HETEROGENEOUS EXPOSURE TO AGGREGATE RISK AND THE MACROECONOMY

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Abstract:

Chapter 1 uses recently available data on the top of the wealth distribution to study the relationship between asset prices and wealth inequality. I document three stylized facts: (1) the share of wealth invested in equity increases sharply in the right tail of the wealth distribution, (2) when stock market returns are high, wealth inequality increases and (3) higher wealth inequality predicts lower future stock returns. These facts correspond to the basic predictions of asset pricing models with heterogeneous agents. Quantitatively, however, standard models with heterogeneous agents cannot fully capture the joint dynamics of asset prices and the wealth distribution. Augmenting the model with additional sources of fluctuations in wealth inequality, namely in the form of time-varying investment opportunities for wealthy households, is crucial to match the observed fluctuations in wealth inequality and in asset prices.

Chapter 2 takes a closer look at recent rise in top wealth inequality. I first derive an analytical formula that decomposes the growth of top wealth shares into three terms: the relative wealth growth of individuals in the top, a term due to idiosyncratic returns, and a term due to population renewal. I then map each term to the data using the annual ranking of Forbes Magazine’s list of the 400 wealthiest Americans. The decomposition reveals that the rise in top wealth shares in 1982-1994 is mostly driven by idiosyncratic returns, while the rise in top wealth shares in 1995-2015 is mostly driven by the relative growth of individuals at the top.

Chapter 3, coauthored with Augustin Landier, David Sraer, and David Thesmar, investigates the heterogeneity of banks’ exposure to interest rate risk. In the cross-section of banks, income gap predicts the sensitivity of cash-flows and lending to interest rates. This analysis also allows us to link banks’ interest risk exposure to firm investment and employment.
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My parents believed in me, cared for me, and guided me. I owe my success to their constant love.
For my grand-parents, for my parents
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CHAPTER 1

Asset Prices and Wealth Inequality

1.1 Introduction

Recent empirical studies have uncovered important fluctuations in wealth inequality over the last century. Volatile stock market returns are a potential candidate to account for these fluctuations. Conversely, a large theoretical literature in asset pricing examines the role of household heterogeneity in shaping asset prices, but seldom considers its implication for wealth inequality. In this paper, I leverage recently available data on wealth inequality to examine empirically and theoretically the interplay between asset prices and the wealth distribution.

I focus on the following mechanism. Risk-tolerant investors hold more risky assets, accumulate more wealth, and disproportionately end up at the top of the wealth distribution. As a consequence, in periods when stocks enjoy large realized returns, investors at the top of the wealth distribution gain more than the rest, i.e. wealth inequality increases. In turn, as a larger share of wealth falls into the hands of risk-tolerant households, the aggregate demand for risk increases, which lowers risk premia and pushes up asset prices, i.e. higher wealth inequality predicts lower future returns. I confirm empirically each step of this mechanism. Wealthy households own more equity: the average household invests 40% of its wealth in equity, while the households in the top 0.01% invest 75%. In line with these magnitudes,

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1The material in this chapter was presented at UCLA Anderson Finance Seminar and at USC Conference on Inequality, Globalization, and Macroeconomics.

2See, for instance, Kopczuk and Saez (2004), Piketty (2014), and Saez and Zucman (2016).
in response to a realized stock return of 10%, the wealth share of the top 0.01% increases by 3.5% (7.5% minus 4%). Consistent with the last step of the mechanism, a one standard deviation increase in the wealth share of the top 0.01% predicts lower future excess returns by 5 percentage points.

I then evaluate whether this mechanism can quantitatively account for asset prices and the wealth distribution in equilibrium. I use the reduced form evidence I documented earlier to estimate a state-of-the-art asset pricing model with heterogeneous agents. I find that the standard model cannot jointly match asset prices and the wealth distribution. Specifically, the model cannot generate the high volatility of asset prices without implying an excessive level of inequality compared to the data. To solve this tension, I propose a parsimonious deviation from the standard model. More precisely, I augment the model with fluctuations in the investment opportunities of the rich relative to the poor. These shocks amplify fluctuations in asset prices without changing the average level of inequality, thereby resolving the tension put forth earlier. Furthermore, these shocks can explain why inequality sometimes increases in time of low asset returns, like in the 2000s.

The paper proceeds in three stages. First, I present three new stylized facts on the relationship between asset prices and the wealth distribution. Using the Survey of Consumer Finances, I document a substantial heterogeneity in portfolio holdings within the right tail of the wealth distribution. While the share of wealth invested in equity is essentially flat over the majority of the wealth distribution, it increases sharply within the top percentiles. As noted above, the average household invests 40% of his wealth in equity while the households in the top 0.01% invest 75% of their wealth in equity. Importantly, the disproportional exposure of the households in the top percentiles matters quantitatively for asset prices because these households hold a large fraction of aggregate wealth. This variation is almost entirely driven by differences in the amount that stockholders invest, rather than by participation decisions. In the time series, this heterogeneity implies that realized stock returns generate changes in wealth inequality. I use top wealth shares series constructed from tax filings and from Forbes 400 to estimate the exposure of the top percentiles to
stock market returns. I find that this exposure is remarkably in line with the estimates from portfolio holdings: in response to a realized stock return of 10%, the average wealth increases by 4%, while the average wealth for the top 0.01% increases by 7.5%. Therefore, the wealth share of the top 0.01% increases by the difference of 3.5%.

The flip side of this relationship is that, in an economy where inequality is high, the share of wealth owned by risk-tolerant investors is high, and therefore, in equilibrium, risk premia are low. Thus, higher inequality should predict lower future returns. Indeed, in the data, the wealth share of the top 0.01% is a robust predictor of stock market returns. A one standard deviation increase in the wealth share of the top 0.01% predicts lower future excess returns by 5 percentage points over the following year. In particular, the decrease in risk premia at the end of the 20th century is concomitant with a large increase in wealth inequality. The predictive power of the top wealth share is robust to the inclusion of other predictors put forward in the literature.

Second, I examine whether those facts are quantitatively consistent with a state-of-the-art asset pricing model with heterogeneous agents. Specifically, I study a continuous-time, overlapping generations framework where agents differ with respect to their risk aversion and intertemporal elasticity of substitution, along the lines of Gărleanu and Panageas (2015). I study the dynamics of asset prices and of the wealth distribution in the model. The model can qualitatively generate my three stylized facts. Moreover, as in the data, the wealth distribution exhibits a Pareto tail, shaped by the growth rate of the wealth of top investors relative to the rest of the economy.

Yet, quantitatively, the model cannot jointly match asset prices and the wealth distribution. Specifically, the model cannot generate volatile asset prices without implying an excessive level of inequality. This is because, to generate volatile asset prices, the model requires a high degree of heterogeneity. Intuitively, large variations in asset prices can come from large variations in the relative wealth shares of different agents or large differences in their demand for assets. Both require a high degree of preference heterogeneity. But this persistent heterogeneity in preferences gives rise to an excessive level of inequality in the long

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3I use the series of top wealth shares constructed by Kopczuk and Saez (2004) and Saez and Zucman (2016).
run: calibrations that fit asset prices generate wealth distributions close to Zipf’s law, i.e. with a power law exponent close to 1, whereas the wealth distribution in the data exhibits a thinner tail, a power law exponent of 1.5. Importantly, this tension arises independently of the source of preference heterogeneity: it is present whether one considers heterogeneity in risk aversion, in intertemporal elasticity of substitution, or in subjective discount rates.

Third, I propose a parsimonious deviation from the standard model. Specifically, I consider the impact of low-frequency changes in the investment opportunities of the rich relative to the poor. These shocks create exogenous changes in wealth inequality, thereby increasing fluctuations in asset prices. However, the transitory nature of the shocks limits their impact on the long run level of inequality. After incorporating these shocks in my framework, I show that the augmented model can match quantitatively asset prices and wealth inequality. Furthermore, these shocks help explain otherwise puzzling periods in the data, like the increase of wealth inequality in the 2000s, a period of low asset returns. During an episode when the wealthy have predictably better investment opportunities — for instance because of the development of a new technology — there is a persistent rise in wealth inequality. At the same time, because of these good future prospects, wealthy households demand more assets today, which gives rise to low asset returns during the episode.

Overall, these results suggest a strong link between asset prices and wealth inequality. While a number of studies focus on the role of the risk-free rate of return in shaping the wealth distribution, I document a more important role for the rate of return on risky assets. Moreover, I show that a simple closed-circuit view, where aggregate endowment shocks feed through heterogeneous preferences to asset prices, does not give the full picture. It is necessary to consider additional, more specific, shocks to wealth inequality to understand asset prices.

Related Literature. This paper lies at the intersection of several strands of literature in finance and macroeconomics. This paper relates to the large asset pricing literature of models with heterogeneous investors, in particular Dumas (1989), Wang (1996), Basak and Cuoco

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4See, for instance, Piketty (2014) and Acemoglu and Robinson (2015).
(1998), Gollier (2001), Chan and Kogan (2002), Gomes and Michaelides (2008), Guvenen (2009), and Gârleanu and Panageas (2015). My paper compares the wealth distribution implied by these models with the data. I argue that the data suggest the existence of additional shocks that redistribute investment opportunities across households. This ties my paper to a growing literature which examines the impact of re-distributive shocks on asset prices, either through technology shocks (Kogan, Papanikolaou, and Stoffman (2013), Gârleanu, Kogan, and Panageas (2012)), fluctuating capital share (Lettau, Ludvigson, and Ma (2016), Greenwald, Lettau, and Ludvigson (2014)), tax rates (Pastor and Veronesi (2016)) or idiosyncratic shocks (Constantinides and Duffie (1996), Storesletten, Telmer, and Yaron (2007), Schmidt (2016)).

This paper also contributes to an important empirical literature in asset pricing that documents the heterogeneity in aggregate exposure across households. A number of papers have documented the heterogeneity in consumption exposure between stockholders and non-stockholders (Mankiw and Zeldes (1991), Brav, Constantinides, and Geczy (2002), Malloy, Moskowitz, and Vissing-Jørgensen (2009), Parker and Vissing-Jørgensen (2009)). This literature shows that the disproportional exposure of stockholders, together with the high volatility of asset prices, can explain the equity premium puzzle. I concentrate on the heterogeneity within the top stockholders and examine whether it can generate a high volatility of asset returns in equilibrium to begin with.

The paper also relates to a literature in household finance which examines the heterogeneity in portfolio choice across the wealth distribution. Guiso, Jappelli, and Terlizzese (1996), Carroll (2000), Wachter and Yogo (2010), Calvet and Sodini (2014) and Bach, Calvet, and Sodini (2015) have documented that the share of wealth invested in risky assets increases with wealth. I show that the increase is entirely accounted for by the top percentiles, and that this heterogeneity generates large fluctuations in wealth inequality over time.

This paper also contributes to the recent literature on wealth inequality. On the empirical side, I rely critically on the recent wealth shares constructed by Kopczuk and Saez (2004) and Saez and Zucman (2016). On the theoretical side, mechanisms generating the Pareto tail of the wealth distribution have been studied in Gabaix (1999) and Gabaix (2009). The relationship between asset prices and the wealth distribution in equilibrium models has been
recently discussed in Benhabib, Bisin, and Zhu (2011), Achdou et al. (2016), Jones (2015) and Cao and Luo (2016). Eisfeldt, Lustig, and Zhang (2016) examine the joint equilibrium of asset prices and of the wealth distribution in an economy populated with investors that differ with respect to their level expertise. Relative to this literature, I explore the case of a stochastic economy, where households have different exposures to aggregate shocks, and therefore, where the wealth distribution is stochastic.

In a contemporaneous working paper, Toda and Walsh (2016) use the series on top income shares from Piketty and Saez (2003) to show that fluctuations in income inequality negatively predict future excess stock returns. In contrast, I show that wealth inequality negatively predicts future excess stock returns, using top wealth shares from Kopczuk and Saez (2004). Empirically, a large share of fluctuations in income inequality is unrelated to fluctuations in wealth inequality.\footnote{In particular, business cycles fluctuations in income inequality are driven by fluctuations in the aggregate level of realized capital gains, which partly reflects the amount of trades. See, for instance, Saez and Zucman (2016).} Furthermore, I examine this relationship within a quantitative model.

**Road Map** The rest of my paper is organized as follows. In Section 1.2, I document three stylized facts consistent with heterogeneous agents models. In Section 1.3, I present a standard asset pricing model with heterogeneous agents to interpret these findings. In Section 1.4, I show that the standard model has difficulty matching asset prices and wealth moments. In Section 1.5, I propose a parsimonious deviation from the standard model. Section 1.6 concludes.

### 1.2 Three Facts on Asset Prices and Wealth Inequality

I now analyze data about the top of the wealth distribution to document three stylized facts predicted by heterogeneous agents models. In particular, I focus on the following mechanism. Risk-tolerant investors hold more risky assets and disproportionately end up at the top of the wealth distribution; thus, richer households own more risky assets. As a consequence, in periods when stocks enjoy large realized returns, investors at the top of the wealth distribution gain more than the rest; thus, inequality increases. In turn, as a larger
share of wealth falls into the hands of risk-tolerant households, the aggregate demand for risk increases, which lowers risk premia and pushes up asset prices; thus, higher inequality predicts lower future returns.

After introducing the data, I document facts reflecting each step of this mechanism.

1.2.1 Data

In order to analyze the investments and the wealth dynamics of households across the wealth distribution, I combine different data sources.

**Equity Investment.** I measure the heterogeneity in investment decisions across households using the Survey of Consumer Finances (SCF). The survey is a repeated cross-section of about 4,000 households per survey year, including a high-wealth sample. The survey is conducted every three years, from 1989 to 2013. The respondents provide information on their networth, including their investment in public and private equity. I define the equity share as the total investment in equity over networth. I define the set of entrepreneurs as the households with equity held in an actively managed business.\(^6\)

It remains difficult for surveys to capture the very top households. In particular, by design, the Survey of Consumer Finance excludes the households who appear on Forbes Magazine’s list of the 400 wealthiest Americans (Kennickell (2009), Saez and Zucman (2016)). Moreover, the SCF is only available every three years since 1989 and it cannot be used to measure business cycle fluctuations in the wealth distribution.

**Wealth Share.** I am interested in measuring changes in the wealth distribution and their relationship to stock returns. Therefore, I need yearly estimates of the wealth distribution that cover several business cycles. I use two datasets that jointly cover most of the last 100 years.

The first wealth series is the annual series of top wealth shares constructed by Kopczuk and Saez (2004). This series is constructed from estate tax returns, which report the wealth of deceased households. The wealth distribution of the deceased is used to capture the wealth\(^6\)The definition follows Moskowitz and Vissing-Jørgensen (2002).
distribution among the living using the mortality multiplier technique, which amounts to weighting each estate tax return by the inverse probability of death (depending on age and gender). The series is constructed using the whole universe of estate tax returns during the 1916-1945 period, and a stratified sample of micro-files for 1965, 1969, 1972, 1975 and 1982-2000.

I supplement the estate tax returns with the list of the wealthiest 400 Americans constructed by Forbes Magazine every year since 1982, which offers an unparalleled view on the right tail of the wealth distribution. The list is created by a dedicated staff of the magazine, based on a mix of public and private information. The total wealth of individuals on the list accounts for approximately 1.5% of total aggregate wealth. By combining the lists over time, I am able to track the wealth of the same individuals over time.

One data series that has continuous coverage between 1917 and 2012 is Saez and Zucman (2016). The series is constructed from income tax returns. The series builds in smoothing over time to focus on low-frequency fluctuations in wealth. Therefore, it is not the most adequate to examine fluctuations in wealth at the business cycle frequency. Still, I show that my results hold qualitatively with this dataset in Section 1.A.2.

**Asset Prices.** I measure stock returns from the value-weighted CRSP index and risk-free rates from the Treasury Bill rate after 1927. For the period before 1927, I obtain stock returns and risk-free rates from Shiller (2015). I also use the set of predictors constructed in Welch and Goyal (2008), which includes, in particular, the price-dividend ratio.

### 1.2.2 Investment in Equity Across the Wealth Distribution

The basic building block of heterogeneous agents models is that there is a group of investors that disproportionately invests in equity. In contrast, if investment in equity were proportional to wealth, movements in stock prices would not generate movements in the

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7Forbes Magazine reports: “We pored over hundreds of Securities Exchange Commission documents, court records, probate records, federal financial disclosures and Web and print stories. We took into account all assets: stakes in public and private companies, real estate, art, yachts, planes, ranches, vineyards, jewelry, car collections and more. We also factored in debt. Of course, we don’t pretend to know what is listed on each billionaire’s private balance sheet, although some candidates do provide paperwork to that effect.”

8This extendsthe construction of Capehart (2014) to the recent years. Recent empirical studies examining the Forbes 400 list also include Klass et al. (2006) and S. N. Kaplan and Rauh (2013).

wealth distribution and, conversely, movements in the wealth distribution would not generate 
movements in stock prices.

Figure 1.1a plots the average equity share within percentile bins across the wealth 
distribution. The average equity share of 0.4 masks a substantial heterogeneity across 
households. The equity share is essentially flat at 0.2 over the majority of the wealth 
distribution, but increases sharply within the top 1%. Figure 1.1b plots the equity share 
with respect to the log top percentiles, showing that the equity share is approximately linear 
in the log percentile at the top of the distribution. The figure suggests that the bulk of the 
heterogeneity is concentrated within the top percentiles. The top percentiles are likely to 
be important for asset prices because they own a large share of wealth: the vertical red line 
in Figure 1.1a shows that half of the total net worth is owned by the households in the top 
3%.

Panel A of Table 1.1 reports the corresponding average equity share in four groups 
of households: all households, households in the top 1% – 0.1%, households in the top 
0.1% – 0.01%, and households in the top 0.01%. The average equity share for households in 
the top 0.01% is 0.75, while the average equity share for all households is 0.4; thus, wealthy 
households hold twice as much equity as the average household.

A stylized fact in the household finance literature is that stock market participation in- 
creases with wealth (Vissing-Jørgensen (2002b)). Therefore, the increase in the equity share 
within the top percentiles could be driven by an increase in the proportion of stockholders 
(i.e. the extensive margin). However, Panel B of Table 1.1 shows that the percentage of 
stockholders is constant within the top percentiles (90%). The increase in the equity share is 
entirely driven by the increase within stockholders. The heterogeneity between stockholders 
and non-stockholders generates a lot of variations at the bottom of the wealth distribution, 
but these variations account for a small share of total wealth.

Investment in risky assets comes mainly in two forms: public equity and private equity. 
Panel A of Table 1.1 decomposes the increase in equity share across the top percentiles 
between the two types of equity. The decomposition reveals that the increase in the equity 
share is mostly driven by an increase in the share of wealth invested in private equity. 
Panel C of Table 1.1 shows that the proportion of entrepreneurs increases sharply in the top
percentile: the proportion of households with an actively managed business is 78.5% in the Top 0.01%, compared to 10% in the general population. The wealth of these entrepreneurs is mostly invested in their private business. A potential concern is that, if entrepreneurs cannot trade or sell their firms easily, the heterogeneity in private equity holdings may have no impact on stock market prices. However, Panel C of Table 1.1 shows that entrepreneurs hold large amounts of public equity (15%). Even with illiquid businesses, entrepreneurs can adjust their overall risky holdings at the margin.

These results show that households at the top of the wealth distribution disproportionately invest in equity. This suggests that they are disproportionately exposed to aggregate risk. An important caveat is that we do not observe the details of the equity positions, public or private, and they might have different characteristics, potentially correlated with wealth. To get around this issue, I now turn to time series evidence on top wealth shares.

1.2.3 Wealth Exposure to the Stock Market Across the Wealth Distribution

Because households across the wealth distribution invest differently in equity, stock market returns generate fluctuations in the wealth distribution. Following a positive return, wealthy households gain more relative to other agents, and therefore wealth inequality increases. I now quantify this mechanism.

To do so, I use wealth series from Kopczuk and Saez (2004) and Forbes 400. I measure the exposure of top households to the stock market by regressing the growth of total wealth in a percentile group on excess stock market returns, i.e.,

\[
\log \frac{W_{t+1}^{p,p'}}{W_{t-1}^{p,p'}} = \alpha + \beta(r^M_t - r^f_t) + \gamma r^f_t + \epsilon_t \quad (1.1)
\]

Similarly, Hurst and Lusardi (2004), using the Panel Study of Income Dynamics (PSID), show that the propensity of entrepreneurship increases sharply with wealth in the top percentiles.

11 For instance, Roussanov (2010) study preferences such that the exposure to idiosyncratic shocks increases with wealth, but the exposure to aggregate shocks decreases with wealth.
where \( W_{t}^{p \rightarrow p'} \) denotes the total wealth of households between the percentiles \( p \) and \( p' \) in year \( t \), \( r_{t}^{M} \) is the stock market return, and \( r_{t}^{f} \) is the risk-free rate.\(^{12}\)

The first four columns in Table 1.2 (Panel A) report the estimates for \( \beta \), the wealth exposure to the stock market, for four groups of households: all households, households in the top \( 1 - 0.1\% \), households in the top \( 0.1\% - 0.01\% \), and households in the top \( 0.01\% \). The estimated exposure \( \beta \) increases monotonically with the top percentiles, from \( \beta = 0.44 \) for the average household, to \( \beta = 0.75 \) for the households in the top \( 0.01\% \). These estimates exactly match the equity shares estimated in Table 1.1, even though the time periods are different. This is what we expect if a dollar invested in equity has the same exposure as a dollar invested in the stock market.

The last two columns of Panel A in Table 1.2 report the wealth exposure of the Top 400 and of the Top 100 from Forbes. The estimates for the households in the extreme tail of the distribution are similar in magnitude to the estimates for the households in the top \( 0.01\% \) from tax data.

Since top households are comparatively more exposed to the stock market, high stock market returns increase inequality. Panel B of Table 1.2 confirms this relationship by regressing top wealth shares on stock market returns. The estimate 0.31 corresponds to the difference of exposure between households at the top and the average household (0.75–0.44).

Some forces might drive a distinction between the exposure of top wealth shares and the relative exposure of individuals in the top percentiles. This is because top percentiles do not necessarily include the same individuals over time. In particular, some fluctuations in top wealth shares may be generated by fluctuations in the size of idiosyncratic shocks. Intuitively, when the variance of idiosyncratic shocks increases, top wealth shares increase through a composition effect. If changes in idiosyncratic variance are positively correlated with stock returns, this results in a positive bias.

I address this bias in two ways. First, I use the panel dimension of Forbes 400 to track the same individuals over time. Appendix Table 1.6 compares the exposure of the wealth of households in the Top 40 to the exposure of the individual households inside the Top

\(^{12}\)The L.H.S. is the growth of \( W_{t}^{p \rightarrow p'} \) between \( t - 1 \) and \( t + 1 \), to avoid overlapping time periods. This is because \( W_{t}^{p \rightarrow p'} \) is the average of wealth owned by the group over the year, rather than the wealth at a given point in time. See also the notes in Table 1.2.
40. The estimates are similar (0.71 vs 0.74). Second, to examine the magnitude of the bias in estate tax returns, I control in regression (1.1) for changes in idiosyncratic variance, as measured by the changes in the idiosyncratic variance of firm level stocks. Appendix Table 1.7 shows that changes in idiosyncratic variance have a positive, non-significant effect on top wealth shares. In particular, the inclusion of this control does not impact the estimate for the exposure of top wealth shares to stock returns, \( \beta \).

The reason why this composition effect turns out to be small empirically is that the wealth distribution is very unequal. Intuitively, because wealth is so concentrated, fluctuations due to entry and exit at the bottom of the percentile are small relative to fluctuations in the wealth of households inside the percentile. Formally, for a wealth distribution Pareto-distributed with power law exponent \( \zeta \), I show in Section 1.B.3 that a rise in idiosyncratic variance \( \Delta \sigma^2 \) increases the growth of top wealth shares by \( (\zeta - 1)\Delta \sigma^2/2 \) in the following year. Because \( \zeta \approx 1.5 \) for the wealth distribution in the U.S., the formula says that a one standard deviation rise in the idiosyncratic variance, \( \Delta \sigma^2 = 0.05 \), increases top wealth shares by 0.5% in the following year. This is much smaller than the impact of a one standard deviation rise in stock prices, \( r^M_t = 0.17 \), which increases top wealth shares by 6%, as measured in Table 1.2. Therefore the potential bias due to changes in idiosyncratic volatility is small.\(^{14}\)

1.2.4 Top Wealth Shares Predict Future Excess Returns

The previous evidence suggests that wealthy households are more willing to take on aggregate risk. As top wealth shares increase, wealth is rebalanced from risk-averse households to risk-tolerant households, and, therefore, the total demand for risk in the economy increases. In equilibrium, the compensation for holding risk decreases. Hence, higher top wealth shares should predict lower future excess returns.

\(^{13}\)To the best of my knowledge, there is no time series on the idiosyncratic variance of the wealth growth of households.

\(^{14}\)This result is consistent with Gabaix et al. (2016). They show that changes in idiosyncratic variance generate slow transition dynamics.
I estimate the predictive power of top wealth shares by regressing excess stock returns on the wealth share of the top 0.01%, i.e.,

\[ \sum_{1 \leq h \leq H} r_{t+h}^{M} - r_{t+h}^{f} = \alpha + \beta_{H} \log \text{Wealth Share Top 0.01\%}_t + \epsilon_{H_t} \]  

(1.2)

where \( h \) denotes the horizon.

The first line in Table 1.3 reports the results of the predictability regression at the one-year and three-year horizons. The estimates are statistically and economically significant. A one standard deviation increase in the log of the wealth share of the top 0.01% is associated with a decrease of excess returns by 5 percentage points over the next year.

Figure 1.2 plots the wealth share of the top 0.01% along with a moving average of excess stock returns over the following eight years. Fluctuations in the wealth share of the top 0.01% do a particularly good job at tracking the low-frequency fluctuations in excess stock returns. Excess stock returns were low when inequality was high in the 1920s. Excess stock returns increased following the decrease in inequality in the 1930s, and decreased following the increase in inequality in the 1980s.

The fact that wealth inequality mostly captures the low-frequency fluctuations in excess returns is not surprising. This is because wealth inequality is persistent. Using the Dickey-Fuller generalized least squares (DF-GLS) test, one cannot reject that the series of the wealth share of the top 0.01% has a unit root. A natural concern is that, in this case, conventional t-statistics are misleading (Elliott and Stock (1994), Stambaugh (1999)). To address this concern, I rely on a test developed in Campbell and Yogo (2006), which is valid when the predictor variable has a largest root close to, or even larger than, one. The results of this test, reported in Table 1.10, show that the wealth share of the top 0.01% still significantly predicts returns, even though the evidence becomes thinner as one allows for explosive dynamics in the predictor.

Finally, I assess whether the information in the wealth share of the top 0.01% is subsumed by other predictors put forward in previous literature. I use the list of predictor variables constructed in Welch and Goyal (2008). For each regressor, I report the \( \beta_1, \beta_2 \) as well as the
$R^2$ corresponding to the following bivariate predictive regression

$$\sum_{1 \leq h \leq H} r_{t+h}^M - r_{t+h}^f = \alpha + \beta H \text{Log Wealth Share Top 0.01} \%_t + \gamma_H \text{Predictor}_t + \epsilon_{Ht} \quad (1.3)$$

Table 1.3 summarizes the results. The first column reports the coefficient on the wealth share of the top 0.01%, the second column reports the coefficient on the other predictor, and the last column reports the adjusted $R^2$ of the regression. While the first three columns report the results with $H = 1$, the last three columns report the results with $H = 3$. To facilitate the comparison between the different predictors, all regressors are normalized to have a standard deviation of one. The table shows that the predictive power of the wealth share of the top 0.01% is robust to the inclusion of other predictors. In particular, the estimate for $\beta_1$ remains stable across the different specifications.

I have shown that households at the top of the wealth distribution invest disproportionately in equity, that fluctuations in stock prices generate fluctuations in inequality, and, in turn, that the level of inequality determines future excess returns. Those three facts are at the heart of asset pricing models with heterogeneous agents. I now examine the quantitative properties of these models.

1.3 An Asset Pricing Model with Heterogeneous Preferences

I consider a continuous-time pure-exchange economy. I present a model where overlapping generations of households differ in their preferences. The baseline model builds on Gârleanu and Panageas (2015). I derive the behavior of asset prices and characterize the properties of the wealth distribution.

1.3.1 Structure

Endowment. I consider a continuous-time pure exchange economy. I assume that the aggregate endowment exhibits i.i.d. growth. Its law of motion is

$$\frac{dY_t}{Y_t} = \mu dt + \sigma dZ_t$$
where $Z_t$ is a standard Brownian motion.

**Demographics.** The specification of demographics follows Blanchard (1985). The economy is populated by a mass one of agents. Each agent faces a constant hazard rate of death $\delta > 0$ throughout his life. During a short time period $dt$, a mass $\delta dt$ of the population dies and a new cohort of mass $\delta dt$ is born, so that the total population stays constant.

**Labor Income.** An agent $i$ born at time $s(i)$ is endowed with the labor income process $L_i = \{L_{it} : t \geq s(i)\}$, given by

$$L_{it} = \omega Y_t \times \xi_i \times G(t - s(i)) \quad (1.4)$$

The first term of this formula, $\omega Y_t$, corresponds to the fraction of the aggregate endowment distributed as labor income. The second term, $\xi_i$, is an individual specific level of income. I assume that it is i.i.d. across agents, with mean 1. The value of $\xi_i$ is realized at birth. This component captures the heterogeneity in labor income within a generation.

The third term, $G(t - s)$, captures the life-cycle profile of earnings of households. The function $G$ is normalized so that aggregate earnings equal $\omega Y_t$ at each point in time, i.e.

$$\int_{-\infty}^{t} \delta e^{-\delta(t-s)} G(t-s) ds = 1$$

The rest of the endowment $(1 - \omega)Y_t$ is paid by claims to the representative firm.

**Preferences.** Agents have recursive preferences as defined by Duffie and Epstein (1992). They are the continuous-time versions of the recursive preferences of Epstein and Zin (1989). For an agent $i$ with a consumption process $C_i = \{C_{it} : t \geq 0\}$, his utility $U_i = \{U_{it} : t \geq 0\}$ is defined recursively by:

$$U_{it} = E_t \int_{t}^{+\infty} f_i(C_{iu}, U_{iu}) du$$

$$f_i(C, U) = \frac{\rho + \delta}{1 - \frac{1}{\gamma_i}} \left( \frac{C^{1 - \frac{1}{\gamma_i}}}{\frac{1}{\gamma_i} - \frac{1}{\gamma_i}(1 - \gamma_i)U} \right)$$
These preferences are characterized by three parameters. The subjective discount factor is $\rho$, the coefficient of relative risk aversion is $\gamma_i$ and the elasticity of intertemporal substitution (EIS) is $\psi_i$.

I assume there are two types of agents, labeled $A$ and $B$, that differ with respect to their coefficient of relative risk aversion $\gamma_i$ and their elasticity of intertemporal substitution $\psi_i$. I denote $A$ the risk-tolerant agent, i.e. $\gamma_A > \gamma_B$. At every point in time a proportion $\pi_A$ of newly born agents are of type $A$.

**Markets.** Households can trade two assets. First, they can trade claims to the representative firm. They can also trade instantaneous risk-free claims in zero net supply. The price of both of those claims is determined in equilibrium. Denote $r_t$ the risk-free rate and $dR_t$ the return of a dollar invested in the representative firm:

$$dR_t = \mu_{Rt} dt + \sigma_{Rt} dZ_t$$

**Household Problem.** Denote $W_{it}$ the financial wealth of agent $i$ at time $t$. As in Blanchard (1985), agents can access a market for annuities. There are life insurance companies that collect the agents’ financial wealth when they die. In exchange, agents receive an income stream equal to $\delta W_{it}$ per unit of time.

The problem of households is as follows. An household $i$ born at time $s(i)$ chooses a consumption path $C_i = \{C_{it} : t \geq s\}$ and an amount of dollars invested in the representative firm $\theta_i = \{\theta_{it} : t \geq s\}$ to maximize his lifetime utility

$$V_{it} = \max_{C_i, \theta_i} U_{it}(C_i)$$

subject to the dynamic budget constraint

$$dW_{it} = (L_{it} - C_{it} + (r_t + \delta)W_{it} + \theta_{it}(\mu_{Rt} - r_t))dt + \theta_{it}\sigma_{Rt}dZ_t$$ for all $t \geq s$
I now make a useful change of variables. Because markets are dynamically complete, there is a unique stochastic discount factor $\eta_t$:

$$\frac{d\eta_t}{\eta_t} = -r_t dt - \kappa_t dZ_t$$

where $r_t$ is the risk free interest rate and $\kappa_t$ is the price of aggregate risk. The expected return of a dollar invested in the representative firm can be written:

$$\mu_{Rt} = r_t + \kappa_t \sigma_{Rt}$$

Households are not subject to liquidity constraints; hence, they can sell their future labor income stream and invest the proceeds in financial claims. Define $N_{it}$, the total wealth of household $i$ as the sum of his financial wealth and his human capital, i.e. the present value of his labor income:

$$N_{it} = W_{it} + E_t\left[\int_{u=t}^{+\infty} e^{-\delta(u-t)} \frac{\eta_u}{\eta_t} L_{ts(i)}\right]$$

In particular, denote $\xi_t \phi_t Y_t$ the wealth of a newborn agent, i.e.,

$$\phi_t = E_t\left[\int_{t}^{+\infty} e^{-\delta(u-t)} \frac{\eta_u}{\eta_t} \omega Y_u \frac{Y_t}{Y_t} G(u-t)\right]$$

The household problem can now be reformulated as follows. Household $i$ chooses a consumption rate $c_i = \{c_{it} = C_{it}/N_{it} : t \geq s(i)\}$ and a wealth exposure to aggregate shocks $\sigma_i = \{\sigma_{it} : t \geq s(i)\}$ such that for all $t \geq s(i)$

$$V_{it} = \max_{c_{it},\sigma_{it}} U_{it}(c_{it}N_{it})$$

s.t. $\frac{dN_{it}}{N_{it}} = \mu_{it} dt + \sigma_{it} dZ_t$

with $\mu_{it} = r_t + \delta + \kappa_t \sigma_{it} - c_{it}$

and $N_{ts(i)} = \xi_i \phi_{s(i)} Y_{s(i)}$
Equilibrium. Informally, an equilibrium is characterized by a map from shock histories $Z_t$ to prices and asset allocations such that, given prices, agents maximize their expected utilities and markets clear. Denote $I_A = [0, \pi_A]$ the set of agents in group $A$ and $I_B = [1 - \pi_A, 1]$ the set of all agents in group $B$.

Denote $p_t Y_t$ the total wealth in the economy. It is the sum of the firm valuation and the human capital of existing agents. Conjecture that $p_t$ follows an Ito process:

$$\frac{dp_t}{p_t} = \mu_p dt + \sigma_p dZ_t$$

Definition 1. An equilibrium is a set of stochastic processes for the interest rate $r = \{r_t; t \geq 0\}$, market price of risk $\kappa = \{\kappa_t; t \geq 0\}$, consumption and investment decisions $c_i = \{c_{it}; t \geq 0\}$, $\sigma_i = \{\sigma_{it}; t \geq 0\}$ such that

1. $(c_i, \sigma_i)$ solve (1.5) given $(r, \kappa)$

2. Markets clear

$$\int_{i \in I_A} N_{it} c_{it} di + \int_{i \in I_B} N_{it} c_{it} di = Y_t \text{ (Consumption)}$$  \hspace{1cm} (1.8)

$$\int_{i \in I_A} N_{it} \sigma_{it} di + \int_{i \in I_B} N_{it} \sigma_{it} di = p_t Y_t (\sigma + \sigma_{pt}) \text{ (Risky asset)}$$  \hspace{1cm} (1.9)

By Walras’s law, the market for risk-free debt clears automatically.

1.3.2 Solution

Solving the Model. All households with the same preference parameters face the same trade-off, irrespective of their wealth or age. This is because their utility function is homogeneous and their death rate is constant over time. In particular, the consumption rate $c_{it}$ and the wealth exposure $\sigma_{it}$ are the same for all agents with the same preferences.

