ESSAYS IN INTERNATIONAL ECONOMICS

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Abstract

This collection of essays examines different topics in international economics. The first two chapters study the role that declines in trade barriers play in shaping the world distribution of net exports, also known as trade imbalances, and sectoral reallocation of economic activity in the U.S., a process also known as structural transformation. The third chapter is co-authored with Gabriel Tenorio and studies optimal capital account policy in small open economies subject to the risks of volatility in international interest rates.

The first chapter proposes a framework that embeds a quantitative multi-country general equilibrium model of international trade into a dynamic framework in which trade imbalances arise endogenously. I calibrate the model and provide a decomposition that shows that 69 percent of the increase in world trade imbalances between 1970 and 2007 can be explained by the decline in trade costs across countries. Moreover, the effect of lower trade costs on trade imbalances is heterogeneous across countries.

The second chapter considers a static general equilibrium open economy model of structural transformation to explore the implications of lower trade costs and trade deficits on structural change in the U.S. The results show that declining trade costs and increasing trade deficits in the U.S. between 1970 and 2007 significantly contributed to the decline in manufacturing’s share in value added. In the absence of declines in trade costs and imbalances, the decline in this share is approximately half of the decline in the baseline calibration of the model.

In the third chapter we study optimal policy responses to shocks in the mean and volatility of the external interest rate in a small open economy with an occasionally binding borrowing constraint. We show that the modeled evolution of interest rates around episodes of sudden stops is consistent with the empirical evidence for a group of emerging markets. We solve the problem of a constrained social planner and show numerically that policy is contingent on the level and volatility of external interest rates.
rate shocks, and that the intensity of the optimal policy is nonmonotonic with respect to the volatility of external shocks.
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Chapter 1

The Role of Trade Costs in the Surge of Trade Imbalances∗

1.1 Introduction

Trade costs affect all forms of trade. Bilateral trade flows at a particular point in time are shaped by the costs associated with shipping goods across countries. Similarly, trade across time periods in the form of trade imbalances, the difference between a country’s total exports and imports, also depends on the levels of these costs at different points in time. Hence, a comprehensive understanding of the forces driving increases in trade imbalances and the risks that they might entail, hinges on identifying how changes in trade costs affect them. Even though this fact is self evident, there is little work exploring the effect of these costs on trade imbalances, or what might be called intertemporal trade.1

∗I wish to thank Brent Neiman and Kei-Mu Yi for very useful comments at various stages of this project. An earlier version of this paper was presented at the Fall 2015 Midwest Macroeconomics Meeting.

1For example, the literature has pointed out the risks associated with rebalancing current accounts around the world [see Blanchard et al. (2005), and Obstfeld and Rogoff (2005)]. For a perspective on the relevance of imbalances after the recent financial crisis, see Blanchard and Milesi-Ferretti (2009) and Obstfeld (2012).

2One contribution that stands out is the work of Obstfeld and Rogoff (2001).
Figure 1.1: Gross Trade Flows and Trade Imbalances (Percent of World GDP)

Notes: The figure plots the evolution of bilateral trade flows aggregated up into world exports as a percentage of world GDP (right axis). The figure also plots the evolution of two measures of trade imbalances normalized to one in 1970 (left axis). The first measure of imbalances is the sum over countries’ absolute values of net exports as a percentage of world GDP. The second measure is the 90-10 percentile difference of net exports in the cross section of countries. The increase in the second measure implies that the increase in imbalances is not being driven by the tails of the cross section distribution of imbalances. All series plotted are the 3-year moving averages (3y-MA) of the original series.

This paper provides a quantitative assessment of the contribution of declining trade costs to the increase in trade imbalances in recent decades. As can be observed in Figure 1.1, there was a steady and sizable increase in bilateral trade flows as well as in the size of trade imbalances, both as a share of world GDP, over the period spanning from 1970 to 2007. I argue in this paper that the decline in trade costs that underlies the increase in observed bilateral trade flows notably contributed to the steady increase in trade imbalances over this period. Specifically, I show that 69 percent of the increase in trade imbalances from 1970 to 2007 can be attributed to lower trade costs in goods markets.
In order to quantify the effects of bilateral trade costs on trade imbalances I propose a theoretical framework that incorporates the main mechanisms driving bilateral trade flows as well as trade imbalances. Specifically, I embed a quantitative multi-country general equilibrium model of international trade into a dynamic framework in which trade imbalances arise endogenously from optimal consumption-saving decisions by economic agents. This model has two main components. First, a static component that builds on the new quantitative multi-country and sector general equilibrium models of international trade based on Ricardian comparative advantages [e.g. Eaton and Kortum (2002), and Caliendo and Parro (2015)]. This part of the model delivers a multi-sector gravity structure of bilateral trade that provides a parsimonious framework to recover the bilateral trade costs that underlie observed bilateral trade flows in the cross section of countries in each year. The second component of the model introduces dynamics that give rise to endogenous trade imbalances based on optimal intertemporal consumption-saving decisions. This part of the model considers a perfect-foresight dynamic framework in which economic agents are able to smooth consumption over time by means of buying and selling one-period bonds in international financial markets. In an equilibrium of the model, perfectly foreseen changes in trade costs, technologies and preferences over time lead to trade imbalances arising from optimal intertemporal decisions. These imbalances, in turn, have to be consistent with those arising from optimal intratemporal trade across countries.

In the model, bilateral trade costs affect equilibrium trade imbalances through two effects. The first is a level effect. Uniformly lower levels of trade costs over all time periods lead to equilibrium goods and factor prices that increase trade imbalances. Intuitively, high bilateral trade costs act as a tax on intertemporal trade due to the fact that this is realized through intratemporal trade of goods and services in different time periods. The seminal work of Obstfeld and Rogoff (2001) points out this mechanism. The authors show how the level effect translates into differences in countries’ real
interest rates depending on their trade balance positions: borrowing countries that run trade deficits pay high real interest rates, while lending countries running trade surpluses get paid low real interest rates, thus, leading to smaller trade imbalances. The link between real interest rates and trade costs arises from the differences in prices due to these costs, together with the fact that these prices determine the real or effective interest rate in each country.

The second effect is associated with the fact that trade costs have been declining over time, which I refer to as the *tilting effect*. This is a general equilibrium effect that arises from the fact that lower trade costs in the future imply that the world economy is becoming richer. This implies that, compared to the case of constant trade costs, equilibrium world real interest rates under declining trade costs are high in initial periods. Therefore, equilibrium imbalances in initial periods are dampened relative to those in the future. Countries that borrow in initial periods borrow less due to high real interest rates, while countries that lend in initial periods lend less due to the positive income effect due to higher interest rates.

In order to quantify the effects of trade costs on trade imbalances, I map the model to observed data for the period 1970-2007 by exploiting the information in bilateral trade flows as well as sectoral and aggregate data on production and prices for a set of 26 countries (including the Rest of the World). Specifically, following Eaton et al. (2011), I rely on the structure of the model’s equilibrium conditions to recover the time series of structural residuals, which I refer to as disturbances, that decompose the forces driving the evolution of the data. The set of disturbances consist of: (i) sector-specific bilateral trade costs, (ii) country and sector-specific productivities, (iii) country and sector-specific demand shifters, and (iv) country-specific intertemporal preference shifters. This set of disturbances accounts for all changes in bilateral trade flows, country and sector-specific prices, country and sector-specific expenditures, and trade imbalances. This procedure allows me to disentangle the effects of bilateral trade
costs from various other forces that affect the realization of trade imbalances. For instance, frictions in international financial markets are captured by the disturbances to intertemporal preferences, \emph{i.e.} wedges in countries’ Euler equations.\footnote{As it will become clear when I describe the model, it incorporates the effects of those financial frictions that manifest themselves as wedges in a country’s Euler equation. Many models with frictions in international financial markets map to this kind of wedges. For example, in Mendoza et al. (2009), frictions show up as wedges in Euler equations.}

Relying on this decomposition, I conduct counterfactual exercises to quantify the consequences of changes in trade costs. My focus is on the contribution of declining trade costs to the surge in trade imbalances. In the main counterfactual exercise, I assume that bilateral trade costs are held fixed at their 1970’s levels and I re-solve for the competitive equilibrium of the world economy. The counterfactual equilibrium is pinned down by the initial net foreign asset distribution of the world economy. Solving for counterfactual equilibria in this fashion is key to providing a quantification that incorporates both the level and tilting effects.\footnote{An alternative would be to solve a planner’s problem. In this case, we could either assign Pareto weights to countries and recover all disturbances based on the planner’s optimality conditions, or abstract from disturbances to intertemporal preferences and recover time-varying weights using the data. However, the issue that arises when conducting counterfactual exercises is that the weights that decentralize the counterfactual equilibrium are unknown. If we were to keep the Pareto weights unchanged in the counterfactual, then we would be ignoring the differential effects across countries due to the ownership of assets. On the other hand, if we were to take a stand on the Pareto weights in the counterfactual, we would not be considering the effects of trade costs in isolation.}

The results of the main counterfactual exercise show that, if trade costs had remained at their 1970’s levels, the increase in world trade imbalances from 1970 to 2007 would have been 69 percent smaller than in the data, which implies that this share of the increase in world trade imbalances is explained by the decline in trade costs across countries. This difference is the result of a decrease in imbalances in 2007 of 41 percent and an increase in 1970 of 28 percent of the actual change that is found in the data. The fact that equilibrium imbalances are greater in the initial years even when trade costs in the counterfactual are the same or similar to the ones in the data is a result of the tilting effect; in the counterfactual, not only do the
levels of trade costs change, but also their entire dynamic path. I confirm this result by conducting an additional exercise in which I isolate the level effect by comparing trade imbalances fixing trade costs at their 1970’s and 2007’s levels. These results highlight the importance of solving for counterfactual competitive equilibria that are pinned down by the initial net foreign asset distribution in the world economy.

In an additional exercise I consider the counterfactual scenario in which agents arrive to the year 1986 and suddenly believe that trade costs will remain constant in all subsequent periods. This exercise is aimed to quantify the effect of the trade liberalizations that came after 1986. In this counterfactual, there is basically no increase in trade imbalances between 1986 and 2007.

In the main counterfactual exercise, I also find that the effects of lower trade costs on trade imbalances are heterogeneous across countries. In particular, trade imbalances in the U.S. and China have been shaped significantly by the fact that trade costs have declined. For example, if trade costs had not decreased, the U.S. would not have experienced the observed increase in its trade deficit from 1970 to 2007. In the case of China, it would have experienced trade surpluses from 1970 until the early 1990s, and deficits thereafter. This is actually exactly the opposite to what we observe in the data. In contrast to these examples, there are other countries whose trade imbalances are mainly driven by forces other than declines in trade costs and therefore do not change significantly in the counterfactual. This is the case, for example, for Japan and Greece.

The reduction in trade costs has important welfare consequences. I compute the welfare gains from lower trade costs in terms of the consumption-equivalent variation in each country. Hence, these measures include not only the static gains from lower trade costs that are usually computed using static models, but also the additional gains due to the ability to trade intertemporally at lower cost. My results show that,

\footnote{By contrast to the equilibrium in the main counterfactual exercise, this equilibrium is pinned down by the net foreign asset distribution of 1986, rather than the one in 1970.}
with the exception of two countries of a sample of 26, all countries benefited from lower trade costs. Furthermore, these gains vary significantly across countries. The median gain is 2 percent of additional consumption per year. For example, Venezuela and Finland suffer welfare losses in the order to 2 percent, whereas Belgium, China and Korea gain additional consumption on the order of 10 percent per year.

These findings highlight the quantitative relevance of lower trade costs for countries’ intertemporal trade decisions. Moreover, they point to the fact that the key for a better understanding of the roots of the steady increase in trade imbalances, and their potential risks, might lie in the fundamental determinants of bilateral trade costs in goods markets rather than other mechanisms.

**Related Literature** This paper contributes to several strands of the literature in international economics. First, it contributes to the literature that explores how trade costs affect international macroeconomic variables [e.g. Backus et al. (1992), Kose and Yi (2006), Fitzgerald (2008), and Barattieri (2014)]. Obstfeld and Rogoff (2001) investigate the potential role of these costs in explaining international macroeconomics puzzles\(^6\). However, their framework is not suitable for a quantitative assessment of these effects.

A limited amount of research has aimed to quantify the importance of trade costs in resolving international macroeconomic puzzles. For example, Fitzgerald (2012) exploits the gravity structure of an Armington-type model of trade in a dynamic-stochastic environment to evaluate how risk-sharing across countries is affected by the degree of incompleteness of financial markets and trade costs in goods markets. Fitzgerald’s results show that trade costs significantly impede risk-sharing, hence, favoring the approach of incorporating these costs when analyzing international macroeconomic variables. In contrast to this paper, her counterfactual exercises do not isolate

\(^6\)The puzzles they consider are: The home-bias-in-trade puzzle, the Feldstein-Horioka puzzle, the home bias in equity holding puzzle, the consumption correlation puzzle, the purchasing-power-parity puzzle, and the exchange rate disconnect puzzle.
the effects of the observed decline in trade costs on trade imbalances. I fill in a gap in this literature by providing a quantitative assessment of the contribution specific to declining trade costs to the evolution of trade imbalances by solving for the counterfactual competitive equilibria that incorporate both the level and tilting effects. Additionally, in contrast to Fitzgerald (2012), I consider multiple sectors in a framework in which intratemporal trade arises due to differences in Ricardian comparative advantage across countries. In recent work, Eaton et al. (2015) propose a theoretical framework similar to the one I propose in this paper to study the effects of trade costs on a number of the puzzles studied by Obstfeld and Rogoff from a quantitative perspective. Their exercise is similar to the one in this paper, but they do not focus on the counterfactual evolution of world trade imbalances absent declines in trade costs. Their results provide further support for the approach taken in this paper.

This paper is also related to the recent literature on new quantitative general equilibrium models of international trade based on gravity-type equations. The seminal work of Eaton and Kortum (2002) provided a micro-foundation based on Ricardian forces for gravity models of trade that led to many of the recent contributions in this literature. Dekle et al. (2007) and Dekle et al. (2008) incorporate trade deficits into this model and develop a procedure for quantitative analysis of counterfactual equilibria that is now standard in the literature. However, their analysis is static, which implies that their framework does not provide an underlying explanation as to why these imbalances arise or might change. Caliendo and Parro (2015) retained the assumption of exogenous trade imbalances and extended the model in Dekle et al. (2007) and Dekle et al. (2008) by incorporating multiple sectors and input-output

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7 Specifically, she solves for the planner’s problem, which implies that in counterfactual exercises she must take a stand on the counterfactual Pareto weights. In her exercises, she focuses on the case in which countries can engage in perfect risk sharing, and does not analyze the counterfactual evolution of trade imbalances.

8 See Costinot and Rodriguez-Clare (2014) for a recent survey of the literature.

9 This paper focuses on the effects of rebalancing current accounts by analyzing a counterfactual world in which all imbalances are eliminated.
linkages in a tractable fashion. This type of models provides now the standard framework for quantitative analysis of gross international trade flows driven by multiple disturbances that can be recovered from observed data. Moreover, recent work has exploited this framework to analyze issues that have been traditionally addressed in macroeconomics.\(^{10}\)

Many recent contributions have enriched these models and the way in which they incorporate imbalances in a static setup \(\textit{e.g.}\) Ossa (2014) and Caliendo et al. (2014), however, basically none of them has considered an intertemporal approach to trade imbalances.\(^{11}\) One exception is Eaton et al. (2016), which incorporates a structure of international trade, similar to the one in my model, into a dynamic model of business-cycles in order to investigate the forces acting on the global economy during the Great Recession and ensuing recovery.\(^{12}\) In their model, trade imbalances also arise from optimal intertemporal decisions by economic agents, hence, linking changes in trade costs to trade imbalances. However, there are substantial differences between their work and mine. First, the focus of my paper is on the forces shaping long run changes in trade imbalances, while their paper focuses on forces at the business-cycle frequency. Therefore, their analysis and model shifts attention to investment and capital accumulation rather than changes in trade imbalances. Second, in terms of methodology, in Eaton et al. (2016) trade imbalances arise from the solution of a planner’s problem that assigns subjective weights to each country. These weights are

\(^{10}\)For example, Parra (2013) explores the effects of trade and capital-skill complementarily on the skill premium; Caselli et al. (2015) analyze how trade affects macroeconomic volatility; and Levchenko and Zhang (2016) exploit the structure of these models to recover the evolution of sectoral productivities and explore changes in comparative advantage over time. Another interesting example is Caliendo et al. (2014) which studies the impact of intersectoral and interregional trade linkages in propagating disaggregated productivity changes in particular locations of the U.S. to the rest of the economy.

\(^{11}\)As discussed in Obstfeld and Rogoff (1995), this approach views trade imbalances, or more precisely current-account imbalances, as the outcome of forward-looking dynamic saving and investment decisions; currently the standard in international macroeconomic models.

\(^{12}\)The methodology for counterfactual analysis in my paper was in part motivated by the first version of their paper Eaton et al. (2011), which did not incorporate endogenous trade imbalances. A subsequent version of their work Eaton et al. (2016), which came out while I had already started this project, does consider trade deficits derived from optimal saving decisions.
held constant across counterfactual exercises. In contrast, I solve for the competitive equilibrium, which implies that weights are mapped to equilibrium outcomes that change across counterfactual equilibria. This difference is key in order to quantify the full effects of trade costs on trade imbalances, like the tilting effect. Third, they incorporate endogenous investment decisions and focus on different sectors for which data on investment is available. Hence, in their counterfactuals, changes in capital stocks over time are driven by endogenous investment decisions which I do not have in my model.\textsuperscript{13}

This paper also contributes to the literature in international macroeconomics that studies the causes and consequences of the observed pattern of external imbalances. Gourinchas and Rey (2014) provide an extensive survey of the literature.\textsuperscript{14} Most of this literature has focused on financial frictions to explain the fundamental causes of the observed pattern of current account imbalances.\textsuperscript{15} For instance, Caballero et al. (2008) and Mendoza et al. (2009) consider the case of differences in the development of financial markets across particular regions or groups of countries. Chang et al. (2013) build on the model with a continuum of countries of Clarida (1990) to quantitatively explore the increase in the dispersion of current account imbalances under uninsurable idiosyncratic risk.\textsuperscript{16} Other papers have explored the interaction between trade and capital flows. The work by Antràs and Caballero (2010) and Jin (2012) provide two

\textsuperscript{13}Extending my theoretical framework to incorporate capital accumulation is relatively standard. In an appendix I present a version of my model with endogenous investment decisions. However, solving for counterfactual equilibria numerically is a computationally complex endeavor, and is currently work in progress.

\textsuperscript{14}Many studies focus on what they refer to as “global imbalances”, which they define as the steady increase in current account balances since the late 1990s, specifically the increase in capital flows from emerging economies to the U.S.; and the fact that net capital inflows tend to be negatively correlated with productivity growth across developing countries, the “allocation puzzle”.

\textsuperscript{15}Bernanke (2005) identifies a number of other potential reasons behind what he calls a “saving-glut”, many of which have also been studied in the literature. For example, Aguier and Amador (2011) study public flows and reserve accumulation and Lane and Milesi-Ferretti (2001) consider demographic factors.

\textsuperscript{16}Bai and Zhang (2010) use a similar framework to study the Feldstein-Horioka puzzle.
examples of this work. However, none of these papers explores the effect of declining trade costs on imbalances.

In contrast to this literature, this paper does not take a stand on a particular fundamental cause for observed trade imbalances; I rather attribute them to a set of disturbances that might be generated by multiple underlying frictions. In this sense, this paper relates to Gourinchas and Jeanne (2013) who rely on disturbances or “wedges” to saving and investment decisions to point to where the underlying causes of the “allocation puzzle” might lie. However, one of the set of disturbances in my model maps directly to the trade costs that arise in micro-founded gravity models of trade, thus, providing a sufficient statistic of frictions in bilateral transactions of goods and services across countries, i.e. trade costs.\textsuperscript{17} By doing so, this paper contributes to the literature in various respects. First, by focusing on frictions in goods rather than financial markets, I show that the effects of the former are quantitatively relevant in the determination of trade imbalances. Second, by considering a multi-country and sector structure and incorporating geography, the model allows me to study the implications for as well as the effects of trade imbalances in particular countries. Third, I contribute by analyzing trade imbalances for a long period of time relative to other studies. As shown in Figure 1.1 and documented in Faruqee and Lee (2009), the increase in imbalances started well before the late 1990s. Lastly, I focus on explaining the evolution of trade rather than current account imbalances. While this is not the standard in the literature on external imbalances, I do so because trade costs primarily affect bilateral trade flows which are directly related to the trade balance rather than the current account. Still, the trade balance accounts for most of the current account in a majority of countries.

\textsuperscript{17}Chari et al. (2000) show how models with different types of frictions map to these shocks or “wedges” in a closed economy RBC model. Kehoe et al. (2013) consider a two country world and rely on a similar accounting procedure to evaluate the contribution of global imbalances to structural transformation in the U.S.
Road Map  The remainder of this paper is organized as follows. Section 2 describes the model and defines an equilibrium. Additionally, it discusses the main mechanisms through which trade costs affect trade imbalances. Section 3 explains how the model is mapped to aggregate data for a set of 26 countries for the period 1970-2007. This section shows how the structure of the model delivers structural residuals that can be identified using the data previously mentioned. Section 5 conducts the counterfactual exercises that lead to the main results of this paper. Section 6 concludes.

1.2 The Model

The main goal of this paper is to provide a quantitative assessment of the role of trade costs as determinants of trade imbalances. In order to do so, this section develops the theoretical framework that will be used to study these effects. The framework embeds a quantitative model of international trade into a dynamic environment in which trade imbalances arise endogenously due to consumption-saving decisions. The static structure of the model builds on the quantitative multi-sector extensions of the work by Eaton and Kortum (2002). Specifically, the static part of my model is closest to the framework in Caliendo and Parro (2015).

Consider an infinite horizon in which time is discrete and indexed by $t = 0, 1, \ldots$. The world consists of $I$ countries indexed by $i = 1, \ldots, I$, each populated by a representative household endowed with $L_{i,t}$ and $K_{i,t}$ units of homogeneous labor and capital in period $t$. Each economy consists of $J$ sectors that I index by $j = 1, \ldots, J$. Hence, in general I will use the letter $t$ to denote time periods, the letter $i$ or $h$ to denote countries, and the letter $j$ to denote sectors. I assume that all economic agents have perfect foresight.
1.2.1 Nontradable Sectoral Goods

Final output in each sector \( j \) is given by an aggregate of a continuum of tradable goods indexed by \( \omega^j \in [0, 1] \). I assume that this aggregation takes on a constant elasticity of substitution (CES) functional form with elasticity of substitution \( \eta > 0 \). Denoting by \( Q_{i,t}^j \) sector \( j \)'s final output in country \( i \) at time \( t \), we have that

\[
Q_{i,t}^j = \left( \int_0^1 d_j^{i,t} (\omega^j) \frac{\eta - 1}{\eta} d\omega^j \right)^{\frac{\eta}{\eta - 1}},
\]

where \( d_j^{i,t} (\omega^j) \) denotes the use in production of intermediate good \( \omega^j \).

The demand for each intermediate good is derived from the cost minimization problem of a price-taking representative firm. Moreover, since good \( \omega^j \) is tradable across countries, the firms producing \( Q_{i,t}^j \) search across all countries for the lowest cost supplier of this good.

The final output in each sector \( j \) is nontradable and can be used either for final consumption or as an intermediate input into the production of the tradable goods. I will denote by \( P_{i,t}^j \) the price of sectoral good \( j \) in country \( i \) at time \( t \). Note that, since sectoral goods are nontradable, these prices can differ across countries. Let us now focus on the technologies available to produce the tradable goods indexed by \( \omega^j \).

1.2.2 Tradable Goods

Consider a particular good \( \omega^j \in [0, 1] \) and let \( q_{i,t}^j (\omega^j) \) denote the production of this good in country \( i \) at time \( t \). The technology to produce each good \( \omega^j \) is given by

\[
q_{i,t}^j (\omega^j) = x_{i,t}^j (\omega^j) \left[ k_{i,t}^j (\omega^j) l_{i,t}^j (\omega^j)^{\varphi_i} M_{i,t}^j (\omega^j) \right]^{\beta_{i,t}^j} [M_{i,t}^j (\omega^j)]^{1-\beta_{i,t}^j},
\]

where \( l_{i,t}^j (\omega^j) \) and \( k_{i,t}^j (\omega^j) \) are the labor and capital respectively used in the production of good \( \omega^j \), and \( M_{i,t}^j (\omega^j) \) denotes the amount of intermediates used in produc-
tion. In particular, I assume that the use of intermediates in production is given by a Cobb-Douglas aggregate of nontradable sectoral goods:

$$M_{i,t}^j (\omega^j) = \prod_{m=1}^J D_{i,t}^{j,m} (\omega^j)^{\nu_{i,m}^{j,m}},$$

(1.3)

where $\sum_{m=1}^J \nu_{i,m}^{j,m} = 1$ for all $j = 1, \ldots, J$ and $\nu_{i,m}^{j,m} \in (0, 1)$ for all $j, m = 1, \ldots, J$. Here, $D_{i,t}^{j,m} (\omega^j)$ denotes the intermediate demand by producers of good $\omega^j$ for sectoral good $m$. The efficiency in the production of good $\omega^j$ is given by $x_{i,t}^{j} (\omega^j)$.

Note that the country and sector specific parameter $\beta_i^j \in (0, 1)$ determines the share of value added in gross production, while $\phi_i \in (0, 1)$ represents the share of capital in value added. Additionally, $\nu_{i,m}^{j,m}$ for all $j, m = 1, \ldots, J$ determine the input-output structure in each country.

I assume that the efficiency in the production of good $\omega^j$, $x_{i,t}^{j} (\omega^j)$, is given by the realization of a random variable, $x_{i,t}^{j} \in (0, \infty)$, distributed conditional on information in period $t$ according to a Fréchet distribution with shape parameter $\theta$ and location parameter $T_{i,t}^j$,

$$F_{i,t}^j (x|t) = \Pr \left[ x_{i,t}^{j} \leq x \right] = e^{-T_{i,t}^j x^{-\theta}}. \quad (1.4)$$

I assume that, conditional on $T_{i,t}^j$, the random variables $x_{i,t}^{j}$ are independently distributed across sectors and countries. In this case, the level of $T_{i,t}^j$ represents a measure of absolute advantage in the production of sector $j$ goods, while a lower $\theta$ implies more dispersion across the realizations of the random variable and a higher scope for gains from comparative advantage differences through specialization.

I will refer to $T_{i,t}^j$ as the sectoral productivity of country $i$ in sector $j$ at time $t$, since their values determine the level of the distribution from which producers draw their efficiencies. These productivities change over time and they represent one of the underlying disturbances that drive the dynamics of the world economy.
1.2.3 Trade Costs and Firms' Optimal Decisions

For each sector \( j = 1, \ldots, J \), goods \( \omega^j \in [0, 1] \) can be traded across countries, but are subject to iceberg type trade costs. Specifically, \( \tau_{ih,t}^j \geq 1 \) denotes the cost of shipping any good \( \omega^j \in [0, 1] \) from country \( h \) to country \( i \) at time \( t \). This means that, in order for one unit of variety \( \omega^j \) to be available in country \( i \) at time \( t \), country \( h \) must ship \( \tau_{ih,t}^j \) units of the good. I assume that \( \tau_{ii,t}^j = 1 \) for all \( i = 1, \ldots, I \), i.e. there are no trade costs associated with trading goods within countries.

Note that these bilateral trade costs are allowed to change over time and that they are sector, but not good specific. Hence, sector specific bilateral trade costs are additional disturbances that drive the dynamics of the model.

Let us now turn to the optimal decisions by firms. In particular, consider first the problem faced by the producer of good \( \omega^j \in [0, 1] \). Assuming perfectly competitive markets and given constant returns to scale in the production of good \( \omega^j \), the free-on-board price (before trade costs) of one unit of this good, if actually produced in country \( i \) at time \( t \), will be equal to its marginal cost, \( c_{i,t}^j x_{i,t}^j(\omega^j) \), where

\[
c_{i,t}^j = \kappa_i^j \left[ (r_{i,t})^\phi_i (w_{i,t})^{1-\phi_i})^{\beta_i^j} \left( \prod_{m=1}^J \left(P_{i,t}^m\right)^{\nu_{i,m}^j} \right)^{1-\beta_i^j} \right]
\]  

(1.5)

is the cost of the input-bundle to produce one unit of \( \omega^j \), \( r_{i,t} \) and \( w_{i,t} \) denote the rental rate and the wage in country \( i \) respectively, and \( \kappa_i^j \) is a constant that depends on production parameters.\(^{18}\)

For a particular sector \( j \), notice that the technologies to produce goods \( \omega^j \in [0, 1] \) differ only by their productivity draw, while \( c_{i,t}^j \) is constant across tradable goods. Hence, following Caliendo and Parro (2015), we can relabel tradable goods by their efficiencies, \( x_{i,t}^j \). Letting \( g^j(x^j|t) \) denote the conditional joint density of the sector specific vector of productivity draws for all countries, \( x^j = (x_{1,t}^j, \ldots, x_{I,t}^j) \), we

\(^{18}\)Specifically, \( \kappa_i^j = (\beta_i^j \phi_i (1-\phi_i)^{(1-\phi_i)} - \beta_i^j ((1-\beta_i^j) \prod_{m=1}^J (\nu_i^{i,m})^{\nu_{i,m}^j} - (1-\beta_i^j)} \).
can define total factor and intermediate input usage from each sector \( m \) in sector \( j \) as

\[
L_{i,t}^j = \int_{R_+^j} l_{i,t}^j(x^j) \varrho_j^j(x^j|t) \, dx^j, \quad (1.6)
\]

\[
K_{i,t}^j = \int_{R_+^j} k_{i,t}^j(x^j) \varrho_j^j(x^j|t) \, dx^j, \quad \text{and} \quad (1.7)
\]

\[
D_{i,t}^{j,m} = \int_{R_+^j} D_{i,t}^{j,m}(x^j) \varrho_j^j(x^j|t) \, dx^j. \quad (1.8)
\]

Let us now turn to the problem faced by the nontradable sectoral goods producers. Given the price of each variety \( \omega_j \in [0,1] \) that the representative firm is faced with, \( p_{i,t}^j(\omega_j) \), the firm solves a cost minimization problem which delivers demand functions, conditional on \( Q_{i,t}^j \), for each tradable good \( \omega_j \in [0,1] \) given by

\[
d_{i,t}^j(\omega_j) = \left( \frac{p_{i,t}^j(\omega_j)}{P_{i,t}^j} \right)^{\eta} Q_{i,t}^j, \quad \text{where} \quad (1.9)
\]

\[
p_{i,t}^j(\omega_j) = \min_h \left\{ p_{h,t}^j(\omega_j) \right\} = \min_h \left\{ \frac{c_{h,t}^j\tau_{h,t}^j}{x_{h,t}^j(\omega_j)} \right\}
\]

and \( P_{i,t}^j \) denotes the price of sectoral good \( j \), which is given by

\[
P_{i,t}^j \equiv \left( \int_0^1 p_{i,t}^j(\omega_j)^{1-\eta} \, d\omega_j \right)^{\frac{1}{1-\eta}}. \quad (1.10)
\]

Note that firms, by minimizing their costs, source tradable good \( \omega_j \) from the lowest cost supplier after taking into account trade costs, as is implied by (1.9). This is an important difference of this model relative to Armington-type models in which each good is origin-specific.
1.2.4 Prices and Trade Shares

Given these distributions of productivities, we can derive an expression for sectoral price indices in equilibrium as functions of all sectoral prices, factor prices, and trade costs around the world. These prices are conditional on the known values of sectoral productivities, $T^j_{i,t}$, and bilateral trade costs, $\tau^j_{ih,t}$, in period $t$. Using (1.10) and the properties of the distribution of efficiencies around the world, we can derive the sectoral prices in each country $i$ and every period $t$. In line with the derivations of Eaton and Kortum (2002) and Caliendo and Parro (2015), these prices are given by

$$P^j_{i,t} = \Gamma \left[ \Phi^j_{i,t} \right]^{-\frac{1}{\theta}}, \quad (1.11)$$

where $\Gamma$ is a constant that only depends on $\eta$ and $\theta$, and

$$\Phi^j_{i,t} = \sum_{h=1}^{I} T^j_{h,t} \left( c^j_{h,t} \tau^j_{ih,t} \right)^{-\theta} \quad (1.12)$$

represents a sufficient statistic for sector $j$ in country $i$ of the state of technologies and trade costs around the globe. Note that as long as there is no free trade, i.e. $\tau^j_{ih,t} \neq 1$ for some countries $i$ and $h$, prices will differ across countries. If there is free trade, it will be the case that $P^j_{i,t} = P^j_{h,t}$ for all $i, h = 1, \ldots, I$.

