Preface

This paper has been prepared for the Manpower Research Seminar with the purpose of stimulating discussion about the optimum course of the disaggregated side of the Systems Analysis of the Labor Market project. It is by no means a finished product, for the issues involved here are extremely complex. They are, however, the most important issues in the construction of a general, interrelated micro model of the labor sector.

Let me first describe what I think is the Ultimate Goal of the labor market project. It is approximately the suggestion of Harbison ("Critical Issues in American Manpower Policy and Practice," IRRA Proceedings, 1964) that a hypothetical group of manpower planners

Build a "model" of the labor force in 1965, setting forth manpower demand by major occupational categories, and making assumptions about the extent of automation and growth rates of the economy. For this model, construct targets for expansion and improvement of general education at all levels. Design a system of continuous education and training for all members of the working forces, in which employers, individuals, communities, as well as state and federal governments would share responsibility. Provide for maximum flexibility of the labor force through a variety of mobility-promoting measures. Create a pool of unskilled jobs with bright futures for that proportion of the labor force expected to be disadvantaged. Invent a scheme for matching men and jobs based upon gatherable information and utilization of both private and public efforts to facilitate the process of placement and transfer. Determine the costs of all this in terms of GNP and availability of strategic manpower, and work out a plan for raising the necessary funds. Then, after this is finished, construct another model or series of models using alternate assumptions and measures. It is just possible that some unifying ideas could emerge from such an exercise!
What would such a model involve? Essentially, there would be a number of
sets of endogenous variables (labor demands and supplies, unemployment and
participation rates, wage and income levels, rates of return to alternative
courses of training, and the like -- broken down by occupation, industry, region,
sex, age, and race) which would depend upon one another and upon a number of sets
of exogenous variables and behavioral parameters (the level and composition of
aggregate demand, labor demand elasticities, inter- and intra-factor elasticities,
wage adjustment and mobility coefficients, supplies of various types of schooling,
and the like -- again broken down by various classifications). Having estimated
the parameters of such a system as well as possible with existing data (and
filling in parameters on an a priori basis when estimation is not possible), the
behavior over time of the endogenous variables after a "shock" or under conditions
of gradual secular change may then be studied by simulation techniques. For
example, suppose we contemplate a government policy to subsidize junior college
education so that expenditure in this area grows X per cent faster than before
period 0. How will this influence the relative wage structure, the dispersion of
unemployment rates, and the aggregate unemployment rate? What is the time pattern
of response of each endogenous variable? And so forth.

Such a model would, we can all agree, be very pretty, but there are some
very severe problems which must be overcome before we will be even close to ready
to start constructing it. First, as the Ultimate Model was described in the last
paragraph, its size and complexity are many times the size and complexity of the
Brookings-SSRC endeavor. (And I gather that this model is proving to be too large
and too complex to be manageable.) One course of action would be to increase the
level of aggregation so that we treat only a few occupational classes, three
regions, and two or three industry groups that would certainly be necessary if
there is a large amount of inter-relation between endogenous variables so that
certain sectors could not be "broken off" from the main part of the system. Of course, such a procedure would destroy much of the detail of the model, and different models would have to be used to tackle different problems. A second possible approach, which in part circumvents the problem of size and complexity, is the treatment only of certain sets of disaggregated data. Some classifications, industry, region, occupation, etc., may not be important. For example, in my dissertation two chapters were spent testing a set of theories on inter-city data, and the principal conclusion with respect to inter-regional labor mobility is that people get where they "should" be according to static neoclassical theory very quickly in most instances. There is a qualification to this. In case the economy ever got to a situation in which there was sustained over-full employment, without price controls, the inter-regional (and inter-firm) adjustment mechanisms should (according to my PSALM Working Paper 3, "Non-Linearities in the Labor Market Adjustment Process") yield a rich variety of behavior, but we do not have observations of this behavior. The only exceptions to this were the relatively few areas in which the skill distribution of the labor force is very low (Providence, Charleston, W.V., and a few others). Thus, although the regional data provide an opportunity to test some simple propositions (and I plan to write up my results in the near future), the classification of the variables by region is fairly unnecessary. Also, on purely labor market considerations, the industry classification is also relatively unimportant. An exception to this may be due to the fact that certain industries -- for whatever the reasons -- pay higher prices for a given quality of labor than other industries, but this too may not be terribly important.

