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TAXATION, HUMAN CAPITAL, AND UNCERTAINTY*

by

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Abstract

This paper analyzes the effect of wage and interest taxation on investment in human capital. It is shown that results derived under the assumption that human capital is a riskless asset fail to obtain when the return on human capital is uncertain. The interaction of the human capital investment decision with savings, consumption and labor-leisure choices are taken into account. An implication of the analysis is that, when the rate of return on human capital is stochastic, efficient taxation requires positive taxation of wage income even when lump-sum taxation is feasible.

I. Introduction

The growth of tax rates on earned income has focused attention upon the impact of such taxes on work effort. The effect of taxes on hours of work has been the subject of both careful theoretical and econometric study.¹ The basic lesson of the theoretical literature is that the impact of taxation on hours of work is logically indeterminate because of the familiar conflict between income and substitution effects. Therefore, an enormous amount of econometric research has been done on this subject. Although there is a disconcertingly high variance in estimates of labor supply elasticities, it would probably be fair to say that the consensus is that the response in hours of work to changes in the net wage is very small for prime age male earners.² This finding has been interpreted by some as evidence that taxes have little influence on work effort. (Pechman, [1977, pp. 68-69].)

Without doubt, theoretical and econometric work to refine estimates of the hours response to changes in the tax structure will continue in the future. However, it must be noted that 'work effort' and 'hours of work' are not synonymous. Even if taxes have little impact upon quantity of hours worked, it is possible that they have an important effect on the quality of number of hours worked. Examination of the impact of taxation upon human capital accumulation may provide insight into the effect of taxes on work effort.

Not very much work has been done on integrating taxation into the analysis of human capital. Boskin [1975] studies a model of individual lifetime earnings maximization, and shows that a proportional wage tax has no effect on

¹ The analysis of the leisure-income choice in the presence of taxation is discussed carefully by Musgrave [1959, Chap. 11] and much of the literature is surveyed in OECD [1975]. Boskin [1976] has reviewed critically empirical studies of the impact of taxes on hours of work.

² See for example, the studies summarized by Cain and Watts [1973, pp. 332-333]. However, the labor supply response of other groups such as married women is considerably higher. See Cain and Watts [1973, pp. 336-337] or Rosen [1976].

human capital accumulation. Heckman [1976] and Driffill [1977] note that income (as opposed to wage) taxes will change the optimal amount of human capital accumulation because they change net discount rates. These results are all obtained from models in which the returns to human capital investment are known with certainty. However, as Levhari and Weiss [1974] emphasize, uncertainty is of paramount importance in human capital investment decisions. Although the taxation of non-human capital in an uncertainty context has been studied extensively (see, e.g., Tobin [1958] or Stiglitz [1969]), it will be explained below that results from this literature do not carry over to the human capital case. One of the main goals of this paper is to analyze theoretically the effect of taxation on human capital accumulation in a model which explicitly allows for uncertainty.

A closely related issue is that of the optimal taxation of wage income. Previous studies of efficient wage taxation have treated returns to human capital as certain. (See, e.g., Stern [1976] and Ordover [1976].) However, to the extent that introducing uncertainty changes behavioral responses to taxes, it may change optimal tax rates as well. A second major purpose of this paper, then, is to explore the problem of efficient taxation in the context of the uncertainty model.

In Part II, we begin by studying a very simple uncertainty model of human capital accumulation, and contrast the tax implications with those which fall out of the corresponding certainty case. We then examine a number of extensions of the basic model, and ascertain how they influence the tax sensitivity of human capital accumulation. It is shown that under reasonable assumptions, the impact of taxation upon human accumulation is very different from that which emerges from the certainty model.

Part III discusses the problem of optimal wage taxation when the returns to human capital are uncertain. We show that under uncertainty the efficient

wage tax may exceed zero even when lump-sum taxation provides an alternative source of revenue. This finding contrasts, of course, with the well-known result from the optimal tax literature that, under certainty, lump-sum taxation provides the most efficient means of raising revenue. The difference arises because the market fails to insure against the risk associated with human capital. The government, via the wage tax, acts as insurer and thus increases welfare.

We follow our discussion of the positive and normative aspects of taxing the returns to human capital with some summary remarks and suggestions for future research in Part IV.

II. The Effect of Taxation on Investment in Human Capital

This part analyzes the effect of income and wage taxation on the accumulation of human capital. Section A develops a model of human capital accumulation under certainty and derives propositions on the effect of taxation on human capital investment in this context. In Section B we consider the effects of taxation when the return on human capital is stochastic.

At the outset, the decision to invest in human capital is treated in isolation from savings decisions and the labor-leisure choice. The effect of taxation on investment in human capital when savings and leisure are endogenous are considered respectively in Sections C and D.

A. The Certainty Case

We begin by analyzing human capital accumulation when its returns are known with certainty. Although some of the certainty results have appeared before, it is useful to review them briefly, both to have a clear statement and to set the stage for the uncertainty case.

