Misallocation in a Model with Endogenous Managerial Capital and Distortions

Jung Eun Yoon

A Dissertation
Presented to the Faculty
of Princeton University
in Candidacy for the Degree
of Doctor of Philosophy

Recommended for Acceptance
by the Department of
Economics
Adviser: Professor Richard Rogerson

June 2017

Abstract

Aggregate total factor productivity (TFP) differences across countries have been widely recognized as the primary source of huge divergence in per capita income across countries. The misallocation literature has found distortions that inefficiently allocate resources across production units can result in a significant aggregate productivity loss, even without deterioration in the underlying productivity distribution. However, with endogenous managerial capital investment decisions, distortions affect the underlying productivity distribution in addition to reallocating resources across production units.

The first chapter quantifies the effect of progressive taxation in a model with endogenous managerial capital investment decisions. Compared to proportional taxation that raises the same tax revenue, progressive taxation distorts the economy more severely. The more progressive is taxation, the less incentive agents have to invest in their managerial capital. This follows because higher managerial capital implies higher profit, which induces higher tax rates. Thus, compared to a proportional tax regime that raises the same tax revenue, under progressive taxation, agents invest less in their managerial capital and the distribution of income is less dispersed. In addition, the equilibrium values of TFP, total output, employment share of large firms are distorted relative to their values under proportional taxation. Hence, progressive taxation improves equality in the economy in exchange for efficiency.

In the second chapter I examine the effects of credit constraint in a model with endogenous managerial capital investment decisions. If agents can optimally invest in their managerial capital, limited access to physical capital will encourage managers to substitute away from physical capital to investment in managerial capital. The accumulation of managerial capital and the change in the underlying productivity distribution will mitigate the adverse effects of misallocation caused by the credit

constraint on the economy. Using calibration, I show that measured TFP could improve with a tighter credit constraint.

The third chapter adds stochastic component to the model with endogenous managerial capital investment decision and credit constraints. I find that with sufficiently large second moment shock to the managerial skill accumulation function, the mitigation effect induced by optimal managerial capital investment decision is itself mitigated.

Acknowledgements

First, I would like to express my gratitude to my advisor Richard Rogerson who has taught me, encouraged me, and led me to the great world of Macroeconomics with his immense knowledge and unbeatable patience. I am also very grateful to Ben Moll for his invaluable support and insightful advices. Moreover, I appreciate Mark Aguiar, Nobuhiro Kiyotaki, Oleg Itskhoki, Ezra Oberfield, Esteban Rossi-Hansberg, Mikhail Golosov for their penetrating comments and guidance inside and outside the Macroeconomics student lunch seminar. I would also like to thank macro group seminar participants for their precious and helpful comments. I give special thanks to Laura M. Hedden for her end-less support and friendliness.

Last but not least, I would like to thank my fellow graduate students in Princeton. SeHyoun Ahn and Sungki Hong would always let my worries go away with their brilliant comments. Yu Zhang, Chang Sun, So Kubota, Daniel Goetz, Justin Weidner and Cristian Alonso were always there to discuss research. Ishita Rajani, Caitlin McGugan, Alexander M. Jakobsen, Nikhil Gupta and Chen Zhao provided not only academic guidance but also emotional support. Seok Min Moon, Donghwa Shin and Jiwon Choi kindly allowed me to use their precious time for several of my practice presentations. The friendships are the most valuable of all I gained in Princeton.

Finally, I would like to thank my old friends Insun Yoon, Sukhwan Choi, Woong Lee, Young Seok Kim and Steve Choi for sharing their time with me and making me laugh even in times of struggle. I am very lucky to have them as my friends.

To my parents, Kyoung Ae Yoon and Tae Soo Yoon.

I am truly blessed to be your daughter.

Contents

Contents

Chapter 1

Endogenous Managerial Capital and Progressive Taxation

1.1 Introduction

A recent literature has explored the role that misallocation of resources across heterogeneous production units can play in generating differences in aggregate TFP. The underlying economics are simple: in an economy with many production units, the output that is generated by a given aggregate supply of factors determines on how the factors are allocated across these establishments. A key issue for researchers is to quantify the magnitude of this effect. The overall effect on TFP can be thought of as the product of two factors: the size of the distortion to the allocation of factors across establishments and the elasticity of TFP with regard to the distortion. Quantifying the effect of misallocation on TFP requires quantifying each of these components.

An early contribution to the literature on misallocation was Restuccia and Rogerson (2008). They assume an exogenous distribution of productivities across establishments and considered establishment specific taxes. One of their main results was that these establishment specific taxes had much larger effects on TFP if the distortions were larger for more productive establishments, though they did not provide any guidance regarding the size and nature of these establishment specific distortions in reality.

In this paper I evaluate the role of one specific institutional feature that implicitly generates establishment specific tax rates that are positively correlated with establishment productivity: progressive income taxes. In particular, I consider a model in which entrepreneurs differ in their managerial ability, thereby leading to differences in measured productivity across establishments. Higher ability entrepreneurs will manage establishments that are more productive and generate higher profits, thereby exposing them to higher marginal tax rates in a progressive income tax system. I adopt the progressive income tax schedule used in Benabou (2002) and calibrate the extent of progressivity for 5 OECD economies using the measures in Guvenen et al (2009).

To assess the impact of this particular distortion on aggregate TFP I also extend the model of Restuccia and Rogerson (2008) so as to amplify the effect of a given set of distortions on aggregate TFP. In particular, whereas they assume that the underlying distribution of productivities across establishments is fixed and exogenous, I follow Battacharya et al (2012) and introduce an endogenous component to this distribution. Specifically, I assume that the distribution of managerial ability at birth is exogenous and fixed, but that managers make an investment in the accumulation of managerial ability. Intuitively, if higher ability managers are taxed at a higher rate due to progressive taxation, then the incentive to accumulate managerial ability is diminished and the overall productivity distribution may be shifted down.

My main quantitative exercise consists of using the benchmark model of Batttacharya et al (2012) to assess the effects for aggregate TFP of empirically reasonable differences in the degree of progressivity. In this model, even proportional taxes will affect TFP through their effect on the incentives to accumulate managerial ability.

To isolate the role of progressivity I will contrast the effects in my benchmark calculations with those that would obtain if tax revenues were held constant but were instead raised by a constant proportional tax on entrepreneurial income.

My main results are such that the more progressive taxation is, the more distortionary it is to an economy. Investment in ability decreases 24 percent to 47 percent, TFP decreases 3 percent to 6 percent, employment share of large firms decreases 13 percent to 32 percent depending on the level of progressivity. Also, the distortionary impact of progressivity per se compared to proportional taxation that raises the same tax revenue increases as taxation becomes more progressive. For instance, under low progressivity which is comparable to that of the U.S, 7 percent of distortion in average firm size is due to progressivity while under high progressivity which is comparable to that of Denmark, 17 percent of distortion is due to progressiveness.

My paper is related to several papers in the literature. First, there are models with exogenous firm level productivity and productivity correlated distortion. Guner, Ventura, Xu(2008) uses Lucas span of control model with exogenous managerial ability that does not change over life cycle and examines firm-size dependent policy distortions on economic outcomes. They restrict production of large establishments while encouraging that of small establishments and find out that this policy will increase number of establishments in the economy and reduce average establishment size.

Tsieh and Klenow(2012) observes difference in life cycle dynamics of firms in U.S, India, Mexico. They observe that firms in Mexico and India grow much slower than firms in the U.S. They argue that this difference is due to lower investments by Indian and Mexican plants compared to those of the U.S. Policy in poor countries is such that it distorts production of large firms and it discourages investments in intangible capital and it results in smaller firm size.

Other set of literature has endogenous component in firm level productivity but without distortion correlated with firm level productivity. Bharttarcharya, Guner, Ventura(2011), from which my model is adopted, uses Lucas span of control model with endogenous investment in managerial ability. They find that under endogenous managerial ability, firms invest less compared exogenous ability with the same level of distortion. Average managerial ability and mean firm size is much smaller when economy is distorted with endogenous managerial ability. Bloom and Reenen(2007) also model managerial input as choice variable of firms. Firms optimally choose how much managerial input to put balancing cost and benefit from investment in those inputs. Gabler and Poschke(2013) evaluates the size of distortionary effects when distortion affects not only the resource allocation but also the evolution of firm-level productivity. To do this, they let firms to engage in risky experiments that takes the form of productivity shocks. They find that endogenous productivity implies twice as large effects of distortions on aggregate consumption.

An outline of this paper is as follows. In section 2, I present a benchmark model which is adopted from BGV. The benchmark model is Lucas span of control model with endogenous managerial ability and it has no tax distortion. In section 3, I show steady state equilibrium of this benchmark model. In section 4, I briefly explain how calibration is done in BGV and show the calibrated parameter values and a set of target fit between model and data. The parameter values remain constant for the rest of this paper. In section 5 I specify tax system and a measure of progressivity. In the same section I show how individual's investment decision is affected by progressive taxation. Numerical results under different levels of progressivity and under proportional tax system that raises the same tax revenue as each progressive taxation raises are presented and analyzed. Section 6 concludes.

1.2 Benchmark Model

In this section, I describe the benchmark model, which is taken from Bhattarcharya, Ventura, Guner's paper. In the benchmark model, there are no taxes. It is life-cycle version of Lucas span- of -control model. Each period, an overlapping generation of heterogenous agents are born an and live for J periods. They work for the first J_R periods, retire and live on their savings for the rest of periods. We assume that each cohort is $1+n$ bigger than the previous cohort. The population structure is stationary in the sense that age j cohort is fraction μ_j of whole population at any time t. The weight is normalized to add up to one. Thus, it satisfies the following.

$$\mu_{j+1} = \mu_j/(1+n) \quad \forall j, \quad \sum_{j=1}^{J} \mu_j = 1 \tag{1.1}$$

where μ_j is measure of age j cohort.

The objective of each agent is to maximize lifetime utility from consumption of the following form.

$$\sum_{j=1}^{J} \beta^{j-1} log(c_j) \tag{1.2}$$

When agents are born they are endowed with managerial ability z which is drawn from an exogenous log normal distribution with mean μ_z and variance σ_z^2. Until retirement, each agent is endowed with 1 unit of time which they spend inelastically as a manager or a worker. Given their managerial capital z, agents decide whether to become a worker or a manager. This decision is irreversible. Labor market and capital market is competitive.

A worker supplies labor inelastically throughout the whole working period and earns the market wage. A worker chooses how much to save and consumes each period to maximize his utility. If an individual becomes a manager, he has to also choose how much physical capital or labor to employ to produce output, and how much to invest in improving managerial skills. Managerial capital z is utilized only if an agent becomes a manager.

1.2.1 Technology

Each manager has access to a span-of-control technology of production. A plant with managerial ability z will produce output with labor and capital with the following production function.

$$y = z^{1-\gamma}(k^{\alpha}n^{1-\alpha})^{\gamma} \tag{1.3}$$

where γ is span of control parameter and α is the share of capital. Managers can enhance their future ability by investing their income into skill accumulation. Skill is accumulated with the function given below.

$$z' = z + g(z, x) = z + z^{\theta_1}x^{\theta_2} \tag{1.4}$$

where z' is next period's ability level and x is investment in skill accumulation. The function g is such that current ability level and investment in future ability display complementarities: $g_{zx} > 0$, i. e., the higher the current level of skill, the more beneficial it is for an agent to invest in skill accumulation. Also, it is assumed that g_{xx} is negative so that there is diminishing returns to skill investment.

1.2.2 Decisions

We focus on a steady state equilibrium with a constant factor prices R and w. Let a denote assets that pay the risk-free rate of return $r = R - \delta$. In a steady state equilibrium, agents born with ability over some threshold ability level $\hat{z}$ will become managers and the rest will become workers. There are no idiosyncratic distortions, so agents with the same ability level will make the same decision regarding their career choice and will end up with exactly the same resource allocation along their life cycle. I next describe the optimization problems for workers and managers.

1.2.3 Managers

The problem of a manager of age j is given by

$$V_j(z,a) = \max_{x,a'}\{log(c) + \beta V_{j+1}(z',a')\} \tag{1.5}$$

subject to

$$c + x + a' = \pi(z; r, w) + (1+r)a \quad \forall 1 \leq j < J_R - 1, \tag{1.6}$$

and

$$z' = z + g(z,x) \quad \forall j < J_R - 1,$$

with

$$V_{J+1}(z,a) \begin{cases} 0 & \text{if } a \geq 0 \\ -\infty & \text{otherwise} \end{cases}$$

Note that managers can freely borrow or lend at market interest rate r. With no borrowing constraints, factor demands and per period managerial income π depends only on managerial ability level z. Managerial income for a manger with ability z is given by

$$\pi(z;r,w) \equiv \max_{n,k}\{z^{1-\gamma}(k^{\alpha}n^{1-\alpha})^{\gamma} - wn - (r+\delta)k\} \tag{1.7}$$

Taking F.O.Cs, factor demands are given by

$$k(z;r,w) = ((1-\alpha)\gamma)^{\frac{1}{1-\gamma}}\frac{\alpha}{1-\alpha}^{\frac{1-\gamma(1-\alpha)}{1-\gamma}}(\frac{1}{r+\delta})^{\frac{1-\gamma(1-\alpha)}{1-\gamma}}(\frac{1}{w})^{\frac{\gamma(1-\alpha)}{1-\gamma}}z \tag{1.8}$$

and

$$n(z;r,w) = ((1-\alpha)\gamma)^{\frac{1}{1-\gamma}}(\frac{\alpha}{1-\alpha})^{\frac{\alpha\gamma}{1-\gamma}}(\frac{1}{r+\delta})^{\frac{\alpha\gamma}{1-\gamma}}(\frac{1}{w})^{\frac{1-\alpha\gamma}{1-\gamma}}z \tag{1.9}$$

Substituting these into the profit function, profits are a linear function of managerial ability, z

$$\pi(z;r,w) = \Omega \quad (\frac{1}{r+\delta})^{\frac{\alpha\gamma}{1-\gamma}}(\frac{1}{w})^{\frac{\gamma(1-\alpha)}{1-\gamma}}z \tag{1.10}$$

Where Ω is a constant given by

$$\Omega \equiv (1-\alpha)^{\frac{\gamma(1-\alpha)}{(1-\gamma)}}\alpha^{\frac{\gamma\alpha}{(1-\gamma)}}\gamma^{\frac{1}{1-\gamma}}(1-\gamma)$$

The solution to the dynamic programming problem is characterized by two conditions. First, the solution for next period's asset level, a', is determined from the standard Euler equation given below:

$$\frac{1}{c_j} = \beta(1+r)\frac{1}{c_j+1}$$

Second, investment is determined by the no arbitrage condition below:

$$(1+r) = \pi_z(z_j; r, w) g_x(z_j, x_j).$$

The left hand side is next period's gain in income from one unit of current savings. The right hand side is the gain in income to the j-period old manger from investing one unit of current consumption in ability accumulation. As assumed before g_{xx} is negative. This implies that the marginal benefit of investing in skill accumulation is monotonically decreasing in the level of skill investment while the marginal cost $(1+r)$ is constant. Thus, a unique interior optimum level of x is determined from the equation above.

