).

If there are substantial adjustment costs when \( \Delta V(t) \neq 0 \), these calculations should indicate those rules which are to be avoided.

Table 3 presents the results of our simulations for Rules I-VI. In discussing them we compare the other rules to Rule II, the standard trigger. Surprisingly, Rule I appears to dominate the current trigger (Rule II) under four of the six sets of assumptions about the duration of training, \( N \), and the relation between the shadow price of trainees'
<table>
<thead>
<tr>
<th>(k, N)</th>
<th>RTD-R</th>
<th>RON</th>
<th>RTD-R</th>
<th>RON</th>
<th>RTD-R</th>
<th>RON</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 4)</td>
<td>.042</td>
<td>4.566</td>
<td>.045</td>
<td>4.573</td>
<td>.011</td>
<td>4.522</td>
</tr>
<tr>
<td>(1, 4)</td>
<td>.369</td>
<td>6.457</td>
<td>.361</td>
<td>6.453</td>
<td>.193</td>
<td>6.418</td>
</tr>
<tr>
<td>(3, 4)</td>
<td>.395</td>
<td>8.099</td>
<td>.378</td>
<td>8.076</td>
<td>.189</td>
<td>7.994</td>
</tr>
<tr>
<td>(0, 8)</td>
<td>.053</td>
<td>3.092</td>
<td>.053</td>
<td>3.095</td>
<td>.025</td>
<td>3.076</td>
</tr>
<tr>
<td>(1, 8)</td>
<td>.450</td>
<td>5.246</td>
<td>.440</td>
<td>5.260</td>
<td>.255</td>
<td>5.270</td>
</tr>
<tr>
<td>(3, 8)</td>
<td>.604</td>
<td>7.702</td>
<td>.575</td>
<td>7.660</td>
<td>.311</td>
<td>7.618</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>IV</th>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPLUS</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>NDUR</td>
<td>33.50</td>
<td>33.00</td>
<td>14.80</td>
</tr>
<tr>
<td>NBRAC</td>
<td>.496</td>
<td>.469</td>
<td>.274</td>
</tr>
</tbody>
</table>

| (0, 4) | .015  | 4.546 | .042  | 4.577 | .044  | 4.570 |
| (1, 4) | .181  | 6.490 | .366  | 6.559 | .371  | 6.472 |
| (3, 4) | .176  | 8.058 | .367  | 8.150 | .392  | 8.105 |
| (0, 8) | .023  | 3.085 | .031  | 3.104 | .054  | 3.096 |
| (1, 8) | .233  | 5.337 | .451  | 5.380 | .452  | 5.265 |
| (3, 8) | .278  | 7.677 | .568  | 7.794 | .600  | 7.712 |

<table>
<thead>
<tr>
<th></th>
<th>IV</th>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPLUS</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>NDUR</td>
<td>10.50</td>
<td>23.40</td>
<td>33.00</td>
</tr>
<tr>
<td>NBRAC</td>
<td>.233</td>
<td>.433</td>
<td>.489</td>
</tr>
</tbody>
</table>
time and their employment rate. Only if that relation is weak does Rule II perform better. This suggests that, at least near the critical value $u = .045$, the cyclical component in the path of unemployment strongly outweighs any random variation. Moreover, the two rules have about the same average duration of the trigger-on periods, and these periods sum to approximately one half the total sample period. Despite the attractiveness of Rule I, the possibility that future variations in the unemployment rate will be more irregular makes it difficult to recommend it as an alternative to the Administration trigger.

Rules III and IV, the only proposals linked mainly to the rate of change of unemployment, never dominate the Administration trigger by criterion (16). This is not surprising, given that $MFRAC$ for each is substantially less than .5. However, if we are just concerned with maximizing the value of $R(t)$ when the trigger is on, Rule IV is superior to Rule II, especially if the duration of training is fairly long. However, if there are adjustment costs, the low values of $NDUR$ would prevent us from recommending either rule as a clearly superior alternative to Rule II.

