One aspect of the disaggregated labor market model will be equations
explaining the supply of labor (in terms of numbers of individuals) to various
"labor markets" of the economy. The purpose of this paper is to review some
rather elementary economic theory relevant to the form of these equations.
Although the issues discussed here are rather obvious, there has been a rather
wide variety of approaches to particular supply curve theory by past investigators.
Does the supply curve of labor to a particular occupation, industry, or region
(or combination of these) look like the curves presented in Figure A, B, or C?
($y_j$ is the relative income level per unit of time -- discounted if appropriate --
and $S_j/S$ is the ratio of supply in the jth market to aggregate supply.)
All of these have been used in recent empirical work, but, although the interpretation of the numbers is very different if another specification of the supply curve is employed, little or no attention is paid to deciding which is appropriate.

The process of the adjustment of the aggregate labor supply between various markets has a very important dynamic dimension (also heretofore relatively unexplored). However, the dynamic aspects of the problem cannot be adequately handled unless the static (or preference) aspects of the process are in order. The first sections of this paper sets out three separate theories of long run particular labor supply and the second examines the interequilibrium properties of each of these. The third section then sees what happens to each if the relative wage structure is assumed to be constant, thus yielding short run theories, and the fourth section introduces some elements of a dynamic theory of supply adjustment. Finally, the fifth section discusses a sample of previous treatments of the problem.
I. Static Labor Market Equilibrium

Consider a closed economy in which $S$ individuals must choose to locate in one of $N$ labor markets ($N > S$). Assume that:

(i) The utility, $Z^i_j$, which the $i^{th}$ individual receives from working in the $j^{th}$ labor market is a function of the income he receives per unit of time, $Y^i_j$, plus the money value of his assessment of the non-pecuniary attributes of being located in that market, $A^i_j$, say $Z^i_j = Z^i (Y^i_j + A^i_j)$, where $dZ^i/dY^i_j > 0$.

(ii) Normal hours of work per unit of time, $Q^j$, are identical in all markets, i.e., $Q^j = Q$.

(iii) The demand for labor (in terms of the number of workers) in the typical market is a function of the money wage rate, $W^j$, in that market, that is $W^j = \Theta D(W^j)$, where $\Theta$ is a shift parameter (reflecting capital stock, technology, product demand, etc.) and $\partial D/\partial W^j < 0$.

(iv) The $i^{th}$ individual moves to or stays in the $j^{th}$ labor market, when $Z^i_j > Z^i_k$, all $k \neq j$.

(v) The income level of the $j^{th}$ market adjusts so that the rate of excess demand for labor in that market (here defined for expositional convenience as $X^j = D^j/S^j$, where $S^j$ is the level of supply in the $j^{th}$ market is unity.

The system is in long run equilibrium when no individual desires to move from the market in which he is located. By assumption (iv) this requires that for the $i^{th}$ market $Z^i_j \geq Z^i_k$, all $k \neq j$, or

\[(1.1)\quad Z^i(Y^i_j + A^i_j) \geq Z^i(Y^i_k + A^i_k),\quad \text{all } k \neq j.\]
To analyze the economic content of these assumptions it is necessary to specify further the nature of the individuals' utility functions or, rather, the nature of the distributions of the $A_{ij}$'s. There are three possible specifications, and these lead to three separate theories of equilibrium. We shall call these the a. general, b. simple, and c. non-pecuniary theories of labor market equilibrium. The general theory is based on the assumption that different individuals have different (or not necessarily similar) tastes for locating in different markets, and nothing may be said a priori about the distributions of the $A_{ij}$'s. The simple theory is based on the assumption that each individual is strictly an income maximizer, so without loss of generality we may say that $A_{ij} = 0$, all $i$ and $j$. The non-pecuniary theory is based on the assumption that all individuals assess the money value of the non-pecuniary attributes of a particular market in the same way but that different markets possess different degrees of desirability. This means that $A_{ij} = A_{ij}$, all $i$ and $j$.

It is now necessary to derive particular labor supply curves by each theory. If we accept the utility function of the general theory, all we know is that, ceteris paribus, each individual is more likely to locate in market $j$ the greater is the income level in that market, and we can thus conclude only that

$$ (1.2) \quad S_j = S_j(y_1, \ldots, y_j, \ldots, y_n; A), $$

where $y_j$ is the relative income level in the $j^{th}$ market, $A$ is the set of $A_{ij}$'s, $\partial S_j / \partial y_j > 0$, and (usually) $\partial S_j / \partial y_k \neq 0$. (On purely mathematical grounds this is rather loose, for the supply function is not strictly differentiable and there undoubtedly exists an $A$ such that the function is not unique. However, it suffices for present purposes, for PSALM is, after all, not especially interested in extending the frontiers of empty geometry.) By the simple theory,
the $A^i_j$'s can be ignored, so (1.1) requires that $Z^i(y_j) \geq Z^i(y_k)$, all $k \neq j$ and each $i$. Obviously this is only satisfied when $y_j = y_k$, all $k \neq j$, so the particular supply curve for the simple theory is infinitely elastic over the relevant range for a relative income level of unity. (This is shown in Figure B.) Finally, the equilibrium condition for the non-pecuniary theory requires that $Z^i(y_j + A_j) \geq Z^i(y_k + A_k)$, all $k \neq j$ and each $i$. This is satisfied only when $y_j = y_k + (A_k - A_j)$, which implies that the labor supply curve for the $j$th market is infinitely elastic over the relevant range for a relative income level of