For the purpose of determining prices, we can abstract from the distribution of wealth within each group: we only need to keep track of the share of aggregate wealth that belongs to the
agent in group $A$:

$$x_t = \frac{\int_{i \in I_A} N_i dt}{\int_{i \in I_A} N_i dt + \int_{i \in I_B} N_i dt}$$  \hspace{1cm} (1.10)$$

I restrict my attention to Markov equilibrium where all processes are functions of $x_t$ only. We have four policy functions $c_{jt}, \sigma_{jt}, j \in \{A, B\}$, two value functions, two prices $\kappa_t$ and $r_t$, and two valuations (firm value and human capital) to solve for. The four first order conditions, the two HJB equations, the two market clearing condition, and the no arbitrage condition for the firm value and for human capital are enough to derive the equilibrium. In the Appendix Section 1.B.2, I reduce the system to a system of PDEs. I solve this system of PDEs using an implicit time-stepping scheme.\textsuperscript{15}

I now turn to two particular parts of the equilibrium.

**Evolution of Household Wealth.** I first characterize the law of motion of households' wealth. This law of motion will determine the law of motion of the wealth distribution, which I observe in the data. Denote $p_{jt}$ the wealth-to-consumption ratio of an agent in group $j \in \{A, B\}$. In equilibrium, the process $p_{jt}$ follows an Ito process

$$\frac{dp_{jt}}{p_{jt}} = \mu_{p_{jt}} dt + \sigma_{p_{jt}} dZ_t$$

The following proposition characterizes the law of motion of the wealth of households within each group:

**Proposition 1** (Law of Motion for Households Wealth). *The wealth of households in group $j \in \{A, B\}$ follows the law of motion*

$$\frac{dN_{jt}}{N_{jt}} = \mu_{jt} dt + \sigma_{jt} dZ_t$$

\textsuperscript{15}More details about the solution method can be found at https://github.com/matthieugomez/EconPDEs.jl.
where $\mu_{jt}$ and $\sigma_{jt}$ are given by

$$\sigma_{jt} = \frac{\kappa_t}{\gamma_j} + \frac{1 - \gamma_j}{\gamma_j(\psi_j - 1)} \sigma_{p,j,t}$$ (1.11)

$$\mu_{jt} = \psi_j (r_t - \rho) + \frac{1 + \psi_j}{2\gamma_j} \kappa_t^2 + \frac{1 - \gamma_j}{\gamma_j(\psi_j - 1)} \kappa_t \sigma_{p,j,t} + \frac{1 - \gamma_j \psi_j}{2(\psi_j - 1)\gamma_j} \sigma_{p,j,t}^2 + \mu_{p,j,t}$$ (1.12)

The volatility of wealth, $\sigma_{jt}$, has two components: a myopic demand and a demand due to time-varying investment opportunities. The myopic demand equals the ratio of the market price of risk to the risk aversion. The lower the risk aversion, the higher the myopic demand. The hedging demand $H_{it}$ captures deviations from the mean-variance portfolio due to variations in investment opportunities. In the calibrations explored below, this term will be positive because expected returns are countercyclical.

The drift of wealth, $\mu_{jt}$ has three components. The first term is the standard term due to intertemporal substitution, determined by the EIS $\psi_i$ and the difference between the interest rate $r_t$ and the subjective discount factor $\rho$: $\psi_j (r_t - \rho)$. The second term comes from risky assets. Agents with a lower risk aversion invest disproportionately in risky assets and therefore earn higher returns. Hence, their wealth grows at a faster rate than the rest of the economy. The third term $\Phi_{it}$ captures changes in investment opportunities.

In short, the proposition suggests that agents with a lower risk aversion invest more in risky assets and grow faster than the rest of the households. Therefore, we naturally obtain my first two stylized facts: agents at the top of the distribution buy more risky assets and households at the top are more exposed to aggregate shocks compared to the rest of the households.

In particular, because agents in group $A$ choose a different wealth exposure compared to agents in group $B$, the share of wealth owned by agents in group $A$, $x_t$, is stochastic. The next proposition characterizes the law of motion of $x_t$:

**Proposition 2.** The law of motion of $x$ is

$$\frac{dx_t}{x_t} = \mu_{x,t} dt + \sigma_{x,t} dZ_t$$
where \(\mu_{xt}\) and \(\sigma_{xt}\) are given by

\[
\sigma_{xt} = (1 - x_t)(\sigma_{At} - \sigma_{Bt})
\]

\[
\mu_{xt} = (1 - x_t)(\mu_{At} - \mu_{Bt}) + (1 - x_t)\frac{\phi_t}{p_t} \delta \left( \frac{\pi_A}{x_t} - \frac{1 - \pi_A}{1 - x_t} \right) - (\sigma + \sigma_{pt})\sigma_{xt}
\]

(1.13)

(1.14)

The volatility of \(x_t\) is directly related to the difference between the wealth volatility of the agents in group \(A\) and the wealth volatility of the agents in group \(B\).

The drift of \(x_t\) is the sum of three terms. The first term is the difference between the wealth drift of the agents in group \(A\) and the wealth drift of the agents in group \(B\). The second term corresponds to the birth of individuals in group \(A\) compared to the birth of individuals in group \(B\). The third term corresponds to an Ito correction term.

**Market price of risk.** The third step of our basic mechanism is that, when more wealth falls into the hands of risk tolerant households, stock prices increase and future returns are lower. To gain some intuition on this relationship in the model, I now consider the determination of the equilibrium price of risk \(\kappa_t\).

Because all agents within the same group choose the same exposure to aggregate shocks, the market clearing for risky assets (1.9) can be written:

\[
x_t\sigma_{At} + (1 - x_t)\sigma_{Bt} = \sigma + \sigma_{pt}
\]

(1.15)

This market clearing simply says that the volatility of aggregate wealth is the wealth-weighted average of the volatility of the wealth of individual agents.

Substituting out the optimal choice \(\sigma_{jt}\) for households in group \(j \in \{A, B\}\) given by (1.11), we obtain the market price of risk \(\kappa_t\):

\[
\kappa_t = \Gamma_t(\sigma + \sigma_{pt} - H_t)
\]

(1.16)
where $\Gamma_t$ corresponds to the aggregate risk aversion and $H_t$ corresponds the total hedging demand

$$\Gamma_t \equiv 1/(\gamma_A + 1 - x_t)$$

$$H_t = x_t H_A + (1 - x_t) H_B$$

The market price of risk $\kappa_t$ is the product of the aggregate risk aversion $\Gamma_t$ times the total quantity of risk $\sigma + \sigma_{pt}$, minus the total demand for risk due to the hedging $H_t$.

The aggregate risk aversion $\Gamma_t$ is a wealth-weighted harmonic mean of individual risk aversions. The higher the share of wealth owned by the agents in group $A$, $x$, the lower the aggregate risk aversion $\Gamma_t$. Therefore, ignoring for a moment the hedging demand, an increase in the fraction hold by $x_t$ decreases the market price of risk $\kappa_t$. This corresponds to the predictive regression of Section 1.2.4.

### 1.3.3 The Wealth Distribution

In this section, I characterize the dynamics of the wealth distribution implied by the model. This step will make it easier to bring the model to the data. More precisely, I analyze the distribution of households’ relative wealth $n_{it}$, defined as

$$n_{it} = \frac{N_{it}}{p_i Y_t}$$

By Ito’s lemma, the law of motion of the relative wealth $n_{it}$ is

$$\frac{dn_{it}}{n_{it}} = \tilde{\mu}_{it} dt + \tilde{\sigma}_{it} dZ_t$$

where $\tilde{\mu}_{it}$ and $\tilde{\sigma}_{it}$ are given by

$$\tilde{\sigma}_{it} = \sigma_{it} - \sigma_{pt}$$

$$\tilde{\mu}_{it} = \mu_{it} - \mu - \sigma_{pt} - (\sigma + \sigma_{pt}) \tilde{\sigma}_{it}$$
Kolmogorov Forward Equation. I first characterize the dynamics of the wealth distribution in the model. Denote $\psi_t$ the wealth distribution of newborn agents.\textsuperscript{16}

**Proposition 3** (Kolmogorov Forward Equation with Aggregate Shocks). Denote $g_{jt}$ the density of relative wealth within each group of agent $j \in \{A, B\}$. The law of motion of $g_{jt}$ is given by

$$dg_{jt}(n) = -\partial_n(\bar{\mu}_{jt}ng_{jt}(n)dt + \bar{\sigma}_{jt}ng_{jt}(n)dZ_t) + \frac{1}{2} \partial_n^2(\bar{\sigma}_{jt}^2n^2g_{jt}(n))dt + \delta(\psi_t(n) - g_{jt}(n))dt$$

Denote $g_t$ the density of relative wealth. We have

$$g_t(n) = \pi_A g_{At}(n) + (1 - \pi_A)g_{Bt}(n)$$

Given $g_{jt}$, the wealth distribution within each group $j \in \{A, B\}$, and the evolution of individual wealth $(\bar{\mu}_{jt}dt, \bar{\sigma}_{jt}dZ_t)$, the Kolmogorov Forward equation gives the wealth distribution tomorrow $g_{j,t+dt}$. The law of motion of individual wealth $(\bar{\mu}_{jt}, \bar{\sigma}_{jt})$ determines the law of motion of the wealth distribution. In particular, because households choose different exposures to aggregate shocks, their relative wealth is stochastic, (i.e. $\bar{\sigma}_{jt} \neq 0$), and therefore the wealth distribution is stochastic.

Top Wealth Shares. The Kolmogorov Forward equation gives the law of motion of the wealth density. I now integrate the Kolmogorov Forward equation to characterize the dynamics of top wealth shares. This makes it easier to relate the model to the data, because I only observe the dynamics of top wealth shares over time. Let $\alpha$ a number between 0 and 1. Denote $q_t$ the $\alpha$-quantile, i.e.,

$$\alpha = \int_{q_t}^{+\infty} g_t(n)dn$$

\textsuperscript{16}It corresponds to the distribution of human capital. Denote $\psi$ the density for $\xi_1$ in (1.4). The expression for $\psi_t$ is:

$$\psi_t(n) = \frac{p_t}{\phi_t} \psi_{\phi_t}(\frac{p_t}{\phi_t}n)$$
and denote $T_t$ the wealth share of the top $\alpha$, i.e.,

$$T_t = \int_{q_t}^{+\infty} n g_t(n) dn$$

For instance, for $\alpha = 1\%$, $q_t$ is the wealth of an agent exactly at the 1% percentile of the distribution and $T_t$ is the wealth share of the top 1%.

The following proposition characterizes the dynamics of $T_t$:

**Proposition 4** (Dynamics of Top Wealth Shares). The law of motion of the top wealth share $T_t$ is

$$\frac{dT_t}{T_t} = \mu_{Tt} dt + \sigma_{Tt} dZ_t$$

where $\mu_{Tt}$ and $\sigma_{Tt}$ are given by

$$\sigma_{Tt} = \int_{q_t}^{+\infty} (\pi_A g_A t \tilde{\sigma}_A t + (1 - \pi_A) g_B t \tilde{\sigma}_B t) n dn$$

$$\mu_{Tt} = \int_{q_t}^{+\infty} (\pi_A g_A t \tilde{\mu}_A t + (1 - \pi_A) g_B t \tilde{\mu}_B t) n dn$$

$$- \delta (1 - \frac{\alpha q_t}{T_t}) + \frac{\delta}{T_t} \int_{q_t}^{+\infty} (n - q_t) \psi_t(n) dn + \frac{1}{2} \frac{q_t^2 g_t(q_t)}{T_t} \text{Var}_t[\tilde{\sigma}_t|n_{it} = q_t]$$

where $\mu_{Tt}$ and $\sigma_{Tt}$ are given by

The volatility of the top wealth share, $\sigma_{Tt}$, is the average, wealth-weighted, volatility of individuals in the top percentile.

The drift of the top wealth share, $\mu_{Tt}$, is the sum of four terms. The first term corresponds to the average drift of individuals at the top. The second term corresponds to the death of individuals at the top. The third term corresponds to the birth of individuals at the top. The last term is due to the heterogeneous exposure of households at the threshold.

I now give an heuristic derivation for the death term. During a short time period $dt$, a mass $\alpha \delta dt$ of households in the top percentile die, which decreases the total wealth in the top percentile by $T_t \delta dt$. Because the population size in the top percentile is held constant, an equal mass of households at the threshold enter the top percentile, with a wealth $q_t$. Therefore, the total change in top wealth share $T_t$ due to death is $-\delta dt(T_t - \alpha q_t)$. 

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I now give an heuristic derivation for the birth term. During a short time period $dt$, a mass $\int_{n_t}^{+\infty} \psi_t(n) dndt$ of households are born in the top percentile, which increases the total wealth in the top percentile by $\int_{n_t}^{+\infty} n\psi_t(n) dndt$. Because the population size in the top percentile is held constant, an equal mass of households at the threshold exit the top percentile, with a wealth $q_t$. Therefore, the total change in top wealth share $T_t$ due to birth is $\delta dt(\int_{n_t}^{+\infty} (n - q_t)\psi_t(n)dn)$.

The fourth term is a Ito correction term. The growth of top wealth shares is nonlinear in the aggregate shock $dZ_t$ due to a composition effect. When a negative shock hits the economy, top wealth shares decrease a little bit less than the wealth of households inside the top percentile, because some households from group $B$ enter the top. Conversely, when a positive shock hits the economy, top wealth shares increase a little bit more than the wealth of households inside the top percentile, because some households from group $A$ enter the top.

As seen in Section 1.2, because wealth is so concentrated, the impact of these fluctuations due to entry and exit is small, in the model as in the data.

The point I emphasize is that death is a key stabilizing force for top wealth shares. Because, in the calibrations below, agents at the top will grow faster than the rest of the economy (i.e. $\tilde{\mu}_{At} \geq 0$), a model without death would feature explosive dynamics for top wealth shares.

**Pareto Tail.** I cannot obtain a closed form characterization of the wealth distribution in the model because the wealth distribution is stochastic. To build intuition on the wealth distribution, I consider a special case of the model in which agents have the same risk aversion (i.e. $\gamma_A = \gamma_B$) but different EIS ($\psi_A \neq \psi_B$). In this case, the economy is essentially deterministic. I show that the wealth distribution is stationary, and that one can characterize its right tail.

The steady-state of this economy is characterized by a constant share of wealth owned by the agents in group $A$, i.e. $\mu_x(x_0) = 0$. Using the expression for $\mu_x(x_0)$ given in

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\footnote{This is true in the model, but also in the data. In Gomez (2016), I show that death is the only force pulling down the share of wealth owned by the top 400 individuals.}
Proposition 2, the steady state is characterized by

\[
0 = \tilde{\mu}_A(x_0) + \delta \left( \frac{\pi_A \phi(x_0)}{x_0 p(x_0)} - 1 \right) \tag{1.17}
\]

For the rest of the section, I assume that \(x_0 \in (0, 1)\) and \(\tilde{\mu}_A(x_0) > 0\).

**Proposition 5** (Pareto Tail). With \(\gamma_A = \gamma_B\), there is a stationary wealth distribution. It exhibits a Pareto tail, that is

\[
P(X \geq x) \sim Cx^{-\zeta_0} \text{ as } x \to +\infty
\]

where the power law exponent \(\zeta_0\) is given by

\[
\zeta_0 = \frac{\delta}{\tilde{\mu}_A(x_0)} \tag{1.18}
\]

as long as the distribution for human capital is less unequal than the wealth distribution\(^{18}\), i.e.,

\[
\lim_{n \to +\infty} \frac{\psi(n)}{n^{-\zeta-1}} = 0
\]

Moreover the fraction of agents that are of type \(B\) tends to zero in the right tail of the distribution.

Equation (1.18) says that the power law exponent of the wealth distribution \(\zeta_0\) is the ratio of the death rate of households, \(\delta\), to the relative wealth growth of the agents in group \(A\), \(\tilde{\mu}_A(x_0)\). This equation can be rewritten as a balance equation for top wealth shares:

\[
0 = \tilde{\mu}_A(x_0) - \frac{\delta}{\zeta_0}
\]

As pointed out in Proposition 4, top wealth shares increase because agents at the top grow faster than the rest of the economy \((\tilde{\mu}_A(x_0) \geq 0)\). On the other hand, top wealth shares are

\(^{18}\text{This is the case empirically. } \zeta \approx 1.5 \text{ for wealth while } \zeta \in (2, 3) \text{ for labor income.}\)
pulled down because agents at the top die and are replaced by households at the bottom threshold. For a Pareto distribution with tail $\zeta$, this force is exactly given by $-\delta/\zeta$.\(^{19}\)

The power law exponent of the wealth distribution does not depend on the distribution of human capital as long as the distribution for human capital has a thinner tail than the wealth distribution. This is the case empirically: the wealth distribution has a power law exponent of 1.5 while the distribution of labor income has a power law exponent between 2 and 3. Therefore, concentrating on the Pareto tail of the wealth distribution as a measure of wealth inequality allows to abstract from labor income inequality.\(^{20}\)

The proposition characterizes the right tail of the wealth distribution in the case where agents have the same risk aversion. What about the general case, where agents have different risk aversion? In the next section, I will show through simulations that the average wealth distribution also exhibits a Pareto tail. Moreover, its power law exponent is still close to $\delta/\bar{\mu}_A(x_0)$, where $x_0$ is the “stochastic” steady state. In short, the intuition for the power law exponent extends beyond the special case $\gamma_A = \gamma_B$.

One can combine the equation (1.18) for the power law exponent $\zeta_0$ and the equation (1.17) for the share of wealth owned by the agents in group $A$ $x_0$ to obtain:

$$
\zeta_0 = \frac{1}{1 - \frac{x_0}{\pi_A} \frac{\phi(x_0)}{p(x_0)}}
$$

(1.19)

This formula expresses the power law exponent $\zeta_0$ as a function of two terms. The first term, $\phi(x_0)/p(x_0)$, is the ratio of the average wealth of newborn agents to the average wealth of existing agents. The second term, $x_0/\pi_A$, is the ratio of the share of wealth held by the agents in group $A$ to their population share. This ratio measures the overall representation of the group $A$ in term of wealth. When agents in group $A$ overtake the economy, i.e. $x_0/\pi_A >> 1$, the Pareto tail thickens and the power law exponent $\zeta_0$ converges to an exponent of 1, i.e. Zipf’s law.

\(^{19}\)For a distribution with a Pareto tail, top wealth shares follow $T(\alpha) \sim \alpha^{1-\frac{\zeta}{\tilde{\mu}}}$ . Therefore, applying Proposition 4, the negative force due to death equals

$$
-\delta(1 - \frac{\alpha q(\alpha)}{T(\alpha)}) = -\delta(1 - \frac{\alpha T''(\alpha)}{T(\alpha)}) = -\frac{\delta}{\zeta}
$$

\(^{20}\)See also Gabaix et al. (2016).
1.4 Matching the Wealth Distribution and Asset Prices

I now bring the model to the data. I find that the model qualitatively generates the three stylized facts documented in Section 1.2. However, to generate volatile asset prices, the model requires a wealth distribution with a much thicker tail than the data.

1.4.1 Estimation Approach

Method. I estimate the parameters of the model by the simulated method of moments (SMM), which minimizes the distance between moments from real data and simulated data. I proceed as follows. I select a vector of moments \( m \) computed from the actual data. Given a candidate set of parameters \( \Theta \), I solve the model, and compute the moments \( \hat{m}(\Theta) \). I search the set of parameters \( \hat{\Theta} \) that minimizes the weighted deviation between the actual and simulated moments

\[
\hat{\Theta} = \arg \min_{\Theta} \{(m - \hat{m}(\Theta))^t W (m - \hat{m}(\Theta))\}
\]

where the weight matrix \( W \) adjusts for the fact that some moments are more precisely estimated than others.\(^{21}\) Details on the simulation method are given in Section 1.B.4.

I use the following set of moments.

Asset Prices Moments. I use four asset prices moments, corresponding to the average and standard deviation of the risk-free rate and of stock market returns, following Gârleanu and Panageas (2015). The data for the average equity premium, the volatility of returns, and the average interest rate are from Shiller (2015). The volatility of the real risk-free rate is inferred from the yields of 5-year constant maturity TIPS, as reported by Gârleanu and Panageas (2015).

Wealth Moments. I consider three moments about the wealth distribution. The first two moments capture the joint dynamics of the wealth distribution and asset returns, which

\[^{21}\text{I use as the weight matrix } W \text{ the variance covariance of the moments in the baseline calibration of the model.}\]
corresponds to the stylized facts of Section 1.2. The third moment captures the average shape of the wealth distribution. As explained in Section 1.3.3, I focus on the right tail of the distribution.

The first moment is the slope coefficient obtained by regressing the share of wealth owned by the top 0.01% on stock returns. The moment was estimated to be $\beta^{Exposure} = 0.35$ in Table 1.2.

The second moment is the slope coefficient obtained by regressing future excess returns on (standardized) log wealth shares. The moment was estimated to be $\beta^{Predictability} = -0.05$ in Table 1.3.

The third moment captures the Pareto tail of the wealth distribution. I use the slope coefficient in a regression of log percentile on log net worth for the households within the top 0.01%. Figure 1.3 plots the log percentile as a function of the log net worth for the U.S. distribution, in the SCF and in Forbes 400 data. The linear slope is characteristic of a distribution with a Pareto tail. I measure a power law exponent of $\zeta = 1.5$, consistent with previous studies.\textsuperscript{22}

**Calibrated Parameters.** The choice of calibrated parameters follows Gärleanu and Panageas (2015). The law of motion of the endowment process is $\mu = 2\%$ and $\sigma = 4.1\%$. The rate of death is $\delta = 2\%$. The share of endowment distributed as capital income is $1 - \omega = 8\%$. It corresponds to the share of total household income received as interest income or dividend income. The life cycle income of households $G(u)$ is a sum of two exponentials approximating the hump shaped pattern of earnings observed in the data:

$$G(u) = B_1 e^{-\delta_1 u} + B_2 e^{-\delta_2 u}$$

with $B_1 = 30.72$, $B_2 = -30.29$.

Following the approach of Barro (2006), I report the stock market returns for a firm with a debt-equity ratio equal to the historically observed debt-equity ratio for the U.S. non financial corporate sector.\textsuperscript{23}

\textsuperscript{22}See, for instance, Klass et al. (2006).

\textsuperscript{23}As Barro (2006), I choose a debt-equity ratio equal to $\lambda \approx 0.5$. 

29
**Estimated Parameters.** The model has 7 remaining parameters. 3 parameters correspond to the preference parameters of each households in group A \( (\rho_A, \gamma_A, \psi_A) \) and 3 parameters correspond to the preference parameters in group B \( (\rho_B, \gamma_B, \psi_B) \). The remaining parameter is the population share of the agents in group A, \( \pi_A \).

### 1.4.2 Estimation Results

For each estimation, I report the parameters and the moments in Table 1.4. I plot the equilibrium functions in Figure 1.4.

**Baseline** I first examine whether a model estimated exclusively on asset prices generates the relationship between asset returns and the wealth distribution measured in Section 1.2. To do so, I first report in Column (1) of Table 1.4 the original calibration of the model by Gârleanu and Panageas (2015), which exclusively targets asset price moments. Qualitatively, the model generates the two stylized facts described in Section 1.2.

First, households in the top percentile are disproportionately invest in the stock market. Therefore, following large stock returns, top wealth shares increase. The model predicts that the exposure of top wealth shares \( \beta^{\text{exposure}} \) to the stock market is 0.50, while it is slightly lower, 0.35, in the data.

Second, when a larger share of wealth falls into the hands of risk-tolerant households, the aggregate demand for risk increases, which lowers risk premia and pushes up asset prices. Thus, a higher top wealth share predicts lower future excess returns. The model yields a predictive coefficient \( \beta^{\text{Predictability}} \) equal to \(-0.03\) while it is \(-0.05\) in the data, well into the standard error in Table 1.3. Therefore, the calibrated model appears to be consistent with the mechanism outlined in Section 1.2.

Does the model also generate a realistic level of inequality? Figure 1.3 compares the log-log relationship between top percentiles and financial wealth in the simulated model and in the data. The red curve represents the wealth distribution in the model. The curve is approximately linear. Therefore, the wealth distribution in the model has approximately a Pareto tail, as discussed in Section 1.3. However, the distribution has a much thicker tail
than in the data. I estimate $\zeta$ equal to 1.0 in the model compared to $\zeta$ equal to 1.5 in the data. The model overestimates the level of inequality.

**Re-estimating the Model** In Column (2), I therefore re-estimate the model, targeting jointly asset returns and the wealth moments. The model now generates a realistic Pareto tail, with a power exponent of 1.4. The exposure of top wealth shares is also lower, equal to 0.38, as in the data. However, the model now misses asset prices. The model completely underestimates the volatility of returns and the equity premium. The volatility of returns in the model, 10.5%, is roughly half of the volatility of returns in the data, 18.2%. Similarly, the equity premium in the model, 2.8%, is roughly half of the equity premium in the data, 5.2%. These two failures suggest a tension between matching the volatility of asset returns and the level of wealth inequality in the data.

To emphasize the conflict between these two moments, Column (3) re-estimates the model, adding exclusively the Pareto tail of the distribution to asset price moments. Similarly to the previous results, the model broadly fits the other distributional moment and the properties of the risk-free rate. However, it still underestimates the volatility of returns, 11.1%, and the equity premium, 2.3%.

**Trade-off between Volatility of Returns and the Tail of the Distribution.** The previous results suggest a trade-off between the volatility of stock returns and the Pareto tail of the distribution. To better understand this trade-off, I examine more precisely the mechanism generating volatile stock returns in the model. Applying Ito’s lemma, the volatility of returns can be written

$$\sigma_R = \sigma + \frac{\partial \log P/D}{\partial \log x} \sigma_x$$

(1.21)

To generate the high volatility of returns, the model must have either large fluctuations in the share of wealth owned by agents $A$, $\sigma_x$, or a large elasticity of the price-dividend ratio to the share of wealth owned by agent $A$, $\frac{\partial \log P/D}{\partial \log x}$. Heterogeneity in preferences may increase the volatility of stock returns by impacting these two terms.
Column (4) re-estimates the model using asset price moments together with the exposure of top wealth shares to stock returns. The latter moment limits the heterogeneity in risk aversion between the two groups.\textsuperscript{24} Still, the model can generate volatile stock returns, with $\sigma_R$ equal to $17.6\%$. Because the supplementary moment decreases the volatility of $x$, $\sigma_x$, relative to the baseline calibration, the model, in order to generate volatile returns, must feature a high elasticity of the price-dividend ratio to $x$, $\frac{\partial \log P/D}{\partial \log x}$. The high elasticity of the price-dividend ratio is obtained by a high heterogeneity in EIS: the estimation yields $\psi_A/\psi_B = 50$. The agents in group $A$ have a much higher propensity to save compared to the households in group $B$. Because of these large differences in saving decisions, the tail of the distribution is much thicker than the data, with a power law exponent of 1.2.

In contrast, Column (5) re-estimates the model by imposing a lower bound on the EIS $\psi_B \geq 0.2$.\textsuperscript{25} This constraint limits the heterogeneity in EIS between the two groups. Still, the model can generate volatile returns, with $\sigma_R$ equal to $17.4\%$. Because the new constraint decreases $\frac{\partial \log P/D}{\partial \log x}$ relative to the baseline calibration, the model, in order to generate volatile returns, must feature a high volatility of $x$, $\sigma_x$. The high volatility $\sigma_x$ is obtained by a high heterogeneity in risk aversion: the estimation yields $\gamma_B/\gamma_A = 50$. This heterogeneity generates large differences in the demand for risky assets between $A$ and $B$, yielding a counter-factually high exposure of top wealth shares to stock returns (1.9, compared to 0.35 in the data). Moreover, the agents in group $A$ earn much higher returns on their wealth compared to the agents in group $B$. Because of these large differences in investment returns, the tail of the distribution is much thicker than the data, with a power law exponent of 1.0.

The last two estimations show that a high degree of preference heterogeneity, whether it comes from differences in EIS or from differences in risk aversion, generates too much wealth inequality compared to what is found in the data. Intuitively, large differences in preferences imply substantial and permanent differences between the wealth growth of the agents in the economy. In the long-run, these differences in growth rates contribute to a thicker right tail of the wealth distribution, as shown in Proposition 5. To generate volatile stock returns, the

\textsuperscript{24}If risk aversion differs too much between the two groups, the rich and the poor differ too much in their risk exposure (Equation (1.11)), and therefore top wealth shares move too much with stock returns (Equation (1.13)).

\textsuperscript{25}This lower bound corresponds to the empirical results of Vissing-Jørgensen (2002a).
model requires a degree of preference heterogeneity so large that the wealth distribution has a power law exponent very close to one. This corresponds to the thickest tail accommodated by the model, Zipf’s law.

1.5 The Role of Heterogeneity in Investment Opportunities

The previous section shows that there is tension in the standard heterogeneous agents model between asset prices and the wealth distribution. The high degree of heterogeneity necessary to generate large fluctuations in asset prices implies more inequality than there is in the data.

I now suggest a parsimonious deviation from the standard model to resolve this tension. I consider the impact of low-frequency changes in the investment opportunities of the rich relative to the poor. These shocks create low-frequency fluctuations in wealth inequality, thereby increasing fluctuations in asset prices. At the same time, since these shocks average to zero, they do not increase the average wealth growth of households at the top. Therefore, these shocks help resolve the tension present in the standard model.

Section 1.5.1 presents the augmented model. Section 1.5.2 estimates the model on asset prices and the wealth distribution. Section 1.5.3 examines additional predictions of the model.

1.5.1 Augmented Model

I now consider a parsimonious departure from the heterogeneous agents model presented in Section 1.4.

In particular, I examine a process $\nu_t$, which generates differences between the financial returns available to the agents in group $A$ and the financial returns available to the agents in group $B$. I introduce these shocks in a way that does not affect the aggregate endowment, i.e. they are purely redistributive. Specifically, I assume that the financial return available to the agents in group $j \in \{A, B\}$ is increased by a group specific term $\nu_{jt}$, i.e. that the budget constraint of an agent in group $j \in \{A, B\}$ becomes

$$
\mu_{jt} = r_t + \delta + \nu_{jt} + \kappa \sigma_{jt} - c_{jt} \quad (1.22)
$$
\( \nu_{At} \) and \( \nu_{Bt} \) are chosen so that the difference between \( \nu_{At} \) and \( \nu_{Bt} \) equals \( \nu_t \) (\( \nu_{At} - \nu_{Bt} = \nu_t \)) and so that the total return on wealth in the economy is left unchanged (\( x_t \nu_{At} + (1-x_t) \nu_{Bt} = 0 \)), that is

\[
\begin{align*}
\nu_{At} &= (1-x_t)\nu_t \\
\nu_{Bt} &= -x_t\nu_t
\end{align*}
\]

While I do not take a stand on the origin of these differential investment opportunities \( \nu_t \), the literature suggests some potential origins. These fluctuations could be generated by changes in technology (Gârleanu, Kogan, and Panageas (2012), Kogan, Papanikolaou, and Stoffman (2013)), changes in financial frictions (Kiyotaki and Moore (1997)), or changes in taxes (Piketty and Zucman (2014), Pastor and Veronesi (2016)). For instance, the following tax policy would exactly generate the specification in my model: the government levies a wealth tax \( -\nu_t \) on agents in group A and redistribute the proceeds to all agents in proportion to their wealth.

I assume that \( \nu_t \) is a mean reverting process which fluctuates around zero. More specifically, its law of motion is

\[
d\nu_t = -\kappa_{\nu}\nu_t dt + \sigma_{\nu} dZ_t
\]

where \( \kappa_{\nu} \) is the mean reversion parameter and \( \sigma_{\nu} \) is the exposure to aggregate shocks.\(^{26}\)

When \( \sigma_{\nu} = 0 \), the model reverts to the baseline model in Section 1.3.

For the agents in group \( j \in \{A, B\} \), fluctuations in \( \nu_{jt} \) have the same effects as fluctuations in the risk-free rate. Hence, the law of motion of their wealth is is the same as in baseline model, after substituting \( r_t - \nu_{jt} \) for the risk-free rate.

**Proposition 6** (Law of Motion for Households Wealth with Fluctuating \( \nu_t \)). Denote \( p_{jt} \) the wealth consumption ratio of each agent, i.e. \( p_{jt} = 1/c_{jt} \). The wealth of households in group

\(^{26}\)In particular I assume that the same shocks drive the aggregate endowment \( Y_t \) and \( \nu_t \) in the economy. This assumption simplifies the presentation. The model can also match asset prices and the wealth distribution when fluctuations in \( \nu_t \) are uncorrelated with aggregate endowment shocks.
\[ j \in \{A, B\} \text{ follows the law of motion} \]

\[ \sigma_{jt} = \frac{\kappa_t}{\gamma_j} + \frac{1 - \gamma_j}{\gamma_j(\psi_j - 1)} \sigma_{pjt} \]  

\[ \mu_{jt} = \psi_j(r_t + \nu_{jt} - \rho) + \frac{1 + \psi_j}{2\gamma_j} \kappa_t^2 + \frac{1 - \gamma_j}{\gamma_j(\psi_j - 1)} \kappa_t \sigma_{pjt} + \frac{1 - \gamma_j \psi_j}{2(\psi_j - 1) \gamma_j} \sigma_{pjt}^2 + \mu_{pjt} \]  

The direct impact of an increase in \( \nu_{jt} \) on \( \mu_{jt} \), the wealth growth of the agents in group \( j \in \{A, B\} \), is given by \( \psi_j \nu_{jt} \). It is the sum of a mechanical increase of their wealth growth (\( \nu_{jt} \)) and of an adjustment in their consumption rate ((\( \psi_j - 1 \nu_{jt} \)).

In equilibrium, fluctuations in \( \nu_t \) generate fluctuations in the price-dividend ratio. First, because the agents have different EISs (\( \psi_A \neq \psi_B \)), a rise in \( \nu_t \) increases the aggregate demand for assets, which pushes up the price-dividend ratio. This is true even though fluctuations in \( \nu_t \) are purely redistributive (\( x_{\nu At} + (1 - x)_{\nu Bt} = 0 \)). Second, a rise in \( \nu_t \) also increases the relative growth rate of the agents in group \( A \), and is therefore associated with an increase in \( x_t \), the share of wealth owned by the agents in group \( A \). This further pushes up asset prices.

Stock returns react both to news about the level of inequality \( x_t \) and to news about the future growth of inequality \( \nu_t \):

\[ \sigma_R = \sigma + \frac{\partial \log pd}{\partial \log x} \sigma_x + \frac{\partial \log pd}{\partial \log \nu} \sigma_\nu \]  

In the baseline model, there is a tension between the high volatility of returns \( \sigma_R \) and the average tail of the wealth distribution. The first requires a high degree of preference heterogeneity while the second is associated with a low degree of preference heterogeneity. In the augmented model, fluctuations in \( \nu_t \) generate additional fluctuations in stock returns, through the additional term \( \frac{\partial \log pd}{\partial \log \nu} \sigma_\nu \) in (1.26). These fluctuations do not change the average tail of the wealth distribution, because they do not change the average value of \( \mu_{At} \). Therefore, fluctuations in \( \nu_t \) help resolve the central tension at the baseline model.

To solve the augmented model, I look for a Markov equilibrium with two state variables \((x_t, \nu_t)\). The law of motion for \( x_t \) is the same as Proposition 2 while the law of motion for \( \nu_t \) is exogenously given by (1.23). Given the law of motion for \( x_t, \nu_t \), one can express the drift
and the volatility of all processes through Ito’s lemma and proceed similarly to the baseline model. I discuss more precisely the solution method in Section 1.B.2.

1.5.2 Estimation Results

I now estimate the model to assess whether it can qualitatively match the stylized facts presented in Section 1.2, but also whether it can generate asset prices and a wealth distribution consistent with the data.

**Parameters.** The model has two new parameters compared to the baseline model, the persistence $\kappa_\nu$ and the volatility $\sigma_\nu$ of the process $\nu_t$.