1.2.4 Workers

The problem of an age j worker is given by following

$$W_j(a) = \max_{a'} \{log(c) + \beta W_{j+1}(a')\} \tag{1.11}$$

subject to

$$c + a' = w + (1+r)a \quad \forall 1 \leq j < J_R - 1, \tag{1.12}$$

and

$$c + a' = (1+r)a \quad \forall j < J_R, \tag{1.13}$$

With

$$W_{J+1}(a) \begin{cases} 0 & \text{if } a \geq 0 \\ -\infty, & \text{otherwise} \end{cases}$$

Like managers, workers can lend and borrow at a given rate r as long as they don't die with negative assets. And again, each worker is born with zero assets.

1.2.5 Occupational Choice

Each agent maximizes their lifetime utility given their ability level z. They choose to become a worker or a manager right after they are born. If their ability level is high enough, they choose to become a manager while when it is not, they choose to become a worker. Let z^*be the ability level at which a new born agent is indifferent between being a worker and a manager. This z^* can be found by the equation below

$$V_1(z*, 0) = W_1(0).$$

$W_1(0)$ is a constant in a steady state equilibrium. V_1 is a continous, strictly increasing function of z so this equation has a well defined solution z^*. Agents with initial ability higher than z^*will choose to become a manager while those under z^* will become a worker.

1.3 Steady State Equilibrium

Let's look into steady state equilibrium with fixed r and w. Managerial abilities are determined endogenously after the first period since each agent optimally invests in their ability level. Therefore, the upper bound for managerial ability is going to be determined endogenously. Let's call this upper bound $\bar{z}$. Then managerial ability take values in a set $Z = [\underline{z}, \bar{z}]$ Similarly, since $A = [0, \bar{a}]$ denote the possible asset levels. Let $\psi_j(a, z)$ be the mass of age-j agents with assets a and ability level z. Given

$\psi_j(a, z)$, let

$$\tilde{f}_j(z) = \int \psi_j(a, z)\,\mathrm{d}a \tag{1.14}$$

be the skill distribution for age j agents. Then, in a steady state equilibrium with given prices (r,w), labor, capital and goods market must clear. The following is the labor market equilibrium condition.

$$\sum_{j=1}^{J_R-1} \mu_j \int_{z^*}^{\bar{z}} n(z; r, w)\tilde{f}_j(z)\,\mathrm{d}z = F(z^*) \sum_{i=1}^{J_R-1} \mu_j \tag{1.15}$$

where μ_j is the total mass of cohort j. The left-hand side is the labor demand from the $J_R - 1$ different cohorts of managers. The right-hand side is the fraction of each cohort employed as workers. For each cohort, those under ability level z^* choose to become workers and there are mass of μ_j in each cohort. Labor supply comes from non-retired cohorts.

In the capital market, there are two sources of demand for savings. Managers demand capital to produce output. They also demand savings to invest in their ability accumulation. Savings comes both from managers and workers of each cohort except for the oldest cohort since they have no incentive to save. Thus, the capital market equilibrium condition can be written as

$$\sum_{j=1}^{J_R-1} \mu_j \int_{z^*}^{\bar{z}} k(z; r, w)\tilde{f}_j(z)\,\mathrm{d}z \tag{1.16}$$

$$+\sum_{j=1}^{J_R-1} \mu_j \int_{z^*}^{\bar{z}} x_j(z, a)\psi_j(z, a)\,\mathrm{d}z\mathrm{d}a = \sum_{j=1}^{J-1} \mu_j \int_{z}^{z^*} a_j^w(a)\psi_j(z, a)\,\mathrm{d}z\mathrm{d}a \tag{1.17}$$

$$+\sum_{j=1}^{J-1} \mu_j \int_{z^*}^{\bar{z}} a_j^m(a)\psi_j(z, a)\,\mathrm{d}z\mathrm{d}a \tag{1.18}$$

The first term of the left-hand side is capital demand from the working cohorts of managers. The second term is the sum of investment of working managers up to one period before they retire. For instance, if they retire at age 4, there are 3 investment periods. These two terms comprise the demand for savings. Each of the right-hand side terms is savings of workers and managers before they die.

The goods market equilibrium condition is that the aggregate output produced in the economy is equal to the sum of aggregate consumption investment in physical capital and skill investments across cohorts by all managers and workers.

1.4 Calibration

Parameter values are the same as those adopted in the benchmark model of Bhattacharya Guner and Ventura. In BGV they assume the U.S economy to be distortion free and calibrate benchmark model parameters to match some features of aggregate U.S data in addition to some aspects of the U.S plant data. Key features of the plant level data are of different sizes. The average size of a plant in the U.S(17.9), and the distribution of employment across plants. A model period corresponds to 10 years. Each cohort enters the model at age 20 and lives until age 80, so they work for 40 years and stay retired for 20 years.

There are 9 parameters to calibrate. The product of the importance of capital(α) and returns to scale(γ) determine the share of capital in output. This is determined from Guner et al (2008) and equals 0.317. Thus, they calibrate γ and set $\alpha = 0.317/\gamma$. The depreciation rate(δ) and population growth(n) are set so that their annual rates are 0.04 and is 0.011 respectively.

So out of 9 parameters, excluding those already determined, α, δ, and n, there are 6 parameters to calibrate: $\gamma, \beta, \theta_1, \theta_2, \mu_z, \sigma_z$. They normalize the mean of the log of the skill distribution to zero and calibrate 5 remaining parameters to match 5 moments of the U.S plant size distribution: mean plant size, fraction of plants with less than 10 workers, fraction of plants with 100 or more workers, fraction of the labor force employed in plants with 100 or more employees and capital output ratio. The calibration successfully replicates multiple features of the U.S plant size distribution.

Tables 1 and 2 show the parameter values thus adopted and the match of data and benchmark model obtained from BGV. The parameter values obtained from this benchmark calibration are going to be used for the remainder of this paper.

Table 1.1: Calibrated parameter values

Parameter	Value
Population Growth Rate (n)	0.011
Depreciation rate (δ)	0.04
Importance of Capital	0.428
Returns to Scale (γ)	0.760
Mean Log-managerial Ability (μ_z)	0
Discount Factor(β)	0.945
Skill accumulation technology (θ_1)	0.953
Skill accumulation technology (θ_2)	0.405

Table 1.2: Fit of the benchmark model and data with parameter values in table 1

Statistic	Data	Model
Average Firm Size	17.9	17.7
Capital Output ratio	2.325	2.304
Fraction of small (0-9 workers) establishments	0.725	0.747
Fraction of large (100+ workers) establishments	0.026	0.027
Employment Share of Large Establishments	0.462	0.472

1.5 Effect of Progressive Taxation

1.5.1 Specification of Progressivity

There were no taxes to now. In this section, I examine how a progressive tax system affects outcomes. I parameterize the tax system as in Benabou(2002). Given a pretax income of y, after tax income y_{AT} is given by

$$y_{AT} = y^{1-\tau^*}\tilde{y}^{\tau^*}$$

where $\tilde{y}$ is an income level at which net taxes become positive. I will call $\tilde{y}$ the tax base. Then the average tax rate for an individual with income y, denoted by $T(y)$ is given by

$$T(y) = 1 - y^{-\tau^*}\tilde{y}^{\tau^*} \tag{1.19}$$

$T(y)$ is increasing in y, implying that both average and marginal tax rates increase as a function of pre tax income y. The parameter τ^* measures the progrogressivity of tax system. A higher τ^* indicates steeper average and marginal tax rate slopes with respect to y.

1.5.2 How it affects decisions

Under this tax system, the average tax rate for a manager depends on his profit level which in turn depends on his current ability level z. Thus, managers have to solve the following problem.

$$\pi_{AT}(z;r,w) \equiv \max_{n,k}\{\tilde{y}^{\tau^*}(z^{1-\gamma}(k^{\alpha}n^{1-\alpha})^{\gamma} - wn - (r+\delta)k)^{(1-\tau^*)}\} \tag{1.20}$$

where π_{AT} denotes after-tax profit.Thus, factor demands under progressive tax system are given by

$$k_p(z;r,w) = ((1-\alpha)\gamma)^{\frac{1}{1-\gamma}} \frac{\alpha}{1-\alpha}^{\frac{1-\gamma(1-\alpha)}{1-\gamma}} (\frac{1}{r+\delta})^{\frac{1-\gamma(1-\alpha)}{1-\gamma}} (\frac{1}{w})^{\frac{\gamma(1-\alpha)}{1-\gamma}} z,$$

and

$$n_p(z;r,w) = ((1-\alpha)\gamma)^{\frac{1}{1-\gamma}} (\frac{\alpha}{1-\alpha})^{\frac{\alpha\gamma}{1-\gamma}} (\frac{1}{r+\delta})^{\frac{\alpha\gamma}{1-\gamma}} (\frac{1}{w})^{\frac{1-\alpha\gamma}{1-\gamma}} z$$

Substituting these into the after tax profit function, profits are still a function of managerial ability z only as before.

$$\pi_{AT} = \tilde{y}^{\tau^*}(\Omega(\frac{1}{r+\delta})^{\frac{\alpha\gamma}{1-\gamma}}(\frac{1}{w})^{\frac{\gamma(1-\alpha)}{1-\gamma}} z)^{1-\tau^*} \tag{1.21}$$

Where Ω is the same constant from proportional tax system case.

$$\Omega \equiv (1-\alpha)^{\frac{\gamma(1-\alpha)}{(1-\gamma)}} \alpha^{\frac{\gamma\alpha}{(1-\gamma)}} \gamma^{\frac{1}{1-\gamma}} (1-\gamma)$$

Savings a' are determined in the same way as above and thus, satisfy the same Euler equation:

$$\frac{1}{c_j} = \beta(1+r)\frac{1}{c_j+1}$$

However, investment in skill decisions will be different since the tax rate is now a function of z. Investment in skill accumulation will increase next period's skill level z' and it will thus increase next period's tax rate. This will reduce manager's incentive to invest in next period's ability level. Thus, investment will satisfy the following equation.

$$(1+r) = g_x(z_j, x_j)\pi_z^{AT}, \tag{1.22}$$

which is implies

$$(1+r) = g_x(z_j, x_j)(1-\tau^*)\frac{\pi^{AT}}{z}. \tag{1.23}$$

Letting T(z) denote the average tax rate for a manager with ability z, the investment decision can be written in the benchmark model comparable form below:

$$(1+r) = g_x(z_j, x_j)\{(1-T(z))\pi_z(z_j; r, w) + (1-T(z))_z\pi(z_j; r, w)\} \tag{1.24}$$

This implies

$$(1+r) = g_x(z_j, x_j)(1-\tau^*)\frac{y(1-T(z))}{z} \tag{1.25}$$

where $y = \Omega(\frac{1}{r+\delta})^{\frac{\alpha\gamma}{1-\gamma}}(\frac{1}{w})^{\frac{\gamma(1-\alpha)}{1-\gamma}}z$ is before-tax profit.

As mentioned in section 5.1, τ^* indicates progressvity of tax system. The higher is τ^*, the steeper is the tax rate increase with respect to the income level.

To see how different levels of τ^* are related to the real world, I set τ^* to match the measure of progressivity for each country studied in Guvenen, Kuruscu and Ozkan (2009) . In GKO, they suggest that the progressivity of a tax system can be measured with the following formula called 'progressivity wedge'.

$$PW_i^*(y_s, y_{s+k}) = 1 - \frac{1 - \tau_m(y_{s+k})}{1 - \tau_m(y_s)}$$

where $\tau_m(y)$ indicates the marginal tax rate at income level y. I adjust my measure of progressivity τ^* to match their measure of progressvity 'PW' for different countries. Typically τ^* is adjusted to match PW of each country between 0.5 times of average wage and 3 times of average wage. As a result, τ equals to 0.09, 0.11, 0.13, 0.15, 0.2. for U.S, France, Germany, Netherlands and Denmark. Following figure shows PW with different levels of tau.