We assume that an individual's productive life is divided into two

periods, each of which contains one unit of productive time. In the first period he allocates a proportion H of his productive time to the accumulation of human capital. He receives no contemporaneous compensation for this effort either in the form of remuneration or utility. These foregone earnings are the only costs of accumulating human capital.³ In the remaining $1-H$ share of his productive time in period 1 the individual earns a pre-tax wage rate w^1 . Pretax labor income in the second period of his working life is given by $h(H)$ where $h' \geq 0$ and $h'' \leq 0$. Thus, human capital investment increases earnings capacity, but at a diminishing rate.⁴

In period 1 the worker earns an after-tax labor income of $(1-t)(1-H)w^1$ where t represents the rate at which wage income is taxed. He consumes an amount C^1 . Individuals are endowed at the beginning of their working lives with an amount w^1 of non-human wealth. Accretion to non-human wealth in the first period is thus $(1-t)(1-H)w^1 - C^1$. In the second period the worker earns, after taxes, $(1-t)h(H)$. In addition, non-human wealth at the beginning of period 2 is given by

$$(2.1) \quad w^2 = (1+\theta r)[w^1 + (1-H)\theta w^1 - C^1] .$$

Here $\theta = 1-t$ and $\phi = 1-\tau$ where τ denotes the rate at which interest income is taxed.

³ This is a common assumption in the human capital literature. See Heckman [1976] and especially Boskin [1975] for its justification. Although it would be possible to generalize the analysis to allow for direct costs as well, this would complicate the analysis without adding any important insights.

⁴ Contrast this to the problem of buying non-human capital, for which the gross rate of return is independent of the amount purchased. Levhari and Weiss [1974] emphasize this distinction, and discuss its consequences.

The worker's problem is to choose non-negative C^1 and H to maximize the smooth, increasing and concave lifetime utility function $u(C^1, C^2)$ where

$$(2.2) \quad C^2 = \theta h(H) + W^2,$$

total resources available for consumption or bequests in period 2.

The first-order conditions for maximum are

$$(2.3a) \quad u_1 - (1+\theta r)u_2 = 0$$

$$(2.3b) \quad \theta u_2 [h' - (1+\theta r)w^1] = 0$$

Equation 2.3a gives the usual condition that discounted marginal utility of consumption be equal in each period. Division of (2.3b) by $\theta u_2 (>0)$ yields an optimality condition which implies that human capital investment takes place up to the point where the increase in the second period wage from an additional investment of one hour is equal to the return on saved earnings from period 1. Optimal H is thus independent of both the utility function and the rate at which wages are taxed, a result very similar to that obtained by Boskin [1975]. This separation obtains because labor is fixed in supply and because human capital has no direct impact on utility.

Differentiating (2.3b) with respect to τ one obtains

$$(2.4) \quad \frac{dH}{d\tau} = - \frac{rw^1}{h''},$$

a positive number; increased taxation of interest income encourages investment in human capital. Since a proportional wage tax has no impact on optimal H while a proportional interest tax increases H , a proportional income tax, which is just a combination of both, must increase H . This result is similar to one found by Heckman [1976] in the context of a life-cycle model

of human capital accumulation.⁵ It may at first seem counter-intuitive that an income tax could actually increase H . However, as Heckman points out, this makes sense once it is realized that: a) an income tax depresses the after-tax rate of interest and, with deductability of interest payments, lowers the cost of borrowing, and b) foregone earnings costs of investment may be written off when incurred [1976, p. 512]. (We have assumed, as did Heckman, that the pre-tax interest rate is unaffected by taxation.)

B. A Simple Uncertainty Case

Given the substantial uncertainty surrounding human capital decisions relative to most investments in non-human capital, it is natural to ask how these results might change when the return to human capital investment is random.⁶ The question of uncertainty in this context is of special interest since the risk associated with investment in human capital is, for reasons discussed below, typically neither insurable nor diversifiable. Let us assume that period 2 pretax wage income is given by $xh(H)$ where x is a random variable with a mean of unity and with support $[a,b]$, where $a \geq 0$. As Levhari and Weiss note, this is a somewhat restrictive way to model the stochastic nature of human capital returns. Nevertheless, the multiplicative specification is a tractable one which yields interesting results.

We will also assume, for the moment, that period 1 consumption c^1 , is given at $\bar{c}^1$. While this assumption does considerable violence to reality it simplifies the analysis and allows us to focus solely on the human capital

⁵ However, Heckman formulates the human capital problem quite differently from us. He views human capital accumulation as augmenting the time endowment. In our model, the time endowment is constant, and human capital increases the reward for spending time at work.

⁶ The impact of uncertainty upon human capital investment has been discussed by Levhari and Weiss [1974] and Williams [1978]. Their analyses ignore taxation.

decision. We study the interaction of the consumption and human capital accumulation decisions under uncertainty in Section C.

In period 1 the individual chooses a level of investment in human capital H to maximize

$$(2.5) \quad \chi = E[u(c^1, c^2)]$$

where

$$(2.6) \quad c^2 = x\theta h(H) + (1+\theta r)[\theta w^1(1-H) + w^1 - \bar{c}^1]$$

The first-order condition for a maximum is given by

$$(2.7) \quad E(u_2 F) = 0$$

where

$$(2.8) \quad F \equiv x\theta h' - (1+\theta r)\theta w^1.$$

Differentiating (2.7) with respect to t and solving for $\frac{dH}{dt}$, we find

$$(2.9) \quad \frac{dH}{dt} = E[u_{22} F I] / E[u_{22} F^2 + u_2 x h'']$$

where

$$(2.10) \quad I \equiv xh(H) + (1+\theta r)w^1(1-H),$$

the sum of labor income in both periods expressed in period 2 units.

