A Regime-Aware Agent-Based Framework for Financial Planning

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Abstract

The vulnerability of individuals planning for retirement has been growing due to the conversion from defined-benefit plans to defined-contribution plans, the steady increase in life longevity, and the uncertainty of asset returns under an ever-changing global environment. A serious problem is the lack of appropriate planning for retirement. How much should an individual save beyond the Social Security tax in order to maintain a reasonable lifestyle after retirement?

This paper designs a framework to facilitate the process of setting realistic goals for financial planning, featuring the concept of agent-based simulations. The framework also provides policy-rule guidelines for the agent to search for an optimal strategy. Additionally, a micro-macro analysis enables us to analyze a cohort of representative agents and aggregate the individual results on the macro-level.

The simulation module employs a regime-based Monte Carlo simulation of multiple asset categories, a factor-based diversifying asset allocation approach, and a collection of dynamic policy-rule-based investment strategies. Empirical results, consisting of a downside risk simulation for university endowments, a sustainability assessment for the Social Security fund, and a personal goal-based retirement planning, demonstrate stylized applications of the planning framework.
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To my family.
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Chapter 1

Overview of the Financial Planning Framework

1.1 Financial Planning

Financial planning problems, also known as financial optimization, is one of the fascinating areas in optimization with uncertainty. It includes various topics such as optimizing a security portfolio for fund managers, asset allocation for institutional investors such as pension funds or university endowments, risk management for financial institutions such as banks, and capital planning for large multi-national companies.

There is a common characteristic shared by the aforementioned topics: the “curse of dimensionality” induced by the multiple stages in consideration. Fund managers need to rebalance their portfolio frequently; consequently, myopic strategies may harm long-term growth. For institutional investors, the planning horizon is always as long as centuries, if not forever. Similar situations apply to large financial institutions and multi-national companies.
In general, the approaches to financial planning can be categorized into four directions:

- **Stochastic control.** First introduced by Samuelson [103] and Merton [84], stochastic control problems involve the solution of Hamilton-Jacobi-Bellman (HJB) partial differential equations.

- **Dynamic programming.** Inspired by the breathtaking innovations in machine learning, there is much literature on applying reinforcement learning to financial planning problems (see [1], [88], for example.)

- **Stochastic programming.** At each decision point, the program generates several branches to represent the uncertainties. The whole program is a large-scale nonlinear program which could be tackled by optimization algorithms (e.g., [48].)

- **Rule-based simulations.** Instead of optimizing for the exact solution (we have witnessed its difficulty), policy rule simulation is applied to the scenarios generated according to the requirement of the system.

In this paper, we employ the rule-based simulation to explore agent-based financial planning with a long horizon. Monte Carlo methods have massive applications in finance; [47] provides an excellent overview of related works. We present below two areas that are pertinent to our research framework.

### 1.1.1 Portfolio Selection

Portfolio selection is a challenging problem in which we seek the “most preferred” allocation of different assets. The idea of modern portfolio optimization theory was first raised by Harry Markowitz in his epic work [77], which was developed
from his doctoral dissertation. In this paper, Markowitz first introduced the influence of risk on decisions and built his theory on modeling uncertainty. At first glance, the application of variance is straightforward; but it does provide a principle for us to measure the risk. Risk measurement had then become a widely discussed topic in quantitative finance, and with his pioneering work \cite{78} and \cite{79} in portfolio theory, Markowitz won the 1990 Nobel Prize in Economics. Following Markowitz’s work, Sharpe developed the capital asset pricing model (CAPM) in his work \cite{105}, which contributes to both pricing theories and capital market theories.

In chapter \ref{ch3.2}, we establish a mean-CVaR portfolio selection model that is based on simulated scenarios. The model provides a portfolio that outperforms the Markowitz portfolio both during and after crash periods, protecting investors from economic downturns.

### 1.1.2 Dynamic Asset Allocation

Merton further pushed the notion of uncertainty in his publications \cite{83} and \cite{84}, where he studied the utility function maximization under continuous-time as an extension to the single-period CAPM model. In addition, Mossin \cite{90} solved a similar problem in discrete settings. Since then, the Merton problem has been extended to include more risky assets and admit various distributions of asset returns.

Herein, we propose a long-term financial planning framework that includes a wide range of asset categories, adopts a regime-based asset return simulator and a stochastic macroeconomic generator and employs agent-based simulations to examine policy rules. We will introduce these features in the subsequent sections of this chapter. The structure of the framework is shown in figure \ref{fig1.1}.
Figure 1.1: Framework for Financial Planning

We highlight several novel aspects of our design:

- Switching regimes identified by historical data;
- Factor-driven performance attribution;
- Realistic economic environments, especially with regard to downside risk;
- Flexible building blocks, with any block ready to be replaced by models pertinent to current goals and resources;
- Application of machine-learning methods such as estimation of factor exposure and identification of economic regimes.

Each subsequent section of this chapter introduces one block in figure 1.1.
1.2 Regime Identification

One critical aspect of financial planning is to identify the current investment environment and estimate possible future courses.

The problem of monitoring the current economic condition has been an interesting and promising topic in economics research. The widely accepted estimate is given by the National Bureau of Economic Research (NBER), who calculates the turning points between “growth” periods and “contraction” periods from historical data since 1978. Although the NBER result is often used in business cycle analysis, it has many drawbacks: for instance, the underlying dataset was not quite transparently disclosed, and NBER was reluctant to update its estimate a posteriori (perhaps NBER is trying to avoid amendment, but this leads to significant lag in their update of turning dates, see [30] for a detailed explanation, for example).

The significantly lagged estimate leads to problems and inconvenience in business decision. Given this circumstance, other methods are also applied to real-time monitoring of the transition. Bry and Boschan [27] established a turning point program that locates turning points as (adjusted) local maximum and minimum in each period, while Hamilton [55] used a discrete-state Markov process combining with an auto-regression model. Layton [76] compared the two models to gain insight into the regime-switching model, whereas Harding and Pagan [34] added a Kalman filter approximation. Some recent works include Chauvet [29] who had proposed a dynamic factor model, and later Chauvet and Piger [30] compared this dynamic factor time series model with the non-parametric method developed by Harding and Pagan.
Similarly, we can separate the historical performance of an asset into distinct clusters that display similar characteristics or follow similar trends, which are called regimes. There have been many pieces of research on regime identifications in different asset categories and characteristics, such as Bae et al. [11], and Bilgili [17]. There has also been much research on capital allocation based on changing economic conditions and regimes, such as Ang and Bekaert [3] and [4], Bauer et al. [14], Collin-Dufresne et al. [32], Guidonlin and Timmermann [53], Kritzman et al. [74], Nystrup et al. [98], and Tu [117].

One widely recognized yet practically neglected fact is that the asset returns are not normally distributed. The Gaussian assumption induces beautiful theoretical results, but historical data informs us that, for instance, the distribution of equity returns is left-skewed and heavy-tailed\textsuperscript{1}. Thus, the probability of a severe loss is much higher than that indicated by (multivariate) normal distributions. There are often occurrences of more than five standard deviations from the mean, for example, the 1987 flash-crash and the 2008-2009 financial crisis.

During these crashes, we have observed a high correlation among equity-like assets, including worldwide equities, real estates, and high-yield bonds. This is a well-known situation called “contagion”. Not only does the volatility of most asset increase significantly, but also the correlation of returns can approach unity, causing the assets to move together as a downward spiral. There are multiple explanations of this situation: investors, especially inexperienced investors and fluttered investors, tend to panic under tensions; equity-like assets also share common factor exposures, so their “ingredients” of risk premia are very similar; there are liquidity constraints and stop-limits that forced institutional investors to sell low and search for safe

\textsuperscript{1}A Q-Q plot is shown in chapter 3.2.1 as illustration.
assets to protect them from future losses. Goldstein and Razin \[49\], and Kyle and Xiong \[75\] discuss various theories of financial crises which are supported by historical evidence.

The important lesson is that the contagion effect disrupts the anticipated diversification built into traditional models, and accordingly, the investors portfolio performs much worse than would be expected. As a result, drawdown can be severe, leading to a major setback for long-term investors who depend upon capital to achieve their goals and pay future liabilities. Therefore, a portfolio model must be able to depict future events with an eye to the worst-case, for example, maximum probability loss or maximum drawdown.

The multi-regime setting provides a natural advantage in the scenario generating system. First, it considers the skewness and excess kurtosis (fat tails) by introducing a “crash” regime. The asset performance under crash regime can be carved as a completely different distribution. The flexible adjustments of parameters also lead to a closer return profile than the multivariate normal distribution.

Next, we propose a Markovian switching mechanism for the regime framework. The consideration is that the markets sometimes turn to crash periods without any caveat. Take the flash-crash in October 1987 for example: the economy is running on the right track; there are no signs of recession. Therefore, when a crash happens, it is difficult to avoid the serious hit to investors’ portfolios. The simulation system needs to address such possibilities for a proper estimate of tail risks. Some previous works have been developing hidden Markov models (HMM) to set up a transition scheme between states, including \[39\] and \[28\].
Finally, empirical evidence is consistent with the contagion environment during crashes. The characteristics of crashes are well considered in the crash regime: the covariance matrix informs high volatility and high correlation of equity-like assets; we also observe high returns of safe assets such as U.S. government bonds and their negative correlation with equities. While it is at least as difficult to predict the timing of a crash as to predict the probability of a recession ahead, a carefully designed financial planning framework should be able to simulate the crash conditions in its scenarios, and especially, advocate policy rules and asset allocation rules that are designed to protect the users capital. The regime-based scenario generator is able to alert the potential risk in current portfolios, as shown in chapter 3.2.

1.3 Introduction to Factor Investing

In this chapter, we give a brief introduction to factor investing. At its core, factor reward investors for bearing the corresponding risks. The first factor model, the Capital Asset Pricing Model (CAPM), is based on the market risk.

1.3.1 CAPM

The CAPM model is a revolutionary model proposed by Sharpe in 1964. In his paper [106], Sharpe raised the opinion that the risk of an asset was not only the intrinsic characteristic of the asset itself but was also the level of co-movement with the market. In the CAPM model, the risk measure of an asset should be determined by the extent to which it is correlated to the market portfolio. An average investor holds the market portfolio, and any risky asset is exposed to the single systematic risk given by the market portfolio. This is the first documented factor in quantitative asset management; before the development of CAPM, investors believe that excess return to the mean-variance portfolio is obtained by management skills (pure alpha).
The CAPM, however, stated that the excess return over the risk-free rates could be explained by the risk of the asset. The risk is not simply defined by the asset’s volatility; rather, it is given by the level of co-movement of the asset with the market. The co-movement is called the beta of the asset, and the expected return of the asset is determined by the beta of the asset as well as the market return in excess to the risk-free rate. Sharpe also pointed out that the market portfolio has the highest risk-adjusted return among all portfolios that are mean-variance efficient. He introduced the famous Sharpe ratio, which is the ratio between excess return and volatility of the asset. We use this ratio for performance comparison in chapter 2.3.

1.3.2 Factor Model

Although CAPM is revolutionary, it is not quite effective practically. It predicts the asset or stock return purely by its exposure to the market risk, but the model relies on only one factor, and empirical results negate this point of view. Nonetheless, the underlying idea of CAPM is outstanding: the risk exposure is the source of return. Investors who are able to weather “bad times” caused by the crash of the factors are compensated with the excess returns when the market is growing.

In order to better explain the excess return, we introduce the multi-factor model that gives a multi-beta relation. The multi-factor investing theory is based on linear factor models. Suppose the return of an asset is \( r_i \), the factor model is given by

\[
   r_i = \alpha_i + \beta^i_1 f_1 + \cdots + \beta^i_n f_n + \epsilon_i. \tag{1.1}
\]

Where the \( \beta = (\beta^i_1, \ldots, \beta^i_n)^T \) represent the factor exposure of the asset, and the \( f_1, \ldots, f_n \) represent factor returns.
A typical factor should satisfy the following criterion:

- A factor should correspond to some risk premium, and thus should have some economic intuition or behavioral explanation to the question of why an investment in it should generate excess return.
- The factor return should be well-documented in academic literature and must persist over time (at least multiple decades).
- The factor return should be steady and pervasive, existing across regions and spanning across asset classes.
- The factor should be directly (or indirectly but conveniently) investable and liquid.

The factors may be divided into two groups: macroeconomic factors (which are more fundamental, sometimes non-investable) and style factors (also called investment factors or dynamic factors). The former consists of factors that reflect economic intuition, as stated in the first bullet point, and the latter mainly focuses on some characteristics shared by the stocks and assets that generate profit over time.

1.3.3 Macroeconomic Factors

The macroeconomic factors, also known as fundamental factors, are strong enough to affect all investors and assets. Though most of the macroeconomic factors are not investable, we may add these factors into our factor model because they have explanatory power.
Macro factors include:

- Economic growth and inflation rate. These factors influence investment decisions; later, we discuss in more details how they affect asset returns.

- Volatility. Under high volatility conditions, investors tend to choose conservative investment styles. In his 2006 paper [6], Ang measured the volatility risk via the VIX index (which is directly investable) and found that during periods of high volatility, the stock prices tend to drop as well. Moreover, stocks that have large exposure to volatility risk perform poorly during turbulence.

- Real Yields and Term Structure. They are related to interest risk and most often considered as indicators of investment conditions.

- Productivity Risk, Demographic Risk, Political Risk, and others. These risks are well documented in Ang’s book [2].

1.3.4 Micro Factors

Different from macroeconomic factors, the micro factors (also called “style factors”) represent different investing styles. For these style factors, we can either build a long-only portfolio (which is always called the smart beta portfolio) or if short-selling is permitted, build a long-short type portfolio (which is called style premia, since we are exploiting the risk premium).

Fama-French Factors

In 1992, Fama and French [42] presented in their best-known paper three factors: value, size, and market. They explained asset returns using these three factors because they discovered that return anomalies could sometimes be attributed to small-cap (size), high book-to-market ratio (value), or the market risk as in the CAPM model.
Later in their 2015 paper [43], they further developed factors such as profitability and level of aggressive investments.

**Momentum**

Price momentum, as documented by Jegadeesh and Timan [67], is the phenomenon whereby the stock with higher past returns generate higher returns over short-term and lower returns in the long run. Hence a strategy of buying winners and selling losers has been proven to generate positive returns over 3 to 12 months holding periods, before reversing after a year. Jagadeesh [68] and Bondt and Thaler [35] explained these results by supporting the argument that biased self-attribution is associated with under-reaction of the investors to market information such as earnings announcements. The positive short-lag autocorrelation is a result of these delayed reactions to the announcements. Momentum has been documented in various asset categories, including stocks, bonds, and surprisingly, micro factors.

Generally, to explain the momentum anomaly or the Fama-French factors, considerable thoughts have been given to behavioral biases of investors, which cause them to over-react or under-react to certain market information. With the significant influence of over- and under-pricing of investors reaction to information, recent studies have recognized attention as a crucial factor in investors reaction to information. Hou, Peng, and Xiong [58] gave a better explanation of the relationship between attention and under- and overreaction suggesting that overreaction-driven price momentum is more pronounced among those stocks that attract more investor attention (stocks with higher attention tend to enjoy the profit of momentum to a more significant extent), while underreaction-driven earning momentum is more pronounced among less attended stocks.
Carry

Carry is a well-known style, particularly in the currency markets. Fundamentally speaking, carry is based on investing (lending) in higher-yielding markets or assets and financing the position by shorting (borrowing) in lower-yielding markets or assets. A simplified description of carry is the return an investor would receive (net of financing) if market conditions remain the same. A classic application is often found in currency markets, where sorting countries by their short term lending rate, and going long the currencies of countries with the highest rates and short the currencies of countries with the lowest rates have been a profitable strategy over several decades. Likewise, carry strategies in fixed income, based on the shape of the yield curve, and commodity futures, where backwardation or contango in the futures maturity curve is exploited across various commodities, have also been profitable over time. In Koijen et al. [72], they define an assets carry as its return, assuming that market conditions stay the same.

The returns to the carry style are related to, but not explained by, other known return predictors: carry generates positive and unexplained alpha within each asset class relative to other known factors in each asset class.

Other Style Factors

This section briefly introduces two recently developed factors. We need time to examine whether they satisfy all the criteria to prevail, but at least myriad literature has been published to endorse these factors.

- Liquidity.

Liquidity is a long-disputed dynamic factor since its performance is not stable sometimes. [60] provides a thorough discussion of the liquidity factor.
• Quality Minus Junk (QMJ).

The quality-minus-junk factor was first introduced by Asness et al. in their paper [10]. The paper establishes a QMJ portfolio that goes long high-quality stocks and short low-quality stocks. The QMJ factor return turns out to be stable and uncorrelated with other factors.

• Betting Against Beta (Defensive).

The BAB (betting against beta) factor, first developed in [46]. This paper proposes a portfolio construction method which is long leveraged low-beta assets and short high-beta assets. Since this strategy prefers low-risk assets, it is sometimes called the defensive or hedging factor.

1.3.5 Summary

For macroeconomic factors, we are more concerned about understanding the performance of assets with the factors. If we view assets as cooked dishes, then the factors are the basic “nutritions” underneath them.

In recent years, alternative assets, including private equities, real assets, and hedge funds, have received much attention from institutional investors. Mulvey and Kim [94] discusses the role of alternatives in asset allocation. Swensen [111] discussed the gradual shift to alternative asset categories among university endowments. Mulvey and Holen [93] survey current asset categories for the largest university endowments in the United States, with focus on the way they are categorized. As such, we need the factor models to utilize their explanatory power and attribute the returns of alternative assets to the basic factors.

For style factors, on the other hand, we are more concerned with their ability
to generate profit. Discussions of the efficacy of factors can be found in [71]. We discuss the macroeconomic sensitivities of assets and portfolios in chapter 3.

1.4 Macroeconomic Scenario Generator

We have mentioned some critical macro factors in the previous section. Perhaps the most inclusive macroeconomic generator is the Federal Reserve scenarios for the Comprehensive Capital Analysis and Review (CCAR) and Dodd-Frank Act stress test exercises required to be performed by banks and supervisors each year.

After the 2008-09 crash, the U.S. and European central banks have established procedure and regulations for the largest global banks, widely known as the Basel Framework. Prakash [101] gives an overview of the evolutions of the framework. As part of these regulations, banks must analyze their performance in forward-looking projections. A major goal of these analyses is to show that the firms current capital is adequate to survive unexpected future events. To this end, the U.S. Federal Reserve (FED) requires the banks to project performance and especially capital preservation under three scenarios: baseline, adverse, and severely adverse.

The FED provides a set of simulated macroeconomic factors under each of these three scenarios. The market and macro-economic factors have a planning horizon of 9 quarters. The banks are required to project their total risk-adjusted capital that is sensitive to these macroeconomic factors over the planning period. For example, the fundamental review of the trading book (FRTB) enforces trading divisions to estimate their profits and losses for each quarter, along with potential capital depletion and contributions. In all of these cases, the divisions must provide clear and convincing evidence that their projections are accurate within the degree possible for
the approval of the regulators.