Figure 1.1: Progressivity Wedge for OECD countries in GKO

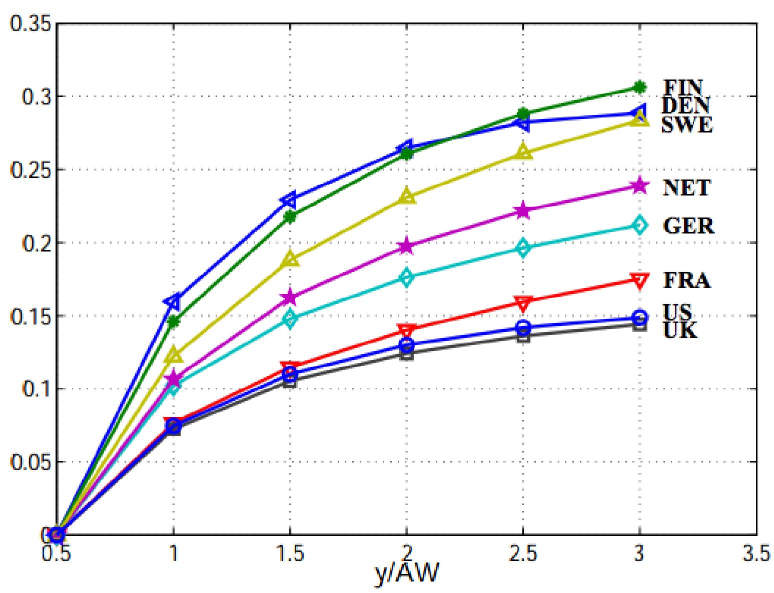
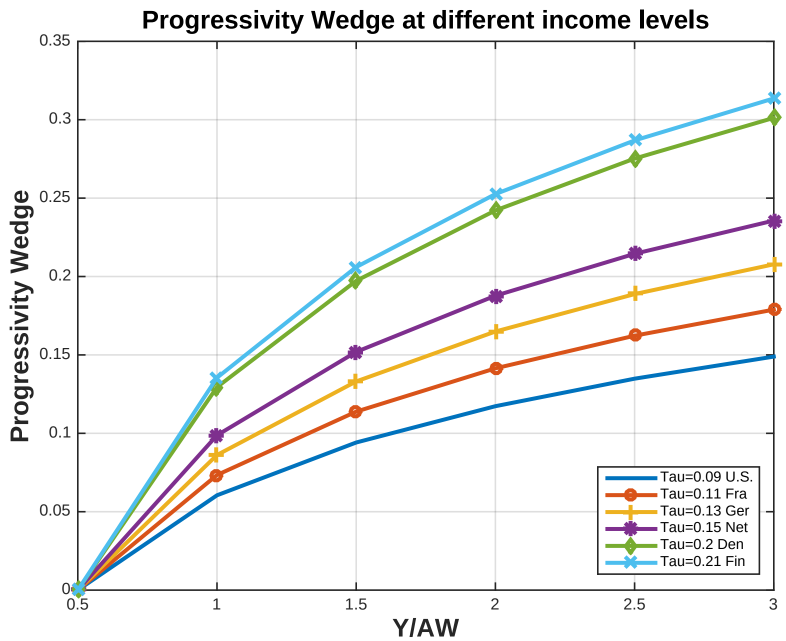


Figure 1.2: Taus that match Progressivity Wedge for different countries from figure 1 using Benabou's tax function



1.5.3 Numerical Results

In this section, I present and discuss the central quantitative findings. First, I present the result of benchmark model with no taxation. I see the effect of progressive taxes compared to benchmark model and how different a higher degree of progressivity affects economic outcomes. I vary the level of progressivity tau to be 0.09, 0.11, 0.13, 0.15, 0.2. These values are progressivity measure for U.S, France, Germany, Netherlands and Denmark obtained from previous section. $\tilde{y}$ is set to equal wage of workers. Tax revenues are redistributed evenly to the agents at the end of each period.

The following tables show the result. Values under the column labelled benchmark are raw numbers and values under each tau are values denoted as fraction of benchmark values. (with the exception of tax rates and tax revenues) Afsize is average firm size, X/Y is investment in ability output ratio, X is total investment in ability, Man bail is average managerial ability, Tax rates is average tax rates of mangers, Total tax is total tax revenue, K/Y is capital output ratio, ES100 is employment share of firms with more than 100 workers, Y is total output and F man is fraction of managers.

Table 1.3: Statistics as fraction of benchmark with different progressivity levels

Variable	Benchmark	$\tau^* = 0.09$	$\tau^* = 0.11$	$\tau^* = 0.13$	$\tau^* = 0.15$	$\tau^* = 0.2$
Afsize	17.67	0.88	0.84	0.80	0.80	0.76
X/Y	0.025	0.58	0.51	0.45	0.4	0.29
X	0.203	0.56	0.48	0.42	0.37	0.27
Man abil	410	0.76	0.70	0.53	0.63	0.56
TFP	3.063	0.96	0.95	0.94	0.94	0.92
Tax rates	0	0.103	0.12	0.139	0.157	0.201
Total tax	0	0.462	0.528	0.587	0.639	0.744
K/Y	0.231	1.05	1.05	1.06	1.07	1.08
ES 100	0.455	0.82	0.77	0.73	0.69	0.60
Y	8.238	0.96	0.95	0.95	0.94	0.92
F man	0.054	1.13	1.18	1.21	1.24	1.30

As progressivity increases, the distortionary effect is bigger. Average firm size drops from 88 percent of the benchmark level to 76 percent as progressivity increases from 0.09 to 0.2. Total investment in ability is cut in halvf, average managerial ability decreases by one third, and TFP decreases by 4 percent.

All of these results are qualitatively consistent with basic intuition. Next I assess the extent to which progressivity per se is generating these results. To do this, for each τ I consider a proportional tax rate that raises the same tax revenue as revenues raised under different taus. The results are in Table 4.

Table 1.4: Statistics as fraction of benchmark with proportional taxation raising the same tax revenue as in progressive taxation

Variable	Benchmark	$\tau^* = 0.09$	$\tau^* = 0.11$	$\tau^* = 0.13$	$\tau^* = 0.15$	$\tau^* = 0.2$
Afsize	17.67	0.95	0.96	0.95	0.93	0.93
X/Y	0.025	0.86	0.84	0.82	0.80	0.77
X	0.203	0.84	0.82	0.80	0.78	0.74
Man abil	410	0.90	0.90	0.89	0.87	0.85
TFP	3.063	0.99	0.98	0.98	0.98	0.98
Tax rates	0	0.0852	0.0979	0.1092	0.1192	0.1397
Total tax	0	0.4617	0.5281	0.5870	0.6387	0.7436
K/Y	0.231	0.99	0.99	0.98	0.98	0.98
ES 100	0.455	0.95	0.94	0.94	0.94	0.92
Y	8.238	0.97	0.97	0.97	0.97	0.96
F man	0.054	1.05	1.05	1.05	1.07	1.07

Proportional taxation is much less distortionary compared to progressive taxation even when raising the same tax revenue. For example, when tau is 0.09, average firm size is 8 percent larger, the investment to output ratio for ability is 48 percent larger, average managerial ability is 18 percent bigger, and TFP is 2.7 percent larger under proportional tax rates compared to progressive taxation that raises same amount of revenue.

1.5.4 Analysis

So how much of distortion is due to the progressiveness of tax system versus the average level of the tax? I can compute the value by subtracting values under proportional taxation and values under progressive taxation. Below shows the result.

Table 1.5: Fraction of distortions explained by progressivity of tax structure: computed as values in table 1-values in table 2

Variable	$\tau^* = 0.09$	$\tau^* = 0.11$	$\tau^* = 0.13$	$\tau^* = 0.15$	$\tau^* = 0.2$
Afsize	0.07	0.12	0.15	0.13	0.17
X/Y	0.28	0.33	0.37	0.4	0.48
X	0.24	0.34	0.38	0.35	0.47
Man abil	0.14	0.2	0.36	0.24	0.29
TFP	0.03	0.04	0.05	0.05	0.06
K/Y	-0.06	-0.06	-0.08	-0.09	-0.1
ES 100	0.13	0. 17	0.21	0.25	0.32
Y	0.01	0.02	0.02	0.03	0.04
F man	-0.08	-0.13	-0.16	-0.14	-0.23

For example, 7 percent of distortion in average firm size is due to progressivity under tau is 0.09, and 28 percent of distortion in investment output ratio, 24 percent of distortion in investment, 14 percent of distortion in average managerial ability is due to progressivity. Note that this rate increases as tau increases. This is as expected since higher tau means more progressivity and more progressive tax system will distort agents' investment in ability decisions more. Negative values for the fraction of managers and capital output ratio means that these values are higher under progressive taxation. Higher absolute values of these indicates they are further away from benchmark so that increase of absolute values as tau increases show that these values are distorted more the higher the tau is. Although progressive taxation is more distortionary, its distortionary effect on TFP is only 3 to 6 percent. Given that progressivity of 0.09 and 0.2 are comparable to the progressivity of U.S and Denmark respectively, it is noteworthy how small an effect it has on TFP. So given that progressive taxation contributes to equality of society, it should be carefully considered whether benefit from more equal income distribution could overwhelm loss of TFP due to progressive taxation.

1.6 Conclusion

Using the version of Lucas' span of control model with endogenous managerial ability proposed by Bhattacharya et al (2011), I study how progressive taxation distorts agents' decisions to invest in their managerial ability and thus, aggregate economic outcomes. To this end, I use the continuous progressive tax function proposed in Benabou(2002). To determine the level of progressivity, I used progressivity measure given by Guvenen et al(2009). For income less than of equilibrium workers' wages, I set the marginal tax rate to zero; this ensures that only managers are taxed. Progressive taxation distorts agents' decisions more severely than the proportional tax schedule that raises the same tax revenue; the distortionary effect is more severe the more progressive the tax schedule. Since agents know that with higher managerial ability and higher income they will face higher tax rates they will invest less under progressive compared to proportional taxation. Thus under progressive taxation, managerial income is less dispersed, total investment in ability is lower, the employment share of large firms is smaller, and TFP is lower compared to proportional taxation. However, the effect on TFP is not substantial, ranging from 3 to 6 percent only. Therefore how the prediction of this model informs government tax policy will depend on how the government values the tradeoff between equity and efficiency.

Chapter 2

Endogenous Managerial Capital and Financial Frictions

2.1 Introduction

Aggregate TFP differences across countries have been widely recognized as the major culprit for huge differences in income per capita across countries.[1] The misallocation literature (Restuccia and Rogerson (2008) and Hsieh and Klenow (2009)) has shown that distortions that inefficiently allocate resources across production units can result in a significant aggregate productivity loss even without deterioration in the underlying productivity distribution. A key challenge is to identify the quantitatively important sources of this misallocation. This will allow us to come up with policy advice that is effective and powerful in promoting income growth of the economies that are suffering from below par aggregate efficiency.

One of the popular sources of distortion that the misallocation literature has focused on is financial frictions. [2] In the existing literature, credit constraints distort

[1]This is a standard result in the development accounting literature. See, for example, Hall and Jones (1999), Caselli (2005), and Hsieh and Klenow (2009).

[2]The most relevant works are, Buera, Kaboski, and Shin (2011), Midrigan and Xu (2014), Moll (2014).

the allocation of physical capital across heterogeneous production units while holding the underlying productivity distribution fixed. This leads to the dispersion of marginal productivity of physical capital across production units, and thus worsens aggregate productivity. In this paper, I examine the effects of credit constraints in a model that features investment in managerial skills as in Bhattacharya et al. (2013). I find that, with optimal managerial capital investment decisions, the adverse effects of credit constraints on an economy are substantially mitigated. In fact, with managerial capital investment decisions, measured TFP could even improve with a tighter credit constraint.

Key to this result is that, in my model, financial frictions affect both the underlying productivity distribution of production units and the resource allocation among those production units. Over the life cycle, managers can optimally invest in managerial capital, which affects the underlying productivity distribution.[3] With endogenous managerial capital investment decisions, if tighter credit constraint restricts the access to physical capital, managers will substitute away from physical capital to investment in managerial capital. The accumulation of managerial capital and the change in the underlying productivity distribution will dampen the adverse effects of the credit constraint on the economy.

To study the quantitative and qualitative implications of credit constraints in a model with endogenous managerial capital investment decisions, I use the Lucas span of control model with optimal managerial capital investment decisions, as in Bhattacharya, Guner, Ventura (2011), and impose collateral constraint in the form of Buera, Kaboski and Shin (2011). I calibrate the parameters of the model assuming that the U.S. is a distortion-free, perfect credit benchmark. The calibration successfully matches U.S. firm size distribution statistics and physical capital output ratio.

[3]In my paper, the production unit is a manager combined with some workers. Thus, hereafter, I will refer to a production unit as a manager and firm-level productivity as managerial capital. I call it managerial capital because it can be utilized only if an agent becomes a manager.

In my main exercise, I hold the calibrated parameters fixed, and vary a single parameter ϕ that governs the tightness of the credit constraint to examine the effects of the constraint on an economy. To isolate the role of endogenous managerial capital investment decisions in the model, I compare the results from this exercise with those from 'exogenous' setup without a managerial capital investment decisions. In the exogenous setup, I assume that agents are forced to invest in their managerial capital the same amount as in the perfect credit benchmark case. Through the comparison, I highlight the extent to which the model with endogenous managerial capital decision differs from the model without it, and the underlying mechanism behind the difference.

My key finding is that, endogenous managerial capital decisions dampen the adverse effects of credit constraints on the economy. In fact, TFP rises with tighter credit constraint; in particular, TFP increases by 2.3% with a credit constraint that lowers the external-finance-to-GDP ratio by 19%. Unlike TFP, output falls, but it falls less than it does in a model without optimal managerial capital investment decisions. This is because the adverse effects of tighter credit constraints through misallocation of physical capital are offset by accumulation of managerial capital. Tighter credit constraint depresses managers' physical capital demand and lowers factor prices. The lower cost of production leads to higher profits for managers and stronger incentive to invest in managerial capital for future profits. As TFP captures both the allocative efficiency among production units and the total amount of managerial capital present in the economy, it improves with tighter constraints.

Another notable feature of my model is that tightness of the credit constraint and firm size dispersion show a non-monotonic relation. Tighter credit constraints and active accumulation of managerial capital by managers lead to a larger mass of more productive managers. However, tighter credit constraint will limit the ability of those managers to increase the size of the firm, and the actual firm size could be bigger or smaller. As a result, the firm size dispersion is non-monotonic to tighter credit constraints.

In my benchmark analysis, I assume that managerial capital is non-stochastic. I also consider a case in which the skill accumulation function has a stochastic component. In this case, I find that an increase in uncertainty coming from the stochastic component discourages managers from accumulating managerial capital, and can wipe away the dampening effect of endogenous managerial capital decisions mentioned above if the uncertainty is sufficiently large.