Diminishing marginal utility of c^2 and concavity of h , which guarantee satisfaction of the second-order condition for a maximum, imply the denominator of (2.9) is negative while the sign of the numerator is in general ambiguous. The sign can, however, be inferred from information on preferences and initial wealth. The following lemma uses a technique suggested by Levhari and Weiss [1974, p. 958] to make such inferences:

Lemma 1: Given a random variable x with p.d.f. $f(x)$ and support $[a, b]$ and given a function $g(x)$ with the property that there exists an $x^* \in [a, b]$ such that $g(x) \geq 0$ for $x \geq x^*$ and $g(x) \leq 0$ for $x \leq x^*$, then, if

$$\int_a^b g(x)f(x)dx = 0 ,$$

the expression

$$\int_a^b \Gamma(x)g(x)f(x)dx > 0 \quad (<0)$$

if $\Gamma(x)$ is uniformly increasing (decreasing).

Proof. Observe that

$$(2.11a) \quad \int_a^{x^*} \Gamma(x)g(x)f(x)dx \gtrless \Gamma(x^*) \int_a^{x^*} g(x)f(x)dx \leq 0$$

and

$$(2.11b) \quad \int_{x^*}^b \Gamma(x)g(x)f(x)dx \gtrless \Gamma(x^*) \int_{x^*}^b g(x)f(x)dx \geq 0$$

as $\Gamma(x)$ is monotonically increasing or decreasing. Thus

$$\int_a^b \Gamma(x)g(x)f(x)dx \gtrless \Gamma(x^*) \int_a^b g(x)f(x)dx = 0$$

as $\Gamma(x)$ increases or decreases monotonically.

End of Proof.

In order to apply the lemma, we rewrite the numerator of expression

(2.9) as

$$(2.12) \quad E[(u_{22}/u_2)Iu_2F] = -E[AIu_2F]$$

where $A \equiv -u_{22}/u_2$, the degree of absolute risk aversion with respect to consumption in period 2. Note that, from (2.7), $E(u_2F)=0$. Furthermore, u_2F satisfies the criterion for $g(x)$ in lemma 1 that there exist an x^* such that, for $x \leq x^*$, $u_2F \leq 0$ while, for $x \geq x^*$, $u_2F \geq 0$. This is true since F is monotonically increasing in x while u_2 is always positive. At x^* , then, $F = 0$. Thus, via lemma 1, expression (2.12) is positive or negative as $d(AI)/dx \lesseqgtr 0$.

Since the denominator of expression (2.9) is negative, $\frac{dH}{dt}$ is positive or negative as $d(AI)/dx \gtrless 0$. We now demonstrate:

Proposition 1. If period 1 consumption is held constant, wage taxation increases investment in human capital if absolute risk aversion is constant.

Under constant absolute risk aversion (CARA) set $\Gamma = AI$ where A is now constant. Thus

$$(2.13) \quad \frac{d\Gamma}{dx} = A \frac{dI}{dx} = Ah(H) \geq 0$$

and the result follows from lemma 1.

Proposition 2. If period 1 consumption is held constant and if relative risk aversion is constant, wage taxation increases or decreases investment in human capital as $W^1 \gtrless \bar{C}^1$.

Under constant relative risk aversion (CRRA) set $\Gamma = RI/C^2$ where the degree of relative risk aversion with respect to period 2 consumption, $R \equiv -u_{22}C^2/u_2$ is constant. Then

$$(2.14) \quad \frac{d\Gamma}{dx} = \frac{Rd(I/C^2)}{dx} = Rh(H) (1+\theta r) (W^1 - \bar{C}^1) \angle (C^2)^2.$$

Since all the terms on the right hand side of (2.14) are unambiguously positive except $(W^1 - \bar{C}^1)$, $d\Gamma/dx$ has the same sign as $(W^1 - \bar{C}^1)$, and Proposition 2 follows from lemma 1.

Propositions 1 and 2 can be understood intuitively. An increase in t reduces both periods' average wages in the same proportion, thus reducing C^2 . At the same time, increasing t reduces the riskiness of period 2 labor income.⁷ The second effect tends to increase investment in the risky asset,

⁷ As t increases, the individual gains less from high realizations of x , but also loses less when low values occur. Thus, risk is decreased. The reasoning here is similar to that employed by Tobin [1958] in his analysis of the effect of proportional taxation of unearned income upon risk-taking.

human capital, while the first, if absolute risk aversion is decreasing, acts to discourage investment in the riskier undertaking. If CARA obtains, only the second effect is operative, which accounts for the unambiguous sign of (2.13). In the case of CRRA, if W^1 exceeds $\bar{C}^1$ then an increase in t reduces C^2 by a smaller proportion than that by which it reduces period 2 wage income and the second effect dominates. The converse holds if W^1 is less than $\bar{C}^1$.