The "nitty-gritty" of micro labor market analysis, then, is the occupational classification, or, more specifically, the quality breakdown. The lags in the adjustment of variables associated with labor quality are quite long, and most
of the "manpower" policies concern the quality rather than the industrial or regional classifications. Demands for different grades of labor, of course, come from the firms in industries and regions, but responses on the supply side come from individuals arrayed by quality levels.

This suggests that the number of variables which must be treated in a disaggregated system is relatively small. The next task is to develop a theoretical structure within which to build the model, and the remainder of this paper represents some thinking about how to go about developing such a structure. Again, it is not really theory, only a preliminary to theorizing. In the seminar, I will be especially happy to hear others' thoughts on what variables and relationships are important in a general, disaggregative model of the labor sector.
Labor Quality in a General Equilibrium System:
Some Initial Notes

If, as has been argued, labor skill is the principal delineator of the boundaries of micro labor markets, many variables which are usually taken as exogenous must be explained within the system. We have a theory (due to Becker et al.) which explains (in part, at least) the determination of the aggregate supplies of individuals with different degrees of general training. The purpose of this essay, which is an initial venture into some very complicated but very important areas, is to discuss the following problems:

1. The determination of the distribution of the demand for different qualities of labor in individual labor markets,

2. The adjustment of the aggregate supplies of different labor quality levels among labor markets, and

3. The inter-relation between the determination of the market distribution of particular quality levels and the determination of the aggregate supplies of the different quality levels.

Very few of the notions expressed here are original, but I am unaware of any discussion which has attempted to integrate them.

A. Determination of the Optimum Labor Quality Levels in Particular Markets

The starting point of the analysis is a micro-economic production function in which output in a particular sector is assumed to be a function of "efficiency units" of labor as well as other factors. This has its counterpart in the aggregate production function literature. Instead of writing the function as \( X = F(L,K) \), where \( X \) is output, \( L \) is labor input, and \( K \) is capital input, the production function is sometimes written as \( X = aF(bL,cK) \). The parameter \( a \) is a reflection of the overall efficiency of production, and its growth over time
causes "neutral" technological change. The parameters \( b \) and \( c \) represent the efficiencies of labor and capital respectively, and their growth over time causes technological change which is "embodied" in these factors. Consider the production function for one of the many labor markets of the economy, the \( i \)th.

Ignoring the \( a \)'s and \( c \)'s, this may be represented as

\[
X_i = P^i(b_iL_i, K_i),
\]

where \( b_i \) is the efficiency of labor in the \( i \)th market. It need not be assumed (and is probably not true) that all production functions are identical. Now on an aggregate level it is often asserted that the efficiency of labor depends on its average "quality," and the same should be true on a micro level. Specifically, let \( Q_i \) represent the average quality level of the labor force of the \( i \)th labor market, where \( Q_i \) is related to innate ability, educational attainment, and the like. Then \( b_i \) may be considered a function of \( Q_i \), say

\[
b_i = g^i(Q_i),
\]

where \( \frac{dg^i}{dQ_i} > 0 \). The relation between efficiency and quality will not be the same for all markets, some markets having high elasticities of efficiency with respect to quality and other markets low elasticities. (Consider the case of academic "firms": The elasticity of efficiency with respect to quality at, say, M.I.T., is somewhat higher than at Southern North Dakota A. & M.) The typical firm must pay a price for employing higher quality workers, for one would expect that better workers would flock toward higher-paying employments. (An exception, one could argue, is the market for young academics, but this might be explained on inter-temporal grounds. That there should be a supply of young academics in the first place is what defies explanation, but ...) Let us express this hypothetical relation between wages and quality levels as

\[
w_i = h(Q_i),
\]
where $\frac{dh}{dQ_1} > 0$. However, we shall assume that the supply of labor of a particular quality to each firm and market is infinitely elastic over the relevant range at the appropriate wage rate.