The structure of the model not only allows for closed form solutions of sectoral price indices, but we can also recover sectoral trade shares for each country in terms of world prices, technologies and trade costs, i.e. we can find expressions for the share of total expenditure on goods produced in sector $j$ that is spent in each country. Let $E^j_{i,t}$ denote total expenditure by country $i$ on sector $j$ goods, and $E^j_{ih,t}$ total expenditure by country $i$ on sector $j$ goods produced in country $h$, so that $E^j_{i,t} = \sum_{h=1}^{I} E^j_{ih,t}$.

Then, the share of total expenditure in sector $j$ by country $i$ in goods produced by

\[19\text{In particular, } \Gamma = \left( \Gamma(1 + \frac{(1-\eta)}{\theta}) \right)^{1/\theta}, \text{ where } \Gamma(\cdot) \text{ denotes the Gamma function evaluated for } z > 0. \text{ Notice this implies that parameters have to be such that } \eta - 1 < \theta. \]
country $h$, $\pi_{ih,t}^j = \frac{E_{ih,t}^j}{E_{i,t}^j}$, is given by

$$\pi_{ih,t}^j = \frac{T_{j,t}^j \left( c_{j,t}^j \tau_{ih,t}^j \right)^{-\theta}}{\Phi_{i,t}^j},$$

(1.13)

and are such that $\sum_{h=1}^I \pi_{ih,t}^j = 1$ for all $i = 1, \ldots, I$ and $j = 1, \ldots, J$. Note that by the expression that we obtained before for equilibrium prices, equation (1.11), we can rewrite this share in terms of the sectoral price in country $i$ as

$$\pi_{ih,t}^j = \left( \Gamma^{-\theta} T_{j,t}^j \left( \frac{c_{j,t}^j \tau_{ih,t}^j}{P_{i,t}^j} \right) \right)^{-\theta}. \quad (1.14)$$

These prices and trade shares fully summarize the optimal decisions by the firms given technologies and factor prices, as well as bilateral trade flows given sectoral expenditure levels in all countries. This can be appreciated in (1.11), which implicitly defines sectoral prices as a function of factor prices, and (1.14), which defines all bilateral trade shares given these sectoral prices.

Up to this point, the structure of the model in a particular period $t$ is very similar to the one in Caliendo and Parro (2015), but adding capital as an additional factor of production. An additional extension relative to their model that is crucial in order to analyze dynamics in the long run is to assume CES preferences rather than Cobb-Douglas. This preference structure will allow us to capture endogenous structural transformation over time due to changes in relative sectoral prices. I now turn to the problem of households in each country and their decisions, which represent the piece of the model that departs from other quantitative general equilibrium models of trade. This will allow us to see how dynamic decisions by the households are affected by trade costs.

18
1.2.5 Households

The dynamic dimension of the model comes entirely from the household’s decisions. The representative household in country $i$ seeks to maximize its discounted lifetime utility given by

$$U_i = \sum_{t=0}^{\infty} \delta^t \phi_{i,t} u(C_{i,t}),$$

(1.15)

where $\delta \in (0, 1)$ is the subjective discount factor, which is common across all countries; $C_{i,t}$ is an aggregate index of sectoral consumption levels; and $\phi_{i,t}$ is an intertemporal preference disturbance that the household in country $i$ experiences in period $t$. I assume that the household aggregates the amounts of nontradable sectoral goods used for consumption in a CES fashion with an elasticity of substitution given by $\psi > 0$. Hence, aggregate consumption is given by

$$C_{i,t} = \left( \sum_{j=1}^{J} (\mu_{i,t}^j)^\frac{1}{\psi} \left( C_{i,t}^j \right)^\frac{\psi-1}{\psi} \right)^{\frac{\psi}{\psi-1}},$$

(1.16)

where $\mu_{i,t}^j > 0$ are sectoral demand disturbance at time $t$, such that $\sum_{j=1}^{J} \mu_{i,t}^j = 1$ for all $t$.20

Note that $\mu_{i,t}^j$ for all $j = 1, \ldots, J$ and $\phi_{i,t}$ for all $i = 1, \ldots, I$ are additional disturbances that are allowed to change over time. These two sets of disturbances to preferences plus those to sectoral productivities and trade costs, aside from changes in the endowment of factors of production, drive the exogenous changes in the world economy over time. I show in the next section how these disturbances imply that

\[\text{20} \text{The use of these type of shifters is common in the international macroeconomics literature. See Tesar and Stockman (1995) or Bai and Ríos-Rull (2015). However, as discussed in the introduction, changes in these shifters can also be interpreted as financial frictions that affect households’ saving decisions.}\]

\[\text{21} \text{This disturbances capture those forces other than changes in relative sectoral prices driving changes in sectoral expenditure shares. For example, if a country’s sectoral expenditure shares depend on its level of income, these effects will be captured by these disturbances.}\]
the model is exactly identified given data on bilateral trade flows, sectoral price for tradable sectors and GDP, sectoral expenditure levels, and net exports.

The representative household in a country has access to international financial markets by means of buying and selling one period bonds that are available around the world in zero net supply. International financial markets are assumed to be frictionless. This implies that the return on bonds in terms of the world currency unit is equalized across countries. Moreover, the fact that all economic agents are assumed to have perfect foresight implies that, in the absence of trade frictions, access to these one period bonds is the only savings vehicle needed for resources to be efficiently allocated around the world in every period. As previously mentioned, I treat capital as a fixed endowment in every period and do not consider endogenous capital accumulation decisions. Even though incorporating this dimension to the model is relatively standard, as I show in an appendix, solving for counterfactual equilibria numerically is challenging and is currently work in progress.\textsuperscript{22} However, given that the focus of this paper is on the long term evolution of trade imbalances rather than changes at the business-cycle frequency, this margin of adjustment might not play such an important role in determining long term changes in imbalances.

Hence, the representative household in country $i$ maximizes (1.15) by choosing bond holdings at the end of period $t$, $B_{t+1}$, and sectoral consumption levels, $C_{j,i,t}$ for all $j = 1, \ldots, I$ subject to the following sequence of budget constraints

$$\sum_{j=1}^{J} P_{i,t}^{j} C_{j,i,t}^{i} + B_{i,t+1} = w_{i,t} L_{i,t} + r_{i,t} K_{i,t} + R_{t} B_{i,t},$$ (1.17)

\textsuperscript{22}The main challenge comes from the fact that with capital accumulation and trade imbalances, the system of equations that determines the world economy competitive equilibrium’s steady state variables is underidentified. This implies that the solution of the transitional dynamics of counterfactual equilibria requires the simultaneous solution of the paths of net foreign asset positions and the net foreign asset position in the counterfactual steady state.
for all $t = 0, 1, \ldots$, and $W_{i,0} \equiv R_0 B_{i,0}$ given for all $i = 1, \ldots, I$ such that $\sum_{i=1}^{I} W_{i,0} = 0$.

Note that the amount of bonds held by country $i$ at the end of period $t$, $B_{i,t+1}$, is denominated in world currency units (or whatever numeraire we choose) and that these bonds have a nominal gross return of $R_t$. I choose world GDP as a numeraire, i.e. $\sum_{i=1}^{I} w_{i,t} L_{i,t} + r_{i,t} K_{i,t} = 1$. This implies that all my result will be in terms of this world GDP, which is in line with other quantitative models of international trade.

1.2.6 Household’s Optimal Decisions

Let us first consider the static problem that the household faces in period $t$ given a choice of $B_{i,t+1}$. In particular, the household optimally chooses sectoral consumption expenditure levels across sectors according to

$$ P_{i,t}^j C_{i,t}^j = P_{i,t}^j \left( \frac{P_{i,t}^j}{P_{i,t}} \right)^{1-\psi} P_{i,t} C_{i,t}, $$

for all $j = 1, \ldots, J$ where $P_{i,t}$ denotes the ideal price index of consumption, which in turn is given by

$$ P_{i,t} = \left( \sum_{j=1}^{J} \mu_{i,t}^j \left( P_{i,t}^j \right)^{1-\psi} \right)^{\frac{1}{1-\psi}}. $$

Therefore, conditional on $P_{i,t} C_{i,t}$, the household choose sectoral consumption expenditures according to (1.18), and we can rewrite total consumption expenditure in country $i$ simply as $P_{i,t} C_{i,t} = \sum_{j=1}^{J} P_{i,t}^j C_{i,t}^j$. The dynamic problem of the household in country $i$ then becomes:

$$ \max_{\{C_{i,t}, B_{i,t+1}\}} \sum_{t=0}^{\infty} \delta^t \phi_{i,t} u(C_{i,t}) $$

s.t. $P_{i,t} C_{i,t} + B_{i,t+1} = w_{i,t} L_{i,t} + r_{i,t} K_{i,t} + R_t B_{i,t}$ for $t = 0, 1, \ldots$
and $R_0 B_{i,0}$ given. This is, the household in country $i$ takes prices as given and chooses consumption, $C_{i,t}$, and bond holdings at the end of period $t$, $B_{t+1}$, so as to maximize its discounted utility stream, $U_i$, subject to its sequence of budget constraints.

The optimality condition derived from solving problem (1.20) is given by the Euler equation that determines the optimal intertemporal consumption choices:

$$u'(C_{i,t}) = \delta \hat{\phi}_{i,t+1} \frac{R_{t+1} P_{i,t}}{P_{i,t+1}} u'(C_{i,t+1}),$$

for all $t = 0, 1, \ldots$ where I have defined $\hat{\phi}_{i,t+t} \equiv \frac{\phi_{i,t+1}}{\phi_{i,t}}$. Note that the real interest rate in country $i$ is then given by $r_{i,t} \equiv \frac{R_{t+1} P_{i,t}}{P_{i,t+1}} - 1$.

Nominal interest rate parity holds in the world economy. There are no frictions in the exchange of bonds denominated in a world currency unit across countries. Therefore, the nominal return on bonds is the same for all countries and determined in equilibrium such that assets are in zero net supply. However, real interest rate parity does not hold because of trade costs that lead to differences in price levels across countries. Specifically, the evolution of the price level in each country determines the real return on bonds. Moreover, price levels include information on all shocks and parameters that determine gross trade flows in a particular time period. Hence, changes in trade costs over time have implications for price levels and, therefore, for the real return on bonds. How do changes in trade costs exactly affect the saving decisions by the household? I address this issue in more detail in the last part of this section. First, I close the description of the model by stating the market clearing conditions.

### 1.2.7 Market Clearing Conditions

Let $Y_{i,j}^t$ denote the value of gross production in sector $j$, and $E_{i,j}^t$ total expenditure by country $i$ on sector $j$ goods. Then, the value of total gross production and total
expenditure in country $i$ and sector $j$ define sectoral net exports, $NX_{i,t}^j = Y_{i,t}^j - E_{i,t}^j$, and aggregate net exports are then simply given by $NX_{i,t} = \sum_{j=1}^J NX_{i,t}^j$.

First, the markets for nontradable sectoral goods and factors must clear in every country and period. These conditions are given by

$$C_{i,t}^j + \sum_{k=1}^J D_{i,t}^{k,j} = Q_{i,t}^j$$ \hspace{1cm} (1.22)

for all $i$ and $j$, and $\sum_{j=1}^J L_{i,t}^j = L_{i,t}$ and $\sum_{j=1}^J K_{i,t}^j = K_{i,t}$ for all $i$. Condition (1.22) states that demand for nontradable goods must equal supply in each country $i$. We can reformulate this condition in terms of expenditures, in which case we can appreciate that total expenditure in goods in sector $j$ in equilibrium must be given by

$$E_{i,t}^j = P_{i,t}^j C_{i,t}^j + \sum_{m=1}^J P_{i,t}^j D_{i,t}^{m,j}.$$ \hspace{1cm} (1.23)

Thus, these equilibrium conditions can be rewritten simply as $E_{i,t}^j = P_{i,t}^j Q_{i,t}^j$.

We now turn to market clearing in tradable goods markets. In terms of expenditure, I refer to these conditions as the flow of goods across countries equilibrium conditions. These conditions are given by

$$Y_{i,t}^j = \sum_{h=1}^I \pi_{h,i,t}^j E_{h,t}^j,$$ \hspace{1cm} (1.24)

and must hold for every country $i$ and sector $j$. This condition states that expenditure by all countries on sector $j$ goods produced in country $i$ must equal the value of total gross production in country $i$. In particular, country $h$ spends $\pi_{h,i,t}^j E_{h,t}^j$ on sector $j$ goods produced in country $i$.

Lastly, there are country-specific resource constraints. This is one of the main differences between a model with endogenous trade imbalances and static trade models. Net exports in goods and services must be consistent with optimal saving decisions.
by the representative household in country $i$. This equilibrium resource constraint is given by

$$B_{i,t+1} - R_t B_{i,t} = \sum_{j=1}^{J} (Y_{i,t}^j - E_{i,t}^j).$$

(1.25)

Another way to interpret this condition is through the balance of payments. This condition is equivalent to the balance of payments identity that is trivially satisfied in most international macroeconomic models and not present in static trade models. This identity can be appreciated by rewriting the previous condition as $NX_{i,t} + (R_t - 1) B_{i,t} + B_{i,t} - B_{i,t+1} = 0$, where $CA_{i,t} \equiv NX_{i,t} + (R_t - 1) B_{i,t}$ denotes the current account in country $i$, and $KA_{i,t} \equiv B_{i,t} - B_{i,t+1}$ denotes the broadly defined capital account.

1.2.8 Equilibrium

Given our previous analysis of the problems of the firms, representative household, and market clearing conditions; we can now proceed to define an equilibrium of the world economy. We will do so for particular sequence of disturbances. Thus, let us define the sequence of disturbances by $\{S_t\}_{t=0}^{\infty}$, where $S_t \equiv \{\tau_{ih,t}, T_{i,t}, \mu_{i,t}, \phi_{i,t}\}_{i,h=1,...,I}$.

**Definition** Given $R_0 B_{i,0}$, $L_{i,t}$ and $K_{i,t}$ for every $i = 1, \ldots, I$ and every $t = 0, 1, \ldots$, and $\{S_t\}_{t=0}^{\infty}$; an equilibrium under disturbances $\{S_t\}_{t=0}^{\infty}$ is defined by sequences of wages, rates of return on capital, gross interest rates and prices,

$$\{\{w_{i,t}, r_{i,t}\}_{i=1}^{I}, R_{t+1}, \{\{P_{i,t}^j\}_{j=1}^{J}\}_{i=1}^{I}, \{P_{i,t}\}_{t=0}^{\infty}\}$$

and allocations, such that given these prices, the allocations satisfy the optimality conditions for the firms and households in every country, and all markets clear.

This definition of an equilibrium differs in one particular and relevant respect from equilibria considered in previous work on quantitative general equilibrium models of international trade. This definition includes the gross interest rate as an equilibrium price, which together with changes in country specific prices over time, determines
the intertemporal allocation of consumption. Households have an endogenous saving decision, and in equilibrium, they optimally chooses how much to save or consume.

Most previous studies relying on new quantitative models of trade do not consider this margin of households’ decisions. Therefore, gains from lower trade costs in my model also include the gains from being able to engage in more intertemporal trade. I now turn to a discussion of how trade costs and the general features of intratemporal trade across countries affect saving decisions by the households in each country.

Effects of Trade Costs on Saving Decisions

As pointed out at the beginning of the paper, the effect of declining trade costs on trade imbalances can be analyzed through the lens of two effects: the level and the tilting effect. I now discuss how these effects lead to changes in trade imbalances.

I consider first the level effect. The basic intuition behind this effect is based on the fact that intertemporal trade is realized through the exchange of goods and services in different time periods. Hence, uniformly high bilateral trade costs act as a tax on intertemporal trade just as they do on intratemporal trade.

In terms of the model, note first that using (1.24) we can rewrite country \( i \)'s net exports in sector \( j \) in terms of countries’ trade shares as

\[ NX_{i,t}^j = Y_{i,t}^j - E_{i,t}^j = \sum_{h=1}^{I} (\pi_{hi,t}^{j} E_{h,t}^{j} - \pi_{ih,t}^{j} E_{i,t}^{j}). \]

Therefore, equation (1.25) becomes

\[ B_{i,t+1} - R_t B_{i,t} = \sum_{j=1}^{J} \left( \sum_{h=1}^{I} (\pi_{hi,t}^{j} E_{h,t}^{j} - \pi_{ih,t}^{j} E_{i,t}^{j}) \right). \]  

(1.26)

Equations (1.26) and (1.21) incorporate the main information regarding the interaction between trade costs and saving decisions.

Consider equation (1.21). As previously stated, real interest rate parity does not hold because of trade costs in goods markets that lead to differences in aggregate prices across countries. Hence, countries’ saving decisions are based on different real interest rates. Obstfeld and Rogoff (2001) show how these differences in real interest
rates due to trade costs in goods markets imply that in equilibrium, trade imbalances are dampened. Their result follows from the following observations. Consider an equilibrium in which country $i$ is running a trade surplus in period $t$ and a deficit in period $t + s$ for $s > 0$. From (1.21) we see that the real return on a unit of consumption saved at $t$ and consumed at $t + s$ is given by $\frac{(R_{t+1} \times \cdots \times R_{t+s})P_{i,t}}{P_{i,t+s}}$. Consider the extreme case in which trade costs are zero. Then, real interest rate parity holds and the real return from period $t$ to $t + s$ is the same for all countries independently of their trade balance position. Now consider the case with positive trade costs to export from $i$ to $h$ for all $h$. In order for country $i$ to run a surplus in period $t$, its equilibrium prices must be low relative to the ones in other countries. This can be inferred from equilibrium condition (1.26). Given prices, positive trade costs lead to a larger home-bias, i.e. greater $\pi^i_{hh,t}$ and smaller $\pi^j_{hi,t}$ for every $h$ and $j$, which implies that the right hand side of (1.26) is lower. Therefore, in order for country $i$ to maintain its trade surplus, production costs must decrease in order for country $i$ to export more goods to other countries. Lower production costs lead to low sectoral prices and, therefore, a low aggregate price, $P_{i,t}$. By the exact same logic, country $i$’s trade deficit in period $t + s$ must be accompanied by high prices relative to those in other countries. Hence, with positive trade costs, real interest rates are high for borrowers and low for lenders. Thus, differences in country-specific real interest rates imply that consumption smoothing is more costly when trade costs are high, and trade imbalances are dampened.

In a general equilibrium model like the one I propose, there is an additional effect. Note that difference in equilibrium production costs are realized through adjustments in factor prices, wages and rental rates, as implied by (1.5). These changes affect a country’s income. Specifically, aggregate prices depend positively on factor prices. Hence, when a country runs a surplus, it also has a low income relative to when it runs
a deficit. This income effect reinforces the effect of real interest rates and dampens trade imbalances further by making consumption smoothing even more costly.

To summarize, there are two basic mechanisms through which the levels of trade costs affect saving decisions. First, the differentials in real interest rates implied by differences in country-specific prices that must be in line with intratemporal trade being consistent with intertemporal trade, including the intertemporal constraint. Second, the income effects generated by adjustments in factor prices in order to be in line with the same country-specific sectoral and aggregate prices. Hence, uniformly higher trade costs imply that trade imbalances are dampened in all time periods.

Let us now turn to the titling effect. In the data, trade costs are declining over time. In order to understand the intuition of this effect, consider the thought experiment in which trade costs remain constant instead of following a declining path. In the case of constant trade costs, these are higher in every period, however, the difference between trade costs in the final years is larger than in the initial ones. Therefore, all countries experience negative income effects in future periods from higher trade costs. Consumption-smoothing motives imply that all countries want to transfer resources from the present to the future by increasing their savings. This leads to an excess in world aggregate savings: countries that under declining trade costs borrow in the initial years want to borrow less and countries that lend in initial years want to lend more. In order to restore the world equilibrium, the world real interest rate must fall. However, as the world real interest rate falls, the burden of future payments by countries that were initially borrowing decreases, leading to a positive income effect in the future that reinforces the desire to decrease savings. This effect acts in the exact opposite direction for countries that were initially saving, thus weakening the effect of the decline in the world real interest rate. In the new equilibrium, the world real interest rate is lower and countries borrowing initially and paying later run

\[\text{23}\]

In Section 4.2, I compare counterfactual trade imbalances in the case of trade costs fixed to their levels in 1970 to those fixed to their levels in 2007. This comparison isolates the level effect.
smaller trade surpluses in the future. Hence, these countries’ future incomes relative to the incomes of countries lending are high, as their terms of trade, and therefore factor prices and income, remain relatively high. In other words, the terms of trade of countries running trade surpluses in the future improve as the world real interest rate decreases. This effect implies that trade imbalances tilt, leading to larger trade imbalances in the initial periods.

1.3 Taking the Model to the Data

We now return to the main question that this paper aims to answer. How much of the increase in trade imbalances in recent decades can be explained by the evolution of bilateral trade costs? To answer this question I proceed in two steps.

First, I will calibrate the model to observed data for the period 1970 to 2007. The calibration requires the identification of the model’s time-invariant parameters and time-varying exogenous variables. Time-varying exogenous variables can be divided into those that are directly observed in the data and those that are not. The set of exogenous variables that are not observed are the ones I call disturbances and previously labeled as $S_t$. I calibrate these disturbances by relying on endogenous outcomes of the model that are observed in the data, specifically, bilateral trade flows, prices for tradable sectors and GDP, sectoral expenditures and net exports. This implies that these disturbances provide a decomposition of the forces underlying the evolution of this data. In other words, given parameter values and observed exogenous variables of the model, I recover a set of structural residuals that rationalizes the data as an equilibrium of the model.

Second, I use the disturbances obtained from this decomposition to carry out counterfactual exercises to provide an answer to the main question of the paper. In this section I describe the procedure followed to identify the parameters, exogenous
variables and disturbances. The counterfactual exercises are left for the following section. Since I consider a perfect foresight economy, it is worth underscoring that it is assumed that at time $t$, all future exogenous variables, including disturbances, are known by economic agents.

Let us recall that recall the set of disturbances that decomposes the data: (i) trade costs, $\tau_{ih,t}$, (ii) sectoral productivities, $T_{ji,t}$, (iii) sectoral demand shifters, $\mu_{ji,t}$, and (iv) intertemporal preference shifters, $\phi_{it,t}$. The trade costs and sectoral productivities affect bilateral trade flows and technologies in each country. The sectoral demand shifters allow us to match the data on sectoral expenditures, which in turn imply that, given trade shares, the model’s outcomes match net exports exactly. In general, these disturbances will capture any mechanism other than changes in relative sectoral prices that lead to shifts in economic activity across sectors over time, also known as structural transformation. I follow the international macroeconomics literature in naming the intertemporal preference disturbance. Even though it can be interpreted as a shock to intertemporal tastes, it can also be associated with many different channels that affect intertemporal decisions, such as financial frictions. We turn to a more detailed discussion of these disturbances later on in this section.

I carry out the calibration for 26 countries, $I = 26$, and three sectors, $J = 3$. The choice of countries and sectors was dictated mainly by the availability of data. In particular, I consider 25 core countries and one aggregate of all other countries for which there is some data available, I call it Rest of the World (ROW). A full list of the countries considered is provided in the data appendix. I assume that the representative household in each country has logarithmic period utility, $u(C) = \ln(C)$.

---

24Introducing uncertainty and solving for a rational expectations equilibrium becomes untractable in this model because of the high dimensionality of the disturbances. Given that I study a long term phenomenon, the choice of a perfect foresight enviroment provides a natual baseline framework given the aim of the paper.
I assume that one of the three sectors is nontradable, that is, this sector must source all its goods from home in order to produce the final nontradable sectoral good. In terms of the model, I model the nontradable sector as other sectors, but impose that bilateral trade costs are infinity, i.e. $\tau_{ih,t}^S = \infty$ for $i \neq h$ or equivalently, $\pi_{ii,t}^S = 1$. This implies that in this sector it is always cheaper to source all tradable goods from domestic suppliers. I consider agriculture and mining (AM) and manufacturing (M) as the two tradable sectors, while all of services (S) are considered nontradable.\footnote{The matrix of bilateral trade costs in this sector is such that it has ones along its diagonal and infinity everywhere else.}

Even though I do not model trade in services, I do take into account trade imbalances in this sector and incorporate them into the model as time-varying exogenous transfers across countries.

There are five sets of time-invariant parameters in the model. Some of these parameters are country and sector specific, others just country-specific or worldwide parameters. I calibrate these parameters either directly from the data or take their values from previous literature. Given these parameters and the exogenous variables of the model that I observe in the data, I can then use the structure of the model to recover the disturbances that drive trade across countries over the period considered.

1.3.1 Time-invariant Parameters

I focus first on the parameters that I recover directly from the data, which are the value added shares, $\beta_{ij}$, and the input-output table coefficients, $\nu_{ik}^{ij}$. These are the two sets of parameters that are country and sector-specific. Using data on gross output and valued added from a combination of data sources that includes EU-KLEMS, UNIDO, GGDC-10 and countries’ official statistical agencies, I set $\beta_{ij}$ to the average of value added divided by gross output in each sector $j \in J \equiv \{AM, MN, S\}$. For the input-

\footnote{Even though many services are nowadays traded across countries, there is no data on bilateral trade flows for most of countries before the mid 1990s. However, the data available shows that international trade in services is still small relative to trade in other sectors.}
output coefficients I use the input-output tables provided by OECD-Stan, World Input-Output Database (WIOD) and countries’ statistical agencies. Depending on the availability of the input-output tables, I recover these coefficients mainly from the tables corresponding to the late 1990s.\footnote{The data appendix provides a detailed and comprehensive list of all data sources.}

I now turn to the parameters that I take from existing literature. For the capital shares in value added, $\varphi_i$, I take the values from Caselli et al. (2007) who calibrate these parameters for a large set of countries. I calibrate the elasticity of substitution across tradable goods $\eta = 2$, in line with Caselli et al. (2015) and Broda and Weinstein (2006). For the case of the trade elasticity, I consider as a baseline $\theta = 4$, which is consistent with the estimates in Simonovska and Waugh (2014) for the case of international trade models.\footnote{Caliendo and Parro (2015) estimate industry-specific trade elasticities and show that these differ across industries. However, there is no clear mapping between their industries and the sectors I consider. Given the broad definition of the two tradable sectors in my model, I choose a uniform trade elasticity as a baseline, as in Costinot et al. (2012). The value I choose for $\theta$ is in line with the aggregate elasticity estimated by Caliendo and Parro (2015).}

For the case of preference parameters, I consider a value of $\psi = 0.4$ for the elasticity of substitution in consumption. This value is consistent with the literature on structural transformation that calibrates values for this parameter to be less than one. The value I consider is in line with Duarte and Restuccia (2010), and in the midrange of estimates for the U.S. and other less developed economies. For the discount factor I set $\delta = 0.95$ which is considered standard in the literature for the case of annual data. Table 1.1 summarizes the baseline values of these parameters as well as the data used to obtain them.

### 1.3.2 Time-varying Exogenous Variables

The model’s endogenous variables are determined given a series of exogenous variables. The time-varying exogenous variables of the model that can be taken directly
Table 1.1: Time-invariant Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Variable</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^j_i$</td>
<td>-</td>
<td>Value added to gross output ratio</td>
<td>Sectoral Data</td>
</tr>
<tr>
<td>$\nu^j_{i,k}$</td>
<td>-</td>
<td>Input-output coefficients</td>
<td>Data, Input-Output Tables</td>
</tr>
<tr>
<td>$\varphi_i$</td>
<td>-</td>
<td>Capital share in value added</td>
<td>Caselli and Feyrer (2007)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>4</td>
<td>Trade elasticity</td>
<td>Range Simonovska and Waugh (2014)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2</td>
<td>Elasticity of substitution in tradable goods</td>
<td>Caselli et al. (2014)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.4</td>
<td>Elasticity of substitution in consumption</td>
<td>Duarte and Restuccia (2010)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.95</td>
<td>Discount factor</td>
<td>Standard for annual data</td>
</tr>
</tbody>
</table>

from the data are the homogeneous labor endowments, $L_{i,t}$, capital stocks, $K_{i,t}$, and net exports in the services sector, $NX_{i,t}^S$.

I construct the series for labor endowments using data on Population, $Pop_{i,t}$, GDP per capita, $rgdpc_{i,t}$, and GDP per worker, $rgdpw_{i,t}$, from the Penn World Tables 7.1. These endowments are then constructed as $L_{i,t} = (rgdpc_{i,t}/rgdpw_{i,t}) \times Pop_{i,t}$. For the capital stock, I use data on capital stocks from the Penn World Tables 7.1 for the year 1970. Then, I use the capital accumulation equation $K_{i,t+1} = (1 - d) K_{i,t} + I_{i,t}$ with $d = 0.05$ and data on investment from the UNStats to construct the stocks from 1971 to 2007.

To construct series for net exports in services, $NX_{i,t}^S$, I consider data on aggregate net export, $NX_{i,t}$, which I obtain from UNStats and bilateral trade flows from country $h$ to country $i$ in tradable sectors, $X_{ih,t}^j$ for $j \in J \setminus S$, which I obtain from the NBER-UN and CEPII-BACI data sets. Then, country $i$’s exports and imports in sector $j$ are given by $\sum_{h=1}^I X_{hi,t}^j$ and $\sum_{h=1}^I X_{ih,t}^j$ respectively, and I construct $NX_{i,t}^S = NX_{i,t} - \sum_{j \in J \setminus S} \left(\sum_{h=1}^I X_{hi,t}^j - \sum_{h=1}^I X_{ih,t}^j\right)$.

1.3.3 Structural Residuals

The endogenous outcomes of the model that I observe in the data and that I use to recover the disturbances in period $t$, $S_t$, for $t = 1970, \ldots, 2007$ are: (i) sectoral bilateral trade flows in tradable sectors, $X_{ih,t}^j$ for $j \in J \setminus S$; (ii) prices for tradable sectors, $P_{i,t}^j$ for $j \in J \setminus S$, and GDP, $P_{i,t}$; (iii) sectoral expenditures, $E_{i,t}^j$ for $j \in J$;
and (iv) net exports, $NX_{i,t}$. In order to obtain sectoral expenditures, I rely on data on bilateral trade flows and sectoral gross output, $Y_{j,i,t}$ for $j \in J$. In addition, I use data on gross domestic product, $GDP_{i,t}$, to recover the factor prices that are consistent with the model instead of relying on data on wages and rental rates of capital that might not be consistent with the labor and capital endowments I recovered in the previous subsection.

Data on bilateral trade flows comes from the same sources that I use to compute net exports for services, and data on sectoral gross output comes from the same sources that I use to calibrate value added to gross output ratios. The data on gross domestic product and net exports comes from UNStats. For aggregate prices I consider gross domestic product prices from the Penn World Tables 7.1. Data on sectoral prices that are comparable across countries for the tradable sectors I consider is not readily available, therefore, I construct these series. In order to do so, first, I fix a base year and estimate relative sectoral prices in that year relying on the static multisector-gravity structure of the model. Then, using data on sectoral price indexes in tradable sectors that I obtain from EU-KLEMS or construct using data from the GGDC-10 database, I construct the entire time series for prices. In the data appendix I provide the details on how I construct these series.