Turning now to the effect of an increase in τ , the tax rate on interest income, we differentiate (2.7) with respect to τ to obtain

$$(2.15) \quad \frac{dH}{d\tau} = \frac{rE[u_2(-\Theta W^1) + u_{22}FG]}{\chi_{HH}}$$

where

$$(2.16) \quad G \equiv \Theta W^1(1-H) + W^1 - \bar{C}^1 ;$$

non-human wealth at the end of period 1. The first term of the numerator is negative while the second, for decreasing absolute risk aversion, is positive. The second order conditions for a maximum guarantee that the denominator is negative. Interest taxation lowers the average return on non-human relative to human wealth and thereby encourages human capital accumulation. By lowering overall after-tax income, however, the tax discourages investment in human capital, the riskier form of wealth. The interest tax, then, like the wage tax, generates a substitution effect which tends to encourage human capital formation and an income effect which discourages it. Contrast this with the certainty result of equation (2.4) in which increases in interest taxation unambiguously increase human capital accumulation. The difference reflects the absence of the separation of production and consumption decisions under uncertainty.

In light of the ambiguity induced by the income effect, it is important to investigate the impact of expected-revenue-compensated changes in τ upon

human capital accumulation. That is, we want to find the effect of an increase in interest taxation accompanied by a sufficient change in t to keep expected tax revenues constant.

Expected total tax revenue in period 2 dollars is

$$(2.17) \quad T = tE(I) + \tau rG.$$

Around the point $T = 0$ the change in t which must accompany a change in τ to keep T constant is

$$(2.18) \quad \left(\frac{dt}{d\tau} \right)_{T=0} = -rG/E(I).$$

Now, the effect of a revenue-compensated increase in τ , near $t=\tau=0$ on optimal human capital accumulation is

$$(2.19) \quad \frac{dH}{d\tau} + \frac{dH}{d\tau} \left(\frac{dt}{d\tau} \right)_{T=0}$$

Substituting (2.9), (2.15), and (2.18) into (2.19) yields

$$(2.19') \quad \left(\frac{dH}{d\tau} \right)_{T=0} = \frac{-rE(u_2 w) + rGE[u_{22} F(E(I)-I)]}{\chi_{HH}}$$

The first term of the numerator is negative while the second term is positive. This is because $u_{22} F(E(I)-I) = -u_2 F A(E(I)-I)$ and, letting $\Gamma = A(E(I)-I)$ and applying lemma 1, Γ is decreasing in x . Thus, replacing a wage tax with an interest income tax yielding equal expected revenue has an ambiguous effect on investment in human capital under uncertainty. This result contrasts with the one which obtains under certainty, that moving from a wage to an income tax necessarily encourages investment in human capital. While, as in the certainty case, the interest tax reduces the expected return on non-human capital relative to human capital, the wage tax makes human capital investments less risky. This second effect acts to encourage investment in human capital by risk-averse individuals and thus a wage tax may provide a greater incentive than an interest tax to invest in human capital.

C. Wage Taxation, Human Capital Accumulation and Savings Under Uncertainty

We now relax the assumption that period 1 consumption is exogenous, giving us essentially the Levhari-Weiss [1974] model augmented with taxes. The

variables H and C^1 are now chosen jointly to maximize $\chi = E[u(C^1, C^2)]$. First order conditions for an interior maximum are

$$(2.20) \quad \chi_C = E[u_1 - (1+\theta r)u_2] = 0$$

$$\chi_H = E[u_2 F] = 0.$$

Where $\chi_C \equiv \partial\chi/\partial C^1$ and, as before, $\chi_H \equiv \partial\chi/\partial H$. Differentiating (2.20) totally with respect to t and appropriate substitution yields

$$(2.21) \quad \frac{dH}{dt} = \left(\frac{dH}{dt}\right)_{C^1} \lambda + \chi_{CH} \chi_{Ct} / \Delta$$

where: (i) $\left(\frac{dH}{dt}\right)_{C^1}$ represents the effect of a change in t on H if C^1 is held constant (i.e., expression (2.9)); (ii) $\Delta \equiv \chi_{HH}\chi_{CC} - (\chi_{HC})^2$ which is positive if the second-order condition for a maximum is satisfied; and (iii) $\lambda \equiv \chi_{CC}\chi_{HH}/\Delta > 1$.

Furthermore

$$(2.22) \quad \chi_{Ct} = E[(1+\theta r)u_{22}I] < 0$$

and

$$(2.23) \quad \chi_{CH} = (1+\theta r)E[Au_2 F] \leq 0.$$

if absolute risk aversion is non-increasing. Thus, if $\left(\frac{dH}{dt}\right)_{C^1} \geq 0$, then interaction with savings tends to increase the positive impact of wage taxation upon human capital accumulation. If, however, $\left(\frac{dH}{dt}\right)_{C^1} < 0$, then permitting C^1 to be endogenous has an ambiguous effect on investment in human capital.

An intuitive explanation of this finding is based on the fact that if absolute risk aversion is diminishing, higher levels of C^2 increase the individual's tolerance of uncertainty in period 2 and therefore increase investment in H , ceteris paribus. In the model with fixed C^1 , when the wage tax is imposed, the decrease in income must come entirely out of C^2 .

Allowing C^1 to be endogenous allows the individual to react to a wage tax increase by raising C^2 at the expense of C^1 and increasing H . Hence, the second term of (2.21) is positive.

