ESSAYS IN INTERNATIONAL ECONOMICS AND
MACROECONOMICS WITH FRICTIONS

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A DISSERTATION
PRESENTED TO THE FACULTY
OF PRINCETON UNIVERSITY
IN CANDIDACY FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

RECOMMENDED FOR ACCEPTANCE
BY THE DEPARTMENT OF ECONOMICS
ADVISOR: OLEG ITSKHOKI

APRIL 2013
Abstract:

This collection of essays investigates the role of frictions in Macroeconomics and International Finance. In the first chapter, we propose a standard international economy model in which a foreign investor has a wedge of return when investing in the domestic economy if compared to the domestic investor. This wedge helps explaining the major patterns of nominal exchange rate, real exchange rate, low correlation of consumption and the forward premium puzzle. We provide three structural interpretations within the class of models that has such wedge: capital controls, debt constraints and noisy agents. By disciplining the models with further data, we find that noisy-induced models are the most promising. In Chapter 2, co-authored with Wei Cui, we provide a Ramsey approach for optimal monetary policy in a model with equity issuance and resaleability constraints with conventional and unconventional policies as instruments. We show that entrepreneurs hold too much liquid asset and, in responding to liquidity shocks, the paths of macroeconomic variables under no policy and optimal policy are sharply different and suggest the need for changing the rate of return on liquid assets. Finally, we prove that the unconventional policy dominates the conventional counterpart, but, quantitatively, the welfare difference of them is negligible. In Chapter 3, we provide an information-based explanation for the existence of nominal rigidity in prices in which we stress the conflicts and the information revelation within the firm. Even if shocks are continuously distributed and there is no observation or menu costs, prices assume only a finite number of values. The mechanism can match already documented micro data moments, such as reference/sales prices, as well as newly-provided data from supermarket brand products and the behavior of sales and length of a price spell. The information-based explanation is fully tractable and we exemplify it by introducing it in a general equilibrium model to study the effects of monetary policy under such environment.
Acknowledgements:

I am deeply indebted to my principal adviser, Oleg Itskhoki, for his invaluable guidance and encouragement. I would also like to thank Mark Aguiar, Patrick Kehoe and Nobu Kiyotaki whose guidance, support, and patience made this dissertation possible.

I have also benefited greatly from advice and support from my teachers and colleagues at various stages of this process. I would especially thank Breno Braga, Qingqing Cao, Wei Cui, Márcio Garcia, Constantinos Kalfarentzos and Thomas Wu.

Finally, I would like to thank my parents, grandparents and Daniela for their unconditional support.
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1.1 Introduction

International economics has been a field in which puzzles related to risk sharing and the exchange rate are abundant. The concomitant high volatility and persistence of the nominal and real exchange rates (Engel (1999)), as well as the disconnect puzzle have been extensively confirmed in the data.

In the risk sharing arena, the so-called Backus-Smith puzzle, the correlation between relative consumptions and the real exchange rate is negative, and the low correlation of consumption, even lower than the correlation of outputs themselves, have also been widely discussed. Another puzzle that has addressed much attention is the forward premium puzzle, i.e., high interest rate currencies tend to appreciate vs. low interest rate currencies, at odds with what models would suggest.

In this paper, we provide a general class of models that can potentially address such puzzles. We start with a wedge-based analysis, in which we show how a difference on the price of a given asset paid by domestic and foreign investors go a long way addressing such
puzzles. We provide closed-form solutions, under complete markets, on the restrictions that the financial-frictions wedge has to satisfy to address each of the puzzles.

Given the wedge-based analysis and an empirical analysis of it, we provide three different structural interpretations for the financial frictions wedges.

The first one is to interpret the wedge as a tax or a capital control, which can be roughly seen as changing the price of the asset itself. Recent evidence on capital controls has showed that it may be optimal to tax (Costinot et al. (2011)) and that those impacts are non-negligible, while not yet well settled in the literature (De Gregorio et al. (2000) and Edwards (1999)). Even though we find that small controls are enough to generate sizable fluctuations, we do not find robust empirical evidence that (1) wedges and capital controls are correlated and (2) capital controls have enough time-variability, which makes it not a good structural candidate.