**Moments.** I introduce a new moment to discipline the law of motion of $\nu_t$. As seen in Proposition 6, fluctuations in $\nu_t$ generate fluctuations in the wealth growth of top households in excess of the observable fluctuations in asset returns.

To capture these fluctuations, I examine, in the data and in the model, the residuals in the regression of top wealth shares on asset returns:

$$\log \text{Wealth Share Top 0.01}\%_{t+1} = \alpha + \rho \log \text{Wealth Share Top 0.01}\%_t + \beta (r_{t+1}^M - r_{t+1}^f) + \gamma r_{t+1}^f + \epsilon_{t+1}$$

(1.27)

Intuitively, low-frequency fluctuations in these residuals help capture fluctuations in $\nu_t$ (Proposition 6). Specifically, I add as a new moment the standard deviation of a five-year moving average of these residuals:

$$\text{std} \left( \sum_{1 \leq i \leq 5} \epsilon_{t+1}/5 \right) = 2.1\%$$

**Results.** Figure 1.5 plots asset prices as a function of the two state variables $x$ and $\nu$. Corresponding to the intuition put forth earlier, a high difference in the investment opportunities of the agents in group $A$ compared to the agents in group $B$, $\nu$, is associated

27I use the five year averages to smooth out the yearly fluctuations of top wealth shares in the data (which also include measurement errors, etc). Averaging residuals at a longer horizon allows to concentrate on the low-frequency fluctuations driven by $\nu_t$. An alternative would be to estimate a state space model.
with a low interest rate (Figure 1.5b) and also with a high drift of households in group A relative to other agents (Figure 1.5e). For both of these reasons, a high \( \nu \) is associated with a high price-dividend ratio (Figure 1.4c).

The last column of Table 1.4 demonstrates that the augmented model can jointly match asset prices and the wealth distribution. In particular, the model can generate a high volatility of returns \( \sigma_R = 17.2\% \) together with a low power law exponent \( \zeta = 1.5 \). In contrast, the baseline model could not jointly match these moments. In the baseline model, there was a tension between the high volatility of returns \( \sigma_R \) and the average tail of the wealth distribution \( \zeta \). The first required a high degree of preference heterogeneity while the second required a low degree of preference heterogeneity.

The augmented model solves this tension through fluctuations in \( \nu \). On the one hand, these fluctuations increase the volatility of returns. Applying the decomposition (1.26), I obtain that 45% of the fluctuations in the price-dividend ratio are driven by fluctuations in the level of inequality \( x_t \), while 55% are driven by fluctuations in \( \nu_t \), the relative investment opportunities of the agents in group A compared to the agents in group B. On the other hand, these fluctuations do not impact the average level of inequality, because they do not change the average wealth growth of top households \( \mu_{At} \). Therefore, fluctuations in \( \nu_t \) allow the model to match a higher volatility of stock returns \( \sigma_R \) together with a lower power law exponent \( \zeta \).

1.5.3 Further evidence

I now examine whether the augmented model can explain additional dimensions of the data.

The Price-dividend Ratio and the Growth of Wealth Inequality. I start by examining the relationship between the price-dividend ratio and the future growth of wealth inequality.

On the one hand, in the model, fluctuations in \( x_t \), the share of wealth owned by the households in group A, generate a negative comovement between the price-dividend ratio and the future growth of wealth inequality. The intuition is as follows. When \( x_t \) is high, the aggregate demand for assets is high. In equilibrium, the price-dividend ratio is high and
future returns are low. These low returns decrease the growth rate of the agents in group A relative to other agents, and therefore, wealth inequality slowly decreases.

On the other hand, fluctuations in \( \nu_t \) generate a positive comovement between the price-dividend ratio and the future growth of wealth inequality. A rise in \( \nu_t \) simultaneously increases the relative wealth growth of households in group A (Figure 1.4e) and the price-dividend ratio (Figure 1.5c).

Therefore, the correlation between price-dividend ratio and the future growth of inequality depends on the relative importance of fluctuations in \( x_t \) and \( \nu_t \). Examining this relationship offers both a qualitative and quantitative test for the augmented model. To do so, I regress the future growth of the share of wealth owned by the top 0.01% on the price-dividend ratio, i.e.

\[
\log \frac{\text{Wealth Share Top 0.01\%}_{t+4}}{\text{Wealth Share Top 0.01\%}_{t+1}} = \alpha + \beta \log \frac{P}{D_t} + \epsilon_t
\]  \hspace{1cm} (1.28)

Table 1.5 reports the result of this regression. In the data, I obtain an estimate for \( \beta \) equal to 0.03 with estate tax returns and equal to 0.11 with income tax returns: the price-dividend ratio tends to forecast positively the future growth of wealth inequality. In simulated data from the augmented model, I obtain a similar positive estimate for \( \beta \) equal to 0.12. In contrast, in the baseline model of Section 1.4, I obtain a strongly negative estimate for \( \beta \) equal to \(-0.10\): the price-dividend ratio forecasts negatively the future growth of wealth inequality. This is because fluctuations in \( x_t \) can only generate a negative relationship between the price dividend and the future growth of inequality. One needs fluctuations in \( \nu_t \) to explain the positive comovement between the price-dividend ratio and the future growth of wealth inequality.

**Episodes of Disconnect.** An additional way to relate the model to the data is to compare the time series of top wealth shares and a running sum of lagged asset returns.

In the baseline model, the differences between the two series are small: fluctuations in top wealth shares are entirely driven by fluctuations in past asset returns. In contrast, in
the augmented model, fluctuations in \( \nu_t \) generate periods of disconnect between top wealth shares and past asset returns (Proposition 6).

Figure 1.6 measures the difference between the two series in the data by comparing the evolution of the share of wealth owned the top 0.01\% and the predicted values from the regression (1.27).

The figure shows large and persistent fluctuations in top wealth shares that cannot be explained by asset returns. In particular, asset returns alone cannot explain the long term decline in inequality in the 1930s. Symmetrically, asset returns cannot explain the increase of inequality after the 2000s, a period of low interest rate and low stock returns. These periods are inconsistent with the baseline model. In the augmented model however, the latter period could be rationalized by a high \( \nu_t \) — a rise in \( \nu_t \) jointly increases wealth inequality and decreases asset returns.

1.6 Conclusion

The results of this paper depict a strong interplay between asset prices and wealth inequality. Because rich households hold more risky assets, realized stock returns generate fluctuations in wealth inequality over time. Conversely, in periods of high inequality, more wealth is in the hands of rich households, the risk-tolerant investors. Therefore, risk premia are low: a high level of inequality predicts low future returns. This interplay is at the heart of heterogeneous agents asset pricing models. I have shown that these models can qualitatively account for these facts. However, the standard models tend to overestimate the thickness of the tail of the wealth distribution.

This difficulty suggests that, while important, heterogeneity in preferences is not sufficient to understand the interplay of inequality and prices. Differences in investment opportunities offer a direction for progress. Augmenting standard models with this feature allows to simultaneously explain the volatility of asset prices and the level of inequality. Further, this approach provides an explanation for temporary disconnects between inequality and asset prices.
To make this point clearly, I use a parsimonious representation of these shocks. But it appears important to go further in understanding the precise source of these differences. The literature suggests promising avenues to answer this question: embodied capital shocks (Papanikolaou (2011), Gârleanu, Kogan, and Panageas (2012)), changes in the capital share (Karabarbounis and Neiman (2014), Lettau, Ludvigson, and Ma (2016)), or taxes and regulation (Lampman (1962)).

The implications of my analysis extend beyond asset pricing. The interplay I put forward can have effects on real quantities as well, through two channels. First, because the level of inequality affects the cost of capital, this can lead to changes in corporate investment policies. Second, a recent literature has also emphasized the role of inequality for aggregate demand (Mian, Rao, and Sufi (2013), G. Kaplan, Moll, and Violante (2016)). Exploring these channels requires moving away from an endowment economy, which I leave for future research.
Table 1.1: The Equity Share Increases Across the Wealth Distribution

<table>
<thead>
<tr>
<th>Groups of Households Defined by Wealth Percentiles</th>
<th>All</th>
<th>1% − 0.1%</th>
<th>0.1% − 0.01%</th>
<th>Top 0.01%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: All Sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity Share</td>
<td>40.8%</td>
<td>55.8%</td>
<td>65.8%</td>
<td>73.9%</td>
</tr>
<tr>
<td>Public Equity</td>
<td>20.2%</td>
<td>22.0%</td>
<td>21.1%</td>
<td>19.6%</td>
</tr>
<tr>
<td>Private Equity</td>
<td>20.6%</td>
<td>33.9%</td>
<td>44.6%</td>
<td>54.4%</td>
</tr>
<tr>
<td>Non Actively Managed</td>
<td>2.4%</td>
<td>4.5%</td>
<td>6.3%</td>
<td>7.8%</td>
</tr>
<tr>
<td>Actively Managed</td>
<td>18.2%</td>
<td>29.4%</td>
<td>38.4%</td>
<td>46.6%</td>
</tr>
<tr>
<td><strong>Panel B: Stockholders</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Is Stockholder</td>
<td>45.9%</td>
<td>90.7%</td>
<td>91.2%</td>
<td>91.0%</td>
</tr>
<tr>
<td>Equity Share among Stockholders</td>
<td>44.7%</td>
<td>56.0%</td>
<td>65.9%</td>
<td>76.0%</td>
</tr>
<tr>
<td><strong>Panel C: Entrepreneurs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Is Entrepreneur</td>
<td>10.5%</td>
<td>62.1%</td>
<td>69.8%</td>
<td>78.5%</td>
</tr>
<tr>
<td>Equity Share among non-Entrepreneurs</td>
<td>26.8%</td>
<td>40.7%</td>
<td>50.0%</td>
<td>57.9%</td>
</tr>
<tr>
<td><strong>Panel D: Stock Options Holders</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Received Stock Options</td>
<td>6.4%</td>
<td>11.2%</td>
<td>11.5%</td>
<td>6.1%</td>
</tr>
<tr>
<td>Equity Share among non Stock Options Holders</td>
<td>44.7%</td>
<td>56.0%</td>
<td>65.9%</td>
<td>76.0%</td>
</tr>
<tr>
<td>Share of Total Wealth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor Income / Wealth</td>
<td>12.6%</td>
<td>2.9%</td>
<td>1.6%</td>
<td>0.7%</td>
</tr>
</tbody>
</table>

Notes. Data from SCF 1989-2013. The variable Equity Share is defined as private equity + public equity over networth: \( \frac{\text{equity} + \text{bus}}{\text{networth}} \). Stockholders are defined as the households that hold public equity. Entrepreneurs are defined as the households with an active management role in one of the company they invest in.
Table 1.2: The Exposure to Stock Returns Increases Across the Wealth Distribution

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Panel A: Wealth</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excess Stock Returns</td>
<td>0.44***</td>
<td>0.52***</td>
<td>0.66***</td>
</tr>
<tr>
<td>Risk Free Rate</td>
<td>0.31</td>
<td>0.61</td>
<td>1.33**</td>
</tr>
<tr>
<td>R²</td>
<td>0.49</td>
<td>0.45</td>
<td>0.58</td>
</tr>
<tr>
<td>N</td>
<td>54</td>
<td>54</td>
<td>54</td>
</tr>
<tr>
<td>Panel B: Wealth Shares</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excess Stock Returns</td>
<td>0.09*</td>
<td>0.22***</td>
<td>0.31**</td>
</tr>
<tr>
<td>Risk Free Rate</td>
<td>−0.26*</td>
<td>0.16</td>
<td>−0.05</td>
</tr>
<tr>
<td>R²</td>
<td>0.20</td>
<td>0.33</td>
<td>0.14</td>
</tr>
<tr>
<td>N</td>
<td>54</td>
<td>54</td>
<td>54</td>
</tr>
</tbody>
</table>

Notes. The table reports the results of the regression of the wealth growth of households in a given percentile group on the excess stock returns and the risk free rate, i.e. Equation (1.1):

\[
\log \frac{W_{p,t+1} - W_{p,t}}{W_{p,t-1} - W_{p,t}} = \alpha + \beta (r^M_{t} - r^f_{t}) + \gamma r^f_{t} + \epsilon_t
\]

The dependent variable is the growth of wealth in Panel A and the growth of wealth shares in Panel B. To avoid overlapping time periods between the regressor and the dependent variable, the timing is as follows:

\[
\begin{align*}
\text{Top Wealth}_{t-1} & \quad r^M_t - r^f_t \quad \text{Top Wealth}_{t+1} \\
\text{t} - 1 & \quad t \quad t + 1 \quad \text{Year} 
\end{align*}
\]

Each column corresponds to a different group of households. The first column corresponds to all U.S households; to measure the wealth of U.S. households, I use data from the Financial Accounts of the United States (Flow of Funds) after 1945. For the period before 1945, I use Kopczuk and Saez (2004). The second to fourth columns correspond to increasing top percentiles in the wealth distribution, using data from Kopczuk and Saez (2004). Columns 5 to 6 correspond to the Top 0.0003% and 0.00008%; the percentiles are chosen so that the group include the 400 wealthiest individuals and the 100 individuals in 2014. Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *, **, *** indicate significance at the 0.1, 0.05, 0.01 levels, respectively.
Table 1.3: The Share of Wealth Owned by the Top 0.01% Predicts Future Excess Returns

\[ \sum_{1 \leq h \leq H} r_{t+h}^M - r_{t+h}^f = \alpha + \beta H \text{Log Top Wealth Shares}_t + \gamma_H \text{Predictor}_t + \epsilon_H \]

<table>
<thead>
<tr>
<th></th>
<th>( \beta_1 )</th>
<th>( \gamma_1 )</th>
<th>Adjusted ( R^2 )</th>
<th>( \beta_3 )</th>
<th>( \gamma_3 )</th>
<th>Adjusted ( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Top Share</td>
<td>-0.053**</td>
<td>-0.057**</td>
<td>0.051</td>
<td>-0.114**</td>
<td>-0.198***</td>
<td>0.196***</td>
</tr>
<tr>
<td>Dividend Price</td>
<td>-0.077***</td>
<td>0.056*</td>
<td>0.089</td>
<td>-0.198***</td>
<td>0.196***</td>
<td>0.265</td>
</tr>
<tr>
<td>cay</td>
<td>-0.044*</td>
<td>0.046*</td>
<td>0.072</td>
<td>-0.106*</td>
<td>0.114**</td>
<td>0.153</td>
</tr>
<tr>
<td>Dividend Payout</td>
<td>-0.066**</td>
<td>0.031</td>
<td>0.054</td>
<td>-0.153**</td>
<td>0.094</td>
<td>0.113</td>
</tr>
<tr>
<td>Long Term Yield</td>
<td>-0.066**</td>
<td>-0.034</td>
<td>0.053</td>
<td>-0.128**</td>
<td>-0.055</td>
<td>0.062</td>
</tr>
<tr>
<td>Default Yield Spread</td>
<td>-0.068**</td>
<td>0.031</td>
<td>0.047</td>
<td>-0.179***</td>
<td>0.146**</td>
<td>0.156</td>
</tr>
<tr>
<td>Treasury Bill Rate</td>
<td>-0.054*</td>
<td>-0.037</td>
<td>0.040</td>
<td>-0.127**</td>
<td>-0.081</td>
<td>0.080</td>
</tr>
<tr>
<td>Stock Variance</td>
<td>-0.058**</td>
<td>0.02</td>
<td>0.045</td>
<td>-0.129**</td>
<td>0.055</td>
<td>0.084</td>
</tr>
<tr>
<td>Inflation</td>
<td>-0.054**</td>
<td>-0.013</td>
<td>0.039</td>
<td>-0.116**</td>
<td>-0.021</td>
<td>0.066</td>
</tr>
<tr>
<td>Default Return Spread</td>
<td>-0.058**</td>
<td>-0.001</td>
<td>0.035</td>
<td>-0.161***</td>
<td>0.014</td>
<td>0.130</td>
</tr>
<tr>
<td>Term Spread</td>
<td>-0.037</td>
<td>0.019</td>
<td>0.017</td>
<td>-0.079</td>
<td>0.066</td>
<td>0.065</td>
</tr>
<tr>
<td>Linear Trend</td>
<td>-0.112***</td>
<td>-0.075*</td>
<td>0.083</td>
<td>-0.254***</td>
<td>-0.178*</td>
<td>0.144</td>
</tr>
</tbody>
</table>

Notes. The table reports the result of the regressions of future excess returns on the share of wealth owned by the Top 0.1% (row 1), along with other predictors from the literature (row 2-11). Each row corresponds to a different set of regressors. Columns (1) (2) (3) report the results when the dependent variable is the one year excess return. Columns (4) (5) (6) report the results when the dependent variable is the three-year excess return. I construct cay in the period 1917-1999 by mirroring the construction in Lettau and Ludvigson (2001) on historical data: wage income from Piketty and Saez (2003), consumption from Shiller (2015), financial wealth from Kopczuk and Saez (2004). All other predictors come from Welch and Goyal (2008). To facilitate the comparison between the different predictors, all regressors are normalized to have a standard deviation of one. Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *, **, *** indicate significance at the 0.1, 0.05, 0.01 levels.
Table 1.4: Matching the Wealth Distribution and Asset Prices

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
<th>Data</th>
<th>Baseline Model Estimated on Asset Prices and...</th>
<th>Augm. Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2) (3) (4) (5) (6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>({}</td>
<td>{} βexp)</td>
<td></td>
</tr>
<tr>
<td>Parameters</td>
<td></td>
<td>(Wealth Dist)</td>
<td>(βExp)</td>
<td>(ψ ≥ 0.2)</td>
</tr>
<tr>
<td>Risk aversion of type-A agents $γ_A$</td>
<td>1.5</td>
<td>2.2</td>
<td>1.6</td>
<td>2.7</td>
</tr>
<tr>
<td>Risk aversion of type-B agents $γ_B$</td>
<td>10.0</td>
<td>13.0</td>
<td>9.0</td>
<td>8.8</td>
</tr>
<tr>
<td>EIS of type-A agents $ψ_A$</td>
<td>0.7</td>
<td>0.4</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>EIS of type-B agents $ψ_B$</td>
<td>0.05</td>
<td>0.01</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>Discount rate $ρ_A$</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Discount rate $ρ_B$</td>
<td>$ρ_A$</td>
<td>5%</td>
<td>1.5%</td>
<td>5%</td>
</tr>
<tr>
<td>Population share $π_A$</td>
<td></td>
<td>1%</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>Persistence $κ_ν$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volatility $σ_ν$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asset Prices</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity Premium</td>
<td></td>
<td>5.2%</td>
<td>5.3%</td>
<td>2.8%</td>
</tr>
<tr>
<td>Volatility of returns</td>
<td></td>
<td>18.2%</td>
<td>19.0%</td>
<td>10.5% 11.1%</td>
</tr>
<tr>
<td>Average interest rate $r$</td>
<td></td>
<td>2.8%</td>
<td>1.2%</td>
<td>2.9%</td>
</tr>
<tr>
<td>Standard deviation interest rate $r$</td>
<td>0.92%</td>
<td>0.4%</td>
<td>1.1%</td>
<td>0.7%</td>
</tr>
<tr>
<td>Wealth Distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exposure Top Wealth Shares</td>
<td>β Exposure†</td>
<td>0.35</td>
<td>0.50</td>
<td>0.38</td>
</tr>
<tr>
<td>Predictability Regression</td>
<td>β Predictability‡</td>
<td>−0.05</td>
<td>−0.03</td>
<td>−0.03</td>
</tr>
<tr>
<td>Power Law Exponent $ζ$</td>
<td></td>
<td>1.5</td>
<td>1.1</td>
<td>1.4</td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard dev. Residuals (5-year horizon)</td>
<td>2.1%</td>
<td>0.8%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. Columns (1) to (5) correspond to different estimations of the baseline model presented in Section 1.3. All estimations include asset price moments but differ with respect to the choice of other moments. Column (1) reports the model estimated on asset prices only. Column (2) reports the model estimated on asset prices and the wealth distribution. Column (3) reports the model estimated on asset prices and the power law exponent $ζ$. Column (4) reports the model estimated on asset prices and the exposure of top wealth shares $β^{Exposure}$. Column (5) reports the model estimated an asset prices with a lower bound on $ψ_B$. Column (6) reports the augmented of Section 1.5 estimated on asset prices, the wealth distribution, and a moment corresponding to the long run standard deviation of the residuals of the regression (1.27). Moments in bold and in red highlight the dimensions of the data that are missed by the model.

† $β^{Exposure}$ is the coefficient obtained by regressing the growth of the share of wealth owned by the top 0.01% on stock returns (Table 1.2).
‡ $β^{Predictability}$ is the coefficient obtained by regressing the future excess stock returns on the log of the share of wealth owned by the top 0.01%, normalized to have a standard deviation of one (Table 1.3).
Table 1.5: The Price-Dividend Ratio and the Future Growth of the Wealth Share of the Top 0.01%

<table>
<thead>
<tr>
<th></th>
<th>Future Growth of the Wealth Share of the Top 0.01%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estate Tax</td>
</tr>
<tr>
<td>Price-Dividend Ratio (log)</td>
<td>0.03 (0.07)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes. The table reports the results of the regression of the future growth of the wealth share of the top 0.01% on the price-dividend ratio, i.e. Equation (1.28):

$\log \frac{\text{Wealth Share Top 0.01\%}_{t+4}}{\text{Wealth Share Top 0.01\%}_{t+1}} = \alpha + \beta \log P/D_t + \epsilon_t$

Each column corresponds to a different dataset. Column (1) corresponds to the wealth share of the Top 0.01% according to Estate Tax Returns (Kopczuk and Saez (2004)). Column (2) corresponds to the wealth share of the top 0.01% according to Income Tax Returns (Saez and Zucman (2016)). Column (3) corresponds to simulated data from the baseline model (parameters in Column (1) of Table 1.4). Column (4) corresponds to simulated data from the augmented model (parameters in Column (6) of Table 1.4).

Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *, **, *** indicate significance at the 0.1, 0.05, 0.01 levels.
Figure 1.1: The Equity Share Increases Across the Wealth Distribution

(a) Top Percentiles Linearly Spaced  
(b) Top Percentiles Log-linearly Spaced

Notes. Figure 1.1a plots the average equity share within 20 linearly spaced percentile bins in the wealth distribution. Figure 1.1b plots the average equity share within 20 logarithmically spaced percentile bins in the wealth distribution. The horizontal line represents the average equity share. The vertical line splits the set of households in two: households on either side of the vertical line own half of total wealth (this corresponds to top percentile ≈ 3%). All averages are wealth-weighted.

Data from the Survey of Consumer Finance (SCF), a cross sectional survey of US households from 1989 to 2013. The equity share is constructed as $(equity + bus) / \text{networth}$. 
Figure 1.2: The Wealth Share of the Top 0.01% and Average Excess Returns

Notes. The figure plots the wealth share of the top 0.01% (log) and the 8-year sum of future excess returns (opposite of). All series are normalized to have a standard deviation of one.

Figure 1.3: The Pareto Tail of the Wealth Distribution in the Data and in the Baseline Model

Notes. The figure compares the log networth (relative to the average networth) to the log percentile in SCF, Forbes, and in the simulated model corresponding to Column (1) of Table 1.4. More precisely, the figure plots the average log networth within 40 logarithmically spaced percentile bins in SCF. The figure plots the average log networth for each position in Forbes 400. The (opposite of) the slope gives $\zeta \approx 1.5$ for SCF and for Forbes 400 but $\zeta \approx 1.1$ for the baseline model.
Figure 1.4: Asset Prices in the Baseline Model

(a) Market Price of Risk $\kappa$

(b) Interest Rate $r$

(c) Price-Dividend Ratio (log)

(d) Volatility Returns $\sigma_R$

(e) Relative Drift $\tilde{\mu}_A$

(f) Relative Exposure $\tilde{\sigma}_A$

Notes. The figure plots equilibrium objects as a function of $x$, the share of wealth owned by the agents in group $A$, for three different estimations of the baseline model. The baseline model, the model with $\{\beta_{\text{exposure}}\}$, the model with $\{\psi_B > 0.2\}$ correspond respectively to Column (1), Column (4) and Column(5) of Table 1.4.
Figure 1.5: Asset Prices in Augmented Model

(a) Market Price of Risk $\kappa$

(b) Interest Rate $r$

(c) Price-Dividend Ratio (log)

(d) Volatility Returns $\sigma_R$

(e) Relative Drift $\tilde{\mu}_A$

(f) Relative Exposure $\tilde{\sigma}_A$

Notes. The figure plots equilibrium objects as a function of $x$, the share of wealth owned by the agents in group $A$, and $\nu$, the difference between the investment opportunities of the rich relative to the poor.
Figure 1.6: Time Series of the $\hat{\text{Top } 0.01\%}$ vs the Top 0.01%

*Notes.* The figure plots the logarithm of the wealth share of the top 0.01%, as well as the “synthetic” values constructed as predicted values by the linear model given in Equation (1.27). More precisely, I estimate the linear model on the series of the wealth share of the top 0.01% and I then construct a synthetic series $\hat{\text{Top } 0.01\%}$ as

$$
\hat{\text{Top } 0.01\%}_{t+1} = \hat{\alpha} + \hat{\beta} \log \hat{\text{Top } 0.01\%}_{t+1} + \hat{\gamma} (r^M_{t+1} - r^f_{t+1}) + \epsilon_{t+1}
$$
Appendix

1.A Empirical Appendix

1.A.1 Data Sources

A direct comparison of estate tax returns and Forbes data by researchers from the IRS Statistics of Income Division (Raub, Johnson, and Newcomb (2010)) finds that actual estates correspond to only about 50 percent of reported Forbes values. This suggests that estate tax returns may underestimate wealth (potentially due to the tax avoidance effect) while Forbes may overestimate wealth (potentially because debts are harder to track than assets). These findings suggest that the main difficulties with measuring top wealth shares primarily pertain to getting the level right. My principal measure, the stock market exposure of the top households, is similar across the two datasets (Table 1.2).

I refer the reader to Kopczuk and Saez (2004) for a detailed descriptions of construction of top wealth shares from estate tax returns.

1.A.2 Measuring the Exposure of Top Households

Robustness w.r.t. Human Capital. Table 1.2 measures the exposure of financial wealth. One may be interested in the exposure of total wealth. However, because human capital is not observable. I now argue that, for households at the top of the wealth distribution, the bias between the exposure of total wealth and the exposure of financial wealth is quantitatively small.

Formally, for a given agent in the economy, denote $w$ his financial wealth, $h$ his human...
Table 1.6: The Exposure to Stock Returns Across the Wealth Distribution: Controlling for the Composition Effect

<table>
<thead>
<tr>
<th></th>
<th>Forbes 40 Top 40</th>
<th>Within Top 40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Excess Stock Returns</td>
<td>0.71***</td>
<td>0.74***</td>
</tr>
<tr>
<td></td>
<td>(0.21)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>Risk Free Rate</td>
<td>1.89</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>(1.16)</td>
<td>(1.15)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.29</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Notes. This table reports the stock market exposure of the total wealth of the households in the Top 40 vs the wealth exposure of the households within the Top 40. Only 4 households in the Top 40 directly exit Forbes 400 (Daniel E. Smith, Gururaj Deshpande, David Huber in 2001 and Robert Pritzker in 2004). I do not know the wealth of these households after they exit the top. I assume that they are just under the threshold for Forbes 400. Quantitatively, the imputation does not matter since the drop already corresponds to a negative return of $-90\%$ (i.e. there is more much more variation between the Top 40 and the Top 400 than between the top 400 and 0).

capital and $\omega = w/(w + h)$ the ratio of financial wealth over total wealth. Following the log linearization in Campbell (1996), the return on total wealth can be written as

$$
\log \frac{w_{t+1} + h_{t+1}}{w_t + h_t} \approx \kappa + \omega \log \frac{w_{t+1}}{w_t} + (1 - \omega) \log \frac{h_{t+1}}{h_t}
$$

Projecting this approximation on stock returns, we obtain the exposure of total wealth as a weighted sum of the exposure of financial wealth and human capital

$$
\beta_{w+h} \approx \omega \beta_w + (1 - \omega) \beta_h
$$

This allows to express the bias due to the omission of human capital

$$
\frac{\beta_{w+h} - \beta_w}{\beta_w} = (1 - \omega) \left( \frac{\beta_h - \beta_w}{\beta_w} \right)
$$

The bias depends on $\omega$ the share of financial wealth in total wealth, and $\frac{\beta_h - \beta_w}{\beta_w}$ the difference between the exposure of financial wealth and of human capital. I now give an order of magnitude for these two terms.

Labor income represents a very small share of total income for households in the top of the
Table 1.7: The Exposure to Stock Returns Across the Wealth Distribution: Controlling for Idiosyncratic Volatility

<table>
<thead>
<tr>
<th>Group of Households Defined by Wealth Percentiles</th>
<th>1 – 0.1%</th>
<th>0.1 – 0.01%</th>
<th>Top 0.01%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Excess Stock Returns</td>
<td>0.58****</td>
<td>0.71***</td>
<td>0.77***</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.18)</td>
<td>(0.24)</td>
</tr>
<tr>
<td>$\sigma^2$ idiosyncratic (firm level)</td>
<td>0.12</td>
<td>0.13</td>
<td>0.27**</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.11)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.51</td>
<td>0.62</td>
<td>0.56</td>
</tr>
<tr>
<td>N</td>
<td>43</td>
<td>43</td>
<td>43</td>
</tr>
</tbody>
</table>

Notes. The table reports the results of a regression of the wealth growth of households in a given percentile group on the excess stock returns, the risk free rate, and the yearly idiosyncratic variance

$$
\log \frac{W_{p+t+1}^{p'}}{W_{p+t}^{p'}} = \alpha + \beta (r_t^M - r_t^f) + \gamma r_t^f + \delta \sigma^2 + \epsilon_t
$$

Idiosyncratic variance $\sigma^2$ is measured as the cross sectional variance of the residual $\epsilon_{it}$ of a regression on firm level stock returns on factors $r_t^f - r_t^f = \alpha_i + \beta_i F_t + \epsilon_{it}$

where $F_t$ includes the three Fama-French factors.

Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *,**,*** indicate significance at the 0.1, 0.05, 0.01 levels, respectively.

distribution (8.5% for the Top 400\(^{28}\)). Assuming the same capitalization rate for human capital and financial capital, this suggests that human capital represents approximately one tenth of financial wealth for the top 400 households.

I proxy the exposure of human capital to the stock market $\beta_h$ as the covariance of labor income growth to stock returns. The approximation is exact when the discount rate associated with human capital are constant over time. Table 1.8 reports the result: I find $\beta = 0.21$, which is smaller than the exposure of financial wealth. Parker and Vissing-Jørgensen (2009) show that the exposure of labor income to aggregate shocks was low before 1982, and increased thereafter.

Joining the estimates for $\omega$ and $\beta_h$, I conclude that the bias is in average negative and represents $0.085 \times (0.2/0.75 - 1) = -6\%$ of the estimated exposure $\beta$, which is much smaller than the standard errors.

Table 1.8: The Exposure of Labor Income Growth Across the Wealth Distribution:

<table>
<thead>
<tr>
<th>Group of Households Defined by Wealth Percentiles</th>
<th>1 - 0.1%</th>
<th>0.1 - 0.01%</th>
<th>Top 0.01%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Excess Stock Returns</td>
<td>0.15</td>
<td>0.20*</td>
<td>0.35**</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.11)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>(2) $R^2$</td>
<td>0.16</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>Period</td>
<td>1918-2010</td>
<td>1918-2010</td>
<td>1918-2010</td>
</tr>
<tr>
<td>N</td>
<td>93</td>
<td>93</td>
<td>93</td>
</tr>
</tbody>
</table>

**Notes.** The table reports the results of the regression of the growth of labor income on asset returns

\[
\log \frac{Y_{t+1}^{p'} - Y_t^{p'}}{Y_{t-1}^{p'}} = \alpha + \beta_1(r_t^{M} - r_t^{f}) + \beta_2 r_t^{f} + \epsilon_t
\]

The total labor income received by within a top percentile is obtained from Saez and Zucman (2016). Estimation is via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *, **, *** indicate significance at the 0.1, 0.05, 0.01 levels, respectively.

Robustness w.r.t. Saez and Zucman (2016) series. Saez and Zucman (2016) have recently proposed a new series for top wealth shares, which relies on Income Tax Returns. Table 1.9 estimates the stock market exposure of the top wealth percentiles by replacing the series of Kopczuk and Saez (2004) by the series of Saez and Zucman (2016). I find that the estimates are now uniformly lower. For instance, the stock market exposure of the Top 0.01% is 0.4 using Income Tax Returns, compared to 0.75 using estate tax returns or Forbes. This suggests that the methodology in Saez and Zucman (2016) may not track well the business cycle frequencies of wealth shares, even though they track more accurately the long run fluctuations in inequality, as argued in Saez and Zucman (2016). A certain number of wealth categories are constructed using trends and interpolations across years, which may bias down the estimate.
Table 1.9: The Exposure to Stock Returns Across the Wealth Distribution: Saez and Zucman Series

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 – 0.1%</td>
</tr>
<tr>
<td></td>
<td>0.1 – 0.01%</td>
</tr>
<tr>
<td></td>
<td>Top 0.01%</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
</tr>
</tbody>
</table>

Panel A: 1960-2011

<table>
<thead>
<tr>
<th>Excess Stock Returns</th>
<th>0.31***</th>
<th>0.38***</th>
<th>0.40***</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Risk Free Rate</td>
<td>−0.34</td>
<td>−0.23</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>(0.42)</td>
<td>(0.58)</td>
<td>(0.62)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.38</td>
<td>0.31</td>
<td>0.27</td>
</tr>
<tr>
<td>$N$</td>
<td>52</td>
<td>52</td>
<td>52</td>
</tr>
</tbody>
</table>

Panel B: 1960-1982

<table>
<thead>
<tr>
<th>Excess Stock Returns</th>
<th>0.23***</th>
<th>0.26***</th>
<th>0.30**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.09)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>Risk Free Rate</td>
<td>0.64*</td>
<td>1.41*</td>
<td>2.09*</td>
</tr>
<tr>
<td></td>
<td>(0.36)</td>
<td>(0.81)</td>
<td>(1.05)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.48</td>
<td>0.33</td>
<td>0.29</td>
</tr>
<tr>
<td>$N$</td>
<td>22</td>
<td>22</td>
<td>22</td>
</tr>
</tbody>
</table>

Panel C: 1982-2011

<table>
<thead>
<tr>
<th>Excess Stock Returns</th>
<th>0.38***</th>
<th>0.47***</th>
<th>0.46***</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.10)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>Risk Free Rate</td>
<td>−0.73</td>
<td>−0.87</td>
<td>−0.73</td>
</tr>
<tr>
<td></td>
<td>(0.52)</td>
<td>(0.54)</td>
<td>(0.57)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.43</td>
<td>0.41</td>
<td>0.38</td>
</tr>
<tr>
<td>$N$</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

Notes. The table reports the results of the regression of the growth of the wealth growth of households in a given percentil group on asset returns. Estimation is via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *, **, *** indicate significance at the 0.1, 0.05, 0.01 levels, respectively.