Given these data, we can recover the sectoral trade shares, sectoral expenditures and factor prices that map directly to the model’s equilibrium conditions laid out in Section 2. These variables provide an explicit mapping between observables and model’s outcomes. To compute sectoral expenditures and trade shares, consider the data on sectoral gross output and bilateral trade flows. Then, expenditure on sector $j$ in country $i$ at time $t$ is given by $E_{j,i,t} = Y_{j,i,t} - NX_{j,i,t}$, where $NX_{j,i,t} = \sum_{h=1}^{I} X_{j,hi,t} - \sum_{h=1}^{I} X_{j,ih,t}$, and the trade share by importer $i$ from exporter $h$ is computed as $\pi_{j,ih,t} = \frac{X_{j,ih,t}}{E_{i,t}}$ for $i \neq h$, and $\pi_{ii,t} = 1 - \sum_{h=1}^{I} \pi_{j,ih,t}$. For factor prices, note

\footnote{Fitzgerald (2012) and Levchenko and Zhang (2016) estimate sectoral prices similarly. However, they do not exploit data on sectoral price indexes.}
that the equilibrium of the model implies that wages and rental rates must be given by \( w_{i,t} = (1 - \varphi_i) \frac{GDP_{i,t}}{L_{i,t}} \) and \( r_{i,t} = \varphi_i \frac{GDP_{i,t}}{K_{i,t}} \) respectively, which implies that we can compute factor prices using the data available.

Armed with these variables, we can now proceed to recover the set of disturbances \( S_t \) for all \( t \). First, let us define

\[
D_t \equiv \left\{ \{ w_{i,t}, r_{i,t}, L_{i,t}, K_{i,t}, NX_{i,t}, NX^S_{i,t}, P_{i,t} \}, \{ E^j_{i,t}, Y^j_{i,t} \}_{j \in \mathcal{J}}, \{ P^j_{i,t} \}_{j \in \mathcal{J} \setminus S}, \{ \pi^j_{i,h,t} \}_{j \in \mathcal{J} \setminus S} \right\}_{t=1970, \ldots, 2007},
\]

the set of data that is observed and used to recover these sets for \( t = 1970, \ldots, 2007 \). For \( t = 2008, \ldots, \) an assumption on the time-varying exogenous variables of the model has to be made in order to conduct counterfactual exercises. I assume that after 2007, the world economy is in a steady state in which all exogenous variables of the model remain at their levels of 2007.

**Sectoral Demand Disturbances**

First, I recover the sectoral demand disturbances, since knowledge of these is necessary to recover other disturbances, but not the opposite. The key equilibrium conditions that allow us to recover the sectoral demand shocks are the optimal static decisions by the households and the firms, and the market clearing conditions for sectoral goods. The following lemma shows how these disturbances are identified.

**Lemma 1.3.1** Given time-invariant parameter values and data \( D_t \) for \( t = 1970, \ldots, 2007 \), there is a one-to-one mapping between observables and sectoral demand shocks, \( \{ \mu^j_{i,t} \}_{j \in \mathcal{J}} \), given by the following equilibrium conditions and model
restrictions:

\[
\mu_{i,t}^j = \left( \frac{P_{i,t}^j}{P_{i,t}} \right)^{(1-\psi)} \left( E_{i,t}^j - \sum_{k=1}^{J} (1 - \beta_k) \nu_k^j Y_{i,t}^k \right) \frac{w_{i,t} L_{i,t} + r_{i,t} K_{i,t} - NX_{i,t}}{w_{i,t} L_{i,t} + r_{i,t} K_{i,t} - NX_{i,t}} \quad \text{for } j \in J \setminus S, \text{ and} \quad (1.27)
\]

\[
\mu_{i,t}^S = 1 - \sum_{j \in J \setminus S} \mu_{i,t}^j. \quad (1.28)
\]

**Proof** See Appendix.

It is worth mentioning that Lemma 2 does not necessarily imply that \( \mu_{i,t}^S > 0 \), which is a restriction of the model. However, the data is such that this restriction holds for every country and period in the sample.

The CES structure of preferences allows me to capture the part of the structural transformation in these economies that can be attributed to changes in relative prices over time. Therefore, if a country’s evolution of prices in tradable sectors is consistent with long term changes in its expenditure shares according to the CES structure of preferences, the time series that we recover for \( \mu_{i,t}^j \) must not show a particular trend over time. This is indeed the case for most developed economies in my core countries. Still, in order for the model to match net exports, these shocks are crucial, since they allow the model to match sectoral expenditures exactly which, together with trade shares, determine net exports.

There are some countries for which sectoral demand shocks show clear trends that are in line with the literature on structural transformation. In particular, less developed countries like China and India show a clear downward trend in \( \mu_{i,t}^{AM} \). This implies that the decline in the expenditure share in \( AM \) in these countries is greater than what can be accounted for solely by changes in prices. The literature on structural transformation has incorporated nonhomotheticities into preferences or technologies in order to explain this kind of trends. These imply that the larger decline in the share could be explained by the fact that expenditure in \( AM \) rises less than propor-
tionately with income. In any case, for our purpose, these disturbances are enough to capture these effects.

Once we have recovered the static demand shocks, we can proceed to back out trade costs and sectoral productivity shocks.

**Trade Costs and Sectoral Productivities**

The next lemma shows how, given the data previously described, sectoral bilateral trade costs and sectoral productivities are uniquely identified by the static equilibrium conditions of the model.

**Lemma 1.3.2** Given \( \{\mu_j^i\}_{j \in J} \) for all \( i \), time-invariant parameter values, and data \( D_t \) for \( t = 1970, \ldots, 2007 \); there is a one-to-one mapping between observables and the shocks \( \{\tau_{jih,t}\}_{j \in J \setminus S} \) and \( \{T_{j,t}\}_{j \in T} \) in period \( t \) given by the following equilibrium conditions:

\[
\tau_{jih,t}^j = \frac{P_{i,t}^j}{P_{h,t}^j} \left( \frac{\pi_{hh,t}^j}{\pi_{ih,t}^j} \right)^{\frac{1}{\theta}} \quad \text{for } j \in J \setminus S, \tag{1.29}
\]

\[
\pi_{ii,t}^j = T_{i,t}^j \left( \frac{C_{i,t}^j}{P_{i,t}^j} \right)^{-\theta} \quad \text{for } j \in J \setminus S \text{ and } \pi_{ii,t}^S = 1 \text{ for } j = S, \text{ and} \tag{1.30}
\]

\[
P_{i,t} = \left( \sum_{j=1}^{J} \mu_{j,t}^i \left( P_{i,t}^j \right)^{1-\psi} \right)^{\frac{1}{1-\psi}} \quad \text{and } \tau_{ih,t}^S = \infty \text{ for all } i \neq h. \tag{1.31}
\]

**Proof** See Appendix.

It is worth taking some time at this point to discuss in more detail the implications of the previous lemma. Let us start with the case of bilateral trade costs. Note from (1.29) that trade costs are uniquely determined given our normalization \( \tau_{ii,t}^j = 1 \) for all \( i \) and \( j \). This equation is obtained by taking the ratio of the trade share of importing country \( i \) and exporter \( h \) to the domestic trade share in country \( h \), \( \tau_{ih,t}^j/\tau_{hh,t}^i \). Given the definition of trade shares in (1.13), this ratio controls for differences in productivity
and production costs across countries and implies that data on sectoral trade shares and prices is sufficient to recover the costs.\footnote{Similar procedures have become standard in the literature on gravity models of trade. See Head and Mayer (2014).}

Figure 1.2 shows the evolution of the average of sectoral bilateral trade costs for all country-pairs trading over time. Note that these trade costs are large, which is in line with the results in the survey by Anderson and van Wincoop (2004). More importantly, note that there is a significant and steady decline in trade costs over the entire period 1970-2007. The evolution of these iceberg-type trade costs is consistent with previous literature, in particular with the aggregate measures of trade costs in Fitzgerald (2012).

One particular feature of the model is that it can only rationalize zero bilateral trade flows for a pair of countries $i$ and $h$, $\pi^i_{ih,t} = 0$, by means of infinite bilateral trade costs, $\tau^i_{ih,t} = \infty$. This can be appreciated in equation (1.29) and is a result of the fact that tradable goods’ producers draw their efficiencies from a probability distribution with an unbounded support.\footnote{Another way to put it is that, in every country there is always a ”superstar” producer that should be shipping goods to all countries.}

The data on bilateral trade flows is such that, even for the broad aggregation of tradable sectors that I consider, there are years, country pairs and sectors for which trade flows are zero. Figure 1.2 drops all observations for which this is the case. However, I need to assign values to these costs for my numerical exercises. Hence, whenever there is a zero, I assign arbitrarily large values to these bilateral trade costs in order to generate negligible bilateral trade shares.\footnote{In the simulations of my model I consider the case in which whenever $\pi^j_{ih,t} = 0$, I set $\tau^j_{ih,t} = \max_{ih,t} \left\{ \tau^j_{ih,t} \right\}$. These trade costs imply that bilateral trade shares are negligible, which is consistent with the data.}

It is worth mentioning that having data on sectoral prices is crucial for the model to match all trade shares exactly. These prices allow us to recover asymmetric trade costs directly from the data instead of having to rely on estimation procedures as other work has done. Another way in which the literature has dealt with this issue is
by imposing a symmetry assumption on trade costs that implies that data on sectoral prices is not needed to recover these. The main drawback of this assumption is that it implies that the model cannot match all bilateral trade flows exactly, *i.e.* the model is overidentified.

Let us now turn to the case of sectoral productivities. First, notice that productivities in the tradable sectors are identified by the equilibrium domestic trade shares. However, these equations do no pin down productivity in the nontradable sector. However, using data on GDP prices, which is equivalent to real exchange rates, I can recover the productivity in this sector. Therefore, sectoral productivities are such that real exchange rates in the data are exactly matched. More importantly, sectoral domestic expenditure shares are also exactly matched.

Figure 1.3 summarizes the evolution of sectoral productivities over the entire time period by plotting the means (dashed dark line) and maximum and minimum bands.
of the log of a measure of average sectoral productivity given by $(T_{ij,t}^j)^{\frac{1}{2}}$. In addition, each plot includes the case of the U.S. (circles) as a reference, as well as the case of particular countries that are either interesting cases on their own, or that follow interesting paths over time (crosses). The figure splits the set of countries into two. The first set of countries includes only developed countries, specifically, it includes all the countries considered in the studies by Bernard and Jones (1996a, 1996b). This set of countries is presented in panel (a). The second set of countries includes all other countries in our sample including ROW. These countries are presented in panel (b) together with the U.S. In line with previous studies, the U.S. represents the technological frontier across developed countries in all sectors except agriculture and mining. My results show that this is also the case for non-developed countries. In addition, as can be appreciated in the figure, for the set of countries in panel (b), the cross section of productivities in each year is more dispersed than for the countries in panel (a). The figure also shows that productivities of countries in panel (b) are lower than those included in panel (a). Moreover, we can appreciate that certain emerging economies experienced significant catch-up growth to the U.S. This is the case of Korea in the Agriculture and Mining sector, or China in the manufacturing sector. In contrast, Portugal has seen a relative decline in its productivity in services that is in line with its real exchange rate appreciation beginning in the 1990s.

**Intertemporal Preference Disturbances**

I now proceed to calibrate the disturbances to intertemporal preferences. These disturbances are calibrated in such a way that observed trade imbalances are matched by the model, which implies that dynamic decisions are also optimal. In particular, recall that optimal dynamic decisions are determined by the Euler equation.
that \( u(C) = \ln(C) \), we have that the Euler equation is given by

\[
\frac{C_{i,t+1}}{C_{i,t}} = \delta \hat{\phi}_{i,t+1} + \frac{R_{t+1}}{P_{i,t+1}/P_{i,t}}.
\]

(1.32)

Now, notice that the equilibrium nominal interest rate, \( R_{t+1} \), must be such that \( \sum_{i=1}^{I} B_{i,t+1} = 0 \) for all \( t \). From the Euler equation we have that

\[
\frac{P_{i,t+1}C_{i,t+1}}{P_{i,t}C_{i,t}} = R_{t+1}\delta \hat{\phi}_{i,t+1},
\]

(1.33)
which implies that given consumption expenditure levels at \( t + 1 \), it must be the case that the equilibrium nominal return is

\[
R_{t+1} = \frac{1}{\delta} \sum_{i=1}^{I} \frac{P_{i,t+1} C_{i,t+1}}{\hat{\phi}_{i,t+1}} \left( \sum_{i=1}^{I} P_{i,t} C_{i,t} \right)^{-1}.
\]  

(1.34)

Hence, note that equations (1.33) and (1.34) define a system of \( I + 1 \) equation and \( I + 1 \) unknowns in \( R_{t+1} \) and \( \hat{\phi}_{i,t+1} \). However, these shocks are only identified up to a normalization, as can be appreciated from (1.33) and (1.34). Therefore, I normalize \( \hat{\phi}_{US,t+1} = 1 \) for all \( t \) and recover only the other \( I \) shocks. In order to do so, I proceed as follows. Using data on net exports, I simulate the model and recover all prices and expenditure levels that are consistent with an equilibrium of the model. Using these values, I then proceed to recover the dynamic preference disturbances. Hence, the model is block recursive in the sense that we can recover all shocks other than the intertemporal preference shock, without knowing the latter. Similarly, given the data, we could recover the intertemporal preference shock in a first step.

Figure 1.4 plots the cross sectional mean and a one standard deviation range around it of the changes in the dynamic preference disturbances over the period 1970-2007. Note that it is not clear from the figure that these shocks or levels are persistently different than one. This provides suggestive evidence that these shocks might not be playing a very relevant role in shaping the long term trend in trade imbalances.

In any case, as long as the sets of disturbances recovered do not systematically correlate with each other, we can isolate the effect of each one of these on equilibrium outcomes of the model by not allowing them to change over time. I turn to this kind of counterfactual exercises in the following section.

\[33\] I choose world GDP as the numeraire in the model. Note that this implies that world consumption is also equal to one in every period. Therefore, I can recover \( R_{t+1} \) having only information consumption expenditure for \( t + 1 \).
1.4 Counterfactual Exercises

In order to quantify the effects of trade costs on trade imbalances I conduct counterfactual exercises in which trade costs are held fixed at specific levels and all other disturbances change according to their original paths. These exercises isolate the effects of trade costs and allow me to quantify their effects by comparing the equilibrium trade imbalances in the counterfactual to those in the data.

Each counterfactual equilibrium is pinned down by the net foreign asset distribution at the time economic agents realize that the path of trade costs will differ from the one in the data. In my main exercise I assume that economic agents realize
the counterfactual evolution of trade costs at the beginning of 1970. Therefore, the counterfactual equilibrium is pinned down by the initial net foreign asset distribution. I use the equilibrium of the model that matches the data to recover this initial distribution. Specifically, given my assumption that the world economy reaches a steady state after 2007, I use the distribution in the steady state that is consistent with the data. Then, I iterate backwards according to the asset accumulation equation, $B_{i,t+1} = NX_{i,t} + R_t B_{i,t}$, using the equilibrium nominal return on bonds of the model, and recover the distribution $\{R_0 B_{i,0}\}_{i=1}^f$, which will remain unchanged across counterfactual equilibria. I compute these equilibria by iterating on the steady state distribution of net foreign assets until convergence of the initial distribution. Specifically, given a steady state distribution of net foreign assets, I recover the initial one by solving the model backwards.$^{34}$

I pin down counterfactual sectoral bilateral trade costs using Head-Ries indices. These indices are defined as

$$HR_{ih,t}^j \equiv \left( \frac{\pi_{ih,t}^j \pi_{hi,t}^j}{\pi_{hh,t}^j \pi_{ii,t}^j} \right)^{-\frac{1}{2}}, \tag{1.35}$$

and provide a measure of country-pair sector-specific bilateral trade frictions that are widely used in the literature on gravity equations.$^{35}$ Note that these indices are symmetric, $HR_{ih,t} = HR_{hi,t}$, and that they are functions of data on bilateral trade shares only. Bilateral trade costs in the model are related to these indices by the fact that, for a particular country pair $(i, h)$, the arithmetic mean $(\tau_{ih,t}^j \tau_{hi,t}^j)^{\frac{1}{2}}$ is equal to this index, as can be seen from equation (1.29). I choose this measure to pin down counterfactual levels of trade costs in order to focus on the effects of the decline in the average levels of bilateral trade costs rather than changes in asymmetries in costs. Therefore, my counterfactual results are not generated by specific changes

$^{34}$More details on my computational algorithm are provided in the appendix.
$^{35}$See Head and Ries (2001) and Head and Mayer (2014).
in asymmetries in bilateral trade costs over time, but rather by the pure effects of the fact that trading goods across borders is easier now than in the past for any country-pair.

1.4.1 Trade Costs Fixed to 1970’s Levels

In my first and main counterfactual exercise, I fix trade costs at 1970’s Head-Ries index values. That is, in the counterfactual, $HR_{ih,t}^j = HR_{ih,1970}^j$ for all $t = 1970, \ldots$ and country pairs. The particular question that I aim to answer by means of this counterfactual exercise is the following: What would trade imbalances have been in the case in which trade costs had remained at their 1970’s levels and all economic agents had been aware of it since the beginning of that year, while keeping all other disturbances constant.

Figure 1.5 plots the evolution of the size of external imbalances, measured as the sum of the absolute values of the trade imbalances as a share of world GDP. The difference between the data and the counterfactual is evident throughout, but beginning in 1992 these differences become larger. This is in line with the fact that low levels of trade costs are reached around that time according to Figure 1.2. Moreover, note how with 1970’s trade costs, the size of trade imbalances remains at significantly lower levels after this year. Hence, Figure 1.5 provides evidence of the quantitative relevance of the decrease in trade costs in the increase in trade imbalances.

Table 1.2 provides some statistics related to Figure 1.5. In particular, if we consider the change in imbalances between 1970 and 2007, Table 1.2 implies that the decrease in trade costs accounts for 69% of the increase in these imbalances. An interesting feature of imbalances in the counterfactual scenario is that they are somewhat larger, but still not considerably different, than trade imbalances in the data previous to 1992. The results show that the 69 percent difference in the change in
imbalances is the result of a decrease in imbalances in 2007 of 41 percent and an increase in 1970 of 28 percent of the overall change in the data.

The reason that these two series are different in the initial periods is because the difference between the set of bilateral trade flows that is consistent with the data and the one in the counterfactual is not only different in terms of their levels, but also the entire path that they follow over time. As previously discussed, the negative income effect in future periods in the counterfactual relative to the data implies that the world real interest rates in the initial periods is lower in the counterfactual than in the baseline equilibrium. These differences in the world real interest rate lead to income effects that act in opposite directions depending on a country’s trade balance positions in the future. In the new equilibrium, borrowers in initial periods borrow more and lenders also lend more, which implies that trade imbalances increase. Hence, the tilting in the evolution of trade imbalances on top of the downward shift. This emphasizes the relevance of the dynamic mechanism as well as the importance of
Table 1.2: Trade Imbalances: Percent of World GDP

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Counterfactual</th>
<th>Decomposition</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1970</td>
<td>2007 Diff.@d</td>
<td>1970 2007 Diff.@d</td>
</tr>
<tr>
<td>1970</td>
<td>0.71%</td>
<td>3.81% 3.1%</td>
<td>1.58% 2.54% 0.96%</td>
</tr>
</tbody>
</table>

solving for counterfactual competitive equilibria that are pinned down by the initial net foreign asset position in the world economy. I go back to this issue in the following subsection when I compare trade imbalances across counterfactual equilibria in which only the levels of trade costs change.

Let us now focus on the case of particular countries. Figure 1.6 shows the evolution of net exports in the U.S. and China over the time period considered. These two countries provide a clear example of what an important role trade costs can play in shaping imbalances. This figure tells us that if trade costs had not fallen as much as they did, we would have seen a much smaller increase in the trade deficit of the U.S. and actual decrease in the trade surplus of China beginning in the 1990s. According to Table 1.3, the U.S. trade deficit in 2007 would have been 0.47 percent of world GDP, rather then the three times larger 1.65 percent observed in the data. In the case of China, it would have experienced a trade deficit in 2007 of 0.27 percent of world GDP rather than the surplus of 0.72 percent observed in the data. Moreover, China would have run large trade surpluses from 1970 to the late 1990s, and then would have started to run trade deficits beginning in the 1990s. Table 1.3 shows that the differences between the data and the counterfactual are quantitatively sizable. In general, our results point in the direction of the existence of substantial effects of the level of trade costs on the magnitude of trade imbalances.

Table 1.3: Trade Imbalances: U.S. and China (Net Exports, Percent of World GDP)

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Counterfactual</th>
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<tbody>
<tr>
<td>US</td>
<td>-0.16%</td>
<td>-1.65%</td>
</tr>
<tr>
<td>CHN</td>
<td>0.00%</td>
<td>0.72%</td>
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</table>
Figure 1.6: Trade Imbalances in U.S. and China: Net Exports

Returning to trade imbalances in all countries, Table 1.4 shows the accumulated trade imbalances for all countries in my sample. I measure accumulated trade imbalances as the sum over the absolute value of net exports as a share of world GDP from 1970 to 2007. Note that accumulated trade imbalances are not greater in the data than in the counterfactual for every country. As previously discussed, this is a result of the tilting effect. Breaking the accumulated imbalances into two subperiods, we can see in the table that for the period 1992-2007, the fact that trade imbalances are dampened by high trade costs is more evident. Countries like Japan and Greece experience small changes in accumulated imbalances relative to their levels in the data. Japan experiences a decrease of 7.5 percent of its original accumulated imbalances, while Greece 4.5 percent.

Lastly, I turn to the welfare gains from lower trade costs in this model. Measuring the welfare gains from trade is a fundamental part of most international trade studies. Hence, I also provide a measure of these gains for our counterfactual exercise.
Table 1.4: Accumulated Trade Imbalances: Percent of World GDP

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</thead>
<tbody>
<tr>
<td>AUS</td>
<td>0.92%</td>
<td>1.04%</td>
<td>-0.12%</td>
<td>0.01%</td>
<td>-0.13%</td>
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<tr>
<td>AUT</td>
<td>0.39%</td>
<td>0.21%</td>
<td>0.18%</td>
<td>0.02%</td>
<td>0.16%</td>
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<tr>
<td>BEL</td>
<td>0.88%</td>
<td>1.47%</td>
<td>-0.59%</td>
<td>-0.53%</td>
<td>-0.06%</td>
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<tr>
<td>BRA</td>
<td>1.53%</td>
<td>2.13%</td>
<td>-0.60%</td>
<td>-0.74%</td>
<td>0.14%</td>
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<tr>
<td>CAN</td>
<td>1.99%</td>
<td>1.73%</td>
<td>0.25%</td>
<td>-0.15%</td>
<td>0.40%</td>
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<tr>
<td>CHN</td>
<td>2.77%</td>
<td>6.72%</td>
<td>-3.95%</td>
<td>-4.2%</td>
<td>0.17%</td>
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<tr>
<td>DEN</td>
<td>0.76%</td>
<td>0.60%</td>
<td>0.16%</td>
<td>0.00%</td>
<td>0.16%</td>
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<tr>
<td>FIN</td>
<td>0.58%</td>
<td>0.65%</td>
<td>-0.06%</td>
<td>-0.16%</td>
<td>0.10%</td>
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<tr>
<td>FRA</td>
<td>2.10%</td>
<td>1.46%</td>
<td>0.64%</td>
<td>0.22%</td>
<td>0.42%</td>
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<tr>
<td>GER</td>
<td>5.44%</td>
<td>3.26%</td>
<td>2.18%</td>
<td>1.24%</td>
<td>0.94%</td>
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<tr>
<td>GRC</td>
<td>1.54%</td>
<td>1.47%</td>
<td>0.07%</td>
<td>-0.16%</td>
<td>0.23%</td>
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<tr>
<td>IND</td>
<td>1.08%</td>
<td>1.53%</td>
<td>-0.45%</td>
<td>-0.56%</td>
<td>0.11%</td>
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<tr>
<td>ITA</td>
<td>1.94%</td>
<td>1.94%</td>
<td>0.00%</td>
<td>0.02%</td>
<td>-0.02%</td>
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<tr>
<td>JAP</td>
<td>6.81%</td>
<td>6.30%</td>
<td>0.51%</td>
<td>0.36%</td>
<td>0.15%</td>
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<tr>
<td>KOR</td>
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<td>2.04%</td>
<td>-0.75%</td>
<td>-0.19%</td>
<td>-0.56%</td>
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<tr>
<td>MEX</td>
<td>2.17%</td>
<td>1.73%</td>
<td>0.45%</td>
<td>0.01%</td>
<td>0.44%</td>
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<tr>
<td>NLD</td>
<td>1.91%</td>
<td>1.85%</td>
<td>0.06%</td>
<td>-0.75%</td>
<td>0.81%</td>
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<tr>
<td>NOR</td>
<td>1.51%</td>
<td>1.40%</td>
<td>0.11%</td>
<td>-0.04%</td>
<td>0.15%</td>
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<tr>
<td>POR</td>
<td>1.05%</td>
<td>1.03%</td>
<td>0.03%</td>
<td>-0.02%</td>
<td>0.05%</td>
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<tr>
<td>SPA</td>
<td>1.64%</td>
<td>1.39%</td>
<td>0.25%</td>
<td>-0.23%</td>
<td>0.48%</td>
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<tr>
<td>SWE</td>
<td>1.21%</td>
<td>1.13%</td>
<td>0.09%</td>
<td>-0.01%</td>
<td>0.10%</td>
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<tr>
<td>SWZ</td>
<td>0.97%</td>
<td>1.02%</td>
<td>-0.04%</td>
<td>-0.32%</td>
<td>0.28%</td>
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<tr>
<td>UK</td>
<td>2.61%</td>
<td>2.26%</td>
<td>0.35%</td>
<td>0.10%</td>
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<tr>
<td>US</td>
<td>21.39%</td>
<td>18.12%</td>
<td>3.27%</td>
<td>-4.09%</td>
<td>7.36%</td>
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<tr>
<td>VEN</td>
<td>1.08%</td>
<td>2.05%</td>
<td>-0.97%</td>
<td>-0.88%</td>
<td>-0.09%</td>
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<tr>
<td>ROW</td>
<td>15.37%</td>
<td>11.39%</td>
<td>3.98%</td>
<td>3.06%</td>
<td>0.92%</td>
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</table>

Given that we now have a dynamic model, a measure of the welfare gains directly comparable with those in Arkolakis et al. (2012) is not available. Hence, in Table 1.5 I measure the welfare gains for each country as the time-invariant percentage increase in consumption that a country would need to receive in the counterfactual in order to be indifferent between this scenario and the benchmark case (consumption-equivalent variation). That is, if $U^D_i$ is the lifetime utility of country $i$ in the benchmark scenario, then we compute the welfare gains from trade as the time-invariant consumption transfer, $x_i$, such that

$$\sum_{t=0}^{\infty} \delta^t \phi_{i,t} \ln \left( C_{i,t}^{CF} \left( \frac{100 + x_i}{100} \right) \right) = U^D_i,$$
where $C_{i,t}^{CF}$ is consumption in the counterfactual in period $t$ and country $i$. As can be appreciated in the table, most countries gain from the decline in trade costs relative to the counterfactual scenario. The cases of Finland and Venezuela are interesting, as according to the model they suffered welfare losses. For the case of Venezuela, it is the case that it was trading more in 1970 than in subsequent years. This fact translates into higher trade costs, mostly driven by policy measures, after 1970. It might also be the case that these countries see a deterioration of their terms of trade in the counterfactual. This seems to be the case for Finland as well.

In line with previous research, small open economies experience significant gains from lower trade costs. Belgium and the Netherlands are two clear examples of this case, with gains of 10.9 and 6.7 percent of additional consumption in every year. However, large emerging economies like China and Korea also experience large gains from lower trade costs. Interestingly, these are two economies that have engaged in export-oriented growth.

### 1.4.2 Effects of Changes in Trade Costs’ Levels: 1970 versus 2007

In the previous counterfactual exercise, trade costs remain fixed at a constant level over time. This implies that relative to actual bilateral trade costs, not only their levels are different, but also the dynamic path they follow. While in the data trade costs are falling, in the counterfactual they are not. This change in the path of bilateral trade costs affects how agents make consumption-saving decisions independently of the changes in trade costs’ levels. In order to understand the implications of changes in the levels of trade costs in isolation, I compare counterfactual equilibria in which bilateral trade costs remain constant over time, but differ in their levels.

Figure 1.7 shows the evolution of trade imbalances for two counterfactual equilibria in which trade costs are constant over time, but differ in their levels. In particular, I
consider the levels of bilateral trade costs in 1970 and 2007 for this comparison. The
equilibrium for 1970 trade costs is the same as in our previous exercise. For the case
of 2007 trade costs, trade imbalances are obtained by fixing trade at 2007’s Head-Ries
index values in every year.

As can be appreciated in Figure 1.7, uniformly lower trade costs across time have
significant level effects on trade imbalances. Trade imbalances for low trade costs are
higher than for high trade costs across the entire time period. This effect is in line
with the level effect explained in Section 2.

In contrast to our first counterfactual, note that uniform differences in the levels
of trade costs do not imply a tilting of trade imbalances. Hence, we can attribute this
effect in our first counterfactual exercise to the fact that trade costs do not follow
a declining path over time. Additionally, this decline seems to be driving part of the increase in observed trade imbalances. We can decompose accumulated trade imbalances from 1970 to 2007 into two effects. The level effect leads to an increase in accumulated trade imbalances from 75.56 to 107.97 that is evenly distributed across time periods. Then, the tilting effect, i.e. making trade costs in the initial periods higher relative to those in the final ones leads to a decrease in accumulated trade imbalances from 107.97 to 80.76. Thus, the overall change in accumulated trade imbalances is $80.76 - 75.56 = (107.97 - 75.56) - (107.97 - 80.76)$, the increase due to level effects, minus the decrease due to the tilting effect.

### 1.4.3 No Trade Liberalizations: Costs of 1986

In the previous exercises I assumed that all agents find out in the year 1970 what the counterfactual path of trade costs will be. Hence, they adjust all their optimal choices thereafter based on perfect knowledge of the future path of changes in disturbances.
I also consider a different exercise in which the agents realize in a later period that trade cost will differ from what they initially expected. I conduct such an exercise in this subsection. Specifically, I assume that after 1986 trade costs remain constant. This can be thought of as the case in which many of the trade liberalizations rounds of the late 1980s had not occur.

Consider the counterfactual scenario in which all agents realize at the beginning of the year 1986 that trade costs will no longer follow their original path, i.e. a declining path. In contrast, trade costs will remain constant at their 1986 levels thereafter. Figure 1.8 plots the evolution of trade imbalances for the data and the counterfactual scenario. In line with our initial exercise, trade imbalances increase at the time that the news arrive, and then follow a much flatter path than in the data. Hence, the tilting in trade imbalances because of the constant trade costs is again very apparent. Moreover, the increase in imbalances after 1990 is less pronounced than in the data.

In the case of no trade liberalizations, post 1986 trade imbalances reach a minimum level of 1.81 percent of world GDP in 1991. A minimum level of 1.22 percent of world GDP is achieved in the same year in the data post 1986. In the data, trade imbalances increase from 1.22 percent in 1991 to 3.81 percent of world GDP in 2007. This increase is 0.38 times the increase over the same period in the scenario of no trade liberalizations.

The main difference between this exercise and the previous ones is that now the net foreign asset position that pins down the equilibrium is the one in 1986. Therefore, trade imbalances in the absence of trade liberalizations are determined based on this initial net foreign asset distribution. Suppose we had assigned Pareto weights to countries, solved a social planner’s problem rather than the actual competitive equilibrium, and backed out intertemporal preference disturbances. If we consider the same weights in the last exercise, then the planner’s allocations do not coincide with
the competitive equilibrium allocations. The weights that decentralize the counterfactual equilibrium should be linked to endogenous equilibrium outcomes after 1986, such as the net foreign asset distribution at the beginning of 1986, rather than those including the period 1970-1985. Hence the relevance of solving for the competitive equilibrium.