The second structural interpretation is based on debt constraints, which can be seen as changing the implicit price of each agent. In that section, we provide two different models: enforcement constraints and borrowing constraints à la Bewley. The purpose is to show that, even though both have the same wedge, they can lead to different cross correlations with respect to the wedge. Despite that in any model in which such wedge shows up we can improve the high-variance puzzle, the cross-correlation puzzles depend on a structural interpretation. Actually, from the mapping suggested from our wedge-based analysis, we show that borrowing constraints have a hard time explaining the puzzles, but enforcement constraints can be helpful theoretically, supporting earlier results from Bodenstein (2008), even though they fall short on some data analysis.

The third structural interpretation is based on exogenous noisy traders, as in Devereux and Engel (2002). We consider a model in which agents do not have access to international asset markets and give the money to dealers who are noisy. In this model, the quantitative implications are in line with what is observed in the data. Such model can, not only explain low consumption correlation and the Backus-Smith puzzle, but it also helps understanding
the high volatility of the exchange rate. Moreover, by considering the fact that floating exchange rate regimes have been found to have much higher noise than pegged regimes, one should observe puzzles to be more pronounced under floating regimes. At the same time, such change could not be blamed to come from monetary policy in a nominal stickiness bare and bones keynesian model, as shown by Devereux and Hnatkovska (2011). Using Penn World Tables, we find that, under pegged regimes, there is more risk-sharing, being robust to different measures of pegged regimes and measures of risk sharing, which confirms the predictions of the model.

The recent attempts of the literature to deal with such puzzles usually rely on changing the utility function. Karabarbounis (2009) investigates how the non separability of leisure and consumption play a role in solving international economy puzzles, while Colacito and Croce (2011) introduce Epstein-Zin preferences in a stylized macro model of endowment. Besides having full tractability of a macro model and suggesting a class of models that solve the puzzles, we contribute in providing a new source to explain the wedges which is in line with data\footnote{Devereux and Hnatkovska (2011) have provided evidence that within-country, some of the puzzles, such as the Backus-Smith are not present, while across countries they are, which is hard to rationalize in a model based purely on different utility functions.}.

The plan of the paper is the following: in the next section, we provide a theoretical and empirical assessment of the wedge introduced, discussing the restrictions that it has to satisfy to address the puzzles. In the second part of the paper, we provide three different interpretations for the wedge. The first interpretation, in section 3, is based on capital controls. In section 4, we discuss debt constraints as the source of the wedge, highlighting the importance of the structural interpretation of the wedge itself. In section 5, we construct a model with noisy traders. Finally, section 6 concludes the paper.
1.2 The Wedge

In this section, we will discuss how the introduction of a financial-frictions wedges in a two-country model can solve some of the open economy puzzles discussed in the literature. In doing this, we will consider a fairly simple model in order to have closed form solutions for the requirements that the wedge has to satisfy under complete markets. We will assume a log utility function throughout the paper for simplicity and, as the puzzles mainly arise on the consumers side, we do not describe in detail the production side of the economy unless when needed for quantitative analysis.

After discussing the theory, we provide some of the characteristics of the wedge and if it is in line with what we observe in the data.

1.2.1 Theoretical Intuition

We consider a standard macro-international specification, decentralized economy, in which firms use labor to produce\(^2\). The only difference is in the financial markets.

The model is then given by:

\[
\max_{E_0} \sum_{t=0}^{\infty} \beta^t u\left(C\left(s^t\right), L\left(s^t\right)\right)
\]

\[s.t.
\]

\[P\left(s^t\right) C\left(s^t\right) + X\left(s^t\right) \sum_{s^{t+1}} \xi\left(s^{t+1}|s^t\right) M^*\left(s^{t+1}|s^t\right) B\left(s^{t+1}\right) \leq W\left(s^t\right) L\left(s^t\right) + X\left(s^t\right) B\left(s^t\right)
\]

where \(B\left(s^{t+1}\right)\) is the amount of state-contingent bonds in foreign currency, \(\xi\left(s^{t+1}|s^t\right)\) is a price wedge, \(M^*\left(s^{t+1}|s^t\right)\) is the Arrow-Debreu price of a security in foreign currency and \(X\left(s^t\right)\) is the nominal exchange rate denoted in local currency/foreign currency.