Similar to risk management practices in banks, an important element in financial planning is the modeling of the random variables (uncertainties) as stochastic processes. The macroeconomic variables affect pension funds and individual investors as well. As Merton and Muralidhar [85] pointed out, monetary policies of the central bank have been (inadvertently) one contributor to the worsening of the funded status of DB pension funds. For individual investors, the growth of their income is considerably influenced by the economic growth and inflation.

The dynamic nature of financial markets also presents a major challenge for individual investors. For example, the total portfolio of individual planning for retirement includes the Social Security benefit and the investment of defined contribution saving accounts. The Social Security benefit, akin to a series of long-term inflation-adjusted bonds\(^2\) depends on the macroeconomic scenarios to a great extent.

A significant issue involves the setting of target projections, both for individuals and institutions. What are reasonable targets, given the need to compete on a global stage, and the importance of becoming a dominate company before its competitors? What is a reasonable amount to save before retirement? In this context, the macroeconomic environment plays a critical role in the future of the individual or organization.

Besides, we propose relevant risk measures that indicate the probability of missing target goals, called Goals-at-Risk (GaR) and Conditional-Goals-at-Risk (CGaR). Since the goal is the most important concern in financial planning, we modify the

\(^2\)As we will see in chapter 5.2
standard risk measures to address these goals.

1.5 Agent-based Modeling

Agent-based models have gained popularity over the past decade, thanks to the increasing availability of microeconomic level data and revolutionary developments in computing. Rather than working solely with general macro-economic statistic (GDP, employment, inflation, industrial production, and interest rates), the agent-based approach focuses on individuals decision units in conjunction with a simulation of a carefully curated set of individuals within a stratified sample of the population. The chapters in [112] and [54] provide an overview of agent-based methods. The approach has been employed over decades in areas such as tax policy analysis and others. The goal is to integrate detailed micro-level data with population projections and macro-econometric time series.

As a simple example, individuals often make financial decisions based on discussions with colleagues, friends, and relatives. Thus, before and during economic crash periods, there are well-documented instances of herding behavior and sentiment modeling based on textual analysis can play a role. For example, the paper [24] has indicated that the use of selected keywords can identify changing economic conditions and possibly regime switches.

As a similar example in forecasting, the International Monetary Authority (IMF) showed in its report [63] that the distribution of loans and debt can have a significant impact on the probability of an upcoming credit crunch. The IMF has created an index called the riskiness of credit allocation, which is computed from micro-level
corporate data. The index aggregates the total level of credit vulnerability, which
takes into account both the private sector and the public sector. Historical data have
shown that in most of the developed countries, the IMF riskiness index spiked during
the period before economic crashes and credit crunches.

A significant element in our framework involves the modeling of micro-level de-
cision agents, individual investors, in our applications. There is enormous research
on individual financial behavior that comes from two main streams: financial decision
making and behavioral decision making. In the former case, the investor utilizes
various tools to optimize their asset allocation and related investment decisions to
achieve desired risk-adjusted performance.

Thaler and Sunstein have shown in their famous book [113] that most people,
as individual decision units, operate with highly subjective and emotional backdrops
when there are major market disruptions. Evidence (such as [15]) also shows that
individuals often manage their portfolios with considerable inertia and even panic
during crash environments, leading to sub-optimal performance. Thus, we build
our design on both normative (for the most educated and motivated investors) and
simplified (for individuals with low levels of experience in financial planning). These
individuals often fall behind the traditional benchmarks, such as the heuristic 60/40
asset mix. A simple comparison of policy rules will “nudge” them back to the right
track.

One of the difficulties in agent-based modeling has been the insufficient compu-
tational power to deal with the enormous interactions of agents. By policy-rule
simulations, our framework limits the computation burden at a reasonable level and
allows instant updates of policy rule alterations.
Chapter 2

Applications of Machine Learning Methods

In this chapter, multiple machine learning models are introduced. We show theoretical properties, discuss algorithm implementations, and present examples in financial planning and business cycle dating that employ these machine learning models.

2.1 Machine Learning Methods in Factor Models

At its core, parameter estimation in factor models can be classified as (linear) regression problems. A common family of problems involves “factor selection”, i.e., picking factors with the most significant impact. Naturally, machine learning methods may be applied to address this concern.

2.1.1 The LASSO Estimator and Factor Selection Model

The Least Absolute Shrinkage and Selection Operator (LASSO), first introduced in 1996 by Robert Tibshirani [114], is a shrinkage method developed upon linear regres-

Some kernel-based factor models are often classified as generalized linear models.
sition models. Consider an ordinary linear regression (OLS) problem

\[ Y_t = \beta_0 + X_t \beta + \epsilon_t \]  \hspace{1cm} (2.1)

where \( X_t = (x_{t1}, \ldots, x_{tp}) \), \( \beta = (\beta_1, \ldots, \beta_p)^T \), i.e. we have \( p \) independent variables.

A shrinkage method adds a penalty on the “absolute size” of the input coefficient to shrink the estimates. The LASSO estimate is defined by

\[ \hat{\beta}^{\text{LASSO}} = \arg \min_{\beta} \left\{ \sum_{t=1}^{n} (y_t - \beta_0 - \sum_{j=1}^{p} x_{tj} \beta_j)^2 + \lambda \sum_{j=1}^{p} |\beta_j| \right\}. \]  \hspace{1cm} (2.2)

By introducing the \( L_1 \)-penalty, the LASSO method results in a sparse estimator. A subset of the coefficients become zero such that the model may serve as the model selection of appropriate factors. In a factor model such as (1.1), replace the OLS estimation by LASSO (2.2) may give us a sparse factor loading matrix, which enables us to pick the “most significant” factors for a certain asset or portfolio.

The \( \lambda \) in (2.2) is called a **hyper-parameter**, which must be determined before the (working) model is trained. One common way to determine the hyper-parameter is k-fold cross-validation. We divide the data into \( k \) folds and create a vector of possible hyper-parameters. For each candidate of hyper-parameter, we train the model on \((k-1)\) folds to obtain an estimate of \( \beta \) and compute the error\(^2\) of linear estimation on the left-out part. We repeat this process until every part has been placed as the training part. Then we compute the average MSE of each hyper-parameter and obtain the best model.

---

\(^2\)Usually we use mean squared error (MSE).
The LASSO estimation (2.2) is a convex problem, so even with cross-validation the computation can be done fairly fast.

2.1.2 Application: Factor-Based Portfolio Allocation

In this section, we present an example in which we build a sample asset allocation portfolio for a university endowment. The LASSO estimator with cross-validation (LASSO-CV) is used for evaluating the sparse factor exposure matrix.

This example takes inspiration from a 2015 paper [21]. We followed their FI-FAA (Flexible Indeterminate Factor-based Asset Allocation) approach and give a sample 4-step procedure in asset allocation, with practical data from Princeton library resources.

The full procedure can be summarized as:

- Factor selection. The appropriate factor set should capture the most risk and return profile but should be parsimonious as well. In this example, we construct a similar factor set with 3 macro factors and 2 micro factors.

- Map asset classes onto factors. Generally known as “determining factor exposures”. In traditional factor analysis, linear regression method is always adopted to calculate factor exposure. In this example, we add LASSO with cross-validation and best subset selection as alternative methods.

\(^3\)Detailed sources of data can be found in Appendix A.
- Determine factor exposures. This step is mainly based on the risk- and return-the objective of the investor. Here we choose a factor exposure for a typical university endowment.

- Tilt the asset portfolio. This step involves another optimization which includes a “preference score” and the factor exposure matrix.

**Factor Selection**

The five factors we choose are listed in table 2.1 below. The first factor is the return of world equities. We choose the world equities index as a benchmark and regional equities indices as investment vehicles. The second factor is the return of the long-term U.S. government bond index. As one of the countries with the best credit rating, the United States can issue bonds with virtually no risk. The third factor, high yield, combines characteristics of equities and bonds and is important in explaining returns of assets such as corporate bonds and real estates. We call these three factors “foundation factors”.

Besides the three asset category factors, we also include two protection factors. They have low returns historically (as seen from figure 2.2 below), but they help to determine the portfolio weights among similar asset categories. For example, the “inflation protection” factor controls investors’ preference between treasury bonds and TIPS.

Finally, the “currency protection” factor is constructed based on weighted average exchange rates between USD and a basket of major foreign currencies. Major currencies index includes the Euro Area, Canada, Japan, United Kingdom, Switzerland, Australia, and Sweden. If an investor has a high demand for currency protection, she will choose more U.S. equities over other global equities.
Table [2.1] summarizes construction mechanism of the factors.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Index Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>World Equities</td>
<td>MSCI all-world cap-weighted index</td>
</tr>
<tr>
<td>US Treasuries</td>
<td>U.S. long-term government bond of different maturities</td>
</tr>
<tr>
<td>High Yield</td>
<td>Also known as “Junk Bonds”, reflects credit risk</td>
</tr>
<tr>
<td>Inflation Protection</td>
<td>Long TIPS, short U.S. Treasury (both 10-year maturity)</td>
</tr>
<tr>
<td>Currency Protection</td>
<td>Trade Weighted U.S. Dollar Index</td>
</tr>
</tbody>
</table>

Table 2.1: Monthly factor data, January 1988 to September 2018

Figure [2.2] plots the cumulative returns of the five factors from 1988 to 2018, all starting at 100:

![Cumulative Factor Returns, 1985=100](image_url)

Figure 2.2: Cumulative Factor Returns, 1985=100
The macro factors generate more return than the protection factors. We also observe that the high yield index had better returns than the equities index or treasury bonds index. It also explains the fact that alternative assets have been outperforming in the past few decades. Notice that the drawdown of high yield index in the 2008-2009 financial crisis was also less than that of equities.

**Factor Exposure Matrix**

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>World Equities</th>
<th>U.S. Treasuries</th>
<th>High Yield</th>
<th>Inflation Protection</th>
<th>Currency Protection</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Equities</td>
<td>0.1%</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>EAFE Equities</td>
<td>-0.1%</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.2</td>
</tr>
<tr>
<td>EM Equities</td>
<td>-0.2%</td>
<td>0.7</td>
<td>0</td>
<td>0.8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PE</td>
<td>-0.1%</td>
<td>1.2</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>U.S. 30-Year Treasuries</td>
<td>-0.1%</td>
<td>0</td>
<td>1.8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>0.1%</td>
<td>0</td>
<td>0.8</td>
<td>0.2</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>Real Estate</td>
<td>0.1%</td>
<td>0.5</td>
<td>0</td>
<td>0.8</td>
<td>0</td>
<td>-0.5</td>
</tr>
<tr>
<td>Commodities</td>
<td>0.4%</td>
<td>-0.1</td>
<td>-0.7</td>
<td>-0.3</td>
<td>2.1</td>
<td>-1.6</td>
</tr>
<tr>
<td>TIPS</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Absolute Returns</td>
<td>0.3%</td>
<td>0.25</td>
<td>0</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.2: Factor Exposure Matrix
Since we have more than 300 months of data points, LASSO with 5-fold cross-validation is appropriate. The sparse factor exposure matrix is shown in table 2.2.

There are many interesting facts illustrated by the factor loading matrix:

- Most equities are combinations of the equity index and the high yield index. U.S. equities have a strong currency protection impact.

- Private equities show “leverage” effect; it has a loading of more than 1 on the equities index.

- Corporate bonds are a combination of treasury bonds index and high yield index, with more loading on the former.

- Commodities are more idiosyncratic (the empirical $R^2$ is low) and contain a mixed exposure to all factors.

Comparing Estimates

To compare the effectiveness of the factor models, we introduce the concept of out-of-sample testing. The cross-validation technique is an in-sample method, which means that we are using the working data to validate the model with the “best” hyper-parameter. We further set aside a portion of data for out-of-sample tests. This portion of data is unavailable until all models are trained and ready to compare. Figure 2.3 illustrates this process.

Figure 2.3: Out-of-sample Testing
Besides the OLS and LASSO-CV model, we also compare a family of linear models with best subset optimization. The best subset problem can be expressed as:

\[
\begin{align*}
\text{minimize} & \quad \|\mathbf{y} - \mathbf{X}\beta\|_2^2 \\
\text{subject to} & \quad \|\beta\|_0 \leq m.
\end{align*}
\]

Here \(\|\beta\|_0\) is the number of non-zero elements in the vector \(\beta\). The best subset optimization selects the best \(m\) predictors for the model. Table 2.3 compares OLS, LASSO along with best subset models where \(m = 2, 3\). We use mean-squared error (MSE) to show goodness-of-fit. Notice that LASSO outperforms in explaining most factors.

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>2-Subset</th>
<th>3-Subset</th>
<th>LASSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Equities</td>
<td>0.126</td>
<td>0.128</td>
<td>0.125</td>
<td>0.123</td>
</tr>
<tr>
<td>EAFE Equities</td>
<td>0.013</td>
<td>0.014</td>
<td>0.013</td>
<td>0.012</td>
</tr>
<tr>
<td>EM Equities</td>
<td>0.168</td>
<td>0.149</td>
<td>0.147</td>
<td>0.145</td>
</tr>
<tr>
<td>Private Equities</td>
<td>0.154</td>
<td>0.152</td>
<td>0.155</td>
<td>0.151</td>
</tr>
<tr>
<td>US Treasuries</td>
<td>0.064</td>
<td>0.063</td>
<td>0.063</td>
<td>0.067</td>
</tr>
<tr>
<td>Corp Bonds</td>
<td>0.03</td>
<td>0.031</td>
<td>0.031</td>
<td>0.03</td>
</tr>
<tr>
<td>Real Estate</td>
<td>0.2</td>
<td>0.204</td>
<td>0.206</td>
<td>0.184</td>
</tr>
<tr>
<td>Commodities</td>
<td>0.267</td>
<td>0.28</td>
<td>0.281</td>
<td>0.267</td>
</tr>
<tr>
<td>TIPS</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.001</td>
</tr>
<tr>
<td>Absolute Return</td>
<td>0.197</td>
<td>0.195</td>
<td>0.198</td>
<td>0.194</td>
</tr>
</tbody>
</table>

Table 2.3: MSE of Factor Models
Factor Allocation

The factor allocation matrix should be adjusted according to the preferences of the investor. Note that the “traditional” allocation is the 60-40 portfolio plus exposure to inflation protection and currency protection.

For university endowments, one primary goal is to pursue the growth of the market. From table 2.2, we learned that the alternative assets have heavy exposure to equities and high-yields, so we adjust the target factor exposure to reflect the preference of these two factors by university endowments. The factor exposure does not necessarily sum to 1; in the optimization problem, we want to have a factor exposure that is proportional to the target exposure.

<table>
<thead>
<tr>
<th></th>
<th>World Equities</th>
<th>U.S. Treasuries</th>
<th>High Yield</th>
<th>Inflation Protection</th>
<th>Currency Protection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td>0.60</td>
<td>0.40</td>
<td>0.00</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Target</td>
<td>0.55</td>
<td>0.25</td>
<td>0.30</td>
<td>0.1</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 2.4: Target Factor Exposure

Portfolio Construction

In the portfolio optimization, we aim to solve the following problem. Here we have \( n \) assets, \( w \) is the target holding weight and \( w^0 \) is some “benchmark” that we give, \( S \) is the preference score vector, \( M \) is the factor exposure matrix, \( F \) is the “target” factor exposure. The last constraint is element-wise. We present the portfolio results in table 2.5.
\[
\begin{align*}
\text{minimize} & \quad w^T S \\
\text{subject to} & \quad w^T 1 = 1 \\
& \quad w^T M \propto F \\
& \quad w \geq 0 \\
& \quad |w - w^0| \leq 15\%.
\end{align*}
\]

<table>
<thead>
<tr>
<th></th>
<th>Preference Score</th>
<th>Original</th>
<th>Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Equities</td>
<td>1</td>
<td>20%</td>
<td>22%</td>
</tr>
<tr>
<td>EAFE Equities</td>
<td>4</td>
<td>5%</td>
<td>0%</td>
</tr>
<tr>
<td>EM Equities</td>
<td>2</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td>Private Equities</td>
<td>1</td>
<td>20%</td>
<td>35%</td>
</tr>
<tr>
<td>U.S. 30-Year Treasuries</td>
<td>1</td>
<td>15%</td>
<td>18%</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>3</td>
<td>5%</td>
<td>0%</td>
</tr>
<tr>
<td>Real Estate</td>
<td>2</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td>Commodities</td>
<td>5</td>
<td>0%</td>
<td>5%</td>
</tr>
<tr>
<td>TIPS</td>
<td>2</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td>Absolute Returns</td>
<td>2</td>
<td>10%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Table 2.5: Final Asset Class Portfolio

We can observe from the table that the weight on private equity is very high (it has reached the upper limit, in fact), as well as the U.S. domestic equities. Private equity has great upward potential and is preferred by university endowments.
Despite their relatively high preference score, the weights of real estate and TIPS are zero in our portfolio. For university endowments, the focus is usually capital accumulation over inflation protection.

Finally, commodities have been under-performing since 2008, so it is not a preferred asset category. However, due to its strong inflation protection characteristics, it replaces TIPS in our final portfolio.

2.2 Machine Learning Models in Regime Identification

We have seen various techniques in regime analysis and regime-based trading/investment models in section 1.2. Turning to machine learning methods, we are looking at a generalized variant of the LASSO estimator - the trend filtering method, which is a non-parametric approach. Before introducing the model, we clarify the term “non-parametric”, since this is one significant difference between the trend-filtering algorithm and LASSO estimation despite their resemblance in forms.

Parametric or Non-parametric?

A parametric model, such as the linear regression factor model (1.1) or the LASSO model (2.2), the parameters of the model (\( \beta \)’s in this case) form a fixed set. Non-parametric models, on the other hand, do not have fixed structures of the model. They may have parameters, but the nature of the parameters is quite flexible. One of the most significant characteristics of non-parametric models is that it does not rely on any assumption of the underlying distribution. As we will see in the subsequent simulations, this is an advantage in financial applications.
The Trend-Filtering Algorithm

The trend-filtering estimation was first introduced by Kim et al. in their paper [70]. Kim introduced $L^1$-filtering, which can smooth a time series by piecewise linear functions of time. The smoothing process is a generalization of the LASSO method given above, with a penalty term in absolute value form.

In a later paper [115], Tibshirani further generalized the trend-filtering method by enabling to smooth by piecewise polynomials of degree $p$, where $p = 1$ has been discussed in [70]. In our report we tend to use piecewise constant functions ($p = 0$) for the “return” series, and piecewise linear functions ($p = 1$) for the index series. The trend-filtering model formulation is given as follows. Given a time-series $x$ and corresponding time $t$, both of length $n$, we would like to find a pointwise correspondence of $x$ and $t$. To be more specific, we aim to find $\beta_j$ such that the following error is minimized:

$$\hat{\beta} = \arg\min_{\beta \in \mathbb{R}^n}(||x - \beta||_2^2 + \lambda||D^{(k+1)}\beta||_1). \quad (2.3)$$

where $x = (x_1, \ldots, x_n)^T$, $\beta = (\beta_1, \ldots, \beta_n)^T$, $\lambda > 0$, and $D^{(k+1)}$ is defined as the discrete difference operator of order $k + 1$. When $k = 0$, we have the first-order difference matrix

$$D^{(1)} = \begin{bmatrix} 1 & -1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & -1 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & -1 & 0 \\ 0 & 0 & 0 & \ldots & 1 & -1 \end{bmatrix} \in \mathbb{R}^{(n-1)\times n}. \quad (2.4)$$
When \( k = 1 \), we have the second-order difference matrix

\[
D^{(2)} = \begin{bmatrix}
1 & -2 & 1 & \ldots & 0 & 0 & 0 \\
0 & 1 & -2 & \ldots & 0 & 0 & 0 \\
\vdots \\
0 & 0 & 0 & \ldots & -2 & 1 & 0 \\
0 & 0 & 0 & \ldots & 1 & -2 & 1
\end{bmatrix} \in \mathbb{R}^{(n-2) \times n}.
\]  

(2.5)

The optimization problem (2.3) is a convex problem since it is sum of norms of affine functions, under the assumption that \( \lambda > 0 \). The function is also strictly convex (given the existence of the quadratic function), therefore the optimized value \( \hat{\beta} \) is unique. We define this \( \hat{\beta} \) as the trend-filtering estimator. In order to obtain a piece-wise constant estimator, we take \( k = 0 \) and plug (2.4) into the optimization problem. If we need a piece-wise linear estimator, we will take \( k = 1 \).