1.5 Conclusions

This paper provides a quantification of the mechanism through which lower trade costs in goods have helped in shaping the surge in trade imbalances over the last four decades. To my knowledge, this is the first attempt to do so by exploiting the
structure of the new quantitative general equilibrium trade models. The results show that 69 percent of the increase in trade imbalances from 1970 to 2007 can be attributed to the fact that trade costs in goods markets declined considerably since 1970. In other words, lower bilateral trade costs across countries have not only allowed for more intratemporal trade, but also for more intertemporal trade around the world. Hence, the key for a better understanding of the roots of the steady increase in trade imbalances might lie in fundamental determinants of bilateral trade costs in goods markets rather than other mechanisms, for example, frictions in international financial markets.

The key mechanism driving these results arises once endogenous trade imbalances driven by consumption-saving decisions are embedded into a framework in which bilateral trade flows are determined by trade costs and comparative advantage across countries. By incorporating these two mechanisms into a unified framework, interesting interactions arise between trade costs and dynamic decisions. These interactions not only show that bilateral trade costs affect trade imbalances, but also that imbalances have implications for bilateral trade flows and the gains from trade. Thus, the paper provides a theoretical framework that incorporates these interactions, but maintains the appealing features of the new quantitative general equilibrium models of trade.

Our results point to the fact that, to the extent that the surge in trade imbalances is a consequence of the process of globalization that has led to lower trade costs, then there is not much room for global welfare improving policies. In other words, the increase in imbalances is simply a result of a more efficient allocation of resources around the world. However, our results also point to the fact that in models in which market imperfections lead to larger imbalances, it is very important to disentangle how much of this increase is simply because of lower trade costs in goods markets.
These types of interaction open up the possibility for a more detailed investigation of how trade policy can affect policies targeted to manage capital flows.

Appendices

A Proof of Lemma 1

Consider equilibrium condition \((1.23)\). This condition can be rewritten as

\[
P_{i,t}^j C_{i,t}^j = E_{i,t}^j - \sum_{m=1}^{J} P_{i,t}^j D_{i,t}^{m,j}.
\]

Notice that conditional on actually producing, firms producing good \(\omega_j^j\) choose intermediate inputs from sector \(m\) such that \(P_{i,t}^m D_{i,t}^{j,m} (\omega^j_i) = \nu_{i,t}^j \beta_{i,t} (\omega^j_i) q_{i,t}^j (\omega^j_i)\).

Now, aggregating over all goods \(\omega^j_i \in [0,1]\), we have that \(P_{i,t}^m D_{i,t}^{j,m} = \nu_{i,t}^j \beta_{i,t} Y_{i,t}^j\) for all \(j, m = 1, \ldots, J\). These conditions together with \((1.18)\) imply that

\[
\mu_{i,t}^j \left( \frac{P_{i,t}^j}{P_{i,t}^j} \right)^{(1-\psi)} P_{i,t} C_{i,t} = E_{i,t}^j - \sum_{k=1}^{J} (1 - \beta_{i,t}^k) \nu_{i,t}^k Y_{i,t}^k
\]

for all \(j = 1, \ldots, J\), which together with the budget constraint in period \(t\) give us the first set of equations of the lemma. Given the data, notice that the shocks recovered for tradable sectors are uniquely pinned down. The restriction that \(\sum_{j=1}^{J} \mu_{i,t}^j = 1\) pins down the shock for the nontradable sector.

B Proof of Lemma 2

Consider sectoral trade shares as given by \((1.14)\). Notice then that

\[
\frac{\pi_{i,h,t}^j}{\pi_{h,h,t}^j} = \left( \frac{P_{i,h,t}^j}{P_{i,t}^j} \right)^{-\theta} \left( \frac{\tau_{i,h,t}^j}{P_{i,t}^j} \right)^{-\theta},
\]
which immediately delivers (1.29). Now, consider $\pi_{ji,t}$ and notice that rearranging terms we obtain (1.30). Lastly, data on $P_{i,t}$ and $P_{i,t}^{j}$ for $j \in J \backslash S$ imply that $P_{i,t}^{S}$ is uniquely pinned down by (1.31). Given $P_{i,t}^{S}$, we can now recover $T_{i,t}^{S}$ using (1.30) as previously.

C Mapping to Armington Model

Consider a model identical to the one outlined in Section 2 with the exception that trade across countries in each period $t$ is done in an Armington fashion. Formally, this model is almost identical to the one previously outlined except for the fact that there is no longer a unit continuum of goods for each sector $j$ that can potentially be produced in each country. Now, each country produces one country-specific good, and sectoral goods producers aggregate these goods across countries in a CES fashion with elasticity of substitution $\rho \geq 0$ in order to produce the final nontradable good in sector $j$:

$$Q_{i,t}^{j} = \left( \sum_{h=1}^{I} \left( \frac{d_{ih,t}^{j}}{P_{i,t}^{j}} \right)^{\frac{\rho - 1}{\rho}} \right)^{\frac{\rho}{\rho - 1}}.$$

This implies that, for the case of sector $j$, expenditure by country $i$ on goods produced in country $h$ is given by

$$p_{ih,t}^{j}d_{ih,t}^{j} = \left( \frac{P_{ih,t}^{j}}{P_{i,t}^{j}} \right)^{1-\rho} P_{i,t}^{j}Q_{i,t}^{j},$$

where $(P_{i,t}^{j})^{1-\rho} = \sum_{h=1}^{I}(P_{ih,t}^{j})^{1-\rho}$ and $p_{ih,t}^{j} = \frac{\tau_{ih,t}^{j}c_{h,t}^{j}}{x_{h,t}^{j}}$. Hence, the share of total expenditure on goods produced in country $h$ is

$$\pi_{ih,t}^{j} = \frac{p_{ih,t}^{j}d_{ih,t}^{j}}{P_{i,t}^{j}Q_{i,t}^{j}} = \left( \frac{P_{ih,t}^{j}}{P_{i,t}^{j}} \right)^{1-\rho} = \left( \frac{\tau_{ih,t}^{j}c_{h,t}^{j}}{x_{h,t}^{j}} \right)^{\rho - 1} \left( \frac{P_{ih,t}^{j}}{P_{i,t}^{j}} \right)^{1-\rho}. \quad (56)$$
Notice then that, given parameters for the model outlined in Section 2, if we let \( \rho = \theta + 1 \) and \( x^j_{h,t} = \Gamma^{-1}\left(T^j_{h,t}\right)^{\frac{1}{\theta}} \), then

\[
\pi^j_{ih,t} = (\Gamma^{\theta}) T^j_{h,t}\left(\frac{c^j_{h,t} x^j_{h,t}}{P^j_{i,t}}\right)^{-\theta}
\]

which is identical to (1.14). Moreover, sectoral prices are also equivalent, since

\[
(P^j_{i,t})^{1-\rho} = \sum_{h=1}^I (p^j_{ih,t})^{1-\rho} = \sum_{h=1}^I \left(\frac{\pi^j_{ih,t} c^j_{h,t}}{x^j_{h,t}}\right)^{1-\rho}.
\]

D Model with Capital Accumulation

Consider the same environment as in the main text, but now there are final non-tradable goods producers that produce the final good by aggregating the sectoral goods in a CES fashion across sectors with an elasticity of substitution of \( \psi > 0 \),

\[
X_{i,t} = \left(\sum_{j=1}^J (\mu^j_{i,t})^{\frac{1}{\psi}} (X^j_{i,t})^{\frac{1-\psi}{\psi}}\right)^{\frac{\psi}{1-\psi}},
\]

where \( X_{i,t} \) denotes production of the final non-tradable good, and \( X^j_{i,t} \) is the conditional demand for input of sector \( j \). Then, perfect competition and cost minimization by these firms implies that sectoral demand is given by \( P^j_{i,t} X^j_{i,t} = \mu^j_{i,t} \left(\frac{p^j_{i,t}}{P^j_{i,t}}\right)^{1-\psi} P_{i,t} X_{i,t} \). Therefore, the equilibrium conditions in the nontradable sector are now given by

\[
X^j_{i,t} + \sum_{k=1}^J D^k_{i,t} x^j_{i,t} = Q^j_{i,t}.
\]

The representative household in country \( i \) now takes \( R_0 B_{i,0}, K_{i,0} \) and prices as given and solves

\[
\max_{\{C_{i,t}, I_{i,t}, K_{i,t+1}, B_{i,t+w}\}_{t=0}^\infty} \sum_{t=0}^\infty \delta^t \phi_{i,t} \ln (C_{i,t})
\]

(1.36)
subject to

\[ P_{i,t}C_{i,t} + P_{i,t}I_{i,t} + B_{i,t+1} = w_{i,t}L_{i,t} + r_{i,t}K_{i,t} + R_{i,t}B_{i,t} \]

(1.37)

\[ K_{i,t+1} = (1 - d) K_{i,t} + \chi_{i,t}I_{i,t} \]

(1.38)

where \( \chi_{i,t} \) is an investment disturbance. Then, the representative household’s optimality conditions are given by:

\[ u'(C_{i,t}) = \beta \hat{\phi}_{i,t+1} \frac{R_{t+1}}{P_{i,t+1}/P_{i,t}} u'(C_{i,t+1}) \text{ and} \]

(1.39)

\[ \frac{R_{t+1}}{P_{i,t+1}/P_{i,t}} = \frac{\chi_{i,t}}{\chi_{i,t+1}} \left( \frac{r_{i,t+1}}{P_{i,t+1}/P_{i,t}} \chi_{i,t+1} + (1 - d) \right) \]

(1.40)

and changes in investment disturbances would provide an additional structural residual that implies that the model can perfectly match data on capital stocks.

E Computation Algorithm

Let \( \{W_i,1970\}_{i=1}^I \) be the net foreign asset distribution in 1970. The following algorithm is used to compute counterfactual equilibria:

1. Guess a steady state net foreign asset distribution, \( \{B_{i,SS}\}_{i=1}^I \), such that \( \sum_{i=1}^I B_{i,SS} = 0 \). Define the vector \( B_{SS} \in \mathbb{R}^I \), and aggregate consumption expenditure in the steady state, \( \{P_{i,SS}C_{i,SS}\}_{i=1}^I \).

2. Iterate backwards as follows: In period \( t \),

(a) Compute the vector of aggregate consumption expenditures according to the Euler equations: Given \( \{P_{i,t+1}C_{i,t+1}\}_{i=1}^I \), compute the nominal interest rate according to (1.34). Then, recover \( \{P_{i,t}C_{i,t}\}_{i=1}^I \) using countries’ Euler equations.
(b) Given $B_{i,t+1}$, guess (update) a vector of wages, $w_t \in \mathbb{R}^I$, and compute the vector of returns on capital, $r_t \in \mathbb{R}^I$, such that $\frac{r_{i,t}}{w_{i,t}} = \frac{\varphi_i}{1-\varphi_i} \frac{L_{i,t}}{K_{i,t}}$. Normalize wages and returns on capital such that world GDP is equal to 1, $\sum_{i=1}^I w_{i,t}L_{i,t} + r_{i,t}K_{i,t} = 1$.

(c) Compute net exports according to the budget constraint: $NX_{i,t}^D = w_{i,t}L_{i,t} + r_{i,t}K_{i,t} - P_{i,t}C_{i,t}$.

(d) Given factor prices, solve for sectoral prices according by solving the system of equations defined by (1.11). Given prices, solve for trade shares according to (#).

(e) Solve for $I \times J$ sectoral expenditures, $E_{i,t}^j$, by solving the following system of equations:

$$E_{i,t}^j = P_{i,t}^j C_{i,t}^j + \sum_{k=1}^J (1 - \beta_k^j) \nu_k^j Y_{i,t}^k,$$

$$Y_{i,t}^j = \sum_{h=1}^I \pi_{hi,t} E_{h,t}^j.$$

(f) Compute net exports according to the intratemporal trade condition:

$$NX_{i,t}^S = \sum_{j=1}^J (Y_{i,t}^j - E_{i,t}^j).$$

(g) Compute $T_t^S = \max_i |NX_{i,t}^D - NX_{i,t}^S|$.

(h) Go back to (b) until $T_t^S$ is close to zero.

(i) If $T_t^S$ is close enough to zero, define $NX_{i,t} = NX_{i,t}^D$, compute $R_t$ according to (1.34) with $P_{i,t}C_{i,t} = w_{i,t}L_{i,t} + r_{i,t}K_{i,t} - NX_{i,t}$, and recover

$$B_{i,t} = \frac{(B_{i,t+1} - NX_{i,t})}{R_t}.$$

3. If $t > 0$, set $t = t - 1$ and proceed to 2. Otherwise, go to next step.
4. Obtain \( R_0 B_{i,0} = B_{i,1} - N X_{i,0} \) and compute \( T^D = \max_i |W_{i,0} - R_0 B_{i,0}| \). If \( T^D \) is greater than a target value close to zero, update on the steady state distribution of net foreign assets according to:

\[
B_{i,SS} = B_{i,SS} + \nu Z_i,
\]

where \( \nu \) is an adjustment factor, and \( Z_i \) a function such that \( \sum_{i=1}^I Z_i = 0 \). And go back to step 1. using the new \( B_{SS} \).

The functions \( Z_i \) are defined as:

\[
Z_i = (W_{i,0} - R_0 B_{i,0}) \left( \prod_{s=1}^T R_s \right).
\]

### F Trade Shares

The model’s specification allows us to recover the probability that country \( i \) imports a particular variety \( \omega_j \) from country \( h \),

\[
\pi_{ih,t}^j (\omega_j) \equiv \Pr \left[ \frac{c_{j,t}^i \tau_{ih,t}^j}{x_{j,t}^i (\omega_j)} \leq \min_{j \neq h} \left\{ \frac{c_{j,t}^j \tau_{ij,t}^j}{x_{j,t}^j (\omega_j)} \right\} \right].
\]

Let \( E_{i,t}^j \) denote total expenditure by country \( i \) on sector \( j \) goods, and \( E_{ih,t}^j \) total expenditure by country \( i \) on sector \( j \) goods produced in country \( h \), so that \( E_{i,t}^j = \sum_{h=1}^I E_{ih,t}^j \). Then, letting \( B_{ih,t}^j \) denote the subset of \( \mathbb{R}_+^I \) over which country \( i \) buys goods from country \( h \), we have that

\[
E_{ih,t}^j = \int_{B_{ih,t}^j} p_{ih,t}^j (x^j) d_{ih,t}^j (x^j) g^j (x^j | t) dx^j.
\]
Now note that, since \( \omega_j \in [0, 1] \), it must be the case that \( \pi_{ih,t}^j (\omega_j) \) for a particular variety is also the share of total expenditure in sector \( j \) by country \( i \) in goods produced by country \( h \), \( \pi_{ih,t}^j = \frac{E_{ih,t}^j}{E_{i,t}^j} \), where \( \sum_{h=1}^{I} \pi_{ih,t}^j = 1 \) for all \( i = 1, \ldots, I \) and \( j = 1, \ldots, J \).

G Data

Countries  The set of countries that I consider as core countries in the sample are: Australia, Austria, Belgium, Brazil, Canada, China, Denmark, Finland, France, Germany, Greece, India, Italy, Japan, Korea, Mexico, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, UK, US, and Venezuela. In addition, I consider an aggregate of other countries to construct the Rest of the World (ROW).

Aggregate Data  Data on GDP, net exports and investment expenditure for all countries comes from the United Nations Statistical Division National Accounts. These data are expressed in current US dollars. For ROW, I compute GDP as the aggregate of all remaining countries that are not part of the core 25 countries.

Gross Output and Value Added I use data on sectoral gross output and value added to compute the average value added shares in gross output from the following sources. For Australia, Austria, Belgium, Denmark, Finland, France, Germany, Greece, Italy, Korea, Netherlands, Portugal, Spain, Sweden and the UK the EU-KLEMS provide data for all year between 1970 and 2007. Data for the following countries also comes from the EU-KLEMS, but some years are missing (years missing in parentheses): Canada (2005-2007), Japan (1970-1972 and 2007) and the US (1970-1976).

For the following countries data on sectoral gross output and value added comes from the United Nations Statistical Division National Accounts (years missing in parentheses): Brazil (1970-1991), India (1970-1998), Mexico (1970-1979) and
Venezuela. For the case of China I obtain data on value added from the GGDC-10 database and on gross output directly from its official statistical agency.

The average value added to gross output shares are computed using these data. In order to construct the actual series on gross output, I consider data on sectoral value added in current dollars provided by the United Nations Statistical Division National Accounts that is available for all countries and years and that is consistent with the source used for aggregate data. Then I use these shares to construct series on gross output. This procedure allows me to recover sectoral value added for ROW using this comprehensive data set, then I consider the average across core countries’ value added in gross output shares to construct series on gross output for the rest of the world.

I define the broad sectors considered based on the following ISIC codes: Agriculture and Mining (ISIC A-C), Manufacturing (ISIC D) and Services (ISIC E-P). This definition is in line with the one provided by the United Nations Statistical Division National Accounts.

**Input-Output Tables** For all core countries except Venezuela I consider data from the OECD Stan Database. The IO table from the mid-1990s were considered for the following 21 countries: Australia, Austria, Belgium, Brazil, Canada, China, Denmark, Finland, France, Germany, Greece, India, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, UK and USA. For the case of Korea, Mexico and Switzerland, the IO tables correspond to the early-2000s. For the case of Venezuela, I use data provided by the Central Bank of Venezuela for the IO table in the year 1997. In order to construct an input-output table for ROW, I consider data from the World Input-Output Database for the year 2000 for the following three countries: Indonesia, Russia and Turkey.
**Bilateral Trade Flows**  In order to construct sectoral bilateral trade flows, I consider data from the NBER-UN for the period 1970-1999 and from CEPII-BACI for the period 1999-2007. These data sets include all core countries and the ROW is constructed by aggregating up all other countries. The data from NBER-UN is only available until the year 2000, while the CEPII-BACI is available from 1999 onwards. The data in these two data sets is not directly comparable in terms of the world sample of countries considered, therefore, I use the growth rates in bilateral trade flows obtained from CEPII-BACI after 1999 to extrapolate the data in the NBER-UN data.

Using this data I also construct net exports in tradable sectors.

**Labor and Capital**  I consider data on GDP per capita, GDP per worker and total population from the Penn World Tables version 7.1. In addition, I consider data on capital stocks in 1970 from the PennWorld Tables version 8.1 to construct capital stocks using data on investment expenditure from the United Nations National Accounts Statistics.

**Prices**  I consider sectoral producer price indexes. The data for gross output prices comes from the EU-KLEMS for the following countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Korea, Netherlands, Portugal, Spain, Sweden, U.K. and the U.S. Data on value added prices for Brazil, China, India and Mexico comes from the GGDC-10 data set. For Switzerland the data on sectoral producer price indexes comes from its official statistical agency. Sectoral prices for ROW are obtained from the estimation procedure described in Appendix H carried out for every year.

**H  Estimation of Sectoral Prices**

In order to construct series on sectoral prices, first I estimate sectoral relative prices in a base year and then use information on producer price indexes. Specifically, I
consider the year 2000 as the base year and estimate sectoral prices relative to the U.S. by exploiting the multisector gravity structure of the model as follows. Note that from (1.14) we have that

\[
\frac{\pi_{ih,t}^j}{\pi_{ii,t}^j} = \frac{T_{h,t}^j (c_{h,t}^j)^{-\theta} (\tau_{ih,t}^j)^{-\theta}}{T_{i,t}^j (c_{i,t}^j)^{-\theta}}.
\]

Taking logs on both sides I obtain that

\[
\ln \left( \frac{\pi_{ih,t}^j}{\pi_{ii,t}^j} \right) = \ln \left( \frac{T_{h,t}^j (c_{h,t}^j)^{-\theta} (\tau_{ih,t}^j)^{-\theta}}{T_{i,t}^j (c_{i,t}^j)^{-\theta}} \right) - \theta \ln (\tau_{ih,t}^j).
\]

Given a value of \(\theta\), suppose that actual trade costs are given by \(\tau_{ih,t}^j = \tilde{\tau}_{ih,t}^j \hat{\tau}_{ih,t}^j \hat{e}_h^j\), where \(\tilde{\tau}_{ih,t}^j = \tilde{\tau}_{hi,t}^j\) is a symmetric multiplicative element of bilateral trade costs, and \(\hat{e}_h^j\) is an export-specific multiplicative element of bilateral trade costs. I assume that the symmetric element is observable in the data and given by \(\tilde{\tau}_{ih,t}^j = \left( \frac{\pi_{hh,t}^j \pi_{ih,t}^j \pi_{ii,t}^j \pi_{hi,t}^j}{\pi_{ih,t}^j \pi_{ii,t}^j} \right)^{\frac{1}{2\theta}}\).

Note that, actually, according to the model, if trade costs were symmetric, then

\[
\tau_{ih,t}^j = \left( \frac{\pi_{hh,t}^j \pi_{ih,t}^j \pi_{ii,t}^j \pi_{hi,t}^j}{\pi_{ih,t}^j \pi_{ii,t}^j} \right)^{\frac{1}{2\theta}} \text{ by (1.29).}
\]

Then, we have that

\[
\ln \left( \frac{\pi_{ih,t}^j}{\pi_{ii,t}^j} \right) = \ln \left( \frac{T_{h,t}^j (c_{h,t}^j)^{-\theta}}{T_{i,t}^j (c_{i,t}^j)^{-\theta}} \right) - \theta \ln (\tilde{\tau}_{ih,t}^j).
\]

Estimating this equation in \(t = 2000\), I can recover \(\hat{S}_{i,t}^j \equiv \left( \frac{T_{i,t}^j}{T_{US,t}^j} \right)^{\frac{1}{\theta}} \left( \frac{c_{i,t}^j}{c_{US,t}^j} \right)^{-1}\). Then, I can estimate sectoral prices relative to the U.S. by noticing that

\[
\frac{\pi_{ii,t}^j}{\pi_{USUS,t}^j} S_{i,t}^j = \left( \frac{\pi_{ii,t}^j}{\pi_{USUS,t}^j} \right)^{\frac{1}{\theta}},
\]

where \(S_{i,t}^j = \left( \hat{S}_{i,t}^j \right)^{\theta}\).
Chapter 2

Globalization and Structural Change in the United States: A Quantitative Assessment

2.1 Introduction

The rapid process of global integration that the world has undergone has affected the U.S. economy. For instance, expenditure by the U.S. on agricultural goods produced abroad as a share of total expenditure on agriculture raised from 7.0 percent in 1970 to 48.9 percent in 2007, while this share in the case of manufacturing goods increased from 4.0 percent to 22.1 percent over the same time period. Furthermore, together with the increase in gross trade flows, the U.S. experienced a significant and steady expansion in its trade deficit. Even though a greater integration of global markets has had a clear impact on the U.S. economy, studies aiming to quantify how much changes in trade flows have affected the reallocation of economic activity across broadly defined economic sectors in the U.S. are scarce. This is, the literature studying this sectoral reallocation in the U.S., also known as structural transformation, has focused on
quantifying the mechanisms driving structural change in a closed economy without taking into account how these mechanisms are influenced by the forces underlying the increase in trade flows.

This paper provides an answer to the following question: how much did the decline in trade costs that underlies the increase in trade flows between the U.S. and its trade partners, and the increase in its trade deficit contribute to the evolution of sectoral value added shares in agriculture, manufacturing and services over the period 1970-2007. Figure 2.1 shows the evolution of these value added shares as well as that of sectoral expenditure shares on goods produced abroad and of the U.S. trade deficit over this time period.

This study is divided into two parts. In the first part, I consider a static general equilibrium model of international trade and structural transformation that includes unbalanced trade in order to assess the quantitative importance of changes in trade costs and trade deficits in shaping structural transformation in value added shares in the U.S. over the period 1970-2007. The model builds on the static structure of the
new quantitative general equilibrium models of international trade and incorporates the main mechanisms that drive structural transformation in closed economy models. I calibrate the model to the data for the case of two countries, the U.S. and the rest of the world (ROW), and recover a set of time series of structural residuals of the model that rationalize international trade data as sequential equilibria of the model. This set of residuals include trade costs, sectoral productivities and net exports. To finish this part of the paper, I consider counterfactual sequential equilibria in which trade flows across countries differ due to the absence of declines in trade costs or to reduced U.S. trade deficits. The main result of these counterfactual exercises is that changes in trade costs and in trade deficits have had sizable effects on structural transformation in the U.S. In the main counterfactual exercise in which trade costs are held fixed at their 1970’s levels and trade is balanced, the manufacturing value added share in the U.S. only decreases by approximately one tenth of the decrease in the baseline calibration of the model. In a different counterfactual scenario in which the trade deficit of the U.S. is adjusted downwards by fifty percent, the decline in the manufacturing value added share in the U.S. is approximately one half of the decrease in the baseline case. These results support the quantitative relevance that changes in trade flows, both gross and net, have in shaping structural change in the U.S.

The results of the first part of this paper imply that in order to provide a correct quantification of the effects of a more integrated world economy, i.e. lower trade costs, on structural transformation in the U.S. we need a framework in which trade deficits are determined endogenously. Therefore, in the second part of this paper I propose an extension of the static model to a dynamic environment in which savings and investment decisions are endogenously determined. Given this extension, I discuss

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1 This extension is similar to the model proposed in Chapter 1 of this dissertation. The main difference with respect to the model in Chapter 1 is the endogenous determination of investment decisions. This additional margin adds a layer of complexity to the model that is discussed in greater detail at the end of this chapter.
how changes in trade costs affect these decisions and the challenges involved in taking
the model to the data and solving it numerically.

There exists an extensive literature on structural transformation. Most recent
work has focused on the interaction between economic growth and structural transfor-
mation in a closed economy setting. This literature has posited two main mechanisms
as the drivers of structural transformation. The first mechanism relies on differences
in income elasticities of demand across sectors, mainly driven by non-homothetic pref-
erences. The works by Caselli and Coleman (2001), Kongsamut et al. (2001), Buera
and Kaboski (2011) and Buera and Kaboski (2012) are only a few of the most recent
contributions emphasizing this mechanism. The second mechanism is a supply-side
mechanism that relies on sectoral biased productivity growth. Baumol (1967) was
the first one to point out how this mechanism could generate structural change, while
Ngai and Pissarides (2007) recently formalized this idea. Most recent work has relied
on both mechanism to try to understand their quantitative relevance. Buera and
Kaboski (2009), Duarte and Restuccia (2010) and Herrendorf et al. (2013) are some
of the most recent contributions. Herrendorf et al. (2014) provide an exhaustive
up-to-date survey of the literature.

Recent work has started to emphasize the role that an open economy can play
in shaping structural transformation. Early studies include the work by Matsuyama
exploited the new quantitative general equilibrium models of international trade to
study structural transformation in an open economy. That paper embeds the two
mechanisms aforementioned into a general equilibrium model of international trade
(2013) rely on this framework to show qualitatively that more access to trade can
generate some features of the data that closed economy models struggle to generate.
The paper then applies this framework to examine structural change in South Korea.
The theoretical framework considered in this paper most closely resembles the one proposed by Uy et al. (2013). However, in this paper I rely on their methodology to provide quantitative evidence on the importance of international trade in shaping structural change in the U.S. In addition, this paper also focuses on the potential role of changes in trade imbalances, which Uy et al. (2013) do not explore. Exploring this additional channel links this paper to recent work that has focused on analyzing quantitative trade models in a fully dynamic setting in order to better understand structural change.\footnote{For example, Sposi (2012), Kehoe et al. (2013) and Chapter 1 of this dissertation.} The second part of this paper focuses on analyzing this link.

The remainder of the paper is organized as follows. Section 2 presents the theoretical model as well as the basic features of the model’s solution. Section 3 takes the model to the data and shows how we can recover the time series of structural residuals of the model. Section 4 presents the quantitative results of counterfactual equilibria. Section 5 presents the dynamic extension of the static model and discusses the challenges in solving for the competitive equilibrium of the model. Section 6 concludes.

### 2.2 The Benchmark Model

Let $t$ denote the time period. In every $t$, the world is formed by two countries indexed by $i \in I \equiv \{US, ROW\}$, each consisting of three sectors indexed by $j \in J \equiv \{a, m, s\}$. The three sectors correspond to agriculture ($a$), manufacturing ($m$) and services ($s$). Each country $i \in I$ is inhabited by a representative household endowed with $L_{i,t}$ units of homogeneous labor and $K_{i,t}$ units of homogenous capital in every period $t$.

There are two types of goods in the model. The first type of goods consists of sector-specific goods produced in each country and which are nontradable across countries. I refer to this type of goods as final sectoral goods. These final sectoral
goods are in turn used either for final consumption by the representative household or as an intermediate input in the production of the second type of goods. The second type of goods are also sector-specific, can be traded across countries and are produced by aggregating up capital, labor and intermediate inputs. These goods are in turn the intermediate goods used in the production of final sectoral goods.

2.2.1 Technologies

Final Sectoral Goods

Final output in each sector \( j \in J \) is given by an aggregate of a continuum of tradable goods indexed by \( \omega^j \in [0, 1] \). Sector \( j \)'s output in country \( i \) at time \( t \) is then given by

\[
Q_{j}^{i,t} = \left( \int_{0}^{1} d_{i,t}^{j} (\omega^j) \frac{\eta - 1}{\eta} d\omega^j \right)^{\frac{\eta}{\eta - 1}}, \tag{2.1}
\]

where \( d_{i,t}^{j} (\omega^j) \) denotes the use in production of intermediate good \( \omega^j \).

The demand for each intermediate good is derived from the cost minimization problem of a price-taking representative firm. Moreover, since good \( \omega^j \) is tradable across countries, the firms producing \( Q_{j}^{i,t} \) search across all countries for the lowest cost supplier of this good.

As previously explained, the final output in each sector \( j \) is nontradable and can be used either for final consumption or as an intermediate input into the production of the tradable goods. I will denote by \( P_{j}^{i,t} \) the price of sectoral good \( j \) in country \( i \) at time \( t \).

 Tradable Goods

Consider a particular good \( \omega^j \in [0, 1] \) and let \( q_{i,t}^{j} (\omega^j) \) denote the production of this good in country \( i \) at time \( t \). The technology to produce each good
\( \omega^j \) is given by

\[
q^j_{i,t}(\omega^j) = x^j_{i,t}(\omega^j) \left[ k^j_{i,t}(\omega^j) \nu^j_{i,t}(\omega^j) \right]^{\beta^j_i} \left[ \prod_{m=1}^M D^j_{i,m}(\omega^j)^{\nu^j_{i,m}} \right]^{1-\beta^j_i},
\]

where \( l^j_{i,t}(\omega^j) \) and \( k^j_{i,t}(\omega^j) \) are the labor and capital respectively used in the production of good \( \omega^j \), and \( D^j_{i,m}(\omega^j) \) denotes the intermediate demand by producers of good \( \omega^j \) for sectoral good \( m \). Here, \( \sum_{m=1}^M \nu^j_{i,m} = 1 \) for all \( j \in J \) and \( \nu^j_{i,m} \in (0,1) \) for all \( j, m \in J \). The efficiency in the production of good \( \omega^j \) is given by \( x^j_{i,t}(\omega^j) \).

I assume that the efficiency in the production of good \( \omega^j \), \( x^j_{i,t}(\omega^j) \), is given by the realization of a random variable, \( x^j_{i,t} \in (0, \infty) \), distributed according to a Fréchet distribution with shape parameter \( \theta \) and location parameter \( T^j_{i,t} \),

\[
F^j_{i,t}(x|t) = \Pr [ x^j_{i,t} \leq x ] = e^{-T^j_{i,t}x^{-\theta}}.
\]

I assume that, conditional on \( T^j_{i,t} \), the random variables \( x^j_{i,t} \) are independently distributed across sectors and countries.