\(^2\)If the firms held capital, the results would still hold.
Analogous framework applies to the foreign economy, except that we normalize so that the wedge is only on the domestic side:

\[
\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u \left( C^* \left( s^t \right), L^* \left( s^t \right) \right)
\]

s.t.

\[
P^* \left( s^t \right) C^* \left( s^t \right) + \sum_{s^{t+1}} M^* \left( s^{t+1} | s^t \right) B^* \left( s^{t+1} \right) + \leq W^* \left( s^t \right) L^* \left( s^t \right) + B^* \left( s^t \right)
\]

Note that for all the analysis on the wedge, we just need to describe the consumers side, but we’ll describe the production side, as well as shocks that describe the general equilibrium, when discussing the structural interpretations. From this simple description, we can provide some statistical requirements that the wedge has to satisfy.

**Real Exchange Rate behavior**

**Persistence and Variance** - Using the first order conditions and imposing non-arbitrage on every asset traded, the real exchange rate behavior

\[
\left( Q \left( s^{t+1} | s^t \right) = \frac{X \left( s^{t+1} | s^t \right) P^* \left( s^{t+1} | s^t \right)}{P \left( s^{t+1} | s^t \right)} \right)
\]

takes the following equation\(^3\):

\[
Q \left( s^{t+1} | s^t \right) = \frac{\tau}{C^* \left( s^{t+1} \right)} \prod_{j=0}^{t+1} \xi \left( s^j \right)
\]  

(1.2.1)

where \( \tau \) is a 0-period constant of relative consumption \( \left( \frac{C^* \left( s^0 \right)}{C \left( s^0 \right)} \right) \) that we normalize to one.

**Lemma 1.** The correlation of the log of the real exchange rate depends on the correlation

\(^3\)We relegate all the proofs to the appendix. Moreover, we use the short notation \( s^j \) to be \( (s^j | s^{j-1}) \), i.e., the state at time \( j \) contingent on a given state at time \( j - 1 \) for the wedge.
of the difference of domestic and foreign log consumption and wedges. The same applies to persistence.

The mechanism proposed, therefore, enables us to generate much higher persistence than the one usually found in models due to the history dependence of wedge.

It should be added that we are being cautious since we do not introduce any of the other mechanisms that help generating variance or persistence such as non-separability of consumption or money in the utility function.

Note, for instance, that under an independent across time wedge, the first order auto correlation of the real exchange rate will asymptotically go to one. The same applies to the variance, that is not even stationary in the iid-wedge case, suggesting that random walkness is not that far from a standard business cycle model.

It is important to stress that the source of persistence and variance is the time-varying pattern of the wedges. It is exactly the history of wedges that define a new component on the exchange rate behavior. One should note, however, that, at this point, we just have a theoretical point, and quantitative and structural explanations will be provided in other sections.

**Backus-Smith puzzle** - Another puzzle to be addressed is the Backus-Smith puzzle, which says that the real exchange rate does not follow the difference in marginal utilities of consumption or, as we suggest here, consumptions themselves. Actually, this correlation turns out to be negative in the data. Under the following requirement, we address the Backus-Smith puzzle:

**Lemma 2.** If $\rho_{\kappa, c-c^*} < -\frac{\sigma_{c-c^*}}{\sigma_{\kappa}}$, the correlation of the log real exchange rate and log consumption is negative, where $\kappa = \sum_{j=0}^{t} \xi_j$, $\sigma$ denotes the standard deviation and, $\rho$, the correlation.
There are two points to highlight. First of all, the Backus-Smith condition is history-dependent even in a complete markets as we have here. The second point is that this constraint is relatively weak. As the denominator increases (history accumulation), we just need a negative covariance between the cumulative wedge and current relative consumption to assure the result.

**Low consumption correlation** - A recurring evidence in international economics has been that the cross-correlation of consumption is smaller than the cross-correlation of outputs. The risk sharing condition suggested before also should help explain that, but a quantitative exercise is needed to assess the magnitude of the mechanism. So far, we just know that we are tilting the risk sharing condition, which should lead to a lower consumption correlation than in a non-wedge model.

Nominal Exchange Rate behavior

As argued by Engel (1999), the nominal exchange rate inherits most of the behavior of the real exchange rate, since prices do not change as much as the nominal exchange rate. And it is quite interesting that our model is able to reproduce high persistence and high variance in the real exchange rate too.

Suppose, therefore, that we have a simple cash in advance constraint to pin down nominal exchange rate, where \( \mu (s^t) \) is the money supply in the economy.