Table 1.10: Accounting for the Persistence of Top Wealth Shares

<table>
<thead>
<tr>
<th>Confidence Interval for $\beta \in [\underline{\beta}, \bar{\beta}]$ in the Predictability Regression (1.2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Case with $\rho = 0.92$</td>
</tr>
<tr>
<td>Log Top Share</td>
</tr>
<tr>
<td>$\underline{\beta}$</td>
</tr>
<tr>
<td>$\bar{\beta}$</td>
</tr>
<tr>
<td>Case with $\underline{\rho} = 1.06$</td>
</tr>
<tr>
<td>Log Top Share</td>
</tr>
<tr>
<td>$\underline{\beta}$</td>
</tr>
<tr>
<td>$\bar{\beta}$</td>
</tr>
</tbody>
</table>

Notes. The time period is 1917-1945, the longest period where the wealth share of the top 0.01% is available without missing years. This table does a Bonferroni test based on the test developed Campbell and Yogo (2006). The test jointly takes into account the persistence of the predictor as well as its correlation with stock returns to compute the confidence interval for $\beta$. The autoregressive lag length for top wealth share is estimated by the Bayes information criterion (BIC), with maximum length equal to four.
1.B Theoretical Appendix

1.B.1 Characterization of the Equilibrium

Proof of Proposition 1. Given the homotheticity assumptions, the value function of the households in group $j \in \{A, B\}$ with wealth $N$ can be written:

$$V_{jt}(N) = \frac{N^{1-\gamma_j}}{1-\gamma_j} p_j^{\frac{1-\gamma}{\psi_j}} (\rho + \delta)^{\frac{1-\gamma_j}{\psi_j}}$$

The HJB equation associated with household’s problem is

$$0 = \max_{c_{jt}, \sigma_jt} \left\{ f(c_{jt}N_{jt}, U) + E\left[ \frac{N^{1-\gamma_j}}{1-\gamma_j} p_j^{\frac{1-\gamma}{\psi_j}} (\rho + \delta)^{\frac{1-\gamma_j}{\psi_j}} \right] \right\}$$

Applying Ito’s lemma

$$0 = \max_{c_{jt}, \sigma_jt} \left\{ \frac{(\rho + \delta)(1 - \gamma_j)}{1 - \frac{1}{\psi_j}} \left( \frac{c_{jt}}{\mu_{jt} N_{jt}} - 1 \right) + (1 - \gamma_j)\mu_i \right. \right.$$  
$$+ \frac{1 - \gamma_j}{\psi_j - 1} \mu_{p_j} - \frac{(1 - \gamma_j)(\gamma_j)^2}{2} \frac{\sigma_j^2}{\psi_j^2} + \frac{(1 - \gamma_j)(2 - \gamma_j - \psi_j)}{2(\psi_j - 1)^2} \sigma_{p_j}^2 + \frac{(1 - \gamma_j)^2}{\psi_j - 1} \sigma_i \sigma_{p_j}$$

Substituting the expression for the wealth drift $\mu_j$ using the budget constraint and dividing by $1 - \gamma_j$

$$0 = \max_{c_{jt}, \sigma_jt} \left\{ \frac{1}{1 - \frac{1}{\psi_j}} \left( \frac{c_{jt}}{\mu_{jt} \psi_j} - \rho - \delta \right) + r_t + \delta + \sigma_j \kappa_t - c_{jt} \right. \right.$$  
$$+ \frac{1}{\psi_j - 1} \mu_{p_j t} - \frac{\gamma_j}{2} \sigma_j^2 + \frac{2 - \gamma_j - \psi_j}{2(\psi_j - 1)^2} \sigma_{p_j t}^2 + \frac{1 - \gamma_j}{\psi_j - 1} \sigma_j \sigma_{p_j t} \right\}$$

The FOC for aggregate risk exposure $\sigma_jt$ gives

$$\sigma_{jt} = \frac{\kappa_t}{\gamma_j} + \frac{1 - \frac{1}{\gamma_j}}{1 - \psi_j} \sigma_{p_j t}$$
The FOC for consumption gives

\[ c_{jt} = 1/p_{jt} \]

that is, \( p_{jt} \) is the wealth / consumption ratio of the household.

Plugging the expression in the HJB equation, we obtain an expression for the wealth drift

\[
\mu_{jt} = r_t + \delta + \sigma_{jt} \kappa_t - c_{jt}
\]

\[
= \psi_j (r_t - \rho) + \mu_{pjt} + \frac{1}{2} \psi_j^2 \kappa_t^2 + \frac{1 - \gamma_j}{\gamma_j (\psi_j - 1)} \kappa_t \sigma_{pjt} + \frac{1 - \gamma_j \psi_j}{2 (\psi_j - 1) \gamma_j} \sigma_{pjt}^2
\]

Proof of Proposition 2. Denote \( N_{Ats} \) the average wealth at time \( t \) of all agents in group \( A \) born at time \( s \). The total wealth owned by agents in group \( A \) is

\[
\int_{i \in I_A} N_{it} di = \int_{-\infty}^t \delta e^{-\delta(t-s)} N_{Ats} ds
\]

Its law of motion is

\[
d[\pi_A \int_{-\infty}^t \delta e^{-\delta(t-s)} N_{Ats} ds] = \pi_A \int_{-\infty}^t \delta e^{-\delta(t-s)} dN_{Ats} ds + \pi_A \delta N_{Att} - \pi_A \int_{-\infty}^t \delta^2 e^{-\delta(t-s)} N_{At,s} ds
\]

Therefore

\[
\frac{d[\pi_A \int_{-\infty}^t \delta e^{-\delta(t-s)} N_{Ats} ds]}{\int_{-\infty}^t \pi_A \delta e^{-\delta(t-s)} N_{Ats} ds} = \mu_A dt + \sigma_A dZ_t + \delta (\frac{\pi_A \phi_t}{x_t} - 1) dt
\]

Similarly

\[
\frac{d[(1 - \pi_A) \int_{-\infty}^t \delta e^{-\delta(t-s)} N_{Bts} ds]}{\int_{-\infty}^t (1 - \pi_A) \delta e^{-\delta(t-s)} N_{Bts} ds} = \mu_B dt + \sigma_B dZ_t + \delta (\frac{1 - \pi_A \phi_t}{1 - x_t} - 1) dt
\]
Applying Ito's lemma on the definition of $x$ (1.10), we obtain

$$\frac{dx_t}{x_t} = \mu_{xt} dt + \sigma_{xt} dZ_t$$

with

$$\sigma_{xt} = (1 - x_t)(\sigma_{At} - \sigma_{Bt})$$

$$\mu_{xt} = (1 - x_t)(\mu_{At} - \mu_{Bt}) + (1 - x) \phi_t \delta \left( \frac{\pi_A}{x_t} - \frac{1 - \pi_A}{1 - x_t} \right) - (\sigma + \sigma_{pt}) \sigma_{xt}$$

\[\square\]

1.B.2 Solving for the Equilibrium

Specifying Life Cycle Function $G$. First, I specify $G(u)$ as a sum of $K$ exponential

$$G(u) = \sum_{1 \leq k \leq K} B_k e^{-\delta_k u}$$

where the coefficients $(B_k)_{1 \leq k \leq K}$ are normalized so that total aggregate earnings equal $\omega Y_t$

$$1 = \sum_{1 \leq k \leq K} B_k \frac{\delta}{\delta + \delta_k}$$

Define $p_k^l$ as the price dividend of a claim with exponentially decreasing endowment at rate $\delta + \delta_k$, for $1 \leq k \leq K$. In particular, $\phi$, the human capital of a newborn agent normalized by total endowment $Y_t$, can be written:

$$\phi = \omega \sum_{1 \leq k \leq K} B_k p_k^l$$

I now write the equilibrium as a system of PDES for the function $p_A, p_B, (p_k^l)_{1 \leq k \leq K}$. 58
Market Clearing for Consumption. Market clearing for consumption (1.8) gives the function $p$ in term of $p_A$ and $p_B$

$$\frac{x}{p_A} + \frac{1-x}{p_B} = \frac{1}{p}$$

In particular, this allows to express the derivatives of $p$ in term of the derivatives of $p_A$ and $p_B$

Market Clearing for Risk. Market clearing for risk (1.9) gives

$$x\sigma_A + (1-x)\sigma_B = \sigma + \sigma_p$$

Plugging the FOC for $\sigma_j$ from (1.11), one obtains the market price of risk (1.16)

Solve for $\sigma_x$. By Ito we have

$$\sigma_{p_j} = \frac{\partial x p_j}{p_j} \sigma_x + \frac{\partial \nu p_j}{p_j} \sigma_{\nu} \text{ for } j \in \{A, B\}$$

$$\sigma_p = \frac{\partial x p}{p} \sigma_x + \frac{\partial \nu p}{p} \sigma_{\nu}$$

Substituting the expression for $\kappa$ in (1.16) in Proposition 2, we can solve for $\sigma_x$:

$$\sigma_x = \frac{x(1-x)\Gamma((\gamma_B - \gamma_A)\sigma + \frac{1-\gamma_A}{\psi_A-1} \frac{\partial \nu p_A}{p_A} \sigma_{\nu} - \frac{1-\gamma_B}{\psi_B-1} \frac{\partial \nu p_B}{p_B} \sigma_{\nu})}{1 - \frac{x(1-x)\Gamma((\gamma_B - \gamma_A) \frac{p_x}{p} + \frac{1-\gamma_A}{\psi_A-1} \frac{\partial \nu p_A}{p_A} - \frac{1-\gamma_B}{\psi_B-1} \frac{\partial \nu p_B}{p_B})}{\gamma_A \gamma_B}}$$

Solve for $\mu_x$. The law of motion Proposition 2 yields $\mu_x$ in term of previously computed quantities:

$$\mu_x = \mu_A - \mu - \mu_p + \delta(\frac{\pi_A}{x} \frac{\phi}{p} - 1) + (\sigma + \sigma_p)\sigma_x$$

$$= \frac{1}{p} - \frac{1}{p_A} + \kappa \sigma_x + \delta(\frac{\pi_A}{x} \frac{p}{p} - 1) + (\sigma + \sigma_p)\sigma_x$$
By Ito, we have
\[
\mu_{p_j} = \frac{\partial_x p_j}{p_j} \mu_x + \frac{\partial_{\nu} p_j}{p_j} \mu_{\nu} + \frac{1}{2} \frac{\partial_{xx} p_j}{p_j} \sigma_{x}^2 + \frac{1}{2} \frac{\partial_{\nu\nu} p_j}{p_j} \sigma_{\nu}^2 + \frac{\partial_{x\nu} p_j}{p_j \sigma_x \sigma_{\nu}} \text{ for } j \in \{A, B\} \\

\mu_p = \frac{\partial_x p}{p} \mu_x + \frac{\partial_{\nu} p}{p} \mu_{\nu} + \frac{1}{2} \frac{\partial_{xx} p}{p} \sigma_{x}^2 + \frac{1}{2} \frac{\partial_{\nu\nu} p}{p} \sigma_{\nu}^2 + \frac{\partial_{x\nu} p}{p \sigma_x \sigma_{\nu}}
\]

**Solve for risk free rate** Combining the market pricing for the price dividend ratio and the market pricing for human capital, we obtain the market pricing for \( p \), the total wealth in the economy

\[
\frac{1}{p} - \delta \phi + \mu + \mu_p + \sigma \sigma_p = r + \kappa (\sigma + \sigma_p)
\]

This gives \( r \).

**System of PDEs.** Given \( r \) and \( \kappa \), we are left with the following system of PDEs

\[
\mu_j = r + \delta - \nu_j + \kappa \sigma_j - \frac{1}{p_j} \text{ for } j \in \{A, B\} \\

\frac{1}{p_k} - \delta - \delta_k + \sigma_{p_k} \sigma = r + \kappa (\sigma + \sigma_{p_k}) \text{ for } 1 \leq k \leq K
\]

### 1.B.3 The Wealth Distribution

For the sake of generality, I study the distribution of a process with both aggregate and idiosyncratic shocks

**Lemma 1** (Kolmogorov Forward). Suppose \( x_t \) is a process evolving according to

\[
dx_t = \mu_t(x) dt + \sigma_t(x) dZ_t + \nu_t(x) dW_t
\]

where \( Z_t \) is a standard aggregate Brownian Motion and \( W_t \) is a standard idiosyncratic Brownian motion and with death rate \( \delta \) and re-injection according to the distribution \( \psi_t \). The
pdf of $x_t$, $g_t$, follows the law of motion

$$
\frac{dg_t}{dt}(x) = -\partial_x(\mu_t(x)g_t(x)) + \sigma_t(x)g_t(x)\frac{dZ_t}{dt} + \frac{1}{2} \partial_x^2(\sigma_t^2(x) + \nu_t^2(x))g_t(x) + \delta(\psi_t(x) - g_t(x))
$$

**Proof for Lemma 1.** I extend the proof in Kredler (2014) for the case of aggregate shocks.

For any function $f$, we have

$$
\int_{-\infty}^{+\infty} f(x)g_{t+dt}(x)dx = \int_{-\infty}^{+\infty} [(f(x) + df(x))g_t(x) + f(x)\delta dt(\psi_t(x) - g_t(x))]dx
$$

Assume that $f$ is a twice differentiable and use Ito’s lemma to obtain

$$
\int_{-\infty}^{+\infty} f(x)dg_t(x)dx = \int_{-\infty}^{+\infty} (\mu_t(x)\partial_x f(x) + \sigma_t(x)\partial_x f(x)dZ_t)g_t(x)dx
$$

$$
+ \int_{-\infty}^{+\infty} \frac{1}{2}(\sigma_t(x)^2 + \nu_t(x)^2)\partial_x f(x)g_t(x)dx
$$

$$
+ \int_{-\infty}^{+\infty} f(x)\delta dt(\psi_t(x) - g_t(x))dx
$$

Assume that $f$ decays fast enough as $|x| \to +\infty$ and use integration by parts to obtain

$$
\int_{-\infty}^{+\infty} f(x)dg_t(x)dx = \int_{-\infty}^{+\infty} f(x)[(-\partial_x(\mu_t(x)g_t(x)) - \partial_x(\sigma_t(x)g_t)dZ_t]dx
$$

$$
+ \int_{-\infty}^{+\infty} f(x)\frac{1}{2} \partial_x^2(\sigma_t^2(x) + \nu_t^2(x))g_t)dt dx
$$

$$
+ \int_{-\infty}^{+\infty} f(x)\delta dt(\psi_t(x) - g_t(x))dx
$$

This equality must hold for all $f$ satisfying the conditions above. Therefore, we obtain

$$
\frac{dg_t}{dt}(x) = -\partial_x(\mu_t(x)g_t(x)) + \sigma_t(x)g_t(x)\frac{dZ_t}{dt} + \frac{1}{2} \partial_x^2(\sigma_t^2(x) + \nu_t^2(x))g_t + \delta(\psi_t(x) - g_t(x))
$$
Proposition 3 is a direct application of Lemma 1 for the particular law of motion of the model. I now derive the evolution of top wealth shares deriving a version of the Kolmogorov Forward equation with both aggregate risk and idiosyncratic risk.

Lemma 2 (Dynamics of Top Wealth Shares). Suppose $x_t$ is a process evolving, for $x$ high enough\textsuperscript{29}, according to

$$\frac{dx_t}{x_t} = \mu_t dt + \sigma_t dZ_t + \nu_t dW_t$$

where $Z_t$ is a standard aggregate Brownian Motion and $W_t$ is a standard idiosyncratic Brownian motion and with death rate $\delta$ and re-injection according to the distribution $\psi_t$.

For a top percentile $\alpha \in (0, 1)$, denote $q_t(\alpha)$ the $\alpha-$quantile, i.e.,

$$\alpha = \int_{q_t}^{+\infty} g_t(n)dn$$

and denote $T_t(\alpha)$ the share of wealth owned by the households in the top percentile $\alpha$, i.e.,

$$T_t = \int_{q_t}^{+\infty} n g_t(n)dn$$

$T_t$ follows the law of motion

$$\frac{dT_t}{T_t} = \mu_t dt + \sigma_t dZ_t - \delta(1 - \frac{q_t(\alpha)}{T_t}) + \frac{\delta}{T_t} \int_{q_t}^{+\infty} (x - q_t) \psi_t(x)dx + \frac{\nu_t^2 q_t^2 g_t(q_t)}{2 T_t}$$

Proof of Lemma 2. In this proof, I use the following notation: for an Ito process $y_t$, $\sigma[y_t]$ denotes the volatility corresponding to Brownian Motion $dZ_t$, i.e.

$$dy_t = E[dy_t] + \sigma[y_t]dZ_t$$

The top quantile $q_t$ is defined as

$$\alpha \equiv \int_{q_t}^{+\infty} g_t(x)dx \quad (1.29)$$

\textsuperscript{29}Formally, for $x$ higher than $q_t(\alpha)$
Applying Ito’s lemma gives the law of motion of the quantile

\[ 0 = -g_t(q_t) \frac{dq_t}{dt} + \int_{q_t}^{+\infty} \frac{dg_t(x)}{dt} dx - \sigma[dg_t(q_t)]\sigma[dq_t] \]

The top share corresponding to the percentile \( p \) is defined as

\[ T_t \equiv \int_{q_t}^{+\infty} xg_t(x)dx. \quad (1.30) \]

Applying Ito’s lemma gives the law of motion of the top share

\[ dT_t = -q_t g_t(q_t) dq_t + \int_{q_t}^{\infty} x dg_t(x) dx - q_t \sigma[dg_t(q_t)]\sigma[dq_t] dt - \frac{1}{2} g_t(q_t)\sigma[dq_t]^2 dt \]

Injecting the law of motion for \( q_t \), we obtain the law of motion for \( T_t \):

\[ dT_t = \int_{q_t}^{\infty} (x - q_t) dg_t(x) dx - \frac{1}{2} g_t(q_t)\sigma[dq_t]^2 dt \]

\[ = \int_{q_t}^{\infty} (x - q_t) dg_t(x) dx - \frac{1}{2} \frac{1}{g_t(q_t)} \left( \int_{q_t}^{\infty} \sigma[dg_t(x)] dx \right)^2 dt \quad (1.31) \]

Substituting the law of motion for \( dg_t \) from the Kolmogorov Forward equation Lemma 1 and integrating by parts:

\[ dT_t = \int_{q_t}^{\infty} (x - q_t) \left( - \partial_x \left( (\mu_t dt + \sigma_t dZ_t) x g_t(x) \right) + \sigma_t^2 \left( \frac{\sigma_t^2 + \nu_t^2}{2} dt x^2 g_t(x) \right) \right) dx \]

\[ + \delta \psi_t(x) dt - g_t(x) dt - \frac{1}{2} \frac{1}{g_t(q_t)} \left( \int_{q_t}^{+\infty} \partial_x (\sigma_t x g_t(x)) dx \right)^2 dt \]

\[ = - \int_{q_t}^{+\infty} \left( \mu_t dt + \sigma_t dZ_t \right) x g_t(x) dx + \partial_x \left( \frac{\sigma_t^2 + \nu_t^2}{2} dt x^2 g_t(x) \right) dx \]

\[ - \delta \int_{q_t}^{\infty} (x - q_t) g_t(x) dt dx \cdot \delta + \delta \int_{q_t}^{\infty} (x - q_t) \psi_t(x) dt dx \]

\[ - \frac{1}{2} \frac{1}{g_t(q_t)} \left( \int_{q_t}^{+\infty} \partial_x (\sigma_t x g_t(x)) dx \right)^2 dt \]

\[ = \left( \mu_t dt + \sigma_t dtdZ_t \right) T_t - \delta dt \left( T_t - q_t \alpha \right) + \delta \int_{q_t}^{+\infty} (x - q_t) \psi_t(x) dt dx + \frac{\nu_t^2}{2} q_t^2 g_t(q_t) dt \]

\( \square \)
Proof of Proposition 4. The proof proceeds similarly to Lemma 2 up to equation (1.31):

\[ dT_t = \int_{q_t}^{\infty} (x - q_t)dg_t(x)dx - \frac{1}{2} \frac{1}{g_t(q_t)}(\int_{q_t}^{\infty} \sigma[dg_t(x)]dx)^2dt \]

Now, \( dg_t(x) \) is given by the Kolmogorov Forward equation in Proposition 3. We obtain:

\[
\begin{align*}
    dT_t &= \sum_{j \in \{A,B\}} \left( \int_{q_t}^{\infty} (x - q_t)(-\partial_x(\pi_j(\mu_j dt + \sigma_j dt dZ_t)xg_j(x)) + \partial^2_x(\frac{\sigma^2_j}{2} dt x^2 g_j(x)))dx \\
    &\quad + \sum_{j \in \{A,B\}} \left( \int_{q_t}^{\infty} (x - q_t)(\delta(\psi_t(x)dt - g_j(x)dt))dx \\
    &\quad - \frac{1}{2} \frac{1}{g_t(q_t)}(\int_{q_t}^{\infty} \partial_x(-\sum_{j \in \{A,B\}} \pi_j \sigma_j x g_j(x)dx))^2dt \\
    \frac{dT_t}{T_t} &= \sum_{j \in \{A,B\}} \int_{q_t}^{\infty} (\pi_j g_j(x) \mu_j dt dx)dt + \sum_{j \in \{A,B\}} \int_{q_t}^{\infty} (\pi_j g_j \sigma_j dt dZ_t) \\
    &\quad - \delta dt (1 - 1 - q_t \frac{\alpha}{T_t}) + \frac{\delta dt}{T_t} (x - q_t)\psi_t(x)dx \\
    &\quad + \frac{1}{2} \frac{q_t^2 g_t(q_t)}{T_t} \left( \sum_{j \in \{A,B\}} \pi_j g_j (\frac{\sigma^2_j}{2}) - \left( \frac{1}{g_t} \sum_{j \in \{A,B\}} \pi_j g_j (\sigma^2_j) \right) \right)dt
\end{align*}
\]

Proof of Proposition 5. I first derive the equilibrium prices when \( \gamma_A = \gamma_B = \gamma \). Households' FOC from Proposition 1 give:

\[
\begin{align*}
    \sigma_j &= \frac{\kappa}{\gamma_j} \\
    \mu_j &= \psi_j (r_0 - \rho) + \frac{1 + \psi_j}{2 \gamma} \kappa_0^2
\end{align*}
\]

for \( j \in \{A, B\} \). Market clearing gives:

\[
\begin{align*}
    x_0 \sigma_{A0} + (1 - x_0) \sigma_{B0} &= \sigma \\
    x_0 \mu_{A0} + (1 - x_0) \mu_{B0} + \delta (\frac{\phi}{p} - 1) &= \mu
\end{align*}
\]
Combining households' FOC with market clearing, we obtain the interest rate $r_0$ and the market price of risk $\kappa_0$:

$$
\kappa_0 = \gamma \sigma^2
$$

$$
r_0 = \rho + \frac{1}{x_0 \psi_A + (1 - x_0) \psi_B} \left( \mu - \gamma \frac{\sigma^2}{2} - \delta \left( \frac{\phi_0}{p_0} - 1 \right) \right) - \gamma \frac{\sigma^2}{2}
$$

(1.32)

The steady state condition $\mu_x(x_0) = 0$ gives, using Proposition 2:

$$
(1 - x_0)(\mu_{A0} - \mu_{B0}) = \delta \frac{\phi_0}{p_0} (1 - \frac{\pi_A}{x_0})
$$

Substituting out $\mu_{A0}, \mu_{B0}$ from the households’ FOC in this expression:

$$
(\psi_A - \psi_B)(r_0 + \frac{\gamma}{2} \sigma^2 - \rho) = \delta \frac{\phi_0}{p_0} \frac{\pi_A - x_0}{(1 - x_0)}
$$

(1.33)

Finally, the no arbitrage condition for total wealth and human capital gives:

$$
\phi_0 = \omega \int_0^{+\infty} e^{-(r + \delta + \gamma \sigma^2 - \mu)u} G(u) du
$$

(1.34)

$$
p_0 = \frac{1 - \delta \phi_0}{r_0 + \gamma \frac{\sigma^2}{2} - \mu}
$$

(1.35)

We are left with a system of 4 unknowns $(r_0, x_0, \phi_0, p_0)$ and 4 equations ((1.32), (1.33), (1.34), (1.35)), which can be solved numerically.

To solve for the stationary wealth distribution, apply Proposition 3:

$$
0 = -\partial_n (\tilde{\mu}_{j0} n g_{j0}(n)) dt + \delta (\psi_0(n) - g_{j0}(n)) dt \text{ for } j \in \{A, B\}
$$

When $\tilde{\mu}_{j0} > 0$, the solution of this equation is a Pareto distribution with right tail $\delta / \mu_{j0}$ (see Gabaix et al. (2016)). Therefore, the wealth distribution has a Pareto tail with power law exponent $\delta / \mu_{A0}$. 

\[\square\]
1.B.4 Model Simulation

I simulate the model over 10000-year long samples, in which I only keep the last 5000 years. I assume that distribution of labor income follows the generalized beta of the second kind, i.e. GB(2), with parameters $a = 3.65, p = 0.3, q = 0.8346$, as estimated in SCF. The parametric form of the distribution does not matter for the results, as suggested in Section 1.3.3.
Bibliography


CHAPTER 2

What Drives the Recent Rise in Top Wealth Shares?

2.1 Introduction

The last forty years have seen a dramatic rise in top wealth shares in the United States.\(^1\) There is an ongoing debate about the source of this phenomenon. One potential explanation, detailed in Piketty (2014), is that the wealth of top households tends to grow at a faster pace than the rest of the population. An alternative hypothesis is that the growth of top wealth shares is driven by a composition effect, i.e. the arrival of successful entrepreneurs which disrupt the existing wealthy households.\(^2\) This paper quantifies the role of these two forces.

I first derive a new formula that decomposes the growth of top wealth shares into three terms: the relative wealth growth of individuals at the top, a term due to idiosyncratic wealth shocks, and a term due to population renewal. The formula is an integrated version of Kolmogorov Forward Equation for top wealth shares. The formula allows to quantify the effect of composition effects and death on top wealth shares. In particular, I show that when the wealth distribution has a Pareto tail $\zeta$, a rise in idiosyncratic variance $\Delta \sigma^2$ increases the growth rate of top wealth shares by $(\zeta - 1)/2\Delta \sigma^2$. A rise in the death rate $\Delta \delta$ decreases the growth rate of top wealth shares by $\Delta \delta/\zeta$.

\(^1\)See, for instance, Saez and Zucman (2016), Piketty (2014).
\(^2\)See, for instance, Summers (2014).
I then map each term of the formula to the data using the annual Forbes Magazine list of the 400 wealthiest Americans. The decomposition reveals that the rise in top inequality in 1982-1994 is mostly driven by an increase of the variance of idiosyncratic wealth shocks, while the rise in top inequality in 1995-2015 is mostly driven by an increase of the wealth growth of individuals at the top.

**Related Literature.** A recent empirical literature has documented the rise in top wealth shares in the last thirty years (Piketty (2014), Piketty and Zucman (2014) and Saez and Zucman (2016)). My contribution is to decompose the rise in top wealth shares in three terms, that correspond to the individual growth of households at the top, a term due to idiosyncratic returns, and a term due to population renewal. I focus on the rise in the wealth owned by individuals in Forbes 400, because the list allows me to follow the same individuals over time.

A recent theoretical literature has used the theoretical framework of random growth process to examine the dynamics of the wealth distribution, in particular Jones and Kim (2016) and Gabaix et al. (2016). My contribution is to derive the law of motion of top wealth shares in term of the law of motion of individual wealth. This formula makes it easier to map the theory to the data. In particular, I show that half of the increase of top wealth shares is driven by idiosyncratic jumps. This constrasts with the results of Gabaix et al. (2016), which argue that idiosyncratic returns cannot explain the increase of top wealth shares.

## 2.2 The Dynamics of Top Shares

### 2.2.1 Law of Motion of Top Shares

I first derive the law of motion of top wealth shares in term of the law of motion for individual wealth. This equaiton can be seen as an integrated version of Kolmogorov Forward equation.

Denote $g_t$ the pdf of wealth relative to the average wealth in the economy. Let $p$ a number between 0 and 1. Denote $q_t$ the the $p$—quantile, i.e.,

$$ p \equiv \int_{q_t}^{+\infty} g_t(x) dx $$

(2.1)
and denote \( T_t \) the wealth share of the top \( p \), i.e.,

\[
T_t \equiv \int_{q_t}^{+\infty} xg_t(x)dx.
\]  \hspace{1cm} (2.2)

For instance, for \( p = 1\% \), \( q_t \) is the wealth of agents at the 1% percentile of the wealth distribution and \( T_t \) is the wealth share of the top 1%.

The next proposition characterizes the law of motion of \( T_t \):

**Proposition 7.** The process \( T_t \) follows the law of motion:

\[
dT_t = \int_{q_t}^{\infty} (x - q_t)dg_t(x)dx.
\]

I now give an heuristic derivation of the proposition. During a small time period \( dt \), a net mass \( \int_{q_t}^{+\infty} dq_t \) of individuals enter the top. Because the population size in the top percentile is held constant, an equal mass of households at the threshold must exit the top percentile, with a wealth \( q_t \). The formula expresses that the total change in \( T_t \) is given by the difference between the wealth change due to entry and the wealth change due to exit.

I start by deriving the law of motion of top wealth shares when individual wealth follows a geometric Levy process. The proposition relates the law of motion of \( T_t \) to the law of motion of \( g_t \). Since Kolmogorov Forward equation gives the law of motion of \( g_t \), we can now express the law of motion of \( T_t \).

**Proposition 8.** Suppose that the process for individual wealth \( x_{it} \) is an exponentiated Levy process, i.e., that it follows the law of motion:

\[
\frac{dx_{it}}{x_{it}} = \mu_t dt + \sigma_t dZ_{it} + (e^{\kappa_{it}} - 1)dN_{it}
\]

where \( Z_{it} \) is a standard Brownian motion and \( N_{it} \) is a jump process with intensity \( \lambda_t \). The innovations \( \kappa_{it} \) are drawn from an exogenous distribution \( \phi_t \) with \( E[e^{\kappa_{it}}] = 1 \). Moreover, assume that individuals die at rate \( \delta_t \), and are born with rate \( \delta_t \), and are reinjected below \( q_t \).

Kolmogorov Forward equation gives the law of motion of \( g_t \).
The process $T_t$ follows the law of motion:

$$
\frac{dT_t}{T_t} = \mu_t dt + \frac{\sigma_t^2 dt}{2} \frac{q_t^2 g_t(q_t)}{T_t} + \lambda_t dt \left[ \frac{E_{(\kappa \geq 0)} \int_{q_t e^{-\kappa}}^{q} (e^\kappa x - q_t) g_t(x)dx}{q_t e^{-\kappa} g_t(x)} - \delta_t dt \left(1 - \frac{q_t p}{T_t}\right) \right]
$$

This proposition relates the dynamics of top wealth shares to the underlying wealth process. It decomposes the growth rate of top shares in three terms: the average wealth growth of individuals at the top relative to the rest of the economy, idiosyncratic returns, and the death rate of individuals at the top. The formula clarifies that idiosyncratic returns and population renewal create a wedge between the growth of top wealth shares and the growth of individuals at the top.\(^3\)

I now give an heuristic derivation of the proposition.\(^4\)

**Death.** Start with the term due to death. During a short time period $dt$, a mass $\delta_t p dt$ of households in the top percentile die, which decreases total wealth in the top percentile by $T_t \delta dt$. Because the population size in the top percentile is held constant, an equal mass of households at the threshold enter the top percentile, with a wealth $q_t$. Therefore, the total change in top wealth share $T_t$ due to death is $-\delta dt(T_t - pq_t)$. I now turn to the term due to idiosyncratic returns.

**Idiosyncratic jumps.** I start with the term due to idiosyncratic jumps. Jumps push up top shares for two reasons. First, some lucky individuals outside the top percentile with positive jumps enter the top. Because the population size in the top percentile is held constant, this displaces marginal individuals at the threshold with wealth $q_t$. This term increases top wealth shares by:

$$
\lambda_t \frac{1}{T_t} E[\kappa \geq 0) \int_{q_t e^{-\kappa}}^{q} (e^\kappa x - q_t) g_t(x)dx]
$$

\(^3\)For instance, the formula clarifies the difference between the synthetic saving rates defined by Saez and Zucman (2016) and the real saving rate of individuals at the top.

\(^4\)The formal proof can be found in Section 2.A.
Second, some unlucky individuals inside the top with negative jumps exit the top. They are replaced by marginal individuals at the threshold with wealth $q_t$. This term increases top wealth shares compared to the average wealth of individuals at the top:

$$\lambda_t \frac{1}{T_t} E[\{\kappa \leq 0\}] \int_{q_t}^{q_t e^{-\kappa}} (q_t - e^{\kappa} x) g_t(x) dx$$

**Idiosyncratic volatility.** Finally, the term due to idiosyncratic volatility can be derived as a special case of the jump term. Indeed, the Brownian motion $dZ_{it}$ can be approximated by a Poisson jump process $dN_{it}$ with $\lambda_t = 1$ and $e^{\kappa_{it}} = (1+\sigma_t)\sqrt{dt}$ with probability $1/2$, and $e^{\kappa_{it}} = (1-\sigma_t)\sqrt{dt}$ with probability $1/2$. Applying the formula seen above for the impact of the jump shocks on top wealth shares, one obtains the total impact of idiosyncratic volatility on the growth of top wealth shares:

$$\frac{q_t(1 + \sigma_t \sqrt{dt}) - q_t}{2T_t} \times \frac{1}{2} g_t(q_t) \sigma_t \sqrt{dt} + \frac{q_t - q_t(1 - \sigma_t \sqrt{dt})}{2T_t} \times \frac{1}{2} g_t(q_t) \sigma_t \sqrt{dt}$$

The terms due to entry and the term due to exit are exactly equal, and they sum to $\sigma^2 q_t^2 g_t(q_t)/T_t$. The formula shows that the impact of idiosyncratic volatility depends on ratio of the wealth of the households at the threshold to the average wealth of households within the top percentiles, i.e. $q_t p / T_t$. The higher wealth inequality, the lower this ratio, and therefore the lower the impact of idiosyncratic volatility on top wealth shares.

In a more general setting where the idiosyncratic volatility depends on wealth, what matters is really only the idiosyncratic volatility of households exactly at the threshold. I prove this formally in Section 2.A.

### 2.2.2 PDE for Top Wealth Shares

Proposition 8 gives the dynamics of top wealth shares in term of top wealth shares $T_t(p)$, the density function $g_t$ and the quantile function $q_t(g)$. These two functions can be expressed as derivatives of $T_t$: $\partial_p T_t = q_t$ and $\partial_{pp} T_t = -1/g_t(q_t)$. Therefore, one can rewrite the dynamics
of top wealth shares as a PDE satisfied by the function $T_t(p)$:

$$\frac{dT_t}{T_t} = \mu_t dt - \frac{\sigma^2_t dt}{2 T_t} \frac{\partial_p T^2_t}{\partial_p \partial T_t} - \lambda_t dt E_t[\int_{\partial_p T_t e^{-\kappa}} e^{\kappa x - \partial_p T_t(x)} dx] - \delta_t dt (1 - \frac{p \partial_p T_t}{T_t})$$

This PDE is an integrated version of the Kolmogorov Forward Equation.\(^5\)

### 2.2.3 Stationary Process

I now turn to an economy where the process for individual wealth is stationary. In this economy, there is a stationary distribution of wealth.

**Stationary Distribution.** Proposition 8 allows to derive the Pareto tail of the wealth distribution.

The stationary distribution is characterized by the fact that top wealth shares stay constant. In light of Proposition 8, this gives the following relationship between $T, q,$ and $g$:

$$0 = \mu dt + \frac{\sigma^2 dt q^2 g(q)}{2 T} + \lambda dt E\left[\int_{q e^{-\kappa}}^q (e^{\kappa x - q}) g(x) dx\right] - \delta dt (1 - \frac{qp}{T})$$

The average wealth growth of people at the top, $\mu$, does not depend on the percentile $p$. Therefore, the wealth distribution is characterized by the fact that the ratios $pq/T$ and $qg/p$ do not depend on $p$. It characterizes a distribution with a Pareto tail.

**Proposition 9.** Suppose that the process for individual wealth $x_{it}$ follows the law of motion

$$\frac{dx_{it}}{x_{it-}} = \mu dt + \sigma dZ_{it} + (e^{\kappa_{it}} - 1)dN_{it}$$

where $Z_{it}$ is a standard Brownian motion, $N_{it}$ is a jump process with intensity $\lambda$. The innovations $\kappa_{it}$ are drawn from an exogenous distribution with pdf $\phi$ with $E[e^{\kappa_{it}}] = 1$.

Moreover, assume that individuals die at rate $\delta$ and are injected below a certain point.