I will refer to \( T^j_{i,t} \) as the sectoral productivity of country \( i \) in sector \( j \) at time \( t \), since their values determine the level of the distribution from which producers draw their efficiencies. These productivities change over time and they represent one of the underlying wedges that drive the dynamics of the world economy.

### 2.2.2 Trade Costs and Firms’ Optimal Decisions

For each sector \( j \in J \), goods \( \omega^j \in [0,1] \) can be traded across countries, but are subject to iceberg type trade costs. Specifically, \( \tau^j_{ih,t} \geq 1 \) denotes the cost of shipping any good \( \omega^j \in [0,1] \) from country \( h \) to country \( i \) at time \( t \). This means that, in order for one unit of variety \( \omega^j \) to be available in country \( i \) at time \( t \), country \( h \) must ship
\(\tau_{ih,t}^j\) units of the good. I assume that \(\tau_{ii,t}^j = 1\) for all \(i\), i.e. there are no trade costs associated with trading goods within countries.

Note that these bilateral trade costs are allowed to change over time and that they are sector, but not good specific. Hence, sector specific bilateral trade costs are additional wedges that drive the dynamics of the model.

Assuming perfectly competitive markets and given constant returns to scale in the production of good \(\omega^j\), the free-on-board price (before trade costs) of one unit of this good, if actually produced in country \(i\) at time \(t\), is equal to its marginal cost, 
\[
c_i^{j,i,t} \rho_j^{j,i,t} \omega_j^{j,i,t}
\]
where \(c_i^{j,i,t}\) is the cost of the input-bundle to produce one unit of \(\omega_j^{j,i,t}\).

Letting \(\varrho_j(x^j|t)\) denote the conditional joint density of the sector specific vector of productivity draws for all countries, \(x_j^j = (x_{US,t}^j, x_{ROW,t}^j)\), we can define total factor and intermediate input usage from each sector \(m\) in sector \(j\) as \(L_j^{i,j,t} = x_{h,t}^j (\omega_j^{j,i,t})\) and \(D_{j,m}^{i,j,t}\). \(L_j^{i,j,t}\) denotes the ideal price index of sectoral good \(j\).

**Prices and Trade Shares** Given these distributions of productivities, we can derive an expression for sectoral price indices in equilibrium as functions of all sectoral

\[
\text{prices and trade shares}
\]

3Specifically, \(c_i^{j,i,t} = \varphi_i^j \left( (r_{i,t})^{\phi_i} \omega_{i,t}^{j,i} \right)^{\beta_i^j} \left( \prod_{m=1}^J \left( P_{m,i,t}^j \right)^{\phi_i^{j,m}} \right)^{1-\beta_i^j} \) where \(r_{i,t}\) and \(w_{i,t}\) denote the rental rate and the wage in country \(i\) respectively, and \(\varphi_i^j\) is a constant that depends on production parameters, \(\varphi_i^j = (\beta_i^j \varphi_i^j (1-\varphi_i^j) \left( (1-\beta_i^j) \prod_{m=1}^J (\varphi_i^j)^{\phi_i^{j,m}} \right) ^{-1-\beta_i^j} \).

4For example, total labor usage would be given by \(L_j^{i,t} = \int_{\mathbb{R}^+} L_j^{i,\varrho^j (x^j|t)} dx^j\).
prices, factor prices, and trade costs around the world. These prices are conditional on the known values of sectoral productivities, $T_{i,t}^j$, and bilateral trade costs, $\tau_{ih,t}^j$, in period $t$. Using the ideal price index corresponding to 2.1 for each sector, and the properties of the distribution of efficiencies around the world, we can derive the sectoral prices in each country $i$ and every period $t$. In line with the derivations of Eaton and Kortum (2002) and Caliendo and Parro (2015), these prices are given by

$$P_{i,t}^j = \Gamma \left[ \Phi_{i,t}^j \right]^{-\frac{1}{\theta}},$$

where $\Gamma$ is a constant that only depends on $\eta$ and $\theta$, and $\Phi_{i,t}^j = \sum_{h=1}^{I} T_{h,t}^j \left( c_{h,t}^j \tau_{ih,t}^j \right)^{-\theta}$ represents a sufficient statistic for sector $j$ in country $i$ of the state of technologies and trade costs around the globe.$^5$ Note that as long as there is no free trade, i.e. $\tau_{ih,t}^j \neq 1$ for some countries $i$ and $h$, prices will differ across countries. If there is free trade, it will be the case that $P_{i,t}^j = P_{h,t}^j$ for all $i, h = US, ROW$.

We can also recover sectoral trade shares for each country in terms of world prices, technologies and trade costs. Let $E_{i,t}^j$ denote total expenditure by country $i$ on sector $j$ goods, and $E_{ih,t}^j$ total expenditure by country $i$ on sector $j$ goods produced in country $h$, so that $E_{i,t}^j = \sum_{h=1}^{I} E_{ih,t}^j$. Then, the share of total expenditure in sector $j$ by country $i$ in goods produced by country $h$, $\pi_{ih,t}^j \equiv \frac{E_{ih,t}^j}{E_{i,t}^j}$, is given by

$$\pi_{ih,t}^j = \frac{T_{h,t}^j \left( c_{h,t}^j \tau_{ih,t}^j \right)^{-\theta}}{\Phi_{i,t}^j},$$

and are such that $\sum_{h=1}^{I} \pi_{ih,t}^j = 1$ for all $i = US, ROW$ and $j \in \mathcal{J}$.

$^5$In particular, $\Gamma = (\Gamma(1 + (1-\eta))^{\frac{1}{\theta}}$, where $\Gamma(\cdot)$ denotes the Gamma function evaluated for $z > 0$. Notice this implies that parameters have to be such that $\eta - 1 < \theta$. 

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2.2.3 Households

The representative household in country \(i\) has period utility given by

\[
u \left( \{C^j_{i,t}\}_{j \in J} \right) = \left( \sum_{j \in J} (\mu^j_i)^{1 \over \psi} (C^j_{i,t} - \bar{C}^j_i)^{\psi - 1 \over \psi} \right)^{\psi - 1 \over \psi}, \tag{2.2}\]

where \(C^j_{i,t}\) for \(j \in J\) represents consumption by the household of final sectoral goods, and \(\bar{C}^j_i\) is the subsistence requirement for sector \(j\) final goods. Preference parameters \(\mu^j_i\) are nonnegative and such that \(\sum_{j=1}^J \mu^j_i = 1\). This type of utility function allows the model to incorporate both mechanisms that the literature has suggested as driving structural change. Differences in subsistence requirements across sectors lead to differences in income elasticities. For instance, a subsistence requirement greater than 0 implies that income elasticity of demand for that sectoral final good is greater than 1. The elasticity of substitution across sectoral consumption is given by \(\psi > 0\). If \(0 < \psi < 1\), then sectoral goods are complements, while \(\psi \geq 1\) implies that they are substitutes. Each case leads to different implications for the effects of changes in relative prices on expenditure shares.

Households then choose sectoral consumption levels \(\{C^j_{i,t}\}_{j \in J}\), to maximize period utility, (2.2), subject to the period budget constraint

\[
\sum_{j \in J} P^j_{i,t} C^j_{i,t} = w_{i,t} L_{i,t} + r_{i,t} K_{i,t} - NX_{i,t},
\]

in each period \(t\).

2.2.4 Household’s Optimal Decisions

This problem leads to the following optimal sectoral expenditure levels for each \(j \in J\):

\[
P^j_{i,t} C^j_{i,t} = \mu^j_i \left( \frac{P^j_{i,t}}{\bar{P}_{i,t}} \right)^{1-\psi} DA_{i,t} + P^j_{i,t} \bar{C}^j_i \tag{2.3}\]
where \( P_{i,t} \) denotes the ideal prices index of aggregate consumption, and

\[
DA_{i,t} = w_{i,t}L_{i,t} + r_{i,t}K_{i,t} - NX_{i,t} - \sum_{j\in J} P_{i,t}^j \bar{C}_i^j
\]

is domestic absorption adjusted by subsistence expenditure.\(^6\) We can immediately observe in (2.3) how changes in relative prices and income growth leads to changes in sectoral consumption shares which are given by

\[
s_{i,t}^j \equiv \frac{P_{i,t}^j \bar{C}_i^j}{w_{i,t}L_{i,t} + r_{i,t}K_{i,t} - NX_{i,t}} = \mu_i^j \left( \frac{P_{i,t}^j}{P_{i,t}} \right)^{1-\psi} \left( 1 - \frac{\sum_{j\in J} P_{i,t}^j \bar{C}_i^j}{w_{i,t}L_{i,t} + r_{i,t}K_{i,t} - NX_{i,t}} \right) + \frac{P_{i,t}^j \bar{C}_i^j}{w_{i,t}L_{i,t} + r_{i,t}K_{i,t} - NX_{i,t}}.
\]

Note that changes in trade costs affect the evolution of relative prices. If declining trade costs lead to a decline in the relative price of more tradable sectors, like agriculture and manufacturing, relative to nontradable sectors, like services, then we would expect the shares in the tradable sectors to decline when \( \psi < 1 \). This is precisely the mechanism emphasized by Ngai and Pissarides (2007). In other words, declines in trade costs lead to growth in sectoral productivity adjusted by trade which is biased towards tradable sectors. Here by sectoral productivity adjusted by trade I am referring to \( A_{i,t}^j \equiv \frac{1}{\pi^i_{i,t}} \left( T_{i,t}^j \right)^{\frac{1}{\theta}} \), which would be measured sectoral productivity assuming that the economy is closed. Income growth also leads to structural transformation due to nonhomotheticities in preferences. For example, if \( \bar{C}_i^j > 0 \), then an increase in income due to an increase in GDP or in trade deficits leads to a less than proportional increase in sectoral final consumption expenditure, leading to a decline in sector \( j \)'s consumption share.

\(^6\)We have that \( P_{i,t} = \left( \mu_i^a \left( P_{i,t}^a \right)^{1-\psi} + \mu_i^m \left( P_{i,t}^m \right)^{1-\psi} + \mu_i^s \left( P_{i,t}^s \right)^{1-\psi} \right)^{\frac{1}{1-\psi}}. \)
2.2.5 Market Clearing Conditions

The following are the market clearing conditions that close the model. First, non-tradable goods’ markets must clear in each country $i$,

$$C_{i,t}^j + \sum_{n \in \mathcal{J}} D_{i,t}^{n,j} = Q_{i,t}^j \text{ for } j \in \mathcal{J},$$

as well as factor markets, $\sum_{n \in \mathcal{J}} L_{i,t}^j = L_{i,t}$ and $\sum_{n \in \mathcal{J}} K_{i,t}^j = K_{i,t}$.

Market clearing for tradable goods can be written in terms of expenditure rather than quantities in the following way. Let $Y_{i,t}^j$ denote the value of production in sector $j$ goods by country $i$ at time $t$. Then, markets for tradable goods clear when if

$$Y_{i,t}^j = \sum_{h=1}^I \pi_{h,i,t}^j E_{h,t}^j \text{ for all } j \in \mathcal{J}.$$ 

This equation simply states that gross production of sector $j$ goods in country $i$ has to be equal to total expenditure around the entire world on those goods.

Lastly, there is a set of country-specific resource constraints that states that exogenously given net exports have to be consistent with world trading decisions:

$$NX_{i,t} = \sum_{j \in \mathcal{J}} (Y_{i,t}^j - E_{i,t}^j).$$

Given this set of market clearing conditions, we can now define an equilibrium of the model.

2.2.6 Equilibrium

Let $\mathcal{S}_t \equiv \{\tau_{i,h,t}^j, T_{i,t}^j, NX_{i,t}\}_{i,h \in \{US, ROW\}}$ be the set of exogenous wedges fed into the model.
Definition Given \( L_{i,t} \) and \( K_{i,t} \) for \( i \in \{US, ROW\} \) and every \( t \); a sequential equilibrium under wedges \( \{S_t\}_{t=0}^\infty \) is defined by sequences of wages, rates of return on capital and prices, \( \left\{ \left\{ w_{i,t}, r_{i,t}\right\} \right\}_{i=1}^I \), \( \left\{ \left\{ P_{j,i,t}\right\} \right\}_{j=1}^J \) \( \left\{ \left\{ P_{i,t}\right\} \right\}_{t=0}^\infty \), and allocations, such that given these prices, the allocations satisfy the optimality conditions for the firms and households in every country, and all markets clear.

2.3 Mapping the Model to the Data

I calibrate the model to observed data for the period 1970 to 2007. The calibration requires the identification of the model’s time-invariant parameters and time-varying exogenous variables. Time-varying exogenous variables can be divided into those that are directly observed in the data and those that are not. The set of exogenous variables that are not observed are the ones I call wedges and previously labeled as \( S_t \).

I recover these disturbances by relying on endogenous outcomes of the model that are observed in the data, specifically, bilateral trade flows and prices for tradable sectors and GDP.

The calibration is carried out for the case of two countries, \( i \in \{US, ROW\} \), and three sector, \( J \equiv \{a, m, s\} \). I assume that the services sector, \( s \), is nontradable, that is, this sector must source all its goods from home in order to produce the final nontradable sectoral good. In terms of the model, this simply implies that bilateral trade costs are infinity in this sector, \( i.e. \tau_{sh,t}^s = \infty \) for \( i \neq h \) or equivalently, \( \pi_{ii,t}^s = 1 \).

Even though I do not model trade in services, I do take into account trade imbalances in this sector and incorporate them into the model as time-varying exogenous transfers across countries.
2.3.1 Time-invariant Parameters

I calibrate time-invariant parameters either directly from the data or take their values from the previous literature. I calibrate value added shares, $\beta^j_i$, using data on gross output and valued added from a combination of data sources that includes EUKLEMS, UNIDO, GGDC-10 and countries’ official statistical agencies. I set $\beta^j_i$ to the average of value added divided by gross output in each sector $j \in J$. For the input-output coefficients, $\nu^j_{ik}$, I use the input-output tables provided by OECD-Stan. Depending on the availability of the input-output tables, I recover these coefficients mainly from the tables corresponding to the late 1990s. I calibrate preference parameters, other than the elasticity of substitution, and consumption subsistence levels following the procedure in Herrendorf et al. (2013).

I now turn to the parameters that I take from existing literature. For the capital shares in value added, $\varphi_i$, I take the values from Caselli et al. (2007) who calibrate these parameters for a large set of countries. I calibrate the elasticity of substitution across tradable goods $\eta = 2$, in line with Caselli et al. (2015) and Broda and Weinstein (2006). For the case of the trade elasticity, I consider as a baseline $\theta = 4$, which is consistent with the estimates in Simonovska and Waugh (2014) for the case of international trade models.

For the case of the elasticity of substitution in consumption, I consider a value of $\psi = 0.4$. This value is consistent with the literature on structural transformation that calibrates values for this parameter to be less than one. The value I consider is in line with Duarte and Restuccia (2010), and in the midrange of estimates for the

\footnote{The data appendix provides a detailed and comprehensive list of all data sources.}
\footnote{I use data on sectoral gross output and net exports to construct sectoral expenditures. Then, I rely on the model’s market clearing conditions to recover sectoral consumption expenditures. Lastly, I choose $\{\mu^j_i, \bar{C}^j_i\}_{j \in J}$ to fit the data conditional on an elasticity of substitution, $\psi$, and the constraints $\bar{C}^m_i = 0$ and $\sum_{j \in J} \mu^j_i = 1$.}
Table 2.1: Time-invariant Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Variable</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_i$</td>
<td>-</td>
<td>Value added to gross output ratio</td>
<td>Sectoral Data</td>
</tr>
<tr>
<td>$\nu_{i,j,k}$</td>
<td>-</td>
<td>Input-output coefficients</td>
<td>Data, Input-Output Tables</td>
</tr>
<tr>
<td>$\varphi_i$</td>
<td>-</td>
<td>Capital share in value added</td>
<td>Caselli and Feyrer (2007)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>4</td>
<td>Trade elasticity</td>
<td>Range Simonovska and Waugh (2014)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2</td>
<td>Elasticity of substitution in tradable goods</td>
<td>Caselli et al. (2014)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.4</td>
<td>Elasticity of substitution in consumption</td>
<td>Duarte and Restuccia (2010)</td>
</tr>
<tr>
<td>$\mu_i^j$</td>
<td>-</td>
<td>Preference parameters</td>
<td>Calibrated [Herrendorf et al. (2013)]</td>
</tr>
<tr>
<td>$\bar{C}_i^j$</td>
<td>-</td>
<td>Consumption subsistence levels</td>
<td>Calibrated [Herrendorf et al. (2013)]</td>
</tr>
</tbody>
</table>

U.S. and other less developed economies. Table 2.1 summarizes the baseline values of these parameters as well as the data used to obtain them.

2.3.2 Time-varying Exogenous Variables

The time-varying exogenous variables of the model are the homogeneous labor endowments, $L_{i,t}$, capital stocks, $K_{i,t}$, and net exports in the services sector, $NX_{i,t}^s$. I construct the series for labor endowments using data on Population, $Pop_{i,t}$, GDP per capita, $rgdpc_{i,t}$, and GDP per worker, $rgdpw_{i,t}$, from the Penn World Tables 7.1. These endowments are then constructed as $L_{i,t} = \left( \frac{rgdpc_{i,t}}{rgdpw_{i,t}} \right) \times Pop_{i,t}$. For the capital stock, I use data on capital stocks from the Penn World Tables 7.1 for the year 1970. Then, I use the capital accumulation equation $K_{i,t+1} = (1 - d) K_{i,t} + I_{i,t}$ with $d = 0.5$ and data on investment from the UNStats to construct the stocks from 1971 to 2007.

To construct series for net exports in services, $NX_{i,t}^s$, I consider data on aggregate net export, $NX_{i,t}$, which I obtain from UNStats and bilateral trade flows from country $h$ to country $i$ in tradable sectors, $X_{ih,t}^j$ for $j \in \mathcal{J} \setminus s$, which I obtain from the NBER-UN and CEPII-BACI data sets. Then, country $i$’s exports and imports in sector $j$ are given by $\sum_{h=1}^I X_{hi,t}^j$ and $\sum_{h=1}^I X_{ih,t}^j$ respectively, and I construct $NX_{i,t}^s = NX_{i,t} - \sum_{j \in \mathcal{J} \setminus s} \left( \sum_{h=1}^I X_{hi,t}^j - \sum_{h=1}^I X_{ih,t}^j \right)$. 79
2.3.3 Structural Residuals

The endogenous outcomes of the model that I observe in the data and that I use to recover the wedges in period \( t \), \( S_t \), for \( t = 1970, \ldots, 2007 \) are sectoral bilateral trade flows in tradable sectors, \( X_{ith,t}^j \) for \( j \in J \setminus S \), and prices for tradable sectors, \( P_{i,t}^j \) for \( j \in J \setminus S \), and GDP. To obtain sectoral trade shares, I rely on data on bilateral trade flows and sectoral gross output, \( Y_{i,t}^j \) for \( j \in J \setminus S \). In addition, I use data on gross domestic product, \( GDP_{i,t} \), to recover the factor prices, \( w_{i,t} \) and \( r_{i,t} \), that are consistent with the model instead of relying on data on wages and rental rates of capital that might not be consistent with the labor and capital endowments I recovered in the previous subsection. For aggregate prices I consider gross domestic product prices from the Penn World Tables 7.1. Data on sectoral prices come from EU-KLEMS and GGDC-10 databases.

For \( t = 1970, \ldots, 2007 \), and \( i, h \in \{US, ROW\} \), define the set of data

\[
D_t = \{(w_{i,t}, r_{i,t}, P_{i,t}) \}, \{P_{i,t}^j\}_{j \in J \setminus S}, \{\pi_{ih,t}^j\}_{j \in J \setminus S}\}.
\]

The following lemma shows how we can recover the model’s wedges given data \( D_t \) in every period \( t \).

**Lemma 2.3.1** Given time-invariant parameter values and data \( D_t \); there is a one-to-one mapping between observables and \( \{\tau_{ih,t}^j\}_{j \in J \setminus S} \) and \( \{T_{i,t}^j\}_{j \in T} \) in period \( t \) given by the following equilibrium conditions:

\[
\tau_{ih,t}^j = \frac{P_{i,t}^j}{P_{h,t}^j} \left( \frac{\pi_{h,t}^j}{\pi_{ih,t}^j} \right) \frac{1}{\theta} \quad \text{for } j \in J \setminus S,
\]

\[
\pi_{i,t}^j = T_{i,t}^j \left( \Gamma_j \pi_{i,t}^j \right)^{-\theta} \quad \text{for } j \in J \setminus S \text{ and } \pi_{i,t}^j = 1 \text{ for } j = s, \text{ and}
\]

\[
P_{i,t} = \left( \sum_{j=1}^{J} \mu_i^j \left( P_{i,t}^j \right)^{1-\psi} \right)^{1/\psi} \quad \text{and } \tau_{i,h,t}^s = \infty \text{ for all } i \neq h.
\]
Figure 2.2: Evolution of Sectoral Trade Costs

Notes: This figure plots the evolution of iceberg trade costs $\tau_{ih,t}^{j} \geq 1$ for tradable sectors $j \in \{a, m\}$.

Figure 2.2 shows the evolution of trade costs to import in each country. In line with the literature, the data shows that trade costs are asymmetric across countries. Specifically, Waugh (2010) shows that in order to rationalize data on trade flows and GDP, it must be that it is more costly for developing countries to import than for developed one. The data precisely shows that it is more costly for ROW to import than it is for the U.S, in both the agriculture and manufacturing sectors. In addition, the data clearly shows the declining path in trade costs in all cases except in the case of trade costs for importing into the U.S. There is a significant amount of volatility in these trade costs over the entire time period.

Figure 2.3 shows the evolution of sectoral productivity in the U.S. that we recovered using the data on factors and sectoral prices. Notice that the, according to the
previous lemma, we can recover these productivities by relying on the dual of the profit maximization problem of the firm. Therefore, we can recover productivities relying on data on sectoral prices rather than on sectoral factors of production and intermediate input usage. The data clearly shows the well established fact that, on average, productivity in agriculture grows at the fastest rate, while that of services at the lowest rate. In the case of productivity in manufacturing, we can see that there are long period of high growth (1982-1986) and other of lower growth (1986-1996).

2.4 Counterfactual Exercises

I now turn to the fit of the model and counterfactual equilibria in which the model’s wedges differ from the values initially recovered. Specifically, I will focus on counter-
factual scenarios in which trade costs are fixed to their level in 1970 and the absolute value of net exports is reduced.

Let us first turn to the actual fit of the model. The model is exactly identified for particular moments of the data targeted in the previous section. For instance, trade shares and prices are exactly identified. However, value added shares are not part of those moments. We proceed now to analyze the modeled evolution of these shares.

Figure 2.4 shows the evolution of value added shares in the data and in the baseline calibration of the model. As can be seen in the figure, the model does a good job in matching the overall trends in value added shares. However, it is clear the model is not able to match the acceleration in the decline of the manufacturing share and in the increase of the services sector after 1980. Recall that the model was not calibrated to match value added shares, but rather sectoral expenditure shares. Thus the difference between the baseline calibration of the model and the data. However, it is worth pointing out that there is room to further improve the calibration of the model in order to better match the data on value added shares. I leave such a recalibration of the model’s parameters for future research.

I now consider the first counterfactual equilibrium and compare the value added shares in such scenarios to those of the baseline calibration of the model. In the first counterfactual, trade costs are held constant and equal to their values in 1970 in every year. All other variables exogenously fed into the model follow the same paths as in the baseline calibration. Figure 2.5 shows the results of this exercise. The exercise reveals that changes in trade costs have contributed to the observed decline in the manufacturing value added share and to a slower decline in the share in agriculture. These results also show that this decline has had a negligible effect on value added in services and the effect on the manufacturing share is quantitatively small. These results are in line with the evolution of the calibrated trade costs presented in Figure 2.2. The steady decline in manufacturing trade costs and high trade costs in agricul-


Notes: This figure plots the evolution of sectoral value added shares in the U.S.

ture have led to changes in manufacturing relative prices leading to reallocation of economic activity towards agriculture.

In a different counterfactual equilibrium I allow trade costs follow their calibrated trajectories, but assume that trade is balanced in every year. This exercise is carried out in order to isolate the effects of the increasing trade deficit in the U.S. on structural change. Figure 2.6 presents the results of this exercise. The results show that the increase in the overall trade deficit in the U.S. has contributed to the decline in the manufacturing share and the increase in the services share. The fact that the U.S. trade deficit is composed in its majority by a deficit in manufacturing implies that closing the overall deficit requires closing the manufacturing deficit. When this is the
Figure 2.5: Counterfactual with Constant Trade Costs of 1970 (Value Added Shares)

Notes: This figure plots the evolution of sectoral value added shares in the U.S.

case, additional expenditure in manufacturing requires more domestic production in that sector, thus, a greater value added share.

Figure 2.7 considers two counterfactual equilibria, one in which balanced trade is imposed and another one in which constant trade costs are accompanied by balanced trade. The second counterfactual scenario presents the case in which trade costs remain unchanged and trade is balanced. In the case in which trade costs remain at their 1970’s levels and net exports are zero, which is very close to their actual values in 1970, we see that the decline in the manufacturing share becomes almost nonexistent.

In the final counterfactual exercises I consider the case in which trade costs stay constant at their levels of 1970 and net exports are reduced by a constant factor across
Figure 2.6: Counterfactual with Balanced Trade (Value Added Shares)

Notes: This figure plots the evolution of sectoral value added shares in the U.S.

all years. Specifically, I consider the case in which trade imbalances are reduced either by seventy five or twenty five percent. Figure 2.8 shows the evolution of value added share under these different assumptions. These exercises convey the relevance of the factor by which net exports are reduced in order to understand the effects of trade on structural transformation in the U.S. However, the static nature of the model does not allow for an explanation of why changes in these imbalances arise. Since the first chapter of this dissertation shows that trade costs in goods markets play an important role in shaping trade imbalances, in the next section I propose a dynamic extension to the current framework that might provide a better understanding of the implications of lower trade frictions on structural transformation.
Figure 2.7: Counterfactual with Trade Costs of 1970 and Balanced Trade (Value Added Shares)

Notes: This figure plots the evolution of sectoral value added shares in the U.S.

Table 2.2 presents the full set of quantitative results of this paper. In the table, $va^j_t$ denotes sector $j$’s value added share in year $t$. The results corresponding to CF 1 are those associated with the counterfactual equilibrium in which trade costs are set to their levels in 1970 and trade is balanced. The case of CF 2 is when we consider the same trade costs as in CF 1, but the trade deficit is adjusted down by fifty percent. I consider this to be a reasonable approximation of how much the trade deficit in the U.S. would be affected if trade costs remained constant at their 1970’s levels.\footnote{This approximation is based on the results obtained in Chapter 1 of this dissertation.} Notice that in this case the decline in the manufacturing sector is almost half of the decline in the baseline equilibrium. Thus, I argue that the evolution of both gross and
Figure 2.8: Counterfactual with Trade Costs of 1970 and Different Net Exports (Value Added Shares)

Notes: This figure plots the evolution of sectoral value added shares in the U.S.

net trade flows have been quantitatively relevant for the determination of structural transformation in the U.S.

2.5 Incorporating Dynamics into the Benchmark Model

In this section I propose a dynamic extension to the benchmark model in order to fully incorporate the effects that changes in trade costs have on dynamic decisions and, therefore, on structural transformation. Consider the case of an infinite horizon economy, \( t = 0, 1, \ldots \), in which all economic agents have perfect foresight. In terms
Table 2.2: Value Added Shares: 1970 and 2007

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Baseline</th>
<th>Trade Costs 1970</th>
<th>Balanced Trade</th>
<th>CF 1</th>
<th>CF 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Agriculture</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$va^{2}_{1970}$</td>
<td>0.029</td>
<td>0.018</td>
<td>0.018</td>
<td>0.018</td>
<td>0.018</td>
<td>0.018</td>
</tr>
<tr>
<td>$va^{2}_{2007}$</td>
<td>0.018</td>
<td>0.013</td>
<td>0.009</td>
<td>0.016</td>
<td>0.012</td>
<td>0.011</td>
</tr>
<tr>
<td>diff $^{a}_{1970-2007}$</td>
<td>-0.010</td>
<td>-0.005</td>
<td>-0.009</td>
<td>-0.002</td>
<td>-0.006</td>
<td>-0.007</td>
</tr>
<tr>
<td><strong>Manufacturing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$va^{m}_{1970}$</td>
<td>0.256</td>
<td>0.258</td>
<td>0.258</td>
<td>0.257</td>
<td>0.257</td>
<td>0.258</td>
</tr>
<tr>
<td>$va^{m}_{2007}$</td>
<td>0.171</td>
<td>0.228</td>
<td>0.231</td>
<td>0.247</td>
<td>0.253</td>
<td>0.241</td>
</tr>
<tr>
<td>diff $^{m}_{1970-2007}$</td>
<td>-0.084</td>
<td>-0.031</td>
<td>-0.027</td>
<td>-0.010</td>
<td>-0.004</td>
<td>-0.017</td>
</tr>
<tr>
<td><strong>Services</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$va^{s}_{1970}$</td>
<td>0.715</td>
<td>0.724</td>
<td>0.724</td>
<td>0.725</td>
<td>0.725</td>
<td>0.724</td>
</tr>
<tr>
<td>$va^{s}_{2007}$</td>
<td>0.809</td>
<td>0.759</td>
<td>0.760</td>
<td>0.737</td>
<td>0.735</td>
<td>0.748</td>
</tr>
<tr>
<td>diff $^{s}_{1970-2007}$</td>
<td>0.094</td>
<td>0.036</td>
<td>0.036</td>
<td>0.012</td>
<td>0.010</td>
<td>0.024</td>
</tr>
</tbody>
</table>

of countries and sectors considered, the dynamic model is identical to the benchmark model.

The representative household in each country has access to frictionless international financial markets by means of buying and selling one period bonds. In terms of endowments, the representative household in country $i$ is endowed with $L_{i,t}$ units of homogeneous labor in every period $t$, and $K_{i,0}$ units of homogenous capital and $W_{i,0}$ units of wealth in terms of net foreign assets in period $t = 0$.

The production side of the economy in this extension is identical to the static benchmark. All dynamic decisions are incorporated into the model by means of households saving and investment decisions. Therefore, in this section I only focus on the household’s problem and the market clearing conditions, which are the pieces of the model that change relative to the benchmark static model.

**Households** The representative household in country $i \in \{US, ROW\}$ has lifetime utility given by

$$U_i = \sum_{t=0}^{\infty} \delta^t \phi_{i,t} u(C_{i,t}),$$
where
\[ C_{i,t} = \left( \sum_{j \in J} \left( \mu_{ij} \right)^{\frac{1}{\sigma}} \left( C_{j,i,t} - \bar{C}_{j,i} \right)^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\sigma-1}{\sigma}}, \]
and \( \mu_{ij} > 0, \sum_{j \in J} \mu_{ij} = 1. \) I consider an additional structural residual of the model given by \( \phi_{i,t}. \) As mentioned in Chapter 1 of this dissertation, this preference wedge is crucial in order to rationalize net exports as an equilibrium outcome of the model. Given wealth in international markets, \( W_{i,0}, \) and physical capital, \( K_{i,0}, \) for \( i \in \{US, ROW\} \) such that \( W_{US,0} + W_{ROW,0} = 0, \) the household in country \( i \) chooses sequences of sectoral consumption and investment levels, physical capital and one period bonds exchanged in international financial markets, \( \{ \{ C_{j,i,t} \}_{j \in J}, \{ I_{j,i,t} \}_{j \in J}, K_{i,t+1}, B_{i,t+1} \}_{t=0}^{\infty} \) to maximize lifetime utility subject to the sequence of budget constraints
\[
\sum_{j \in J} P_{j,i,t} C_{j,i,t} + \sum_{j \in J} P_{j,i,t} I_{j,i,t} + B_{i,t+1} = w_{i,t} L_{i,t} + r_{i,t} K_{i,t} + R_{i} B_{i,t}
\]
for every \( t, \) where \( R_{i} B_{i,0} = W_{i,0} \) for \( i \in \{US, ROW\}, \) and the law of motion for capital
\[
K_{i,t+1} = (1 - d_{i}) K_{i,t} + \chi_{i,t} I_{i,t}
\]
where \( d_{i} \) denotes the depreciation rate of physical capital, and aggregate investment is given by \( I_{i,t} = (I_{i,t}^m)^{\iota_{i}} (I_{i,t}^m)_{km} (I_{i,t}^s)^{\iota_{i}} \), where \( \iota_{i} > 0 \) and \( \sum_{j \in J} I_{j,i,t} = 1. \) For the case of investment I consider an additional structural residual, \( \chi_{i,t}, \) which can be recovered using observed data on investment expenditure.