Examining the results for Rule VI, we see that it dominates Rule II by all criteria except for $RON$ when $j = 0$ and $N = 4$. Rule V dominates Rule II by criterion (16) only when $j = 1$, but under it the value of $RON$ is always higher than under Rule II. Rule VI spends extra funds as often as Rule II, and it appears to be more closely centered around $t^*$. If fixed rules only are permitted, Rule VI appears to be more attractive than the current trigger by the criteria we have
constructed. Because it avoids extra expenditures when unemployment is still high but falling rapidly from its cyclical peak, it avoids the necessity of incurring large opportunity costs when workers are in training programs in a tightening labor market.

The proportional rules we evaluate increase the number of units subsidized above its value when the trigger is off by:

\[
\begin{align*}
x & \quad \text{if } 0.045 \leq U(t) < 0.050, \ V(t) = 1 \\
\vdots & \\
6x & \quad 0.070 \leq U(t), \quad V(t) = 1,
\end{align*}
\]

where \( x \) is some arbitrary constant. We use an evaluation criterion like (15):

\[
(17) \quad \text{SWTD} = \frac{1}{270} \left\{ \sum_{t \in S} R(t) [1-yx] + \sum_{i=1}^{6} \sum_{t \in N_i} [1-yx + ix] R'(t) \right\},
\]

where \( N_{1m}, \ldots, N_{6m} \) are the number of periods the extra subsidy takes the values \( x, \ldots, 6x \) for Rule \( m \); and:

\[
y = \frac{1}{270} \sum_{i=1}^{6} \frac{1}{i} N_{im},
\]

is a constant embodying the constraint that the total number of units subsidized is the same under each rule. Rearranging (17), we derive:

\[
(18) \quad \text{SWTD} - \bar{R} = x \left\{ \frac{1}{270} \sum_{i=1}^{6} \sum_{t \in N_i} iR(t) - yR \right\}.
\]

Equation (18) is used to compare the proportional rules and is analogous to (16) used in the discussion of the fixed rules.
As before we use a weighted average of $\bar{R}(t)$ when the trigger is on, SON, to supplement criterion (18) in evaluating the rules. The figures W1F, ..., W6F are the fractions of the sample period for which the extra number of units subsidized are $x$, ..., $6x$ respectively. In Table 4 we present the results of these simulations for four of the six rules discussed in Section IV; Rules III and IV are not included because their trigger points are not linked to levels of $U$ for both $\Delta V(t) = 1$ and $\Delta V(t) = -1$.

As with the fixed rules, Rule II appears best only if the relation between the unemployment rate of workers with the characteristics of the trainees and the opportunity cost of trainees' time is weak. The stronger this relationship, the greater the dominance of Rule VI over Rule II. Interestingly, Rule I dominates all other rules by criterion (18) except if $j = 0$ and $N = 4$. However, it performs fairly poorly if the rankings of SON are used. Given the chance that Rule I might in the future trigger on and off very frequently, its superiority here must be discounted.

Our theoretical discussion predicted that the sum of the benefit-cost ratios over the cycle weighted by the number of units subsidized would be greater for proportional than for fixed rules. Comparing SMTD - $\bar{R}$ in Table 4 to its analogue RPTD - $\bar{R}$ in Table 3, we see that this prediction is borne out strongly. There is a substantial gain in program efficiency to using a proportional rule, and Rule VI is probably the best such rule for use in subsidizing training.
Table 4