$$1 + \frac{A_k - A_j}{y},$$

where $A$ and $y$ are the average non-pecuniary and money incomes ($A \equiv \frac{1}{J} \sum_{j=1}^{J} S_j^i$) and $y \equiv \sum_{j} S_j$, where $S \equiv \sum_{j} S_j$. (See Figure C.) In equilibrium, then, the more desirable is a market on non-pecuniary grounds the lower will be its income level. An Aside: by the non-pecuniary theory wage levels for the same quality work should be low in California, high in Michigan, and about average in New Jersey and Massachusetts. The predicted and actual ranks are given in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Predicted</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Michigan</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>New Jersey</td>
<td>2.5</td>
<td>3</td>
</tr>
<tr>
<td>Massachusetts</td>
<td>2.5</td>
<td>4</td>
</tr>
</tbody>
</table>

The rank correlation between the predicted and actual wage levels is -0.562, which is inconsistent with the non-pecuniary theory. Purists may argue that a sample
size of four is insufficient to refute a theory, but I haven't lived anywhere else (and see Eckstein and Wilson, *Quarterly Journal of Economics*, 1962).

Since the rate of excess demand in each market is always unity, we have that

\[(1.3) \ \sum_j D(y_j/Y) - s = 0,\]

and, given the equilibrium relative wage levels, \(Y\) must adjust to satisfy this equation. It is important to stress that the preceding analysis is only valid if the \(y_j\)'s and \(Y\) are flexible. The case in which they are not is handled in Section III.
II. Comparative Statics

The three theories will now be investigated to see what they predict in and between equilibrium states. The simple and non-pecuniary theories offer very straightforward predictions. For the former, the relative income level in each market is always unity, and changes in the composition of demand (represented by changes in the $\Theta_j$'s) have no influence on the relative wage structure. Similarly, for the non-pecuniary theory relative income levels are always fixed at a certain level depending on the relative desirability of the market. The actual level of supply in the $j^{th}$ market is equal to the level of demand. Thus, for the simple theory, we see that

$$S_j = \Theta_j D\left(\frac{Q}{Y}\right)$$

and

$$\frac{S_j}{S} = \frac{\Theta_j}{\Theta}$$

and a similar but slightly more complicated pair of expressions explains $S_j$ and $(S_j/S)$ for the non-pecuniary theory. For the non-pecuniary theory, however, one would have to specify what observable phenomena make the $j^{th}$ market relatively desirable or undesirable (the variation of air temperature of a region, ease of work of an industry, social status of an occupation, or whatever). Otherwise, the theory is really only an "alibi" for ignorance of the process of market income determination.

The general theory does not offer any predictions concerning the equilibrium structure of market income levels, for variations in returns are due to differences in tastes which, of course, are unobservable. Between equilibrium states, however, changes in relative income levels will be
positively related to changes in employment—assuming that the supply
functions (tastes) do not shift. If tastes do change, we have no prediction
concerning the behavior of any of the variables.

section

It is necessary for the next/to express these results, which are
intuitively obvious, in relatively rigorous form. For the sake of simplicity
we shall first rewrite the supply function according to the general theory,
(1.2), as

\[ S_j = \lambda_j S(y_j)y, \]

where \( \lambda_j \) is a shift parameter reflecting tastes of the aggregate labor force
for the \( j \)th market and, since \( \sum_j \lambda_j S(y_j)y = 1 \), certain aggregation effects.

Now the equilibrium condition becomes

\[ Y \frac{\partial D(y_j)}{\partial y_j} \lambda_j S(y_j)y = 0. \]

Treating \( y \) as fixed (and equal to whatever is necessary to equate aggregate
demand and aggregate supply, we differentiate the equilibrium condition
totally with respect to \( y_j, \Theta_j \), and \( \lambda_j \), or

\[ \frac{\partial Y}{\partial y_j} \left( \frac{\partial D}{\partial y_j} \lambda_j S(y_j)y \right) \] + \[ \frac{\partial D}{\partial y_j} \lambda_j S(y_j)y \] = 0,

so

\[ \frac{\partial y_j}{\partial \Theta_j} = - \frac{\frac{\partial D}{\partial \Theta_j} \lambda_j S(y_j)y}{\Theta_j D' - \lambda_j S(y_j)y} > 0 \]
and

$$\frac{\delta y_j}{\delta \lambda_j} = \frac{S_j/\lambda_j}{[\Theta \frac{Y}{\delta} b^t - \lambda_j s^t s]} < 0.$$