**Lemma 3.** Under complete markets with a financial wedge,

\[
\frac{X (s^{t+1}|s^t)}{X (s^t|s^{t-1})} = \frac{\mu^* (s^t)}{\mu^* (s^{t+1})} \frac{\mu (s^{t+1})}{\mu (s^t)} \xi (s^{t+1}|s^t)
\]

\[
X (s^{t+1}|s^t) = \frac{\mu (s^{t+1})}{\mu^* (s^{t+1})} \prod_{j=0}^{t+1} \xi (s^j)
\]
Since part of the nominal and the real exchange rate determination is on the wedges, the nominal exchange rate does inherit part of the behavior of the real exchange rate as observed in data. Therefore, variance and persistence should increase as discussed before\(^4\). Moreover, the determination of the exchange rate also becomes history-dependent.

**Interest Parity**

The introduction of a wedge in the internation asset allocation also helps explaining the uncovered interest parity puzzle.

Consider the case of a domestic investor deciding whether to invest money in its own currency or in the foreign one.

Aggregating over all states of the world, in order to have a non-state contingent rate, under a first order approximation, we have:

\[
R_t = \frac{R_t^*}{E_t(\xi_{t+1})} E_t \left( \frac{X_{t+1}}{X_t} \right) (1.2.2)
\]

where we assume that the wedge is on the return of the bond denominated in foreign currency, just as before.

What is important to stress is that this wedge changes the uncovered interest parity in a way that could help explaining such major puzzle.

Taking logs, we have:

\[
r_t - r_t^* + \log [E_t (\xi_{t+1})] = E_t (x_{t+1} - x_t)
\]

But one should bear in mind that most models give \( Cov (r_t - r_t^*, E_t (x_{t+1} - x_t)) = 1 \), while in data this covariance is negative. The literature has suggested some possibilities for

\(^4\)Such fact is dependent on the statistical properties of the wedge, that could be exogenous or endogenous to the model.
that; Burnside et al. (2009), Lustig and Verdelhan (2007) and Alvarez et al. (2009) are recent examples. We follow earlier suggestions from Burnside (2007) in that we do not put much emphasis on risk.

With the introduction of such wedge, we can address this puzzle depending on the correlation of the wedge and monetary policies.

**Lemma 4.** If $\rho_{\Delta \mu, E_t \xi_{t+1}} < -\frac{\sigma_{\Delta \mu}}{\sigma_{E_t \xi_{t+1}}}$, the coefficient of a UIP regression that neglects $\xi$ is negative, where $\Delta \mu = E_t (\mu_{t+1} - \mu^*_t) - (\mu_t - \mu^*_t)$

Different from the Backus-Smith condition, the UIP is a stricter condition, since it is a period-by-period condition. Furthermore, the restriction shown, which is derived from the cash in advance constraint, is a restriction on the primitives, while the Backus-Smith result is a general equilibrium condition.

Another important point is that the absolute value of $\xi$ is not important in itself; its covariance with interest rates and exchange rates does matter though.

**What this is and what this is not**

The stylized model addresses some of the puzzles in international economics, but it asks varying requirements in order to do so. Even though the requirements to attain variance and persistence are weak, achieving cross-correlation results demand more. In the Backus-Smith puzzle, a sufficient condition is that the stream of wedges is negatively related to contemporaneous consumption, evidencing the history-dependence of such condition. In the UIP, the condition is stronger as it depends on a period-by-period condition, relating money supply and wedges.

The point to stress is that it is an intertemporal misallocation and, therefore, it cannot
be a trade cost or a change in utility\(^5\). The argument is that for those, we would need huge shocks because there is no amplification mechanism through auto-correlation of the resulting shocks. However, in the case of intertemporal wedges, it generates an amplification mechanism and history-dependence.

In a more practical sense, this wedge is related to anything that distorts the optimal allocation of assets. It is by distorting the savings decisions that one can get such wedge. In providing three different interpretations of the wedge, such mechanisms will be clearer.

This wedge should not be considered necessary to address the puzzles. We argue, though, that under the statistical requirements showed before, it should be sufficient. In the next part, we discuss a little how this wedge can be found in the data and what does it suggest as a guidance for the discussion.

### 1.2.2 Empirical Counterpart of the Wedge

The empirical evaluation of the wedge comes from the fact that we have two equations that pin down the wedge: the Backus-Smith and the UIP. In our specification, it should be the case that they give similar results, which can be used to discuss the empirical relevance of the wedge.

We’ll first assume a complete markets framework, such that equation 1.2.1 holds and we can evaluate the wedges directly.