2.2.1 Properties of the Trend Filtering Estimator

We can observe from the optimization problem (2.3) some properties of the trend filtering estimator:

- The estimator converges to \( \mathbf{x} \) as \( \lambda \to 0 \).

- The estimator converges to the best constant fit (to be specific, the estimator is the mean of the series \( \mathbf{x} \) throughout the whole time) as \( \lambda \to \infty \). In fact, the characteristic of the LASSO estimator enables the convergence as \( \lambda \geq \lambda_{\text{max}} \) for some \( \lambda_{\text{max}} \) large enough.
The estimator is piecewise constant in $t$ for $k = 0$. To observe this, notice that our optimization problem can be written as:

$$
\arg\min_{\theta \in \mathbb{R}^n} ||x - L\theta||_2^2 + \lambda \sum_{i=2}^{n} |\theta_i|.
$$

(2.6)

where $\theta = (\theta_1, \ldots, \theta_n)$ is the new decision variable and $L$ satisfies $L\theta = \beta$.

Here $L$ is a lower triangular matrix that serves as the “inverse” of $D^{(1)}$ (but not exactly the inverse since $D^{(1)}$ is not a square matrix). By the properties of LASSO estimator, we know that the solution $\theta$ will likely to be sparse. Therefore, for many $j$, we will have $\beta_j = \beta_{j+1}$. The estimator is piecewise constant, and we call the points $t_i$ *kink points* if $\beta_j \neq \beta_{j+1}$.

The estimator is also piecewise constant, as a function of $\lambda$ for $k = 0$. In other words, there exist $0 = \lambda_1 < \cdots < \lambda_n$ such that

$$
\hat{\beta} = \beta_i, \quad \lambda_i < \lambda < \lambda_{i+1},
$$

where $\beta_i$ is $\hat{\beta}$ with $\lambda = \lambda_i$.

For $k = 1$, the estimator will be piecewise linear. Similar to the reasoning above, for most $j$, we will have $\beta_j - 2\beta_{j+1} + \beta_{j+2} = 0$, i.e. $\beta_j - \beta_{j+1} = \beta_{j+1} - \beta_{j+2}$.

When the kink points are fixed, the trend filtering algorithm is just a linear spline method. However, one of the advantage of trend filtering method is that the kink points can be adaptive to data input. Therefore, if we change the time period of input time series, the kink points may change.

### 2.2.2 Software Implementation

Numerically, $\hat{\beta}$ can be computed in linear time $O(n)$. (Worst case is about $O(n^{1.5})$, as given in [70].) There is a package “genlasso” in R that implements the trend
filtering method, while the programs in this paper are coded in Python. Since this is a standard convex problem, we can formulate in the cvxpy solver and solve it efficiently. The results are presented in the next section, in which we will introduce regime identification analysis, including division of “normal” and “crash” periods. We will see that under our specific requirements, the trend filtering estimator is more suitable for investment purposes.

Figure 2.4 shows the monthly return series and fitted series by the trend-filtering algorithm, which is indeed piecewise constant. Figure 2.5 shows the regimes between 1986 and 2018, with 0 as the threshold for fitted values.

![Figure 2.4: Monthly Return and Fitted Series of S&P 500 Index](image)
Figure 2.5: Regimes of the S&P 500 Index

Figure 2.6: Regimes of the Wilshire 5000 Index
One advantage of the trend-filtering algorithm is the stability with respect to scale. Figure 2.6 shows the results of trend-filtering algorithm on another equity index in the U.S.: the Wilshire 5000 index. The time period is also different: we choose a longer period, 1971-2018, to test the stability of our algorithm. We used the same hyper-parameter\(^4\) for the two series.

It turns out that the regime identifications between 1985 and 2018 are exactly the same for two different datasets. This stability of regime will be crucial when we are generating long-term asset return paths, especially when we compute the transition matrix for historical regimes. The stability also helps in real-time regime identification in which we manage to determine the current regime given the most recent stock index data.

Table 2.6 compares historical performance when we separate by the trend-filtering method (S&P 500 regime) or by NBER dates of peak and trough periods. We show annualized return and standard deviation under normal or crash regime.

Apparently, the regime identified by the S&P 500 index is more successful in separating performance for all assets shown in table 2.6 including the treasuries index (which has a massive 17% annualized return during crash periods). Developing our own regime-based mean and standard deviation parameters is crucial for a realistic simulation of asset returns; the NBER regimes will significantly underestimate the magnitude of crashes (only -13% annually, versus the -34% from the S&P 500 regime).

\(^4\lambda = 16\) for percentage return
<table>
<thead>
<tr>
<th>SP500 Regime</th>
<th>US Equities</th>
<th>EAFE Equities</th>
<th>EM Equities</th>
<th>Long Treasuries</th>
<th>Real Estate</th>
<th>Commodities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean-Normal</td>
<td>21.00%</td>
<td>14.87%</td>
<td>22.59%</td>
<td>6.35%</td>
<td>17.23%</td>
<td>7.54%</td>
</tr>
<tr>
<td>STD-Normal</td>
<td>12.12%</td>
<td>14.70%</td>
<td>20.16%</td>
<td>9.45%</td>
<td>14.45%</td>
<td>18.62%</td>
</tr>
<tr>
<td>Mean-Crash</td>
<td>-34.33%</td>
<td>-35.27%</td>
<td>-39.43%</td>
<td>17.04%</td>
<td>-26.68%</td>
<td>-18.65%</td>
</tr>
<tr>
<td>STD-Crash</td>
<td>21.06%</td>
<td>23.41%</td>
<td>30.62%</td>
<td>13.23%</td>
<td>36.16%</td>
<td>30.40%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NBER Regime</th>
<th>US Equities</th>
<th>EAFE Equities</th>
<th>EM Equities</th>
<th>Long Treasuries</th>
<th>Real Estate</th>
<th>Commodities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean-Expansion</td>
<td>13.90%</td>
<td>9.29%</td>
<td>14.20%</td>
<td>7.96%</td>
<td>12.41%</td>
<td>5.87%</td>
</tr>
<tr>
<td>STD-Contraction</td>
<td>12.80%</td>
<td>14.80%</td>
<td>20.65%</td>
<td>9.55%</td>
<td>13.81%</td>
<td>18.24%</td>
</tr>
<tr>
<td>Mean-Expansion</td>
<td>-13.26%</td>
<td>-21.05%</td>
<td>-15.65%</td>
<td>6.57%</td>
<td>-12.77%</td>
<td>-16.98%</td>
</tr>
<tr>
<td>STD-Contraction</td>
<td>27.21%</td>
<td>30.38%</td>
<td>37.52%</td>
<td>14.37%</td>
<td>46.05%</td>
<td>36.69%</td>
</tr>
</tbody>
</table>

Table 2.6: Annualized Asset Statistics Under Different Regimes, 1988-2018

2.3 Combination of Macroeconomic Variables and Factor Analysis

We observe from table 2.6 that all assets have quite the opposite performance under different regimes. Previous works, including [61] and [56], also pointed out that during adverse conditions of the market, the promised diversification benefits provided by traditional asset classes, industrial sectors or investments in different geographic locations are not satisfactory. The actual outcome is that assets returns became highly correlated, which led to co-crashes between stock and bond markets,
between different industrial sectors and between developed and emerging economies, also known as the “contagion” effect.

In a 2015 paper [62] by Ilmanen et al., the authors explore macro sensitivities of asset classes, portfolios and factor (style) investing by providing a framework in which five major macroeconomic dimensions are identified, and portfolio performances are examined in terms of Sharpe ratio. We follow this procedure, but the economic dimensions are determined by our regime analysis. We take the most critical macroeconomic environment indicator - growth and inflation - and construct a 4-regime framework with the trend-filtering algorithm.

2.3.1 Growth Indicator

We select the Chicago Fed National Activity Index (CFNAI) for industrial production growth of the United States. The CFNAI is a weighted average of 85 monthly indicators of national economic activity. Specifically, we use the CFNAI-MA3 index to track expansions and recessions of the U.S. economy.

NBER identified seven economic recessions during the period that we choose, 1970-2015. We conduct trend filtering with the tuning parameter based on this economic fact. The business cycles we detected are very close to the NBER certificate of expansion and recession periods (which happened during 1974, 1979, 1981, 1990, 2001, 2008). Comparing with the “turning points” disclosed by NBER that are neither real-time nor adaptive, our trend filtering algorithm provides real-time monitoring of turning points and is well fitted to the underlying time series.
2.3.2 Inflation Indicator

Similarly, we aim to find an indicator of the U.S. inflation status. The CPI index is convenient, albeit an “index” series. We run the trend-filtering algorithm with a piecewise linear series, which turned out to be unsatisfactory. Instead, we use the inflation rate series, calculated by year-on-year CPI (for instance, the inflation rate of May 2016 was calculated by the increase in CPI from May 2015 to May 2016).

<table>
<thead>
<tr>
<th>Regime #</th>
<th>Definition of Regime</th>
<th>Number of Months</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>High Growth, low inflation (best)</td>
<td>153</td>
</tr>
<tr>
<td>1</td>
<td>High Growth, high inflation</td>
<td>160</td>
</tr>
<tr>
<td>2</td>
<td>Low Growth, low inflation</td>
<td>107</td>
</tr>
<tr>
<td>3</td>
<td>Low Growth, high inflation (worst)</td>
<td>132</td>
</tr>
</tbody>
</table>

Table 2.7: Summary of regimes

2.3.3 Regime Summary

From table 2.8 we observe that regime 0 is indeed most suitable for investment. The asset classes are much more sensitive to macroeconomic shocks than the factors; the equity index and the industry sectors suffer from low growth, and the bonds suffer from high inflation. Factor portfolios, on the other hand, are less sensitive to macro risks. This is the main idea that the paper 62 wants to convey. Although the Sharpe ratio of factor portfolios are not significantly higher than that in the top table, their steady performance in other regimes enables them to be a good diversification tool.
<table>
<thead>
<tr>
<th>Name</th>
<th>Regime.0</th>
<th>Regime.1</th>
<th>Regime.2</th>
<th>Regime.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Equity</td>
<td>0.98</td>
<td>-0.04</td>
<td>0.49</td>
<td>-0.36</td>
</tr>
<tr>
<td>Government Bonds</td>
<td>0.64</td>
<td>0.15</td>
<td>0.67</td>
<td>0.49</td>
</tr>
<tr>
<td>Consumer Sector</td>
<td>0.81</td>
<td>0.16</td>
<td>0.95</td>
<td>0.24</td>
</tr>
<tr>
<td>Manufactory Sector</td>
<td>1.06</td>
<td>0.33</td>
<td>0.71</td>
<td>-0.07</td>
</tr>
<tr>
<td>Hi Tech Sector</td>
<td>1.07</td>
<td>0.17</td>
<td>0.55</td>
<td>-0.34</td>
</tr>
<tr>
<td>Health Sector</td>
<td>0.90</td>
<td>0.14</td>
<td>0.68</td>
<td>0.22</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Factor Portfolios</th>
<th>Regime.0</th>
<th>Regime.1</th>
<th>Regime.2</th>
<th>Regime.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size (SMB)</td>
<td>1.46</td>
<td>0.66</td>
<td>0.70</td>
<td>0.20</td>
</tr>
<tr>
<td>Value (HML)</td>
<td>1.57</td>
<td>0.97</td>
<td>0.65</td>
<td>0.43</td>
</tr>
<tr>
<td>Profitability (RMW)</td>
<td>1.39</td>
<td>0.67</td>
<td>0.86</td>
<td>0.36</td>
</tr>
<tr>
<td>Investment (CMA)</td>
<td>1.44</td>
<td>0.73</td>
<td>1.00</td>
<td>0.43</td>
</tr>
<tr>
<td>Momentum (MoM)</td>
<td>1.24</td>
<td>0.89</td>
<td>0.99</td>
<td>0.23</td>
</tr>
<tr>
<td>Size and momentum</td>
<td>1.13</td>
<td>0.89</td>
<td>1.09</td>
<td>0.45</td>
</tr>
<tr>
<td>Size and value</td>
<td>1.36</td>
<td>0.82</td>
<td>0.98</td>
<td>0.33</td>
</tr>
<tr>
<td>Size, value, profitability</td>
<td>1.46</td>
<td>0.95</td>
<td>0.86</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 2.8: Sharpe Ratio of Portfolios
Chapter 3

Scenario Generator for Financial Planning

It is critical to build a scenario generator that admits historical input but includes prospect of the economy in the future. We utilize stochastic modeling, the vector autoregressive approach, regime-based simulations, and the factor model to build our scenario generator.

3.1 Macroeconomic Generator

Our financial planning framework requires a simulator for macroeconomic environments to determine key variables pertinent to the planning goal. This section compares various techniques in macroeconomic forecasting and scenario generating.

The book edited by Elliott and Timmermann [38] gives a brilliant overview of macroeconomic forecasting models. Here we present two major directions for scenario generators.
3.1.1 Stochastic Modeling Approach

The stochastic approach employs a hierarchical structure and is inspired by the cas-
cade structure in Mulvey [91]. We start from generating “key” variables, including
the short interest rate:

\[ dr_t = f_1(r_u - r_t)dt + f_2(r_u, r_t, l_u, l_t, i_u, i_t, g_u, g_t)dt + f_3(r_t)dZ_t^1 \]  \hspace{1cm} (3.1)

Long interest rate:

\[ dl_t = f_4(l_u - l_t)dt + f_5(r_u, r_t, l_u, l_t, i_u, i_t, g_u, g_t)dt + f_6(r_t, l_t)dZ_t^2 \]  \hspace{1cm} (3.2)

Inflation:

\[ di_t = f_7(i_u - i_t)dt + f_8(r_u, r_t, l_u, l_t, i_u, i_t, g_u, g_t)dt + f_9(i_t, r_t)dZ_t^3 \]  \hspace{1cm} (3.3)

Economic growth:

\[ dg_t = f_{10}(g_u - g_t)dt + f_{11}(r_u, r_t, l_u, l_t, i_u, i_t, g_u, g_t)dt + f_{12}(g_t)dZ_t^4 \]  \hspace{1cm} (3.4)

Where the \( Z \)'s are Brownian motions generated from the covariance matrix:

\[ \mathbb{E} \left( dZ_i^idZ_j^j \right) = \sigma_{ij}dt. \]  \hspace{1cm} (3.5)

The next layer includes variables that depend on the key variables. For multi-national
financial planning, we can generate exchange rates with a similar structure. The
combination of short- and long-term interest rates help to price the future liabilities
for a long time period. The yield curve can also be generated from the short and
long interest rates.
We can use the Granger causality to determine the hierarchical relationship, see [51] and [52] for a detailed description. The Granger causality is available in most statistical packages.

On the next layer generates asset prices. This method may be used as complementary to the asset return simulator. For example, when we are simulating scenarios for the Chinese economy, we may separately generate the real estate price, which depends heavily on the interest rate as well as economic growth and inflation:

\[ de_t = e_1 dt + e_2(e_t, p_t, g_t)dt + e_3(e_t, g_t)dZ^5_t. \] (3.6)

The functions \( f_j \) provides flexibility to our modeling approach; we could simply use a constant (for mean-reverting approach) or the square-root function (when we choose the Cox-Ingersoll-Ross (CIR) model for interest rates.)

The parameter estimation can be implemented via maximum likelihood estimation (MLE). For instance, when estimating an Ornstein-Uhlenbeck model

\[ dX_t = \lambda(\mu - X_t)dt + \sigma dZ_t, \] (3.7)

we compute the mean and variance at time \( t \):

\[ \mathbb{E}\theta(X_t|X_0 = x) = \mu + (x - \mu) \exp(-\lambda t) \] (3.8)

\[ \text{var}_\theta(X_t|X_0 = x) = \frac{\sigma^2}{2\lambda}[1 - \exp(-2\lambda t)] \] (3.9)
Then the log-likelihood function can be expressed as

\[
\ell_n(X; \theta) = \sum_{i=1}^{n} \log p_\theta (X_i|X_{i-1}) + \log (p_\theta (X_0)) \\
= -\frac{n}{2} \log \left( \frac{\sigma^2}{2\lambda} \right) - \frac{1}{2} \sum_{i=1}^{n} \log (1 - e^{-2\lambda\Delta}) \\
- \frac{\lambda}{\sigma^2} \sum_{i=1}^{n} \frac{(X_i - \mu - (X_{i-1} - \mu)e^{-\lambda\Delta})^2}{1 - e^{-2\lambda\Delta}}
\]

which can be numerically solved. (\(\Delta\) is the discretized time interval)

Other methods, as described in \cite{95}, include method of (generated) moments and its extensions.

### 3.1.2 Econometric Approach

Alternatively, we can use econometric models to generate the macroeconomic series. A family of linear models is introduced in \cite{38}: vector autoregressive (VAR) models. The basic VAR model admits linear relationship between the key variables \(Y\):

\[
y_t = A_0 + A_1 y_{t-1} + A_2 y_{t-2} + u_t, u_t \sim \mathcal{N}(0, \Sigma) \quad (3.10)
\]

\[
Y = \begin{pmatrix}
y_1 \\
 \vdots \\
y_T
\end{pmatrix} \quad (3.11)
\]

The parameters \(A_0, A_1, \ldots\) can be estimated via regression models or Bayesian approach. When the latter is used, the model becomes a Bayesian vector autoregression (BVAR) model. Higgins, Zha and Zhong \cite{57} implement the BVAR model to forecast and generate scenarios for Chinese macroeconomy.
3.2 Regime-based Simulator for Portfolio Models

We begin from the one-period model. In this section, a multi-regime scenario-based portfolio optimization model is built and compared with the traditional mean-variance portfolio. The development of a scenario-based portfolio model is quite general, allowing for almost any type of distribution or range of events. We start with an introduction of the mean-variance optimization problem and show scenario-equivalent formulation. Then we update the scenarios with our regime settings and propose the mean-CVaR portfolio optimization. Finally, we run an empirical backtest with historical asset return data. This chapter was adapted from an award-winning presentation at the 2016 New Jersey Chapter Student Contest of the INFORMS meeting and the subsequent publication [92].