My paper is related to several literatures. Restuccia and Rogerson (2008) show that idiosyncratic policy distortions could lead to a substantial decrease in aggregate production. Using Chinese and Indian manufacturing firm data, Hsieh and Klenow (2009) show that reallocating resources within those countries to equalize marginal products to the same extent as in the U.S., would result in TFP gains of at least 30 to 40% in those countries. In these models, distortions do not affect the underlying productivity distributions. Bhattacharya et al. (2011) assume that the distortions affect not only the allocation of resources across production units but also the underlying productivity distribution through investment in managerial capital. They show that, if distortions are correlated with the size of production units, endogenous managerial capital investment decisions amplify the distortive effects. However, they don't examine financial frictions as a source of miallocation. The literature is unclear about whether or not losses from misallocation generated by financial frictions are big. Using a two-sector model, Buera, Kaboski and Shin (2011) shows that financial friction alone can bring down aggregate TFP by 36% and can account for a substantial part of TFP and income differences across countries. Midrigan and Xu (2014), using Korean plant level data, present that financial friction does not generate much losses in TFP from misallocation. They show that losses come from low levels of entry and technology adoption when there is credit constraint. Moll (2014) shows that, if productivity shock is persistent, steady state TFP loss from credit constraint is small, as agents save out of their credit constraints. In my paper, losses from financial frictions are further mitigated by accumulation of managerial capital.

My paper is closest to Fattal-Jaef (2015). In his paper he shows that the output gains from relaxing miallocation are reduced because firm entry and exit decision offsets them. The number of firms in his model is comparable to the amount of aggregate managerial capital accumulated in my model.

My paper is also related to the literature identifying the importance of management practice for the productivity of a firm. Bloom and Van Reenen (2007) show that management practices display significant cross-country differences and are strongly associated with firm-level productivity. Caselli and Gennaioli (2012) show that aggregate TFP might be negatively affected by dynastic management, with which less developed countries are more comfortable. They find that poor management correlated to dynastic management could account for a large part of TFP losses in those countries.

An outline of this paper is as follows. In section 2, I present a benchmark model with endogenous managerial capital investment decisions and collateral constraints. In section 3, I show steady state equilibrium of the benchmark. In section 4, I calibrate the model. In section 5, I present the main results. In section 6, I add a stochastic component to the model. In section 7, I conclude.

2.2 Benchmark Model

In this section, I describe the benchmark model, which is taken from Bhattarcharya et al. (2013). It is life-cycle version of the Lucas span of control model. Each period, an overlapping generation of heterogeneous agents are born and live for J periods. They work for the first J_R periods, retire, and live on their savings for the rest of their life. In the benchmark model, there are no financial frictions. Agents can borrow and save freely at the market interest rate. We assume that each cohort is $1+n$ larger than the previous cohort. The population structure is stationary in the sense that the age j cohort is a fraction μ_j of the whole population at any time t, with

$$\mu_{j+1} = \mu_j/(1+n) \quad \forall j, \quad \sum_{j=1}^{J} \mu_j = 1 \tag{2.1}$$

The objective of each agent is to maximize lifetime utility from consumption of the following form.

$$\sum_{j=1}^{J} \beta^{j-1} log(c_j) \tag{2.2}$$

When agents are born, they are endowed with managerial capital z, which is drawn from an exogenous log normal distribution with mean μ_z and variance σ_z^2. Until retirement, each agent is endowed with 1 unit of time which they spend inelastically as a manager or a worker. Agents are born with zero assets. At the beginning of each period, given their capital level z and asset level a, agents decide whether to become a worker or a manager. After production, an agent decides how much to save and consume, and how much to invest in their managerial capital. Only managers can invest in managerial capital. Labor and capital markets are competitive.

A worker supplies labor inelastically throughout the whole working period and earns the market wage. All individuals are equally productive as workers. A worker chooses how much to save and consume each period to maximize his utility. If an individual becomes a manager, he also has to choose how much capital or labor to employ to produce output, and how much to invest in improving managerial skills.

2.2.1 Technology

Each manager has access to a span-of-control technology of production. A plant with managerial capital z will produce output using labor and capital with the following production function.

$$y = z^{1-\gamma}(k^{\alpha}n^{1-\alpha})^{\gamma} \tag{2.3}$$

where γ is the span of control parameter and α is the share of capital. Managers can enhance their future ability by investing their income into managerial capital accumulation. Managerial capital is accumulated with the function given below.

$$z' = z + g(z,x) = z + z^{\theta_z}x^{\theta_x} \tag{2.4}$$

where z' is next period's managerial capital level and x is investment in skill accumulation. The function g is such that current managerial capital level and investment in future managerial capital display complementarities: $g_{zx} > 0$, i. e., the higher the current level of skill, the more beneficial it is for an agent to invest in skill accumulation. Also, it is assumed that g_{xx} is negative so that there are diminishing returns to skill investment.

2.2.2 Decisions

I focus on a steady state equilibrium with constant factor prices R and w. Let a denote assets that pay the risk-free rate of return $r = R - \delta$, where δ is the depreciation rate for capital. In a steady state equilibrium, agents born with ability over some threshold ability level $\hat{z}$ will become managers and the rest will become workers. Agents with the same ability level will make the same decision regarding their career choice and will end up with exactly the same resource allocation along their life cycle. I next describe the optimization problems for workers and managers.

2.2.3 Managers

The problem of a manager of age j is given by

$$M_j(z,a) = \max_{x,a'}\{log(c) + \beta V_{j+1}(z',a')\} \tag{2.5}$$

subject to

$$c + x + a' = \pi(z;r,w) + (1+r)a \quad \forall 1 \leq j < J_R - 1, \tag{2.6}$$

$$a' \geq 0 \tag{2.7}$$

and

$$z' = z + g(z,x) \quad \forall j < J_R - 1,$$

with

$$V_{J+1}(z,a) \begin{cases} 0 & \text{if } a \geq 0 \\ -\infty & \text{otherwise} \end{cases}$$

where $V_j(z,a)$ is a value function at period j defined as the maximum continuation value of becoming a manager at age j and becoming a worker at age j.

Note that managers can save but not borrow from the future (a is nonnegative). This is assumed so that one-to-one comparison between a perfect credit benchmark case and a less-than-perfect capital rental market easier. (In a less than perfect credit market, a cannot be negative.) In the absence of financial frictions, when managers can freely rent physical capital at market rental rate R, a manager's optimal demand for inputs depends on their managerial capital only and does not depend on their savings. Managerial income for a manger with ability z is given by

$$\pi\big(z;r,w\big) \equiv \max_{n,k}\Big[z^{1-\gamma}(k^{\alpha}n^{1-\alpha})^{\gamma} - wn - (r+\delta)k\Big] \tag{2.8}$$

Taking F.O.Cs, factor demands are given by

$$k\big(z;r,w\big) = \big((1-\alpha)\gamma\big)^{\frac{1}{1-\gamma}}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1-\gamma(1-\alpha)}{1-\gamma}}\left(\frac{1}{r+\delta}\right)^{\frac{1-\gamma(1-\alpha)}{1-\gamma}}\left(\frac{1}{w}\right)^{\frac{\gamma(1-\alpha)}{1-\gamma}} z \tag{2.9}$$

and

$$n\big(z;r,w\big) = \big((1-\alpha)\gamma\big)^{\frac{1}{1-\gamma}}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{\alpha\gamma}{1-\gamma}}\left(\frac{1}{r+\delta}\right)^{\frac{\alpha\gamma}{1-\gamma}}\left(\frac{1}{w}\right)^{\frac{1-\alpha\gamma}{1-\gamma}} z \tag{2.10}$$

Substituting these into the profit function, profits are shown to be a linear function of managerial ability, z

$$\pi\big(z;r,w\big) = \Omega \quad \left(\frac{1}{r+\delta}\right)^{\frac{\alpha\gamma}{1-\gamma}}\left(\frac{1}{w}\right)^{\frac{\gamma(1-\alpha)}{1-\gamma}} z \tag{2.11}$$

Where Ω is a constant given by

$$\Omega \equiv \big(1-\alpha\big)^{\frac{\gamma(1-\alpha)}{(1-\gamma)}}\alpha^{\frac{\gamma\alpha}{(1-\gamma)}}\gamma^{\frac{1}{1-\gamma}}\big(1-\gamma\big)$$

The solution to the dynamic programming problem is characterized by two conditions. First, the solution for next period's asset level, a', equation given below:

$$\frac{1}{c_j} \geq \beta\big(1+r\big)\frac{1}{c_{j+1}}$$

Second, investment is determined by the no arbitrage condition below:

$$(1+r) = \pi_z\big(z_j;r,w\big)g_x(z_j,x_j) \tag{2.12}$$

The left-hand side represents the next period's gain in income from one unit of current savings. The right-hand side is the gain in income to the j-period-old manger from investing one unit of current consumption in managerial capital accumulation. As noted previously, g_{xx} is negative. This implies that the marginal benefit of investing in skill accumulation is monotonically decreasing in the level of skill investment while the marginal cost $(1+r)$ is constant. Thus, a unique interior optimum level of x is determined from the equation above.

2.2.4 Workers

The problem of an age j worker is given by following

$$W_j(a) = \max_{a'}\{log(c) + \beta V_{j+1}(z, a')\} \tag{2.13}$$

subject to

$$c + a' = w + (1+r)a \quad \forall 1 \leq j < J_R - 1, \tag{2.14}$$

and

$$c + a' = (1+r)a \quad \forall j < J_R, \tag{2.15}$$

With

$$W_{J+1}(a) \begin{cases} 0 & \text{if } a \geq 0 \\ -\infty, & \text{otherwise} \end{cases}$$

Like managers, workers cannot borrow.

And finally,

$$V_j(z,a) = \max[W_j(a), M_j(z,a)] \tag{2.16}$$

2.2.5 Financial Friction

Now, assume that renting of physical capital is limited by imperfect enforceability of contracts, as in Buera, Kabokski, and Shin (2009). After production, agents can renege, in which case they can keep a fraction $1-\phi$ of undepreciated capital and revenue net of labor payments, but all financial assets deposited in the bank are confiscated. Thus, ϕ is the strength of an economy's legal institutions for enforcing contracts. Banks will rent capital only if agents will repay; thus, agents can borrow up to the amount that makes them to abide by the contract than to defalut on it. Also, agents gain access to the next period's financial market without any penalty. Agents choose to abide the contract if and only if

$$\max_n\{z^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma - wn - (r+\delta)k\} + (1+r)a \geq \max_n(1-\phi)\{z^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma - wn + (1-\delta)k\} \tag{2.17}$$

Which simplifies to

$$(1+r)a \geq -\phi\left[\max_n(z^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma - wn) - \frac{(1-\phi+r+\phi\delta)}{\phi}k\right] \tag{2.18}$$

This inequality decides the limit to the amount of physical capital each agent can borrow. The right-hand-side is minimized for some $\hat{k}(z;\phi)$ less than the unconstrained optimal demand of k. One can think of the k that maximizes the value in the bracket in the inequality above as an optimal capital demand under a higher rental rate $\frac{(1-\phi+r+\phi\delta)}{\phi} > r+\delta$. This eliminates the case that capital constraint only allows a greater amount of capital rental than is desired. Let the upper bound on capital that is consistent with entrepreneurs to abide the contract is $\bar{k}(a, z; \phi)$.

Then $\bar{k}(a, z; \phi)$ is given by the max of 0 and largest root of the equation

$$(1+r)a = -\phi\left[\max_n (z^{1-\gamma}(\bar{k}^\alpha n^{1-\alpha})^\gamma - wn) - \frac{(1-\phi+r+\phi\delta)}{\phi}\bar{k}(a, z; \phi)\right] \tag{2.19}$$

The capital constraint thus reduces to $k \leq \bar{k}(a, z; \phi)$.
It is obvious that larger the amount of assets (a) held by the entrepreneur, higher the current managerial capital z, and larger the ϕ fraction taken away by the contract enforcing intermediaries, the entrepreneur's collateral is more valuable and thus he can borrow more physical capital to put into production. The proof is the same as in Buera, Kaboksi, and Shin (2009).
The problem of a manager of age j with a capital constraint is then given by

$$M_j(z, a) = \max_{x, a'}\{log(c) + \beta V_{j+1}(z', a')\} \tag{2.20}$$

subject to

$$\begin{aligned} c + x + a' = \pi(a, z; r, w) + (1+r)a \quad \forall 1 \leq j < J_R - 1 \\ c + a' = (1+r)a \quad \text{for} \quad j \geq J_R - 1 \end{aligned} \tag{2.21}$$

$$\bar{k}(a, z; \phi) \geq k \tag{2.22}$$

and

$$z' = z + g(z, x) \quad \forall j < J_R - 1,$$

with

$$V_{J+1}(z,a)\begin{cases} 0 & \text{if } a \geq 0 \\ -\infty & \text{otherwise} \end{cases}$$

Thus, capital and labor demand with financial constraints are a function of (z, a) instead of z only.

2.2.6 Occupational Choice

Agents maximize their lifetime utilities given the ability level z and assets a. Agents freely choose to become a worker or a manager at the beginning of each period. Let ${z^*}_{(j,a)}$ be the ability level at which an age j agent is indifferent between being a worker and a manager if he has an assets a. This ${z^*}_{(j,a)}$ can be found by the equation below

$$M_j\big({z^*}_{(j,a)}, a\big) = W_j\big(a\big). \quad \forall a, j$$

$W_j\big(a\big)$ is a constant in a steady state equilibrium. M_j is a continuous, strictly increasing function of z and a, so this equation has a well defined solution $z^*_{(j,a)}$. At each period j, given their assets a, agents with managerial capital higher than $z^*_{(j,a)}$ will choose to become a manager, while those under $z^*_{(j,a)}$ will become a worker.