The profit equation for the typical market (acting like a firm) is

$$
\pi_i = P_i F^i \left( g(Q_i), L_i, K_i \right) - h(Q_i)L_i - rK_i
$$

where $P_i$ is the product price and $r$ the rental price of capital. Holding capital constant, the first-order conditions for profit maximization require that

$$
P_i F^i \left( \frac{dg_i}{dQ_i} \right) = \left( \frac{dh}{dQ_i} \right).
$$

By solving for $F^i_E$ in the first expression and substituting this into the second expression, the conditions are seen to reduce to

$$
\left( \frac{dg_i}{dQ_i} \right) \left( Q_ig_i \right) = \left( \frac{dh}{dQ_i} \right) \left( Q_iN_i \right),
$$

which implies that the elasticity of labor efficiency with respect to labor quality, $E^i_1$, must equal the elasticity of the wage rate with respect to labor quality, $E_2$. The second-order conditions for profit maximization are satisfied if $E^i_1 - E_2$ decreases as $Q_1$ is increased.

Thus, the determinants of the optimum quality level prevailing in the $i^{th}$ market are the elasticities of efficiency and the wage rate with respect to the quality level. This scheme is shown geometrically in Figures 1-3 below. In Figure 1 two hypothetical relations between efficiency and quality are displayed. In market 1 efficiency increases sharply at first but then further increases in quality have no further influence on efficiency. This is reflected in the shape of the relation between elasticity and quality, $E^i_1$, in Figure 2. For market 2 it has been assumed that there is a linear relation between efficiency and quality,
so $E_1^2$ is always unity. The relation between $w_1$ and $Q_1$ will be investigated in more detail later, but the assumption of Figure 3 is that the wage rate for a zero quality level is positive (reflecting a social minimum wage rate) and then the wage rate rises at an increasing rate as $Q_1$ increases. At the point where the slope of $h(Q_1)$ is equal to $w_1/Q_1$ (represented by the tangency of $h$ and the dotted line in Figure 3), $E_2 = 1$. The equilibrium $b_1$'s, $Q_1$'s, and $w_1$'s are determined by the intersection of the curves in Figure 2. Notice that, given the shapes of $E^1$ and $E^2$, $b_1 > b_2$ although $w_2 > w_1$. This sort of result might be eliminated by restriction of the $g^i$'s to a certain class of shapes, but I think that this would be overly restrictive.

B. Adjustment of a Given Quality Distribution Between Markets

The assumption that the "right" quality is always matched with the wage rate is, among other things, not true. In a dynamic economy wage levels and market demand levels are constantly changing, and this requires that we provide a theory of the adjustment of the quality distribution of the total labor force among individual labor markets. Recall that we assumed that each firm in a given labor market can obtain all the labor of a particular quality it wants at the going wage rate for that quality level. Suppose that the firm is not paying wages high enough to retain its present employees or to attract new labor of the desired quality level. Then the quality level should fall as the firm must replace its previous labor force with one of lower quality. In terms of an adjustment mechanism, this is

$$Q_1 = \psi(h(w_1) - Q_1),$$

where $\psi(0) = 0$ and $\psi' > 0$. In other words, there is a "tendency for men with particular qualities to move towards those employers who can make the best use of their qualities." (Hicks, Theory of Wages, p. 35). At the same time, firms will
tend to increase (decrease) wage rates when quality levels are below (above) desired quality levels. The desired quality level, $Q_i^*$, is the individual firm's assessment of what is the profit maximizing quality level given the wage structure, and in terms of a differential equation the hypothesis is

$$\dot{\psi}_i = w(Q_i^* - Q_i),$$

where $w(0) = 0$ and $w > 0$. The system is stable if for any initial values of $Q_i$ and $\psi_i$ the variables go to their equilibrium values over time. This requires that

1. $$\frac{\partial \psi_i}{\partial Q_i} + \frac{\partial \dot{Q}_i}{\partial Q_i} < 0$$

and

2. $$\frac{\partial \dot{Q}_i}{\partial Q_i} \frac{\partial \psi_i}{\partial Q_i} - \frac{\partial \dot{Q}_i}{\partial Q_i} \frac{\partial \psi_i}{\partial Q_i} > 0$$

For all values of $Q_i$ and $\psi_i$. In terms of $\psi$ and $\dot{\psi}$, these conditions become

1. $$\dot{\psi} > 0$$

and

2. $$\ddot{\psi} h w > 0,$$

both of which are always true by hypothesis. Thus, $Q_i$ tends toward $Q_i^*$ and $\psi_i$ toward $h(Q_i^*)$.