By taking logs of equation 1.2.1, and subsequently taking the difference:

\[
\log \left( Q \left( s^{t+1} | s^t \right) \right) = \log \left( C \left( s^{t+1} \right) \right) - \log \left( C^* \left( s^{t+1} \right) \right) + \sum_{j=0}^{t+1} \log \left[ \xi \left( s^j \right) \right] \\
q_{t+1} - q_t = [c_{t+1} - c_t] - [c^*_t - c^*_t] + (\xi_{t+1} - 1)
\]

We use World Development Indicators from the World Bank using annual growth on

\(^5\)For trade costs, see Obstfeld and Rogoff (2001)
consumption expenditures\textsuperscript{6}. For the real exchange rate, we consider all the countries vs. the United States, so we construct the changes in the real exchange rate from nominal exchange rates and price levels\textsuperscript{7}.

The wedges are then defined by the difference of the growth on real exchange rate and the growth on relative consumptions.

The first stylized fact is that the wedges are time-series and cross-section varying, which is important if we are to rely on them as a source of explanation (the summary statistics of the wedges by country are in the appendix). Even though the mean of the wedge on the sample is just $-0.4\%$, the standard deviation is high ($13\%$)\textsuperscript{8}. Moreover, the volatility of the wedge is correlated with the volatility of depreciation, just as theoretically found before. In Figure 1.2.1, we compare the average depreciation and wedge for each country in the sample.

\textbf{Figure 1.2.1: Depreciation and Wedges (country standard deviations)}

![Depreciation and Wedges (country standard deviations)](image)

The results can be better seen in graphs for a selection of countries in Figure 1.2.2.

Such wedges were found from the risk-sharing condition under complete markets. In order to sustain the argument that this is not related to the utilities, but due to returns, we

\textsuperscript{6}We consider, henceforth, annual growth instead of log differences relying on approximations that are accurate if such changes are not very large.

\textsuperscript{7}$\log (q_{t+1}) - \log (q_t) = \log \left(\frac{q_{t+1}}{q_t}\right) = \log \left(\frac{E_{t+1}P_t^*}{P_{t+1}E_t}\right) = \Delta e_{t+1} + \pi_{t+1}^* - \pi_{t+1}$. We use realized inflation instead of expected inflation because it allows us to span a broader span of countries and also puts us in the same base of comparison as the previous literature. For a detailed description of the data used, see the Appendix.

\textsuperscript{8}A cross-country average gives us that $\frac{sd(\xi)}{mean(x_t)} = 3.7\%$ and $\frac{sd(\xi)}{sd(x_t/x_{t-1})} = 1.3$. 

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can compare with the wedges found from the uncovered interest parity equation (incomplete markets model). By assuming perfect foresight and a first order approximation, we have:

\[ r_t - r^*_t + \kappa_{t+1} = e_{t+1} - e_t \]

Therefore, we show, for the same countries we discussed, a comparison between a complete and an incomplete market wedge in Figure 1.2.3.

In a model in which one has distortions on the utility function, the wedge on the first order approximation of the UIP should be null. However, we find a striking result supporting our interpretation. In the same venue, velocity and cash-in-advance shocks should not show up in the Backus-Smith. Furthermore, the puzzles are independent of capital or labor wedges given that these are separable from consumption. Another way of seeing it is that the UIP and the Backus-Smith are just two ways of seeing the same puzzle and, therefore, the same source of risk-sharing limitation should show up in both conditions.
If anything, what we find is that it should be an intertemporal wedge that is responsible for almost all changes in the exchange rate.

As we have already laid out the stochastic properties of the wedges, we can construct processes disciplined by how the wedge is in data. In the next section, we discuss a capital control approach for the wedge, followed by a borrowing constraint model and a noisy traders model.

1.3 Capital Controls

When one thinks of different prices for a foreign and a domestic resident for the same bond, the first thing that comes to mind is a price-based capital control. Even though there are other mechanisms, we start by describing this one. Moreover, recently, there has been a revival of capital controls as evidenced in media (The Economist, issue of August 2010).
As we computed the financial frictions wedge for a set of countries, we can evaluate the correlation of such wedge with capital controls. We start by motivating our proxy of capital controls. Capital controls are not easy to measure, since some are quantity-based, others are price-based and they are not always explicitly defined. As Magud et al. (2011) and Ostry et al. (2011) argue, the capital controls literature has serious "apple-to-orange" problems. To circumvent this, we use the index of capital account openness from Chinn and Ito (2006), which is an index based on the IMF’s Annual Report on Exchange Arrangements and Exchange Restrictions, that evaluates the presence of multiple exchange rates, restrictions on current account transactions, restrictions on capital account transactions and requirement of the surrender of export proceeds.