2.3 Steady State Equilibrium

I focus on a steady state equilibrium in which r and w are constant over time. Managerial capitals are determined endogenously after the first period, since each agent optimally invests in their managerial capital level. Therefore, the upper bound for managerial capital is going to be determined endogenously. Let's call this upper bound $\bar{z}$. Then managerial capital takes values in a set $Z = [\underline{z}, \bar{z}]$. Similarly, let $A = [0, \bar{a}]$ denote the possible asset levels. Let $\psi_j(a, z)$ be the mass of age-j agents with assets a and ability level z. Given $\psi_j(a,z)$, let

$$\tilde{f}_j(z) = \int \psi_j(a,z)\,\mathrm{d}a \tag{2.23}$$

be the skill distribution for age-j agents. In a steady state equilibrium, labor, capital, and goods markets must clear given the prices (r,w). The labor market equilibrium condition is given by.

$$\sum_{j=1}^{J_R-1} \mu_j \int_{\underline{a}}^{\bar{a}} \int_{z^*_{(j,a)}}^{\bar{z}} n(z,a;r,w)\psi_j(a,z)\,\mathrm{d}z\mathrm{d}a = F(z^*_{(j,a)},a) \sum_{i=1}^{J_R-1} \mu_j \tag{2.24}$$

where μ_j is the total mass of cohort j. The left-hand side is the labor demand from the $J_R - 1$ different cohorts of managers. The right-hand side is the fraction of each cohort employed as workers. For each cohort, given a, those under ability level $z^*_{(j,a)}$ choose to become workers, and there are mass of μ_j in each cohort. Labor supply comes from non-retired cohorts.

In the capital market, there are two sources of demand for savings. Managers demand capital to produce output. They also demand savings to invest in their managerial capital accumulation. Savings comes both from managers and workers of each cohort except for the oldest cohort, since they have no incentive to save. Thus, the capital market equilibrium condition can be written as :

$$\begin{aligned}
&\sum_{j=1}^{J_R-1} \mu_j \int_{\underline{a}}^{\bar{a}} \int_{z^*_{(j,a)}}^{\bar{z}} k(z,a;r,w)\psi_j(a,z)\,\mathrm{d}z\mathrm{d}a \\
+ \sum_{j=1}^{J_R-1} \mu_j \int_{\underline{a}}^{\bar{a}} \int_{z^*_{(j,a)}}^{\bar{z}} x_j(z,a)\psi_j(a,z)\,\mathrm{d}z\mathrm{d}a = &\sum_{j=1}^{J-1} \mu_j \int_{\underline{a}}^{\bar{a}} \int_{z^*_{(j,a)}}^{\bar{z}} a^w_j(a)\psi_j(a,z)\,\mathrm{d}z\mathrm{d}a \\
&+ \sum_{j=1}^{J-1} \mu_j \int_{\underline{a}}^{\bar{a}} \int_{z^*_{(j,a)}}^{\bar{z}} a^m_j(a)\psi_j(a,z)\,\mathrm{d}z\mathrm{d}a
\end{aligned} \tag{2.25}$$

The first term of the left-hand side is physical capital demand from the working cohorts of managers. The second term is the sum of investment in managerial capital of working managers up to one period before they retire. For instance, if they retire at age 4, there are 3 investment periods. These two terms comprise the demand for savings. The right-hand side terms are savings of workers and managers before they die.

The goods market equilibrium condition is that the aggregate output produced in the economy is equal to the sum of aggregate consumption plus investment in physical capital and managerial capital investments across cohorts by all managers and workers.

2.4 Quantitative Analysis

In this section, I calibrate the parameters of the model so that the steady state equilibrium of the model matches key features of the U.S. economy assuming no credit constraints. I vary the credit constraint parameter ϕ to see the effects of different levels of credit constraints on an economy if agents can optimally invest in their managerial capital over the life cycle. To assess the importance of endogenous managerial capital, I also consider a model with exogenous managerial capital. In the exogenous setup, managers dont have an option to optimally invest in their managerial capital. Instead, they are forced to invest as much as they do in the perfect credit benchmark.

2.4.1 Calibration

Parameter values in the benchmark model are calibrated so that the steady state equilibrium of the model matches features of U.S. firm size distribution and aggregate physical capital output ratio. In my calibration, I assume that the U.S .has a perfect credit market[4] as in Buera, Kaboski and Shin (2011). One period in the model corresponds to 10 years. Each cohort enters the model at age 20 and lives until 80. They work for 40 years, and during working periods they supply their labor inelastically. They stay retired for the remaining 20 years.

There are 9 parameters to calibrate. The share of physical capital in output is set at 0.317, as in Guner et al. (2008). Since the product of the importance of capital(α) and returns to scale(γ) responds to the share of physical capital in the model, α is determined from γ as $\alpha = 0.317/\gamma$. The depreciation rate (δ) and population growth(n) are set so that their annual rates are 0.06 and 0.011 respectively.

This leaves 6 parameters to calibrate: γ, β, θ_1, θ_2, μ_z, σ_z. I normalize the mean of the log of the skill distribution to zero and calibrate the 5 remaining parameters to match 4 moments of the U.S plant size distribution and the physical capital to output ratio: mean plant size, fraction of plants with less than 10 workers, fraction of plants with 100 or more workers, fraction of the labor force employed in plants with 100 or more employees and the physical capital to output ratio. The calibration successfully replicates the features of the U.S. plant size distribution. The fraction of small establishments is large(73%) but a substantial part(46%) of employment is at the large establishments. Tables 1 and 2 show the calibrated parameter values and the match to the U.S. data with perfect credit. The parameter values obtained from this calibration are used for the benchmark model and the exogenous setups.

[4] $\phi = 1$ corresponds to perfect credit, $\phi = 0$ corresponds to no credit

Table 2.1: Calibrated Parameter Values

Parameter	**Value**
Population Growth Rate (yearly) (n)	0.011
Depreciation Rate (yearly) (δ)	0.06
Importance of Capital (α)	0.417
Returns to Scale (γ)	0.7601
SD of log of Managerial Captial (σ_z)	2.2731
Discount Factor (yearly) (β)	0.94
Skill accumulation technology (θ_z)	0.9102
Skill accumulation technology (θ_x)	0.5172

Table 2.2: Fit of the benchmark model and data with parameter values in table 1

Statistic	**Data**	**Model**
Average Firm Size	17.9	17.9
Physical Capital (yearly) to Output ratio	0.23	0.23
Fraction of small (0-9 workers) Firms	0.73	0.74
Fraction of large (100+ workers) Firms	0.026	0.022
Employment Share of Large Firms	0.46	0.46

2.5 Results

Having calibrated the model to match the firm size distribution of the U.S. and the physical capital output ratio, I now use the calibrated parameter values and vary the parameter ϕ that governs the strictness of credit constraints. First, I will look at the steady state equilibrium statistics at different values of ϕ and addresss the effect of credit constraint on an economy when agents can optimally invest in their managerial capital over the life cycle. Results are presented in Table 3. TFP is measured as follwing: $TFP = Y/(K^{\alpha}L^{(1-\alpha)})^{\gamma}$.[5]

Table 2.3: **Financial friction with managerial capital investment decisions: Denoted as % of $\phi = 1$ value**

Statistic	$\phi = 1$	$\phi = 0.5$	$\phi = 0.4$	$\phi = 0.3$
TFP	100	102.5	102.6	102.7
Y	100	99.3	95.4	96.2
K/Y	100	91	83	85
X/Y	100	127	136	144
K+X	100	104	87	90
H	100	114.8	117.7	121.5
Mean Firm Size	100	84	73	65
Mean Profit	100	115	118	118
Manager fraction	100	117	134	149
Mean ability	100	98	88	88
MPK variance(level)	0	0.087	0.243	0.350
EF/Y	100	90	79	77

Y	Total output
K	Total amount of physical capital
X	Total investment in managerial capital
H	Total amount of managerial capital held by managers
MPK	Marginal productivity of physical capital
EF/Y	External finance to GDP ratio

[5] I tried with different TFP measures. The direction of change is robust to many of those measures. For detail, see section 5.2

In a model without managerial capital investment decisions, tighter credit constraint misallocates resources across production units, lowering aggregate output, aggregate physical capital, and aggregate measured TFP. However, in my model, aggregate TFP increases with tighter credit constraint. Quantitatively, the TFP measure increases by 2.7% with a credit constraint that reduces external finance to GDP ratio by 23%. With endogenous managerial capital investment decisions, tighter credit constraints worsen the allocation of physical capital across production units but improve underlying productivity distribution by encouraging managers to substitute away from physical capital to investment in managerial capital.[6] While the total amount of physcial capital falls with tighter credit constraint, the total amount of managerial capital increases.[7] The traditional TFP measure improves because the accumulation of managerial capital offsets the adverse effect of credit constraint that is caused by misallocation. Quantitatively, the total amount of investment in managerial capital increases by more than 38%, while total amount of physical capital decreases by 13.8%, with the credit constraint that lowers the external-finance-to-GDP ratio by 23%.[8]

In the model, there are two distinct incentives for a manager to invest in his managerial capital. First, having more managerial capital in the next period will allow him to borrow more physical capital in the next period, since collateral is

[6] Managers are able to channel their resources toward managerial capital with tighter constraint because credit constraint does not restrict investment in managerial capital, while it limits the borrowing of physical capital directly. Managerial capital investment is also restricted indirectly through inter-temporal borrowing constraint since borrowing is not allowed in my model; managers cannot borrow to invest in their managerial capital. However, this indirect restriction is not positively correlated with the strictness of the physical credit constraints.

[7] Although the total amount of managerial capital increases, average managerial capital falls as less productive managers enter the market with tighter credit constraints. The average is taken using mass of firms at each managerial capital level as weight. However, large firms with more than 100 workers produce more than 45% of total production. Using the mass of production of firms as weight, average managerial capital rises and then decreases with tighter credit constraints.

[8] However, the investment in managerial capital is much smaller than the amount of physical capital in level. Thus, the sum of physical capital and managerial capital investment decreases (by 9.9%) with tighter credit constraints. Managers are substituting away from physical capital to managerial capital but by less than one-to-one.

a function of productivity. Second, with the same amount of physical capital, he can have higher profits with higher productivity, and managerial capital investment improves one's productivity. I call the first incentive the *collateral incentive* and the latter the *profit incentive*.

The collateral incentive is stronger the higher ϕ is because ϕ governs the fraction of a manager's profit that can be redeemed for collateral. If $\phi = 0$, the collateral incentive is gone and only the productivity incentive is present. Therefore, other things being equal, tighter credit constraint will discourage managers from investing in managerial capital. However, credit constraint will also depress factor prices as it restricts total demand for inputs at a given underlying productivity distribution. Lower costs of production lead to higher profits for managers holding other state variables and credit constraint parameter constant. Therefore, in a general equilibrium in which factor prices adjust to clear capital markets, the profit incentive grows stronger with tighter credit constant.[9] Thus, depending on which incentive dominates, managerial capital investment of a manager of a particular characteristic (age, asset, current managerial capital), could either increase or decrease with tighter credit constraints. In my model, profit incentive dominates and managers engage in more active investment in managerial capital as credit tightens.

Another important feature of my model is that credit constraint and dispersion of firm size distribution has a non-monotonic relation. Credit constraints limit the size of firms and induce the entry of less productive agents into entrepreneurship. As a result, there is a larger mass of smaller firms with tighter credit constraints. At the same time, lower factor costs encourages managers to invest more in their managerial

[9] If I set the price level equal to that of perfect credit benchmark and tightens the credit market, I can erase the profit incentive and factor out how credit constraint discourages investment in managerial capital through weaker collateral incentives. Under this partial equilibrium, the managerial-capital-to-output ratio falls by more than 20%, while physical-capital-to-output ratio falls by more than 34%. Hence, the general equilibrium has substantially different implications from partial equilibrium regarding investment in managerial capital. Managerial capital investment increases in a general equilibrium while it falls in a partial equilibrium.

capital and there is a larger mass of very productive managers in the economy. The dispersion in the underlying productivity distribution has the potential to give rise to a larger dispersion in firm size with tighter credit constraints. However, severe credit constraints induce productive managers to optimally choose to reduce the number of workers they hire in spite of their higher managerial capital, and dispersion in actual firm size could also shrink with tighter credit constraint. With the calibrated parameters that I have, as credit tightens, the mass of large firms (with more than 100 employees) increases first and then decreases.

An economy with credit constraints could have a larger amount of managerial capital and have higher measured TFP but will still produce less than the distortion free economy. Therefore, when investment in managerial capital decision is endogenous, and is thus affected by distortions in the economy, traditional TFP measure is incapable of capturing the true inefficiency of the market because it does not take into account of the allocation of resources between managerial capital and physical capital. The implication is that an economy with lower measured TFP could be producing more efficiently with better allocation of resources across tangible and non-tangible capital, such as managerial capital. Large TFP could be the result of sub-optimal choice of agents in the economy to invest more in non-tangible (managerial) capital as they are restricted from investing in tangible (physical) capital.

2.5.1 Different TFP measures

If the non-monotonicity of TFP is caused only by the fact that the TFP measure that I used is not sufficient to capture the misallocative effect of credit constraint, particularly because it includes resources used for managerial capital investment as part of its output while those resources cannot be consumed, then, it is a mere problem of definition of TFP measure that is leading to this non-monotonic pattern of aggregate productivity as credit constraints tighten. To verify this, I used several different TFP measures, one of which is the following:

$$TFP = (Y - X)/(K^{\alpha} L^{(1-\alpha)})^{\gamma} Z^{1-\gamma}$$

Where X = Total amount of investment in managerial capital. It showed a similar non-monotonic pattern as credit tightens. Thus, even after taking account of the fact that resources used in managerial capital investment cannot be turned into consumption, total factor productivity improves with credit constraints if investment in managerial capital is endogenous and the dampening effect of managerial capital accumulation of credit constraints is robust.[10]

2.5.2 Endogenous vs. Exogenous

Having seen the features of the benchmark model, I will compare the benchmark model to a model without optimal managerial capital investment decisions. Through the comparison, I try to quantify the importance of endogenous managerial capital investment decisions in the model. Credit constraints quantitatively and qualitatively have different implications with and without endogenous managerial capital investment decisions. In the endogenous investment economy, as credit tightens, managers can substitute away from physical capital to managerial capital. Tighter credit constraints reduce the demand for physical capital and depress factor prices for both. And lower costs of production lead to larger average profits of managers.