D. Wage Taxation, Investment in Human Capital and the Consumption of Leisure under Uncertainty

We now assume that the number of hours worked in period 2 is a choice variable. Total time available for work in period 2 is one, and the share of this time consumed as leisure is denoted L . The after-tax wage in period 2 is $\theta x h(H)$ so that period 2 labor income is $\theta x h(H)(1-L)$. The worker chooses L with knowledge of the realized value of x .⁸ His objective is to maximize the expected value of a utility function.

$$(2.24) \quad u(C^1, C^2, L)$$

which is concave and increasing in each argument.

Given values of C^1 and H , chosen prior to knowledge of x , and the value of x itself, the individual has only to choose a value of L . The first-order condition for a maximum is

$$(2.25) \quad -u_2(C^1, C^2, L) \times \theta h(H) + u_3 = 0.$$

Implicitly, (2.25) defines a leisure demand function $L(w^2, W^2)$ where $w^2 \equiv \theta x h(H)$. Since $C^2 = \theta x h(H)(1-L) + W^2$ we may define the function

$$(2.26) \quad v(C^1, w^2, W^2) = \max_L u(C^1, C^2, L)$$

For simplicity we revert to the assumption that $C^1 = \bar{C}^1$ and so remove C^1 as an argument of v . In this case, v is just the indirect

⁸ This procedure of viewing the economic agent as optimizing after the realization of a random process is quite common in the trade literature. See, e.g., Turnovsky [1974] and Ruffin [1974].

utility function showing maximum utility given some choice of human capital investment and period 1 consumption.

As before, the individual allocates time in period 1 between investment in human capital and working for wage w^1 . He selects a portion H of his time in period 1 for human capital investment to maximize expected utility $\psi \equiv E[V(w^2, W^2)]$. The first-order condition for a maximum is given by setting $\partial\psi/\partial H$ equal zero:

$$(2.27) \quad \psi_H = E[V_1 \frac{\partial w^2}{\partial H} - V_2 (1+\phi r) w^1] = 0$$

where $V_1 \equiv \partial V / \partial w^2$ and where $V_2 \equiv \partial V / \partial W^2$.

Substituting Roy's identity,⁹ $V_1 = (1-L)V_2$, into (2.27) gives

$$(2.27') \quad E(V_2 B) = 0$$

where

$$(2.28) \quad B \equiv xh'(H)(1-L) - (1+\phi r)w^1$$

Note that under certainty, $B = 0$ at the optimum. This is because equation (2.27') is the analogue of the first-order condition (2.3b): the marginal benefit of human capital accumulation equals its marginal costs. Note that the return on the human capital investment now depends on its rate of utilization, i.e., the amount of labor supplied in period 2, $(1-L)$.

The effect of an increase in t on optimal investment in human capital is given by totally differentiating (2.27') with respect to H and t and re-arranging to obtain

$$(2.29) \quad \frac{dH}{dt} = -\frac{\psi_{Ht}}{\psi_{HH}}$$

⁹ Roy's identity relates the demand for a commodity to the partial derivatives of the indirect utility function. Its interpretation when leisure is one of the commodities is discussed by Abbott and Ashenfelter [1976].

where

$$(2.30a) \quad \psi_{Ht} \equiv E\{-V_2[(\frac{\partial w^2}{\partial H})(\frac{dL}{dt})] - V_2B(\frac{dw^2}{dt})L_W + V_{22}B[\frac{dw^2}{dt}(1-L) + \frac{dw^2}{dt}]\}$$

and

$$(2.30b) \quad \psi_{HH} = E\{-V_2(\frac{\partial w^2}{\partial H})^2L_W - 2L_W(\frac{\partial w^2}{\partial H})V_2B + V_2\frac{\partial^2 w^2}{\partial H^2} + V_{22}B^2\};$$

$$L_W = \partial L / \partial w^2; \quad \text{and} \quad L_W = \partial L / \partial w^2.$$

Taken together, equations (2.29), (2.30a) and (2.30b) indicate that, when leisure is endogenous, wage taxation is not neutral in its effect on human capital investment even under certainty. To see this, set $B = 0$ in (2.30a) and (2.30b) and substitute into (2.29):

$$(2.31) \quad \frac{dH}{dt} = \frac{(\frac{dL}{dt})(\frac{\partial w^2}{\partial H})}{(\frac{\partial^2 w^2}{\partial H^2}) - (\frac{\partial w^2}{\partial H})(\frac{dL}{dt})}.$$

Assuming that ψ_{HH} , the denominator of (2.22b), is negative,¹⁰ then dH/dt is opposite in sign to dL/dt . If taxing wages increases leisure then the expected return to human capital and therefore optimal investment in human capital is diminished. The converse holds if taxation reduces leisure. In other words, changes in leisure, by changing the rate of utilization of human capital investment, affect its profitability.

Further insights with respect to the sign of (2.29) is gained if dL/dt is signed. Note that

$$(2.32) \quad \frac{dL}{dt} = L_W(-\frac{dw^2}{dt}) + L_W(\frac{dw^2}{dt}) \quad 11$$

¹⁰ This is implied by the second order condition.

¹¹ It is reasonable in general to assume that $dw^2/dt < 0$. In particular, if first period's leisure is fixed as we assume, it must be so.