Relating such index to the wedge found before may not be a simple linear function. The wedge is a function that depends on the capital controls of the pair of countries in a possibly non-trivial way. However, the analysis benefits from the fact that USA had its index constant throughout the sample period (1970 to 2009), and we assume that it didn’t have capital controls, linking directly to the model shown before.

First, we look at the cross-section relationship analogue based on the average across time of the capital openness index and the wedges in Figure 1.3.1. Due to the possibility of nonlinearities, we compute the locally weighted estimator of the relationship, allowing, therefore, for nonlinearities.

Apart from the outliers with very small openness index, there seems to be a positive relationship. It is also interesting that for high values of the index, in which the country doesn’t have much controls, the wedge is close to zero, just as expected. However, as our mechanism relies on time-varying movements, it is hard to sustain that exchange rate movements come from capital controls, since they are very slow moving. In order to discuss such criticism in more details, we consider a country fixed-effect specification of the model,
to highlight the time-varying perspective, which is given in Figure 1.3.2.

By looking at a parametrized equation with linear and quadratic functions, we do not find linear or even quadratic robust relationship between the wedge and the capital account openness\textsuperscript{10}. It should also be stressed that the explained portion of the variance of the wedge attributed to capital controls is very small.

The empirical results, therefore, suggest that capital account can not be assumed to be the source of the time-varying pattern in the wedge. As imagined, capital controls lack the appropriate time-varying pattern and ask for another source of explanation. We further investigate some other mechanisms that show up like the wedge discussed before.

\textsuperscript{9}We only look at pairs against USA, following the analysis of the wedge.

\textsuperscript{10}When using the lag of the dependent variable as a regressor, we use Arellano-Bond estimator.
1.4 Debt Constraints

The price wedge mechanism discussed before can show up explicitly, as in the case of capital controls, but it can show up implicitly as in a borrowing constraint. If one country has a slacker borrowing constraint than another, they implicitly price debt differently. Therefore, just as we showed in Section 1.2, such wedge generates the results discussed before, with the caveat that it is now endogenous so the correlations are found from the equilibrium.

We first present a model with borrowing constraints to investigate the relationship between wedges and borrowing constraints. Based on that, we compare how would the model fit the data. From such comparison, we can find the conditions that a borrowing constraint model has to satisfy to fit the data. Identifying borrowing constraints is not a simple task since a given level of debt can be due to unconstrained optimality or due to a constraint, but we can disentangle them in our empirical exercise. We can then evaluate which type of borrowing-constraint model is more promising, based on the ones already suggested in the literature\textsuperscript{11}.

1.4.1 Evidence of Borrowing Constraints

For simplicity in this part, assume a complete markets benchmark, but we discuss an incomplete analogue in the appendix. The borrowing constraint is stated in very general terms and just says that the total amount of debt held cannot be larger than a given state-contingent value (that we assume that does not depend on the consumption or the asset allocation). Apart from that change, all the other margins are the same as before\textsuperscript{12}.

\textsuperscript{11}Alvarez and Jermann (2000), Mendoza (2002), Bianchi (2009), Bianchi (2010) are examples

\textsuperscript{12}In this specification, we opt for including capital, just to highlight that the results do not change whether we specify with or without capital.
\[
\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t)
\]

s.t.
\[
P(s^t) C(s^t) + I(s^t) + X(s^t) \sum_{s^{t+1}} M^*(s^{t+1}|s^t) B(s^{t+1}) \\
\leq W(s^t) L(s^t) + X(s^t) B(s^t) + r_{k,t} K_t \\
K_{t+1} = (1 - \delta) K_t + I_t \\
X(s^t) \sum_{s^{t+1}} M^*(s^{t+1}|s^t) B(s^{t+1}) \geq \circ(s^t)
\]

By using the appropriate first order conditions, we have:

\textbf{Lemma 5.} The non-arbitrage condition can be written as

\[
q_t = \frac{c_t}{c_t^*} \prod_{j=0}^{t} \left( \frac{1 - \chi_j}{1 - \chi_j^*} \right)
\]

where \(\chi_j = \zeta_j c_j p_j\), and \(\zeta_j\) is the Lagrange multiplier on the borrowing constraint.