Evaluation of Proportional Rules I, II, V and VI

<table>
<thead>
<tr>
<th>(k_{ij})</th>
<th>I</th>
<th></th>
<th></th>
<th>II</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S\text{STD-R}</td>
<td>S\text{ON}</td>
<td>S\text{STD-R}</td>
<td>S\text{ON}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,4)</td>
<td>.102</td>
<td>4.553</td>
<td>.106</td>
<td>4.557</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,4)</td>
<td>1.434</td>
<td>6.736</td>
<td>1.411</td>
<td>6.744</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,4)</td>
<td>1.138</td>
<td>8.123</td>
<td>1.121</td>
<td>8.122</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,8)</td>
<td>.158</td>
<td>3.099</td>
<td>.155</td>
<td>3.099</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,8)</td>
<td>1.888</td>
<td>5.685</td>
<td>1.857</td>
<td>5.695</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,8)</td>
<td>1.911</td>
<td>7.846</td>
<td>1.847</td>
<td>7.831</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(k_{ij})</th>
<th>V</th>
<th></th>
<th></th>
<th>VI</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S\text{STD-R}</td>
<td>S\text{ON}</td>
<td>S\text{STD-R}</td>
<td>S\text{ON}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,4)</td>
<td>.101</td>
<td>4.550</td>
<td>.103</td>
<td>4.554</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,4)</td>
<td>1.415</td>
<td>6.793</td>
<td>1.429</td>
<td>6.740</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,4)</td>
<td>1.109</td>
<td>8.149</td>
<td>1.145</td>
<td>3.126</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,8)</td>
<td>.152</td>
<td>3.103</td>
<td>.157</td>
<td>3.099</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,8)</td>
<td>1.866</td>
<td>5.764</td>
<td>1.883</td>
<td>5.692</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,8)</td>
<td>1.837</td>
<td>7.885</td>
<td>1.892</td>
<td>7.843</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

N1F  .093  .104  
N2F  .143  .137  
N3F  .115  .107  
N4F  .059  .059  
N5F  .056  .056  
N6F  .026  .026  
N1F  .052  .089  
N2F  .133  .144  
N3F  .107  .115  
N4F  .059  .059  
N5F  .056  .056  
N6F  .026  .026  


V. Qualifications and Conclusions

Although our theoretical model was based partly on the need to account for the allocation of overhead costs, this consideration was ignored in the simulation work on training programs. To some extent the overhead costs of training at all times will be increased because of the need to maintain the administrative structure to handle the peak load of enrollees during the trigger-on periods. There may also be some slight increase in instructional costs as instructors demand higher wage rates to compensate for what may be the increased variability of employment induced by the variations in spending under the trigger mechanism. While these factors may have some effect on costs, simple calculations suggest that its magnitude is likely to be quite small, as will be the effect on the ranking of the trigger rules.13/

The greatest difficulty with our model stems from the lack of precise estimates of \( N \) and the relation between the shadow price of labor and the state of the labor market. While we do have some data on the duration of training, we do not know the extent to which shorter training periods can be traded off for more intensive or expensive training.

---

13/ Assume that the additional funds triggered average fifty percent of spending during trigger-off periods and also that the trigger is on one half the time. Evidence in [6, pp. 124, 130], the most detailed accounting available in the benefit-cost literature, suggests that non-instructional costs total only twenty-six percent of \( K \). Even if the entire extra cost also had to be borne during the trigger-off period, the increase in \( K \) during that period attributable to the trigger would only be thirteen percent. The increase in total costs would be less than five percent, for opportunity costs are the major component of the total during the trigger-off period.
training to achieve the same resulting increase in skill levels. If there is flexibility in the production of trainees, training periods could be lengthened during the early part of a recession to increase even further the number of trainees when the opportunity cost of workers' time is low. It is probably impossible to estimate how rapidly the shadow price of the average trainee's time declines as the unemployment rate increases. One can only hope that our assumptions have bracketed the range of variation of the parameters describing this relation.

In this study we have demonstrated that a mechanism which triggers off extra training expenditures at an unemployment rate above that used to trigger them on produces better results than one that uses the same unemployment rate for both purposes. Since the proportional rules produce better results than do the fixed triggers, the most successful approach would appear to combine proportional increases with an asymmetric trigger. Our results suggest that this proposal would capture most of the benefits of using a proportional rule, while at the same time avoiding the administrative costs of adding many small increments to training expenditures.

More important than this specific empirical result is the set of simple theoretical results we have derived to guide the construction of time-varying subsidies for any program with benefits or costs that vary cyclically. The exact simulation technique we have used to evaluate triggers for training expenditures would have to be modified to be applied to other programs. For each subsidy both the nature of the lags between expenditures and benefits and the factors causing cyclical variations in costs and benefits would need to be determined. Nonetheless, the
framework we have presented should be useful in the construction and
testing of formulae to evaluate the performance of many subsidy programs.
References