The stationary distribution has a Pareto tail, i.e

$$P(x_{it} \geq x) \sim Cx^{-\zeta}$$

---

\(^5\)A PDE for the quantile function has been derived by Steinbrecher and Shaw (2008).
Moreover, its power law exponent $\zeta$ given by

$$0 = \mu + \frac{\zeta - 1}{2} \sigma^2 + \frac{\lambda}{\zeta} (\hat{\phi}(-\zeta) - 1) - \frac{\delta}{\zeta}$$

where $\hat{\phi}$ denotes the Laplace transform of $\phi$.

This proposition is usually proven using the Kolmogorov Forward equation (see, for instance, Gabaix et al. (2016)). In light of Proposition 8, I want to emphasize that the formula can be interpreted as a balance equation for top wealth shares.

The first term in the law of motion of top wealth shares is the wealth growth of agents at the top. It does not depend on $\zeta$. The other terms increase in $\zeta$. Therefore, $\zeta$ is pinpointed by the condition that the (positive) force due to idiosyncratic returns and the (negative) force due to death are low enough to counterbalance the wealth growth of individuals at the top.

**Growth of Top Wealth Shares after a Shock.** The next proposition characterizes the law of motion of top wealth shares following a change in the law of motion for the individual wealth process.

**Proposition 10.** Consider the stationary distribution in an economy where individual wealth follows the process given in (2.3). Following a change in the process for individual wealth (e.g. $\Delta \mu, \Delta \sigma^2, \Delta \lambda \ldots$), the instantaneous change in top wealth shares is given by:

$$\frac{1}{T} \frac{dT}{dt} = \Delta \mu + \frac{\zeta - 1}{2} \Delta \sigma^2 + \frac{1}{\zeta} \Delta (\lambda \hat{\phi}(-\kappa)) - \frac{\Delta \delta}{\zeta}$$

In particular, when $\zeta$ is close to one, changes in idiosyncratic volatility cannot account for large fluctuations in top wealth shares in the short term. For instance, start from a stationary distribution with $\mu = 0.5\%, \sigma = 20\%, \delta = 0.03$, i.e. with a power law exponent 1.65. A (large) change of idiosyncratic volatility from 0.2 to 0.3 means that top wealth shares only grow by $(0.3^2 - 0.2^2)/2 \times (1.5 - 1) = 1.2\%$ in the next year.

Moreover, note that the instantaneous growth of top wealth shares does not depend on $\rho$. In contrast, because the wealth distribution converges to a distribution with a new power
law exponent, the long run change of top wealth shares converges to infinity as $p$ tends to 0. This means that the ratio between the long run growth and the instantaneous growth tends to infinity as $p$ tends to 0. This suggests that the convergence to the new stationary distribution is very slow in the right tail. This property is proven formally in Gabaix et al. (2016).

### 2.3 Mapping the Formula to the Data

#### 2.3.1 Data

**Forbes 400** I use the list of the wealthiest 400 Americans constructed by Forbes Magazine every year since 1982. The list is created by a dedicated staff of the magazine, based on a mix of public and private information. The total wealth of individuals on the list accounts for approximately 1.5% of total aggregate wealth. By combining the lists over time, I am able to track the wealth of the same individuals over time.

I concentrate on the Forbes list because it makes it possible to track the same individuals over time, which makes it possible to measure directly the growth of top wealth shares due to idiosyncratic returns. In contrast, Kopczuk and Saez (2004), Saez and Zucman (2016) rely on repeated cross section to construct the top wealth shares at different percentiles.

A potential problem with Forbes 400 is that the list may report a biased estimate of the wealth of top individuals. As argued by Atkinson (2008), the magazine may overestimate the wealth of top individuals, since debts which are harder to track than assets. Another concern with Forbes 400 is that it tracks only a small subset of the population (i.e. external validity). To address both concern, I plot in Figure 2.1 the cumulative growth in the Top 400 and in different percentiles, based on Saez and Zucman (2016). The figure shows that the dynamics of the top Forbes 400 mirror reasonably well the dynamics of the top 0.01%, which suggests that the growth of top wealth shares are driven by similar forces. As stressed

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6Forbes Magazine reports: “We pored over hundreds of Securities Exchange Commission documents, court records, probate records, federal financial disclosures and Web and print stories. We took into account all assets: stakes in public and private companies, real estate, art, yachts, planes, ranches, vineyards, jewelry, car collections and more. We also factored in debt. Of course, we don’t pretend to know what is listed on each billionaire’s private balance sheet, although some candidates do provide paperwork to that effect.”

7This extends the construction of Capehart (2014) to the recent years. Recent empirical studies examining the Forbes 400 list also include Klass et al. (2006) and Kaplan and Rauh (2013).
by Saez and Zucman (2016) “the wealth share of the top 400 has increased from 1% in the early 1980s to over 3% in 2012-3, on par with the tripling of our top 0.01% wealth share.”

**Other Data Sources** I use summary statistics from the IRS to estimate the wage income and the tax received by the top 400\(^8\). Since this information is only available after 1992, I input the tax rate and the labor income rate in 1982-1992 by using their values in 1992-2000.

The aggregate wealth of US individuals is obtained from the Financial Accounts (Flow of Funds). The population of households is obtained from the US Census Bureau. Asset returns are obtained from the Fama-French Data Library.

### 2.3.2 Methodology

**Accounting Decomposition** I now apply this theoretical framework to decompose the dynamics of top wealth shares in the data. The decomposition of Proposition Proposition 8 has a clear mapping to the data. Denote \(T\) the top and \(T'\) the group of individuals in the following period. Denote \(DEATH\) the group of individuals dropping due to death. Denote \(EXIT\) the group of individuals that exit the top. Denote \(ENTRY\) the group of arriving individuals. The total wealth at the top can be decomposed as follows:

\[
\sum_{i \in T'} W_{i,t+1} = \sum_{i \in T \setminus \text{DEATH}} W_{i,t+1} + \sum_{i \in ENTRY} W_{i,t+1} - \sum_{i \in EXIT} W_{i,t+1}
\]

Therefore, the growth of top wealth share can be decomposed as follows

\[
\frac{\sum_{i \in T'} W_{i,t+1}}{\sum_{i \in T} W_{i,t}} = \frac{\sum_{i \in T \setminus \text{DEATH}} W_{i,t+1} + \sum_{i \in ENTRY} W_{i,t+1} - \sum_{i \in EXIT} W_{i,t+1}}{\sum_{i \in T} W_{i,t}}
\]

We can linearize the first term when the wealth of dying people is small compared to the total wealth in the top

\[
\sum_{i \in T \setminus \text{Death}} W_{i,t+1} \sum_{i \in T} W_{i,t} = \sum_{i \in T \setminus \text{Death}} W_{i,t} + \sum_{i \in \text{Death}} W'_{i,t} - \sum_{i \in \text{Death}} W_{i,t+1} - \sum_{i \in T} W_{i,t}
\]

Putting all the terms together, we obtain the following decomposition

\[
\sum_{i \in T'} W_{i,t+1} \approx \sum_{i \in T \setminus \text{Death}} W_{i,t} + \sum_{i \in \text{ENTRY}} (W_{i,t+1} - q_{t+1}) + \sum_{i \in \text{EXIT}} (q_{t+1} - W_{i,t+1})
\]

\[
\sum_{i \in \text{Death}} W_{i,t+1} \sum_{i \in T} W_{i,t} + \sum_{i \in T} W_{i,t} + \sum_{i \in \text{ENTRY}} q_{t+1} - \sum_{i \in \text{EXIT} \cup \text{Death}} q_{t+1}
\]

\[
\sum_{i \in T} W_{i,t}
\]

**Accounting for Individuals Dropping from the Top.** I apply this decomposition to the dynamics of the Top 400. The only difficulty is to measure the wealth of individuals dropping from the top, which is required to construct the individual growth term and in the exit term. For this reason, I concentrate on the top \(1^{-6}\), a subset of the top 400. Only 13 individuals in the top \(1e^{-6}\) have dropped directly out of the top 400. I input the return of these drop-offs by maximum likelihood. The results are not sensitive to the details of the imputation method \(^9\)

\(^9\)This robustness is due to the Pareto property of the wealth distribution. Intuitively, a drop from the Top \(1^{-6}\) to outside the Top 400 corresponds in average to a negative wealth return of \(-90\%\). Therefore, the remaining uncertainty on the total return is lower than 10 percentage points. Since there is an average of 100 individuals in the top \(1e^{-6}\), the resulting uncertainty on the average wealth return is lower than 0.05 percentage points.
2.3.3 Results

The result of this decomposition can be found in Table 2.2 and Figure 2.2. The increase in top shares since the 1980s has been equally driven by the growth of individuals at the top and idiosyncratic returns. In the time series, the importance of idiosyncratic returns has declined over the years, while the importance of the wealth growth of individuals at the top has increased. At the business cycle frequency, most of the fluctuations in top shares are due to fluctuations in the growth rate of existing individuals, rather than fluctuations in idiosyncratic returns.

Decomposing Individual Growth

The relative growth rate of existing individuals can be decomposed into a growth rate due to portfolio return, labor income / tax rate, and the growth of average wealth in the U.S.

Term due to Individual Growth

\[ T_{t+1} = r_{T_t}^T + y - \tau - U.S. \text{ Wealth growth per capita}_t \]

where \( \tau \) is the tax rate and \( y \) is the income / wealth ratio.

The portfolio returns can itself be decomposed using Fama-French three factors model Fama and French (1993)

\[ r_{T_t}^T - r_{t}^f = \alpha + \beta_M \times (r_{t}^m - r_{t}^f) + \beta_{hml} \times hml_t + \beta_{smb} \times smb_t + \epsilon_t \]

where \( r_{t}^f \) is the risk free rate and \( r_{t}^m \) is the stock market return. To estimate this decomposition, I regress \( r_{t}^T - r_{t}^f \) on excess returns, \( hml \) and \( smb \). The intercept and residual of this regression \( \alpha + \epsilon_t \) can be interpreted as a mix of consumption rate and abnormal portfolio returns. I find that the wealth growth of individuals at the top is positively related to the market (\( \beta_{erm} = 0.7, t_{erm} = 5.7 \)), negatively related to \( hml \) (insignificantly: \( \beta_{hml} = -0.17, t_{hml} = -1.1 \)) and negatively to \( smb \) (significantly: \( \beta_{smb} = -0.58, t_{smb} = -2.47 \)) - intuitively, the biggest fortunes own the biggest firms.
The results of this decomposition can be found in Table 2.2 and Figure 2.3. The decomposition reveals that part of the high wealth growth of individuals at the top comes from a compensation for their risk exposures (i.e. their $\beta$). However, the intercept also suggests that the top households have earned abnormal portfolio returns.

**Decomposing Idiosyncratic Returns**

The term due to idiosyncratic returns can be more finely decomposed into a term due to individuals entering the top and a term due individuals exiting the top, as in Proposition 8

$$\text{Term due to Idiosyncratic returns}_{t+1} = \frac{\sum_{i \in \text{ENTRY}} (W_{i,t+1} - q_{t+1})}{\sum_{i \in T} W_{i,t}} + \frac{\sum_{i \in \text{EXIT}} (q_{t+1} - W_{i,t+1})}{\sum_{i \in T} W_{i,t}}$$

The results of this decomposition can be found in Table 2.2 and Figure 2.4. The entry term (which is due to the upward jumps of individuals outside the top) is quantitatively more important than the exit term (which is due to the fact that the size of the downward jumps of individuals exiting the top does not impact top shares). Remember that the two would be equal if there were no jumps (i.e. case of idiosyncratic volatility).

A further way to decompose each term is to write it as the product of the number of entry time the average growth per entry:

$$\frac{\sum_{i \in \text{ENTRY}} (W_{i,t+1} - q_{t+1})}{\sum_{i \in T} W_{i,t}} = \frac{\sum_{i \in \text{ENTRY}}}{\sum_{i \in I} \text{Intensity}} \times \frac{\sum_{i \in \text{ENTRY}} (W_{i,t+1} - q_{t+1})}{\sum_{i \in \text{ENTRY}} \sum_{i \in I} \sum_{i \in \text{ENTRY}} \text{Average Growth}}$$

Section 2.4 plots this decomposition. The two terms co-move together: in times where there is more entry in the top 400, the average growth per entry is also higher. Quantitatively, however, changes in the average growth per entry are much more important than the changes in the number of entry.

---

10 Using Proposition Proposition 8, one can compute the average return due to idiosyncratic volatility. I estimate $\sigma(q) \approx 0.25$ as the average standard deviation of the residual of log returns regressed on year dummies for households close to the quantile. I measure $\zeta \approx 1.5$ as the Pareto exponent in the tail. These two estimates mean that idiosyncratic volatility accounts for an annual growth rate of 1%. In contrast, the annual growth due to idiosyncratic returns is 5% before 1995. Idiosyncratic volatility has a much smaller impact than jumps for the dynamics of top wealth shares.
The term to idiosyncratic returns is particularly high at the end of the 90s, a period which corresponds to large development in computing and telecommunications.

2.4 Conclusion

I develop a new methodology to decompose the growth in top wealth shares. The methodology is very general and could be applied to other settings.

I find that the rise in inequality since 1982 has been equally driven by two forces: an increase of idiosyncratic returns and an increase in the average wealth growth of people at the top. In the first period, (1982-1995), the rise is mostly driven by an increase of idiosyncratic returns. In the second period (1995-2014), the rise is mostly driven by an increase of the average wealth growth of individuals at the top. In particular, the data suggests that, in recent years, individuals at the top have earned abnormal portfolio returns.
Table 2.1: Decomposing the Annual Growth of Top Shares (Ten-year-period)

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Individual Growth</th>
<th>Idiosyncratic returns</th>
<th>Pop. Renew.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Market Exp.</td>
<td>Labor/Tax</td>
<td>Res.</td>
</tr>
<tr>
<td>1982-1994</td>
<td>5.9</td>
<td>3.2</td>
<td>11.6</td>
<td>0.7</td>
</tr>
<tr>
<td>1995-2004</td>
<td>5.7</td>
<td>4.2</td>
<td>6.3</td>
<td>0.6</td>
</tr>
<tr>
<td>2005-2014</td>
<td>4.1</td>
<td>3.6</td>
<td>5.8</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 2.2: Decomposing the Annual Growth of Top Shares

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Individual Growth</th>
<th>Idiosyncratic returns</th>
<th>Pop. Renew.</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Total</td>
<td>Market Exp.</td>
<td>Labor/Tax</td>
<td>Res.</td>
</tr>
<tr>
<td>1984</td>
<td>-3.2</td>
<td>-7.0</td>
<td>16.8</td>
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</tr>
<tr>
<td>1985</td>
<td>-7.5</td>
<td>-9.8</td>
<td>19.8</td>
<td>0.7</td>
</tr>
<tr>
<td>1986</td>
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<td>22.4</td>
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</tr>
<tr>
<td>1987</td>
<td>-9.7</td>
<td>-9.8</td>
<td>14.8</td>
<td>0.7</td>
</tr>
<tr>
<td>1988</td>
<td>7.7</td>
<td>7.2</td>
<td>10.9</td>
<td>0.7</td>
</tr>
<tr>
<td>1989</td>
<td>-2.1</td>
<td>-4.6</td>
<td>12.1</td>
<td>0.7</td>
</tr>
<tr>
<td>1990</td>
<td>6.2</td>
<td>3.5</td>
<td>7.5</td>
<td>0.7</td>
</tr>
<tr>
<td>1991</td>
<td>4.8</td>
<td>1.2</td>
<td>6.8</td>
<td>0.7</td>
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<tr>
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</tr>
<tr>
<td>1993</td>
<td>4.1</td>
<td>4.8</td>
<td>2.3</td>
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<tr>
<td>1994</td>
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<td>7.6</td>
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</tr>
<tr>
<td>1995</td>
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<tr>
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</tr>
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<td>2.7</td>
<td>-0.5</td>
</tr>
<tr>
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<td>-0.8</td>
<td>6.7</td>
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</tr>
<tr>
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<td>2.6</td>
<td>3.8</td>
<td>-0.8</td>
</tr>
<tr>
<td>2006</td>
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<td>17.9</td>
<td>14.9</td>
<td>-0.8</td>
</tr>
<tr>
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<td>12.6</td>
<td>1.6</td>
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<td>2010</td>
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<td>6.5</td>
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<td>2011</td>
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<td>6.0</td>
<td>10.3</td>
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<tr>
<td>2012</td>
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<td>6.6</td>
<td>6.8</td>
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<tr>
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<td>15.4</td>
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<tr>
<td>2014</td>
<td>-3.6</td>
<td>-3.9</td>
<td>9.2</td>
<td>-0.7</td>
</tr>
</tbody>
</table>

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Figure 2.1: Cumulative Growth of Top Wealth Shares
Figure 2.2: Decomposing the Growth of Forbes 400 Wealth Share

(a) Returns

(b) Cumulative Returns

Figure 2.3: Decomposing the Growth of Individuals at the Top

(a) Returns

(b) Cumulative Returns

Figure 2.4: Decomposing Idiosyncratic returns

(a) Returns

(b) Cumulative Returns
(a) Decomposition Entry

(b) Decomposition Exit
Appendix

2.A The Dynamics of Top Shares

Proof of Proposition 7. Derive (2.1) to obtain \( \frac{dq_t}{dt} \)

\[ 0 = -q_t(q_t) \frac{dq_t}{dt} + \int_{q_t}^{+\infty} \frac{dg_t(x)}{dt} dx. \]

Derive (2.2) to obtain \( \frac{dT_t}{dt} \)

\[
\frac{dT_t}{dt} = -q_t g_t(q_t) \frac{dq_t}{dt} + \int_{q_t}^{+\infty} \frac{x}{dt} d\int g_t(x) dx
\]

\[ = \int_{q_t}^{+\infty} (x - q_t) \frac{d\int g_t(x)}{dt} dx. \]

Proof of Proposition 8. The Kolmogorov Forward equation corresponding to the wealth process is

\[ \frac{dg_t}{dt} = -\partial_x (\mu x g_t) + \partial^2_x \left( \frac{\sigma^2 x^2}{2} g_t \right) - \delta g_t + \lambda E[e^{-\kappa} g_t(xe^{-\kappa}) - g_t(x)]. \]

Substituting \( \frac{dg_t}{dt} \) into Proposition 7

\[
\frac{dT_t}{dt} = \int_{q_t}^{+\infty} (x - q_t) \frac{d\int g_t(x)}{dt} dx
\]

\[ = \mu T_t + \frac{\sigma^2 q_t^2}{2} g_t(q_t) - \delta (T_t - q_t p) + \lambda \int_{q_t}^{+\infty} (x - q_t) e^{-\kappa} E[g_t(xe^{-\kappa}) - g_t(x)] dx \]
The new jump term can be rewritten by inverting the summation over $\kappa$ and $x$:

$$\lambda \int_{q_t}^{+\infty} (x - q_t)e^{-\kappa}E[g_t(xe^{-\kappa}) - g_t(x)]dx$$

$$= \lambda \int_{x=q_t}^{+\infty} (x - q_t) \int_{\kappa=-\infty}^{+\infty} (e^{-\kappa}g_t(xe^{-\kappa}) - g_t(x))\phi(\kappa)d\kappa$$

$$= \lambda \int_{\kappa=-\infty}^{+\infty} \phi(\kappa)d\kappa \int_{x=q_t}^{+\infty} (x - q_t)(e^{-\kappa}g_t(xe^{-\kappa}) - g_t(x))dx$$

$$= \lambda \int_{\kappa=-\infty}^{+\infty} \phi(\kappa)d\kappa \left( \int_{q_t e^{-\kappa}}^{q_t} (e^\kappa x - q_t)g_t(x)dx + \int_{x=q_t}^{\infty} (e^\kappa - 1)xg_t(x)dx \right)$$

$$= \lambda E\left[ \int_{q_t e^{-\kappa}}^{q_t} (e^\kappa x - q_t)g_t(x)dx \right].$$

Proof of Proposition 9. We have $q_t(p) = \partial_p T(p)$ and $g_t(q_t(p)) = -1/\partial_{pp} T(p)$. A power law is characterized by $T(p) \sim p^{1-1/\zeta}$ as $p$ tends to zero. This means that

$$\lim_{x \to +\infty} \frac{q_t p}{T_t} = 1 - \frac{1}{\zeta}$$

$$\lim_{x \to +\infty} \frac{g_t p}{q_t} = \frac{1}{\zeta}$$

Plugging this guess into Proposition 8, and setting the growth of top wealth shares to zero, one obtains the result.

Proof of Proposition 10. Similar to the proof of Proposition 9.

I now derive a formula in the case of diffusive Markov process with re-injection according to the pdf $\psi$.

**Proposition 11 (Markov Process).** Take a wealth process that follows

$$dx = \mu(x)dt + \sigma(x)dZ_{it}$$

where $Z_{it}$ is a standard Brownian motion, and with death rate $\delta$ and re-injection with density $\psi$. 

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The evolution of $T_t$ is

$$\frac{dT_t}{dt} = \int_q^\infty \mu(x)g_t(x)dx + \frac{\sigma^2(q_t)}{2}g_t(q_t)$$

$$\quad + \delta \int_q^{+\infty} (x - q_t)\psi(x)dx - \delta(T_t - q_tP).$$

Proof of Proposition 11. The Kolmogorov Forward equation corresponding to the wealth process is

$$\frac{dg_t}{dt} = \partial_x(\mu(x)g_t) + \frac{\sigma^2(x)}{2}g_t + \delta(\psi(x) - g_t(x)) + \lambda\int(g_t(x - \kappa) - g_t(x))\phi(\kappa).$$

Substitute this equation into Proposition 7 and integrate by part to obtain the result. □

I now prove that formula Proposition 11 holds with aggregate risk, after substituting $\mu$ by the average geometric return $\mu + \sigma\frac{dZ_t}{dt}$.

**Proposition 12 (Aggregate Risk).** Take a wealth process that follows

$$dx = \mu(x)dt + \sigma_{idio}(x)dZ_{it} + \sigma(x)dZ_t$$

where $Z_{it}$ is an idiosyncratic standard Brownian motion, $Z_t$ is an aggregate standard Brownian motion, and with death rate $\delta$ and reinjection with density $\psi$.

The evolution of $T_t$ follows

$$\frac{dT_t}{dt} = \int_q^\infty (\mu(x) + \sigma(xdZ_t)g_t(x)dx + \frac{\sigma^2_{idio}(q_t)}{2}g_t(q_t) + \delta \int_q^{+\infty} (x - q_t)\psi(x)dx - \delta(T_t - q_tP).$$

Proof of Proposition 12. First, derive the change in top share with aggregate risk using Ito

$$0 = -g_t(q_t)\frac{dq_t}{dt} + \int_q^{+\infty} \frac{dg_t(x)}{dt}dx - \sigma[dg_t(q_t)]\sigma[dq_t]$$

$$\frac{dT_t}{dt} = -q_tg_t(q_t)\frac{dq_t}{dt} + \int_q^\infty x\frac{dg_t(x)}{dt}dx - q_t\sigma[dg_t(q_t)]\sigma[dq_t] - \frac{1}{2}g_t(q_t)\sigma[dq_t]^2.$$
Therefore

\[ dT_t = \int_{q_t}^{\infty} (x - q_t)dg_t(x)dx - \frac{1}{2} \frac{1}{g_t(q_t)} \left( \int_{q_t}^{\infty} \sigma[dg_t] \right)^2. \]

The Kolmogorov Forward equation corresponding to the wealth process is

\[ \frac{dg_t}{dt} = \partial_x((\mu(x) + \sigma(x) \frac{dZ}{dt} g_t) \partial_x^2 \left( \frac{\sigma^2(x) + \sigma_{idio}^2(x)}{2} g_t \right) + \delta(\psi(x) - g_t(x)), \]

Substitute this equation into Proposition 7 and integrate by part to obtain the result. □
Bibliography


CHAPTER 3

Banks’ Exposure to Interest Rate Risk and the Transmission of Monetary Policy

3.1 Introduction

Bank profits are exposed to interest rates movements. Some banks issue interest bearing deposits so that their profits shrink when rates go up as they have to increase the compensation of depositors. Other banks tend to lend at variable interest rate so that their profits go up when the short rate increases. The gap between the interest rate sensitivities of assets and liabilities is called the “income gap”: it measures the extent to which banking profits respond to monetary policy tightening Flannery (1983). If the Modigliani-Miller proposition for banks does not hold and banking profits affect lending activity Kashyap and J. C. Stein (1995), the income gap of banks may affect the extent to which monetary policy is transmitted to the real economy. This paper documents empirically that banks with a larger income gap respond to a monetary policy tightening by lending relatively more, which in turn affect the ability of their borrowers to invest and hire. Our income gap effect is not in contradiction with the lending channel view of monetary policy: Our focus is the banks’ relative propensity to lend in reaction to interest rate shocks, not their aggregate propensity to lend. Our analysis has however aggregate consequences, as it implies that a

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1The material for this chapter was co-authored with Augustin Landier, David Sraer, and David Thesmar. The material in this chapter was presented at St Gallen (2012), Humboldt University (2013), Bank of England (2013), AFA Meetings (2014)
high aggregate positive income gap of the financial sector is likely to act as a mitigating force to the traditional lending channel.

We start by documenting the exposure of banks’ cash flows to interest rate risk. Using bank holding company (BHC) data on U.S. banks – available quarterly from 1986 to 2013 – we measure a bank’s income gap as the difference between the dollar amount of the bank’s assets that re-price or mature within a year and the dollar amount of liabilities that re-price or mature within a year, normalized by total assets. We show substantial variations in the measured income gap, both in the cross-section and in the time-series. We also document that banks do not fully hedge their interest rate exposure. In our data, a bank’s income gap strongly predicts the sensitivity of the bank’s future profits to interest rates. This result echoes earlier work by Flannery and James (1984), who show that income gap explains how S&Ls’ stock returns react to changes in interest rates. It is also consistent with recent findings by Begeneau, Piazzesi, and Schneider (2012), who show that for the four largest US banks, net derivative positions tend to amplify, not offset, balance sheet exposure to interest rate risk. This incomplete hedging is also consistent with English, Heuvel, and Zakrajsek (2012), who document that unexpected increases in interest rates cause bank share prices to drop, especially for banks with a low maturity mismatch.

We then provide evidence that income gap strongly predicts how bank-level lending reacts to interest rate movements. Since interest rate risk exposure affects bank cash-flows, it may affect their ability to lend if external funding is costly. Quantitatively, we find that a 100 basis point increase in the Fed funds rate leads a bank at the 75th percentile of the income gap distribution to increase its lending by about 0.4 ppt more than a bank at the 25th percentile. This magnitude is to be compared to quarterly loan growth in our data, which equals 1.8%. The estimated effect is thus large in spite of potential measurement errors in our income gap measure. It also resists a battery of robustness checks. In particular, our estimation is unchanged after controlling for factors previously identified in the literature as determining the sensitivity of lending to interest rates like leverage, bank size and asset liquidity. In particular, income gap is a much stronger determinant of the sensitivity of lending to interest rates than leverage. We find the estimated effect to be larger for smaller banks and for banks that report no hedging of interest-rate risk, which is consistent with
the idea that smaller banks are more financially constrained and that income gap is better measured for banks that do not hedge. We also report evidence that the effect propagates through internal capital markets from the income gap of the bank holding company to banking subsidiaries.

The rest of our analysis exploits loan-level data from Dealscan. A potential concern with the previous results is that they are driven by the endogenous matching of banks and firms. As is standard in the banking literature, we alleviate this concern by using loan-level data, which allows us to control for credit demand shocks through the inclusion of borrower-time fixed effects Khwaja and Mian (2008) and Gan (2007). We find that the effect of income gap on the sensitivity of bank lending to interest rates remains essentially similar, both in terms of magnitude and statistical significance, after including these additional fixed-effects. This result, consistent with random matching between banks and firms, is reminiscent of earlier findings in the literature Khwaja and Mian (2008), Jiménez et al. (2012), and Iyer et al. (2014).

We further exploit this loan level dataset, by linking lenders’ income gap to the real behavior of their borrowers, both in terms of investment and hiring. This analysis relies on the identifying assumption that banks and firms are randomly matched, an assumption warranted by our loan-level findings as well as additional analyses discussed in the paper. We find that firms borrowing from higher gap banks tend to borrow relatively more when interest rates go up, suggesting—in line with the relationship banking literature—very little substitution between sources of outside borrowing. We then document that the income gap of a firm’s main lender affects the firm’s sensitivity of investment and hiring to interest rates. For instance, assuming the average bank has a gap of 30% (this corresponds to the 2013 asset weighted average), a 100bp increase in Fed funds rate would lead the average firm to increase employment by .6% more than average. We find a similar effect for capital expenditures.

The mechanism we document at the micro-level has potentially important effect for the transmission of monetary policy. In the data, banks’ average income gap is positive. Part of this surprising finding is due to the fact that we treat transaction deposits or savings deposits as liabilities that do not reprice immediately with the short-term rate, though having a
zero contractual maturity. In doing so, we follow the literature that finds that interest rates on these core deposits adjust only sluggishly to changes in short-term market rates (English, Heuvel, and Zakrajsek, 2012, Hannan and Berger, 1991, Neumark and Sharpe, 1992, Drechsler, Savov, and Schnabl, 2014). Given an aggregate positive income gap, our cross-sectional estimate suggests that the exposure of the banking sector to interest rate movement may potentially dampen the effect of monetary policy shocks coming from the traditional bank lending channel, both in terms of lending activity and real effects.

Our paper is mainly related to the literature on the bank lending channel of transmission of monetary policy. This literature seeks to find evidence that monetary policy affects the economy via credit supply. The bank lending channel is based on a failure of the Modigliani-Miller proposition for banks. Consistent with this argument, monetary tightening has been shown to reduce lending by banks that are smaller (Kashyap and J. C. Stein (1995)), unrelated to a large banking group (Campello (2002)), hold less liquid assets (Kashyap and J. C. Stein (2000)) or have higher leverage (Kishan and Opiela (2000), Gambacorta and Mistrulli (2004)). We find that the “income gap” effect we document is essentially orthogonal to these effects, robust across specifications. In line with the more recent banking literature, we are also able to document that these effects are not driven by endogenous matching of banks to borrowers Khwaja and Mian (2008), Jiménez et al. (2012), and Iyer et al. (2014). This allows us to trace the impact of interest rate shocks onto firm growth. Via its focus on interest risk exposure, our paper also relates to the emerging literature on interest rate risk in banking and corporate finance (Flannery and James (1984), Chava and Purnanandam (2007), Purnanandam (2007), and Begeneau, Piazzesi, and Schneider (2012), Vickery (2008)). These papers are mostly concerned with the analysis of banks’ risk-management and its implication for stock returns. We complement this literature by focusing on bank lending and borrower behavior.

The rest of the paper is organized as follows. Section 3.2 presents the datasets used in the paper. Section 3.3 examines the link between income gap, profits and lending policy, using bank level data only. Section 3.4 shifts the focus to loan-level data: We control for non-random matching of banks to firms and investigate the effect of income gap on real
corporate behavior. Section 3.5 further discusses interpretations of our results. Section 3.6 concludes.

3.2 Data and Descriptive statistics

3.2.1 Data construction

Bank-level data

We use quarterly Consolidated Financial Statements for Bank Holding Companies (BHC) available from WRDS (form FR Y-9C). These reports have to be filed with the Federal Reserve by all US bank holding companies with total consolidated assets of $500 million or more. Our data covers the period going from 1986:1 to 2013:4. We restrict our analysis to all BHCs with more than $1bn of assets in 2010 dollars. The advantage of BHC-level consolidated statements is that they report measures of the bank’s income gap every year from 1986 to 2011 (see Section 3.2.2). Commercial bank-level data that have been used in the literature (Kashyap and J. C. Stein, 2000; Campello, 2002) do not have a consistent measure of income gap over such a long period.

For each of these BHCs, we use the data to construct a set of dependent and control variables. We report summary statistics for these variables in Table 3.1 and provide details on the construction of these variables in Appendix 3.A.\(^2\) We use two sets of dependent variables in our analysis. First are income-related variables that we expect to be affected by movements in interest rates: net interest income and net profits. We also use non-interest income as a “placebo” variable, since non-interest income should not be directly affected by variations in interest rates and income gap. We normalize all these variables by total assets.\(^3\) Second, we look at two variables measuring credit growth at the bank level: (1) the quarterly change in log commercial & and industrial loans and (2) the quarterly change in log total loans.

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\(^2\)We describe the “income gap” measure in Section 3.2.2 in detail.

\(^3\)All ratios are trimmed by removing observations that are more than five interquartile ranges away from the median. Our results are qualitatively similar when trimming at the 5th or the 1st percentile of the distribution.
As shown in Table 3.1, the quarterly change in interest income is small compared to total assets as interest rates do not change much from quarter to quarter. On average, quarterly net interest income accounts for about 0.9% of total assets, while the bottomline (earnings) is less than 0.2%. Non-interest income is as large as interest income on average (1% of assets compared to 0.9%), but much more variable (s.d. of 0.023 vs 0.003).

Our analysis uses as control variables the determinants of the sensitivity of bank lending to interest rates that have been discussed in the previous literature. In line with Kashyap and J. C. Stein (2000), we control for equity normalized by total assets, size (log of total assets) and the share of liquid securities. The share of liquid securities variable differs somewhat from Kashyap and J. C. Stein (2000)’s definition (Fed funds sold + AFS securities) due to differences between BHC consolidated data and call reports. In our data, available-for-sale securities are only available after 1993 and Fed funds sold are only available after 2001. To construct our measure of liquid securities, we thus deviate from Kashyap and J. C. Stein (2000)’s definition and take all AFS securities normalized by total assets. Consequently, liquidity measure is available for the 1994-2013 sub-period only.

These control variables, obtained from accounts consolidated at the BHC-level, have orders of magnitudes that are similar to existing studies on commercial bank-level data: Average equity-to-asset ratio is 8.7% in our data, compared to 9.5% in Campello (2002)’s sample (which covers the 1981-1997 period). The share of liquid assets is 27% in our sample, compared to 32% in his sample.

Loan-level Data

We also use more granular loan-level data. These data allow us to connect banks to individual borrowers. Our source here is the Dealscan database which contains publicly available information on over 100,000 corporate loans booked since 1987. It contains detailed transaction-level information on the size, maturity and the terms (fees, rates) of individual loan deals, as well as the identity of the borrowers and the lenders. For the majority of originations, DealScan reports the syndicate structure but not the actual loan shares. Such information loss is, however, negligible: For the sample of syndicates with non missing loan shares, the syndicate structure explains most of the variations in lender shares, with an
explanatory power of 94.15%. For observations for which loan shares are missing, we thus impute loan shares from the average lender shares of syndicates with a similar structure (i.e. same number of lead lenders and participants).

We then construct the panel using the following procedure. DealScan contains the origination date, amount, and termination date of individual loan deals (see e.g. Chava and Roberts, 2008b for a thorough presentation of the DealScan dataset). We combine this information with loan shares to construct a yearly panel of outstanding loans for each borrower-lender pair that is active – i.e. for which there is at least one outstanding loan. Thus, we work on the loan-level data at the annual frequency instead of quarterly because loan-level data are too granular to be useful at such high frequency (i.e. there would be too many observations corresponding to zero lending). Since the coverage of banks by Dealscan tends to be volatile, we drop lenders for which aggregate lending growth is higher than 200% or lower than -50%. Finally, we drop lenders with a different top-holder BHC in year \( t \) or \( t+1 \). Our key dependent variable is then the symmetric loan growth measure \( \Delta L_{i \rightarrow j, t} \) which is the change in loan outstanding from bank \( i \) for firm \( j \) normalized by the across period average (see Appendix 3.A). This growth rate measure has the advantage of being bounded by -2 and +2. It also accommodates initiation and terminations of lending relationships: It is equal to +2 when the bank starts lending and -2 when it stops lending to a given firm.