A number of time series studies¹² of labor supply suggest that L_w is close to zero. If such is the case, then the first term of (2.32) is zero, while the second term is negative if leisure is a normal good. Together, these results imply a positive relationship between wage taxation and investment in human capital in the certainty model with endogenous leisure.

Turning now to the uncertainty case, note that the third term of ψ_{Ht} of equation (2.30a) is equivalent to the numerator of expression (2.9) and thus is ambiguous in sign. It is, as indicated by Propositions 1 and 2, positive under CARA or, if $w^1 > c^1$, under CRRA. The second term of ψ_{Ht} is equal to

$$(2.33) \quad L_w h(H) \int_a^b x V_2 B f(x) dx$$

which is positive if $L_w > 0$,¹³ i.e., if leisure is normal.

Thus, under uncertainty, a positive response of work effort in period 2 to increased wage taxation is not sufficient to guarantee that a worker will invest more in human capital. It is true that if he works more hours as a result of the tax, a given increase in second period wages will have a bigger payoff, ceteris paribus. But, if absolute risk aversion is diminishing, increased wage taxation may result in a preference for investment in less risky assets. Again, the introduction of uncertainty clouds the results from the analogous certainty model.

¹² See, for example, Lucas and Rapping [1970]. In addition, cross section studies of prime age males also suggest negligible wage elasticities. See the studies in Cain and Watts [1973].

¹³ The expression inside the integral is positive as may be shown by applying lemma 1, letting $\Gamma(x) = x$.

III. Optimal Taxation and Investment in Human Capital

Part II dealt with the effect of wage and interest rate taxation on investment in human capital in a variety of contexts. The analysis demonstrated that while the direction of the effect of taxation is in general ambiguous, its sign can be determined from information on initial wealth and attitudes towards risk and leisure. In this section we consider an optimal tax system embodying wage and lump-sum taxes.* The two-period model developed in Part II serves as a starting point for the analysis.

The government's objective is to raise from each individual an expected per capita revenue $\bar{R}$ through a combination of wage and lump-sum taxes.¹⁴ In doing so it wishes to maximize the expected after-tax utility of the individual or, equivalently, to minimize the excess burden of the tax. As in Section D of Part II, we assume that period 1 consumption is pre-determined while hours worked in period 2 is subject to choice. We choose this formulation both for reasons of tractability and because much of the previous analysis of the excess burden of wage taxation has focused on the labor-leisure question.¹⁵ Finally, we assume that there is no pre-existing interest tax so that $\phi = 1$.

The government's problem, then, is to choose t and a per capita lump sum tax T to raise expected tax revenue

$$(3.1) \quad \bar{R} = t[(1-L)\bar{x}h(H) + (1+r)(1-H)w^1] + T$$

in such a way as to maximize

¹⁴ Our assumption that the government is only concerned with expected revenue need not imply that the government is risk neutral. If the risk associated with human capital investment is independent across individuals and if the number of individuals is large, then the law of large numbers will guarantee the government a constant total revenue despite the uncertainty with regard to revenue collected from each individual. The government is, in this case, simply a more efficient risk-pooler than the individual.

¹⁵ See, for instance, Mirrlees [1971] and Fair [1971].

* Diamond, Helms and Mirrlees [1978] study optimal or distributive taxation in an uncertainty context, but do not consider explicitly the human capital aspects of the problem.

$$(3.2) \quad \psi = E\{V[\Theta x h(H), (1+r)((1-H)\Theta w^1 + w^1 - \bar{C}) - T]\}$$

where $V(\cdot)$ is the indirect utility function.¹⁶ The first-order condition for a maximum is

$$(3.3) \quad \frac{d\psi}{dt} = E\left\{V_1 \frac{dw^2}{dt} + V_2 \left(\frac{dw^2}{dt} - \frac{dT}{dt}\right) + \left[V_1 \frac{\partial w^2}{\partial H} + V_2 \left(\frac{\partial w^2}{\partial H} - \frac{\partial T}{\partial H}\right)\right] \left(\frac{dH}{dt}\right)_Y - V_2 \frac{\partial T}{\partial L} \left(\frac{dL}{dt}\right)_Y\right\} = 0 \quad 17$$

The individual's problem is to choose H to maximize (3.2) taking T and t as given. A first-order condition for this maximum is that

$$(3.4) \quad E\left[V_1 \frac{\partial w^2}{\partial H} + V_2 \frac{\partial w^2}{\partial H}\right] = 0$$

Substituting (3.1), (3.2), (3.4), and Roy's identity into (3.3) gives

$$(3.5) \quad -E[V_2(1-L)h(H)(\bar{x}-x)] + tE\{[V_2[(1-L)\bar{x}h'(H) - (1+r)w^1]](dH/dt)_Y \\ + tE[V_2\bar{x}h(H)](dL/dt)_Y = 0$$

Subtracting $0 = E\{V_2[(1-L)\bar{x}h'(H) - (1+r)w^1]\}$ from (3.5) allows us to rewrite (3.5) as

$$(3.6) \quad \frac{d\psi}{dt} = [h(H) + th'(H)(dH/dt)_Y](1-L)E[V_2(\bar{x}-x)] - tE[V_2\bar{x}h(H)](dL/dt)_Y = 0$$

Under certainty $x = \bar{x}$, and (3.6) is satisfied at $t = 0$. This is just another version of the well-known result that lump-sum taxation is efficient under certainty. However, we will now demonstrate that when the return to human capital is uncertain, ψ is not maximized at the point $t = 0$.