3.2.1 Introduction to Asset Class Portfolio Optimization

We view a portfolio as an investment policy involving allocating weights on \( N \) different assets. For simplicity, we will assume total amount invested be 1 unit. Write \( x_n \) as amount invested in asset \( n \). Then:

\[
x_1 + \ldots + x_N = 1.
\]  

(3.12)

Define \( R_n, n = 1, 2, \ldots \) as the return of asset \( n \). If the expected return of asset \( n \) is \( r_n \) and our anticipated return rate is \( r_0 \), then the expected return constraint is:

\[
x_1 r_1 + \ldots + x_N r_N \geq r_0.
\]  

(3.13)
Also define the covariance matrix:

\[
Q = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\
\sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN}
\end{bmatrix},
\]

where \(\sigma_{ij}\) stands for the covariance between asset \(i\) and \(j\):

\[
\sigma_{ij} = E[(R_i - r_i)(R_j - r_j)].
\]

Then, one natural estimator of uncertainty is the variance of our portfolio:

\[
s^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x'_ix_jq_{ij} = x'^{T}Qx.
\]

where \(x = (x_1, \ldots, x_N)^T\). The Markowitz optimization problem (as described in [77]) becomes:

\[
\begin{align*}
\text{minimize} & \quad x'^{T}Qx \\
\text{subject to} & \quad (3.12) \text{ and } (3.13)
\end{align*}
\]

Problem (3.14) is a convex quadratic optimization problem and can be easily solved by any software. Or, we can formulate in another way, maximizing the expected return while posing the risk constraint. However the latter formulation is not as popular as minimizing the risk, since we prefer to have linear constraints and quadratic objectives.
In this basic portfolio optimization model, we have the following specifications:

- **Risky or risk-free assets**

  If all assets are risky, the matrix $Q$ is typically non-zero elementwise. On the other hand, if we include a risk-free asset (and let it be asset 1, without loss of generality), then $R_1$ is a constant, and $\sigma_{1n} = \sigma_{n1} = 0$ for any $n = 1, \ldots, N$. Therefore, $Q$ becomes a singular matrix, and may affect optimization algorithms.

  In the scenario of risk-free asset, we will use $y$ to denote the amount invested in this asset. Since the risk-free asset does not affect variance, (3.1) and (3.2) becomes:

  $$x_1 + \ldots + x_N + y = 1,$$

  (3.15)

  and

  $$x_1 r_1 + \ldots + x_N r_N + y r_{\text{free}} \geq r_0.$$  

  (3.16)

- **Short-selling**

  Short selling is interpreted as negative values of $x_n$. If short selling is permitted, $x_n$ can take any real value. However, large values of $x_n$ will yield large variance, so the optimal allocation will be modestly leveraged at most. In asset-class optimization, short selling is usually not permitted. For example, a university endowment may not participate in long-short strategies (although it can invest in long-short funds, that can be attributed to the “Absolute Returns” category.)

The basic form of portfolio optimization (3.14) can also be expressed as the scenario-equivalent portfolio model:
\[
\begin{align*}
\text{minimize } & \quad \text{variance}(\mathbf{x}^T \mathbf{r}_s) \\
\text{subject to } & \quad x_1 + \ldots + x_N = 1, x \geq 0 \\
& \quad \frac{1}{|S|} \sum_{s \in S} \mathbf{x}^T \mathbf{r}_s \geq r_0.
\end{align*}
\] (3.17)

Here \( S \) is a set of scenarios with return \( \mathbf{r}_s \) of each asset following the same mean vector and covariance matrix as historical data\(^1\) and we are minimizing the variance of the “outcome” in all scenarios.

**Result:** Matrix containing \( M \) scenarios: \( S \)

Initialize with empty matrix;

Compute probability of crash \( p \);

Compute historical 2-regime mean: \( \mathbf{r}_{\text{normal}}, \mathbf{r}_{\text{crash}} \);

Compute historical 2-regime covariance \( Q_{\text{normal}}, Q_{\text{crash}} \);

\begin{algorithm}
while \(|S| < M\) do
  generate uniform random number \( q \in [0, 1] \);
  if \( q > p \) then
    generate \( s \sim \mathcal{N}(\mathbf{r}_{\text{normal}}, Q_{\text{normal}}) \);
    \( \text{S.append}(s) \);
  else
    generate \( s \sim \mathcal{N}(\mathbf{r}_{\text{crash}}, Q_{\text{crash}}) \);
    \( \text{S.append}(s) \);
  end
end

**Algorithm 1:** Generating Two-Regime Scenarios

\(^1\)We usually generate multivariate normal distribution \( \mathcal{N}(\mathbf{r}, Q) \).
The scenario generator of this model for $r_s$ is easily adaptable for multiple regimes. One advantage of the regime-based simulation is that the two-regime setting addresses the tail risk of assets. Figure 3.1 and 3.2 show the Q-Q plots of historical returns of U.S. equities index and the two-regime simulations of the index.

The Q-Q plot compares an empirical distribution with the normal distribution. From figure 3.1 we observe a heavy left tail of the equities index, implying more severe worst scenarios than the normal distribution would suggest.

The tail risk is addressed in the two-regime simulation, as seen from figure 3.2 where the tail has a similar shape as the historical data.

![Q-Q Plot of Asset: US Equities](image)

*Figure 3.1: Q-Q plot of US Equity Index*
On the other hand, the mean-variance problem (3.14) or its scenario-equivalent version (3.17) cares only about the mean and covariance matrix. Therefore we need a formulation of asset allocation optimization which emphasizes the “worst-case scenario”.

3.2.2 Mean-CVaR optimization

One possible modification involves minimizing the conditional value-at-risk (CVaR). CVaR, as a coherent measure of risk, is developed upon value-at-risk (VaR).

The definition of VaR is

$$\text{VaR}_h(X) = \min\{v | P(-X > v) \leq h\}. \quad (3.18)$$

It is the “cutoff” boundary of the worst $h$ proportion of the scenarios. For example, if there are 10,000 possible scenarios, VaR$_{0.05}$ (also called VaR with 95% confidence) describes the loss of the worst 500th scenario.
Based on VaR, the definition of CVaR is

$$\text{CVaR}_h(X) = -\mathbb{E}[X | X \leq -\text{VaR}_h(X)]$$

(3.19)

CVaR reflects the average loss of these worst cases that has probability $h$ to happen. It is known to be a consistent risk measure. Rockafellar and Uryasev et al. (102) provide further details of optimizing under VaR and CVaR via a scenario-based portfolio model. We can modify the mean-CVaR portfolio optimization problem as:

$$\min_{v, \mathbf{x}} \quad v + \frac{\sum_{s \in S} u_s}{M(1 - h)}$$

subject to

$$x_1 + \ldots + x_N = 1, \quad \mathbf{x} \geq 0$$

$$\frac{1}{M} \sum_{s \in S} \mathbf{x}^T \mathbf{r}_s \geq r_0$$

$$u_s \geq 0, \quad \mathbf{x}^T \mathbf{r}_s + v + u_s \geq 0$$

Here we construct auxiliary variables $v$ for value-at-risk and $u_s$ for excess loss under scenario $s$ (0 if no excess loss). Now we have a convex (linear) portfolio model that is scenario-based and more adaptable to the minimization of tail risks. We could also show that minimizing VaR will not yield a convex program and might be hard to solve.

### 3.2.3 Empirical Test

Based on mean-CVaR optimization, we construct portfolios with empirical data. The parameters of portfolios are estimated from historical data from 1973 to 2006, and portfolios are evaluated based on data from 2007 to 2015 in an out-of-sample test. The data include eight asset categories (U.S. Equity, International Equity, U.S. Treasury, Corporate Bond, Real Estate, Commodity, TIPS, and risk-free) and the
data sources are listed in Appendix A. All returns are in real terms, accounting for inflation as computed from CPI index.

We set target real returns from 3.5% to 6% annually, with an increment of 0.5%. Only private equity achieved an average real return of more than 5.5%, so the 6% target will be mostly focused on private equity in all portfolios. The lower bound, 3.5%, is set as the average of bond returns.

The goal of these out of sample empirical tests is to evaluate the performance of portfolios during the 2007-2009 global crash (with maximum drawdown as criteria) and the subsequent performance until 2015. As such, we do not rebalance or re-optimize during the evaluation period.

Table 3.1 and 3.2 shows standard risk and return evaluation metrics for mean-variance portfolios (traditional) and mean-CVaR portfolios (with scenarios based on two-regime simulations). We compare mean, volatility, VaR, CVaR, maximum drawdown, and Sharpe ratio.

<table>
<thead>
<tr>
<th>Portfolio Target Return $r_0$</th>
<th>3.50%</th>
<th>4.00%</th>
<th>4.50%</th>
<th>5.00%</th>
<th>5.50%</th>
<th>6.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (Annualized)</td>
<td>1.00%</td>
<td>1.43%</td>
<td>1.70%</td>
<td>1.98%</td>
<td>1.36%</td>
<td>0.95%</td>
</tr>
<tr>
<td>VaR (5%)</td>
<td>4.34%</td>
<td>5.28%</td>
<td>6.49%</td>
<td>7.78%</td>
<td>10.51%</td>
<td>13.37%</td>
</tr>
<tr>
<td>CVaR (5%)</td>
<td>5.73%</td>
<td>7.32%</td>
<td>9.06%</td>
<td>11.25%</td>
<td>15.83%</td>
<td>21.44%</td>
</tr>
<tr>
<td>Volatility (Annualized)</td>
<td>7.27%</td>
<td>9.18%</td>
<td>11.19%</td>
<td>13.63%</td>
<td>18.33%</td>
<td>23.74%</td>
</tr>
<tr>
<td>Sharpe Ratio (Annualized)</td>
<td>0.0421</td>
<td>0.0454</td>
<td>0.0465</td>
<td>0.0437</td>
<td>0.0274</td>
<td>0.0171</td>
</tr>
<tr>
<td>Max Drawdown</td>
<td>24.46%</td>
<td>29.83%</td>
<td>34.98%</td>
<td>40.81%</td>
<td>50.39%</td>
<td>58.38%</td>
</tr>
</tbody>
</table>

Table 3.1: Portfolio Performance between 2007 and 2015, Mean-Variance Portfolio
Table 3.2: Portfolio Performance between 2007 and 2015, Mean-CVaR Portfolio

We observe that risk metrics (VaR, CVaR, volatility, and maximum drawdowns) increase when target returns increase because when target return is higher, allocation in assets that provide high returns and high risks, such as equities, is higher, and downside risks inevitably increase. Sharpe ratios for portfolio with target return of 4.5% is the highest.

Comparing to table 3.1 portfolios in table 3.2 have better risk and return performance. Taking portfolios with 4.5% target return as an example, the mean-CVaR portfolio yields higher geometric mean of return (2.13% vs 1.70%, annualized), lower 5% VaR, higher Sharpe ratio, and lower maximum drawdown (29.15% vs. 34.98%) during the 2008-2009 crash period.

Such results are as expected because the mean-CVaR portfolios address the tail risk with the aid of the regime-based scenario generator. Comparatively speaking, regime-based portfolio construction captures information of different regimes and thus can adapt to different economic environments more easily.

\[\text{Table 3.2: Portfolio Performance between 2007 and 2015, Mean-CVaR Portfolio}\]

<table>
<thead>
<tr>
<th>Portfolio Target Return $r_0$</th>
<th>3.50%</th>
<th>4.00%</th>
<th>4.50%</th>
<th>5.00%</th>
<th>5.50%</th>
<th>6.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (Annualized)</td>
<td>1.17%</td>
<td>1.68%</td>
<td>2.13%</td>
<td>2.35%</td>
<td>1.54%</td>
<td>0.08%</td>
</tr>
<tr>
<td>VaR (5%)</td>
<td>3.42%</td>
<td>4.06%</td>
<td>4.74%</td>
<td>6.18%</td>
<td>9.40%</td>
<td>13.92%</td>
</tr>
<tr>
<td>CVaR (5%)</td>
<td>4.88%</td>
<td>6.17%</td>
<td>7.53%</td>
<td>9.44%</td>
<td>14.86%</td>
<td>23.30%</td>
</tr>
<tr>
<td>Volatility (Annualized)</td>
<td>6.09%</td>
<td>7.62%</td>
<td>9.25%</td>
<td>11.50%</td>
<td>17.19%</td>
<td>25.33%</td>
</tr>
<tr>
<td>Sharpe Ratio (Annualized)</td>
<td>0.0520</td>
<td>0.0554</td>
<td>0.0563</td>
<td>0.0535</td>
<td>0.0322</td>
<td>0.0014</td>
</tr>
<tr>
<td>Max Drawdown</td>
<td>20.46%</td>
<td>24.82%</td>
<td>29.15%</td>
<td>35.14%</td>
<td>47.88%</td>
<td>60.32%</td>
</tr>
</tbody>
</table>

\[\text{Except for the portfolio with a target return of 6\%, which might be too high as the real return.}\]
3.2.4 Summary

In this example, we observe that traditional mean-variance portfolio optimization methods and related downside risk measurements tend to underestimate the frequency of worst-case events. Accordingly, we can improve the downside risk estimation through a regime-based approach. Furthermore, for the example, mean-CVaR optimization with multiple regimes is superior to mean-variance optimization in the way that it leads to portfolios that protect investors against severe loss during sharp drawdown periods. We will see another example that actually estimates the downside risk in the next section.
3.3 Application: Downside Risk Control

In this section, we extend the regime-based asset return generator to multi-period and use our two-regime environment to evaluate the downward risk of a university’s endowment. One important reason we choose to study a university endowment is that the endowment is always responsible for the most of the university’s expenditure. Thus it is of vital importance for the endowment to weather major crashes so that the university can provide constant and steady level of service to its faculty and students, especially during severe economic conditions such as the financial crisis in 2008. A possible procedure adopted by some of the universities is Monte Carlo simulation. The university examines the current portfolio of the endowment, set the return vector and the variance-covariance matrix and run forward-looking simulations based on the portfolio.

We provide two improvements to this simulation. First, we introduce the regime-switching simulation approach, which involves distinct return vectors and covariance matrices under each regime and a transition matrix to switch between the two regimes. Regime-based modeling leads to more realistic scenarios, as we have seen in the previous section. Next, we replicate the strategies of some universities during the 2008 financial crisis period; we cut the spending rate by 25% and observe the improvement of performance. We focus, in particular, on the “worst case” probability: we computed the probability of losing 25% of the total capital in 5 years, as well as the probability of losing 50% of the total capital in 50 years.
Algorithm 2 describe the regime-switching mechanism for asset return simulations.

**Result:** 3-D Matrix containing M scenarios: \( S \)

Compute transition matrix \( p_{nn}, p_{nc}, p_{cn}, p_{cc} \);

Initialize current regime \( R \) (1: normal, -1: crash);

**while** \(|S| < M\) **do**

    Initialize one scenario \( s \) with empty matrix;

    **while** \( t < T \) **do**

        if \( R = 1 \) then
            generate uniform random number \( q \in [0, 1] \);
            if \( q < p_{nn} \) then generate \( s_t \sim \mathcal{N}(\mu_{normal}, \Sigma_{normal}) \);
            s.append(\( s_t \)), \( R \leftarrow 1 \) ;
            else generate \( s_t \sim \mathcal{N}(\mu_{crash}, \Sigma_{crash}) \);
            s.append(\( s_t \)), \( R \leftarrow -1 \) ;
        else
            generate uniform random number \( q \in [0, 1] \);
            if \( q < p_{cn} \) then generate \( s_t \sim \mathcal{N}(\mu_{normal}, \Sigma_{normal}) \);
            s.append(\( s_t \)), \( R \leftarrow 1 \) ;
            else generate \( s_t \sim \mathcal{N}(\mu_{crash}, \Sigma_{crash}) \);
            s.append(\( s_t \)), \( R \leftarrow -1 \) ;
        end
    end

    if \( \bar{r}_{stock} \in [r_l, r_u] \) then S.append(\( s \));
end

**Algorithm 2:** Generating Multi-Period Scenarios
3.3.1 Settings of the Simulation Problem

We build our portfolio based on the allocation determined in chapter 2.1.2.

<table>
<thead>
<tr>
<th>Asset</th>
<th>Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Equities</td>
<td>22%</td>
</tr>
<tr>
<td>EAFE Equities</td>
<td>0%</td>
</tr>
<tr>
<td>EM Equities</td>
<td>10%</td>
</tr>
<tr>
<td>Private Equities</td>
<td>35%</td>
</tr>
<tr>
<td>US Treasuries</td>
<td>18%</td>
</tr>
<tr>
<td>Corp Bonds</td>
<td>0%</td>
</tr>
<tr>
<td>Real Estate</td>
<td>0%</td>
</tr>
<tr>
<td>Commodities</td>
<td>5%</td>
</tr>
<tr>
<td>TIPS</td>
<td>0%</td>
</tr>
<tr>
<td>Absolute Return</td>
<td>10%</td>
</tr>
</tbody>
</table>

Table 3.3: Portfolio Construction

The next step is to determine the spending rule of the university, which contributes to the major difference between our forward-looking simulation and the typical financial simulation by investment institutions. Each university endowment has its specific spending rule, following the operational requirements of the university. Here we apply a simple rule with adjustments according to our experience. We consider a university that receives 0.5% of its total wealth as gifts and donations. For the first four years, the spending is controlled at a fixed value (which is 4% in our case, and 3.5% after considering the givings). For the following years, we add a variable denoting the endowment’s financial status:
(a) If the current wealth is more than 80% of the original wealth, we define the current status as “good”. In this case, the current spending amount is set as the average spending of the previous four years. If this amount is more than a predetermined “ceiling” percentage of current wealth or less than the “floor,” then the spending is set at the ceiling or floor rate. Else, the potential spending amount is accepted. In our simulation, the pre-set range is \([3.5\%, 4.75\%]\), and after considering the donations, the spending rate is controlled within the range \([3\%, 4.25\%]\).

(b) If the current wealth is less than 80% of the original wealth, we label the status as “adverse” and cut the spendings. We then use the same rule as (a) but cut the spending by 25% in each case. Therefore the new range after considering the spending (the spending, though, is assumed not to be affected by the adverse condition) is set as \([2.1\%, 3.1\%]\). Moreover, this “adverse” label lasts for four years, so it is not removed until four consecutive years of “good” status.

3.3.2 Monte Carlo Simulation

The one-regime simulation is a simple mean-covariance simulation with the historical data from 1994 to 2015. Moreover, we do not introduce the spending rule for comparison purpose, so the spending rate always falls in the range \([3\%, 4.25\%]\).

Upon assuming the total initial capital as 1, we have the key statistics in the simulation (table 3.4). We find that the probability of a major drawdown (50%) in 50 years is less than 5%, indicating that the Value at Risk at 95% confidence interval is just about 50% in this case. This low level of risk may lead to the conclusion that the university is not likely to suffer from a severe loss.
In the two-regime analysis, we split the monthly return series into the “normal” regime and the “crash” regime according to the trend-filtering algorithm based on the S&P index. Then we compute the historical geometric mean as well as covariance-variance matrices for each period and use a transition matrix to switch between the states, which is also computed by historical data. Finally, we ensure that the total average return is consistent with the one-regime simulation (the average return under two regimes is the same). Table 3.5 below shows the key statistics.