We then complement these data with information about the lender, most importantly the income gap of its current top-holder BHC. To do so, we manually match the biggest lenders in DealScan to US commercial banks, based on the names and cities reported in DealScan on the one hand and in the Reports of Condition and Income (Call Reports) on the other hand. US commercial banks report their current top-holder BHC in the Call Reports (rssd9348) : this allows us to construct a match table between Dealscan lenders and top-holder BHCs. In total, we match 300 lenders to 139 BHCs. The matched lenders represent 3.86% of the total number of US lenders in DealScan but they account for 44.91% of the total number of loans. The remaining lenders are either institutions that are not part of a BHC or smaller commercial banks that could not be matched. The BHCs identified through our procedure

\[ \Delta \] Aggregate lending growth is obtained by summing loans across all of the bank borrowers. A very high or very low growth may reflect merger activities or alternatively abrupt increase of the bank coverage in DealScan.
in Dealscan are bigger than the average BHC in the bank level data (we report the results of this comparison in Appendix Table 3.1). This is not a surprise given that we manually matched only the biggest banks in Dealscan (another reason is that DealScan only covers the biggest deals). These banks also tend to have a bigger average gap. We find, however, that the standard deviation of income gap among the matched BHCs (.16) is similar to the s.d. in the overall BHC sample (.19).

DealScan contains coarse information on the borrower: debt, zip code, industry SIC code, and total sales at origination. In order to investigate real effects of bank lending, we add accounting information about publicly listed borrowers from COMPUSTAT. To implement this, we use a updated version of the crosswalk provided by Chava and Roberts (2008a). This crosswalk allows to match 9,767 borrowers out of the 25,481 borrowers in our sample. For these firms, we retrieve from COMPUSTAT information on employment (compustat item EMP), total assets (AT) and total financial debt (DLC+DLTT). These three variables are winsorized, based on the median plus or minus 5 times the interquartile range.

**Interest Rates**

We use three time-series of interest rates. In most of our regressions, we use the Fed funds rate as our measure of short-term interest rate, available monthly from the Federal Reserve’s website. We define quarterly interest rates as the interest rate prevailing in the last month of the quarter. In the penultimate Section, we also use the spread on the 10-year treasury bond, available from the Fed’s website. Finally, we construct a measure of expected short interest rates using the Fama and Bliss (1987) series of zero coupon bond prices. For each quarter $t$, we use as our measure of expected future short rate the 1-year forward rate as of $t - 8$.

\[ \frac{p_{2,t-s}}{p_{3,t-s}} - 1 \]

This forward is calculated using the zero coupon bond prices according to the formula $p_{2,t-s}/p_{3,t-s} - 1$, where $p_{j,s}$ is the price of the discount bond of maturity $j$ at date $s$. 

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3.2.2 Exposure to Interest Rate Risk

Income Gap: Definition and Measurement

We use the definition of the income gap of a financial institution in Mishkin and Eakins (2009):

\[
\text{Income Gap} = RSA - RSL
\]  

(3.1)

where RSA is a measure of the amount of assets that either reprice, or mature, within one year, and RSL the amount of the liabilities that mature or reprice within a year. RSA (RSL) is the number of dollars of assets (liability) that will pay (cost) variable interest rate. Hence, the income gap measures the extent to which a bank’s net interest income are sensitive to interest rates changes. Because the income gap is a measure of exposure to interest rate risk, Mishkin and Eakins (2009) propose to assess the impact of a potential change in short rates \( \Delta r \) on bank income by calculating: Income Gap \( \times \Delta r \).

However, this relation has no reason to hold exactly. The income gap measures a bank’s exposure to interest rate risk imperfectly. First, the cost of debt rollover may differ from the short rate. New short-term lending/borrowing will also be connected to the improving/worsening position of the bank on financial markets (for liabilities) and on the lending market (for assets). This introduces some noise in the relationship between banks income and Income Gap \( \times \Delta r \). Second, depending on their repricing frequency, assets or liabilities that reprice may do so at times where short rates are not moving. This will weaken the correlation between change in interest income and Income Gap \( \times \Delta r \). To see this, imagine that a bank holds a $100 loan, financed with fixed rate debt, that reprices every year on June 1. This bank has an income gap of $100 (RSA=100, RSL=0). Now, assume that the short rate increases by 100bp on February 20. Then, in the first quarter of the year, bank interest income is not changing at all, while the bank has a $100 income gap and interest rates have risen by 100bp. During the second quarter, the short rate is flat, but bank interest income is now increasing by $1 = 1\% \times $100. For these two consecutive quarters, the correlation between gap-weighted rate changes and interest income is in fact negative. This represents another source of noise in the relation between banks’ income and
Income Gap × ∆r. Finally, banks might be hedging some of their interest rate exposure, which would also weaken the link between income and Income Gap × ∆r. Despite these sources of measurement error, we believe that this measure of income gap is particularly attractive for our purposes, given its simplicity and direct availability from BHC data.

In the data, we construct the income gap using variables from schedule HC-H of form FR Y-9C, which is specifically dedicated to the interest sensitivity of the balance sheet. RSA is directly provided (item bhck3197). RSL is decomposed into four elements: Long-term debt that reprices within one year (item bhck3298); long-term debt that matures within one year (bhck3409); variable-rate preferred stock (bhck3408); interest-bearing deposit liabilities that reprice or mature within one year (bhck3296), such as certificates of deposits. Empirically, the latter is by far the most important determinant of the liability-side sensitivity to interest rates. All these items are available every year from 1986 to 2011. We scale these variables by total assets, and report summary statistics in Table 3.2.

The average income gap is 12.6% of total assets. This means that, for the average bank, an increase in the short rate by 100bp will raise bank revenues by 0.126 percentage points of assets. There is significant cross-sectional dispersion in income gap across banks, which is crucial for our empirical analysis. About 78% of observations correspond to banks with a positive income gap. For these banks, an increase in interest rates yields an increase in cash flows. A second salient feature of Table 3.2 is that RSL (interest rate-sensitive liabilities) mostly consists of variable rate deposits, that either mature or reprice within a year. Long term debt typically carries a fixed rate.

That the gap is positive can seem surprising because one expects banks to be maturity transformers, thus borrowing short and lending long. The explanation for this seemingly counterintuitive result comes partially from our treatment of deposits. In the BHC data, the item corresponding to short-term deposit liabilities (bck3296) does not include transaction deposits or savings deposits. As mentioned in English, Heuvel, and Zakrajsek (2012), interest rates on these “core” deposits, while having a zero contractual maturity, are known to adjust quite sluggishly to changes in short-term market rates (Hannan and Berger (1991), Neumark and Sharpe (1992)). Therefore, despite their short maturity, it is natural to exclude

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them from our measure of income gap, as they will not induce direct cash-flow changes when interest rates change. However, if these “core” deposits adjust slightly to changes in Fed funds rates, our income gap measure will over-estimate the real income gap.

This treatment of deposits, however, can only partially explain the non-negative average income gap measured in our data. If we were to make the assumption that all non-interest bearing deposits had short maturity as in English, Heuvel, and Zakrajsek, 2012, we would still find that the average income gap is zero, not negative. A reason for this is that we measure the income gap and not the duration gap. The income gap is a cash-flow concept. It measures the extent to which a bank’s interest income is sensitive to the short rate. The duration gap, by contrast, is a value concept. It measures the extent to which equity value is sensitive to the short rate. For a given bank, the duration gap can be negative while the income gap is positive, implying opposite elasticities of cash-flows and equity values to interest rates. This happens for instance when the maturity of long-term assets is substantially longer than that of long-term deposits (which account for most of the liabilities). In fact, the positive average elasticity of banks’ earnings to interest rates has already been observed in the literature. In its seminal contribution, Flannery, 1981 shows that the average income gap for large banks is either close to zero or positive. Flannery, 1983 extends the result to small banks. More recently, English, Heuvel, and Zakrajsek, 2012 shows that a 100 basis point increase in interest rates increases the median bank’s net interest income relative to assets by almost 9 basis points and decreases its market value of equity by 7%. These numbers are very much in line with our average income gap of 12.6%. Although understanding the average income gap in the economy is important, we should emphasize that the sign of the average income gap is irrelevant for our empirical analysis, since the identification strategy exploits cross-sectional differences in income gap.

**Direct evidence on Interest Rate Risk Hedging**

In this section, we ask whether banks use derivatives to neutralize their “natural” exposure to interest rate risk. We can check this directly in the data. The schedule HC-L of the form FR Y-9C reports, starting in 1995, the notional amounts in interest derivatives contracted by banks. Five kinds of derivative contracts are separately reported: Futures (bchk8693),
Forwards (bhck8697), Written options that are exchange traded (bhck8701), Purchased options that are exchange traded (bhck8705), Written options traded over the counter (bhck8709), Purchased options traded over the counter (bhck8713), and Swaps (bhck3450).

We scale all these variables by assets, and report summary statistics in Table 3.3. Swaps turn out to be the most prevalent form of hedge used by banks. For the average bank, they account for about 18% of total assets. This number, however, conceals the presence of large outliers: a handful of banks—between 10 and 20 depending on the year—have total notional amount of swaps greater than their assets. These banks are presumably dealers. Taking out these outliers, the average notional amount is only 4% of total assets, a smaller number than the average income gap. 40% of the observations are banks with no derivative exposure.

Unfortunately, the data only provide us directly with notional exposures. Notional amounts may conceal offsetting positions (see Begeneau, Piazzesi, and Schneider, 2012 for an inference of net exposure using public data). To deal with this issue, we directly look at the sensitivity of each bank’s revenue to interest rate movement and show that it is related to the income gap (see Section 3.3.3).

### 3.3 Bank-level Evidence

In this section, we provide evidence based on bank-level data. We first validate the income gap measure: We find that the income gap is strongly correlated with the sensitivity of interest income and profits to interest rates. Second, we move to the focus of the paper, i.e. that the income gap explains the sensitivity of bank lending to interest rates.

#### 3.3.1 Methodology

In this Section, we follow the specification typically used in the literature (Kashyap and J. C. Stein (1995), Kashyap and J. C. Stein (2000) or Campello (2002)), and estimate the
following linear model for bank $i$ in quarter $t$:

$$
\Delta Y_{it} = \sum_{k=0}^{k=4} \alpha_k (\text{gap}_{it-1} \times \Delta \text{fed funds}_{t-k}) + \sum_{x \in \text{Control}} \gamma_{x,k} (x_{it-1} \times \Delta \text{fed funds}_{t-k})
+ \sum_{k=0}^{k=4} \eta_k \Delta Y_{it-1-k} + \phi \text{gap}_{it-1} + \sum_{x \in \text{Control}} \mu_x x_{it-1} + \delta_t + \epsilon_{it}
$$

(3.2)

where the control variables are $\text{Control} = \{\text{Size}, \text{Equity}, \text{Liquidity}\}$, and standard errors are clustered at the BHC level. All variables are scaled by total assets. $Y_{it}$ stands for the different outcome variables we explore in this Section: Profits and its component, and lending activity. These variables are formally defined in Appendix 3.A. $\sum_{k=0}^{k=4} \alpha_k$ is the cumulative effect of interest rate changes on change in the outcome variable $Y$, given the income gap of bank $i$ and is the coefficient of interest in Equation (3.2). If the income gap variable contains information on banks’ interest rate exposure and if banks do not fully hedge this risk, we expect $\sum_{k=0}^{k=4} \alpha_k > 0$. Consistent with the literature, we control for known determinants of the sensitivity of bank lending to interest rates: bank size, bank equity and bank liquidity (as described in Section 3.2). All the controls are included directly, as well as interacted with current and four lags of interest rate changes. These controls have been shown to explain how bank lending reacts to changes in interest rates. Their economic justification in a profit equation is less clear, but since our ultimate goal is to explain the cross-section of bank lending, we include these controls in the profit equations for consistency.

### 3.3.2 Interest Rate Shocks and Interest Income

Our first test is a sanity check: We use net interest income (interest income minus interest expenses) normalized by lagged assets as our first explanatory variable. We report the estimation results in Table 3.4, column (1) to (5). The bottom panel reports the coefficient of interest, $\sum_{k=0}^{k=4} \alpha_k$, i.e. the cumulative effect of a change in interest rate on the change in net interest income, as well as the p-value of the F-test for $\sum_{k=0}^{k=4} \alpha_k = 0$. Column (1) provides estimation results over the whole sample. $\sum_{k=0}^{k=4} \alpha_k = 0$ is significantly different from 0 at the 1% confidence level (p value < .01). Quantitatively, a $1 increase in \text{Gap}_{it-1} \times \Delta \text{FedFunds}_{t},
after 5 consecutive quarters, raises interest income by about 0.06 dollars. The income gap captures a BHC’s interest rate exposure in a statistically meaningful way.

As expected, the effect uncovered in Column (1) of Table 3.4 is observed across bank sizes and is unaffected by the use of hedging by BHCs. This is because the link between the gap and interest income is a near accounting identity. It may fail to hold exactly for the reasons mentioned in Section 3.2.2 but should not be affected by bank size of hedging policy. Columns (2)-(3) split the sample into large and small banks. “Large banks” are defined as the 50 largest BHCs each quarter in terms of total assets. The effect of the income gap on BHCs’ sensitivity of net interest income to interest rates is similar across large and small banks. For both groups, \( \sum_{k=0}^{4} \alpha_k \) is estimated at .06 and is statistically different from 0 at the 1% confidence level. A test of equality of \( \sum_{k=0}^{4} \alpha_k \) across the two groups yields a p-value of 0.83, so that we cannot reject the null hypothesis that these coefficients are in fact equal, as they should be. Columns (4)-(5) split the sample into banks that report some notional exposure to interest rate derivatives and banks that report no interest rate derivative exposure. This sample split reduces the period of estimation to 1995-2013, as notional amounts of interest rate derivatives are not available in the data before 1995. The sample size drops accordingly from 37,888 BHCs-quarter to 26,322 BHCs-quarter. For both groups of banks, we find that the net interest income of banks with a larger income gap increases significantly more following an increase in interest rates, with a cumulated effect of .05 (resp. .07) for banks with (resp. without) derivative exposure. As expected, across the two groups yields a p-value of 0.19.\(^7\)

If our measure perfectly captured a BHC’s income exposure to interest rates, we would expect \( \sum_{k=0}^{4} \alpha_k \) to be estimated at .25, since our interest rates are annualized but income is measured quarterly. Instead, Table 3.4 reports a coefficient estimate of 0.06. As mentioned in Flannery, 1981 and explained in Section 3.2.2, beyond measurement error, there are several reasons to expect a coefficient estimate below .25. Explicit or implicit commitments to renew existing loans without a full pass-through of rate changes to the customer might add noise to the relationship between net interest income and changes in interest rates.

\(^7\)In non-reported regressions, we further restrict the sample to BHCs whose notional interest rate derivative exposure exceeds 10% of total assets (some 4,000 observations): even on this smaller sample, the income gap effect remains strongly significant and has the same order of magnitude.
Reset dates of variable rate loans do not exactly occur at the beginning of each quarter. ARMs, which make up a big fraction of variable rate exposure of banks, have lifetime and periodic caps and floors that reduce the sensitivity of their cash flows to interest rates. All these effects can dampen the elasticity of net interest income to interest rates and probably explain why our estimate of \( \sum_{k=0}^{4} \alpha_k \) is significantly lower than .25.

To further assess the validity of our income gap measure, we run a “placebo” test in Columns (6)-(10) of Table 3.4. We use non-interest income as a dependent variable in Equation (3.2). Non-interest income includes servicing fees, securitization fees, management fees and trading revenue. While non-interest income may be sensitive to interest rate fluctuations, there is no reason why this sensitivity should depend on a BHC’s income gap. Columns (6)-(10) of Table 3.4 reproduce the analysis of Columns (1)-(5) using non-interest income as a dependent variable. In all these specifications, the estimated \( \sum_{k=0}^{4} \alpha_k \) is lower than .01 and statistically indistinguishable from 0.

### 3.3.3 Interest Rate Shocks, Earnings and Value

We show in this section that banks with larger income gap experience a larger relative increase in total earnings and market value following an increase in interest rates. We first estimate Equation (3.2) using total earnings scaled by total assets as a dependent variable. The coefficient estimates are presented on Columns (1) to (5) of Table 3.5. The estimated \( \sum_{k=0}^{4} \alpha_k \) is positive and significantly different from 0 at the 1% confidence level across these specifications. The coefficient estimates in Column (1) imply that a $1 increase in \( \text{Gap}_{t-1} \times \Delta \text{FedFunds}_t \) after 5 consecutive quarters raises earnings by about $0.07. This order of magnitude is similar to the effect on net interest income estimated in Columns (1)-(5) of Table 3.4. This is not surprising since we know from Columns (6)-(10) of Table 3.4 that the income gap has no effect on non-interest income. We obtain a similar estimate for \( \sum_{k=0}^{4} \alpha_k \), when restricting the sample to large banks (Column (2)), small banks (Column (3)), banks with no notional exposure to interest rate derivatives (Column (4)) and banks with some notional exposure to interest rate derivatives (Column (5)).

We then estimate Equation (3.2) using BHCs market value scaled by total assets as a dependent variable. The coefficient estimates are presented on Columns (6) to (10) of
Table 3.5. The estimated $\sum_{k=0}^{4} \alpha_k$ is positive and significantly different from 0 at the 1% confidence level for all specifications except when the sample is restricted to large banks, in which case $\sum_{k=0}^{4} \alpha_k$ is not significantly different from 0. Quantitatively, a $1 increase in $\text{Gap}_{it-1} \times \Delta \text{FedFunds}_t$ raises the market value of banks equity by about $1.8. Since the same shock to $\text{Gap}_{it-1} \times \Delta \text{FedFunds}_t$ raises total earnings by $0.07, this implies an earnings multiple of approximately 25, which is large but plausible. The estimated $\sum_{k=0}^{4} \alpha_k$ is significantly smaller when estimated over the sample of large banks (Column (8)), but is otherwise comparable to its full sample estimate when estimated over the sample of small banks (Column (7)), banks with no derivatives exposure (Column (9)) and banks with derivative exposure (Column (10)).

These results indicate that for most banks in our sample, interest rate hedging does not significantly reduce banks’ balance sheet exposure to interest rate risk. This conclusion seems to hold even for the largest banks, which is consistent with Vickery (2008) and Begeneau, Piazzesi, and Schneider (2012).

### 3.3.4 Interest Risk and Lending

The response of banks’ earnings to monetary policy shocks depend on their income gap. In the presence of financing frictions, the response of banks’ lending to monetary policy should also depend on their income gap Kashyap and J. C. Stein (1995). To test this hypothesis, we follow Kashyap and J. C. Stein (2000) and estimate equation (3.2) using the quarterly change in log-lending as the dependent variable.

We control for bank size and bank equity.\(^8\) In all regressions, we include these controls directly as well as interacted with current and four lags of interest rate changes. These interaction terms help to measure the sensitivity of lending to interest rates. For instance, we expect high equity banks and large banks to be less sensitive to interest rate fluctuations (Kashyap and J. C. Stein (1995)). This is because changes in the cost of funding affect cash flows, which reduces lending by financially constrained banks. We also expect banks with liquid assets to lend relatively more when rates increase (Kashyap and J. C. Stein (2000)).

---

\(^8\)Asset liquidity is available only since 1993. Thus, our main specifications do not include this control, but we include it in robustness Table 3.7 and find our estimates to be unchanged.
We report the results in Table 3.6. Columns (1) to (5) use C&I loan growth as a dependent variable. Columns (6) to (10) use total lending growth as a dependent variable. Columns (1) and (6) use the whole sample. Columns (2) and (7) restrict the sample to small banks, Columns (3) and (8) to large banks, Columns (4) and (7) to banks that report no notional exposure to interest rate derivatives, Columns (5) and (8) to banks reporting no interest rate derivatives exposure. We focus first on the results using total lending growth as a dependent variable. $\sum_{k=0}^{4} \alpha_k$ is estimated over the whole sample at 1.1 and is statistically significant at the 1% level. This effect is economically significant. If we compare a bank at the 25th percentile of the income gap distribution (0.01) and a bank at the 75th percentile (0.24), and if the economy experiences a 100 basis point increase in the Fed funds rate, total loans in the latter bank will grow by about .25 percentage points more. This has to be compared with a sample average quarterly loan growth of about 1.7%.

The estimate of $\sum_{k=0}^{4} \alpha_k$ increases when estimated on the sample of banks with no notional exposure to interest rate derivatives: it is then 1.7 and is significantly different from 0 at the 1% confidence level (Column (9)). When estimated on the sample of banks reporting some notional exposure, $\sum_{k=0}^{4} \alpha_k$ is only .89 and is marginally significant (p-value of .11) (Column (10)). The difference between the point estimates across these two samples is, however, not significantly different from 0 (p-value = 0.29).

The estimate of $\sum_{k=0}^{4} \alpha_k$ is smaller when estimated on the sample of large banks (Column (8)) than when estimated on the sample of small banks (.8 vs. 1.2). While the small-bank estimate is statistically significant at the 1% confidence level, the large-bank estimate is insignificant. However, the two estimates are not statistically different (p-value = 0.72).

The results obtained when using C&I lending as a dependent variable are essentially similar, except that the estimate of $\sum_{k=0}^{4} \alpha_k$ on the sample of banks reporting some hedging is insignificant (Column (5)) and that the large-bank estimate is negative and insignificant (Column (3)).

In contrast to the income gap, the equity ratio control we include in the regressions has no impact on the sensitivity of lending to rates, but the size control goes in the right direction. In row 4 of the bottom panel of Table 3.6, we report the sum of the coefficients on interaction terms with size, measured as banks total assets. We do indeed find that larger
banks are significantly less likely to reduce lending when interest rates increase. This effect is significant for both types of lending measures. In rows 6, we report the same estimates for the coefficients on interaction terms with equity. In most specifications, our results show that equity rich banks are not less likely to cut lending when rates increase. All in all, this analysis suggests that the income gap is a more relevant predictor of bank lending sensitivity to rates than the equity ratio.

One last useful exercise consists in comparing our cross-sectional estimate with numbers obtained on aggregate time series about the transmission of monetary policy shocks. The exercise cannot be exact as it amounts to comparing numbers obtained with different methodologies, but it is still helpful in order to assess the meaningfulness of the income gap channel that we highlight in this paper. We use Bernanke and Blinder, 1992 as a benchmark. Using a VAR methodology, they find that, in response a 1 s.d. shock to Fed Funds rates, aggregate bank loans decrease by some 0.5% after 12 months. Our effect goes opposite to the aggregate transmission effect of monetary policy, as the average gap is positive. In our data, a one s.d. of change in Fed Funds rates equals 1.2%. The average income gap is 13% of total assets. The cumulative impact on total loans is 1.1 (Table 3.6, column 6). Thus, our long-term (12 months) response to a 1 s.d. shock to Fed Funds rates is equal to $1.1 \times 1.3 \times 1.2 = .2$ which is, in absolute value, nearly half of the aggregate response estimated by Bernanke and Blinder, 1992. This exercise should be taken with a grain of salt, as our point estimate (1.1) fluctuates somewhat across specifications (it is actually much bigger when estimated on loan-level data). Taking our micro estimates to the macro also involves ignoring general equilibrium effects (expanding banks crowding out shrinking ones) which we should be taking into account. It however suggests that the income gap channel has the potential to be sizable and significantly dampening to the aggregate response of the economy to monetary policy shocks. Such dampening occurs because, in the data, the income gap is positive as discussed in Section 3.2.2.

3.3.5 Robustness

First, our definition of the income gap treats all core deposits as fixed-rate liabilities (see discussion in Section 3.2.2). This choice is justified from the abundant evidence from the
banking literature documenting that deposits are sluggish, and respond little to interest rates. In unreported regressions, we re-estimate Equation (3.2) using alternative definitions of the income gap that subtract 0, 25, 50, 75 and 100% of all non-interest bearing deposits. The coefficient estimates obtained with these alternative definitions are similar to our main estimates. This result is not surprising since non-interest bearing deposits are a relatively small fraction of total liabilities (about 12.6% of liabilities on average).

Second, we have re-estimated our regressions controlling for bank asset liquidity interacted with interest rates movements, in the spirit of Kashyap and J. C. Stein (2000). We do not have this control in our main specification because it restricts the sample to 1993-2013. Table 3.7 contains the regression results with the new controls. Despite the smaller sample size, the estimate of \( \sum_{k=0}^{k=4} \alpha_k \) are similar to those obtained in our main analysis of Table 3.6. In our sample, asset liquidity does not significantly predict how BHCs’ lending react to changes in interest rates. If anything, the estimated effects have the “wrong” economic sign: banks with more liquid assets tend to reduce their lending more when interest rates increase. The discrepancy with Kashyap and J. C. Stein (2000) originates from our use of BHC-level data instead of commercial bank data. As previously discussed, we focus on BHC data because they contain the income gap over a much longer period (1986-2013).

Third, in Appendix 3.B, we use an alternative technique, also used in the literature, to estimate Equation (3.2). This estimation proceeds in two steps. First, each quarter \( t \), we estimate the sensitivity of \( \Delta Y_{it} \) to the gap \( \text{gap}_{it-1} \) in the cross-section of banks. Then, we ask whether the time series of this slope coefficient is correlated with changes in interest rates. We show in Table 3.1 that the estimates using this alternative method are very similar to the ones we discuss below.

Last, we exploit internal capital markets to somewhat alleviate the concern that borrowers are not randomly assigned to banks. We fully deal with this concern in the next Section, where we exploit loan-level data that allow us to control for demand shocks. But internal capital markets also allow us to test the robustness of our results. The intuition is the following: Commercial banks belonging to larger bank holding companies have two income gaps: their own as well as the one of the BHC. The income gap of the BHC is arguably more exogenous to the commercial bank. In Appendix 3.C, we describe how we re-estimate
a version of equation (3.2) at the commercial bank level, introducing both the income gap of the bank itself and the gap of its top holding BHC. The sample size is reduced because income gaps of commercial banks are only available from 1997 to 2013. We find, however, that while the CB level gap is insignificant in our regressions, the BHC level gap remains statistically significant. We also show that our effect is present in banking groups with at least 3 or even 5 subsidiaries.

3.4 Loan-Level Evidence

In this Section, we confirm and extend the results above on loan level data. This approach has two main advantages. First, it helps to control for the fact that banks may be non-randomly matched to borrowers. This would be a concern if, for instance, banks were choosing their income gap in response to the expected sensitivity of their borrowers to interest rates. Loan level data alleviate this reverse causality concern because we can control for firm level credit demand shocks, as in Khwaja and Mian, 2008. The second advantage of using loan level data is that it allows us to investigate the real effects of credit supply shocks on firm investment and employment.

3.4.1 Testing for Sorting

In this Section, we first provide several pieces of evidence that banks’ income gaps are orthogonal to borrower characteristics. Loan-level data allow us to do this because we can observe borrowers directly. The main concern here is that banks may choose their income gap to match the sensitivity of loan demand to interest rate, i.e. that high gap may be an endogenous response to the fact that customers borrow more when rates are high (Froot, Scharstein, and J. Stein, 1993, Vickery, 2006).

We start with borrowers’ observable characteristics. We use information on borrowers available from Dealscan – hence for the entire sample and not just the matched sample with Compustat. To directly test Vickery, 2006’s hypothesis, we use information from Dealscan on the entire sample of loans to compute the sensitivity (β) of total debt growth to interest
rate changes in each zip code and SIC code (both at the 3 digits level). We also construct average characteristics at the bank level by directly averaging the characteristics of all its borrowers (weighted by loan shares): Sales, debt, age (i.e. number of years since the borrower enters Dealscan), public status and loan maturity. These determinants may be argued to be correlated with loan demand sensitivity to interest rates in a manner not fully captured by our β's. Table 3.8 displays the average of these bank-level covariates after splitting banks into four quantiles of income gap. We also report the sample s.d. for comparison, as well as the p-value of the regression coefficient of each average borrower characteristic on the bank’s gap. We find that no borrower characteristic is significantly related to the income gap.

Another test of this sorting hypothesis comes from comparing banks lending to the same firm. If high income gap banks were to lend to specific firms, one would find that that two banks lending to the same firm should have correlated gaps. We implement this intuition using two approaches. First, for firms with multiple lenders in the same year, we regress the income gap of the firm’s largest lender on the income gap of the firm’s second largest lender, as in Greenstone, Mas, and Nguyen (2015). Second, for firms issuing multiple loans sequentially, we regress the income gap of their future main lenders on the income gap of their current main lenders. None of these test allows us to reject the hypothesis that banks lending to the same firms have uncorrelated gap policies (see Table 3.2).

### 3.4.2 Interest rate shocks and lending supply shocks

We now examine whether banks with high income gap increase more their lending when interest rates go up. The advantage of loan-level data is that we can include a borrower-year fixed effect in such a regression, thus controlling for borrower specific demand shocks as in Khwaja and Mian (2008). More precisely, we estimate the following linear model for bank i and firm j in year t:

\[
\Delta L_{i \rightarrow j, t} = \alpha (\text{gap}_{it-1} \times \Delta \text{fed funds}_t) + \sum_{x \in \text{Control}} \gamma_x (x_{it-1} \times \Delta \text{fed funds}_t) + \eta_k \Delta L_{i \rightarrow j, t-1} + \phi \cdot \text{gap}_{it-1} + \sum_{x \in \text{Control}} \mu_x \cdot x_{it-1} + \delta_{jt} + \epsilon_{it},
\]

(3.3)

We remove zip codes and industries with less than 10 borrowers.
where $L_{i\rightarrow j,t}$ denotes the outstanding loans from bank $i$ to firm $j$. Compared to the baseline model (3.2), the unit of observation is a lender-borrower relationship. When a borrower-year fixed effect $\delta_{jt}$ is included, identification of $\alpha$ effectively comes from the differential lending response to interest rates of banks that lend to the same borrower. $\Delta L_{i\rightarrow j,t}$ is the symmetric bank-firm specific growth rate of loans described in Section 3.2.1, designed to account for relationship terminations and initiations and bounded by -2 and +2.

We report regression results in Table 3.9; We find slightly bigger estimates that in bank level data but the order of magnitude is similar. In Column (1), we first run this specification without borrower x year fixed effect. We find a coefficient of 5.3, very comparable to the aggregate lending effect shown in Table 3.6 of 1.7. Such similarity is reassuring given that we are using very different datasets for lending activity (BHC data in the previous section and Dealscan here). In Column (2), we restrict the sample to firms matched with multiple banks; These firms are the ones on which the specification with borrower-year fixed effect will be identified. We find no material change in the point estimate (4.9) and it remains significant at 5%. In Column (3), we add borrower × year fixed effects. The effect of income gap on lending growth does not lose significance, and keeps the same value (4.7) even though the inclusion of borrower fixed effects doubles the $R^2$ to more than 60%. The investigation of Table 3.9 suggests that controlling for credit demand shocks does not change our estimates, or put differently, that bank-firm matching is not endogenous to the problem at hand. This result is standard in the banking literature (Khwaja and Mian, 2008; Jiménez et al., 2012; Iyer et al., 2014) but it is comforting that it also applies in our set-up. It also allows us to move to the investigation of real effects.

### 3.4.3 Real Effects

We test here whether firms matched to banks with, say, high income gap, tend to be able to invest and grow more when interest rates are high. The idea behind this test is that bank-firm relationships are sticky, and that it is difficult for a firm which faces a reduction in lending to quickly find alternative sources of finance. This would lead the firm to potentially scale down total borrowing, and therefore investment and employment.
To examine this question, we estimate the following linear model for firm $j$ in year $t$:

$$
\Delta \log Y_{jt} = \sum_{k=1}^{k=0} \alpha (\text{gap}_{it-1} \times \Delta \text{fed funds}_{t-k})
+ \sum_{k=0}^{k=1} \sum_{x \in BankControl} \gamma_{kx} (x_{it-1} \times \Delta \text{fed funds}_{t-k}) + \sum_{x \in BankControl} \mu_{x} x_{it-1} + \sum_{x \in FirmControl} \mu_{x} x_{jt-1} + \phi \text{gap}_{jt-1} + \sum_{k=0}^{k=1} \eta \Delta Y_{jt-k-1} + \delta_{t} + \epsilon_{it}
$$

(3.4)

The dependent variables are alternatively the firm debt growth, employment growth and total asset growth described in Section 3.2.1. In these regressions, $\text{gap}_{it-1}$ is the income gap of the firm’s lead arranger. The idea is that the banking relationship is mostly built on information exchange between the borrower and the lead arranger. We also need to restrict ourselves to firms for which accounting information is available in Compustat. We only keep firm-year observations with a fiscal year ending in December, so that their balance sheet data match the reporting period of banks. We show in Table 3.3 that our main loan-level results do not change when we focus on the subsample of Compustat-matched firms.

In terms of identification, our analysis of real effects requires that we work at the firm level, which prevents us from incorporating firm-year dummies as in the previous loan level analysis. The identification strategy here thus relies on the assumption that banks with higher income gap do not sort into firms with different sensitivity to interest rates. This hypothesis is supported by the loan-level evidence of Section 3.4.1, where we established that controlling for borrower-year fixed effect did not affect lending response to income gap. Still, we include several borrower controls available in Compustat that may determine firm decisions: four size bin dummies (based on quartiles of assets in previous years), four-digit SIC code dummies and state dummies, all interacted with year dummies. Finally, errors are two-way clustered by firm and bank.

Results are presented in Table ?? for total debt, total assets and employment. For each variable we experiment specifications with firm- and bank-level controls interacted with the full set of year dummies. Columns 1-3 use total financial debt as the $Y$ variable. We obtain an annual effect 2.9, very similar in magnitude to the measured effect on individual loans.
from Table 3.9—which was 4.7. Such evidence is indicative that very little substitution, if any, is going on between potential lenders. This is in line with the existing banking literature (e.g. Khwaja and Mian, 2008 or Iyer et al., 2014). We then move to “real variables”: Columns 4-9 indicate that total assets or employment have an elasticity of about 1/2 to the change in debt induced by the interest rate exposure of lenders. The effects on employment and total assets are of similar sizes, indicative of a roughly constant capital to labor ratio. Results also suggest that the progressive inclusion of lender and borrower controls does not affect the point estimate too much. Comparing firms in the top (.3) and bottom quartile (.15) of effective income gap, and assuming an annual increase in short rates by 100bp, we would expect a differential response in asset growth by some .2 percentage points, and a differential employment growth by some .3 percentage points. In Appendix Table 3.4, we replicate this analysis on quarterly data, using debt and assets from Compustat quarterly (unfortunately employment is not available in quarterly accounts). We find similar estimates.

To conclude this Section, we discuss whether our micro estimates have the power to matter at the aggregate level. To implement this exercise, we ignore general equilibrium effects and directly extrapolate our micro number to the entire economy (GE effects would spontaneously dampen the aggregate impact of our micro estimate). We then make the assumption that the average firm faces a bank income gap of 18%, which corresponds to the average in our sample. Last, we take the most saturated estimates of columns 3, 6, and 9. Let us now assume a one standard deviation increase in interest rates (in these annual data, over 1986-2013, the volatility of the Fed Funds rate is 1.6% ). Compared to the average bank, our loan-level estimates of the income gap effect suggest that aggregate firm borrowing would decrease by $2.9 \times .18 \times 1.6\% = .8\%$. This number may be compared with the volatility of aggregate bank credit growth, which is equal to 4.6% during the 1986-2013 period (Source: Federal Reserve). Similarly, we estimate the response of total assets and employment to a 1 s.d. shock to short rates to be equal to respectively .4% and .6%. Again we can compare this to the s.d. of aggregate capital growth rate and employment growths which are both around 2% (computations made using BLS and BEA data). Overall, the real effects induced by the income gap channel of monetary policy are sizable and have the power to significantly
contribute to the transmission of monetary policy. Given that the average gap is positive, they go in the direction of dampening the macro effect of changes in interest rates.

3.5 Discussion

3.5.1 Credit Multiplier

This Section uses interest rate shocks to identify the credit multiplier of banks in our sample. We estimate equation (3.2) using as a dependent variable the quarterly increase in the amount of loans normalized by lagged assets, instead of the change on log loans. This scaling by assets allows us to directly interpret the sum of the interacted coefficients $\sum_{k=0}^{4} \alpha_k$ as the impact on lending of a $1$ increase in the interest-sensitive income, i.e. $\text{gap} \times \Delta r$. $\sum_{k=0}^{4} \alpha_k$ is estimated at .81 and is significantly different from 0 at the 1% confidence level. Thus, a $1$ increase in $\text{gap} \times \Delta r$ leads to an increase of lending by $.81$. At the same time, Table 3.4 shows that the same $1$ increase in $\text{gap} \times \Delta r$ generates an increase in total earnings of about $.07$. Assuming that the sensitivity of lending to interest rates comes only through this income shock, these estimates imply a credit multiplier of $0.81/0.07=11.5$: a $1$ increase in income leads to an increase in lending by $11.5$.