In the endogenous case, managers use the profits to invest in their managerial capital, increasing the total amount of investment in managerial capital, and also the total amount of managerial capital present in the economy. Thus, the economy

[10]The only measure of TFP that showed monotonically decreasing level with tighter credit constraint was the measure that fully takes in to account the total managerial capital held by managers:

$$TFP = Y/(K^{\alpha} L^{(1-\alpha)})^{\gamma} H^{1-\gamma})$$

Where H = Total amount of managerial capital held by managers. However, traditional TFP measures do not capture managerial capital or intangible capital as thoroughly as this measure.

can maintain higher average managerial capital in spite of the entry of less productive marginal managers with tighter constraint. Unlike managers in an endogenous economy, in which managers can increase their profits through solely investing in their managerial capital and use external financing to borrow physical capital, managers in the exogenous case are more likely to save and use less external finance as their managerial capital is fixed over time and the only way to increase their profits/consumption in the future is by saving more.

Table 2.4: **Effect of Financial frictions, Endogenous vs. Exogenous: Denoted as % of pefect credit** $\phi = 1$

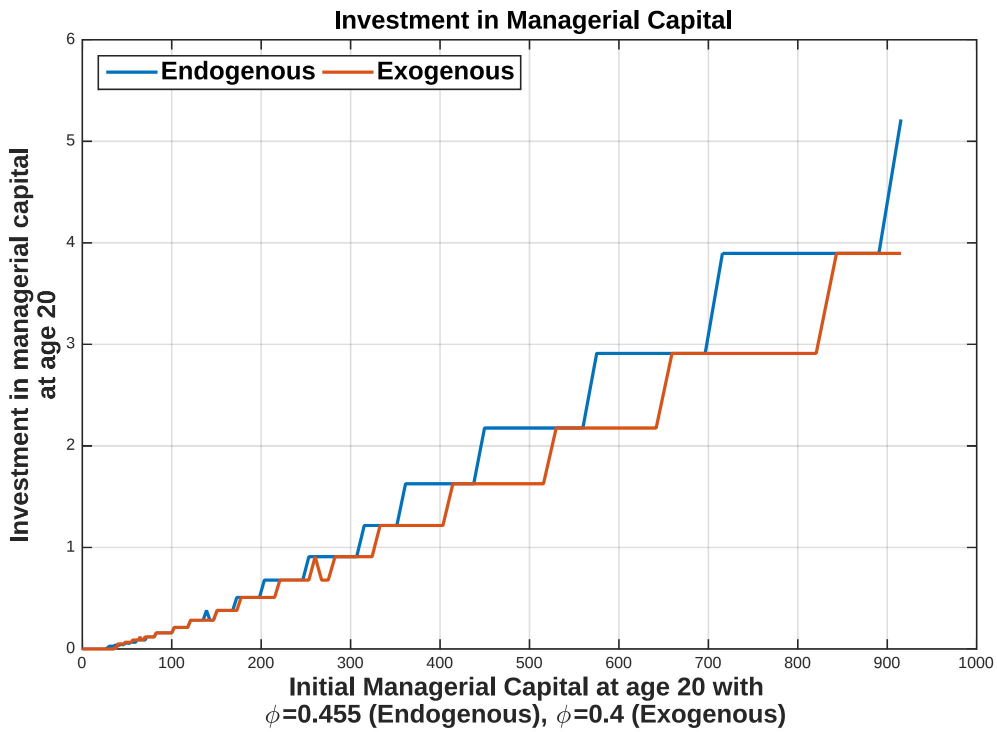
Statistic	Endogenous	Exogenous
TFP	102.5	99.7
Y	95.1	93.3
K/Y	83	87
X/Y	130	111
X	123	103
H	114.6	104.2
Mean Firm Size	78	70
Manager fraction	127	140
Mean ability	90	75
EF/Y	80.6	80.8

Endogenous	Optimal investment in managerial capital, Constrained, $\phi = 0.455$
Exogenous	Forced investment in managerial capital at the level of perfect credit investment, Constrained, $\phi =$
Y	Total output
K	Total amount of physical capital
X	Total investment in managerial capital
H	Total amount of managerial capital held by managers
EF/Y	External-finance-to-GDP ratio

In the exogenous setup, agents invest in managerial capital, but they are forced to invest the same amount as in the distortion-free benchmark model. Through the comparison I document some notable quantitative and qualitative differences between the two setups. Specifically, I will set credit constraint parameters for each case so that the external-finance-to-GDP ratio drops by the same proportion with respect to that of the perfect credit case for each. Then I compare the proportional change with respect to the distortion-free benchmark case of each for several aggregate statistics.[11]

The result shows that accumulation of additional managerial capital induced by credit constraint in the endogenous case dampens the adverse effect of credit constraint on the economy. Output falls less in endogenous case (4.9% vs. 6.7%), whereas physical capital output ratio falls more(16.5% vs. 13%) as managers substitute away from physical capital to managerial capital. Average managerial ability falls much less for the endogenous case (9.5% vs. 25%); total investment in managerial capital increases substantially more (23% vs. 3.8%). TFP increases by 2.3% in the endogenous case with tighter credit constraints while it decreases slightly (0.7%) in the exogenous case. Endogenous managerial capital decisions also contribute to larger dispersion in managerial income by inducing more productive agents to invest more in their managerial capital and thereby increase their productivity more than the less productive managers. Large dispersion in managerial ability leads to large dispersion in managerial income and also larger income gap between workers and managers. So, the adverse effect of credit constraint on income inequality is more pronounced with endogenous managerial investment decisions.

[11] In this particular exercise, I matched the drop to 19% for each



If a manager's managerial capital was fixed over time, a managers will try to save out of the credit constraint because his profit is constant over time if he didn't save. He needs to save to obtain higher profits in the future and to smooth his consumptions. However, with growing managerial capital over the life cycle and growing profits, managers find it not optimal to save. In particular, the more productive the manager is, the less attractive an option saving becomes compared to investment in managerial capital. Complementarity between current managerial capital and investment allows the more productive managers to achieve greater increases in their productivity with the same amount of managerial capital investment. As a result, their profits/managerial capital increase much more rapidly than less productive managers over the life cycle. In addition, consumption smoothing motives make more productive agents less eager to save than to consume. Therefore, the marginal productivity of physical capital of productive managers remains high throughout their working periods. They choose instead to increase managerial capital and stay constrained than to save. Under my calibration, the top 50% managers don't save at all, except for the last working period in which they have to save to consume during the retired periods. On the other hand, less productive managers save and do get out of credit constraints at the end of their working periods, and their marginal productivity of physical capital decreases over time.

To sum up, despite the higher value of TFP with a tighter credit constraint in endogenous case, the adverse impact of credit constraints on aggregate economy in terms of real production and consumption is still substantial. In my model, an economy with a credit constraint that lowers the external-finance-to-GDP ratio by 18.6%, has a measured TFP that is 2.3% higher than that of the distortion-free, first-best benchmark economy. Alternatively, in the exogenous case, measured TFP falls by 1.8% relative to perfect-credit TFP.

2.6 Conclusion

In a model in which the underlying productivity distribution is also affected by the credit constraints, measured TFP does not monotonically decrease with tighter credit constraints. Limited access to physical capital will encourage agents to substitute away from physical capital investment to investment in managerial capital. The accumulation of managerial capital will change the underlying productivity distribution and increase aggregate measured TFP. Also, improvement in the underlying productivity distribution will alleviate the adverse effects of a credit constraint on the economy and thus lead to a less radical fall in output and average firm size compared to a model without optimal managerial capital investement decisions. However, under a tighter credit constraint, dispersion in the firm size and the firm profits could be more pronounced with optimal managerial capital investment decision compared to a model without it.

Chapter 3

Endogenous Managerial Capital with Stochastic Skill Accumulation and Financial Frictions

3.1 Introduction

In the previous section, the model assumed that there is no stochastic factor in skill accumulation. Agents knew how much his next period's managerial capital would grow in relation to resources he invested in the previous period. However, if the skill accumulation process has a stochastic component, investment in managerial capital becomes less attractive and it mitigates the mitigation effect of endogenous managerial capital on distorted economy. I analyze the effect of uncertainty in skill accumulation on an economy when it has financial friction.

In section 2, I re-introduce model. In section 3, I compare the model with stochastic component and the model without stochastic component. Then I proceed to see the role of optimal managerial capital investment decision by comparing endogenous case with two different exogenous-type model when skill accumulation function has a stochastic component. In section 4, I vary stochastic parameter in the model and verify how large the second moment of shock should be to eliminate the mitigating effect of optimal managerial capital investment decisions in the model. I find that with sufficiently large second moment of shock (that could increase 1 percent of population's managerial capital by more than 6 times) in skill accumulation function, despite endogenous managerial capital investment, agents optimally choose not to invest in managerial capital even with tighter credit constraints. As a result aggregate measured TFP falls as much as in a model without endogenous managerial capital investment decisions.

3.2 Model with Stochastic Skill Accumulation

I add a stochasticity element to the model by assuming the managerial capital accumulation process has a random part. I refer to this setup as the *stochastic case* and the benchmark setup without stochastic component as the *non-stochastic case*. Specifically, managerial capital accumulation is assumed to follow the formula below.

$$z' = z + z^{\theta_z} x^{\theta_x} \epsilon \qquad \epsilon \sim lnN(-\frac{1}{2}\sigma_e^2, \sigma_e^2)$$

Due to the stochastic component in the managerial capital accumulation function, the next period's managerial capital will not be perfectly correlated with current period's investment in managerial capital. This uncertainty will discourage managers from investing in managerial capital. If managers are accumulating managerial capital less actively, its' dampening effect on credit constraint will diminish. Other than the skill accumulation function, the model setup is identical to the model setup in chapter 2 and only managers' problem and its' solution will be different from chapter 2. I am presenting the part that is different from chapter 2.

The problem of a manager of age j is given by

$$M_j(z,a) = \max_{x,a'} E\{log(c) + \beta V_{j+1}(z',a')\} \tag{3.1}$$

subject to

$$c + x + a' = \pi(z;r,w) + (1+r)a \quad \forall 1 \leq j < J_R - 1, \tag{3.2}$$

$$a' \geq 0 \tag{3.3}$$

and

$$z' = z + g(z,x) \quad \forall j < J_R - 1,$$

with

$$V_{J+1}(z,a) \begin{cases} 0 & \text{if } a \geq 0 \\ -\infty & \text{otherwise} \end{cases}$$

where $V_j(z,a)$ is a value function at period j defined as the maximum continuation value of becoming a manager at age j and becoming a worker at age j.

The solution to the dynamic programming problem is characterized by two conditions. First, the solution for next period's asset level, a', equation given below:

$$\frac{1}{c_j} \geq \beta(1+r)E\left[\frac{1}{c_{j+1}}\right]$$

Second, investment is determined by the no arbitrage condition below:

$$(1+r) = \pi_z\big(z_j;r,w\big)E\big[g_x(z_j,x_j)\big] \tag{3.4}$$

E is over next period's managerial capital.

3.3 Stochastic vs. Non-stochastic Managerial Capital Accumulation

I recalibrate parameters to target the same statistics that I used for calibrating the benchmark non-stochastic case. I will compare the effect of credit constraints in the stochastic case with the benchmark without stochastic component and address how the result is different if there is uncertainty in skill accumulation. In this section, I set the credit constraint parameter in each case so that external finance to GDP ratio in both cases falls by approximately 20%.

3.3.1 Calibration

Parameters are recalibrated to match the same target as the non-stochastic case. I set the variance of the log of stochastic component σ_e equals to 1 and set the mean of the log of ϵ so that expected value of ϵ equals 1.[1]

[1] Since the level of variance of shock that I chose is arbitrary, it is important to know the right way to decide the level of uncertainty in a model. I have tried different σ_es ranging from 1/3 to 1. Depending on the level of uncertainty agents face, the extent to which credit constraint distorts and lowers TFP and how investment in managerial capital dampens those effects is different.

Table 3.1: **Calibrated Parameter Values: Stochasitc**

Parameter	**Value**
Population Growth Rate (yearly) (n)	0.011
Depreciation rate (yearly) (δ)	0.06
Importance of Physical Capital (α)	0.419
Returns to Scale (γ)	0.7558
SD of Log-Managerial Capital (σ_z)	2.1132
Discount Factor (yearly)(β)	0.95
Skill accumulation technology (θ_z)	0.7715
Skill accumulation technology (θ_x)	0.5666
SD of shock (σ_ϵ)	1

Table 3.2: **Fit of the Benchmark Model and Data Statistics with the Calibrated Paramters**

Statistic	**Data**	**Model**
Average Firm Size	17.9	17.9
Physical Capital to Output Ratio	0.23	0.25
Fraction of small (0-9 workers) Firms	0.73	0.75
Fraction of large (100+ workers) Firms	0.026	0.02
Employment Share of Large Firms	0.46	0.46

3.3.2 Stochastic Case vs. Non-stochastic Case

With uncertainty in managerial capital accumulation, as credit tightens, managers will not invest in managerial capital as actively as in the case without a stochastic component. As a result, the dampening effect of endogenous managerial capital investment on credit constraints is itself dampened.

In my model, increase in investment in managerial capital in the stochastic case (17% increase) is 6% point lower than the increase investment in managerial capital in non-stochastic case (23% increase). As a result, the physical-capital-to-output ratio falls less in the stochastic case (12% vs. 17%) as managers are not substituting away from physical capital as much as in the non-stochastic case. Since the accumulation of managerial capital is less pronounced, the underlying productivity distribution changes less, and TFP increase is insignificant in the stochastic-case. Quantitatively, measured TFP increases less than 1% in the stochastic case, while it increases by 2.3% in the non-stochastic case.

In the stochastic case managers are more reluctant to invest in managerial capital and as a result, the total amount of capital (both managerial capital and physical capital) present in the economy is also lower(78% vs.82%). The larger drop in total amount of capital in the economy leads to a bigger drop in aggregate output (8.8% vs. 5.2%) for the stochastic case. Furthermore, with uncertainty in skill accumulation, becoming a manager is a less attractive option despite the lower cost of operation with tighter credit constraints. Therefore, the number of managers increases less than in the non-stochastic case and average firm size drops less. Since the marginal worker who becomes a manager in the stochastic case has higher productivity than one in the non-stochastic case, average managerial ability should fall less in the stochastic case, but because existing managers are not investing as much as they would have done in the non-stochastic case, the average managerial ability falls (by 8.6%) in the stochastic case while it increases (by 14.3%) in the non-stochastic case.