<table>
<thead>
<tr>
<th>Probability of losing 25% in 5 years</th>
<th>10.3%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of losing 50% in 50 years</td>
<td>4.9%</td>
</tr>
<tr>
<td>Average total asset worth, 5 years</td>
<td>1.0644</td>
</tr>
<tr>
<td>Average total asset worth, 10 years</td>
<td>1.1635</td>
</tr>
<tr>
<td>Average total asset worth, 20 years</td>
<td>1.3998</td>
</tr>
<tr>
<td>Average total asset worth, 50 years</td>
<td>2.6630</td>
</tr>
</tbody>
</table>

Table 3.4: Baseline Results, One-Regime

<table>
<thead>
<tr>
<th>Probability of losing 25% in 5 years</th>
<th>18.4%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of losing 50% in 50 years</td>
<td>19.9%</td>
</tr>
<tr>
<td>Average total asset worth, 5 years</td>
<td>1.0780</td>
</tr>
<tr>
<td>Average total asset worth, 10 years</td>
<td>1.1793</td>
</tr>
<tr>
<td>Average total asset worth, 20 years</td>
<td>1.4230</td>
</tr>
<tr>
<td>Average total asset worth, 50 years</td>
<td>2.6197</td>
</tr>
</tbody>
</table>

Table 3.5: Baseline Results, Two-Regime
We discover that the average total asset worth under the two-regime setting is almost the same as that in the one-regime, but the “worst-case” probabilities become much higher.

This fact suggests that the two-regime analysis may alert the university endowment about the true probability of the “extreme” events, which can be higher than what we have expected. In order to confirm that this advantage, we rerun the two simulations with reduced spending in “adverse” conditions.

### 3.3.3 Statistics With the Spending Cut

In this section, we simulate the spending rule (b). Therefore the spending is reduced by 25% in bad conditions. This time, we add a variable that reflects the percentage of time in the “adverse” condition and control the spending rule by this status variable. We expect an increase in the average total asset worth after 5, 10, 20, and 50 years as well as a decrease in the probability of significant loss.

<table>
<thead>
<tr>
<th>Probability of losing 25% in 5 years</th>
<th>10.3%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of losing 50% in 50 years</td>
<td>1.7%</td>
</tr>
<tr>
<td>Average total asset worth, 5 years</td>
<td>1.0644</td>
</tr>
<tr>
<td>Average total asset worth, 10 years</td>
<td>1.1711</td>
</tr>
<tr>
<td>Average total asset worth, 20 years</td>
<td>1.4239</td>
</tr>
<tr>
<td>Average total asset worth, 50 years</td>
<td>2.7350</td>
</tr>
<tr>
<td>Percentage of time in “adverse” period</td>
<td>2.03%</td>
</tr>
</tbody>
</table>

Table 3.6: Spending Cut, One-Regime
<table>
<thead>
<tr>
<th>Probability of losing 25% in 5 years</th>
<th>18.4%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of losing 50% in 50 years</td>
<td>11.7%</td>
</tr>
<tr>
<td>Average total asset worth, 5 years</td>
<td>1.0780</td>
</tr>
<tr>
<td>Average total asset worth, 10 years</td>
<td>1.1904</td>
</tr>
<tr>
<td>Average total asset worth, 20 years</td>
<td>1.4606</td>
</tr>
<tr>
<td>Average total asset worth, 50 years</td>
<td>2.7462</td>
</tr>
<tr>
<td>Percentage of time in “adverse” period</td>
<td>4.30%</td>
</tr>
</tbody>
</table>

Table 3.7: Spending Cut, Two-Regime

Comparing the results in table 3.6 and 3.7:

1. The spending rule (b) does not affect short-time risk analysis very much, since the “crash” probability in 5 years are both 10% and 18%, for one- and mixed-regime, respectively. On the other hand, this rule helps a lot in long-term risk management, as we are observing a significant drop in “crash” probability in 50 years, from 4.9% to 1.7% in one regime, and from 19.9% to 11.7% in the mixed regime. One reason may be that in longer time period, the chance of spending control will increase.

2. The average total wealth after 50 years is a little higher in the mixed regime under the “spending cut” rule because the simulation using the regime switch method is able to capture the events that the total wealth grow significantly, by the same reason the crash probabilities are also higher since the events that the total wealth drops dramatically are also more frequent. However, the “crash” situations are protected by the cut-spending rule, so this rule makes the average 50-year wealth higher.
3. The spending rule is helpful in risk management, and the mixed-regime analysis can better model the extremely adverse events since under such conditions the correlation between assets becomes high and tends to “crash” together. Therefore combining the spending rule and the mix-regime analysis in the simulation will help the trustee of the university in controlling the unfavorable phenomenon.
Chapter 4

Agent-based Modeling

In this chapter, we propose that agent-based modeling be applied to set realistic goals for global pension funds or government-owned funds such as the Social Security in the U.S. The core of our model, agents, are a group of representative individuals at a specific age. By evaluating the discounted total inflows and total liabilities of the whole group, we can search for a balance point where the Social Security can maintain sustainability by successfully matching between the current benefit and contribution amounts. Starting from an introduction of the background, we then briefly summarize the characteristics of the Social Security fund, the research target of this chapter. Scenario-based simulations and sensitivity analysis follow, and a short comment is made based on the results.

This chapter is developed upon a previous publication [82] collaborated with Martellini, Mulvey, and Li.

In the next chapter, we further develop the “agent” by building an integrated example of a personal investment planning problem.
4.1 Setting Realistic Goals for Pension Funds

The main cause of the funding issues among pension funds is over-promises and low interest rates. There are also arguments about the investment decisions of some DB pensions funds, as well as unfavorable macroeconomic policies and demographic challenges. Still, the mismatch between contributions and benefits plays a significant role.

The usual order of business for pension funds is to compute its funding ratio (defined as the total present value of assets divided by the total present value of liabilities) annually with a specified discount rate, amortize any deficits over multiple years, and render required contributions. This process has been unable to preserve the long-term health of DB pension systems in the United States. (Mulvey [96]) It leads to under contributions and a slow realization that the survivability of the pension system is threatened. Also, the volatility of contributions can be high and thereby difficult to manage from the standpoint of the sponsor.

One innovative aspect of our agent-based modeling approach is to evaluate the macroeconomic impacts and adequacy of goals at the individual level and conduct a sensitivity analysis on the impact of contribution rate and benefit amount to the sustainability of the fund. The Social Security fund is the starting point due to steady income flow (social security tax) and less volatile asset returns (most returns are generated from interest rates). The goals are also simple to define: the level of contribution that maintains the balance of the fund.

4.2 The Social Security System

As one significant pillar of the U.S. retirement system, the Social Security program was started by the U.S. government in 1935 and had been paying on a timely schedule
ever since. The program is funded by tax from both employers and employees in the form of payroll taxes or self-employment taxes. It pays for retirement and other benefits such as disability.

The mechanism of Social Security benefit is straightforward: workers and their employers pay the Social Security tax\(^1\) and qualified workers\(^2\) can start to enjoy the Social Security benefit when they reach a certain age. Applying for the benefit before the full retirement age (FRA) will incur a penalty, while deferred retirement will grant 8% more for each year deferred.

Since 1941, it is required that the Social Security Board of Trustees presents to the Congress a financial report and detailed actuarial estimates of the fund. According to \([50]\), the board has six members, including the Secretary of the Treasury as the managing trustee, the Secretary of Labor, the Secretary of Health and Human Services, and the Commissioner of Social Security, plus two public trustees appointed by the president and confirmed by the senate. The annual report \([22]\) shows the current financial statement of the fund.

### 4.3 Micro-Macro Model for the Social Security Fund

In this section, we create a highly stylized agent-based model in the case of calculating the proper level of support of the Social Security Fund. An effective micro-macro model should be able to select realistic pension benefits and show the impact of a

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\(^1\)The tax shown on the payroll is actually called FICA tax; it includes 6.2% Social Security tax and 1.45% Medicare tax.

\(^2\)To qualify, an individual needs at least 10 full working years.
potential policy change. We discuss model assumptions, detailed model structure, and compare benefit decisions in terms of FRAs.

4.3.1 Model Assumptions

The model is built on three major categories of assumptions: demographic assumptions, macroeconomic assumptions, and program policy assumptions. We follow the assumptions mentioned in the OASDI annual report \[22\] and adjust with our macroeconomic scenario generator in chapter 3.

Demographic Assumptions

We simulate a population of 1,000,000 people who just joined the workforce at age 25. Since the age of this workforce is fixed, the model does not consider demographic changes except the mortality rate. (For example, the expansion/contraction of the workforce is not modeled because the new workforce is modeled every year.) The proportion of male and female is based on current workforce distribution. Per the mortality table in [22], the model simulates male and female employees separately based on their expected mortality rate.

The current mortality rate is taken from the Social Security website from ages 25 to 120. However, the mortality rate declines as time goes by. In the three possible scenarios given by the board of trustee (2018), the rate will decline each year at the speed of 0.41, 0.77, and 1.15 percent on average. To be consistent with the board’s assumptions, we apply a rate of decline at 0.65 and 0.85 percent per year, respectively, for men and women.

\[^3\text{Roughly 53:47.}\]
Macroeconomic Assumptions

The main economic assumptions are the interest rate $r_t$, the inflation rate $i_t$, and the cost-of-living (COLA) rate $C_t$. We will apply the tools described in chapter 3.1 to generate these three series. The stylized system can be expressed as:

$$dr_t = \lambda_1 (\mu_1 - r_t) dt + \sigma_1 r_t \frac{1}{2} dZ_t^1 \quad (4.1)$$

$$di_t = \lambda_2 (\mu_2 - i_t) dt + \sigma_2 dZ_t^2 \quad (4.2)$$

$$C_t = \beta_1 r_t + \beta_2 i_t + \beta_3 i_{t-1} + \sigma_3 dZ_t^3 \quad (4.3)$$

According to historical data, the long-term mean of interest rate (after expenses) is estimated to be around 2.4%. We use the interest rate as the discount rate for present value calculations. The interest rate is simulated by a Cox-Ingersoll-Ross (CIR) model with a long-term mean ($\mu_1$) and standard deviation ($\sigma_1$) estimated via maximum likelihood methods mentioned in 3.1 and adjusted with current trends of Fed regulations.

We assume the wage increases at the inflation rate. There is a myriad of modeling techniques to simulate the inflation rate. Under the modern monetary theory, the Fed has set the inflation rate as its core variable to control. Thus it is reasonable to model the inflation rate with a mean-reverting process such as the Ornstein-Uhlenbeck process. We choose a large mean-reversion coefficient ($\lambda_2 = 0.5$) to represent the policy effectiveness of the Fed and compute the volatility coefficient to reflect the historical volatility of the inflation rate (which is around 1% over the last 25 years).

The rise in Social Security benefits has been following the “cost-of-living” adjustments (or COLAs)\textsuperscript{[4]} From historical data, we found that the average of COLA

\textsuperscript{[4]} Detailed description of COLA can be found in [https://www.ssa.gov/oact/cola/colaserie](https://www.ssa.gov/oact/cola/colaserie).
is about 0.05% higher than inflation with higher standard deviation as well. The correlation between inflation rate and COLA is 0.7, a high level as expected.

In observation of the official description that the COLA follows the interest rate as well as the inflation rate of the current and previous year, we run a regression on historical data of COLA, interest rate, and inflation rate:

<table>
<thead>
<tr>
<th>Dep. Variable:</th>
<th>COLA</th>
<th>R-squared:</th>
<th>0.907</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model:</td>
<td>OLS</td>
<td>Adj. R-squared:</td>
<td>0.897</td>
</tr>
<tr>
<td>Method:</td>
<td>Least Squares</td>
<td>F-statistic:</td>
<td>94.38</td>
</tr>
<tr>
<td>Df Residuals:</td>
<td>29</td>
<td>Prob (F-statistic):</td>
<td>4.58e-15</td>
</tr>
<tr>
<td>Df Model:</td>
<td>3</td>
<td>Log-Likelihood:</td>
<td>-42.201</td>
</tr>
<tr>
<td>No. Observations:</td>
<td>32</td>
<td>AIC:</td>
<td>90.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BIC:</td>
<td>94.80</td>
</tr>
</tbody>
</table>

| coef          | std err | t    | P>|t| | [0.025 0.975] |
|---------------|---------|------|-------|----------------|
| Inflation     | 0.1373  | 0.168| 0.816 | 0.421 -0.207 0.481 |
| Inflation-prev| 0.6061  | 0.132| 4.588 | 0.000 0.336 0.876 |
| Treasury-adj  | 0.1432  | 0.108| 1.320 | 0.197 -0.079 0.365 |

<table>
<thead>
<tr>
<th>Omnibus:</th>
<th>7.824</th>
<th>Durbin-Watson:</th>
<th>2.294</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob(Omnibus):</td>
<td>0.020</td>
<td>Jarque-Bera (JB):</td>
<td>6.852</td>
</tr>
<tr>
<td>Skew:</td>
<td>0.757</td>
<td>Prob(JB):</td>
<td>0.0325</td>
</tr>
<tr>
<td>Kurtosis:</td>
<td>4.688</td>
<td>Cond. No.</td>
<td>7.42</td>
</tr>
</tbody>
</table>

Table 4.1: OLS Regression Results
We use the linear relationship in equation (4.3) to simulate annual COLA values, since it is mostly determined by inflation rate, inflation rate of the previous year, and the interest rate. The linear coefficients will follow the results in table 4.1. A sample 100-year path of rates is shown in figure 4.1. The inflation rates have higher volatility than the interest rates, and the COLA rates follow closely with the inflation rate. This concludes our system of macroeconomic variables.

Program-specific Assumptions

The Social Security benefit is roughly proportional to the salary of the individual up to an upper limit, so we simulate our 1 million workforces at the cap of social security tax, and correspondingly the maximum full benefit employees will receive upon retirement.

As for 2019, the cap of income that is taxable for Social Security is $132900. The effective tax rate now is 12.4%, equally divided between the employer and
employee. This means in real term, each person in the new workforce (together with the employer) is paying a fixed amount of $16479.6 per year.

The maximum monthly Social Security benefit payment for the employee at full retirement age (FRA) is $2,861 if the employee has paid the maximum tax for at least 35 years. The average for the all retired workers is estimated at $1360. This benefit increases by the cost-of-living adjustments (COLAs) each year. Finally, deferred retirement earns an additional 8% per year over FRA, up to age 70. The current FRA is set between 66 years old (for people born before 1944) and 67 years old (for people born after 1960), but we have been witnessing an increasing trend. We assume the maximum amount ($34332 annually) for all workers and that this value increases at our simulated COLA rates every year.

4.3.2 Model Overview

The ultimate goal of the Social Security fund is that across time, it should balance the sum of expected contributions plus investment returns with the expected outflows to the retirees. Thus, on average, the contributions by the employees and employer plus investment returns should equal expected benefit payments. For the group of 1 million people, their total lifespan is divided into a contribution period and a withdrawal period.

Notations

We use the following notations:

- $t$: time. Starting from 25.
- $T$: full retirement age (FRA).
- $p_{t_1|t_2}$: the survival probability at time $t_1$, given the person is alive at time $t_2$. 
\( s \in S \): scenario. We generate \(|S| = 10000\) scenarios.

- \( C_{t,s} \): total capital at the end of year \( t \), in scenario \( s \). Note that \( C_{25,s} = 0 \).

- \( r_{t,s} \): interest rate generated per macroeconomic assumptions.

- \( i_{t,s} \): inflation rate generated per macroeconomic assumptions.

- \( \text{COLA}_{t,s} \): COLA rate generated per macroeconomic assumptions.

- \( y_{t,s} \): inflow (positive) or outflow (negative) of capital.

- \( y_0^{\text{cont}}, y_0^{\text{bene}} \): current contribution and benefit. The numbers are 16479.6 and 34332 dollars, respectively.

### Contribution Period

Per our assumptions in the above section, for each year \( t = 26, \ldots, T \):

\[
C_{t,s} = (1 + r_{t,s})C_{t-1,s} + y_{t,s}p_{t|26}
\]

where

\[
y_{t,s} = y_0^{\text{cont}} \cdot \prod_{u=26}^{t} i_{u,s}.
\]

### Withdrawal Period

Here \( t = T + 1, \ldots, 125 \). The upper bound reflects 100 years of macroeconomic data.

\[
C_{t,s} = (1 + r_{t,s})C_{t-1,s} - y_{t,s}p_{t|26}
\]

where

\[
y_{t,s} = y_0^{\text{bene}} \cdot \prod_{u=26}^{t} \text{COLA}_{u,s}.
\]

### 4.3.3 Simulation Results - Baseline

We generate 10,000 independent scenarios with macroeconomic variables. Due to the randomness of the scenario, for each individual, the total (discounted) contribution
and total (discounted) benefit may not be consistent. For example, a person could live as long as 100 years old and reap much more net benefits.

On the aggregate level, we are able to estimate the final capital \( C_{125,s} \) in each scenario, according to equation (4.4) and (4.6). Table 4.2 below shows the probability that \( C_{125} > 0 \), i.e., the proportion of scenarios in which the total contribution of the workers can cover their Social Security benefits.

The contribution rate ranges from 12% to 13%, as the probability ranges from 56% to 85%. On the 12% level, a mere 0.2% increase in the contribution rate will grant more than 7% boost of the fund’s survivability. This baseline result is useful for policymakers and program analysts when they consider the appropriate contribution level and its impact.

<table>
<thead>
<tr>
<th>Contribution Rate</th>
<th>12.0%</th>
<th>12.2%</th>
<th>12.4%</th>
<th>12.6%</th>
<th>12.8%</th>
<th>13.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>56.2%</td>
<td>63.3%</td>
<td>70.5%</td>
<td>76.2%</td>
<td>81.7%</td>
<td>84.9%</td>
</tr>
</tbody>
</table>

Table 4.2: Baseline Results: Probability with a positive net balance

### 4.3.4 Policy Adjustments

In the last section of the chapter, we run a sensitivity analysis on the assumptions we have made. The goal is to estimate the effect of policy adjustments.

**Interest Rate Sensitivity Analysis**

The first set of sensitivity analysis focuses on the interest rate. We keep the FRA at 69 years old and adjust the long-term mean of the simulated interest rate. The probability of self-sustainability is presented in table 4.3 below. Notice that a small change in long-term estimation of interest rate could result in enormous difference in
self-sustainability. Therefore, it is crucial to set a realistic relationship of inflation, interest rate, and COLA to obtain reasonable simulation results.

<table>
<thead>
<tr>
<th></th>
<th>12.0%</th>
<th>12.2%</th>
<th>12.4%</th>
<th>12.6%</th>
<th>12.8%</th>
<th>13.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>56.2%</td>
<td>63.3%</td>
<td>70.5%</td>
<td>76.2%</td>
<td>81.7%</td>
<td>84.9%</td>
</tr>
<tr>
<td>Long-term +0.25%</td>
<td>83.5%</td>
<td>87.6%</td>
<td>90.6%</td>
<td>93.0%</td>
<td>94.7%</td>
<td>96.6%</td>
</tr>
<tr>
<td>Long-term -0.25%</td>
<td>27.2%</td>
<td>32.8%</td>
<td>40.3%</td>
<td>47.8%</td>
<td>54.0%</td>
<td>60.1%</td>
</tr>
</tbody>
</table>

Table 4.3: Probability with a positive net balance under interest rate change

**Postponing Retirement**

Next, we consider the situation in which some people choose to retire later so that they can maximize their Social Security benefits. While people are allowed to take Social Security benefit as early as age 62, the early withdrawal penalty applies to an individual before FRA. On the other hand, people will receive delayed retirement credits for deferring retirement. The current rate is 8 percent for each deferred year. For instance, deferring to retire at 70 years old while the FRA is 67 years old will grant the individual 24% more benefit.