This credit multiplier is slightly lower than what bank leverage suggests: In our sample, the average asset-to-equity ratio is 13.1. Given that net income also represents additional reserves, the credit multiplier we obtain is consistent with existing reserve requirements in the US, which are around 10 for large banks. These estimates do, however, need to be taken with caution since lending may be affected by $\text{gap} \times \Delta r$ through channels other than net income, as we discuss in the next section.

3.5.2 Short vs Long Rates: Cash flow vs Collateral Channel

An alternative interpretation of our results is that the income gap is a noisy measure of the duration gap. The duration gap measures the difference of interest rate sensitivity between the value of assets and the value of liabilities (Mishkin and Eakins (2009)). Changes in interest rates may therefore affect the value of a bank’s equity. Changes in the value of equity may in turn have an impact on how much future income a bank can pledge to its
investors. For a bank with a positive duration gap, an increase in interest rates raises the value of equity and therefore its debt capacity: it can lend more. This alternative channel also relies on a failure of the Modigliani-Miller theorem for banks, but it goes through bank value instead of banks net income. This is akin to a balance sheet channel, à la Bernanke and Gertler (1989), but for banks.

Directly measuring the duration gap is difficult as it essentially relies on strong assumptions about the duration of assets and liabilities. Instead, to distinguish income effects and balance sheet effects, we exploit the fact that the effect of interest rates on bank value partly comes from long-term rates. To see this, notice that the present value at $t$ of a safe cash-flow $C$ at time $t + T$ is $\frac{C}{(1 + r_{t,T})^T}$, where $r_{t,T}$ is the risk-free yield between $t$ and $t + T$. Thus, as long as there are shocks to long-term yields that are not proportional to shocks to short-term yields, we can identify a balance sheet channel separately from an income channel. Consider for instance an increase in long-term rates, keeping short-term rates constant. If our income gap measure affects lending through shocks to asset values, we should observe empirically that firms with lower income gap lend relatively less following this long-term interest rates increase. By contrast, if our income gap measure affects lending only through contemporaneous or short-term changes in income, such long-term rate shock should not impact differently the lending of high vs. low income gap banks.

We implement this test in Table 3.10. We add to our benchmark equation (3.2) interaction terms between the income gap – as a proxy for the duration gap – and five lags of changes in long term interest rates, measured as the yield on 10 years treasuries. The coefficients on these interaction terms are reported in the lower part of the top panel of Table 3.10. In the bottom panel, we report the sum of these coefficients (the cumulative impact of interest rates) as well as their p-value. We use as dependent variables interest income (Column (1)), market value of equity (Column (2)), C&I lending growth (Column (3)) and total lending growth (Column (4)).

We find no evidence that long term interest rates affect BHCs’ cash flows, value or lending. If anything, the cumulative effect goes in the opposite direction to what would be expected if the income gap was a proxy for the duration gap. Additionally, estimates of the income gap effect on BHCs lending sensitivity to interest rates are unaffected by the
inclusion of the long rate interaction terms. This test suggests that monetary policy affects bank lending via income gap induced income shocks much more than through shocks to the relative value of banks’ assets and liabilities. However, it is important to emphasize that the power of test is limited by the fact that we do not directly measure the duration gap.

3.6 Conclusion

This paper shows that banks retain significant exposure to interest rate risk. Our sample consists of quarterly data on US bank holding companies from 1986 to 2013. We measure a bank’s income exposure to interest rates through its income gap, defined as the difference between assets and liabilities that mature in less than one year. The average income gap in our sample is 12.6% of total assets (28% in asset-weighted terms), but it exhibits significant cross-sectional variation. The income gap strongly predicts how banks’ profits respond to changes in interest rates.

We also find that banks exposure to interest rate risk has implications for the transmission of monetary policy. An increase in the short rate directly affects banks’ incomes through their income gap and, in the presence of credit frictions, their lending policy. We are able to confirm these results on loan level data, and are even able to trace the impact of these shocks on actual firm growth: Firms that are linked with high gap banks tend to grow faster (in terms of assets and employment) when interest rates go up. Given that the average gap is positive, the income gap effect tends to dampen the receisionary effect of monetary policy on bank lending and the real economy. The income gap has a significant explanatory power on the sensitivity of lending to changes in interest rates, larger or similar in magnitudes than previously identified factors, such as leverage, bank size or even asset liquidity. Finally, we report evidence consistent with the hypothesis that our main channel is an income effect, as opposed to a collateral channel. Interest rates affect lending because they affect banks’ net income, not because they affect the market value of equity.

Our results suggest that the allocation of interest rate exposure across agents (banks, households, firms, government) is an important variable to understand how an economy responds to monetary policy. In particular, the distribution of interest rate risk across
agents is crucial to analyze the redistributive effects of monetary policy and thus to trace the roots of the transmission of monetary policy.
## Tables

Table 3.1: Summary Statistics: Dependent and Control Variables

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>sd</th>
<th>p25</th>
<th>p75</th>
<th>count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net interest income / assets</td>
<td>0.009</td>
<td>0.004</td>
<td>0.008</td>
<td>0.010</td>
<td>47394</td>
</tr>
<tr>
<td>Non interest income / assets</td>
<td>0.004</td>
<td>0.011</td>
<td>0.002</td>
<td>0.004</td>
<td>47419</td>
</tr>
<tr>
<td>Earnings / assets</td>
<td>0.002</td>
<td>0.005</td>
<td>0.002</td>
<td>0.003</td>
<td>47419</td>
</tr>
<tr>
<td>Market value of equity / assets</td>
<td>0.152</td>
<td>0.193</td>
<td>0.092</td>
<td>0.183</td>
<td>23243</td>
</tr>
<tr>
<td>$\Delta$ Interest</td>
<td>0.000</td>
<td>0.001</td>
<td>-0.000</td>
<td>0.000</td>
<td>44161</td>
</tr>
<tr>
<td>$\Delta$ Non-interest</td>
<td>0.000</td>
<td>0.001</td>
<td>-0.000</td>
<td>0.000</td>
<td>42858</td>
</tr>
<tr>
<td>$\Delta$ Earnings</td>
<td>0.000</td>
<td>0.001</td>
<td>-0.000</td>
<td>0.000</td>
<td>42805</td>
</tr>
<tr>
<td>$\Delta$ Market Value</td>
<td>0.005</td>
<td>0.023</td>
<td>-0.007</td>
<td>0.015</td>
<td>22104</td>
</tr>
<tr>
<td>$\Delta$ log(C&amp;I loans)</td>
<td>0.015</td>
<td>0.090</td>
<td>-0.028</td>
<td>0.055</td>
<td>44631</td>
</tr>
<tr>
<td>$\Delta$ log(total loans)</td>
<td>0.017</td>
<td>0.046</td>
<td>-0.006</td>
<td>0.037</td>
<td>45185</td>
</tr>
<tr>
<td>Log of assets</td>
<td>15.033</td>
<td>1.403</td>
<td>14.009</td>
<td>15.665</td>
<td>47427</td>
</tr>
<tr>
<td>Equity to assets ratio</td>
<td>0.090</td>
<td>0.045</td>
<td>0.070</td>
<td>0.100</td>
<td>47427</td>
</tr>
<tr>
<td>Fraction Liquid assets</td>
<td>0.225</td>
<td>0.124</td>
<td>0.141</td>
<td>0.287</td>
<td>35205</td>
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<tr>
<td>Mean local % ARM</td>
<td>0.220</td>
<td>0.086</td>
<td>0.164</td>
<td>0.234</td>
<td>47427</td>
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<tr>
<td>Local $\beta_{Debt / Int. rate}$</td>
<td>6.434</td>
<td>1.021</td>
<td>6.015</td>
<td>6.691</td>
<td>47427</td>
</tr>
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</table>

Note: Summary statistics are based on the quarterly Consolidated Financial Statements (Files FR Y-9C) between 1986 and 2013 restricted to US bank holding companies with total consolidated assets of $1Bil or more in 2010 dollars. All variables are quarterly.
Table 3.2: Income Gap and Its Components

<table>
<thead>
<tr>
<th></th>
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<th>p75</th>
<th>count</th>
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</thead>
<tbody>
<tr>
<td>Income Gap =</td>
<td>0.126</td>
<td>0.187</td>
<td>0.012</td>
<td>0.243</td>
<td>47043</td>
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<tr>
<td>Assets maturing/resetting &lt; 1 year</td>
<td>0.426</td>
<td>0.152</td>
<td>0.325</td>
<td>0.525</td>
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<tr>
<td>Liabilities maturing/resetting &lt; 1 year =</td>
<td>0.299</td>
<td>0.157</td>
<td>0.191</td>
<td>0.384</td>
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</tr>
<tr>
<td>Short Term Liabilities</td>
<td>0.288</td>
<td>0.157</td>
<td>0.180</td>
<td>0.374</td>
<td>47420</td>
</tr>
<tr>
<td>+ Variable Rate Long Term Debt</td>
<td>0.010</td>
<td>0.027</td>
<td>0.000</td>
<td>0.008</td>
<td>47253</td>
</tr>
<tr>
<td>+ Short Maturity Long Term Debt</td>
<td>0.001</td>
<td>0.005</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>+ Preferred Stock</td>
<td>0.000</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>47063</td>
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Note: Summary statistics are based on the quarterly Consolidated Financial Statements (Files FR Y-9C) between 1986 and 2013 restricted to US bank holding companies with total consolidated assets of $1Bil or more in 2010 dollars. The variables are all scaled by total consolidated assets (bhck2170) and are defined as follows:

- Interest Sensitive Liabilities = (bhck3296+bhck3298+bhck3409+bhck3408)/bhck2170;
- Interest Sensitive Assets = (bhck3197)/bhck2170;
- Short Term Liabilities = bhck3296/bhck2170;
- Variable Rate Long Term Debt = bhck3298/bhck2170;
- Short Maturity Long Term Debt = bhck3409/bhck2170;
- Preferred Stock = bhck3408/bhck2170
Table 3.3: Summary Statistics: Derivatives Hedges of Interest Rate Risk

<table>
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<tr>
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<th>count</th>
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<tr>
<td>Futures</td>
<td>0.023</td>
<td>0.156</td>
<td>0.000</td>
<td>0.000</td>
<td>33127</td>
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<tr>
<td>Forward Contracts</td>
<td>0.040</td>
<td>0.296</td>
<td>0.000</td>
<td>0.002</td>
<td>33146</td>
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<tr>
<td>Written Options (Exchange Traded)</td>
<td>0.008</td>
<td>0.069</td>
<td>0.000</td>
<td>0.000</td>
<td>33112</td>
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<tr>
<td>Purchased Options (Exchange Traded)</td>
<td>0.010</td>
<td>0.086</td>
<td>0.000</td>
<td>0.000</td>
<td>33110</td>
</tr>
<tr>
<td>Written Options (OTC)</td>
<td>0.029</td>
<td>0.193</td>
<td>0.000</td>
<td>0.003</td>
<td>33139</td>
</tr>
<tr>
<td>Purchased Options (OTC)</td>
<td>0.028</td>
<td>0.185</td>
<td>0.000</td>
<td>0.000</td>
<td>33161</td>
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<tr>
<td>Swaps</td>
<td>0.177</td>
<td>1.499</td>
<td>0.000</td>
<td>0.034</td>
<td>46689</td>
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<tr>
<td>At least some I.R. hedging</td>
<td>0.586</td>
<td>0.493</td>
<td>0.000</td>
<td>1.000</td>
<td>33107</td>
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</tbody>
</table>

Note: Summary statistics are based on Schedule HC-L of the quarterly Consolidated Financial Statements (Files FR Y-9C) between 1995 and 2013 restricted to US bank holding companies with total consolidated assets of $1Bil or more in 2010 dollars. Schedule HC-L is not available prior to 1995. The variables report notional amounts in each kind of derivatives at the bank holding-quarter level and are all scaled by total consolidated assets (bhck2170). Variables are defined as follows: Futures contracts = bhck8693/bhck2170; Forward contracts = bhck8697/bhck2170; Written options (exchange traded) = bhck8701/bhck2170; Purchased options (exchange traded) = bhck8705/bhck2170; written options (OTC) = bhck8709/bhck2170; Purchased options (OTC) = bhck8713/bhck2170; Swaps = bhck3450/bhck2170. HEDGED is a dummy equal to one if a bank has a positive notional amount in any of the seven types of interest hedging derivatives in a given quarter.
### Table 3.4: Income Gap, Interest rates, and Interest Income

<table>
<thead>
<tr>
<th></th>
<th>∆Interest&lt;sub&gt;it&lt;/sub&gt;</th>
<th></th>
<th>∆Non Interest Income&lt;sub&gt;it&lt;/sub&gt;</th>
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<tbody>
<tr>
<td></td>
<td>All</td>
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</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Gap&lt;sub&gt;it−1&lt;/sub&gt; × ∆FedFunds&lt;sub&gt;it&lt;/sub&gt;</td>
<td>.019***</td>
<td>.019***</td>
<td>.018</td>
</tr>
<tr>
<td></td>
<td>(3.4)</td>
<td>(3.1)</td>
<td>(1.5)</td>
</tr>
<tr>
<td>Gap&lt;sub&gt;it−1&lt;/sub&gt; × ∆FedFunds&lt;sub&gt;it−1&lt;/sub&gt;</td>
<td>.038***</td>
<td>.038***</td>
<td>.041***</td>
</tr>
<tr>
<td></td>
<td>(6.7)</td>
<td>(6.2)</td>
<td>(3.1)</td>
</tr>
<tr>
<td>Gap&lt;sub&gt;it−1&lt;/sub&gt; × ∆FedFunds&lt;sub&gt;it−2&lt;/sub&gt;</td>
<td>.0035</td>
<td>.0057</td>
<td>-.012</td>
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<tr>
<td></td>
<td>(.8)</td>
<td>(1.3)</td>
<td>(.87)</td>
</tr>
<tr>
<td>Gap&lt;sub&gt;it−1&lt;/sub&gt; × ∆FedFunds&lt;sub&gt;it−3&lt;/sub&gt;</td>
<td>.0072</td>
<td>.0051</td>
<td>.024*</td>
</tr>
<tr>
<td></td>
<td>(1.6)</td>
<td>(1.1)</td>
<td>(1.8)</td>
</tr>
<tr>
<td>Gap&lt;sub&gt;it−1&lt;/sub&gt; × ∆FedFunds&lt;sub&gt;it−4&lt;/sub&gt;</td>
<td>-.006</td>
<td>-.0059</td>
<td>-.00072</td>
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<tr>
<td></td>
<td>(-1.4)</td>
<td>(-1.3)</td>
<td>(.05)</td>
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<td>3847</td>
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<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
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<td>.11</td>
<td>.16</td>
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<tr>
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<td>.06</td>
<td>.06</td>
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<td>p-value of gap coefficients</td>
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<td>p-value of equality test</td>
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<td>0</td>
<td>-.16</td>
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<td>p-value of size coefficients</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Sum of equity coefficients</td>
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<td>.03</td>
<td>.05</td>
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<tr>
<td>p-value of equity coefficients</td>
<td>.24</td>
<td>.28</td>
<td>.72</td>
</tr>
</tbody>
</table>

Note: This table estimates:

\[
\Delta Y_{it} = \sum_{k=0}^{4} \alpha_k (\text{gap}_{it-1} \times \Delta \text{fed funds}_{t-k}) + \sum_{x \in \text{Control}} \sum_{k=0}^{4} \gamma_k (x_{it-1} \times \Delta \text{fed funds}_{t-k}) + \sum_{k=0}^{4} \eta_k \Delta Y_{it-1-k} + \phi \text{gap}_{it-1} + \sum_{x \in \text{Control}} \mu_k x_{it-1} + \delta_t + \epsilon_{it}
\]

\(\Delta Y\) is the quarterly change in interest income divided by lagged total assets \((\text{Interest}_{it} - \text{Interest}_{it-1}) / (\text{Assets}_{it-1})\) in Columns (1)-(5) and change in non interest income normalized by lagged assets in Columns (6)-(10). Columns (1) and (6) report estimates for the entire sample. Columns (2)-(3) and (6)-(7) break down the sample into small and large banks. Columns (4)-(5) and (9)-(10) break down the sample into banks reporting some positive notional exposure to interest rate derivatives and banks with no such exposure. The controls are \(\log(\text{assets}_{it-1})\), book equity \(\text{it}/\text{assets}_{it-1}\), Local % Debt / Int. rate, Local % ARM 1988. s quarter fixed effects. Standard errors are clustered at the BHC level. "Sum of gap coefficients" report the coefficient estimate for \(\sum_{k=0}^{4} \alpha_k\). We also report the p-value of a test of significance for \(\sum_{k=0}^{4} \alpha_k\), as well as a test of equality of these sums across subsamples (large vs small banks, hedged vs unhedged banks). These equality tests use the SURE procedure to nest the two equations in a single model.
Table 3.5: Income Gap and Interest rates: Earnings and Market Value

<table>
<thead>
<tr>
<th></th>
<th>All Small</th>
<th>Big</th>
<th>No Hedge</th>
<th>Some Hedge</th>
<th>All Small</th>
<th>Big</th>
<th>No Hedge</th>
<th>Some Hedge</th>
</tr>
</thead>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\text{Gap}<em>{it-1} \times \Delta \text{Fed Funds}</em>{it} )</td>
<td>0.029***</td>
<td>0.031***</td>
<td>0.022</td>
<td>0.03**</td>
<td>0.038**</td>
<td>.5*</td>
<td>.47*</td>
<td>.71</td>
</tr>
<tr>
<td></td>
<td>(3.7)</td>
<td>(3.6)</td>
<td>(1.1)</td>
<td>(2.2)</td>
<td>(2.4)</td>
<td>(1.9)</td>
<td>(1.7)</td>
<td>(2.2)</td>
</tr>
<tr>
<td>(\text{Gap}<em>{it-1} \times \Delta \text{Fed Funds}</em>{it-1} )</td>
<td>0.03***</td>
<td>0.033***</td>
<td>0.0025</td>
<td>0.059***</td>
<td>0.029*</td>
<td>0.65*</td>
<td>.72***</td>
<td>-2.6</td>
</tr>
<tr>
<td></td>
<td>(3.4)</td>
<td>(3.4)</td>
<td>(1.2)</td>
<td>(3.5)</td>
<td>(1.9)</td>
<td>(2.5)</td>
<td>(2.7)</td>
<td>(-2.8)</td>
</tr>
<tr>
<td>(\text{Gap}<em>{it-1} \times \Delta \text{Fed Funds}</em>{it-2} )</td>
<td>0.0057</td>
<td>0.0019</td>
<td>0.035</td>
<td>-0.01</td>
<td>0.021</td>
<td>0.15</td>
<td>0.16</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>(.69)</td>
<td>(.22)</td>
<td>(1.5)</td>
<td>(-.66)</td>
<td>(1.5)</td>
<td>(5.6)</td>
<td>(.57)</td>
<td>(.86)</td>
</tr>
<tr>
<td>(\text{Gap}<em>{it-1} \times \Delta \text{Fed Funds}</em>{it-3} )</td>
<td>0.0051</td>
<td>0.003</td>
<td>0.0087</td>
<td>0.015</td>
<td>-.0014</td>
<td>.2</td>
<td>.28</td>
<td>-1.1</td>
</tr>
<tr>
<td></td>
<td>(.7)</td>
<td>(.38)</td>
<td>(.49)</td>
<td>(.97)</td>
<td>(-1)</td>
<td>(.86)</td>
<td>(1.2)</td>
<td>(-1.4)</td>
</tr>
<tr>
<td>(\text{Gap}<em>{it-1} \times \Delta \text{Fed Funds}</em>{it-4} )</td>
<td>0.0055</td>
<td>0.0054</td>
<td>0.019</td>
<td>0.0061</td>
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<td>.11</td>
<td>.074</td>
<td>.79</td>
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<tr>
<td></td>
<td>(.76)</td>
<td>(.7)</td>
<td>(1.2)</td>
<td>(.45)</td>
<td>(-.78)</td>
<td>(.51)</td>
<td>(.32)</td>
<td>(1.2)</td>
</tr>
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<td>3628</td>
<td>11094</td>
<td>15228</td>
<td>19498</td>
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<td>1312</td>
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<td>(R^2)</td>
<td>.22</td>
<td>.22</td>
<td>.27</td>
<td>.24</td>
<td>.22</td>
<td>.32</td>
<td>.33</td>
<td>.29</td>
</tr>
</tbody>
</table>

|                  |           |     |          |            |           |     |          |            |
| Sum of gap coefficients | 0.07 | 0.07 | .08 | .09 | .07 | 1.6 | 1.7 | .9 | 2.3 | 1.7 |
| p-value of gap coefficients | 0 | 0 | 0 | 0 | 0 | 0.22 | 0 | 0 | 0 | 0 |
| p-value of equality test | .55 | .27 | .28 | .39 |
| Sum of size coefficients | 0 | 0 | -.06 | 0 | 0 | .04 | .03 | -3.4 | .02 | .08 |
| p-value of size coefficients | 0 | 0 | .46 | .82 | .04 | .05 | .12 | .19 | .8 | .01 |
| Sum of equity coefficients | .13 | .14 | -.11 | -.07 | .24 | 3.1 | 2.7 | 6.6 | 5.5 | 1.9 |
| p-value of equity coefficients | .2 | .16 | .61 | .46 | .22 | .23 | .32 | .3 | .29 | .51 |

Note: This table estimates:

\[
\Delta Y_t = \sum_{k=0}^{4} \alpha_k \text{gap}_{it-1} \times \Delta \text{fed funds}_{it-k} + \sum_{x \in \text{Control}} \gamma_{x,k} (x_{it-1} \times \Delta \text{fed funds}_{it-k}) + \sum_{k=0}^{4} \eta_k \Delta Y_{it-1-k} + \phi \text{gap}_{it-1} + \sum_{x \in \text{Control}} \mu_x x_{it-1} + \delta_t + \epsilon_{it}
\]

\(\Delta Y\) is the quarterly change in Earnings divided by lagged total assets \((\text{Earnings}_{it} - \text{Earnings}_{it-1}) / (\text{Assets}_{it} - \text{Assets}_{it-1})\) in Columns (1)-(5) and change in market value of equity normalized by lagged assets in Columns (6)-(10). Columns (1) and (6) report estimates for the entire sample. Columns (2)-(3) and (6)-(7) break down the sample into small and large banks. Columns (4)-(5) and (9)-(10) break down the sample into banks reporting some positive notional exposure to interest rate derivatives and banks with no such exposure. The controls are log(assets_{it-1}), book equity_{it-1}/assets_{it-1}, Local Debt / Int. rate, Local % ARM_bars. The regression also includes quarter fixed effects, standard errors are clustered at the BHC level. ‘Sum of gap coefficients’ report the coefficient estimate for \(\sum_{k=0}^{4} \alpha_k\). We also report the p-value of a test of significance for \(\sum_{k=0}^{4} \alpha_k\), as well as a test of equality of these sums across subsamples (large vs small banks, hedged vs unhedged banks). These equality tests use the SURE procedure to nest the two equations in a single model.
### Table 3.6: Income Gap, Interest Rates and Lending

<table>
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<tr>
<th></th>
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<th>Small</th>
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<th>Some Hedge</th>
<th>All</th>
<th>Small</th>
<th>Big</th>
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<th>Some Hedge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
<td>(10)</td>
</tr>
<tr>
<td>( \Delta \log(\text{C&amp;I}) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( \text{Gap}_{t-1} \times \Delta \text{FedFunds}_t )</td>
<td>-0.27</td>
<td>-0.023</td>
<td>-1.5</td>
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<td>-0.35</td>
<td>-0.5</td>
<td>0.76</td>
<td>-0.23</td>
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<td>( \text{Gap}<em>{t-1} \times \Delta \text{FedFunds}</em>{t-1} )</td>
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<td>0.77</td>
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<td>0.51</td>
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<td>1**</td>
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<td>(2)</td>
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<td>( \text{Gap}<em>{t-1} \times \Delta \text{FedFunds}</em>{t-2} )</td>
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<td>1.6**</td>
<td>2.9</td>
<td>3.5**</td>
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<td>0.91*</td>
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<td>(1.8)</td>
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<td>( \text{Gap}<em>{t-1} \times \Delta \text{FedFunds}</em>{t-4} )</td>
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<td>38848</td>
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<td>0.27</td>
<td>0.25</td>
<td>0.22</td>
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<td>-1.2</td>
<td>2.4</td>
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<td>0.0</td>
<td>0.11</td>
<td>0.9</td>
<td>0.11</td>
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<td>Sum of size coefficients</td>
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<td>-0.01</td>
<td>-0.8</td>
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<td>0.78</td>
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<td>0.27</td>
<td>0.27</td>
</tr>
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<td>Sum of equity coefficients</td>
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<td>-9.8</td>
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<td>-1.9</td>
<td>-8.1</td>
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<td>p-value of equity coefficients</td>
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<td>0.94</td>
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<td>0.32</td>
<td>0.95</td>
<td>0.42</td>
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</table>

Note: This table estimates:

\[
\Delta Y_t = \sum_{k=0}^{k=4} \alpha_k (\text{gap}_{t-1} \times \Delta \text{FedFunds}_{t-k}) + \sum_{x \in \text{Control}} \sum_{k=0}^{k=4} \gamma_x \sum_{t=1}^{t-1} \Delta \text{FedFunds}_{t-k} + \sum_{k=0}^{k=4} \eta_k \Delta Y_{t-1-k} + \phi \text{gap}_{t-1} + \sum_{x \in \text{Control}} \mu_x x_{t-1} + \delta_t + \epsilon_{it}
\]

\( \Delta Y \) is the quarterly change in log C&I lending divided by lagged total assets in Columns (1)-(5) and quarterly change in log total lending normalized by lagged assets in Columns (6)-(10). Columns (1) and (6) report estimates for the entire sample. Columns (2)-(3) and (6)-(7) break down the sample into small and large banks. Columns (4)-(5) and (9)-(10) break down the sample into banks reporting some positive notional exposure to interest rate derivatives and banks with no such exposure. The controls are \( \log(\text{assets}_{t-1}) \), book equity / assets, \( \beta_{\text{Debt} / \text{Int. rate}} \), Local % ARM. The regression also includes quarter fixed effects. Standard errors are clustered at the BHC level. "Sum of gap coefficients" report the coefficient estimate for \( \sum_{k=0}^{k=4} \alpha_k \). We also report the p-value of a test of significance for \( \sum_{k=0}^{k=4} \alpha_k \), as well as a test of equality of these sums across subsamples (large vs small banks, hedged vs unhedged banks). These equality tests use the SURE procedure to nest the two equations in a single model.
### Table 3.7: Robustness: Controlling for Liquidity

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<th>Some Hedge</th>
<th>All Small</th>
<th>Big</th>
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<th>Some Hedge</th>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta \log (\text{C&amp;I}) )</td>
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<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
</tr>
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<td>( \Delta \log (\text{Total Loans}) )</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{Gap}<em>{it} \times \Delta \text{FedFunds}</em>{t-1} )</td>
<td>(0.039)</td>
<td>(0.055)</td>
<td>(-0.52)</td>
<td>(0.43)</td>
<td>(-0.74)</td>
<td>(0.014)</td>
<td>(-1.7)</td>
<td>(1.3)</td>
</tr>
<tr>
<td></td>
<td>(.05)</td>
<td>(.05)</td>
<td>(-.27)</td>
<td>(.4)</td>
<td>(-.73)</td>
<td>(.038)</td>
<td>(-.43)</td>
<td>(1.2)</td>
</tr>
<tr>
<td>( \text{Gap}<em>{it} \times \Delta \text{FedFunds}</em>{t-2} )</td>
<td>(0.64)</td>
<td>(0.71)</td>
<td>(-3.3)</td>
<td>(.26)</td>
<td>(0.41)</td>
<td>(0.69)</td>
<td>(1.5)</td>
<td>(.43)</td>
</tr>
<tr>
<td></td>
<td>(.15)</td>
<td>(.95)</td>
<td>(-0.71)</td>
<td>(1.4)</td>
<td>(.35)</td>
<td>(1.2)</td>
<td>(-2.5)</td>
<td>(.13)</td>
</tr>
<tr>
<td>( \text{Gap}<em>{it} \times \Delta \text{FedFunds}</em>{t-3} )</td>
<td>(2.6**)</td>
<td>(2.5***)</td>
<td>(3.1)</td>
<td>(3**)</td>
<td>(1.9)</td>
<td>(1.6)</td>
<td>(1.7)</td>
<td>(.94)</td>
</tr>
<tr>
<td></td>
<td>(.3)</td>
<td>(.3)</td>
<td>(.21)</td>
<td>(1.2)</td>
<td>(.69)</td>
<td>(1)</td>
<td>(1.7)</td>
<td>(.32)</td>
</tr>
<tr>
<td>( \text{Gap}<em>{it} \times \Delta \text{FedFunds}</em>{t-4} )</td>
<td>(-2***)</td>
<td>(-2.5***)</td>
<td>(1.8)</td>
<td>(-1.2)</td>
<td>(1.6)</td>
<td>(.43)</td>
<td>(1.5*)</td>
<td>(.65)</td>
</tr>
<tr>
<td></td>
<td>(-.71)</td>
<td>(.95)</td>
<td>(-.92)</td>
<td>(-2.7)</td>
<td>(-2.1)</td>
<td>(-.62)</td>
<td>(.95)</td>
<td>(-.3)</td>
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<td>Observations</td>
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<td>27377</td>
<td>3089</td>
<td>11921</td>
<td>16845</td>
<td>30006</td>
<td>26968</td>
<td>3038</td>
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<tr>
<td>( R^2 )</td>
<td>(.092)</td>
<td>(.098)</td>
<td>(.11)</td>
<td>(.082)</td>
<td>(.12)</td>
<td>(.22)</td>
<td>(.22)</td>
<td>(.31)</td>
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<tr>
<td>Sum of gap coefficients</td>
<td>(1.8)</td>
<td>(2.3)</td>
<td>(-5.8)</td>
<td>(2.3)</td>
<td>(1.2)</td>
<td>(1.3)</td>
<td>(.81)</td>
<td>(.17)</td>
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<tr>
<td>p-value of gap coefficients</td>
<td>(.01)</td>
<td>(.01)</td>
<td>(.05)</td>
<td>(.05)</td>
<td>(.68)</td>
<td>(.0)</td>
<td>(0.54)</td>
<td>(.0)</td>
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<tr>
<td>p-value of equality test</td>
<td>(.27)</td>
<td>(.27)</td>
<td>(.24)</td>
<td>(.24)</td>
<td>(.72)</td>
<td>(.3)</td>
<td>(.24)</td>
<td>(.72)</td>
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<tr>
<td>Sum of size coefficients</td>
<td>(.37)</td>
<td>(.38)</td>
<td>(-6.6)</td>
<td>(.12)</td>
<td>(.22)</td>
<td>(.05)</td>
<td>(.03)</td>
<td>(-5.8)</td>
</tr>
<tr>
<td>p-value of size coefficients</td>
<td>(.0)</td>
<td>(.0)</td>
<td>(.57)</td>
<td>(.76)</td>
<td>(.05)</td>
<td>(.25)</td>
<td>(.46)</td>
<td>(.32)</td>
</tr>
<tr>
<td>Sum of equity coefficients</td>
<td>(-.9)</td>
<td>(-9.4)</td>
<td>(14)</td>
<td>(.53)</td>
<td>(-20)</td>
<td>(-1.5)</td>
<td>(-1.9)</td>
<td>(.22)</td>
</tr>
<tr>
<td>p-value of equity coefficients</td>
<td>(.22)</td>
<td>(.22)</td>
<td>(.42)</td>
<td>(.53)</td>
<td>(.22)</td>
<td>(.69)</td>
<td>(.62)</td>
<td>(.82)</td>
</tr>
<tr>
<td>Sum of liquidity coefficients</td>
<td>(-1.3)</td>
<td>(-2.1)</td>
<td>(1.3)</td>
<td>(-2.4)</td>
<td>(-3)</td>
<td>(-.69)</td>
<td>(-2.2)</td>
<td>(.43)</td>
</tr>
<tr>
<td>p-value of liquidity coefficients</td>
<td>(.31)</td>
<td>(.11)</td>
<td>(.76)</td>
<td>(.89)</td>
<td>(.14)</td>
<td>(.3)</td>
<td>(.33)</td>
<td>(.19)</td>
</tr>
</tbody>
</table>

Note: This table estimates:

\[
\Delta Y_{it} = \sum_{k=0}^{4} \alpha_k (\text{gap}_{it-1} \times \Delta \text{FedFunds}_{t-k}) + \sum_{x \in \text{Control}} \sum_{k=0}^{4} \gamma_{x,k} (x_{it-1} \times \Delta \text{FedFunds}_{t-k}) + \sum_{k=0}^{4} \eta_k \Delta Y_{it-1-k} + \phi \text{gap}_{it-1} + \sum_{x \in \text{Control}} \mu_x x_{it-1} + \delta_t + \epsilon_{it}
\]

\( \Delta Y \) is the quarterly change in log C&I lending divided by lagged total assets in Columns (1)-(5) and quarterly change in log total lending normalized by lagged assets in Columns (6)-(10). Columns (1) and (6) report estimates for the entire sample. Columns (2)-(3) and (6)-(7) break down the sample into small and large banks. Columns (4)-(5) and (9)-(10) break down the sample into banks reporting some positive notional exposure to interest rate derivatives and banks with no such exposure. The controls are log(assets\(_{it-1}\)), book equity\(_{it-1}/\text{assets}_{it-1}\), Local \(\beta_{\text{Debt/Int. rate}}\), Local % ARM\(_{1988}\) and liquid assets\(_{it-1}/\text{assets}_{it-1}\). The regression also includes quarter fixed effects. Standard errors are clustered at the BHC level. "Sum of gap coefficients" report the coefficient estimate for \(\sum_{k=0}^{4} \alpha_k\). We also report the p-value of a test of significance for \(\sum_{k=0}^{4} \alpha_k\), as well as a test of equality of these sums across subsamples (large vs small banks, hedged vs unhedged banks). These equality tests use the SURE procedure to nest the two equations in a single model.
### Table 3.8: Testing for Sorting along Observable Characteristics

<table>
<thead>
<tr>
<th></th>
<th>Income Gap Quatile</th>
<th>Overall</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Income Gap</td>
<td>-.0221</td>
<td>.0844</td>
<td>.186</td>
</tr>
<tr>
<td>SIC code $\beta_{\text{Debt / Int. rate}}$</td>
<td>5.39</td>
<td>5.38</td>
<td>5.41</td>
</tr>
<tr>
<td>Zipcode $\beta_{\text{Debt / Int. rate}}$</td>
<td>5.07</td>
<td>4.84</td>
<td>4.89</td>
</tr>
<tr>
<td>Share public</td>
<td>.54</td>
<td>.559</td>
<td>.6</td>
</tr>
<tr>
<td>Sales at close (millions)</td>
<td>6,800</td>
<td>8,771</td>
<td>8,693</td>
</tr>
<tr>
<td>Total Debt (millions)</td>
<td>1,376</td>
<td>1,819</td>
<td>1,933</td>
</tr>
<tr>
<td>Loan maturity (months)</td>
<td>45.9</td>
<td>47.6</td>
<td>46.4</td>
</tr>
<tr>
<td>Age</td>
<td>6.48</td>
<td>6.82</td>
<td>7.16</td>
</tr>
</tbody>
</table>

Note: We construct average characteristics at the bank level by averaging the characteristics of all its borrowers (weighted by loan shares) for each bank-year observation of our loan level data. The table displays the average of these bank-level covariates after splitting banks into four quantiles of income gap. All variables are demeaned with respect to year. The fifth column reports the standard deviation of the variable summarized in each row. The sixth column reports the $p$-value for the regression of the row variable on the income gap. Standard errors are clustered at the bank level.
Table 3.9: The Impact of Income Gap on Lending Within Borrowers