Table 3.3: **Financial friction with managerial capital investment decisions with/without stochastic component: Values are denoted as % of perfect credit;** $\phi = 1$

Statistic	BM, Constrained	Stochastic, Constrained
TFP	102.3	100.7
Y	95	91
K/Y	83	88
X	123	117
H	105.5	114.7
Mean Firm Size	77	86
Manager Fraction	128	115
Mean Managerial Capital	114	91
MPK variance(level)	0.1842	0.1322
EF/Y	79.9	79.9

Y	Total output
K	Total amount of physical capital
X	Total investment in managerial capital
H	Total amount of managerial capital held by managers
MPK	Marginal productivity of physical capital
EF/Y	External finance to GDP ratio
BM, Constrained	Benchmark case without stochastic component, $\phi = 0.45$
Stochastic, Constrained	With stochastic compoenent, $\phi = 0.44$

Variance of marginal productivity of physical capital is larger for the non-stochastic case. Less productive managers are more likely to save for the future. If less productive managers become more productive in the next period because of a shock, they can use those savings to be free of the credit constraint and the marginal productivity of physical capital for those managers would be lower than it would be without a shock. On the other hand, If a more productive manager becomes less productive because of a shock, he has not saved and he will be constrained. The marginal productivity of physical capital for those managers will increase as a result of the shock. Given that the marginal productivity of physical capital is higher for more productive managers, if managers hold the same amount of assets, the shock will work to reduce the dispersion of marginal productivity of physical capital across managers. Thus, in the stochastic case, the variance of marginal productivity is much lower than in the non-stochastic case.

3.3.3 Stochastic: Endogenous Case vs. Exogenous Case

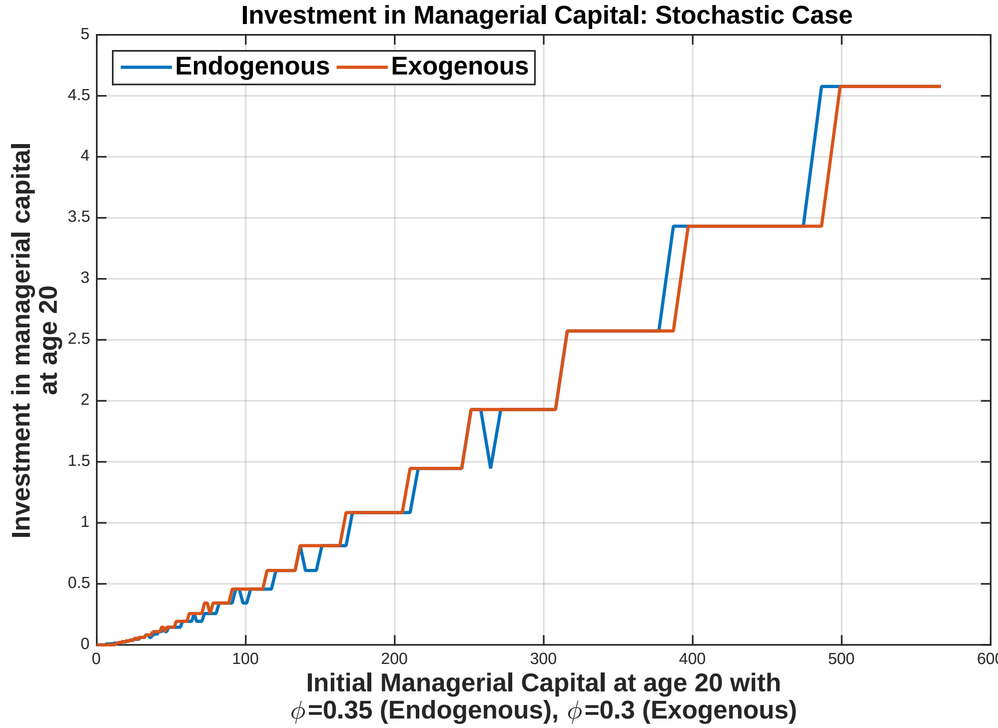
In this section, I am going to compare the two setups. Both have stochastic component in the managerial capital accumulation function, but one has optimal managerial capital investment decisions and the other has forced investment in managerial capital at the level identical to that in the perfect-credit endogenous case. By comparing these two setups, I find that even if opportunity to optimally invest in managerial capital is present in the economy, as the friction that prevents managers from actively investing in managerial capital becomes larger, the offsetting effect of optimal investment decision on credit constraint weakens and could become non-existent. The following results (Table 8) show that endogenous setup and fixed investment setup do not differ in aggregate measure of productivity if the variance of shock in managerial accumulation process is sufficiently large.[2]

[2]The variance of shock used is 1 ($\sigma_e^2 = 1$).

Table 3.4: **Financial friction with managerial capital Investment decisions vs. Fixed investment in managerial capital: Both with stochastic element. Denoted as % of perfect credit;** $\phi = 1$

Statistic	Endogenous	Exogenous
TFP	99.9	99.7
Y	86	87
K/Y	73	74
X	117	112
H	104	104
Mean Firm Size	81	79
Manager Fraction	122	125
Mean Managerial Capital	85	83
MPK Variance (level)	0.3295	0.3568
EF/Y	71.7	71.5

Y	Total output
K	Total amount of physical capital
X	Total investment in managerial capital
H	Total amount of managerial capital held by managers
MPK	Marginal productivity of physical capital
EF/Y	External finance to GDP ratio
Endogenuos	Optimal investment in managerial capital, $\phi = 0.35$
Exogenous	Forced investment in managerail capital at the level of perfect credit case, $\phi = 0.3$



In the endogenous case, agents can optimally invest in managerial capital. However, uncertainty prevents them from investing actively in managerial capital. Still, managers in the endogenous case invest slightly more than in the exogenous case (117% of the constraint-free economy, while in the exogenous set up, investment in managerial capital is 112% of the constraint-free economy), in which agents are forced to invest as much as the constraint-free economy. In the fixed investment case, agents depend less on external finance and choose to save. Therefore, the external-fianance-to-GDP ratio falls less rapidly than the endogenous case. Therefore, to have the same proportional drop in the external-finance-to-GDP ratio, the credit constraint parameter ϕ has to be smaller(stricter). With a stricter credit constraint parameter, firms will hire a smaller number of workers, reducing average firm size and encouraging a larger fraction of workers to become a manager. Therefore, the fraction of managers is higher in the constrained exogenous case than in the constrained endogenous case. This additional entry of managers will offset the lower total investment in managerial capital in the constrained, exogenous case and, in the end, the total amount of managerial capital in the economy increases the same amount in the two cases as credit tightens. Therefore, the magnitude of the fall in output is similar in the two constrained cases. The TFP fall is also similar in the two cases. However, in the constrained exogenous case, there is a larger fraction of firms with low productivity and smaller size (with less than 10 workers) than in the constrained endogenous case and the TFP is slightly lower in the exogenous case.

3.3.4 Endogenous vs. Fixed managerial capital distribution

In this section, I am going to compare the benchmark economy with the economy in which investment in managerial capital is not allowed and thus productivity of managers is fixed over the life-cycle. Through the comparison I want to address differences in the results if I had used the model without growth of managerial capital over the life cycle. As is done in the previous section, I change the credit constraint parameter for each case so that external-finance-to-GDP ratio drop matches each other.[3]

The first period distribution of managerial capital for the exogenous case comes from the steady state distribution of endogenous managerial capital investment model without credit constraint. Thus, the exogenous economy starts from a more dispersed managerial capital distribution than the beginning managerial capital distribution of the endogenous case. The steady state distribution of managerial capital for each setup under perfect credit is identical. In both cases agents start with the same amount of asset equals to zero.

[3] I set the credit constraints parameter in each case so that external finance to GDP ratio falls by 18.6% compared to that of the economy with no credit constraints. With perfect credit ($\phi = 1$), external finance to GDP ratio is 17% in the endogenous case and 13.9% in the exogenous case. According to Buera, Kaboski and Shin(2011), external dependence is 21% for manufacturing and 9% for services. Both numbers are within this range. To obtain an 18.6% decrease in external-finance-to-GDP ratio in each case, I set $\phi = 0.4578$ (endogenous), $\phi = 0.3$ (exogenous) for each.

Table 3.5: **Financial Friction with and without Managerial Capital Investment decisions: Denoted as % of Perfect Credit $\phi = 1$**

Statistic	**Endogenous**	**Fixed**
TFP	102.3	98.2
Y	95.4	96.3
K/Y	84.1	97.9
H	115	102
Mean Firm Size	78	76
Manager Fraction	127	130
Mean Managerial Capital	91	79
EF/Y	81.4	81.4
X/Y	130	
X	124	

Y	Total output
K	Total amount of physical capital
H	Total amount of managerial capital held by managers
Endogenous	Optimal investment in managerial capital, Constrained, $\phi = 0.4578$
Fixed	Without investment in managerial capital, Constrained, $\phi = 0.3$
EF/Y	External finance to GDP ratio
X	Total investment in managerial capital

In both setups, managers are almost always constrained, but the managers in the endogenous case are relying more on external finance while they focus on improving their productivity instead of saving.[4] The external-finance-to-GDP ratio of the ex-

[4] I compare the endogenous setup and exogenous setup at different ϕ so that the amount of proportional change of external finance to GDP ratio from the constraint free benchmark is equal at 18.6%. The corresponding ϕ that achieves this fall in each case is $\phi = 0.4578$ (endogenous) and

ogenous case is smaller than that of the endogenous case for a given credit constraint level ϕ, since managers are saving to get out of credit constraint.

The total amount of physical capital is much higher in the exogenous case. This is partly because investment in physical capital is substituted by investment in managerial capital in the endogenous case. Managers decide to channel their resources in managerial capital investment instead of physical capital investment. However, even after taking into account the increase in managerial capital, physical capital falls more drastically in the endogenous case compared to the exogenous case. To depress the external-finance-to-GDP ratio down to 11% so that it is 18.6% lower than the already low external finance to GDP ratio of perfect credit exogenous case(14%), the credit constraint parameter ϕ has to be much lower (0.3 compared to 0.4578) in endogenous case. A stricter parameter results in a large fall in physical capital price and much larger profit for managers in the exogenous case. As profit is also part of a manager's collateral, higher profit means that a manager can borrow a larger amount of physical capital given his managerial capital and assets and it offsets some of drop in physical capital from the constraint. Therefore, the fall in aggregate physical capital as a result of credit constraints is much larger in the endogenous case where factor prices do not have to drop as much to clear the capital market.

Despite the increase in investment in managerial capital with tighter credit constraint in the endogenous case, drastic fall in physical capital results in lower output and output falls 0.9% points more in the endogenous case compared to the exogenous case. This is partly because under my calibration, the importance of physical capital in production($\alpha * \gamma = 0.317$) is larger than the importance managerial capital in production ($1 - \gamma = 0.2442$). Considering that some outputs are used for investment in managerial capital which cannot be used for consumption, the fall in output available

$\phi = 0.3$ (exogenous) respectively. External -finance-to-GDP ratio in the perfect credit benchmark is 17% in the endogenous case and 14% in the exogenous case. Setting $\phi = 0.3$ for the endogenous case and comparing it with the exogenous case with $\phi = 0.3$ still displays qualitatively similar results.

for consumption is even larger in the endogenous case. Subtracting total investment in managerial capital from total output, endogenous case output available for consumptions falls by 5.2% with the credit constraint and it is 1.5% points larger than the fall in output due to credit constraint in the exogenous case.

3.4 Sensitivity to the size of Stochastic Component

In this section, I vary parameter σ_ϵ which governs the level of uncertainty in skill accumulation in the model and calibrate the rest of parameters so that steady state equilibrium of the model matches key features of the U.S. economy assuming there is no credit constraint(ϕ is fixed at 1). Then, for each model with different σ_ϵ, I depress the credit constraint parameter until the external finance to GDP ratiol drops by the same proportion across models. Then I anaylze the effects of credit constraints on economy at different levels of uncertainty with optimal investment decision in managerial capital.

3.4.1 Calibration

I calibrate models for different values of σ_ϵ. Parameter values in a model with given σ_ϵ are calibrated so that the steady state equilibrium of the model matches features of U.S. firm size distribution data and aggregate physical capital output ratio. In my calibration, I assume that the U.S .has a perfect credit market[5] as in Buera, Kaboski and Shin (2011). One period in the model corresponds to 10 years. Each cohort enters the model at age 20 and lives until 80. They work for 40 years, and during working periods they supply their labor inelastically. They stay retired for the remaining 20 years.

[5] $\phi = 1$ corresponds to perfect credit, $\phi = 0$ corresponds to no credit

There are 9 parameters to calibrate in each case. The physical capital to output ratio is set at 0.317, as in Guner et al. (2008). Since the product of the importance of capital(α) and returns to scale(γ) responds to the share of physical capital in the model, α is determined from γ as $\alpha = 0.317/\gamma$. The depreciation rate (δ) and population growth(n) are set so that their annual rates are 0.06 and 0.011 respectively.

This leaves 6 parameters to calibrate: γ, β, θ_1, θ_2, μ_z, σ_z. I normalize the mean of the log of the skill distribution to zero and calibrate the 5 remaining parameters given σ_ϵ to match 4 moments of the U.S plant size distribution and the physical capital to output ratio: mean plant size, fraction of plants with less than 10 workers, fraction of plants with 100 or more workers, fraction of the labor force employed in plants with 100 or more employees and the physical capital to output ratio. The calibration successfully replicates the features of the U.S. plant size distribution. In the U.S, the fraction of small establishments is large(73%) but a substantial part(46%) of employment is at the large establishments. Tables in the next section show the calibrated parameter values for a given stochastic paramter σ_ϵ and the match to the U.S. data with perfect credit. The parameter values obtained from this calibration are used for the benchmark case with perfect credit for each σ_ϵ.