C. The Relation Between Wages and Quality

So far little has been said on the distribution of labor qualities and labor demands for different qualities. We now discuss the former. First, suppose all individuals had the same endowments (were of the same quality) at some early age, say 14. They would then look out at the world and solve the following problem: "I want to maximize the present value of my future income stream, and I will train for S more years before entering the labor force. Given my discount
rate, the costs of each additional year of training, and the income I can expect to receive in each year after I stop training—for each year of training, what S will accomplish this? Since these 14-year olds all have (i) the same discount rates, (ii) perfect knowledge of demand and supply conditions in the future, (iii) awareness of the choices which other 14-year olds are making concerning amounts of training (and of the choices made by previous 14-year olds who have not completed their training periods), and (iv) possession of very facile minds, it should be that the present value of all alternative courses of action are always equal.

Assuming further that the cost, C, of each year of schooling is constant, regardless of its form, and that the wage rate (actually income) for each skill level is the same throughout each individual's working life, we have that the present value of the income stream associated with S years of schooling,

\[
V_s = -C \int_0^T e^{-rt} dt + w_s \int_s^T e^{-rt} dt,
\]

where T is the number of years the individual expects to remain in the labor force, must be independent of S. We are interested in the relation between S and w_s for a given C and r. To obtain this, first integrate the expression of the present value of S years of schooling to obtain

\[
V_s = \frac{C}{r} \left[ e^{-rS} - 1 \right] - \frac{w_s}{r} \left[ e^{-rT} - e^{-rS} \right].
\]

Then differentiate this totally with respect to S and w_s, that is

\[
dV_s = - (C + w_s) e^{-rS} \frac{1}{r} e^{-rT} - e^{-rS} dw_s.
\]

Thus, the derivative of w_s with respect to S is

\[
\frac{\partial w_s}{\partial S} = \frac{r (C + w_s) e^{-rS}}{[e^{-rS} - e^{-rT}]} e^{-rS},
\]
which, since \( S < T \), is positive. If \( r \) and/or \( T \) are sufficiently large so that income in \( T \) is worth practically nothing in the decision period, this reduces simply to \( r[C+\omega_s] \), and it can be shown that \( \frac{\partial^2 \omega_s}{\partial S^2} = r^2 [C+\omega_s] > 0 \). \( \omega_s \) does, in fact, depend upon \( C \) and \( r \) as well as \( S \), and both \( \frac{\partial \omega_s}{\partial C} \) and \( \frac{\partial \omega_s}{\partial r} \) are positive. Further, \( \frac{\partial^2 \omega_s}{\partial S \partial C} > 0 \). (This is the basis of Becker's discussion of "Wage Differentials and Secular Changes" in Human Capital, pp. 52-55. In this he argues that the secular decline of occupational wage differentials is due not to the increased supply of free education but rather to the decline in \( C \) due to long-run technological change. One would imagine that it is due to both factors -- plus some institutional reasons, and it would be interesting to try to measure the relative contribution of each.) The implication of this analysis is that the relation between wages and schooling is positively sloped and convex, as in Figure 4. There remains, then, the problem of relating quality to schooling.

![Figure 4](image-url)
Since we have assumed that all individuals are the same in the year of decision, quality differences arise only because of differences in additional years of schooling, and it can probably be safely assumed that the relation is roughly linear. In other terms, the quality level of the typical individual is

\[ Q = Q_0 + aS, \]

where \( Q_0 \) is the endowment of quality at the decision age, and the relation between wages and quality level would look approximately like that depicted in Figure 5 (which is similar to that shown in Figure 2).

![Figure 5](image)

Of course, the assumptions at the beginning of this section are not necessarily true. For example, Schultz does not agree with (ii) and (iii) ("The Rate of Return in Allocating Investment Resources to Education," *Journal of Human Resources*, Summer 1967, esp. pp. 303-4) (i), identical discount rates, (iv), intelligence of 14-year olds, and the assumption concerning identical initial endowments are also open to question. However, to the extent that there
is anything to the human capital approach, this relation should exist -- at least as an equilibrium condition. Later we shall discuss the problem of the adjustment of the quantities of individuals with different quality levels in response to "shocks" in the system and steady secular change in the distribution of demand levels.