<table>
<thead>
<tr>
<th></th>
<th>All Firms</th>
<th>Firms with Multiple Banks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$\Delta L_{i\rightarrow j,t}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{Gap}_{it-1} \times \Delta \text{FedFunds}_t$</td>
<td>5.3***</td>
<td>4.9**</td>
</tr>
<tr>
<td></td>
<td>(2.4)</td>
<td>(2.2)</td>
</tr>
<tr>
<td>Observations</td>
<td>290167</td>
<td>273502</td>
</tr>
<tr>
<td>Borrowers</td>
<td>11174</td>
<td>8378</td>
</tr>
<tr>
<td>Lenders</td>
<td>287</td>
<td>287</td>
</tr>
<tr>
<td>BHC top-holders</td>
<td>130</td>
<td>130</td>
</tr>
<tr>
<td>FE</td>
<td>Year</td>
<td>Year</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.27</td>
<td>.28</td>
</tr>
</tbody>
</table>

Note: This table estimates:

$$\Delta L_{i\rightarrow j,t} = \alpha (\text{gap}_{it} \times \Delta \text{fed funds}_t) + \sum_{x \in \text{Control}} \gamma_x (x_{it-1} \times \Delta \text{fed funds}_t) + \eta \Delta L_{i\rightarrow j,t-1} + \phi \text{gap}_{it} + \sum_{x \in \text{Control}} \mu_x x_{it-1} + \delta_j + \epsilon_{it}$$

$\Delta L_{i\rightarrow j,t}$ is the change in loans outstanding from bank $i$ to firm $j$ normalized by the across period average. The controls are $\log(\text{assets}_{it-1})$, book equity$_{it-1}$/assets$_{it-1}$, included directly, but also interacted with the full set of year dummies. The first column includes all firms. The second column only includes firms matched with multiple banks. The third columns adds borrower-year fixed effects. Standard errors are clustered at the bank level.
\[
\Delta Y_{it} = \alpha (\text{gap}_{it-1} \times \Delta \text{FedFunds}_{t}) + \phi \text{gap}_{it-1} + \sum_{x \in \text{BankControl}} \gamma_x (x_{it-1} \times \Delta \text{FedFunds}_{t}) + \sum_{x \in \text{BankControl}} \mu_x x_{it-1} + \sum_{x \in \text{FirmControl}} \mu_{xt} x_{it-1} + \eta \Delta Y_{it-1} + \delta_t + \epsilon_{it}
\]

\[
\Delta Y \text{ is alternatively the annual change in total debt (Column (1) (2) (3)), annual change in employment (Column (4) (5) (6)) and annual change in investment (Column (7) (8) (9)). The lender controls are } \log(\text{assets}_{it-1}) \text{ and book equity}_{it-1}/\text{assets}_{it-1}, \text{ included directly but also interacted with the full set of year dummies. The borrower controls are four size bin dummies, state dummies and four-digit SIC code dummies, all interacted with year dummies. Standard errors are two-way clustered at the firm and bank level.}
\]

Note: This table estimates for a firm \( i \) in year \( t \) with lead arranger \( b \):

\[
\Delta \text{Dep. variable} = 0.063 \quad 0.063 \quad 0.18 \quad 0.18 \quad 0.18 \quad 0.04 \quad 0.04 \quad 0.04 \quad 0.04
\]

\[
R^2 = 0.031 \quad 0.031 \quad 0.4 \quad 0.078 \quad 0.078 \quad 0.44 \quad 0.094 \quad 0.094 \quad 0.46
\]

---

<table>
<thead>
<tr>
<th></th>
<th>( \Delta \text{Debt} )</th>
<th>( \Delta \text{Employment} )</th>
<th>( \Delta \text{Assets} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>( \text{Gap}<em>{it-1} \times \Delta \text{FedFunds}</em>{t} )</td>
<td>4.7**</td>
<td>5.1***</td>
<td>2.9**</td>
</tr>
<tr>
<td>Lender controls</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Borrower Controls</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>37511</td>
<td>37511</td>
<td>37511</td>
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<tr>
<td>Borrowers</td>
<td>3417</td>
<td>3417</td>
<td>3417</td>
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<tr>
<td>Lenders</td>
<td>104</td>
<td>104</td>
<td>104</td>
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<tr>
<td>( E[\Delta \text{dep. variable}] )</td>
<td>.063</td>
<td>.063</td>
<td>.063</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>.031</td>
<td>.031</td>
<td>.4</td>
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Table 3.10: Short vs. Long-Term Rates

<table>
<thead>
<tr>
<th></th>
<th>∆Interest Income</th>
<th>∆Market Value</th>
<th>∆log(C&amp;I)</th>
<th>∆log(Total Loans)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Gap_{it-1} × ∆FedFunds_{it}</td>
<td>.022***</td>
<td>.38</td>
<td>.057</td>
<td>.37</td>
</tr>
<tr>
<td></td>
<td>(.4)</td>
<td>(1.3)</td>
<td>(.077)</td>
<td>(.98)</td>
</tr>
<tr>
<td>Gap_{it-1} × ∆FedFunds_{it-1}</td>
<td>.039***</td>
<td>.77***</td>
<td>.029</td>
<td>.12</td>
</tr>
<tr>
<td></td>
<td>(6.5)</td>
<td>(2.7)</td>
<td>(.039)</td>
<td>(.36)</td>
</tr>
<tr>
<td>Gap_{it-1} × ∆FedFunds_{it-2}</td>
<td>-.00075</td>
<td>.38</td>
<td>.59</td>
<td>.56</td>
</tr>
<tr>
<td></td>
<td>(-.16)</td>
<td>(1.3)</td>
<td>(.75)</td>
<td>(1.5)</td>
</tr>
<tr>
<td>Gap_{it-1} × ∆FedFunds_{it-3}</td>
<td>.0048</td>
<td>.3</td>
<td>2.2***</td>
<td>.48</td>
</tr>
<tr>
<td></td>
<td>(.97)</td>
<td>(1.1)</td>
<td>(3)</td>
<td>(1.4)</td>
</tr>
<tr>
<td>Gap_{it-1} × ∆FedFunds_{it-4}</td>
<td>-.0034</td>
<td>-.043</td>
<td>-1.2*</td>
<td>.042</td>
</tr>
<tr>
<td></td>
<td>(-.77)</td>
<td>(-.17)</td>
<td>(-1.7)</td>
<td>(.13)</td>
</tr>
<tr>
<td>Gap_{it-1} × ∆10years_{t-1}</td>
<td>-.00072</td>
<td>.18</td>
<td>-.74</td>
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</tr>
<tr>
<td></td>
<td>(-.17)</td>
<td>(.88)</td>
<td>(-1.3)</td>
<td>(-3.7)</td>
</tr>
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<td>Gap_{it-1} × ∆10years_{t-1}</td>
<td>-.0063</td>
<td>.029</td>
<td>.31</td>
<td>-.75**</td>
</tr>
<tr>
<td></td>
<td>(-1.5)</td>
<td>(.13)</td>
<td>(.49)</td>
<td>(-2.2)</td>
</tr>
<tr>
<td>Gap_{it-1} × ∆10years_{t-2}</td>
<td>.0029</td>
<td>-.22</td>
<td>.65</td>
<td>-.23</td>
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<td></td>
<td>(.72)</td>
<td>(-.95)</td>
<td>(1.1)</td>
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<tr>
<td>Gap_{it-1} × ∆10years_{t-3}</td>
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<td>.097</td>
<td>-.16</td>
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<td></td>
<td>(.82)</td>
<td>(-1.6)</td>
<td>(.17)</td>
<td>(-.53)</td>
</tr>
<tr>
<td>Gap_{it-1} × ∆10years_{t-4}</td>
<td>.0014</td>
<td>-.21</td>
<td>-.39</td>
<td>.29</td>
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<td>(-1)</td>
<td>(-.68)</td>
<td>(1)</td>
</tr>
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<td>35561</td>
<td>35277</td>
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<td>R²</td>
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<td>.33</td>
<td>.097</td>
<td>.22</td>
</tr>
<tr>
<td>Sum of gap coefficients (Fed Funds)</td>
<td>.36</td>
<td>1.8</td>
<td>1.7</td>
<td>1.5</td>
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<tr>
<td>p-value</td>
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<td>0</td>
<td>.04</td>
<td>0</td>
</tr>
<tr>
<td>Sum of gap coefficients (10years)</td>
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<td>-.57</td>
<td>-.07</td>
<td>-.2</td>
</tr>
<tr>
<td>p-value</td>
<td>.96</td>
<td>.36</td>
<td>.96</td>
<td>.03</td>
</tr>
</tbody>
</table>

Note: This table estimates:

\[
\Delta Y_{it} = \sum_{k=0}^{4} \alpha_k (\text{gap}_{it-1} \times \Delta \text{FedFunds}_{t-k}) + \sum_{x \in \text{Control}} \sum_{k=0}^{4} \gamma_{x,k} (x_{it-1} \times \Delta \text{FedFunds}_{t-k}) \\
+ \sum_{k=0}^{4} \sigma_k (\text{gap}_{it-1} \times \Delta \text{10years}_{t-k}) + \sum_{x \in \text{Control}} \sum_{k=0}^{4} \theta_{x,k} (x_{it-1} \times \Delta \text{10years}_{t-k}) \\
+ \sum_{k=0}^{4} \eta_k \Delta Y_{it-1-k} + \phi \text{gap}_{it-1} + \sum_{x \in \text{Control}} \mu_x x_{it-1} + \delta_t + \epsilon_{it}
\]

\(\Delta Y\) is the quarterly change in net interest income (Column (1)), quarterly change in market value (Column (2)), quarterly change in log C&I lending (Column (3)), quarterly change in log total lending (Column (4)), divided by lagged total assets. The controls are \(\log(\text{assets}_{it-1})\), book equity_{it-1}/\text{assets}_{it-1}, \text{Local } \beta_{\text{Debt} / \text{Int. rate}}, \text{Local } \% \text{ ARM}_{1988}$. The regression also includes quarter fixed effects. Standard errors are clustered at the BHC level. “Sum of gap coefficients (Fed Funds)” reports the coefficient estimate for \(\sum_{k=0}^{4} \alpha_k\). “Sum of gap coefficients (10years)” reports the coefficient estimate for \(\sum_{k=0}^{4} \sigma_k\). p-value corresponds to the p-value of a test of significance for the estimated coefficients.
Appendix

3.A Variable Definitions

This Section describes the construction of all variables in detail. \( i \) is an index for the bank, \( t \) for the quarter.

3.A.1 Bank-level Variables

This Section gathers the variables constructed using the Consolidated Financial Statements of Bank Holding Companies (form FR Y-9C). Note that flow variables (interest and non-interest income, earnings) are defined each quarter “year to date”. Hence, each time we refer to a flow variable, we mean the \textit{quarterly}, not year-to-date, flow. To transform a year-to-date variable into a quarterly one, we take the variable as it is for the first quarter of each year. For each quarter \( q = 2, 3, 4 \), we take the difference in the year-to-date variable between \( q \) and \( q - 1 \).

- \( \Delta \text{Interest}_{it} \): Change in interest income = \[ \text{interest income (bhck4107) at } t + \text{interest expense (bhck4073) at } t - 1 - \text{interest income (bhck4107) at } t - 1 - \text{interest expense (bhck4073) at } t \] / \[ \text{total assets (bhck2170) taken in } t - 1 \]. Note that bhck4073 and bhck4107 have to be converted from year-to-date to quarterly as explained above.

- \( \Delta \text{Non Interest}_{it} \): Change in non interest income = \[ \text{non interest income (bhck4079) at } t - \text{non interest income (bhck4079) at } t - 1 \] / \[ \text{total assets (bhck2170) taken in } t - 1 \]. Note that bhck4079 has to be converted from year-to-date to quarterly as explained above.
• $\Delta \text{Earnings}_{it}$: Change in earnings = \[ \text{earnings (bhck4340) at } t - \text{earnings (bhck4340) at } t - 1 \] / ( total assets (bhck2170) taken in $t - 1$ ). Note that bhck4340 has to be converted from year-to-date to quarterly as explained above.

• $\Delta \text{Value}_{it}$: Change in interest income = \[ \text{Equity market value at } t - \text{Equity market value at } t - 1 \] / ( total assets (bhck2170) taken in $t - 1$ ). Equity market value is obtained for publicly listed banks after matching with stock prices from CRSP. It is equal to the number of shares outstanding (shrout) × the end-of-quarter closing price (absolute value of prc).

• $\Delta \log(\text{C&I loans}_{it})$: commercial and industrial loan growth = log\[ C\&I loans to US adresseses (bhck1763) at $t$ + C\&I loans to foreign adresseses (bhck1764) at $t$ \] - log\[ C\&I loans to US adresseses (bhck1763) at $t - 1$ + C\&I loans to foreign adresseses (bhck1764) at $t - 1$ \].

• $\Delta \log(\text{Total loans}_{it})$: Total loan growth = log \[ \text{Total loans (bhck2122) at } t \] - log\[ \text{Total loans (bhck2122) at } t - 1 \].

• $\Delta \text{Earnings}_{it}$: Change in earnings = \[ \text{earnings (bhck4340) at } t - \text{earnings (bhck4340) at } t - 1 \] / ( total assets (bhck2170) taken in $t - 1$ ). Note that bhck4340 has to be converted from year-to-date to quarterly as explained above.

• $\text{Gap}_{it-1}$: Income gap = \[ \text{assets that reprice or mature within one year (bhck31970) - interest bearing deposits that reprice or mature within one year (bhck3296) - long term debt that reprices within one year (bhck3298) - long term debt that matures within one year (bhck3409) - variable rate preferred stock (bhck3408) } \] / total assets (bhck2170)

• $\text{Equity}_{it-1}$: Equity ratio = 1 - \[ \text{total liabilities (bhck2948) / total assets (bhck2170) } \]

• $\text{Size}_{it-1}$: log ( total assets (bhck2170) )

• $\text{Liquidity}_{it-1}$: Liquidity ratio = \[ \text{Available for sale securities (bhck1773) + Held to Maturity Securities (bhck1754) } \] / total assets (bhck2170)
3.A.2 Times series Variables

This Section gathers different measures of interest rates used in the paper.

- $\Delta \text{Fed Funds}_t$: First difference between “effective federal funds” rate at $t$ and $t-1$. Fed funds rates are available monthly from the Federal Reserve’s website: each quarter, we take the observation corresponding to the last month.

- $\Delta 10\text{yrs}_t$: First difference between yields of 10 year treasury securities at $t$ and $t-1$, available from the Federal Reserve’s website.

- $\Delta \text{Expected FF}_t$: Change in past “expected” 1 year interest rate between $t-1$ and $t$. Expected 1 year rate at $t$ is obtained from the forward rate taken at $t-8$ (two years ago), for a loan between $t$ and $t+3$ (for the coming year). This forward rate is computed using the Fama-Bliss discount bond prices. At date $t-8$, we take the ratio of the price of the 2-year to the 3-year zero-coupon bond, minus 1.

3.A.3 Loan level Variables

This Section explains how variables used in loan level regression are constructed.

- $\Delta L_{i\rightarrow j,t}$ is the growth of total outstanding loans made by bank $i$ to firm $j$ between $t$ and $t+1$:

$$\Delta L_{i\rightarrow j,t} = \frac{2L_{i\rightarrow j,t+1} - L_{i\rightarrow j,t}}{L_{i\rightarrow j,t+1} + L_{i\rightarrow j,t}}$$

This measure is always bounded by -2 and 2. It also accommodates initiation and terminations of lending relationships. We have also experimented specifications with changes in logs, and obtained similar results.

- BHC level variables are obtained from the BHC data for the top holder of bank $i$ at date $t$, most importantly the income gap, but also BHC size etc.
3.B Time-Series Regressions

We provide here estimates using an alternative specification also used in the literature
(Kashyap and J. C. Stein (2000), Campello (2002)).

3.B.1 Methodology

We proceed in two steps. First, we run, separately for each quarter, the following regression:

\[ X_{it} = \gamma_{gap_{it-1}} + controls_{it} + \epsilon_{it} \] (3.6)

where \( X_{it} \) is a cash flow or lending LHS variable. \( controls_{it} \) include: \( X_{it-1}, ..., X_{it-4}, \log(assets_{it-1}), \)
\( equity_{it-1}, \) \( assets_{it-1} \). From this first step, we obtain a time-series of \( X \) to gap sensitivity \( \gamma_{it} \).

In our second step, we regress \( \gamma_{it} \) on change in fed funds rate and four lags of it, as well as four quarter dummies:

\[ \gamma_{it} = \sum_{k=0}^{k=4} \alpha_k \Delta fedfunds_{t-k} + quarterdummies_{t} + \epsilon_{it} \] (3.7)

Again, we expect that \( \sum_{k=0}^{k=4} \alpha_k > 0 \): in periods where interest rates increase, high income gap firms tend to make more profits, or lend more.

We report the results using this methodology in Tables 3.1 and 3.2. Results are a little bit weaker using this approach, but have the same order of magnitude. Results on profits and cash flows are still all significant at the 1% confidence level, and have the same order of magnitude. Results on lending, controlling for size and leverage, but not for liquidity, remain significant at the 1 or 5% level for total lending growth. They become a bit weaker, albeit still significant at the 5% level for the whole sample, for C&I loans. Controlling for liquidity restricts the sample to 1994-2011 (BHC data do not report liquidity holdings before 1994) and reduces the sample size by a third. Significance weakens, but income gap effects on total lending remains statistically significant at the 5% level for the whole sample and small firms, as well as firms with some interest rate derivative exposure. This alternative estimation procedure provides estimates with very similar orders of magnitude.
We run each quarter the following regression:

\[ X_{it} = \gamma_t \text{gap}_{it-1} + \text{controls}_{it} + \epsilon_{it} \]

where \( X_{it} \) is net interest income (Columns (1)-(5)) and total earnings (Columns (6)-(10)). \( \text{controls}_{it} \) include: \( X_{it-1}, \ldots, X_{it-4}, \text{log}(\text{assets}_{it-1}), \frac{\text{equity}_{it-4}}{\text{assets}_{it-4}}. \) We then estimate:

\[ \gamma_t = \sum_{k=0}^{4} \alpha_k \text{FedFunds}_{it-k} + \text{quarterdummies}_{it} + \epsilon_{it} \]

“Sum of coefficients” reports the coefficient estimate for \( \sum_{k=0}^{4} \alpha_k \). p-value is the p-value from a significance test of the estimate of \( \sum_{k=0}^{4} \alpha_k. \)
We run each quarter the following regression:

\[ X_{it} = \gamma_t \text{gap}_{it} - 1 + controls_{it} + \epsilon_{it} \]

where \( X_{it} \) is log C&I lending (Columns (1)-(5)) and total lending (Columns (6)-(10)). \( controls_{it} \) include: \( X_{it-1}, \ldots, X_{it-4}, \log(\text{assets}_{it-1}), \frac{\text{equity}_{it-1}}{\text{assets}_{it-1}} \). We then estimate:

\[ \gamma_t = \sum_{k=0}^{4} \alpha_k \Delta \text{fedfunds}_{it-k} + \text{quarterdummies}_{it} + \epsilon_{it} \]

"Sum of coefficients" reports the coefficient estimate for \( \sum_{k=0}^{4} \alpha_k \). p-value is the p-value from a significance test of the estimate of \( \sum_{k=0}^{4} \alpha_k \).
3.C Internal Capital Markets

This Appendix exploits the existence of internal capital markets within BHC. As we showed in Section 3.3.2, a BHC with a higher income gap receive a larger net income shock following an increase in monetary policy. Through internal capital markets, this liquidity shock will propagate to the BHC divisions (Rosengren and Peek, 2000, Gilje, Loutskina, and Strahan, 2013, Cetorelli and Goldberg, 2012). Using commercial bank level data, we can then investigate whether, controlling for the commercial bank own income gap, this liquidity shock leads to increased lending. This strategy is valid under the identifying assumption that, conditional on a commercial bank own income gap, the BHC choice of income gap is unrelated to the sensitivity of the commercial bank lending opportunities to interest rates. This identification strategy is analogous in spirit to that used in the seminal contribution by Lamont, 1997, who uses shocks to oil-divisions in conglomerates as an exogenous source of variation in internal financing available to non-oil divisions, and in the context of bank conglomerates by Campello, 2002.

We estimate the following equation for a commercial bank $i$, belonging to a BHC $j$ in quarter $t$:

$$
\Delta Y_{i,j,t} = \sum_{k=0}^{4} \alpha_k (\text{BHC gap}_{j,t-1} \times \Delta \text{fed funds}_{t-k}) + \sum_{k=0}^{4} \zeta_k (\text{CB gap}_{i,j,t} \times \Delta \text{fed funds}_{t-k}) + \phi \text{BHC gap}_{j,t-1} + \psi \text{CB gap}_{i,j,t-1} + \sum_{x \in \text{Control}} \sum_{k=0}^{4} (\mu_x \cdot x_{i,j,t-1} + \gamma_{x,k} (x_{i,j,t-1} \times \Delta \text{fed funds}_{t-k})) + \sum_{k=0}^{4} \eta_k \Delta Y_{i,j,t-1-k} + \delta_t + \epsilon_{i,j,t}, \quad (3.8)
$$

where $\text{Control} = \{\text{Size, Equity, Liquidity, Local } \beta_{\text{Debt / Int. rate, Local } \% \text{ ARM}_{1988}\}$, and standard errors are clustered at the BHC level. All variables are scaled by total assets. $Y_{it}$ is total lending growth or C&I lending growth. $\sum_{k=0}^{4} \alpha_k$ is our coefficient of interest. A positive and significant $\sum_{k=0}^{4} \alpha_k$ implies that commercial banks belonging to BHCs with a large income gap cut lending less following a monetary tightening than commercial banks belonging to
BHCs with a small income gap, even if the commercial banks all have the same own income gap.

One issue with estimating equation (3.8) is that we need to compute the income gap at the commercial bank level, which restricts the sample period to 1997-2013. Table 3.1 presents the estimation of equation (3.8) on the sample of commercial banks with more than $500m in total assets. Panel A uses total lending growth as a dependent variable; Panel B uses C&I lending growth as a dependent variable. The explanatory variables in Column (1) are the commercial bank-level income gap, equity and size interacted with change in interest rates. Column (2) includes the same variables, but measured at the BHC level. Columns (3)-(5) have both set of controls. Column (3) restricts the sample to BHCs with more than 1 commercial bank, Column (4) to BHCs with more than 2 commercial banks and Column (5) to BHCs with more than 3 commercial banks.

Throughout the specifications, we see that the commercial bank-level income gap has little explanatory power on the sensitivity of the commercial bank lending to interest rates. This can be interpreted as a validation that capital budgeting decisions are taken at the BHC level, which validates our main approach of using BHC-level data. The commercial bank-level income gap may however still be an important control in that it may reflects the loan demand faced by the commercial bank. Panel A shows that when interest rates increases, a commercial bank belonging to a BHC with a large income gap will cut its total lending significantly less than a commercial bank with a comparable income gap, but belonging to a BHC with a smaller income gap. Quantitatively, consider two commercial banks with a similar income gap, a similar capitalization ratio and of a similar size; one of these commercial banks belongs to a BHC with an income gap at the 75th percentile of the income gap distribution, while the other belongs to a BHC with an income gap at the 25th percentile. Assume that the two BHCs that own these two commercial banks have more than three commercial banks. Following a 100 basis point increase in interest rates, the first commercial bank will increase lending by 1.2 percentage points more (Panel A, Column (5) of Table 3.1). All the estimates of $\sum_{k=0}^{4} \alpha_k$ in Table 3.1 are significant at the 1% confidence level.
The estimates obtained when using C&I lending as a dependent variable are similar once we restrict the sample to BHCs with more than two commercial banks. Column (5) of Panel B reports an estimated $\sum_{k=0}^{4} \alpha_k$ of 4.4, significantly different from 0 at the 5% confidence level. The corresponding estimate when looking at total lending is 5.5, so that the economic magnitudes are very close.
Table 3.1: Internal Capital Markets

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Total Loans</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of CB gap coefficients</td>
<td>.07</td>
<td>-.96</td>
<td>-1.7</td>
<td>-3.1</td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>.91</td>
<td>.43</td>
<td>.24</td>
<td>.04</td>
<td></td>
</tr>
<tr>
<td>Sum of BHC gap coefficients</td>
<td>.12</td>
<td>2.7</td>
<td>3.7</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>.01</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>26782</td>
<td>26782</td>
<td>9423</td>
<td>6838</td>
<td>5012</td>
</tr>
<tr>
<td><strong>Panel B: C&amp;I Loans</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of CB gap coefficients</td>
<td>1.3</td>
<td>.26</td>
<td>-.68</td>
<td>-2.5</td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>.22</td>
<td>.88</td>
<td>.73</td>
<td>.27</td>
<td></td>
</tr>
<tr>
<td>Sum of BHC gap coefficients</td>
<td>.11</td>
<td>1.9</td>
<td>5.4</td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>.22</td>
<td>.32</td>
<td>.03</td>
<td>.06</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>27217</td>
<td>27217</td>
<td>9610</td>
<td>6920</td>
<td>5123</td>
</tr>
</tbody>
</table>

Note: This table estimates $\Delta Y_{i,j,t} = \sum_{k=0}^{4} a_k (\text{BHC gap}_{j,t-1} \times \Delta \text{fed funds}_{t-k} + \sum_{k=0}^{4} \zeta_k (\text{CB gap}_{i,j,t-1} \times \Delta \text{fed funds}_{t-k}) + \psi \cdot \text{BHC gap}_{j,t-1}$

$+ \psi \cdot \text{CB gap}_{i,j,t-1} + \sum_{x \in \text{Control}} \sum_{k=0}^{4} (\mu_{x,i,j,t-1} + \gamma_{x,k} (x_{i,j,t-1} \times \Delta \text{fed funds}_{t-k}))$

$+ \sum_{k=0}^{4} \eta_k \Delta Y_{i,j,t-1-k} + \delta_t + \epsilon_{i,j,t},$

where $i$ is a commercial bank, $j$ is the BHC it belongs to, and $t$ is a quarter. $\Delta Y$ is the quarterly change in log total lending normalized by lagged assets (Panel A) and quarterly change in log C&I lending divided by lagged total assets (Panel B). The controls are $\log(\text{assets}_{i,t-1}), \text{book equity}_{i,t-1}/\text{assets}_{i,t-1}$, Local $\beta_{\text{Debt}} / \text{int. rate}$, Local % ARM1998. Column (1) includes only the commercial-bank level controls. Column (2) include only the BHC-level controls. Columns (3)-(5) include both. Columns (1)-(5) are estimation run on the entire sample of commercial banks with total assets above $500m$. In Column (3), the sample is further restricted to BHCs with more than 1 commercial bank; in Column (4) more than 2 commercial banks; in Column (5) more than 3 commercial banks.
### 3.D Supplementary Tables

Table 3.1: Comparison of the subsample of BHC matched with Dealscan

<table>
<thead>
<tr>
<th></th>
<th>BHC Sample</th>
<th>DealScan match</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of assets</td>
<td>15.033</td>
<td>16.854</td>
<td>15.784</td>
</tr>
<tr>
<td>Income Gap</td>
<td>0.126</td>
<td>0.218</td>
<td>9.853</td>
</tr>
<tr>
<td>Δ Non-interest</td>
<td>0.000</td>
<td>0.000</td>
<td>6.618</td>
</tr>
<tr>
<td>Δ Earnings</td>
<td>0.000</td>
<td>0.000</td>
<td>5.487</td>
</tr>
<tr>
<td>Earnings / assets</td>
<td>0.002</td>
<td>0.002</td>
<td>3.658</td>
</tr>
<tr>
<td>Net interest income / assets</td>
<td>0.009</td>
<td>0.009</td>
<td>-3.136</td>
</tr>
<tr>
<td>Non interest income / assets</td>
<td>0.004</td>
<td>0.005</td>
<td>2.706</td>
</tr>
<tr>
<td>Δ Market Value</td>
<td>0.005</td>
<td>0.005</td>
<td>2.186</td>
</tr>
<tr>
<td>Fraction Liquid assets</td>
<td>0.225</td>
<td>0.210</td>
<td>-1.876</td>
</tr>
<tr>
<td>Δ log(C&amp;I loans)</td>
<td>0.015</td>
<td>0.017</td>
<td>1.415</td>
</tr>
<tr>
<td>Market value of equity / assets</td>
<td>0.152</td>
<td>0.145</td>
<td>-0.948</td>
</tr>
<tr>
<td>Equity to assets ratio</td>
<td>0.090</td>
<td>0.088</td>
<td>-0.824</td>
</tr>
<tr>
<td>Local β Debt / Int. rate</td>
<td>6.434</td>
<td>6.499</td>
<td>0.728</td>
</tr>
<tr>
<td>Mean local % ARM</td>
<td>0.220</td>
<td>0.224</td>
<td>0.536</td>
</tr>
<tr>
<td>Δ Interest</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.109</td>
</tr>
<tr>
<td>Δ log(total loans)</td>
<td>0.017</td>
<td>0.017</td>
<td>-0.029</td>
</tr>
</tbody>
</table>

Note: The table splits the BHC sample in the subsample matched with Dealscan and the subsample not matched with Dealscan. Each variable in the first column is regressed on a dummy indicated whether the BHC is matched in DealScan in a given year and the t-statistics is reported. Standard errors are clustered at the BHC level.
Table 3.2: Testing for Sorting along Unobservable Characteristics

<table>
<thead>
<tr>
<th></th>
<th>Income Gap (Main Lender)</th>
<th>Income Gap (Second Lender)</th>
<th>Income Gap (Future Lender)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td></td>
</tr>
<tr>
<td>Income Gap (Main Lender)</td>
<td>-.0064</td>
<td>-.022</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-.069)</td>
<td>(-.48)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>29303</td>
<td>4210</td>
<td></td>
</tr>
<tr>
<td>Borrowers</td>
<td>8772</td>
<td>2683</td>
<td></td>
</tr>
<tr>
<td>Lenders</td>
<td>249</td>
<td>217</td>
<td></td>
</tr>
<tr>
<td>BHC top-holders</td>
<td>110</td>
<td>102</td>
<td></td>
</tr>
<tr>
<td>FE</td>
<td>Year</td>
<td>Year</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>.17</td>
<td>.19</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table estimates for a firm $j$ in year $t$:

$$
\text{gap}_{jt} = \delta_t + \alpha \text{gap}_{jt} + \epsilon_{jt}
$$

The regressor is the income gap of the bank with the largest outstanding loan to the firm. In Column (1), the dependent variable is the income gap of the bank with the second largest loan. In Column (2), the dependent variable is the income gap of the bank with the future largest loan. The sample is composed of firms for which the two banks are matched with different BHCs. Standard errors are two-way clustered with respect to the two BHCs.
Table 3.3: The Impact of Income Gap on Lending Within Borrowers (Compustat Firms)

<table>
<thead>
<tr>
<th></th>
<th>All Firms</th>
<th>Firms with Multiple Banks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$\Delta Loan$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Gap_{it-1} \times \Delta FedFunds_t$</td>
<td>5.7***</td>
<td>5.3**</td>
</tr>
<tr>
<td></td>
<td>(2.6)</td>
<td>(2.4)</td>
</tr>
<tr>
<td>Observations</td>
<td>213668</td>
<td>204967</td>
</tr>
<tr>
<td>Borrowers</td>
<td>6276</td>
<td>4928</td>
</tr>
<tr>
<td>Lenders</td>
<td>286</td>
<td>285</td>
</tr>
<tr>
<td>BHC top-holders</td>
<td>129</td>
<td>129</td>
</tr>
<tr>
<td>FE</td>
<td>Year</td>
<td>Year</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.28</td>
<td>.28</td>
</tr>
</tbody>
</table>

Note: This table estimates:

$$\Delta L_{i\rightarrow j,t} = \alpha (gap_{it} \times \Delta fed\ funds_t) + \sum_{x \in Control} \gamma_x (x_{it-1} \times \Delta fed\ funds_t)$$

$$+ \eta \Delta L_{i\rightarrow j,t-1} + \phi \cdot gap_{it} + \sum_{x \in Control} \mu_x \cdot x_{it-1} + \delta_{jt} + \epsilon_{it}$$

The controls are $\log(assets_{it-1})$, book equity$_{it-1}$/assets$_{it-1}$. The sample is composed of firms matched with Compustat. The first column includes all firms. The second column only includes firms matched with multiple banks. The third column adds borrower-year fixed effects. Standard errors are clustered at the bank level.
Table 3.4: The Effect of Income Gap on Borrowers Decisions: Quarterly Regressions

<table>
<thead>
<tr>
<th></th>
<th>(\Delta) Debt</th>
<th>(\Delta) Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>(Gap_{it-1} \times \Delta FedFunds_{t-1})</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.6***</td>
<td>.36</td>
</tr>
<tr>
<td></td>
<td>(.6)</td>
<td>(.3)</td>
</tr>
<tr>
<td></td>
<td>(.96)</td>
<td>(.59)</td>
</tr>
<tr>
<td>(Gap_{it-1} \times \Delta FedFunds_{t-2})</td>
<td>-.6</td>
<td>-.29</td>
</tr>
<tr>
<td></td>
<td>(-.69)</td>
<td>(-.19)</td>
</tr>
<tr>
<td>(Gap_{it-1} \times \Delta FedFunds_{t-3})</td>
<td>3.4***</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>(3.8)</td>
<td>(1.5)</td>
</tr>
<tr>
<td>(Gap_{it-1} \times \Delta FedFunds_{t-4})</td>
<td>.75</td>
<td>-.0081</td>
</tr>
<tr>
<td></td>
<td>(.1)</td>
<td>(.005)</td>
</tr>
<tr>
<td>Sum of gap coefficients</td>
<td>5.3</td>
<td>3.3</td>
</tr>
<tr>
<td>p-value of gap coefficients</td>
<td>0</td>
<td>.11</td>
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<tr>
<td>Lender controls</td>
<td>No</td>
<td>Yes</td>
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<tr>
<td>Borrower Controls</td>
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<td>No</td>
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<tr>
<td>Observations</td>
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<tr>
<td>Borrowers</td>
<td>3884</td>
<td>3884</td>
</tr>
<tr>
<td>Lenders</td>
<td>97</td>
<td>97</td>
</tr>
<tr>
<td>(E[\Delta \text{ dep. variable}])</td>
<td>.0051</td>
<td>.0051</td>
</tr>
<tr>
<td>(R^2)</td>
<td>.04</td>
<td>.16</td>
</tr>
</tbody>
</table>

Note: This table estimates for a firm \(i\) in year \(t\) with lead arranger \(b\):

\[
\Delta Y_{it} = \sum_{0 \leq k \leq 4} \alpha_k (gap_{it-1} \times \Delta \text{fed funds}_{t-k}) + \phi \cdot gap_{it-1} \\
+ \sum_{x \in \text{BankControl}} \sum_{0 \leq k \leq 4} \gamma_k x_{it-1} \times \Delta \text{fed funds}_{t} + \sum_{x \in \text{BankControl}} \sum_{0 \leq k \leq 1} \mu_k x_{it-1} \\
+ \sum_{x \in \text{FirmControl}} \sum_{0 \leq k \leq 4} \nu_k x_{it-1} + \sum_{0 \leq k \leq 4} \eta_k \Delta Y_{it-1-k} + \delta_t + \epsilon_{it}
\]

\(\Delta Y\) is alternatively the quarterly change in total debt (Column (1) (2) (3)) and quarterly change in investment (Column (4) (5) (6)). The dependent variable is constructed using Compustat Quarterly Data, which contains quarterly information on debt and asset (while information on employment is only available in the Annual Data). The lender controls are \(\log(\text{assets}_{it-1})\) and \(\text{book equity}_{it-1}/\text{assets}_{it-1}\). The borrower controls are four size bin dummies, state dummies and four-digit SIC code dummies, all interacted with year dummies. Standard errors are two-way clustered at the firm and bank level.
Bibliography