¹⁶ We assume, in defining V , that L is chosen optimally, ex post, on the basis of the actual outcome of x and the level of H determined ex ante.

¹⁷ Here $\left(\frac{dH}{dt}\right)_Y$ and $\left(\frac{dL}{dt}\right)_Y$ represent the expected-income-compensated effects of an increase in t on H and L respectively.

Indeed, it will be shown that under reasonable conditions, $d\psi/dt$ is positive at this point, implying that at least partial reliance on a wage tax is more efficient than a pure lump sum tax.

Under uncertainty, near the point $t = 0$, expression (3.6) has the same sign as $E[V_2(\bar{x}-x)]$. In order to sign this expression, we note that

$$(3.7) \quad E[\bar{x}-x] = 0$$

and apply lemma 1 to determine whether $dV_2/dx > 0$. The result thus hinges on the sign of $dV_2/dx = V_{12} \partial h(H)$. But by differentiating Roy's identity with respect to w^2 , and rearranging, we find that

$$(3.8) \quad V_{12} = -L_W V_2 + (1-L)V_{22}.$$

The first term is negative as long as the demand for leisure increases with non-human wealth, while risk aversion implies that the second term is negative as well. An increase in x , then, tends to lower the marginal utility of non-human wealth. Thus V_2 and $(\bar{x}-x)$ are positively correlated.

Therefore, at $t = 0$ expression (3.6) is positive. Welfare is increased by raising t and lowering T to maintain expected revenue constant. In contrast with the certainty case, sole reliance upon lump-sum taxation is inefficient. Intuitively, allocative efficiency is enhanced by a tax which reduces the riskiness of investment in human capital. Through the wage tax the risk-neutral government provides insurance of the risk-averse individual's uncertain undertaking. Note, furthermore, that the optimality of taxing wages at a positive level is independent of the effect of wage taxation on investment in human capital or on the consumption of leisure.

It has been assumed, of course, that the market does not provide insurance against the vagaries of wage rates. We feel that this is typically the case. Problems of moral hazard associated with insurance in general are especially

pervasive in the insurance of human capital.¹⁸ The private insurer, unable to distinguish clearly between external events and the endogenous behavior of the insured party, would provide an incentive for an insured worker to work less hard, spend less time seeking a higher-paying job, or otherwise earn an income below potential. In such a situation the market is unlikely to provide insurance. Taxation of wage income at a positive rate partially compensates for this failure.

Another way of stating this finding is that, near $t = 0$, wage taxation imposes a negative excess burden. Taxing wages and returning the revenue as a lump-sum enhances expected welfare.¹⁹ Pushing this argument to an extreme, it could be argued that full government insurance of human capital investment via a 100 percent wage tax is optimal.* However, such a result is unlikely. Since $\left(\frac{dL}{dt}\right)_y$ represents the expected-income-compensated effect on leisure of an increase in the wage tax, it must be positive in sign. The negativity of the last term of (3.6) near $t = 1$ is likely to make the sign of (3.6) negative in this region. Thus, optimal taxation involves wage taxation at a level strictly between 0 and 100 percent with additional lump-sum taxes or transfers to meet revenue requirements.²⁰

¹⁸ Arrow [1970] discusses problems of moral hazard as they arise in insurance markets.

¹⁹ A similar exercise can be performed to determine the optimal taxation of interest income. Since in this model the return to non-human capital is certain, the interest tax only falls on non-stochastic components of income and does not reduce the riskiness of human capital. It can be shown that when lump-sum taxation is feasible the optimal interest tax is zero whether or not the return to non-human capital is uncertain.

²⁰ Although we have shown the non-optimality of pure lump-sum taxation in an uncertainty model with endogenous human capital, this result would continue to hold even if H were fixed.

* A result somewhat similar to this has been obtained by Mayshar [1977], who studies the subsidization of risky private projects by the government.

IV. Summary

Although optimal human capital accumulation has been studied intensively, little attention has been devoted to the impact of taxes on the human capital decision. This paper has studied the effect of wage and income taxes on human capital using a model in which the returns to human capital investment are uncertain. We began by studying a simple, two-period model in which leisure and first-period consumption were exogenously determined. It was shown that the well-known neutrality of wage taxation in such a model is broken when the returns to human capital are uncertain. The tax decreases the risk associated with human capital investment, tending to increase its desirability. At the same time wage taxation lowers wealth, which reduces the demand for human capital, ceteris paribus, when the degree of absolute risk aversion is decreasing. The overall effect depends upon the relative strengths of these two tendencies. Similarly, under uncertainty it is no longer the case that an income tax unambiguously expands human capital accumulation. Quite possibly, income taxation will discourage human capital.

We then explored the consequences of wage taxation in models which allowed for endogenous first-period consumption and endogenous leisure, respectively. It was shown how the behavioral parameters in these more complicated models interact with risk aversion in order to determine how human capital accumulation is influenced by the wage tax.