In observance of the policy above, we run a set of sensitivity analysis based on the distribution of retirement age. In the benchmark model, we assumed a unified retirement at age 69. Here we consider two distributions of retirement ages, one of which includes slightly delayed retirements:
In the other distribution, people are leaning towards later retirement:

Table 4.4 below provides the probability of successful maintenance of the fund. We observe that deferred retirement has minimal effect on the sustainability of the fund. This sensitivity analysis suggests that the determination of appropriate pension benefits should be evaluated on several considerations.
Table 4.4: Probability with a positive net balance, deferred retirements

<table>
<thead>
<tr>
<th></th>
<th>12.00%</th>
<th>12.20%</th>
<th>12.40%</th>
<th>12.60%</th>
<th>12.80%</th>
<th>13.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark</strong></td>
<td>56.2%</td>
<td>63.3%</td>
<td>70.5%</td>
<td>76.2%</td>
<td>81.7%</td>
<td>84.9%</td>
</tr>
<tr>
<td><strong>Dist1</strong></td>
<td>53.3%</td>
<td>60.2%</td>
<td>66.8%</td>
<td>73.4%</td>
<td>78.8%</td>
<td>83.1%</td>
</tr>
<tr>
<td><strong>Dist2</strong></td>
<td>50.1%</td>
<td>57.0%</td>
<td>63.0%</td>
<td>70.8%</td>
<td>76.2%</td>
<td>81.8%</td>
</tr>
</tbody>
</table>

FRA Policy Change

In consideration of the estimated FRA at 69 for this group of people, we would like to run another sensitivity test of changing the FRA to 68 or 70 years old. From table 4.5, we observe that the early FRA significantly reduces the self-sustainability of the fund and vice versa.

Table 4.5: Probability with a positive net balance, FRA change

<table>
<thead>
<tr>
<th></th>
<th>12.00%</th>
<th>12.20%</th>
<th>12.40%</th>
<th>12.60%</th>
<th>12.80%</th>
<th>13.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark</strong></td>
<td>56.2%</td>
<td>63.3%</td>
<td>70.5%</td>
<td>76.2%</td>
<td>81.7%</td>
<td>84.9%</td>
</tr>
<tr>
<td><strong>FRA = 68</strong></td>
<td>30.6%</td>
<td>37.0%</td>
<td>45.2%</td>
<td>51.5%</td>
<td>57.4%</td>
<td>64.5%</td>
</tr>
<tr>
<td><strong>FRA = 70</strong></td>
<td>78.4%</td>
<td>83.2%</td>
<td>88.0%</td>
<td>90.5%</td>
<td>93.7%</td>
<td>95.8%</td>
</tr>
</tbody>
</table>

4.4 Summary: Social Security Fund

These sensitivity analyses suggest that the determination of appropriate pension benefits should be evaluated on several considerations.

First, the benefits need to be reasonable for both the fund and the beneficiaries. Under most circumstances, a long-term contribution rate below the current threshold (12.4%) is unsustainable. Also, there should be careful modeling of the retirement age distribution. It is becoming evident that the traditional retirement
age 65 will need to be increased going forward (the current policy is age 67, to be implemented in 2027). According to table 4.5, even one year difference in FRA results in around 20% change in survivability. Finally, the modeling of various policy rates, such as the interest rate and inflation rate, should be handled with utmost care. Monetary policies always have the power to change the funding status of pensions.

Our micro-macro analysis provides a framework for these meaningful policy discussions. As an essential part, we have been modeling a target group of individuals at the maximum level of contributions to the Social Security fund. It is relatively straightforward to model these individuals since the Social Security system has a cap on the level of payout during retirement. We show that the agent-based model provides information to inform policy analysts about the survivability of the pension plans under a set of stochastic scenarios for an economy.
Chapter 5
Regime-aware Goal-based
Financial Planning Models

The agent-based model in the previous chapter has demonstrated the importance of setting realistic goals for a large public welfare fund. The same framework could also be applied to goal-based personal planning problems, in which we identify the goal as saving enough money for retirement.

To simulate the agent-based model for an individual, we apply the full planning framework shown in figure 1.1. It combines the factor- and regime-based asset return simulator with the macroeconomic time series generator, as well as the policy rules estimator for the agent that takes into account the simplest “buy-and-hold” strategies and dynamically adjusted allocations based on age and current goal completion status.

We understand that limiting risks is at least equally important as promising asset growth in retirement planning. Besides the probability of achieving the goal, we also provide a couple of risk measures with respect to the goal.
5.1 Overview of Retirement Planning

One of the primary concerns for the new century is that people may have inadequate savings for retirement. Naturally, the first pillar of retirement income is the Social Security benefit. It provides a stable and predictable benefit to the retiree, only subject to the uncertainty of inflation rates. We apply our macroeconomic generator and agent-based Social Security simulator in chapter 4 to estimate the total amount of Social Security benefits. Comparing Social Security benefit with the same savings amount that invests in a portfolio with 100% inflation-linked government bonds, we discover that the former promises similar total income as the latter.

Traditionally, individuals have relied on defined-benefit (DB) pension funds as their primary source of income during retirement. However, a significant portion of DB pension funds has remained underfunded after hitting by the tech bubble in 2001-2002 and the global financial crisis between 2007 and 2009. It becomes extremely tough for the DB pension funds to catch up due to the inability to reduce promised payments and the diminishing workforce.

In recent years, an increasing number of companies have been switching from DB plans to defined-contribution (DC) plans, which lightens the burden of managing a pension fund for the company. On the other hand, this requires the employees to plan their investments carefully. As such, the retirement planning process should be a goal-based asset-liability management problem which could be solved by utilizing optimization tools (details described in [120], for example). The same scenarios apply to pension funds, as the discussions presented in [96].

For many people, target-date funds (TDFs) has been a benchmark for solving their problem, especially after the Pension Protection Act of 2006 which designated
TDF as one of the three possible Qualified Default Investment options. Investors held, in total, about $1.4 trillion in TDFs and lifestyle funds at the end of 2018, compared to a mere $2.75 billion at the end of 1995 (ICI Handbook [64]). The basis of TDFs is the human capital theory that first proposed by Shiller [108] and Cocco [31]. Since human capital usually has a negative correlation with total savings, the investors should diversify the total portfolio by more allocation on equities when they are young. A couple of literature, such as [23] and [65], also confirms the strategies of life-cycle funds.

Nevertheless, we have been witnessing mismanagement of individual retirees, even for those who utilized target-date funds (TDFs). “… (A) net $532 billion in investor money poured in between 2005 and 2007” towards TDFs that mature in 2010, reported by Morningstar [89], only to lost almost one quarter during the financial crisis. According to Towers Watson [116], investors planning to retire in 15 years or more had experienced declines of 27% to 37% in the value of their plans during 2008-2010. We also discuss the flaws of TDFs in section 5.4.

In this chapter, we recommend that a first and foremost step in retirement planning is to set up a realistic goal and consistently update the probability to reach the goal. For example, the goal of funds that promises constant income stream for retirees (such as DB pension funds or the Social Security fund) should be a well-crafted balance between providing maximum benefits for participants and maintaining the sustainability of the fund. Similar situations apply to individuals: under the current U.S. pension system, the personal retirement income will be a combination of Social Security benefit and personal investment account withdrawal, so the target would be the most important factor in deciding the annual savings amount and the asset allocation. Here, we follow the report by United States Government Accountability
Office and set the goal as the percentage of the first post-retirement income to final salary, referred to as the “replacement rate.” More discussions can be found in Asad-Syed et al. and Barney.

With a benchmark probability to achieve the retirement goal, the subsequent concern is to improve the risk-adjusted performance. We propose multiple dynamic policy rules as guidelines and use the regime-based asset return simulator to calculate the risk measures of reaching various investment goals for a couple of portfolio allocations. The two-regime setting provides more realistic simulations, as discussed in and . According to Sheikh and Sun, the regime-switching mechanism also demonstrates the effectiveness of dynamic asset allocation strategies over fixed strategies.

The ultimate goal is that the agent-based simulation helps the individual to establish a reasonable retirement goal and choose asset allocations that maximize the chance to achieve her goal.

5.2 Retirement Analysis: Replacement Income

As one significant pillar of the retirement system, the Social Security fund provides solid benefits to people at or after the full retirement age (FRA). To estimate the help of Social Security benefit, let us consider a personal planning problem in which a person saves as much as the Social Security tax and calculate the fraction of income this investment can provide at retirement, i.e., the replacement rate.

Consider Ms. Lisa Li, born in 1994 (age 25), who starts working with a total salary of $132900 per year (the cap of current Social Security taxable amount). Her
salary will grow at the inflation rate during her career, and she plans to work until
the estimated FRA, age 69. She has a “personal security” account in which she
puts 12.4% of her salary each year, and the account invests in inflation-linked bonds,
namely the Treasury Inflation-Protected Securities (TIPS) that is assumed to yield
0.25% over inflation rate each year. This model implies that the total amount Lisa
can save with this account is solely determined by the inflation rate.

We follow the settings in chapter 4.3.2. The total amount Lisa saves with this
low-risk account in scenario $s$, at year $t$ is

$$C_{t,s} = (1 + i_{t,s} + 0.25\%)C_{t-1,s} + y_{t,s} \quad (5.1)$$

where

$$y_{t,s} = y_0^\text{cont} \cdot \prod_{u=26}^{t} i_{u,s}. \quad (5.2)$$

To calculate the replacement rate, we need to estimate the expected total spending
for 100% replacement income (i.e. if she withdraws 100% of her last income $W_T$
during the first year of retirement, and this amount grows at the inflation rate):

$$S_{t,s} = \sum_{t=T+1} W_T \cdot p_{t|T} \cdot \prod_{u=T+1}^{t} \left( 1 + r_{u,s} \right) \quad (5.3)$$

where

$$W_T = W_0 \cdot \prod_{u=26}^{T} i_{u,s}. \quad (5.4)$$

We follow the survival probability $p_{t|T}$ of the agent-based model in the last chapter.
According to the adjusted mortality table, Lisas expected remaining life is 20.92
years at 2063 (age 69), about 3-4 years longer than a 69-year-old female today.

The simulation results for the replacement rate $\frac{C_{t,s}}{S_{t,s}}$ are listed in the table 5.1
below; the median replacement is around 27.4%. We will then estimate the Social
Security benefit with the same framework.
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected longevity</td>
<td>20.92</td>
</tr>
<tr>
<td>Median final salary</td>
<td>$359,162</td>
</tr>
<tr>
<td>Median total amount saved</td>
<td>$1,862,927</td>
</tr>
<tr>
<td>Median amount needed for 100% income</td>
<td>$6,859,645</td>
</tr>
<tr>
<td>Median replacement rate</td>
<td>27.4%</td>
</tr>
</tbody>
</table>

Table 5.1: Replacement Analysis for Lisa Li

For Lisa’s expected Social Security benefit, the total amount of benefits are computed with simulated COLA values and initial benefit $b_0 = $34,332:

$$B_{t,s} = \sum_{t=T+1} \left( b_T \cdot p_{t|T} \cdot \prod_{u=T+1}^{t} \frac{1 + \text{COLA}_{u,s}}{1 + r_{u,s}} \right)$$  \tag{5.5}

where

$$b_T = b_0 \cdot \prod_{u=26}^{T} \text{COLA}_{u,s}.$$  \tag{5.6}

Table 5.2 informs us that the Social Security benefit will cover 26.3% of pre-retirement income, close to what we have calculated for Lisa when she invests individually, thus exhibiting a micro-macro consistency. This consistency hints the effectiveness of agent-based modeling; the model focuses on individuals decision units in conjunction with a simulation of a carefully curated set of individuals within a stratified “sample” of the population.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected longevity</td>
<td>20.92</td>
</tr>
<tr>
<td>Median final salary</td>
<td>$359,162</td>
</tr>
<tr>
<td>Median Social Security benefit</td>
<td>$1,805,390</td>
</tr>
<tr>
<td>Median amount needed for 100% income</td>
<td>$6,859,645</td>
</tr>
<tr>
<td>Median replacement rate</td>
<td>26.3%</td>
</tr>
</tbody>
</table>

Table 5.2: Social Security benefit analysis for Lisa Li
5.3 The Retirement Simulator

Here we apply the full financial planning framework that we described in chapter 1.1. As for the planning problem of Lisa, we assume that in addition to the Social Security benefit, she has a personal retirement investment account (either a 401(k) account, an IRA account, or a Roth IRA account). We consider all income before tax so that we will hold on to the potential tax benefits of each account.

According to our planning framework, there are three core blocks of the framework: asset return simulator, macro-economic time series generator, and policy rule simulator. We describe the three blocks in detail below.

5.3.1 Macroeconomic Environments

The macroeconomic time series simulation of the retirement planning problem is the same as the Social Security fund model in chapter 4: 10,000 scenarios with interest rates, inflation rates, and COLA rates. The data before retirement determine the increase of wage and the risk-free interest rates, while the data after retirement will be applied to the discounted cash flow model.

\[
\begin{align*}
    dr_t &= \lambda_1 (\mu_1 - r_t) \, dt + \sigma_1 r_t \, dz_1^t \\
    di_t &= \lambda_2 (\mu_2 - i_t) \, dt + \sigma_2 dZ^2_t \\
    C_t &= \beta_1 r_t + \beta_2 i_t + \beta_3 i_{t-1} + \sigma_3 dZ^3_t
\end{align*}
\]

(5.7) (5.8) (5.9)

A sample macroeconomic scenario is presented in figure 4.1 for reference.
5.3.2 Asset Return Simulator

For the personal investment account, each of the 10,000 scenarios will also contain asset return paths for a universe of assets. Here we employ the two-regime Markov switching scenario generator in chapter 3, following algorithm 2. The data sources are listed in Appendix A.

The expected returns under normal and crash regimes are computed via the 5-factor model; the factor returns are adjusted (shrunk) to meet the long-term expectations. The first consideration is a lower return perspective, in accordance with the estimates of Siegel [109]. Another concern is the Stein’s paradox, mentioned in various literature such as [36]. The computed regime-based monthly returns are shown in table 5.3 and table 5.4.

The covariance matrices under two regimes are estimated via exponentially weighted moving average (EWMA) models with the decay parameter $\lambda = 0.995$ per month. The historical data can be found in table 2.6.

For equity-like assets (domestic and foreign equity, high-yield bonds, real estate, commodities), the performance under the normal regime is significantly higher than its long-term average. Although crash periods do not last long, the impact on equities could be severe. Treasury bonds, however, serve as protection under crash regimes with a positive annualized return. Corporate bonds can also provide partial protection but are subject to excessive credit risk during economic downturns.
<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>World Equities</th>
<th>Treasury Bonds</th>
<th>High Yield</th>
<th>Inflation Hedge</th>
<th>Currency Hedge</th>
<th>Monthly Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Return</td>
<td>1.00%</td>
<td>0.20%</td>
<td>0.80%</td>
<td>0.05%</td>
<td>-0.10%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Equities</td>
<td>0.1</td>
<td>0.6</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0.4</td>
<td>1.06%</td>
</tr>
<tr>
<td>EAFE Equities</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.00%</td>
</tr>
<tr>
<td>EM Equities</td>
<td>0.1</td>
<td>0.9</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>-0.4</td>
<td>1.20%</td>
</tr>
<tr>
<td>US Treasuries</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.20%</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>0</td>
<td>0</td>
<td>0.9</td>
<td>0.3</td>
<td>0</td>
<td>0</td>
<td>0.42%</td>
</tr>
<tr>
<td>TIPS</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.25%</td>
</tr>
<tr>
<td>Real Estates</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>0.9</td>
<td>0.5</td>
<td>0</td>
<td>1.05%</td>
</tr>
<tr>
<td>Commodities</td>
<td>0.4</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>1.5</td>
<td>0</td>
<td>0.68%</td>
</tr>
<tr>
<td>T-bills</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.20%</td>
</tr>
</tbody>
</table>

Table 5.3: Expected Return of Assets, Normal Regime
<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>World Equities</th>
<th>Treasury bonds</th>
<th>High Yield</th>
<th>Inflation Hedge</th>
<th>Currency Hedge</th>
<th>Monthly Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Return</td>
<td>-2.50%</td>
<td>0.80%</td>
<td>-0.90%</td>
<td>0.00%</td>
<td>0.20%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Equities</td>
<td>-0.7</td>
<td>0.5</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>0.5</td>
<td>-2.21%</td>
</tr>
<tr>
<td>EAFE Equities</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-2.50%</td>
</tr>
<tr>
<td>EM Equities</td>
<td>-0.4</td>
<td>0.7</td>
<td>0</td>
<td>0.8</td>
<td>0</td>
<td>0</td>
<td>-2.87%</td>
</tr>
<tr>
<td>US Treasuries</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.80%</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>0</td>
<td>0</td>
<td>0.8</td>
<td>0.2</td>
<td>0.2</td>
<td>0</td>
<td>0.46%</td>
</tr>
<tr>
<td>TIPS</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.80%</td>
</tr>
<tr>
<td>Real Estates</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0.8</td>
<td>0</td>
<td>-0.5</td>
<td>-2.07%</td>
</tr>
<tr>
<td>Commodities</td>
<td>-0.6</td>
<td>0</td>
<td>-0.7</td>
<td>-0.3</td>
<td>2.1</td>
<td>-1.6</td>
<td>-1.21%</td>
</tr>
<tr>
<td>T-bills</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.20%</td>
</tr>
</tbody>
</table>

Table 5.4: Expected Return of Assets, Crash Regime
5.3.3 Portfolios

From table 5.2 we understand that on average, Social Security benefit will cover 26% of Lisas full income before retirement. Munnell and Sunden estimated an average of around 9% contribution rates to 401(k) accounts in their paper \[97\]. If Lisa saves another 10-15% of her salary\[1\] she can aim at 60-90% replacement income after retirement.

The first goal is to estimate the approximate probability range that Lisa reaches her goal. We consider two types of fixed holding strategies, keeping the assumption as simple as possible:

- A conservative bond portfolio that consists of Treasury bonds, corporate bonds, TIPS, and Treasury bills only;
- A fixed portfolio with significant inclination on equity-like assets, including US domestic equities, developed country and emerging market equities, and real estate investment trusts (REITs).

Branch and Qiu \[25\] have verified with historical data that a portfolio with high fixed stock allocation will result in high mean and median wealth accumulations, even after risk-adjustments such as the Sharpe ratio. Therefore, an equity-focused portfolio will be a qualified indicator of the upper bound of Lisas chance to reach her goal, while the conservative bond portfolio provides the lower bound.