If the borrowing constraint in the domestic economy tightens (\textit{ceteris paribus}), the real exchange rate appreciates. This is a fairly direct implication that can be tested in the data.

From equation 1.2.1, we know that there should be a negative correlation between the wedge and the consumption to explain risk-sharing puzzles such as the Backus-Smith puzzle. From the borrowing constraint, we know that when the wedge decreases, the borrowing constraint tightens\(^{13}\). Therefore, we need a model in which the borrowing constraint tightens when consumption is high. From this chain of events, models based on incomplete markets or borrowing constraints à la Bewley do not seem appropriate to explain risk sharing puzzles\(^ {13}\).

\(^{13}\)We assume that \(\xi\) dominates on the term \(\chi\) since consumption and price levels are slow moving objects.
since borrowing constraints bind when a bad shock hits.

The other possibility of borrowing constraint models that seem more appropriate are based on the work of Alvarez and Jermann (2000), where the economy can be decentralized with state-contingent borrowing constraints, the so-called "endogenous solvency constraints". This is in line with the work of Bodenstein (2008) that find that a Kehoe and Levine (1993) decentralized economy can help explain the Backus-Smith puzzle. The question is whether data supports such assertion.

The issue, however, is that we do not observe the borrowing constraint in the data. Therefore, what we do is to evaluate the relationship between debt (net foreign assets) and the wedge. The relationship of the net foreign assets with the wedge will hold only if $\chi_j$ is different than zero, which means that we have a constrained optimal problem. Therefore, if we find that there is a relationship between the net foreign assets and the wedge, given our model, this suggests that borrowing constraints do matter.

If an endogenous borrowing constraint is the correct specification, we should expect a negative relationship between them, since a decrease in wedges (tightening of borrowing constraint) should be correlated with an increase of the net foreign assets\(^{14}\).

\(^{14}\)Note that the map between the wedge and the borrowing constraint is simple:

$$\kappa_j = \frac{1 - \chi_j}{1 - \chi_j^*}$$

And we take logs and approximate to have a linear function:

$$\log(\kappa_j) = -\chi_j + \chi_j^*$$

However, as we do not know the Lagrange multiplier, we have to consider a proxy and impose a relationship. We will investigate the relationship between net foreign assets and the wedge.

$$\log(\kappa_j) = \beta_0 - 1_{\{\zeta_j \geq 0\}} \beta_1 [NFA_j]$$

For debt, we consider the net foreign assets over GDP from Lane and Milesi-Ferretti (2001) and Lane and Milesi-Ferretti (2007). The use of the growth on net foreign assets is due to the fact that we have only one-period bonds in our model and, furthermore, we cannot reject the null hypothesis that the net foreign assets is non-stationary on a Fisher-type unit root test. As usual, we use Arellano Bond when we have the lagged dependent variable in the specification.
However, as shown in Figure 1.4.1, the opposite pattern happens in the data. The result is not robust to introducing consumption in the specification, but, even though we may not be able to distinguish with clarity between a nonconstrained problem or one in which we have a positive coefficient, it is clear that we do not have strong evidence towards negative coefficient\textsuperscript{15}.

If anything that one can take from such section is the fact that the models suggested in the borrowing constraints literature are at odds with the data or the statistical requirements to fit some of the international puzzles.

\textsuperscript{15}By doing a regression country by country, we find that Australia, Germany, Dominican Republic, Iceland, New Zealand and Portugal are the countries for which we have positive coefficients significant at 10\%, which can be thought as some of the more borrow constrained in our OECD sample. None of the countries in our sample have negative significant coefficients. However, adding consumption does bring an endogeneity issue because the wedge is constructed from consumption.
1.5 Exogenous Noise Model

1.5.1 Introduction

The last structural model that we consider assumes that there is noise in the foreign bonds market. We construct a fully fledged incomplete markets model with noise. As far as we are concerned, there is no quantitative evaluation of a noisy model in the international macro literature, so we calibrate the model to see how far can such mechanism explain the puzzles\(^\text{16}\). The data that supports such model is pervasive, like the disconnect puzzle literature, or excessive exchange rate volatility and we just point to references herein (Shleifer and Summers (1990), De Long et al. (1990) and Jeanne and Rose (2002)).