**Market Clearing Conditions** In terms of the market clearing conditions of the model some adjustments are needed. First, the market clearing condition for nontradable final sectoral goods change in order to incorporate the resources used for
investment:
\[ C_{i,t} + I_{i,t}^j + \sum_{n \in \mathcal{N}} D_{n,j}^{i,t} = Q_{i,t}^j \text{ for } j \in \mathcal{J}. \]

In addition, the country-specific resource constraints must now be such that intertemporal trade decisions are consistent with intratemporal trade,
\[ B_{i,t+1} - R_t B_{i,t} = \sum_{j \in \mathcal{J}} \left( Y_{i,t}^j - E_{i,t}^j \right), \]
and international bond markets must clear, \( B_{US,t+1} + B_{R,t+1} = 0 \) for all \( t \).

**Household’s Optimal Decision and Structural Transformation** The household’s problem now deliver two additional optimality conditions that must hold in an equilibrium. First, the Euler equation for international bonds,
\[ u'(C_{i,t}) = \delta \hat{\phi}_{i,t+1} \frac{R_{i,t+1}}{P_{i,t+1}^C/P_{i,t}^C} u'(C_{i,t+1}), \quad (2.4) \]
where \( P_{i,t}^C \) denotes the ideal price index for final aggregate consumption, and second, the Euler equation corresponding to investment decisions,
\[ \frac{R_{i,t+1}}{P_{i,t+1}^C/P_{i,t}^C} = \frac{\chi_{i,t}}{\chi_{i,t+1}} \left( \frac{r_{i,t+1}}{P_{i,t+1}} + (1 - d_i) \right), \quad (2.5) \]
where \( P_{i,t}^I \) denotes the ideal price index for final aggregate investment.

If we focus on the case of the effects of trade costs on structural transformation, we see that in addition to the static effects identified previously, trade costs affect dynamic decisions in two ways. First, high trade costs dampen trade imbalances through (2.4) as it was explained in Chapter 1. Second, high trade costs reduce investment by decreasing the return on capital adjusted by the price of investment, \( r_{i,t+1}/P_{i,t+1}^I \). The last channel has the potential to affect the evolution of income across countries in such a way that structural change would be affected.
Solving the Model Numerically  The main challenge in solving this model numerically comes from the fact that its steady state is not uniquely determined by parameters of the model. The steady state depends on the steady state distribution of wealth across countries. In other words, the model has a continuum of steady states indexed by the steady state distribution of wealth. However, the model has a unique equilibrium corresponding to a given initial distribution of wealth, \( \{W_{US,0}, W_{ROW,0}\} \). Therefore, in order to solve this model numerically we need to solve for the steady-state of the model and the entire transition path simultaneously in order to ensure that the steady-state and transition path are consistent with the initial distribution of wealth in the data.

Existing literature has relied on a planner’s problem to circumvent this issue. By choosing particular Pareto weights, the steady state distribution of net foreign assets is uniquely pinned down by these weights, and the steady state of the model is uniquely determined. However, one issue with this procedure arises once one wants to consider counterfactual equilibria. This happens due to the fact that there is no well defined criterion to choose Pareto weights in the counterfactual equilibrium. The literature has thus relied on making assumptions to pin down counterfactual Pareto weights (Fitzgerald (2012)), or on keeping Pareto weights unchanged (Eaton et al. (2016)). However, relying on any of these approaches implies that the actual mechanism that we are aiming to quantitatively assess is contaminated by any other mechanisms leading to the assumed Pareto weights. Therefore, any study aiming to provide a clear assessment of the effects of globalization on structural change will need to face this issue. I leave such an endeavor for future research.
2.6 Conclusions

This paper quantitatively assesses the relevance of declining trade costs and increasing trade deficits in shaping structural transformation in the U.S. over the period 1970-2007. The main result of the paper is that these two mechanisms substantially contributed to the decline of manufacturings share in value added. The decline in trade costs by itself only accounts for approximately one tenth of the decline in the manufacturing share delivered by the baseline calibration of the model. However, when taking into account both the decline in trade costs and the increase in trade deficits, these two forces can account for over 85 percent of the decline in the share of manufacturing in value added. In other words, the observed increase in gross and net trade flows between the U.S. and its trading partners has substantially contributed to shaping structural change in the U.S. over the last four decades.

These results underscore the importance of considering an open economy framework when studying sectoral reallocation of economic activity. In order to better understand or forecast the process of structural transformation in the U.S., and therefore its implications for workers in different sectors, we need to take into account the fact that the world economy is going through a continuous process of globalization. This paper takes a first step in this direction by proposing a static framework that is suitable for a quantitative assessment of structural change in an open economy. However, this framework also has important limitations. Specifically, the static nature of the model does not provide an explanation as to why changes in trade deficits might arise. This limitation calls for a dynamic quantitative model of international trade and structural transformation. This issue is addressed in last section of this chapter. Specifically, that section proposes a dynamic framework suitable for a full quantitative analysis of structural change and discusses the challenges posed in order to take such a model to the data. The ability to circumvent these challenges will allow us to better understand structural transformation in the U.S.
Chapter 3

Managing Capital Flows in the Presence of External Risks

3.1 Introduction

Large and volatile capital flows across countries carry risks. The global financial crisis of 2008 provided additional evidence of how risky these flows can be in the short run. This episode led to reemergence of the debate regarding the optimal use of different policy instruments, such as capital controls as an example, to reduce these risks. Furthermore, new theoretical contributions have established the grounds underlying the use of these instruments. However, even though the risks associated with higher volatility of capital flows—in part due to higher uncertainty in the world economy—have been identified as relevant, the recent theoretical literature has not yet studied the influence of uncertainty in external shocks in the design of optimal policy.

In this paper, we show how optimal policy should respond to external shocks by introducing them into a benchmark model of sudden stops that has been recently used to study the design of optimal macroprudential policy. The baseline theoretical

*I wish to thank Gianluca Benigno, Jesús Fernández-Villaverde and Enrique Mendoza for very useful comments at various stages of this project.
framework considers a small open economy that faces an external borrowing limit that depends on the value of a domestic non-tradable asset. We introduce external risk by means of shocks to the mean and variance of interest rates at which the small country borrows and lends, and we ask how a benevolent social planer would set an optimal policy in response to external risks. We show that the optimal policy is contingent not only on the level, but also on the volatility of external shocks, and that the macroprudential tax on debt introduced by the social planner is non-monotonic with respect to the level of the volatility of the external shocks. Our results shed light on the optimal use of macroprudential controls in a particularly relevant moment, since many emerging economies have shown recent concerns about the volatility in global markets, partly due to the uncertainty in the decisions of advanced economies regarding their countercyclical policies.

The use of policy tools to manage capital flows across countries has repeatedly been at the center of the debate in international macroeconomics. Until very recently, the benefits of liberalizing capital flows were considered greater than their intrinsic risks; thus, under this view, no management of capital flows should be called upon. However, the global financial crisis of 2008 and the associated transfers of capital across countries have generated a reemergence of the interest in the policy and academic agenda on the use of different policy tools to either prevent or minimize the costs associated with capital flows. A number of policy studies that call for a more active management of capital flows have been published since the crisis. These studies identify two sources of risk associated with capital flows across countries: (i) the actual size, and (ii) the volatility of these flows. In particular, the policy agenda has identified both sources of risk as posing significant policy challenges for all countries, but especially for emerging economies. Hence, a new policy paradigm has emerged that includes policies such as capital controls or other type of restrictions on capital

\footnote{See Ostry et al. (2011) and Dell’Ariccia et al. (2012) as analytical background for IMF (2012), the organization’s institutional view, and the policy proposals in IMF (2013), Chapter 4.}
flows as potential policy tools when it comes to prevent or minimize the ex post costs associated with the risks carried by capital flows.

Paralleling the changes in the policy agenda, a theoretical literature has emerged that provides theoretical grounds, in terms of a country’s welfare, for the implementation of many of the aforementioned policy recommendations[^2] A theoretical framework based on dynamic-stochastic general equilibrium models traditionally exploited to study the positive side of large and abrupt capital outflows in emerging economies—also known as the sudden stops phenomenon—has emerged as a benchmark framework to analyze the normative aspects of optimal policy in the face of the risks carried by capital flows[^3] In particular, the rationale for policy intervention in this framework emerges due to the presence of a pecuniary externality in the domestic agents’ borrowing decisions. The externality arises from the fact that the external borrowing constraint of the economy hinges on an endogenously determined price—either the price of an asset or the real exchange rate—that depends itself upon the aggregate level of external indebtedness.

Most theoretical frameworks used to study capital flows and optimal policy share two fundamental features. First, the shocks leading to a sudden stop—which is typically interpreted as a binding borrowing constraint—are either particular to a country’s fundamentals or zero-probability events exogenously imposed to the model. Hence, optimal policy in these studies does not depend on potentially relevant external shocks. Second, the possibility of a sudden stop relies most importantly on large capital inflows that increase the level of a country’s leverage, and this has come to be appreciated in the literature as the economy “overborrowing” relative to a social planner’s borrowing decisions. Thus, the normative implications of these variants of the model are most directly related to the policy challenge imposed by the size of

[^2]: Korinek (2011) and Korinek and Mendoza (2013) provide extensive surveys of this literature.
[^3]: Mendoza and Smith (2002) and Mendoza (2010) explore the positive aspects of this framework.
capital flows rather than their volatility, the latter being recently emphasized in the policy arena.

The literature on emerging economies has documented that these countries not only face the risk of a sudden stop due to weak fundamentals, but also significant risks associated to external shocks, which are independent of a country’s fundamentals. A strand of the literature has focused on the effects of international interest rates on these countries, and has by now clearly documented that there are significant effects of this type of shocks on real economic activity in emerging markets. By considering the shocks to interest rates at which these economies borrow as driven by external factors, these studies have identified that not only the first, but also the second moment of these shocks matter for emerging market business cycles. Moreover, recent studies have also shown empirically that these shocks have direct effects on capital flows across countries.

In addition, there exists an empirical association between sudden capital flow reversals and external interest rate volatility. Reyes-Heroles and Tenorio (2015) document the empirical patterns of interest rates and output faced by emerging markets around the beginning of sudden stops. Their main findings are that: (i) sudden stops are preceded by periods of below-normal interest rates, which rise when the sudden stop occurs, and revert to their normal levels in the following years; (ii) sudden stops are preceded by periods of slowly-increasing interest rate volatility, which spikes sharply both at the beginning and the end of the sudden stop; and (iii) sudden stops are preceded by economic expansions, which abruptly turn into output drops at the beginning of the episode, and are followed by slow recoveries. Given the associations between output, interest rates, and volatility with capital flow reversals in the data, it

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4See Mackowiak (2007) and Chang and Fernández (2013).
6See Ahmed and Zlate (2013).
becomes relevant to consider the role of these external factors in the design of optimal policy.\footnote{Furthermore, recent studies have shown that an environment of high volatility is most likely to continue for emerging economies throughout the unwinding of the countercyclical policies implemented in advanced economies, which stresses the importance of considering volatility shocks in policy (e.g., Aizenman et al. 2014; Eichengreen and Gupta 2014).}

For this reason, in this paper we introduce these types of shocks to the simplest benchmark theoretical framework that has been used to study the qualitative and quantitative features of optimal policy regarding capital flows, and we analyze the quantitative implications of these shocks. We do so by considering a variant of the small open economy model proposed by Jeanne and Korinek (2010) and Bianchi and Mendoza (2013). We extend their model by letting the interest rate at which the economy borrows follow a stochastic process with time-varying volatility. In our model, a small open economy is populated by a continuum of households, whose only source of income is the payoff of a risky asset. The asset’s shares cannot be traded across borders, but the households can lend or borrow from abroad in the form of non-contingent riskless bonds. Borrowing is subject to a collateral constraint, and the amount of collateral available depends on the value of the households’ holdings of the risky asset. The households also face a refinancing risk, because the international interest rate is stochastic, so they take this into account when making consumption and saving decisions. A sudden stop occurs when a long enough series of negative shocks drives the households to borrow up to the point where the borrowing constraint binds. This forces an abrupt deleveraging of the households, which reduces current consumption and causes a drop in asset prices, further reducing the value of collateral and tightening the borrowing constraint. By simulating the model, we show that the evolution of the interest rate during sudden stop events is similar to the empirical event windows that we document for a sample of emerging markets.

After setting up the model and discussing the main features of the competitive equilibrium, we compare the equilibrium allocation to that of a social planner that
internalizes the effects of borrowing on the price of the asset. Even though the planner internalizes the effects of borrowing on the asset price and on the borrowing constraint, he cannot choose a price directly and acts according to this price being consistent with equilibrium conditions. We solve the model numerically using global methods, and investigate the implications of the external shocks on optimal policy. The use of global methods is necessary in this type of models in order to fully characterize the nonlinearities that arise in the region where the collateral constraint binds.

Even though we consider a simple model that lacks some of the mechanisms that the literature has identified as relevant in order for external shocks to significantly affect business cycles, the simplicity of the model clarifies the central mechanisms at play that help us answer one key question in the paper: does greater external volatility call for higher taxes on capital flows? Since this model allows us to focus on the effects of the pecuniary externality, we disregard other mechanisms that imply a negative effect of higher volatility on the economy. Hence, our results can be interpreted as a lower bound on the responsiveness of the social planner’s optimal policy with respect to external shocks.

The main results of our numerical exercises can be summarized as follows: (i) the optimal policy is indeed contingent on the size of external shocks; (ii) even in this very simplified framework, optimal policy depends on the volatility of the external shock; and (iii) the level of capital flow taxation that decentralizes the planner’s allocation is non-monotone in the level of external volatility. This last result should be underscored, as common intuition tells us that higher volatility should lead to more stringent capital controls, as the probability of a binding collateral constraint increases. However, as we discuss in the paper, this intuition does not take into account the effects of interest rates on the pecuniary externality, which might have an offsetting effect on the planner’s decision.
3.1.1 Related literature

This paper is mainly related to a relatively recent strand of literature that explores optimal policy, in particular the use of capital controls, to mitigate the risks associated with capital flows across countries. The methodology we follow is most closely related to Jeanne and Korinek (2010). They focus on a simple framework in order to analyze the implications of the pecuniary externality that drives the amplification mechanism that opens up the possibility for second-best type of policies. This amplification mechanism was initially introduced to the positive study of sudden stops in Mendoza (2002), Mendoza and Smith (2002) Mendoza and Smith (2006) and Mendoza (2010). However, the events associated with the global financial crisis of 2008 have fostered the studies focusing on the normative aspects of these mechanisms. Within this framework, the literature has focused on two different aspects of optimal policy, either its “prudential” features, in the sense that policy is undertaken ex ante in order to reduce the probability of a crisis, or its ex post characteristics, once a crisis has occurred. Jeanne and Korinek (2010), Bianchi (2011), Bianchi and Mendoza (2011) and Bianchi and Mendoza (2013) focus on the former, while Benigno et al. (2011) and Benigno et al. (2013b) focus on the latter. Most recently, other studies like Jeanne and Korinek (2013) and Benigno et al. (2013a) have focused on the use of both, ex ante as well as ex post policies in order to mitigate the risks associated to capital flows.

Another strand of the literature has studied the effects of external shocks on emerging market business cycles. More precisely, most of these studies have looked at shocks to the interest rate at which emerging markets borrow as a potential source of variation in real economic activity. Uribe and Yue (2006) and Neumeyer and Perri (2005) study the effect of interest rate shocks on emerging markets business cycles. Fernández-Villaverde et al. (2011) show that not only the first, but also the second moment of the shocks to interest rates have implications on real economic activity in
emerging markets. This paper follows the methodology of these studies to introduce external shocks in our model of sudden stops. In a recent study, Carrière-Swallow and Cépedes (2013), have further emphasized that global uncertainty has important effects on real economic activity in emerging economies.

The rest of this paper is organized as follows. In Section 3.2 we introduce the theoretical model of a small open economy that is borrowing constrained, and that faces domestic and external risks. We describe the competitive equilibrium and discuss the presence of a pecuniary externality that motivates the intervention of a social planner to increase welfare in the economy. In Section 3.3 we present the results of our numerical exercises. We show that the dynamics of interest rates around episodes of sudden stop in the model are consistent with their empirical counterparts. Moreover, we explain how the optimal response of the planner is shaped by the possibility of future binding borrowing constraints and by the size of pecuniary externalities. In Section 3.4 we conclude with the main implications of our exercise for macroeconomic policy.

3.2 A model of endogenous sudden stops with external interest rate risk

3.2.1 Framework

Our framework is closely related to Jeanne and Korinek (2010) and Bianchi and Mendoza (2013). There is an open economy inhabited by a continuum of unit measure of identical households that have preferences for streams of a consumption good, $c_t$, given by:

$$E_0 = \sum_{t=0}^{\infty} \beta^t u(c_t),$$
where \( u \) is an increasing, concave, and differentiable function that satisfies the usual Inada conditions.

There is a Lucas tree that yields a stochastic flow of consumption goods of \( d_t = d \exp(z_t) \) per period. The flow of goods provided by the tree can be traded period by period with the rest of the world, but the stocks of the tree can only be held by domestic owners. A possible explanation is that this arrangement arises from drastic (unmodeled) asymmetries of information between domestic managers and international investors that impede foreigners to earn profit from holding stocks of the tree. We denote by \( q_t \) the market value of the tree at time \( t \), and by \( s_t \) the holdings of the asset chosen by the representative household.

Households have access to debt financing in international financial markets in order to smooth their consumption and fund their stock purchases. The bonds issued by households in international markets have a maturity of one period, and they pay an exogenous gross return of \( R_t = R \exp(r_t) \). We let the external interest rate have a stochastic transition, but debt contracts are locally risk free: a household knows at time \( t \) the interest rate that it must pay next period for its outstanding bonds, but it does not know the interest rate that it will face next period if it decides to refinance its stock of debt.

Motivated by the findings of [Reyes-Heroles and Tenorio (2015)], we allow for contemporaneous correlation and dynamic feedbacks between the exogenous output and interest rate processes. The random vector \((z_t, r_t)’\) has the following VAR specification:

$$
\begin{pmatrix}
z_t \\
r_t
\end{pmatrix}
= A_0 + A_1 \begin{pmatrix}
z_{t-1} \\
r_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
\varepsilon^z_t \\
\varepsilon^r_t
\end{pmatrix}.
$$

(3.1)

The draws of the shock vector \((\varepsilon^z_t, \varepsilon^r_t)’\) are independent across time, and they have a Gaussian distribution with zero mean and a covariance matrix that has itself a
stochastic evolution:

\[ \Sigma_t = \begin{pmatrix} (\sigma_z)^2 & \rho \cdot \sigma_z \cdot \sigma_r \\ \rho \cdot \sigma_z \cdot \sigma_r & (\sigma_r^2) \end{pmatrix} . \]

As in Reyes-Heroles and Tenorio (2015), we allow the external interest rate volatility to take on two values, \( \sigma_r^L, \sigma_r^H \), with \( \sigma_r^H > \sigma_r^L > 0 \). The switching between these regimes is governed by a first-order Markov process with transition matrix \( \Pi \).

Let us denote by \( b_t \) the face value of bonds that are held by the households at the beginning of period \( t \). Throughout the paper, we follow the convention that a positive \( b_t \) represents savings of the households overseas, whereas negative positions represent external household debt. The time \( t \) budget constraint faced by a household is:

\[ c_t + q_t s_{t+1} + \frac{b_{t+1}}{R_t} = (q_t + d_t) s_t + b_t. \] (3.2)

The key friction in this economy is that the amount of borrowing that households can undertake is limited by the value of their asset holdings. More specifically, the market value of debt issued by a representative household at time \( t \), \( -\frac{b_{t+1}}{R_t} \), is constrained to be less than or equal to the valuation of their holdings of stocks of the tree, \( q_t c_t s_{t+1} \), multiplied by a constant \( \kappa \) that determines how stringent the financial frictions are:

\[ -\frac{b_{t+1}}{R_t} \leq \kappa q_t c_t s_{t+1}. \] (3.3)

We are making explicit the fact that the price used to value asset holdings as collateral at time \( t \), \( q_t^c \), is not necessarily the same as the market price, \( q_t \). In Appendix A we provide a microeconomic foundation of the collateral constraint that is based in contractual imperfections, as is common in the literature of financial frictions (e.g., Kiyotaki and Moore, 1997; Bernanke et al., 1999). The main idea is that within each period, there is a time in which households can divert a fraction \((1 - \kappa)\) of the assets previously posted as collateral, sell them off at the prevailing price \( q_t^c \), and default
on their outstanding loans. After this, the foreign lender is entitled to the remaining
fraction $\kappa$ of collateral assets, which must be sold in the domestic market at the
prevailing price $q_c^t$. In the appendix, we show that the market price of the tree and
its resale value need not be the same, and we also derive the relationship that has to
hold in equilibrium between them.

3.2.2 Competitive equilibrium

A competitive equilibrium is a sequence of allocations $\{c_t, b_{t+1}, s_{t+1}\}_{t=0}^\infty$ for every
household and a prices of the tree $\{g_t, q_t^c\}_{t=0}^\infty$ (market and collateral valuations) such
that households optimize their utility, subject to the budget and borrowing con-
straints, and the market for stocks of the tree clears. Given the fact that all the
households are identical and they only face aggregate shocks, market clearing implies
that $s_t = 1$ in every period.

We rewrite the problem of the representative household in recursive form in order
to highlight the role of pecuniary externalities in the competitive equilibrium. The
aggregate states in the household’s problem are the aggregate level of savings $B$, and
the current realization of the stochastic shocks, which we denote $X \equiv (z, r, \sigma^r)$. The
individual states of a household are its holdings of bonds $b$, and stocks of the tree $s$.
We denote by $V(b, s, B, X)$ the value of the problem for a household with portfolio
$(b, s)$ when the aggregate states are $B$ and $X$. Households take as given a perceived
law of motion for aggregate bonds, $B' = B(B, X)$, in order to form expectations on
future prices. Then, the Bellman equation of the problem is:

$$V(b, s, B, X) = \max_{c, b', s'} u(c) + \beta E[V(b', s', B', X') | X]$$  (3.4)
subject to:
\[ c + Q(B, X)s' + \frac{b'}{R(X)} = [Q(B, X) + d(X)]s + b, \]
\[ -\frac{b'}{R(X)} \leq \kappa Q'(B, X)s', \]
\[ B' = B(B, X). \]

In the previous expression, \( Q(B, X) \) is the market value of the tree, and \( Q'(B, X) \) is the value of the asset when employed as collateral. These two prices are determined in equilibrium and depend on the aggregate states of the economy. In a recursive competitive equilibrium, it must be the case that \( B \) is consistent with optimal individual decision rules, and that \( Q \) and \( Q' \) ensure the clearing of the market for stocks of the tree, in the different trading cycles.

In Appendix A we show that the solution to the household’s problem satisfies the following Euler equations for bonds and stocks of the tree, respectively:

\[ u'(c(b, s, B, X)) - \mu(b, s, B, X) \]
\[ = R(X)\beta E\{u'(c(b', s', B(B, X), X'))|X\}, \]
\[ Q(B, X)u'(c(b, s, B, X)) \cdot \left(1 + \frac{\kappa\mu(b, s, B, X)}{u'(c(b, s, B, X))}\right)^{-1} \]
\[ = \beta E\{u'(c(b', s', B(B, X), X'))[Q(B(B, X), X') + d(X')]|X\}, \]

where \( \mu \geq 0 \) is the multiplier on the borrowing constraint. The left hand side of the Euler equation for bonds is the marginal cost of saving an additional unit of consumption good at time \( t \): the household loses utility \( u'(c_t) \) in the margin and, if the borrowing constraint is binding, an additional unit of saving relaxes the constraint, with a shadow value of \( \mu_t \), thus reducing the marginal cost of saving. The right hand side represents the gains obtained by the household next period: for the additional unit saved in the margin, the household gets \( R_t \) goods in the next period, which are
valued at the expected marginal utility $\mathbb{E}_t[u'(c_{t+1})]$, and discounted by the subjective discount factor $\beta$.

Similarly, the left hand side of the Euler equation for stocks shows the marginal cost faced by a household that is buying additional shares of the tree: for each stock, the household must pay a price of $q_t$, and it has a marginal utility loss of $q_t u'(c_t)$. The factor at the end of the left hand side is the wedge between the market price of stocks of the tree and their collateral value (see Appendix A). This wedge is non-zero only when the borrowing constraint is binding, which means that the household values the additional service that their asset holding brings by increasing its borrowing opportunities. In turn, the right hand side is the expected benefit received by the household, which is the resale value of the stock, $q_{t+1}$, and the dividend, $d_{t+1}$, as valued by the marginal utility of the household, $u'(c_{t+1})$, and discounted by $\beta$.

Alternative specifications of the household’s problem, such as Jeanne and Korinek (2010), assume that the household’s borrowing is constrained by the aggregate number of stocks in the economy, rather than the household’s individual holdings. This eliminates the effect of relaxing the borrowing constraint through an increase of the value of collateral in the Euler equation for stocks (i.e., the wedge between the market and collateral values of the tree). The authors claim that the quantitative results of this alternative formulation do not vary significantly with respect to the problem that we are solving.

In our framework, a sudden stop in external financing arises endogenously as a consequence of the households’ borrowing decisions. For high levels of leverage, if the borrowing constraint binds, the households are forced to have a fast reduction of debt, which is only possible through drastic declines in consumption. This causes falls in asset prices by increasing today’s marginal utility of consumption and discounting more heavily future cash flows. In turn, this reduces the value of collateral, which further tightens the borrowing constraint, and induces more deleveraging. The feedback
between asset price reductions, forced deleveraging, and consumption drops, follows
ad infinitum, generating a sudden reversal of the capital flows into the country.

Korinek and Mendoza (2013) highlight that when the external borrowing rate is
lower than the households’ discount factor, the households face a fundamental tradeoff
between impatience and insurance. They have an incentive to borrow from overseas
in order to consume in advance because interest rates are low. Nonetheless, for high
levels of borrowing, a sudden stop is more likely to happen and, given that it is
accompanied by a drastic decline in consumption, households have the incentive to
save out of the sudden stops region. In the next section, we illustrate the interaction
between these two motives using a numerical solution to our model.

3.2.3 Constrained efficient allocation

The fact that the aggregate level of debt determines asset prices, and this in turn
affects the borrowing capacity of the households, creates a pecuniary externality in
the economy. Individual households do not internalize the effect of their indebtedness
on the borrowing possibilities of the rest of the households, which results in Pareto
inefficient allocations. In this section, we study the problem of a social planner that
internalizes the effect of external indebtedness on the value of collateral and, hence,
on the borrowing capacity of the country. In particular, we consider a social planner
that can only choose the level of aggregate debt, subject to the economy’s borrowing
constraint. The planner cannot intervene directly in the trading of the asset that
takes place between households, so it tries to affect the equilibrium value of collateral
indirectly, by altering the economy’s borrowing decisions. We assume that the planner
cannot commit to future policies, and we solve for the constrained efficient allocation
that he would implement through time-consistent policies.

We follow Klein et al. (2005) in laying out the social planner’s problem and in
finding its time-consistent solution. In particular, we restrict attention to the case in
which policy rules only depend on the current state variables of the economy. This restriction implies that the policy rule of the planner is given by a simple function of the current states, \((B, X)\), that maps them into levels of aggregate bonds, \(B' = \Psi (B, X)\). In Appendix [A] we show that the problem that is being solved by the social planner can be stated as follows. Given an arbitrary future policy rule, \(\Psi (B, X)\), and the associated asset pricing function, \(Q(B, X)\), the social planner chooses \(c\) and \(B'\) that solves the following Bellman equation:

\[
W(B, X) = \max_{c, B'} \{u(c) + \beta \mathbb{E}[W(B', X') | X]\}
\]

subject to

\[
c + \frac{B'}{R(X)} = d(X) + B,
\]

\[
-\frac{B'}{R(X)} \leq \kappa Q(B, B', X),
\]

and the valuation of collateral is consistent with the household’s trading of the stocks of the tree:

\[
Q(B, B', X) = \beta \mathbb{E} \left[ \frac{u'(d(X') + B' - \frac{\Psi(B', X')}{R(X')}) [Q(B', X') + d(X')]}{u'(d(X) + B - \frac{B'}{R(X)})} \right] | X. \tag{3.5}
\]

In the appendix we prove that this is the relevant equilibrium pricing condition that the planner faces, given the microeconomic foundations that give rise to our collateral constraint.\(^8\)

Different authors have defined the planner’s problem in alternative ways. Bianchi and Mendoza (2011) use the competitive equilibrium price schedule \(Q(B, X)\) and do not allow it to satisfy the frictionless asset pricing condition of the households. They

\(^8\)Following the literature on optimal taxation under commitment, this condition has been referred to as an implementability constraint.
call this the problem of the “financial regulator”, and use it to argue that sudden stops are preceded by overborrowing, and there is a role for policy to improve upon competitive equilibrium allocations by reducing external borrowing. Our description of the planner’s problem is essentially the same as Bianchi and Mendoza (2013) because we also study time-consistent policies in which the planner affects the value of collateral only through the choice of indebtedness, but households update their valuation of the tree consistent with the planner’s policies.

The planner’s decision now internalizes the fact that increasing households’ savings affects equilibrium asset prices, which in turn alters the value of collateral in the borrowing constraint. In particular, the functions that solve the planner’s problem, \( c = \hat{C}(B, X) \) and \( B' = \Psi(B, X) \), must satisfy the following condition:

\[
\begin{align*}
  u'(\hat{C}(B, X)) - \mu(B, X) [1 + \kappa R(X) \xi(B, X)] \\
  = R(X) \beta \mathbb{E} [u'(C(B', X')) + \kappa \mu(B', X') \psi(B', X')] ,
\end{align*}
\]

where

\[
\psi(B, X) = \frac{\partial \bar{Q}(B, \Psi(B, X), X)}{\partial B}, \quad \xi(B, X) = \frac{\partial \bar{Q}(B, \Psi(B, X), X)}{\partial B'},
\]

and \( C(B, X) = B + d(X) - \frac{\Psi(B, X)}{R(X)} \).