One common wisdom involves the traditional 60-40 portfolio: 60% equities and 40% bonds. Herein, as seen from the Social Security section, Lisa already has roughly 25-30% of retirement income in a TIPS-like portfolio, so a personal in-

\[1\]Which is attainable since employers are matching 401(k) savings up to 3-6% per year.
vestment portfolio with 80% equities (including real estates) and 20% bonds will approximate the 60-40 rule when Lisa combines all accounts. This portfolio promises extensive growth considering the stellar long-term performance of equities. Table 5.5 shows the allocation of the two portfolios for simulations.

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Equities</td>
<td>40%</td>
<td></td>
</tr>
<tr>
<td>EAFE Equities</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>EM Equities</td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td>US Treasuries</td>
<td>10%</td>
<td>30%</td>
</tr>
<tr>
<td>Corp Bonds</td>
<td></td>
<td>30%</td>
</tr>
<tr>
<td>TIPS</td>
<td>10%</td>
<td>30%</td>
</tr>
<tr>
<td>Real Estate</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>Commodities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T-bill</td>
<td></td>
<td>10%</td>
</tr>
</tbody>
</table>

Table 5.5: Sample Asset Allocation for Fixed Mix

5.3.4 Results: Fixed Portfolios

The simulation results are provided in tables 5.6 and 5.7. The bonds portfolio has a high probability of achieving the retirement goal only if the saving rate is high (14-15 percent) and the goal is modest (60% pre-retirement income, which is considered insufficient in official documents published by the Government Accountability Office.)

On the other hand, the equity-inclined portfolio promises a better chance of reaching the goal, even if it is as high as 80%-90% of pre-retirement income. For
example, when the goal is to maintain 80% of pre-retirement income, the equity portfolio with a saving rate of 12% will succeed in more than two-thirds of the scenarios, while the conservative bond portfolio hits the line under only 3.6% of cases. Therefore, in order to achieve higher income after retirement, Lisa needs a major allocation to equities that has higher long-run growth rate than bonds. The next step for us, therefore, is to determine the optimal proportion of equities and bonds at different periods.

<table>
<thead>
<tr>
<th>Savings</th>
<th>8%</th>
<th>9%</th>
<th>10%</th>
<th>11%</th>
<th>12%</th>
<th>13%</th>
<th>14%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>60%</td>
<td>72.8%</td>
<td>79.8%</td>
<td>84.9%</td>
<td>88.7%</td>
<td>91.5%</td>
<td>93.9%</td>
<td>95.4%</td>
<td>96.6%</td>
</tr>
<tr>
<td>70%</td>
<td>53.2%</td>
<td>61.7%</td>
<td>69.6%</td>
<td>75.8%</td>
<td>80.9%</td>
<td>84.7%</td>
<td>87.8%</td>
<td>90.1%</td>
</tr>
<tr>
<td>80%</td>
<td>36.9%</td>
<td>45.6%</td>
<td>53.6%</td>
<td>60.8%</td>
<td>67.4%</td>
<td>73.0%</td>
<td>77.5%</td>
<td>81.5%</td>
</tr>
<tr>
<td>90%</td>
<td>25.0%</td>
<td>32.9%</td>
<td>40.4%</td>
<td>47.4%</td>
<td>54.1%</td>
<td>60.1%</td>
<td>65.9%</td>
<td>70.8%</td>
</tr>
</tbody>
</table>

Table 5.6: Probability of achieving 60-90% replacement income at different saving levels, equity-focused portfolio

<table>
<thead>
<tr>
<th>Savings</th>
<th>8%</th>
<th>9%</th>
<th>10%</th>
<th>11%</th>
<th>12%</th>
<th>13%</th>
<th>14%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>60%</td>
<td>7.6%</td>
<td>19.1%</td>
<td>34.5%</td>
<td>52.3%</td>
<td>68.1%</td>
<td>80.4%</td>
<td>89.3%</td>
<td>94.5%</td>
</tr>
<tr>
<td>70%</td>
<td>0.4%</td>
<td>1.7%</td>
<td>4.9%</td>
<td>11.6%</td>
<td>21.6%</td>
<td>34.0%</td>
<td>47.7%</td>
<td>60.5%</td>
</tr>
<tr>
<td>80%</td>
<td>0.0%</td>
<td>0.1%</td>
<td>0.4%</td>
<td>1.5%</td>
<td>3.6%</td>
<td>7.8%</td>
<td>14.7%</td>
<td>23.4%</td>
</tr>
<tr>
<td>90%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.2%</td>
<td>0.5%</td>
<td>1.3%</td>
<td>2.9%</td>
<td>5.9%</td>
</tr>
</tbody>
</table>

Table 5.7: Probability of achieving 60-90% replacement income at different saving levels, bond portfolio
5.4 Dynamic Policy Rules

There are also extensive discussions on non-fixed policy rules. The core strategy of target-date funds is the so-called glide path strategy. TDFs allocate more equities for the early years and shift the allocation to safer assets (treasury bonds) towards retirement, claiming that this helps to preserve the earned capital and avoid serious losses at later stages. Such a strategy is called age-based asset allocation since the portfolio weights depend only on the age (or remaining years to retirement) of the investor.

5.4.1 Age-Based Asset Allocation

The benchmark that we consider in this chapter is the standard glide path (SGP) strategy as adopted by target-date funds. For Lisa, the corresponding product is the Vanguard Target Retirement 2060 Fund (VTTSX), with the allocation illustrated below.

Figure 5.1: Sample Glide Path for the 2060 Target Date Fund (Copyright: Vanguard)

Surprisingly, perhaps, the reversed glide path (RGP) strategies also received much attention. Completely contrary to SGP, the RGP tends to be conservative in the early years and switch gradually to equities later. We could categorize the strategies by the “slope” of the equity allocation along the path:
• Negative slope: Standard Glide Path (SGP);
• Zero slope: Fixed mix;
• Positive slope: Reversed Glide Path (RGP).

There is significant literature that questions the adequacy of SGP strategies in terms of both return and risk measures. Johnson et al. [69] found little, if any, improvement of performance by target-date mutual funds over simple strategies on risk measures such as Sharpe ratios and Treynor ratios. Singh provided a complete survey in [110] and concluded that there should be diversified target-date funds that offer different risk profiles. Arnott et al. [7] have shown that SGP is an inferior strategy by comparing various versions of glide-path strategies, including SGP, RGP, and flat paths (i.e., fixed portfolio). In his later paper [8], the authors summarized the flaws in glide-path strategies and pointed out that policy-based dynamic allocations can improve the retirement outcome. Estrada [40] utilized a set of samples from 19 countries spanning 110 years to demonstrate that TDF glide path strategies seriously limit the upside of the terminal wealth. He further concludes in his 2016 paper [41] that both an all-equity portfolio and a 60/40 stock/bond allocation are simple and very effective strategies for retirees to implement.

Most of the literature above believes that the “sin” of TDFs is that they appear to be both over-aggressive when the investment amount is small such that steady growth is preferred and over-conservative when a market downturn happens at later stages, limiting the capital accumulation.

On the other hand, proponents of RGP strategies have shown that the RGP provides the more upward potential for investors because of the allocation to equities near or even after retirement. Early literature such as [16] and [18] found that SGPs
might not work as well as RGPs. Most recently, Schleef and Eisinger [104] updated earlier findings and reinforced the argument that high equity allocations near retirement will promise higher withdrawal rates. Pfau and Kitces [100] compared the shortfall risk measures and lifetime utilities of “reversed life-cycle” portfolios versus traditional life-cycle portfolios. Wiafe et al. [118] used Monte Carlo simulations with three sets of assumptions to show that RGP strategies have the potential to reduce both the chances of failure and the magnitude of deficits. Similar discussions can be found in Mladina [86].

5.4.2 Dynamically Adjusted Asset Allocation

The ultimate question for investors boils down to how the risk is defined and what their risk appetite is. The defenders of TDFs pointed out that the goal of TDFs is not to pursue extraordinary capital growth at the cost of higher risks. Under the liability-driven investing (LDI) framework, the portfolio should fully support withdrawals after retirement.

As such, researchers have been actively searching for appropriate dynamic strategies to improve risk-adjusted performance in retirement planning. Basu et al. [13] extended the target date funds to dynamic strategies that lower equity allocation when performance has reached desired growth. They noticed that the optimal dynamic strategies should be able to assess the current situation relative to the goal. Janssen et al. [66] advocated goal-based investing that adjusts the portfolio according to the investors’ risk and return appetite. Blanchett and Straehl [19] placed alternative assets into the portfolio and provided guidance on glide path designs. Fraser and Payne [45] inflate the bond allocation towards retirement and argued that this non-linear path is superior over linear paths. Finally, Forsyth and Vetzal [44] applied

\footnote{Discussions of LDI can be found in [5] and [80].}
control theory to develop strategies that are mean-variance and quadratic shortfall optimal that significantly outperform the TDFs when the stock indices are assumed to follow the jump-diffusion process.

In this paper, we include most of these strategies to compare:

- The “Standard Glide Path” (SGP) portfolio that allocates 90% in equities initially and reduces (by 1% per year) to 46% equity at retirement, similar to a TDF.

- The “Reversed Glide Path” (RGP) as described by Pfau [100]. It allocates 25% equity at the beginning and increases the equity allocation by 1.5% per year.

- A “Catching-up” dynamic strategy that follows SGP most of the time, but adds up to 20% in equities allocation when falling behind at least 5% with respect to a “lower bound”. The maximum allocation of equity is 95%.

- A “Play-safe” dynamic strategy that follows SGP most of the time, but switches to the fixed bonds allocation when reaching a “safe bound” such that the common goal is satisfied with only 2% annual return in the remaining period.

- A “Dynamic Zone” strategy that combines the two bounds in “Catching-up” and “Play-safe”. It protects subsequent crashes happening to lucky people who have already generated enough income for early retirement and gives people a chance to catch up when they are trailing behind.

- A Dynamic Zone-save strategy that is the same as the Dynamic Zone strategy but saves an extra 3% when falling behind. The extra savings could reduce the downside for the investor.

- A “Reversed Dynamic Zone” strategy that applies dynamic zone to RGP.
• A “Reversed Dynamic ZoneSave” strategy that applies dynamic zone to RGP, with extra 3% savings.

A sample boundary when the goal is to achieve 80% pre-retirement income is shown in figure 5.2. The unit of the y-axis is million $.

![Figure 5.2: Boundary for SGP](image)

The asset allocations for dynamic strategies are shown in table 5.8 below. Since we have a heavy focus on equity most of the time, we pick only Treasury bonds and TIPS in our portfolios. The maximum total equity allocation for catch-up is 95%, and the minimum equity allocation for play-safe is 20%.
Table 5.8: Allocation for Dynamic Strategies

<table>
<thead>
<tr>
<th></th>
<th>SGP (First)</th>
<th>SGP (Last)</th>
<th>RGP (First)</th>
<th>RGP (Last)</th>
<th>Catch-up (Max)</th>
<th>Play-safe (Min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Equities</td>
<td>40%</td>
<td>20%</td>
<td>15%</td>
<td>54%</td>
<td>42.5%</td>
<td>10%</td>
</tr>
<tr>
<td>EAFE Equities</td>
<td>10%</td>
<td>5%</td>
<td>2.5%</td>
<td>8%</td>
<td>10.5%</td>
<td>2%</td>
</tr>
<tr>
<td>EM Equities</td>
<td>30%</td>
<td>15%</td>
<td>5%</td>
<td>18%</td>
<td>31.5%</td>
<td>5%</td>
</tr>
<tr>
<td>US Treasuries</td>
<td>5%</td>
<td>27.5%</td>
<td>25%</td>
<td>4%</td>
<td>2.5%</td>
<td>40%</td>
</tr>
<tr>
<td>TIPS</td>
<td>5%</td>
<td>27.5%</td>
<td>50%</td>
<td>8%</td>
<td>2.5%</td>
<td>40%</td>
</tr>
<tr>
<td>Real Estate</td>
<td>10%</td>
<td>5%</td>
<td>2.5%</td>
<td>8%</td>
<td>10.5%</td>
<td>3%</td>
</tr>
</tbody>
</table>

5.4.3 Risk Measures

Huxley and Burns [59] and Branning and Grubbs [26] introduced probability-based “safety-first” measures for retirement planning. Herein, we employ various risk measures to compare fixed portfolios and dynamic strategies:

- Median wealth among the 10,000 scenarios.
- Probability of achieving the goal.
- 5% Goal-at-Risk (GaR). This is similar to VaR, except that we are computing the borderline “shortage” to the goal in the 5% worst scenarios.
- 5% Conditional Goal-at-Risk (CGaR). This is the average “shortage” to the goal in the 5% worst scenarios.
- Portion of time that Lisa falls behind the target path (lower bound).
- Portion of time that Lisa could switch to a safer portfolio.
If the goal is $G$, we have

$$GaR_h(X) = \min\{v | P(G - X > v) \leq h\}, \quad (5.10)$$

and

$$CGaR_h(X) = \mathbb{E}[G - X | X \leq -GaR_h(X)]. \quad (5.11)$$

### 5.5 Evaluation of Dynamic Rules

In this section, we study the strategies in a unique setting. The aggregation of risk measures provides a full picture of retirement planning:

1. The median wealth represents the upside.

2. The probability of achieving the goal is what attracts investors’ attention. We focus on the ability of dynamic strategies to improve this measure.

3. Goal-at-Risk (GaR) and conditional Goal-at-Risk (CGaR) illustrate what happens under the worst cases.

4. Proportion of time that Lisa crosses the borderline (% Lag and % Safe) give information about the effectiveness of dynamic strategies. For “Zone-save” strategies, this also provides an estimate for the total extra amount saved.
Note: “R-Zone” and “R-ZoneSave” are corresponding dynamic rules that applied to the reversed glide path. The unit of numbers is million $.

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
<th>Success%</th>
<th>GaR</th>
<th>CGaR</th>
<th>% Lag</th>
<th>% Safe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>5.00</td>
<td>80.9%</td>
<td>1.11</td>
<td>1.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td>2.77</td>
<td>21.6%</td>
<td>1.46</td>
<td>1.72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGP</td>
<td>4.55</td>
<td>81.7%</td>
<td>0.81</td>
<td>1.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RGP</td>
<td>4.59</td>
<td>76.0%</td>
<td>1.34</td>
<td>1.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catch-Up</td>
<td>4.72</td>
<td>86.9%</td>
<td>0.76</td>
<td>1.23</td>
<td>10.7%</td>
<td></td>
</tr>
<tr>
<td>Safe-Play</td>
<td>4.41</td>
<td>84.5%</td>
<td>0.77</td>
<td>1.16</td>
<td></td>
<td>28.4%</td>
</tr>
<tr>
<td>Zone</td>
<td>4.50</td>
<td>89.2%</td>
<td>0.67</td>
<td>1.19</td>
<td>10.3%</td>
<td>29.6%</td>
</tr>
<tr>
<td>ZoneSave</td>
<td>4.54</td>
<td>91.6%</td>
<td>0.40</td>
<td>0.94</td>
<td>9.0%</td>
<td>30.3%</td>
</tr>
<tr>
<td>R-Zone</td>
<td>4.74</td>
<td>85.0%</td>
<td>1.20</td>
<td>1.74</td>
<td>11.7%</td>
<td>25.7%</td>
</tr>
<tr>
<td>R-ZoneSave</td>
<td>4.79</td>
<td>87.2%</td>
<td>1.03</td>
<td>1.58</td>
<td>10.5%</td>
<td>26.3%</td>
</tr>
</tbody>
</table>

Table 5.9: Comparison of Strategies, Setting 1

We do not include 60% income as the goal because it is considered too low by the Government Accountability Office [99].

When Lisa saves an extra 12% per year, but her goal is only 70%, the equity portfolio already provides a satisfactory probability of achieving the goal and the highest median wealth. Still, the tail-risk measure is not pleasant, as she will be more than 1 million dollars short in the worst 5% cases.

Comparing the standard glide path versus the reversed glide path, we find that RGP results in a slightly higher median wealth and similar success probability, as mentioned by [8] and other papers discussed in chapter 5.4. However, the tail-risk
measures of RGP are even worse than the fixed equity portfolio. The discrepancies in our results and most RGP papers stem from the simulation method, specifically the regime-based settings. Under the regime-switching model, the occurrence of a crash significantly erode the capital; thus, it is not optimal to increase equity allocation at later stages. This is contrary to what Estrada found in his paper [40]. Blanchett [20] pointed out that one of the critical reasons in the disagreement among glide paths is the asset return simulation procedure.

The one-sided dynamic strategies, “Catch-Up” that focus on the lower bound and “Safe-Play” that monitors the upper bound provides limited if any, performance boost from SGP. The combination of them, on the other hand, renders superior performance in terms of both success probability and tail risk measures.

The main contribution of the “ZoneSave” strategy is the reduction of tail risk. On average, this strategy saves an extra amount of less than $30,000 (computed from simulations) but reduces the tail risk by $250,000. This is also the only strategy that reaches the goal of more than 90% of the time, with absolute safety in more than 30% of the scenarios.

The “zone” strategies are not well-adapted to reversed glide path strategies, however. They still suffer from high tail-risks and lackluster probabilities of achieving the retirement goal.
Lisa (Female, age 25): Save 12%, Goal 80%, Salary 132900

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Median</th>
<th>Success%</th>
<th>GaR</th>
<th>CGaR</th>
<th>% Lag</th>
<th>% Safe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>5.00</td>
<td>67.4%</td>
<td>1.90</td>
<td>2.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td>2.77</td>
<td>3.6%</td>
<td>2.35</td>
<td>2.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGP</td>
<td>4.55</td>
<td>62.9%</td>
<td>1.63</td>
<td>2.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RGP</td>
<td>4.59</td>
<td>61.2%</td>
<td>2.15</td>
<td>2.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catch-Up</td>
<td>4.92</td>
<td>73.2%</td>
<td>1.69</td>
<td>2.19</td>
<td>24.3%</td>
<td></td>
</tr>
<tr>
<td>Safe-Play</td>
<td>4.63</td>
<td>66.5%</td>
<td>1.62</td>
<td>2.03</td>
<td>20.2%</td>
<td></td>
</tr>
<tr>
<td>Zone</td>
<td>4.92</td>
<td>76.5%</td>
<td>1.67</td>
<td>2.18</td>
<td>23.8%</td>
<td>22.6%</td>
</tr>
<tr>
<td>ZoneSave</td>
<td>5.04</td>
<td>81.8%</td>
<td>1.30</td>
<td>1.84</td>
<td>20.5%</td>
<td>24.1%</td>
</tr>
<tr>
<td>R-Zone</td>
<td>5.11</td>
<td>73.4%</td>
<td>2.13</td>
<td>2.67</td>
<td>28.0%</td>
<td>20.1%</td>
</tr>
<tr>
<td>R-ZoneSave</td>
<td>5.29</td>
<td>78.0%</td>
<td>1.87</td>
<td>2.42</td>
<td>24.4%</td>
<td>21.5%</td>
</tr>
</tbody>
</table>

Table 5.10: Comparison of Strategies, Setting 2

Table 5.10 shows the risk measures when the goal is 80% replacement income. The “ZoneSave” strategy still has the highest success probability as well as the lowest tail risk.