Finally, attention was focused upon the problem of efficiently taxing an individual whose human capital yields a random return. An interesting result emerged. Pure lump-sum taxation does not necessarily minimize excess burden. Given risk aversion, partial reliance on a wage tax improves allocative efficiency because it reduces the riskiness of investment in human capital.

Our models are very simple, and there are a number of ways in which they could be extended. It would be interesting to analyze cases in which returns to non-human capital are also stochastic. Study of models in which the random component affects the return to human capital non-multiplicatively would also be useful. Our guess, however, is that comparative statics results on the impact of taxes from more complicated models will not yield many new insights. If this paper is right in its assertion that the unambiguous results of the certainty models do not generalize to the uncertainty case, then empirical work is probably the most fruitful way to carry on research of this important question.

The results of Part III suggest, furthermore, the value of additional theoretical analysis of the design of optimal tax systems. The familiar result that, under certainty, commodity or income taxation is necessarily second best does not generalize to situations characterized by the presence of uncertainty which is neither diversifiable nor insurable. The uninsurability of most investments in human capital suggests that departures from lump-sum taxation may be optimal more frequently than previously realized.

Our conclusions also have implications for optimal redistributive income taxation as developed for the certainty case by Mirrlees [1971], Fair [1971] and Feldstein [1973], for example. It may be the case, for instance, that introducing uncertainty of the type hypothesized in our model will, ceteris paribus, raise the optimal marginal tax rate of more risk-averse income groups. These issues are worth further inquiry.

References

- Arrow, K. J., Essays in the Theory of Risk Bearing, Amsterdam: North Holland, 1970.
- Abbott, M. and O. Ashenfelter, "Labour Supply, Commodity Demand, and the Allocation of Time," The Review of Economic Studies, vol. 43, (October 1976), pp.389-411.
- Boskin, M. J., "Notes on the Tax Treatment of Human Capital," Stanford University, mimeo, 1975.
- _____, "On Some Recent Econometric Research in Public Finance," American Economic Review Papers and Proceedings, vol.66, no.2 (May 1976), pp. 102-109.
- Cain, E. and H. Watts (eds.), Income Maintenance and Labor Supply, Chicago: Rand McNally, 1973.
- Diamond, P., J. Helms and J. Mirrlees, "Optimal Taxation in a Stochastic Economy: A Cobb-Douglas Example," mimeo, MIT, 1978.
- Driffill, J., "A Simulation Analysis of the Effects of Taxes," mimeo, University of Southampton, 1977.
- Fair, R. C., "The Optimal Distribution of Income," Quarterly Journal of Economics, 85 (November 1971), pp. 551-579.
- Feldstein, M. S., "On the Optimal Progressivity of the Income Tax," Journal of Public Economics, 2 (November 1973), pp. 357-376.
- Heckman, J., "A Life-cycle Model of Earnings, Learning, and Consumption," Journal of Political Economy, vol. 84, no. 4, Part 2 (August 1976), pp. 511-544.
- Levhari, D. and Y. Weiss, "The Effect of Risk on the Investment in Human Capital," American Economic Review, vol. 64, no. 6 (December 1974), pp. 950-963.
- Lucas, R. and L. Rapping, "Real Wages, Employment and Inflation," in Micro-economic Foundations of Employment and Inflation Theory, Edmund Phelps (ed.), New York: W. W. Norton and Co., 1970.
- Mayshar, J., "Should Government Subsidize Risky Private Projects?" American Economic Review, vol. 67, no. 2 (March 1977), pp. 20-28.
- Mirrlees, J. G., "An Exploration in the Theory of Optimum Income Taxation," Review of Economic Studies, vol. 38, (1971), pp. 179-208.
- Musgrave, R. A., The Theory of Public Finance, New York: McGraw-Hill Book Company, 1959.
- Ordover, J. A., "Distributive Justice and Optimal Taxation of Wages and Interest in a Growing Economy," Journal of Public Economics, 5 (January-February 1976), pp. 139-160.
- Organization for Economic Cooperation and Development, Theoretical and Empirical Aspects of the Effects of Taxation on the Supply of Labour (1975).
- Pechman, J., Federal Tax Policy (3rd edition), The Brookings Institution, 1977.

- Rosen, H., "Taxes in a Labor Supply Model with Joint Wage-Hours Determination," Econometrica, vol. 44, no. 3 (June 1976), pp. 485-507.
- Ruffin, R. J., "Comparative Advantage and Trade Under Uncertainty," Journal of International Economics, 4 (1974), pp. 261-273.
- Stern, N. H., "On the Specification of Models of Optimum Income Taxation," Journal of Public Economics, 5 (July-August 1976), pp. 123-162.
- Stiglitz, J., "The Effects of Income, Wealth and Capital Gains Taxation on Risk Taking," Quarterly Journal of Economics, 83 (May 1969), pp. 263-283.
- Tobin, J., "Liquidity Preference as Attitude Toward Risk," Review of Economic Studies, vol. 25 (February 1958), pp. 65-86.
- Turnovsky, S. J., "Technological and Price Uncertainty in a Ricardian Model of International Trade," Review of Economic Studies, 41 (April 1974), pp. 201-217.
- Williams, J. T., "Risk, Human Capital, and the Investor's Portfolio," Journal of Business, 51 (January 1978), pp. 65-89.