D. Adjustment of Aggregate Demand-Supply For Quality Levels

We now see that there is an equilibrium relation between $w_i$ and $Q_i$ on the supply side. The function $h(Q_i)$, which was employed in Section A, may be considered a short run relation, given the quality distribution of the labor force, while the relation $H(Q_i)$ will be used to refer to the long run equilibrium relation which was derived in Section C and summarized in Figure 5. Suppose at some moment in time the relation between wages and quality levels, $h(Q_i)$, as compared to the equilibrium relation, is as depicted in Figure 6. In zones A and C the present
value of the corresponding amounts of schooling is greater than for zones B and D. Efficient adjustment would be such that the parameters of h tended to become equal to the parameters of H.

At the same time, there would be an adjustment in the demands for various skills. Some firms in zone B, for example, would note that the elasticity of the relation between quality and wages, $E_2$, was either too great (those in C with a steep wage-quality relation) or too low (those in the right-hand section of zone C) and would decrease (increase) their optimum quality levels accordingly. This, along with the tendency of new entrants to train for A and C rather than B and D, would cause the short run relation, h, to move toward the equilibrium relation, H.

Since the equilibrium wage-quality elasticity is determined in the human capital market, the primary determinant of the equilibrium demand-supply levels is the distribution of the efficiency elasticities with respect to quality, the $E_1^i$'s. These are exogenous to the system, and if there is a sudden increase in a substantial number of them (due, say, to a subtle technological discovery), many firms will revise their optimum quality levels upward, and this will be only partially offset by the increase in the supply elasticities in the higher ranges. Thus, the short run relation between quality and wages will be tilted with respect to the long run relation as in Figure 7. In this case, only the gradual process of new entrants into those occupations with high present values will bring about the return of h to H.

Even if the $E_1^i$'s remain constant over time, secular increases in productivity due to neutral technological change will cause an increase in the demand for higher quality levels. As the general level of real wages rises, the cost of training diminishes relative to the returns, and the dispersion of quality wage rates diminishes. This is shown in Figure 8 where $H(Q)$ shifts down for all
positive schooling levels. The elasticity of $w_4$ with respect to $Q_4$ increases as $Q_4$ increases, but the shift downward of $H$ is greater for larger values of $Q_4$. 

Figure 8
This implies that the elasticity of \( w_i \) with respect to \( Q_i \) for larger values of \( Q_i \) declines at a more rapid rate than for smaller values of \( Q_i \). Ceteris Paribus, then, neutral technological change will be accompanied by a shift in the composition of labor demand to more skilled occupations. (Notice the implications for an aggregate theory of growth. Neutral technological change causes an increase in overall labor quality which would -- if it could be identified -- be technological change embodied in labor. But it would not have occurred in the first place had there not been neutral technological change.)

E. The Distribution of Income and Labor Supplies by Quality Levels

The schema offers no predictions concerning the distribution of the aggregate supplies of the various quality levels. At any moment in time all variables are in full equilibrium if \( w_i = H(Q_i) \), \( Q_i = Q_i^* \), and \( Q_i^* \) is such that \( E_i^1 = D_2 \), for all markets. That is, in equilibrium there is no tendency for:

(i) the distribution by quality level of new entrants to the labor force to be different from the existing distribution by quality level of the aggregate labor force,

(ii) wage rates for particular markets to change due to a deviation of the actual from the optimum quality level,

(iii) firms to adjust optimum quality levels due to the inequality of \( E_i^1 \) and \( E_2 \),

(iv) individuals in the labor force to change labor markets.

If these four stability characteristics are met, the aggregate supply of skill levels will reflect the distribution of the \( E_i^1 \)'s, the elasticities of efficiency with respect to quality. These are exogenous, technical parameters, and we have presented no explanation of their distribution. Hence, we can only say that the levels of the demand and supply by quality levels depend upon the distribution of the \( E_i^1 \)'s. The distribution of wages by quality levels, on the other hand, is given
by $H(Q_i)$, and (conceptually) we know the parameters of this relation. The size distribution of income is, of course, a combination of both these distributions but, again, its parameters depend upon the unobservable distribution of the $E_i$'s.

One thing we do know, however, is that there will be a secular upward shift in the quality composition of the labor force due to forces discussed in the last section and that the distribution of wages with respect to quality will compress over time. Each of these results will have an opposite affect on the size distribution of income, and at this point there is no basis for speculating about which is stronger.