If we do not permit extra savings, however, the regular “dynamic zone” strategy is also agreeable. The median wealth is close to the full-equity allocation, but Lisa has about 10% more chance to achieve her goal. In the best 22.6% scenarios, she could switch to a safe bond portfolio and enjoy the promised lifestyle after retirement.
Lisa (Female, age 25): Save 12%, Goal 90%, Salary 132900

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Median</th>
<th>Success%</th>
<th>GaR</th>
<th>CGaR</th>
<th>% Lag</th>
<th>% Safe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>5.00</td>
<td>54.1%</td>
<td>2.71</td>
<td>3.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td>2.77</td>
<td>0.5%</td>
<td>3.26</td>
<td>3.61</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGP</td>
<td>4.55</td>
<td>45.3%</td>
<td>2.45</td>
<td>2.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RGP</td>
<td>4.59</td>
<td>47.3%</td>
<td>2.96</td>
<td>3.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catch-Up</td>
<td>5.10</td>
<td>59.3%</td>
<td>2.55</td>
<td>3.09</td>
<td>41.1%</td>
<td></td>
</tr>
<tr>
<td>Safe-Play</td>
<td>4.67</td>
<td>48.5%</td>
<td>2.45</td>
<td>2.91</td>
<td>14.1%</td>
<td></td>
</tr>
<tr>
<td>Zone</td>
<td>5.24</td>
<td>63.1%</td>
<td>2.54</td>
<td>3.08</td>
<td>40.5%</td>
<td>17.8%</td>
</tr>
<tr>
<td>ZoneSave</td>
<td>5.52</td>
<td>72.0%</td>
<td>2.09</td>
<td>2.65</td>
<td>34.7%</td>
<td>20.2%</td>
</tr>
<tr>
<td>R-Zone</td>
<td>5.39</td>
<td>61.9%</td>
<td>2.96</td>
<td>3.51</td>
<td>48.2%</td>
<td>16.4%</td>
</tr>
<tr>
<td>R-ZoneSave</td>
<td>5.84</td>
<td>70.7%</td>
<td>2.56</td>
<td>3.15</td>
<td>41.4%</td>
<td>19.0%</td>
</tr>
</tbody>
</table>

Table 5.11: Comparison of Strategies, Setting 3

If Lisa has a high goal of 90% income, she had better adopt the dynamic zone strategy, as seen from table 5.11.

We noticed that in our Dynamic “Zone-Save” strategy, Lisa saves an extra 3% when she is falling behind. Considering the extra savings happens 30%-40% of the time under the goal of 90% replacement income, this yields an average saving level at 13%. Therefore, it is fair to compare the “ZoneSave” strategies at 12% savings with other strategies at 13% savings to reflect the difference. The risk measures are presented in table 5.12.
Lisa (Female, age 25): Goal 90%, Salary 132900

<table>
<thead>
<tr>
<th></th>
<th>Save 13%</th>
<th></th>
<th>Save 12%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>SGP</td>
<td>RGP</td>
<td>Zone</td>
</tr>
<tr>
<td>Median</td>
<td>5.40</td>
<td>4.91</td>
<td>4.97</td>
</tr>
<tr>
<td>Success%</td>
<td>60.1%</td>
<td>53.1%</td>
<td>53.8%</td>
</tr>
<tr>
<td>GaR</td>
<td>2.51</td>
<td>2.22</td>
<td>2.77</td>
</tr>
<tr>
<td>CGaR</td>
<td>3.06</td>
<td>2.69</td>
<td>3.33</td>
</tr>
<tr>
<td>% Lag</td>
<td></td>
<td></td>
<td>32.3%</td>
</tr>
<tr>
<td>% Safe</td>
<td></td>
<td></td>
<td>20.1%</td>
</tr>
</tbody>
</table>

Table 5.12: Saving Efficiency

We conclude that the “ZoneSave” strategy still outperforms other strategies in the table. The probability of success is the highest, and the tail risks are within control. This fact implies the efficiency of savings for the “ZoneSave” strategy; during the underperforming periods (presumably caused by a stock market crash), it is recommended to purchase more equities since they tend to be under-valued.

The upper and lower bounds promise the capital growth (median wealth and probability of reaching the goal) and the extra savings at the correct times reduces the tail risk. Even if caught in an elongated and unprecedented economy downturn (worst 5% case), Lisa could still obtain about $4 million (including discounted cash inflows of Social Security payments) for retirement.
5.6 Planning Beyond the Social Security

In this section, we consider a situation in which Lisa’s initial salary is $200,000, higher than the Social Security contribution maximum base of $132,900. We also start from the goal of 70%. The probability of reaching the goal in table 5.13 is lower than the corresponding values in table 5.9 since the Social Security benefit covers a smaller portion of replacement income.

Note: “R-Zone” and “R-ZoneSave” are corresponding dynamic rules that applied to the reversed glide path. The unit of numbers is million $.

<table>
<thead>
<tr>
<th>Lisa (Female, age 25): Save 12%, Goal 70%, Salary 200000</th>
<th>Median</th>
<th>Success%</th>
<th>GaR</th>
<th>CGaR</th>
<th>% Lag</th>
<th>% Safe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>7.44</td>
<td>68.4%</td>
<td>2.75</td>
<td>3.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td>4.14</td>
<td>4.1%</td>
<td>3.38</td>
<td>3.82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGP</td>
<td>6.77</td>
<td>64.3%</td>
<td>2.34</td>
<td>2.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RGP</td>
<td>6.85</td>
<td>62.2%</td>
<td>3.10</td>
<td>3.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catch-Up</td>
<td>7.29</td>
<td>74.0%</td>
<td>2.44</td>
<td>3.16</td>
<td>23.3%</td>
<td></td>
</tr>
<tr>
<td>Safe-Play</td>
<td>6.86</td>
<td>68.8%</td>
<td>2.30</td>
<td>2.92</td>
<td>22.3%</td>
<td></td>
</tr>
<tr>
<td>Zone</td>
<td>7.18</td>
<td>78.3%</td>
<td>2.37</td>
<td>3.13</td>
<td>22.6%</td>
<td>24.7%</td>
</tr>
<tr>
<td>ZoneSave</td>
<td>7.35</td>
<td>83.1%</td>
<td>1.83</td>
<td>2.64</td>
<td>19.5%</td>
<td>26.1%</td>
</tr>
<tr>
<td>R-Zone</td>
<td>7.51</td>
<td>75.3%</td>
<td>3.06</td>
<td>3.86</td>
<td>26.4%</td>
<td>21.8%</td>
</tr>
<tr>
<td>R-ZoneSave</td>
<td>7.77</td>
<td>79.8%</td>
<td>2.67</td>
<td>3.50</td>
<td>23.0%</td>
<td>23.2%</td>
</tr>
</tbody>
</table>

Table 5.13: Comparison of Strategies, High Salary and Medium Goal
Lisa (Female, age 25): Save 12%, Goal 80%, Salary 200000

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Median</th>
<th>Success%</th>
<th>GaR</th>
<th>CGaR</th>
<th>% Lag</th>
<th>% Safe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>7.44</td>
<td>54.7%</td>
<td>3.96</td>
<td>4.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td>4.14</td>
<td>0.6%</td>
<td>4.73</td>
<td>5.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGP</td>
<td>6.77</td>
<td>46.0%</td>
<td>3.58</td>
<td>4.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RGP</td>
<td>6.85</td>
<td>48.0%</td>
<td>4.32</td>
<td>5.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catch-Up</td>
<td>7.60</td>
<td>59.9%</td>
<td>3.72</td>
<td>4.51</td>
<td>40.3%</td>
<td></td>
</tr>
<tr>
<td>Safe-Play</td>
<td>6.95</td>
<td>50.0%</td>
<td>3.56</td>
<td>4.24</td>
<td>15.6%</td>
<td></td>
</tr>
<tr>
<td>Zone</td>
<td>7.73</td>
<td>64.3%</td>
<td>3.69</td>
<td>4.49</td>
<td>39.5%</td>
<td>19.3%</td>
</tr>
<tr>
<td>ZoneSave</td>
<td>8.13</td>
<td><strong>73.3%</strong></td>
<td><strong>3.02</strong></td>
<td><strong>3.86</strong></td>
<td>33.8%</td>
<td>21.8%</td>
</tr>
<tr>
<td>R-Zone</td>
<td>8.00</td>
<td>63.6%</td>
<td>4.30</td>
<td>5.14</td>
<td>46.9%</td>
<td>17.7%</td>
</tr>
<tr>
<td>R-ZoneSave</td>
<td>8.59</td>
<td>72.1%</td>
<td>3.70</td>
<td>4.60</td>
<td>40.2%</td>
<td>20.3%</td>
</tr>
</tbody>
</table>

Table 5.14: Comparison of Strategies, High Salary and Medium-High Goal

We have observed a significantly lower probability of achieving the medium-high goal at 80% replacement income in table 5.14. Still, the dynamic “Zone” strategy outperforms other strategies and could be further enhanced by the extra savings.

The reversed “ZoneSave” strategy has the highest median wealth in both table 5.14 and 5.15. It also has the highest chance of success in the latter case. Nonetheless, it produces significantly higher tail-risk and requires the investor to consistently save more, potentially reducing Lisa’s life quality before retirement.
Lisa (Female, age 25): Save 12%, Goal 90%, Salary 200000

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
<th>Success%</th>
<th>GaR</th>
<th>CGaR</th>
<th>% Lag</th>
<th>% Safe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>7.44</td>
<td>43.1%</td>
<td>5.23</td>
<td>6.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td>4.14</td>
<td>0.1%</td>
<td>6.10</td>
<td>6.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGP</td>
<td>6.77</td>
<td>31.2%</td>
<td>4.84</td>
<td>5.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RGP</td>
<td>6.85</td>
<td>35.8%</td>
<td>5.60</td>
<td>6.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catch-Up</td>
<td>7.84</td>
<td>48.2%</td>
<td>4.98</td>
<td>5.84</td>
<td>57.9%</td>
<td></td>
</tr>
<tr>
<td>Safe-Play</td>
<td>6.90</td>
<td>35.2%</td>
<td>4.83</td>
<td>5.58</td>
<td>10.8%</td>
<td></td>
</tr>
<tr>
<td>Zone</td>
<td>8.08</td>
<td>52.7%</td>
<td>4.97</td>
<td>5.84</td>
<td>57.2%</td>
<td>15.3%</td>
</tr>
<tr>
<td>ZoneSave</td>
<td>8.90</td>
<td>64.5%</td>
<td>4.10</td>
<td>5.02</td>
<td>48.6%</td>
<td>18.9%</td>
</tr>
<tr>
<td>R-Zone</td>
<td>8.25</td>
<td>52.5%</td>
<td>5.55</td>
<td>6.43</td>
<td>65.0%</td>
<td>14.5%</td>
</tr>
<tr>
<td>R-ZoneSave</td>
<td>9.38</td>
<td>65.4%</td>
<td>4.76</td>
<td>5.74</td>
<td>56.0%</td>
<td>18.2%</td>
</tr>
</tbody>
</table>

Table 5.15: Comparison of Strategies, High Salary and High Goal

### 5.7 Summary

A critical part of the financial planning problem is to set up a realistic goal relative to future liabilities and estimate the chance of achieving such a goal. The same problem applies to both the macro-level, as the example of the Social Security fund demonstrated, and the micro-level, as the simulation of Ms. Lisa Li illustrated. The financial planning framework should be made aware to the robo-advising industry, as most prevalent robo-advising websites still estimate the future outcome by a fixed set of asset allocations and limited availability to the full asset universe.
Chapter 6

Future Work and Conclusion

6.1 Conclusion

In this paper, we build a financial planning framework that contains multiple machine learning, optimization, and Monte Carlo simulation tools, with adaptability to different problem backgrounds.

As the first empirical example, we apply regime-based multi-period asset simulation to study the downside risk of a university endowment whose asset allocation is determined via a factor exposure matrix and factor risk appetites. The regime-switching setting contributes to the tail-risk analyses.

This paper also shows the benefits of an agent-based approach. Our modeling framework consists of macroeconomic time series, regime-based asset return simulations, and agent-based simulations. With this framework, we are able to assess possible impacts of policy change, such as the determination of Full Retirement Age (FRA) for the Social Security fund.
There is a global trend, especially in East Asian countries such as South Korea and China, for governments to establish a sophisticated retirement system. For people that start planning retirement, one of the crucial steps is to assess the replacement income that public welfare grants upon retirement and determine the total replacement objective accordingly. As expected, we have seen that increasing the savings rate from 10% to 15% will significantly boost the probability that Lisa reaches her financial goal if she adopts an appropriate strategy such as the “Dynamic Zone”, even when the goal is as high as 90% replacement income.

On the aggregate level, the government should carefully estimate the trends of macro-economy, demographics, and monetary policy in order to make an appropriate promise to its citizen. The micro-macro approach in this paper provides a systematic starting point.
6.2 Future Work

In chapter 5, we witness the power of the regime-aware, macroeconomic variable-driven, agent-based framework in analyzing various strategies under differentiated model assumptions. We propose here a couple of future work directions to enhance this framework.

6.2.1 Forward-looking Scenario Generator

Currently, the scenario generator has two primary ingredients: a macroeconomic time series generator and a regime-based asset return generator. We are employing a Markov switching regime simulator, which means the probability of a crash in the next regime depends only on the status of the current regime.

Machine learning techniques have been applied to macroeconomic forecasting problems. Coulombe et al. give an overview of multiple approaches in their paper [33]. Models such as support vector machine (SVM), random forest, and XGBoost have increased the accuracy of forecasting a crash. It would be thrilling to see the improvement of portfolio performance if we have a regime forecasting model with decent reliability.

6.2.2 Choice of Defensive Assets

In this paper, we used US treasury bonds (including inflation-adjusted bonds) as the conservative part of our portfolio. Researchers have found more effective defensive assets tailored to retirement needs, as well as overlay strategies such as factor momentum [37]. These assets and overlays could be an excellent complement to our dynamic “zone” strategies.
6.2.3 Agent-based Simulation for DB Pension Funds

This will be a combination of chapter 4 and 5. We simulated 1 million people to estimate the sustainability of the Social Security fund and established the micro-macro consistency. For Lisa as an individual, the Social Security benefit is only (a small) part of her retirement income.

Now assume that we simulate 1 million people with similar characteristics as Lisa, each possessing a personal investment account. The agent-based simulation can be readily implemented as the funding ratio analysis of a DB pension fund. Indeed, the agent-based simulation of pension funds involves more complicated policy rules, since pension funds actively adjust their portfolio. Nonetheless, the next section provides a path to solve the optimal allocation of the pension fund.

6.2.4 Numerical Solver of Optimal Strategy

We proposed a good number of strategies in chapter 5.4, including dynamic strategies that are inspired by the “No-Trade Zone” (see 119 for a detailed discussion) and the strategies that invest money at correct times.

The next step will naturally be searching for the best strategy under each scenario. We believe that reinforcement learning techniques could be used to search for the optimal allocation at any point along the path. For example, the deep Q-network (DQN) can be trained to learn the optimal action-value function for any scenario (also known as Q-function.) The state may consist of current wealth, the current regime, and estimated distance to the goal. More details of the DQN algorithm can be found in 87.
6.2.5 Integration of Multiple Goals

This framework can be further extended: for personal planning, more goals can be added to the retirement planning, for example saving the down payment of a condo in 10 years or saving for children’s college tuition. The optimal allocation of an individual should be closely related to his/her unique situation, including risk aversion, income flow consistency, and, of course—the goals to achieve. More discussions can be found in [81] and [66].

For institutional investors, liability can be added to the framework and estimated by the macroeconomy generator. The ALM for DB pension funds and insurance companies are of great resemblance to the goal-based planning framework in this paper.
Bibliography


Appendix A

Data Sources

A.1 Database

The databases we use in this dissertation are the Datastream (available in Princeton Library), WRDS (available online), CRSP and FRED at St. Louis. Most asset categories have monthly data from January 1988 to September 2018 (369 months in total).

A.2 Asset Universe

Here we give detailed information of the assets we choose:

<table>
<thead>
<tr>
<th>Name</th>
<th>Index</th>
<th>Ticker</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Equities</td>
<td>Wilshire 5000</td>
<td>WIL5TMK(RI)</td>
</tr>
<tr>
<td>EAFE Equities</td>
<td>MSCI EAFE USD</td>
<td>MSEAFE$(MSRI)</td>
</tr>
<tr>
<td>EM Equities</td>
<td>MSCI Emerging Market USD</td>
<td>MSEMKF$(MSRI)</td>
</tr>
<tr>
<td>Long Treasuries</td>
<td>FTSE US Government Bond</td>
<td>SBUS10L(RI)</td>
</tr>
<tr>
<td>Corp Bonds</td>
<td>Barclays U.S. Corporate Investment Grade</td>
<td>LHCCORP(IN)</td>
</tr>
<tr>
<td>TIPS</td>
<td>Barclays U.S. Treasury: U.S. TIPS USD</td>
<td>LHTRINF(IN)</td>
</tr>
<tr>
<td>Real Estate</td>
<td>FTSE/NAREIT All REITS</td>
<td>NARALL$(RI)</td>
</tr>
<tr>
<td>Commodities</td>
<td>S&amp;P GSCI Commodity</td>
<td>GSCITOT(TR)</td>
</tr>
<tr>
<td>T-bill</td>
<td>US 3-month T-bill</td>
<td>FRTBS3M(IR)</td>
</tr>
</tbody>
</table>

Table A.1: Monthly asset data from Datastream, January 1988 to September 2018
### A.3 Alternative Assets

<table>
<thead>
<tr>
<th>Name</th>
<th>Index</th>
<th>Ticker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Equities</td>
<td>S&amp;P Listed Private Equity Index</td>
<td>SPLPEI$(RI)</td>
</tr>
<tr>
<td>Absolute Returns</td>
<td>Princeton Private Data</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table A.2: Monthly alternative asset data, January 2004 to September 2018

### A.4 Industry Sector Data

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>US Equity</td>
<td>S&amp;P 500 Index</td>
<td>10.27%</td>
<td>15.27%</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>Government Bonds</td>
<td>Barclays Index</td>
<td>7.40%</td>
<td>5.11%</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>Consumer Sector</td>
<td>DataStream</td>
<td>11.67%</td>
<td>16.02%</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>Manufactory Sector</td>
<td>DataStream</td>
<td>10.81%</td>
<td>15.37%</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>Hi Tech Sector</td>
<td>DataStream</td>
<td>9.47%</td>
<td>19.68%</td>
<td>0.29</td>
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<tr>
<td></td>
<td>Health Sector</td>
<td>DataStream</td>
<td>11.72%</td>
<td>17.15%</td>
<td>0.43</td>
</tr>
<tr>
<td>Factor Portfolios</td>
<td>Size (SMB)</td>
<td>Fama&amp;French</td>
<td>10.00%</td>
<td>15.19%</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>Value (HML)</td>
<td>Fama&amp;French</td>
<td>14.03%</td>
<td>17.63%</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>Profitability (RMW)</td>
<td>Fama&amp;French</td>
<td>11.52%</td>
<td>15.73%</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>Investment (CMA)</td>
<td>Fama&amp;French</td>
<td>13.89%</td>
<td>17.24%</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>Momentum (MoM)</td>
<td>Fama&amp;French</td>
<td>15.78%</td>
<td>21.47%</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table A.3: Variables and Basic Statistics, January 1970 to December 2016