In order to gain some intuition on how the planner internalizes the pecuniary externality, let us first focus on the case in which the collateral constraint is not binding in the current period, \( \mu(B, X) = 0 \). In this case, equation (3.6) becomes:

\[
\begin{align*}
  u'(\hat{C}(B, X)) = R(X) \beta \mathbb{E} [u'(C(B', X')) - \kappa \mu(B', X') \psi(B', X')] .
\end{align*}
\]

Klein et al. (2005) call this a “generalized Euler equation” because it is a functional equation of an unknown equilibrium object, in this case \( \bar{Q} \).
The planner’s intervention considers not only the possibility of a binding borrowing constraint and how tight it is through the $\mu(B', X')$ term, but also the risk associated with the size of the price externality through the $\kappa\psi(B', X')$ term. Conditional on today’s collateral constraint being non-binding, if the future price schedule were constant with respect to debt, the planner would not intervene, regardless of the possibility of the borrowing constraint being binding. Likewise, if there were an externality from borrowing but the planner did not expect the borrowing constraint to bind in the following period, he would not have a reason to distort the households’ borrowing decisions. In the appendix, we show that:

$$\psi(B, X) = -\frac{u''(C(B, X))}{u'(C(B, X))} Q(B, X),$$

(3.7)

which implies that the price externality depends on the level of asset prices and the coefficient of absolute risk aversion of the representative household.\(^{10}\)

Let us now consider the case in which the collateral constraint is binding in the current period. In this case, $\mu(B, X) > 0$, and equation (3.6) now includes an additional term related to a partial derivative of an unknown function, $\bar{Q}$. Notice that this is the relevant case in which a time inconsistency problem arises for the planner. The term $\xi(B, X) = \frac{\partial Q(B, B', X)}{\partial B'}$ shows that if the borrowing constraint is currently binding, the planner has an incentive to affect current asset prices by making future promises that would not be time consistent for a committed planner.\(^{11}\) In the problem of the planner, we assumed that an arbitrary future policy rule, $\Psi(B, X)$, and its implied asset pricing function, $Q(B, X)$, are taken as given. Hence, the current planner can only affect the pricing function by choosing $B'$ and then having the future planner make his decision based on $\Psi(B', X')$, rather than committing to $B'$ and $B''$.

\(^{10}\)The fact that $\psi(B, X)$ can be written in terms of unknown functions, rather than partial derivatives of unknown functions simplifies the analysis of the functional equation.

\(^{11}\)See Bianchi and Mendoza (2013) for a detailed explanation of the difference between a planner with and without commitment.
In Appendix A we provide an expression for $\xi(B, X)$ that shows explicitly how it relates to the planner taking future policy rules as given.

Given the characteristics of the social planner’s problem, it is straightforward to define a recursive constrained efficient allocation, conditional on arbitrary future planners’ policy rules. Our definition of a constrained efficient allocation further requires that the these policy rules be time consistent. In other words, we require that the policy that solves the strategic game being played by sequential planners is a fixed point, deriving in a Markov stationary policy rule. We provide further details and formal definitions of these concepts in the appendix.

### 3.3 The dynamics of sudden stops, optimal capital flow management, and external interest rates

#### 3.3.1 Parameterization and numerical solution

In order to illustrate the general equilibrium interaction of the borrowing constraint and the external shocks, we present a numerical solution of the model. We use a utility function from the constant relative risk aversion family:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

Table 3.1 presents the baseline parameterization of the model for an annual time frequency. The parameters for preferences are standard in the literature of small open economies. Our choice of the relative risk aversion, $\gamma = 2$, lies in the lower end of the values used for emerging economies in the open economy business cycle literature. Hence, the quantitative effects of volatility on real allocations and asset prices that we show are, in principle, conservative. The mean of the dividends process, $d$, is normalized to one, so we can easily interpret the measurements of consumption,
Table 3.1: Baseline parameterization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time discount</td>
<td>$\beta$</td>
<td>0.96</td>
</tr>
<tr>
<td>Relative risk aversion</td>
<td>$\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>Dividends</td>
<td>$d$</td>
<td>1</td>
</tr>
<tr>
<td>Collateral constraint</td>
<td>$\kappa$</td>
<td>0.04</td>
</tr>
</tbody>
</table>

savings, and asset prices relative to the mean annual income. The parameter of the collateral constraint, $\kappa = 0.04$, is chosen to match the ratio of foreign liabilities to GDP observed in a sample of emerging markets in the 1990-2011 period, which averaged 66.7%.$^{12}$ In the model, the ergodic mean of the debt-to-output ratio is 65.6%.

We estimate the parameters that rule the regime-switching VAR given by (3.1) for a group of emerging markets using the maximum likelihood approach of Reyes-Heroles and Tenorio (2015), with the data corresponding to Sample 1. The only difference with respect to our estimations in the referred paper is that here we use annual data, which better corresponds to the timing of our model. Quarterly GDP figures were annualized and then log-linearly detrended, and monthly interest rate data was averaged arithmetically. The estimated process is:

$$
\begin{pmatrix}
z_t \\
r_t
\end{pmatrix} = 
\begin{pmatrix}
0.0052 \\
0.0025
\end{pmatrix} + 
\begin{pmatrix}
0.6079 & -0.1321 \\
0.1289 & 0.8261
\end{pmatrix}
\begin{pmatrix}
z_{t-1} \\
r_{t-1}
\end{pmatrix} + 
\begin{pmatrix}
\epsilon^z_t \\
\epsilon^r_t
\end{pmatrix},
$$

(3.8)

and the covariance and transition matrices are composed of:

$$
\sigma^z = 0.0312, \quad \rho = -0.4048, \quad \pi_L = 0.9610,
$$

$$
\sigma^r_L = 0.0150, \quad \sigma^r_H = 0.0661, \quad \pi_H = 0.7468.
$$

$^{12}$This is calculated using data from the updated and extended External Wealth of Nations database of Lane and Milesi-Feretti (2007). The figure corresponds to the countries in Sample 1 described in Reyes-Heroles and Tenorio (2015). As a reference, an alternative calibration target could have been the average net foreign asset to GDP ratio, which amounts to 27.8% of GDP in our sample.
This features similar properties to the models estimated at a monthly frequency in Reyes-Heroles and Tenorio (2015). A further description of the business cycle implications is provided in that paper.

The ergodic mean of the output and the interest rate processes can be obtained by inverting the VAR as follows:

\[
E \begin{pmatrix} z_t \\ r_t \end{pmatrix} = (I - \hat{A}_1)^{-1}\hat{A}_0 = \begin{pmatrix} 0.0066 \\ 0.0196 \end{pmatrix},
\]

where \( \hat{A}_0 \) and \( \hat{A}_1 \) denote the estimated matrices in (3.8). The long-run average of the external interest rate is, thus, 1.96%, which is considerably below the households’ discount rate of \( (\beta^{-1} - 1) \approx 4\% \). This gives the households an incentive to borrow from the exterior in order to consume upfront.

The two regimes of the VAR have considerably different interest rate volatilities. In the low volatility regime, the standard deviation of interest rate shocks is small, \( \sigma_L = 1.50\% \), leading to a very low refinancing risk for bond holdings. In contrast, in the high volatility state, the standard deviation is 4.4 times higher, \( \sigma_H = 6.61\% \), which induces a large uncertainty in the future access to debt financing for the economy. The transition matrix between the two volatility states has a high persistence: the mean duration of low and high volatility episodes is 25.6 and 3.9 years, respectively. In the long run, the system spends 86.6% of the time in the low volatility state.

Our estimates for the variance of the external interest rate are consistent with the findings in the literature (e.g., Fernández-Villaverde et al. 2011). A limitation in our estimated process is that the shocks to the interest rate are symmetric: when volatility increases, it is equally likely for it to reach high deviations above or below the mean. We opt not to introduce asymmetries in our estimation for the sake of parsimony and simplicity. However, an estimation of the VAR model with additional degrees of freedom can be conducted to assess the quantitative relevance of asymmetric shocks.
We use a global solution method to characterize the recursive competitive equilibrium of the economy in a discretized version of the aggregate state space. We use a grid of 300 points for household savings, placing 80% of them around the region where the borrowing constraint binds, in order to better capture the nonlinearities of the model. We discretize the estimated VAR process using a two-dimensional variation to the Tauchen (1986) method that allows for different levels of variance of the shocks. We use a grid of 7 points for output shocks and 15 points for the interest rate, to better capture the effects of changing volatility of the latter variable. We truncate the grids in order to include 95% of the probability mass of shocks at the ergodic distribution, which was approximated by simulating the VAR for one million periods. To solve the system of rational expectations with occasionally binding constraints, we use an adaptation of the endogenous grid method of Carroll (2006). Appendix B describes in detail our algorithm and its numerical accuracy.

3.3.2 Description of the competitive equilibrium

Figure 3.1 depicts the numerical solution to the recursive competitive equilibrium. In the first panel, we show the representative household’s savings rule $B(B, X)$ as a function of the initial level of aggregate savings $B$. This decision rule is non-monotonic: for high levels of wealth, the savings rule is upward sloping, as expected. Given that the average interest rate is below the households’ discount factor, there is an incentive to increase the economy’s indebtedness, which is reflected on the fact that the savings rule lies below the 45 degree line. If the amount of debt reaches high enough levels, the borrowing constraint becomes binding. In this situation, the households must reduce their consumption in order to lower their stock of debt, as displayed in the second panel of the figure. This causes an increase in the marginal utility of contemporaneous consumption, which in turn induces a higher discount of future cash flows and a consequent drop in asset prices. This is shown in the third
Figure 3.1: Recursive competitive equilibrium: savings rule, consumption, asset prices, and multiplier on the borrowing constraint.

panel, which depicts the equilibrium asset prices, $Q(B, X)$, as a function of savings $B$. The sharp drop in the value of collateral forces a large deleveraging, as shown in the first panel, which feeds back into further consumption cuts and asset price falls, ad infinitum.

Episodes with binding borrowing constraints in our economy are accompanied with sharp declines in consumption, and since the households are inelastic in terms of intertemporal substitution, this fast deleveraging entails high utility losses. Therefore, households have a precautionary savings motive around the region in which borrowing constraints bind. The first panel of Figure 3.1 shows that the rate of indebtedness is lower around this region: the slope of the savings rule slowly decreases as the level of debt increases, before hitting the borrowing constraint. Hence, the
precautionary motive gains an increasing importance vis-à-vis the impatience motive in the households’ problem.

In Figure 3.2, we compare the savings decision rule for two different levels of the contemporaneous endowment. When there is a high level of output in the period (green dashed line), there tend to be greater savings from the households in the region where borrowing constraints do not bind. This occurs because households wish to smooth consumption across time, and since the process for the endowment is mean reverting, it is likely that in future periods there will be a lower output than in the present. However, since the endowment process is persistent, a high level of contemporaneous output predicts high levels of output in the near future, which in turn increases the value of the Lucas tree for the household. This causes an increase in the value of the collateral available in the economy, which raises the borrowing capacity of the households. Hence, the borrowing constraint starts binding at higher levels of debt, as the green dashed line shows.

Figure 3.3 compares the savings decision rules for two different levels of the external interest rate. Away from the borrowing constraint, the interest rate has the usual
impact on the economy: when the country faces a higher cost of borrowing (green
dashed line), it tends to increase its savings. However, in the vicinity of the borrowing
constraint, changes in the interest rate have an additional effect: an increase in the
interest rate causes a decline in the stochastic discount factor (in expectation), which
in turn reduces the value of the tree because its future flows are discounted more
heavily. Hence, when the country faces higher interest rates, the value of collateral is
lower, and the borrowing constraint starts binding for lower levels of debt.

In Figure 3.4 we compare the decision rules in the economy for two different
levels of the variance of the external interest rate, $\sigma^r$. We keep constant the level of
interest rates, but only compare the decision rules for the two levels of such variance.
The figure shows that the savings rule for the high volatility state lies slightly above
the one for low volatility. This was the expected outcome, because the households
should have a higher precautionary saving motive when they face a world with higher
uncertainty. Nonetheless, the magnitude of the difference between both decision
rules is considerably small, so these shocks do not modify the household’s saving
substantially.
In our model, the small effect of external volatility on equilibrium allocations arises from the absence of a complete production economy with capital accumulation. Fernández-Villaverde et al. (2011) provide, to our knowledge, the first solution of an open economy business cycle model that faces shocks to the volatility of the external interest rates. In their model, there is a significant response of the economy to these shocks due mainly to a capital accumulation motive. The mechanism that they highlight is the importance of external debt as a hedge for domestic income shocks: in the real business cycle framework, most of the risk in the households’ consumption arises from shocks to the domestic productivity level. The external locally-risk-free debt is a good hedge for domestic risk arising from productivity fluctuations. However, when the rollover risk of external debt increases, foreign bonds are less useful as a hedge, which implies that the economy must cut on their holdings of capital to reduce their exposure to domestic risk. As they do so, they cause a decrease in output in the subsequent periods, which reduces the wealth of the economy, and induces a reduction in consumption and foreign indebtedness.
Unlike Fernández-Villaverde et al. (2011), we do not incorporate capital accumulation and production in our framework, since we are interested in isolating the policy response of a planner that only cares about the incidence and consequences of binding borrowing constraints along the business cycle. By introducing the production mechanism, we would potentially be increasing the planner’s incentives to engage in ex ante and ex post interventions to reduce the incidence of crises and the size of their effects. In this sense, our exercise is conservative when it comes to the reasons that a social planner would have to prevent the occurrence of binding borrowing constraint episodes.

We now turn to describe the nonlinear dynamics of the economy. In Figure 3.5 we show our simulated impulse-responses around the steady state where the economy would remain if the level of output from the tree remained permanently constant at 2 standard deviations below its mean, the interest rate remained at 0.6%, and the variance of the interest rate remained permanently at 6.6%, i.e., in the high volatility regime. We then give a ±5.2% shock to the interest rate for one period, and bring the interest rate to 0.6% thereafter. First, we explain the effects of the interest rate decrease on the rest of the economy (green dashed line). The immediate effect is an incentive for the households to consume in advance. Therefore, they increase their consumption by 4.8% on the first period, without significant changes in the net savings of the economy. Asset prices show a 10.6% increase in the first period because the households are discounting future cash flows less, but they revert close to their long-run level in the following period.

In contrast, the economy responds very differently to an increase in the interest rate of the same magnitude. The immediate effect of the shock is a decline in asset prices, as shown in the last panel of the figure (blue solid line). The decline in the value of collateral causes the borrowing constraint to bind for a period, which forces a reduction in consumption in order to cut off the level of debt. As mentioned before,
the feedback between deleveraging and the decline of asset prices amplifies the initial shock: consumption initially falls by 12.6%, and asset prices drop by 23.9%. This carries a sharp reduction in foreign debt: it goes from 70.9% of average output to 62.1% in just one period. In addition, as the graphs show, the sudden deleveraging has long-lasting effects: given that there is a lower level of debt, asset prices remain high because there is a low probability of hitting the borrowing constraint again in the near future. Moreover, since the country has accumulated more savings, the household increases its consumption in the subsequent periods because it remains relatively impatient with respect to the rest of the world, until the stock of debt converges back to its long-run level. This exercise exemplifies the nonlinear and asymmetric dynamics of the model that arise from the presence of an occasionally binding borrowing constraint.
We simulate the model for one hundred thousand periods to study the prevalence of binding borrowing constraints and their effects around these events. We find that in our baseline parameterization, a binding borrowing constraint is a rare event: it only takes place in 1.82% of the periods. Even in the periods preceding the actual occurrence of a binding constraint, the model assigns conditional probabilities to this event below 10% on average.

In Figure 3.6, we present event studies by averaging the equilibrium variables around the period in which the borrowing constraint binds. All the variables are divided by their average value in "normal times", i.e., in periods in which the borrowing constraint is non-binding. The only exception is the window for interest rate volatility, which shows the fraction of episodes in which the high volatility regime is prevailing. Each panel shows the normalized average of the variable from \( t - 3 \) to \( t + 3 \), where \( t \) is the moment in which the borrowing constraint binds. In the first panel, we see that binding constraints arise from periods in which the economy has a relatively large stock of debt: the average level of debt before sudden stop periods is almost 10% higher to the average debt in non-binding periods. In the panels of the
second row, we can see that binding borrowing constraints are typically accompanied by low levels of the endowment, $z$, and drastic increases in the interest rate, $r$.

To contrast our model with the empirical evidence, we follow the literature in associating a period in which a borrowing constraint binds in the model with the occurrence of a sudden stop in the data. From this perspective, the prevalence of sudden stops in the model is considerably lower than in the data. Under the definition of sudden stops adopted in Reyes-Heroles and Tenorio (2015), the prevalence of these episodes in the emerging markets studied lies between 14.6% and 15.21% of the periods (measured in months), depending on the sample of countries that is considered.

Nonetheless, the evolution of the modeled economy around sudden stops is consistent with the empirical evidence presented in Reyes-Heroles and Tenorio (2015) regarding the dynamics of the external interest rate. Both in the model and in the data, a sudden stop is associated with a sharp increase in the interest rate: the model predicts that sudden stops happen when the interest rate increases on average 1.5 percentage points with respect to the normal times mean, whereas in the data, the interest rate increases between 1 and 2 percentage points in the 12 months that follow the beginning of such episodes. In addition, the model predicts that sudden stops take place after periods of relatively low interest rate volatility, in the moment in which volatility switches to the high regime, allowing for large upward shocks in the level of the interest rate. Again, this pattern is consistent with the sudden rise in volatility in the year of the sudden stop that we observed in the data.

The fourth panel of Figure 3.6 shows that a binding borrowing constraint is typically preceded by a sequence of negative output shocks, and an abnormally large negative shock in the period in which the constraint binds, that brings the level of output almost 8% below its normal times level. This contrasts with the empirical evidence in two respects. First, sudden stops are typically preceded by economic expansions, of around 1%, in the sample studied by Reyes-Heroles and Tenorio (2015).
Second, the empirical output declines after the episode begins are relatively modest, of around 2% relative to its normal times level. In terms of consumption and asset prices, the dynamics of the model agree with the empirical patterns of balance of payment crises: these are usually accompanied with sharp declines in consumption and asset prices. However, the fall in consumption that arises in our model, of about 20% below the normal times level, is considerably higher than its empirical counterpart, of about 2 or 3% in the countries studied by Korinek and Mendoza (2013).

3.3.3 The constrained efficient allocation and optimal capital flow management

We use the same parameterization of the previous section to characterize quantitatively the solution to the planner’s problem. In this section, we follow Jeanne and Korinek (2010) in postulating that the following condition holds:

**Assumption** The parameters and stochastic processes of the economy are such that

\[ 1 + \kappa R(X)\xi(B, X) > 0. \]

This condition guarantees that there exists a unique level of future savings in the planner’s problem, \( B' \), for which the collateral constraints holds with equality. If this condition were not true, it could be the case that an increase in household debt relaxes the constraint by increasing the value of collateral. This is in principle a counterintuitive outcome, but it is possible to have a negative derivative of the \( \bar{Q} \) schedule of equation (3.5) with respect to \( B' \), due to the concavity of the utility function (see Appendix A for an expression of \( \xi(B, X) \) based on marginal utilities and equilibrium objects). Jeanne and Korinek (2010) prove that under this assumption,
the Euler equation for the planner’s problem (3.6) simplifies to:

\[ u'(C(B, X)) - \mu(B, X) = R(X)\beta\mathbb{E}[u'(C(B', X')) + \kappa\mu(B', X')\psi(B', X')] \]

For the remainder of this section, we describe the optimal decision rule of the planner, and the associated equilibrium outcomes, based on this version of the Euler equation.

Figure 3.7 compares the savings rules for the households in the competitive equilibrium and the solution to the planner’s problem. As [Bianchi and Mendoza (2011)] have previously noted, the savings rule in both problems are similar in most of the state space, but they differ considerably in what they call the “high externality region”, where the borrowing constraint has a high probability of binding, and the asset price schedule becomes steeper as a function of savings.

Even though the savings rules do not show large differences between the competitive equilibrium and the planner’s problem, there are indeed some differences in the dynamics of both problems. First, we find that the planner is able to reduce the frequency of sudden stops from 1.82% of the periods in the competitive equilibrium,
to 1.61% in the constrained efficient allocation. However, as we observe in Figure 3.8, the amount of leverage in the planner’s economy does not change considerably with respect to the competitive equilibrium. Here, we define leverage as the discount value of debt divided by the market value of the Lucas tree, $-b_{t+1}/R_t q_t$. The red line marks the level of leverage where the borrowing constraint binds, given by $\kappa = 0.04$ in our numerical example. Both histograms of leverage have a similar mean of around 0.028 and the same dispersion, of 0.0048.

Nevertheless, the planner’s actions do have an effect in the severity of the sudden stops that the economy faces. In Figure 3.9, we show event studies around the periods in which the borrowing constraint binds in the planner’s economy. The outcomes corresponding to the planner’s problem are depicted in green dashed lines. We observe that the consequences of a binding borrowing constraint are considerably milder in the planner’s allocation, compared to the laissez faire competitive equilibrium: consumption decreases by less, asset prices remain higher, and the deleveraging
is slower. Even though the average decline in the endowment is roughly the same in both economies, it takes a larger interest rate shock to hit a borrowing constraint in the constrained efficient economy. This is accompanied by a sudden increase in volatility, that enables the interest rate shock to reach high realizations.

**Decentralization**

We now explore how the planner responds to the exogenous shocks that the economy faces. To do so, we use the fact pointed out by [Jeanne and Korinek (2010)](JeanneKorinek2010) and [Bianchi and Mendoza (2011)](BianchiMendoza2011) that the constrained efficient allocation that solves the planner's problem can be decentralized with a state contingent “macroprudential” tax on debt. These authors show that the wedge on the households’ gross interest
rate that implements the allocation of the planner’s problem is:

\[ \tau(B, X) = \frac{E[\kappa \psi(B', X') \mu(B', X') | X]}{E[u'(c(B', X')) | X]}, \]

(3.9)

where \( B' = \Psi(B, X) \) is the optimal level of savings chosen by the planer when initial savings are \( B \), and shocks \( X \) are realized. The size of the planner’s intervention is, thus, determined by the expected marginal welfare gain of reducing households’ indebtedness: the value of reducing households’ debt by a unit is equal to the increase in the value of collateral, \( \kappa \psi(B', X') \), times the marginal value of relaxing the collateral constraint, \( \mu(B', X') \).

In Figure 3.10 we depict the optimal tax on debt, \( \tau(B, X) \), as a function of the initial savings of the country, \( B \), for two different levels of the endowment shock. Focus first on the green dashed line, corresponding to a high realization of the endowment. For high levels of household savings (to the right of the graph), the borrowing constraint is less likely to bind, which makes the planner’s intervention small or even null. Then, as debt starts accumulating, two things happen: (i) the borrowing constraint is more likely to bind, and it becomes more stringent, which derives in a higher
multiplier \( \mu(B',X') \), and (ii) the size of the pecuniary externality \( \psi(B',X') \) is higher because consumption is lower. Both of these effects call for a larger intervention by the planner, reflected in a higher macroprudential tax. In our numerical example, the tax rate amounts to a few percentage points over the gross interest rate, which considerably increases the after-tax interest rate paid by the households. In the figure, we also see that for higher levels of debt, the borrowing constraint binds and the households are forced to delever drastically by the price-debt mechanisms of the model. This brings the stock of debt away from the borrowing constraint for the immediate future. In this case, the tax on debt is zero because the economy is not borrowing-constrained in the upcoming period. Thus, this model has no space for ex post intervention; the planner’s actions to eliminate pecuniary externalities are only necessary before a borrowing constraint binds.

Figure 3.10 also shows that the macroprudential intervention is always non-negative. The planner taxes debt whenever he expects that reducing households’ borrowing has a positive effect on welfare through the internalization of the price effect of debt. From equation (3.7), we see that the pecuniary externality of debt is always non-negative because the utility function is strictly increasing and concave, and asset prices are non-negative throughout the state-space. On the other hand, the effect of relaxing the collateral constraint is non-negative, because it necessarily increases welfare when the constraint binds, and has a null effect otherwise. The tax on debt is thus given by the expected product of two non-negative random variables, so it must itself be non-negative.

We now analyze how the planner’s intervention responds to endowment shocks. The solid blue line in Figure 3.10 depicts the optimal tax on debt as a function of households’ savings for a low realization of the endowment shock, \( z \). In the region where borrowing is unconstrained, the tax on debt is typically higher for lower realizations of the endowment, which is explained by the fact that low levels of dividend
reduce the value of the Lucas tree, which in turn decreases the value of collateral available, and increases the probability of a binding borrowing constraint in the near future. In addition, the same reasoning explains why the planner’s intervention becomes null for lower levels of debt, compared with the intervention for high endowment realizations.

In Figure 3.11, we describe the dependence of the optimal macroprudential tax on interest rate shocks. The first thing we observe is that the macroprudential tax is almost uniformly lower for high levels of the interest rate, which is consistent with the findings of Jeanne and Korinek (2010). The authors make a comparative statics exercise on how the macroprudential tax changes with different values of the external interest rate. In their exercise, the interest rate is a fixed parameter in the planner’s problem, and they compare the steady state value of the tax when the value of the endowment is kept constant. The authors find that the steady state level of the macroprudential tax is decreasing with respect to the external interest rate: as the interest rate increases, the planner has a lower need to reduce households’ borrowing because they do so themselves as a response of a higher cost of credit. Our analysis, in
Figure 3.12: Tax on debt as a function of savings: different variances of the interest rate

contrast, studies the response of taxes to interest rate shocks off the steady state. As Figure 3.11 shows, for some levels of debt, the tax that results after a high interest rate shock is actually larger than the one corresponding to a low interest rate shock, which is explained by the fact that higher interest rates depress asset prices and reduce the value of collateral, which increases the probability of a borrowing constraint binding and calls for a larger intervention.

Next, we study whether an increase in the volatility of the external interest rate calls for a larger intervention of the social planner. Figure 3.12 depicts the schedule of tax on debt as a function of household savings, for the two different regimes of interest rate variability. We draw two main conclusions from the effect of interest rate volatility on the planner’s problem. First, the planner does indeed have a volatility-contingent optimal policy. This contrasts with the result that the savings rules in the competitive equilibrium do not differ considerably between high and low volatility states (see Figure 3.4). In the constrained efficient allocation, the planner’s policy is affected by the volatility of interest rates because, as the variability increases, the
Figure 3.13: Histograms of tax on debt conditional on variance states

- Low volatility of interest rates (mean = 0.0196, s.d. = 0.0179)
- High volatility of interest rates (mean = 0.0171, s.d. = 0.0166)

The economy is more likely to hit states in which the borrowing constraint binds, which usually calls for larger intervention of the social planner.

Our second conclusion is that the size of the optimal planner’s intervention is non-monotonic with respect to the volatility of the interest rate. Figure 3.12 shows that for certain levels of savings, the planner intervenes more when the volatility is high, but in other levels of savings the planner has a smaller intervention. This follows from the fact that the planner is weighing two criteria while choosing the optimal tax on debt: the incidence of sudden stops, and the size of the pecuniary externality. In the following section we show that the interaction between these two factors shapes the response of the planner to volatility shocks.

We now study whether the non-monotonic effect of volatility on taxes is also present in the simulated economy. First, we find that the share of states in which the planner chooses a zero tax on debt is larger when there is high variance than low variance: the planner sets a tax of zero in 59.6% of the periods of high volatility,
versus 55.3% of low volatility periods. This is due, partly, to the fact that the economy is more likely to be hit by very low interest rates when the variance is high, and in those states the planner is unlikely to intervene. In Figure 3.13, we look at the ergodic distribution of the tax on debt, conditioning on low and high volatility states, and ignoring the periods of zero intervention. We see that the positive interventions in the low volatility state have an average of 1.96%, which is larger than the average positive intervention in high volatility periods, of 1.71%. Moreover, the highest interventions in our simulations reach 10.7%, and take place only in the low volatility state. In contrast, the highest intervention in the high volatility state is 8.92%.

Finally, we go back to Figure 3.9, and we observe in the last panel the evolution of the macroprudential tax around the occurrence of a sudden stop. We find that prior to hitting the borrowing constraint, the planner charges on average a tax on debt of around 3.5%, which significantly raises borrowing costs for households, because the average interest rate they face is just 1.96%. Nonetheless, as we previously discussed, the planner does not engage in ex post macroprudential policies: the tax on debt when the borrowing constraint binds is close to zero, given the fact that there is a fast deleveraging taking place that makes it unlikely for a subsequent period to observe a binding borrowing constraint. Therefore, there is no motive for the planner to intervene once the borrowing constraint is already binding.

**Decomposition of the optimal policy**

In this section, we further study the planner’s response to the different shocks in the modeled economy. In preliminary numerical exercises, we have found that most of the response of the macroprudential tax to exogenous shocks comes from the numerator of (3.9), because the denominator remains fairly constant across different states, due to the planner’s tendency to smooth households’ consumption. Hence, the natural way to proceed is to decompose the numerator of the macroprudential tax in a product
term and a covariance term:

\[ E[\kappa \psi(B', X')\mu(B', X')] = E[\kappa \psi(B', X')] : E[\mu(B', X')] + \text{Cov}(\kappa \psi(B', X'), \mu(B', X')) \]

where all the moments are conditional on the contemporaneous vector of shocks, \( X \). Thus, the planner’s intervention is higher either: (i) when he expects a higher prevalence of binding borrowing constraints and a higher stringency when they bind, through the expectation of the \( \mu(B', X') \) term; (ii) when he expects a high degree of pecuniary externalities taking place in the following period, through the expectation of \( \kappa \psi(B', X') \) term; or (iii) when he expects these two factors to have a large covariance in the following period.

This last effect is less intuitive, but it has the following rationale: the larger the conditional covariance between the pecuniary externality, \( \kappa \psi(B', X') \), and the shadow valuation of the borrowing constraint, \( \mu(B', X') \), the larger the planner’s efforts will be to reduce the households’ borrowing by increasing the tax on debt. This in an effective measure to increase household welfare: if the size of the pecuniary externality were uncorrelated with the shadow valuation of the borrowing constraint, then the planner would not have much ability to improve the households’ utility by reducing their borrowing, so his optimal intervention would be small. As the covariance between both criteria increases, the expected welfare effect of the planner’s intervention is higher: inducing a reduction in households’ borrowing diminishes the pecuniary externality more in the states in which the borrowing constraint is tighter. Thus, the planner finds it optimal to intervene more when these two criteria are correlated.

Figure 3.14 shows the decomposition of the numerator of \( \tau(B, X) \), for two different levels of the endowment shock. The fact that the planner intervenes more after low endowment realizations is driven by the fact that the borrowing constraint is
expected to be more stringent in the following period, because current reductions of the endowment cause a decline in asset prices due to the persistence of the shock. The externality effect, $\psi$, has the opposite direction, since the planner is expecting lower externalities when the realization of the endowment is lower. The covariance effect is negative in this case, but it does not respond significantly to different endowment realizations. Thus, the effect of binding borrowing constraints, $\mu$, is the one driving the increase in macroprudential intervention after low endowment realizations.

In Figure 3.15 we perform a similar exercise, decomposing the planner’s tax on debt for two different levels of the interest rate shock. We wish to explain why the planner’s intervention is lower when interest rates increase. From the figure, we learn that both the stringency of future borrowing constraints, and future externalities are expected to be lower when the interest rate rises. On one hand, the increase in interest
rates decreases asset prices, which directly lowers the derivative $\psi$, as can be seen in expression (3.7). On the other hand, the reduction of interest rates relaxes the left hand side of the borrowing constraint, which reduces its multiplier. The covariance between these two effects is higher (or less negative) when the interest rate takes on high levels. However, over most of the points in the savings grid, the first two effects dominate the planner’s decision, so there is a higher intervention when interest rates are low.

Finally, in Figure 3.16, we present the decomposition for two different levels of interest rate volatility. The effects are less clear here: we can only observe a slightly higher expectation of the externality $\psi$ when the high volatility regime prevails. This could be associated to a possible concavity of $\psi$ with respect to interest rates, that would lead to a Jensen-inequality type of effect (i.e., if the variance of the interest
rate increases, then the expectation of a concave function of the interest rate would fall). However, the stringency of the borrowing constraint, and the covariance term show a non-monotonic response with respect to volatility, that cause the non-uniform response of the macroprudential tax with respect to this shock.

The decomposition of the optimal tax on debt shows that the relations between external shocks and the incentives on the planner’s problem are complex. The dynamics of the pecuniary externality and the multiplier on the borrowing constraint are determined in general equilibrium and in response to forward-looking factors, and the ultimate policy prescriptions depend upon the different forces acting in the economy. The lesson of this exercise is that simple policy prescriptions based on partial equilibrium rationales are insufficient to internalize the effect of overborrowing on asset prices and households’ borrowing capacity, and they might lead to counterpro-
ducted outcomes. A better understanding the equilibrium behavior of the factors in the planner’s solution would provide further insights about the operation of the economy and on the determination of appropriate policy prescriptions.