Table 3.6: Parameter values when $\sigma_\epsilon = \frac{1}{3}$, $\phi = 1$

Calibrated Parameters	**Value**
SD of initial log-managerial capital (σ_z)	2.0526
Skill accumulation technology (θ_z)	0.9009
Skill accumulation technology (θ_x)	0.5132
Returns to Scale (γ)	0.7564
Discount Factor (yearly) (β)	0.9411
Assigned Paramters	**Value**
Stochasticity Parameter (σ_ϵ)	0.33
Depreciation Rate (yearly) (δ)	0.06
Population Growth Rate (yearly) (n)	0.011
Importance of Capital (α)	0.4191
Mean of log initial skill distribution (μ_z)	0

Table 3.7: Parameter values when $\sigma_\epsilon = \frac{2}{3}$, $\phi = 1$

Calibrated Parameters	**Value**
SD of initial log-managerial capital (σ_z)	2.08
Skill accumulation technology (θ_z)	0.8013
Skill accumulation technology (θ_x)	0.5779
Returns to Scale (γ)	0.756
Discount Factor (yearly) (β)	0.9459
Assigned Paramters	**Value**
Stochasticity Parameter (σ_ϵ)	0.67
Depreciation Rate (yearly) (δ)	0.06
Population Growth Rate (yearly) (n)	0.011
Importance of Capital (α)	0.4193
Mean of log initial skill distribution (μ_z)	0

Table 3.8: Parameter values when $\sigma_\epsilon = 1$, $\phi = 1$

Calibrated Parameters	**Value**
SD of initial log-managerial capital (σ_z)	2.2971
Skill accumulation technology (θ_z)	0.6293
Skill accumulation technology (θ_x)	0.6966
Returns to Scale (γ)	0.7672
Discount Factor (yearly) (β)	0.9372
Assigned Paramters	**Value**
Stochasticity Parameter (σ_ϵ)	1
Depreciation Rate (yearly) (δ)	0.06
Population Growth Rate (yearly) (n)	0.011
Importance of Capital (α)	0.4132
Mean of log initial skill distribution (μ_z)	0

Table 3.9: Fit of the model and U.S. firm data for different values of σ_ϵ, with $\phi = 1$

Statistic	**Data**	$\sigma_\epsilon = \frac{1}{3}$	$\sigma_\epsilon = \frac{2}{3}$	$\sigma_\epsilon = 1$
Average Firm Size	17.9	17.8	17.9	17.9
Physical Capital (yearly) to Output ratio	0.23	0.24	0.25	0.22
Fraction of small (0-9 workers) Firms	0.73	0.78	0.72	0.69
Fraction of large (100+ workers) Firms	0.026	0.028	0.026	0.030
Employment Share of Large Firms	0.46	0.46	0.46	0.42

Standard deviation of initial log-managerial capital (σ_z), importance of current managerial capital in skill accumulation (θ_z), importance of current investment in managerial capital in skill accumulation (θ_x), span of control parameter (γ) are all very critical parameters that jointly determine firm size distribution. As the variance of shock becomes bigger, agents are less willing to become a manager and fraction of managers shrinks. Furthermore, the steady state dispersion of managerial capital of operating managers will be larger with higher shock as some are hit by consecutive positive shocks and become very productive regardless of their investment in managerial capital. Greater dispersion of managerial capital (productivity) will lead to a larger employment share of large firms. If the variance of shock overpowers the initial managerial capital distribution, only very few who were lucky to get consecutive positive shocks will operate as manager and employment share of larger firms will be too high (higher than 46% which was the target). To lean against the effect of larger shock, initial managerial capital distribution has to be more dispersed to match the employment share distribution as variance of shock increases. Also, larger shock in skill accumulation will deter investment in managerial capital. To offset this and encourage managers to invest in managerial capital despite of higher risk, θ_x should increase while θ_z should drop. This will lead to the parameter values that I have.[6] $1 - \gamma$ will decide how important managerial capital is in production and thus will affect calibration of all other parameter values. [7]

3.4.2 Results

Using the calibrated parameters for each σ_ϵ, I now vary the credit constraint parameter ϕ that governs the strictness of credit constraint and analyze how credit constraint affects economy at each level of stochastic shock on skill accumulation. I will compare

[6] Although there could be completely different set of parameter values than the ones presented here that still matches the statistic from data, the parameter values presented here are robust to around 10 percent of fluctuation in initial values. i.e. even if I set the initial values of parameters either above/below the numbers found here, the parameters will converge to similar numbers that are presented here.

[7] In this paper, parameter values are found when γ is calibrated around 0.75(i.e importance of managerial capital in production is approximately around 0.25) and is not very radical. However analyzing the extent to which quantitative results could change with higher/lower γ specification would be interesting. I shall do it as a robustness check in the future.

the proportional change in steady state equilibrium statistics as credit tightens to depress external finance to GDP ratio by the same amount across models with different stochasticity parameter σ_ϵ. For all specifications, agents invest in managerial capital optimally.

Table 3.10: **Effects of Financial friction at different levels of σ_ϵ: Denoted as % of perfect credit ($\phi = 1$) value**

Statistic	$\sigma_\epsilon = \frac{1}{3}$	$\sigma_\epsilon = \frac{2}{3}$	$\sigma_\epsilon = 1$
TFP	102.8	100	98.6
Y	91.7	87.6	85.2
K/Y	76	75.5	74
X/Y	151	138	157
H	118.5	107.1	102.4
Mean Firm Size	73	70	81
Manager fraction	133	139.4	121.7
EF/Y	72	72	72
ϕ	30	30	23

Y	Total output
K	Total amount of physical capital
X	Total investment in managerial capital
H	Total amount of managerial capital held by managers
EF/Y	External finance to GDP ratio
ϕ	Credit constraint parameter,

Figures in the table are denoted as percentage of the case with perfect credit ($\phi = 1$). With low levels of uncertainty ($\sigma_\epsilon^2 = \frac{1}{3}$), measured TFP increased with a tighter credit constraint that depressed external finance to GDP ratio by 28%. This is because as credit tightens, managers can only use fraction of their current period profit as collateral and this limits managers ability to operate at the efficient level of production. Tighter credit constraint will lower factor price since the demand for both labor and physical capital is suppressed by the constraint. Ceteris paribus, lower factor costs will increase the profit of a firm operating with a given skill level. Instead of saving, firms will optimally decide to use these profits to accumulate managerial capital for the following reasons. First, holding the same amount of physical capital, larger amount of managerial capital will bring higher profits because higher managerial capital leads to higher productivity of a firm operated by the manager. Second, the increased profit will allow managers to borrow more physical capital and get out of credit constraints. While investing in physical capital (saving) will only bring the second benefit, investing in managerial capital begets higher return by boosting up productivity as well as allowing firms to get out of credit constraints.

As a result, with endogenous managerial capital investment decisions tighter credit constraint encourages accumulation of managerial capital and this will improve the underlying productivity distribution of firms. This in turn will offset part of adverse effects of credit constraints caused by distorting allocation of resources across production units. Sometimes the offsetting force will be so large as to overpower the adverse effects caused by credit constraints and measured TFP in a constrained economy could be even larger than measured TFP in a constrain-free economy.

This is because measured TFP only captures tangible capital as input. (If we take total amount of managerial capital present in the economy as input as well, the measured TFP will monotonically decrease with tighter credit constraints.) We observe that the measured TFP increases by 2.8% with a credit constraint that depresses physical capital to output ratio by 28% in a model with low uncertainty in skill accumulation($\sigma_\epsilon^2 = \frac{1}{3}$).

However, bigger uncertainty (larger σ_ϵ) in skill accumulation will offset the offsetting effects of managerial capital accumulation on credit constrained economy. If a manger faces higher level of uncertainty in skill accumulation he will accumulate less managerial capital (H). Quantitatively, at a low level of uncertainty ($\sigma_\epsilon^2 = \frac{1}{3}$), managers accumulated 18.5 percent more managerial capital with tighter credit constraint. On the other hand, at a higher level of uncertainty($\sigma_\epsilon^2 = \frac{2}{3}$), managerial capital H only increases by 7.1%. with tighter credit constraint and if the uncertainty is very large ($\sigma_\epsilon^2 = 1$), H increases by 2.4 If we used a different TFP measure which captures total amount of managerial capital present in the economy as a form of input, [8] then the TFP decreases monotonically with tighter credit constraints and it decreases slightly more with higher uncertainty. This is because with higher uncertainty in skill accumulation, agents cannot prepare for the future credit constraint as they would do in a economy with less uncertainty. Hence people will be constrained unexpectedly and this will exacerbate the effect of credit constraint in the economy lowering efficiency measured by aggregate TFP. This will also lead to more drastic fall in aggregate output in a economy with higher uncertainty (15% when $\sigma_\epsilon^2 = 1$ vs. 8.4% when $\sigma_\epsilon^2 = \frac{1}{3}$).

However, if the level of shock becomes very large it stops to exert any further negative interaction with credit constraints. The major channel through which uncertainty harms economy is by preventing accumulation of managerial capital. From certain threshold of uncertainty, agents stop to invest in managerial capital at all. Hence higher uncertainty in skill accumulation will only negatively affect the economy by preventing consumption smoothing of an individual.

Quantitatively, under my calibration, $\sigma_\epsilon^2 = \frac{1}{2}$ is large enough to wash away all the mitigation impact of endogenous managerial capital investment decision if credit constraint is such that external finance to GDP ratio falls by 28 percent. At this level of uncertainty, the measured TFP in a constrained economy is almost the same as the measured TFP in a constraint-free economy.

[8] TFP=$\frac{Y}{K^{\alpha\gamma}H^{1-\gamma}}$

Under $\sigma_\epsilon^2 = \frac{1}{3}$ each period, approximately 1 percent of agents will be hit by a shock that will more than double their current managerial capital. On the other hand, approximately 1 percent will be hit by a shock that will erode his current managerial capital by more than 60 percent. On the other hand if $\sigma_\epsilon^2 = \frac{2}{3}$, 1 percent of agents will be hit by a shock each period that will more than quadruple his managerial capital while unfortunate 1 percent will lose more than 80percent of his managerial capital.

If $\sigma_\epsilon^2 = 1$, 1 percent will be hit by a shock that raises the managerial capital more than 6 times while 1 percent will lose 94 percent of the current managerial capital. The huge uncertainty in skill accumulation will prevent managers from investing both in managerial capital and physical capital and thus measured TFP will fall with tighter credit constraint just as in a model without endogenous managerial capital investment decisions.

3.5 Conclusion

Optimal investment in managerial capital will offset the adverse effect of credit constraint on an economy by encouraging accumulation of managerial capital and shifting underlying productivity distribution. However, the existence of uncertainty in skill accumulation process will mitigate the effect by hindering investment in managerial capital. Larger the second moment of the shock, smaller the mitigation effect will be and after certain threshold, the distortionary effect of a credit constraint on allocative efficiency in a model with endogenous managerial capital investment decision will be the same as in a model without endogenous managerial capital investment decision.

Bibliography

[1] Banerjee, A. V. and E. Duflo 2005, *Growth Theory through the Lens of Development Economics*, vol. 1 of Handbook of Economic Growth, chap. 7, pp. 473552. Elsevier

[2] Benabou,R. 2002. *Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?*, Econometrica, 70(2), 481-517, March.

[3] Bloom, N., J. Van Reenen 2007, *Measuring and Explaining Management Practices Across Firms and Countries. Quarterly Journal of Economics*, 122(4), 1451-1408

[4] Buera, F. J., J. P. Kaboski, and Y. Shin 2011, *Finance and Development: A Tale of Two Sectors, American Economic Review*, 101(5), 19642002.

[5] Buera, F. J. and B. Moll 2012, *Aggregate Implications of a Credit Crunch*, NBER Working Papers No. 17905

[6] Buera, F. J., B. Moll, and Y. Shin 2013, *Well-Intended Policies, Review of Economic Dynamics*,16(1), 216230.

[7] Buera, F. J., J. P. Kaboski, and Y. Shin 2012, *The Macroeconomics of Microfinance*

[8] Caselli, F., 2005, *Accounting for Cross-Country Income Differences Handbook of Economic Growth* Volume 1, Part A, 2005, Pages 679741

[9] Caselli, F., and N. Gennaioli 2013, *Dynastic Management, Economic Inquiry*, 51(1), 971996.

[10] Bhattacharya, D., N. Guner, G. Ventura 2013, *Distortions, Endogenous Managerial Skills and Productivity Differences, Review of Economic Dynamics*,16(1),11-25, January.

[11] Fattal-Jaef, R. N 2012, *Entry, Exit and Misallocation Frictions*, Job market paper

[12] Guner, N., G.Ventura, and Y. Xu 2008, *Macroeconomic Implications of Size-Dependent Policies , Review of Economic Dynamics*, 11(4), 721-744

[13] Guvenen, F., B. Kuruscu and S. Ozkan, 2009, *Taxation of Human Capital and Wage Inequality: A Cross-Country Analysis*, NBER working paper 15526

[14] Hall, Robert E. and Charles I. Jones 1999, *Why do Some Countries Produce So Much More Output Per Worker than Others?*, The Quarterly Journal of Economics (1999) 114 (1): 83-116.

[15] Hsieh, C. and P. Klenow 2009, *Misallocation and Manufacturing TFP in China and India*, Quarterly Journal of Economics, 124(4): 1403-1448, November.

[16] Hsieh, C. and P. Klenow 2012, *The life-cycle of plants in India and Mexico*, NBER working paper 18133.

[17] Jeong, H. and R. Townsend 2007, *Sources of TFP Growth: Occupational Choice and Financial Deepening*, Economic Theory, 32, 179221.

[18] Midrigan, V., and D. Y. Xu 2014 *Finance and Misallocation: Evidence from Plant-Level Data*, American Economic Review, Vol. 104 No. 2 February 2014

[19] Moll, B. 2014, *Productivity Losses from Financial Frictions: Can Self-financing Undo Capital Misallocation?*, American Economic Review, Vol. 104 No. 10 October 2014: 3186-3221

[20] Poschke, M. and A. Gabler, 2011, *Growth through Experimentation , Society for Economic Dynamics*, 2011 Meeting Papers 643

[21] Restuccia, D and R. Rogerson, 2008, *Policy distortions and aggregate productivity with heterogenous establishments* , Review of Economic Dynamics, 11(4), 707-720

[22] Restuccia, D and R. Rogerson 2013, *Misallocation and Productivity*, Review of Economic Dynamics, 16(1), 1-10