3.4 Conclusions

In recent years, the international capital flows entering small open economies have become larger in volume and more volatile. The uncertainty regarding policy actions in industrialized economies, as well as other underlying institutional and financial risks, have made the timing and direction of capital flows unpredictable. Policy makers around the world have grown concerned about the potential consequences of sudden reversals over their domestic financial sectors and ultimately on the real economic activity. This has motivated the surge of a myriad of unconventional policy tools to moderate the movement and regulate the composition of transborder capital flows. The international community has recognized that the risks carried by the volatility of international flows call for a more thorough analysis of the design and implementation of macroprudential capital account policies (see IMF 2012). This work makes a contribution in our understanding of the direction and intensity of macroprudential interventions that should be undertaken when a borrowing constrained economy faces external shocks.

In the paper, we extend the small open economy framework of Jeanne and Korinek (2010) and Bianchi and Mendoza (2011) to include shocks to the level and volatility of the interest rate faced by the economy, in the spirit of Fernández-Villaverde et al. (2011). We show that the dynamics of interest rates around episodes of sudden stop generated by the model have a similar behavior to the one observed empirically in a group of emerging markets. In the model, there is scope for a Pareto improving intervention that internalizes the effect of household borrowing in the value of
domestic assets, and thus in the collateral that can be used for external borrowing. The planner’s intervention dictates increasing the cost of households’ borrowing when it is likely that both a collateral constraint might be binding in the near future and pecuniary externalities are high. We show that the planner tends to increase his intervention as a response to low realizations of the endowment shock to offset the negative effect on the value of collateral and the tightening of the borrowing constraint that accompanies negative output realizations. The planner, on the other hand, tends to increase his intervention as a response to low interest rates shocks to offset the increase in the size of pecuniary externalities, despite the fact that there is a lower possibility of hitting a borrowing constraint. Moreover, we show that, keeping the level of interest rates constant, the planner has a non-monotonic response to interest rate volatility shocks. The degree of his intervention depends on how the changes in external volatility affect the expectations of the shadow value of collateral, pecuniary externalities, and the covariance between these two factors.

The lessons of this exercise for policy makers facing a rise in external risks are not clear-cut. We conclude that a mere increase in the volatility of external interest rates, like the one observed in recent months as the international financial markets adjust to expected policy changes in industrialized economies, does not necessarily call for a higher macroprudential intervention and the imposition of more stringent controls on the capital account. Policy makers should not only weigh the possibility of current account reversals to shape their interventions; they should also consider how external shocks affect the size of pecuniary externalities and the borrowing capacity of the country.
Appendices

A  Microeconomic foundations of the model

The timing of borrowing and asset trading

We denote the individual and aggregate household choice variables with lowercase and uppercase letters, respectively. We divide any given period in three sub periods: morning, afternoon, and night.

The period begins in the morning, with aggregate asset holdings \((B, S)\) carried from the previous period. The realization of the external shocks \(X = (z, r, \sigma^r)\) takes place at the beginning of the morning, and individual households receive the dividends from their holdings of the tree, \(s \cdot d \exp(z)\). Each household makes an optimal consumption and portfolio decision \((b', s', c)\) subject to its budget and borrowing constraints \((3.2)\) and \((3.3)\), taking the morning price \(Q(B, X)\) and interest rate \(R \exp(r)\) as given. In this sub period, there is perfect enforcement of debt contracts, so the household fully repays its outstanding debt \(b\) before consuming. At this point, the choice of \(c\) is just a plan; every household carries the physical goods it has designated to consume into the following subperiods. In addition, we assume that the household undertakes borrowing \(b'\) with just one foreign lender. This can be justified by introducing an infinitesimally small fixed cost of borrowing with each additional competitive lender.

In the afternoon, an individual household is holding a portfolio of assets \((b', s')\), and has \(c\) units of consumption good to eat later at night. At this point, the household has the possibility of diverting the stocks that it holds by selling them to the rest of the households in the economy and defaulting on his outstanding debt with the foreign lender. The defaulting household, however, cannot steal the entirety of the asset; it can only take a fraction \((1 - \kappa) \in [0, 1]\) away, and it leaves behind the remaining of its holdings. We denote by \(Q^c(B, X)\) the prevailing price for this transaction in the
afternoon market. Since we assume that households compete à la Bertrand for the
stocks of the tree, then the market price of the tree in the afternoon is as high as the
representative household prices the dividend payouts and resale value next period,
according to the following Euler equation:

\[ Q^c(B, X) = \beta E \left[ \frac{u'(C(B(X), X'))}{u'(C(B, X))} \right] Q(B(X), X') + d(X') \bigg| X \right], \]

where \( C \) and \( B \) denote the aggregate decision rules of the economy.

At night, the international lender finds out whether he has been defaulted or not.
If he has, he is entitled to obtain the fraction \( \kappa \) of the household’s stockholdings that
were not diverted. The lender, nevertheless, cannot directly receive dividends from
the tree, so he must necessarily sell it to the local households in order to obtain a
profit. Again, households compete à la Bertrand to buy the banker’s tree holdings,
so the value at which the transaction takes place is the prevailing market price,
\( Q^c(B, X) \). The lender then proceeds to loan the receipts of the transaction in the
international financial market, at the prevailing risk-free interest rate, \( R \exp(r) \).
Since the interest rate is positive, and the evolution of the stock prices does not in general
have a positive trend, the lender has incentives to immediately sell the stocks and
lend the revenue in the overnight market.\(^{13}\) After these transactions take place, the
non-defaulting households are able to consume what they had originally planned, \( c \).

In order to avoid losses from household default, lenders constrain the amount that
they lend, \(-b'/R \exp(r)\), to be less than or equal to the market value of the household’s
asset holdings that cannot be diverted, \( \kappa Q^c(B, X)s' \). This justifies the presence of
the borrowing constraint \( 3.3 \) in the problem of the representative household.

It only remains to explain the relation between the morning and the afternoon
prices, \( Q \) and \( Q^c \). Suppose that the borrowing constraint is binding, so \( \mu(B, X) > 0 \).

\(^{13}\)Otherwise, we can assume that the holdings of the tree depreciate overnight when held by the
lender, so he has incentives to immediately sell them.
For every additional stock of the tree that the household buys in the morning, it must sacrifice $Q(B, X)$ units of consumption, that are valued at the marginal utility $u'(B, X)$. On the other hand, by buying more stocks of the tree, the representative household relaxes the borrowing constraint, and obtains a marginal benefit of $\kappa \mu(B, X) Q^e(B, X)$, in the same sub period. Thus, the net marginal cost of saving in stocks of the tree in the morning is:

$$Q(B, X)u'(C(B, X)) - \kappa \mu(B, X) Q^e(B, X).$$

In the afternoon, the household can sell these stocks at the prevailing price, $Q^e(B, X)$, which is valued at the marginal utility of consumption $u'(C(B, X))$. Thus, for the household demand of stocks to be optimal, it must be the case that the marginal cost in the morning equates the marginal benefit in the afternoon:

$$Q(B, X)u'(C(B, X)) - \kappa \mu(B, X) Q^e(B, X) = Q^e(B, X)u'(C(B, X)).$$

From this expression, it is easy to see that whenever the borrowing constraint binds, the value of the tree in the morning will be higher than in the afternoon, because it helps the households relax the borrowing constraint and increase their debt. The decrease in prices from the morning to the afternoon is perfectly foreseen by every agent in the economy, but there are no opportunities of arbitrage because it is forbidden to hold the asset in short positions.

**The planner’s intervention** The social planner understands that the current aggregate level of debt, $B$, and the choice of future indebtedness $B'$ affect the value of collateral available in the economy, and thus constrain the borrowing possibilities of the households. In order to internalize this pecuniary externality, the planner can control the households’ borrowing decisions that take place in the morning, $B$. 

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Nonetheless, the planner cannot overcome the fact that households can divert their asset holdings in the afternoon and default on their outstanding debt. Moreover, the planner cannot intervene in the night stock market, in which the defaulted foreign lenders sell the remaining fractions of the diverted asset. Thus, the planner faces the same borrowing constraint as the households (3.3), and the price of the assets must be consistent with the household Euler equation of stocks:

\[ Q(B, X) = \beta E \left[ \frac{u'(C(B(X), X'))[Q(B(B, X), X') + d(X')]}{u'(C(B))} \right] X. \]

In this case, the market price of the stocks is the same throughout the day, because households do not internalize the effect of their savings in stocks on the borrowing possibilities for the planner’s problem.

**Competitive equilibrium**

Consider the recursive formulation of the household’s problem, expressed in program (3.4). The solution to the household’s problem is characterized by a pair of optimal decision rules for bonds and stocks, \( \hat{b}(b, s, B, X) \) and \( \hat{s}(b, s, B, X) \) respectively, that satisfy the following set of equations:

\[
\begin{align*}
  u'(c) &= \mu(b, s, B, X) + \beta R(X)E[u'(c') | X], \text{ and} \\
  Q(B, X)u'(c) &= \beta E[u'(c')(Q(B(B, X), X') + d(X')) | X] \\
  &+ Q_c(B, X)\mu(b, s, B, X)\kappa,
\end{align*}
\]

the budget constraint of the household in each period, and the collateral constraint

\[
-\frac{\hat{b}(b, s, B, X)}{R(X)} \leq \kappa Q_c(B, X) \hat{s}(b, s, B, X).
\]

We now proceed to define a recursive competitive equilibrium.
**Definition** A recursive competitive equilibrium of this economy consists of pricing functions $\hat{Q}(B,X)$ and $\hat{Q}^c(B,X)$, a perceived law of motion for aggregate bond holdings, $\hat{B}(B,X)$, and decision rules for households, $\hat{b}(b,s,B,X)$ and $\hat{s}(b,s,B,X)$, with associated value function $\hat{V}(b,s,B,X)$ such that:

1. Given $\hat{Q}(B,X)$, $\hat{Q}^c(B,X)$ and $\hat{B}(B,X)$, households’ decision rules, $\hat{b}(b,s,B,X)$ and $\hat{s}(b,s,B,X)$, and the associated value function $\hat{V}(b,s,B,X)$ solve the recursive problem of the household given by (3.4).

2. $\hat{B}(B,X)$ is consistent with the actual law of motion for bond holdings; $\hat{B}(B,X) = \hat{b}(B,1,B,X)$.

3. Markets must clear. In particular, $\hat{Q}(B,X)$ and $\hat{Q}^c(B,X)$ are such that $\hat{s}(1,B,X) = 1$.

Given the definition of the equilibrium, notice that the equilibrium level of bonds can be characterized by a simple function of the aggregate state variables, $B' = \hat{B}(B,X)$, which together with the resource constraint defines consumption as a function of aggregate state variables, $\hat{C}(B,X)$.

**Social planner’s recursive problem**

We consider a social planner that lacks commitment and that can only choose aggregate bond holdings for households, but is still subject to the borrowing constraint. Following [Klein et al. (2005)](), in order to solve for the time consistent policy, we focus on Markov stationary policy rules that only depend on the current state of the economy. In particular, they only depend on the aggregate state of the economy, $(B,X)$. We solve for the constrained efficient allocation following the three steps described in [Klein et al. (2005)](): (i) we first define a recursive competitive equilibrium for arbitrary policy rules; (ii) we then proceed to define a constrained-efficient allocation for arbitrary policy rules of future planners; and (iii) we define the constrained efficient
allocation for the case in which such policies are time consistent, i.e., we solve for
the fixed point of the game being played by successive planners. In this problem,
the social planner makes the borrowing decisions for the households, so he is the one
facing the collateral constraint. Households are allowed to trade stocks of the tree
freely, without government intervention.

Let us consider a planner who chooses an arbitrary sequence of state-contingent
lump-sum transfers, \( \{T_t\}_{t=0}^{\infty} \). Given this sequence of transfers, we can write down the
Bellman equation for the household’s problem as follows:

\[
V^A (s, T, X) = \max_{c, s'} \{ u(c) + \beta \mathbb{E} [V^A (s', T', X') | X] \}
\]

subject to

\[
c + Q^A (T, X) s' = [Q^A (T, X) + d(X)] s + T.
\]

When solving this problem, the household takes the pricing function, \( Q^A (T, X) \),
and the sequence of transfers as given. The solution to this problem is characterized
by a policy rule for stock holdings, \( s^A (s, T, X) \), such that Euler equation for stock
holdings holds,

\[
Q^A (T, X) = \frac{\beta \mathbb{E} [u'(c') (Q^A (T', X') + d(X')) | X]}{u'(c)},
\]

where

\[
c + Q^A (T, X) s^A (s, T, X) = [Q^A (T, X) + d(X)] s + T.
\]

Notice that the resource constraint of the economy implies that \( T = B - \frac{B'}{R(X)} \).
Hence, given \( B \), the planner actually chooses \( T \) by choosing \( B' \). Therefore, we can
rewrite the planner’s policy rule as one that dictates \( B' \) as a function of the current
aggregate state, \( (B, X) \). Call this policy rule \( \Psi (B, X) \), and define the following
functions:

\[ Q(B, X) \equiv Q^A \left( B - \frac{\Psi(B, X)}{R(X)}, X \right), \]
\[ s(s, B, X) \equiv s^A \left( s, B - \frac{\Psi(B, X)}{R(X)}, X \right), \text{ and} \]
\[ V(s, B, X) \equiv V^A \left( s, B - \frac{\Psi(B, X)}{R(X)}, X \right). \]

Hence, we can rewrite the optimality conditions for the household’s problem as follows:

\[ Q(B, X) = \beta \mathbb{E} \left[ u'(c') (Q(B', X') + d(X')) | X \right], \]

where

\[ c + Q(B, X) \hat{s}(s, T, X) = [Q(B, X) + d(X)] s + B - \frac{\Psi(B, X)}{R(X)}. \]

**Definition** A recursive competitive equilibrium for an arbitrary policy rule \( \Psi(B, X) \) consists of a pricing function, \( \hat{Q}(B, X) \), and decision rules for households, \( \hat{s}(s, B, X) \), with associated value function \( \hat{V}(s, B, X) \) such that:

1. Given \( \Psi(B, X) \) and \( \hat{Q}(B, X) \), households’ decision rules, \( \hat{s}(s, B, X) \), and the associated value function \( \hat{V}(s, B, X) \) solve the recursive problem of the household.

2. Markets clear: \( \hat{Q}(B, X) \) is such that \( \hat{s}(s, B, X) = 1 \) and the resource constraint holds, \( c + \frac{B'}{R(X)} = B + d(X) \), where \( B' = \Psi(B, X) \).
Therefore, in such an equilibrium, we have that the following set of equations must be satisfied:

\[
\hat{Q}(B, X) = \frac{\beta \mathbb{E} \left[ u' \left( B' + d(X) - \frac{B''}{R(X)} \right) \left[ \hat{Q}(B', X') + d(X') \right] \right] X}{u' \left( B + d(X) - \frac{B'}{R(X)} \right)},
\]

\[B' = \Psi(B, X) \text{ and } B'' = \Psi(\Psi(B, X), X').\]

Given that the planner we consider can only affect the allocation of bond holdings, but cannot directly intervene in the markets for stocks, the pricing condition for \(\hat{Q}(B, X)\) has to hold in a constrained efficient allocation, in particular, this condition defines the price at which lenders value collateral in the current period borrowing constraint. Taking into account this kind of implementability constraint for the planner, we can now define the problem to be solved by a planner that takes as given the policy functions of future planners. Given future policy rules, \(\Psi(B, X)\), associated pricing function \(\hat{Q}(B, X)\), and consumption rule \(C(B, X)\), the current planner chooses current consumption, \(c\), and future bond holdings to solve the following Bellman equation:

\[
W(B, X) = \max_{c, B'} \{ u(c) + \beta \mathbb{E} [W(B', X') | X] \}
\]

subject to

\[
c + \frac{B'}{R(X)} = d(X) + B,
\]

\[
-\frac{B'}{R(X)} \leq \kappa \hat{Q}(c, B', X),
\]

where

\[
\hat{Q}(c, B', X) = \frac{\beta \mathbb{E} \left[ u' \left( C(B', X') \right) \left( \hat{Q}(B', X') + d(X') \right) \right] X}{u'(c)},
\]

and \(C(B', X') = d(X') + B' - \frac{\Psi(B', X')}{R(X')}\).
Definition A constrained efficient allocation given a policy rule for future planners \( \Psi (B, X) \), with associated pricing function \( \hat{Q} (B, X) \) and consumption rule \( C (B, X) \), consists of an optimal policy rule, \( \hat{\Psi} (B, X) \), such that given functions \( \Psi (B, X) \), \( \hat{Q} (B, X) \) and \( C (B, X) \), the current policy rule \( B' = \hat{\Psi} (B, X) \) and associated value function, \( \hat{W} (B, X) \), solve the recursive problem of the current planner.

Let us define the following function,

\[
\bar{Q} (B, B', X) = \beta \mathbb{E} \left[ \frac{u' \left( B' + d(X') - \frac{\Psi (B', X')}{R(X')} \right) \left[ \hat{Q} (B', X') + d(X') \right]}{u' \left( d(X) + B - \frac{B'}{R(X)} \right)} \right] X.
\]

Then, \( \hat{\Psi} (B, X) \) has to be such that the generalized Euler equation holds:

\[
u'(\hat{C} (B, X)) - \hat{\mu} (B, X) [1 + \kappa R(X) \xi (B, X)] = R(X) \beta \mathbb{E} [u'(\hat{C} (B', X')) + \kappa \hat{\mu} (B', X') \psi (B', X') | X],
\]

where \( \psi (B, X) = \frac{\partial Q(B, \psi (B, X), X)}{\partial B} \), \( \xi (B, X) = \frac{\partial Q(B, \psi (B, X), X)}{\partial B'} \), and \( \hat{C} (B, X) = B + d(X) - \frac{\Psi (B, X)}{R(X)} \). The multiplier on the collateral constraint is given by:

\[
\hat{\mu} (B, X) = \max \left\{ 0, \frac{1}{1 + \kappa R(X) \xi (B, X)} \left[ u' \left( B + d(X) - \frac{\hat{\Psi} (B, X)}{R(X)} \right) \right] \right\} ,
\]

where \( \hat{\Psi} (B, X) = -R(X) \kappa \hat{Q} (B, \psi (B, X), X) \). After this characterization of the allocation, we can now define a recursive constrained efficient allocation as follows.

Definition The recursive constrained efficient allocation consists of functions \( \Psi (B, X) \), \( \hat{Q} (B, X) \), \( C (B, X) \), and \( \hat{\Psi} (B, X) \) with associated value function, \( \hat{W} (B, X) \), such that:
1. $\hat{Q}(B, X), \mathcal{C}(B, X), \hat{\Psi}(B, X)$, and the associated value function $\hat{W}(B, X)$, constitute a constrained efficient allocation, given a policy rule for future planners, $\Psi(B, X)$.

2. The planner’s plans are time-consistent: $\hat{\Psi}(B, X) = \Psi(B, X)$ and

$$\hat{Q}(B, \hat{\Psi}(B, X), X) = \hat{Q}(B, X).$$

**Non-binding current collateral constraint:** $\mu(B, X) = 0$ Let us consider first the case in which $\mu(B, X) = 0$. Given our definition of $\hat{Q}(B, B', X)$, notice that:

$$\frac{\partial \hat{Q}(B, B', X)}{\partial B} = \beta \mathbb{E} \left\{ -\frac{u'(\mathcal{C}(B', X))}{u'(c)} \left( \frac{\hat{Q}(B', X) + d(X')}{u'(c)} \right) \right\}$$

$$= -\frac{u''(c)}{u'(c)} \hat{Q}(B, B', X),$$

which implies that:

$$\psi(B, X) = -\frac{u''(\mathcal{C}(B, X))}{u'(\mathcal{C}(B, X))} \hat{Q}(B, X).$$

Therefore, when $\mu(B, X) = 0$, condition 10 becomes a regular Euler equation (with a $\mu$ wedge):

$$u' \left( \hat{\mathcal{C}}(B, X) \right) = R(X) \beta \mathbb{E} \left[ u'(\mathcal{C}(B', X')) - \kappa \hat{\mu}(B', X') \frac{u''(\mathcal{C}(B', X'))}{u'(\mathcal{C}(B', X'))} \hat{Q}(B', X') | X \right].$$

**Binding current collateral constraint:** $\mu(B, X) > 0$ Let us first notice that the current planner has to choose $B'$ subject to the collateral constraint

$$\frac{B'}{R(X)} + \kappa \hat{Q}(B, B', X) \geq 0.$$
If the left hand side of the previous inequality is strictly increasing in $B'$, then, given $B$, there is a unique $B'$ such that this equation holds with equality. Hence, when the current collateral constraint is binding, the optimal policy rule by the current planner must solve $\frac{\hat{\psi}(B, X)}{R(X)} + \kappa \tilde{Q}(B, \hat{\psi}(B, X), X) = 0$, and this policy rule is unique. Notice that that left hand side if strictly increasing if and only if

$$1 + \kappa R(X) \xi(B, X) > 0.$$ 

In equilibrium, $\xi(B, X) < 0$, therefore we expect this condition to hold whenever $\kappa$ is a small number. Given the definition of $\tilde{Q}(B, B', X)$, notice that

$$\frac{\partial \tilde{Q}(B, B', X)}{\partial B'} = \beta E_{\omega} \left[ \Omega(B, B', X) \right] + \frac{u''(c) \tilde{Q}(B, B', X)}{u'(c) R(X)} \tag{11}$$

where

$$\Omega(B, B', X) = u''(C(B', X')) \frac{\partial C(B', X')}{\partial B} \left[ Q(B', X') + d(X') \right] + u'(C(B', X')) \frac{\partial Q(B', X')}{\partial B}.$$ 

This last expression shows how the current planner takes into account how his decision affect future planners actions by changing $B'$.

B Numerical solution of the model

Competitive equilibrium

Let us denote by $B$ the aggregate equilibrium savings of the economy, and by $X = (z, r, \sigma^r)$ the realization of exogenous shocks. We wish to find functions $B(B, X)$,

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14Notice that when $\kappa$ is small enough, then the term $\kappa \mu(B', X') \hat{\psi}(B', X')$ also becomes very small and $\hat{\psi}(B, X)$ is also unique in the case in which $\mu(B, X) = 0$. 

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\( \mathcal{C}(B, X), \mathcal{Q}(B, X), \mathcal{Q}^e(B, X) \) and \( \mu(B, X) \) that satisfy:

\[
\frac{u'(\mathcal{C}(B, X))}{\beta} = \mathbb{E}\left[u'(\mathcal{C}(\mathcal{B}(B, X), X'))|X\right] + \mu(B, X), \tag{12}
\]

\[
\mathcal{C}(B, X) + \frac{\mathcal{B}(B, X)}{R(X)} = d(X) + B, \tag{13}
\]

\[
-\frac{\mathcal{B}(B, X)}{R(X)} \leq \kappa \mathcal{Q}(B, X), \tag{14}
\]

\[
\mathcal{Q}^e(B, X) = \beta \mathbb{E}\left[\frac{u'(\mathcal{C}(\mathcal{B}(B, X), X'))}{u'(\mathcal{C}(B, X))} \mathcal{Q}(\mathcal{B}(B, X), X') + d(X')|X\right], \tag{15}
\]

\[
\mathcal{Q}(B, X) = \left(1 + \frac{\kappa \mu(B, X)}{u'(\mathcal{C}(B, X))}\right) \mathcal{Q}^e(B, X). \tag{16}
\]

We extend the endogenous grid method (EGM) of [Carroll (2006)] to our framework where there is a borrowing constraint that binds occasionally:

1. For each \( \sigma^r \in \{\sigma^r_L, \sigma^r_H\} \equiv \mathcal{S} \), calculate the transition matrix for a discrete approximation to the VAR(1) process of \((z, r)\) over \(\mathcal{Z} \times \mathcal{R}\), with \(\mathcal{Z} = \{z_1, \ldots, z_{N_z}\}\) and \(\mathcal{R} = \{r_1, \ldots, r_{N_r}\}\).

2. Generate a grid \( \mathcal{B} = \{b_1, b_2, \ldots, b_N\} \), and an extended grid

\[
\mathcal{B} = \mathcal{B} \cup \{b_{N+1}, b_{N+2}, \ldots, b_{N+M}\},
\]

where \(b_{N+M}\) is chosen such that the resulting \( \max X \mathcal{B}(b_N, X) \leq b_{N+M} \) (to be verified in the end).

3. Guess functions \( \mathcal{C}_1(B, X), \mathcal{Q}_1(B, X) \) and \( \mathcal{Q}^e_1(B, X) \), for every \((B, X) \in \mathcal{B} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}\). The initial guess we use is:

\[
\mathcal{C}_1(B, X) = d(X) + B \left(1 - \frac{1}{R(X)}\right),
\]

\[
\mathcal{Q}_1(B, X) = \frac{\beta}{1 - \beta} d(X),
\]

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and \( Q_c(B, X) = Q_1(B, X) \), which corresponds to the assumption that \( B(B, X) = B \), \( z' = z \) and \( r' = r \) for all \((B, X)\).

4. Set \( C_0(B, X) = C_1(B, X) \), \( Q_0(B, X) = Q_1(B, X) \) and \( Q_c^0(B, X) = Q_c^1(B, X) \) for each \((B, X) \in \bar{B} \times Z \times R \times S\).

5. Assume that (14) does not bind. Use (12) and (13) to calculate:

\[
\hat{C}(B', X) = u'^{-1} (\beta R(X) \mathbb{E}[u'(C_0(B', X')) | X]) ,
\hat{B}(B', X) = \hat{C}(B', X) + \frac{B'}{R(X)} - d(X).
\]

Notice that \( \hat{B} \) is the level of contemporaneous savings that yield an optimal savings decision \( B' \) when the realization of shocks is \( X \) and the borrowing constraint does not bind.

6. For each \( X \), let us denote by \( \bar{B}(X) \) the endogenous grid of points generated by \( \hat{B}(B', X) \). For every \( X \), interpolate \( B' \) from \( \hat{B}(B', X) \) to \( \bar{B} \), and denote the resulting function \( \check{B}(B, X) \).

7. Calculate \( \check{B}(B, X) = \max\{\check{B}(B, X), -\kappa R(X)Q_0^c(B, X)\} \), and the corresponding consumption:

\[
\check{C}(B, X) = d(X) + B - \frac{\check{B}(B, X)}{R(X)}.
\]

8. Find \( B^*(B, X) = \min\{B \in \bar{B} : B \geq \check{B}(B, X)\} \). Using (12), (15) and (16), find:

\[
\check{\mu}(B, X) = u'(\check{C}(B, X)) - \beta R(X) \mathbb{E}[u'(C_0(B^*(B, X), X')) | X] ,
\check{Q}^c(B, X) = \beta \mathbb{E}\left[ \frac{u'(C_0(B^*(B, X), X')) [Q_0(B^*(B, X), X') + d(X')]}{u'(\check{C}(B, X)) - \kappa \check{\mu}(B, X)} \bigg| X \right] ,
\check{Q}(B, X) = \left( 1 + \frac{\kappa \check{\mu}(B, X)}{u'(\check{C}(B, X))} \right) \check{Q}^c(B, X).
\]
9. For every \((B, X) \in \bar{B} \times Z \times \mathcal{R} \times \mathcal{S}\), update:

\[
\begin{align*}
C_1(B, X) &= \alpha \tilde{C}(B, X) + (1 - \alpha) C_0(B, X), \\
Q_1(B, X) &= \alpha \tilde{Q}(B, X) + (1 - \alpha) Q_0(B, X), \\
Q^c_1(B, X) &= \alpha \tilde{Q}^c(B, X) + (1 - \alpha) Q^c_0(B, X).
\end{align*}
\]

for some \(\alpha \in (0, 1]\). For \(B \in \bar{B} \setminus \bar{B}\), set \(C_1(B, X) = C_1(b_N, X)\), \(Q_1(B, X) = Q_1(b_N, X)\) and \(Q^c_1(B, X) = Q^c_1(b_N, X)\).

10. Repeat steps 4-9 until convergence.

**Constrained efficient allocation**

The constrained efficient allocation satisfies:

\[
\begin{align*}
\left. u'(C(B, X)) - \mu(B, X) \left[1 + \kappa R(X) \xi(B, X)\right] \right|_{X} \\
&= R(X) \beta \mathbb{E} \left[ u'(C(B', X')) + \kappa \mu(B', X') \psi(B', X') \right|_{X}, \\
Q(B, X) &= \beta \mathbb{E} \left[ \frac{u'(C(B, X), X') \left[ Q(B(B, X), X') + d(X') \right]}{u'(C(B, X)) - \kappa \mu(B, X)} \right|_{X},
\end{align*}
\]

(17)

(18)

together with (13) and (14). Some steps of the EGM algorithm change with respect to the solution of the competitive equilibrium:

3. Guess functions \(C_1(B, X), Q_1(B, X)\) and \(\mu_1(B, X)\) for every \((B, X) \in \bar{B} \times Z \times \mathcal{R} \times \mathcal{S}\). The initial guess we use is: \(\mu_1(B, X) = 0\).

4. Set \(C_0(B, X) = C_1(B, X), Q_0(B, X) = Q_1(B, X)\) and \(\mu_0(B, X) = \mu_1(B, X)\) for each \((B, X) \in \bar{B} \times Z \times \mathcal{R} \times \mathcal{S}\).

Calculate:

\[
\psi(B, X) = -\frac{u''(C_0(B, X))}{u'(C_0(B, X))} Q_0(B, X).
\]
Use the numerical derivatives of $C_0$ and $Q_0$ with respect to $B$ to calculate $\xi(B, X)$ using equation (11) of Appendix A.

5. Assume that (14) does not bind. Use (17) and (13) to calculate:

$$\hat{C}(B', X) = u'(B', X') + \kappa \mu_0(B', X') \psi(B', X') | X|,$$
$$\hat{B}(B', X) = \hat{C}(B', X) + \frac{B'}{R(X)} - d(X).$$

8. Find $B^*(B, X) = \min\{B \in \bar{B} : B \geq \hat{B}(B, X)\}$. Using (17) and (18), find:

$$\tilde{\mu}(B, X) = \frac{1}{1 + \kappa R(X) \xi(B, X)} \left\{ u'(\hat{C}(B, X)) - \beta R(X) \mathbb{E}[u'(C_0(B^*(B, X), X')) + \kappa \mu_0(B^*(B, X), X') \psi(B^*(B, X), X') | X]\right\},$$
$$\tilde{Q}(B, X) = \beta \mathbb{E} \left[ \frac{u'(C_0(B^*(B, X), X'))) [Q_0(B^*(B, X), X') + d(X')] - \kappa \tilde{\mu}(B, X) \right] | X,$$

9. For every $(B, X) \in \bar{B} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$, update:

$$C_1(B, X) = \alpha \hat{C}(B, X) + (1 - \alpha) C_0(B, X),$$
$$Q_1(B, X) = \alpha \tilde{Q}(B, X) + (1 - \alpha) Q_0(B, X)$$
$$\mu_1(B, X) = \alpha \tilde{\mu}(B, X) + (1 - \alpha) \mu_0(B, X).$$

for some $\alpha \in (0, 1]$. For $B \in \bar{B} \setminus \bar{B}$, set $C_1(B, X) = C_1(b_N, X)$, $Q_1(B, X) = Q_1(b_N, X)$ and $\mu_1(B, X) = \mu_1(b_N, X)$.

10. Repeat steps 4-9 until convergence.
Accuracy of the approximation

We compute the Euler equation errors following Aruoba et al. (2006) to assess the accuracy of our solution. The histograms in Figure 17 show that the errors remain below $10^{-2}$ units of consumption in most of the state space. The maximum levels of the errors are reached around the region where the borrowing constraint binds. The errors are modestly higher in the solution to the constrained efficient allocation, but they remain within a reasonable level.

Figure 17: Euler equation errors: ergodic distributions
Bibliography