1.5.2 Model

The quantitative counterpart asks us to describe the equilibrium fully, including the structure of goods and firms. The main difference, if compared to the theoretical benchmark we have provided, is that we introduce capital and we consider an incomplete markets model. We provide the setup of the model, shocks and the parametrization in this section, but a more detailed description of the model, with the derivation of it and the equilibrium, can be found in the appendix.

Agents:

We depart from the usual specification of the financial markets by the fact that we assume that agents do not have access to the international financial market. It is through foreign exchange dealers that agents have access to the financial market. Note, however, that such structural model is isomorphic to the one presented in Section 1.2.

\(^{16}\)Xu (2010), which seems to be the closest paper, does a quantitative analysis only on the exchange rate disconnect.
\[
\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u (C_t, L_t)
\]

\[s.t.
P_t C_t + d_t B_{t+1} + I_t = W_t L_t + \Pi_t + B_t + V_t + r_t K_t \]

\[K_{t+1} = (1 - \delta) K_t + I_t\]

where \(B_{t+1}\) is the amount of domestic holding on domestic bonds, \(\Pi_t^f\) is the profit from firms and \(\Pi_t\) is the profit obtained from foreign exchange dealers, and \(d_t\) is the price of the non-contingent domestic bond.

Foreign agents have an analogous problem, in which they only have access to the foreign bonds market.

The source for dynamics of the real exchange rate is the home bias on the consumption goods, that we assume to be fully tradeable.

Imports and Exports are chosen by each household as usual given the consumption aggregator:

\[
\max \left[ \left( \frac{\nu}{2} \right) C_{h,t}^\rho + \left( 1 - \frac{\nu}{2} \right) C_{f,t}^\rho \right]^{\frac{1}{\rho}}
\]

\[s.t.
P_{f,t} C_{f,t} + P_{h,t} C_{h,t} = M\]

**Foreign Exchange Dealers**

We follow earlier work from Jeanne and Rose (2002) and Devereux and Engel (2002). We assume that all foreign exchange dealers are noisy and that dealers are risk-neutral,
maximizing the expected returns

\[ E_t^n \left[ \Phi X_{t+1} B^*_{h,t+1} - d^*_t X_{t} B^*_{h,t+1} \right] \]

where \( \Phi \) is the state-contingent stochastic discount factor of the domestic agent. The expected return shows how much a domestic agent will get next period \( (X_{t+1} B^*_{h,t+1}) \) if he invests the amount \( (d^*_t X_{t} B^*_{h,t+1}) \), where \( d^*_t \) is the price of a foreign bond.

The noise is on the first moment of the implied expectation of the log-exchange rate (it is the rational expectation plus a disturbance):

\[ E_t^n x_{t+1} = E_t x_{t+1} - v_t \]
\[ V_t^n (x_{t+1}) = V_t (x_{t+1}) \]

where \( v_t \sim N (0, \sigma^2) \). Under free-entry:

\[ d^*_t = \mathbb{E}_t^n \Phi_t \frac{X_{t+1}}{X_t} \]

By doing a log-approximation, we have the usual UIP equation with an exogenous disturbance given by the noise shock.

**Firms**

A constant elasticity of substitution production function is defined by:

\[ Y_t = A_t \left[ \nu (K_t)^{1-\theta} + (L_t)^{1-\theta} \right]^{\frac{1}{1-\theta}} \]

where \( A_t \) is the technology shock, which is one of the shocks that we’ll consider\textsuperscript{17}.

\textsuperscript{17}Results do not change by considering a Cobb-Douglas production function.
Shocks

We consider, under the benchmark model, just productivity shocks and we introduce the noise afterward to stress its effects. The productivity shocks are standard:

$$A^i_t = \rho_A A^i_{t-1} + \varepsilon^A_i, i = 1, 2$$

For the noise shock, to highlight the mechanism, we consider a purely normal iid shock and we provide results for different variances, but always with zero mean.

Parametrization

We'll start by the production function. $\theta$ is found so that the elasticity of substitution between capital and labor is .95\(^{18}\). Furthermore, $\nu$ is found such that the capital share in the economy is $\frac{1}{3}$.

The utility function we consider is $u(c, l) = \log(c) - \psi_0 \frac{(1-l)^{1-\psi}}{1-\psi}$, where $\psi_0$ is found such that the steady state labor supply is .4 and $\psi$ is such that the Frisch elasticity is $3^{19}$.

Usual values for the discount factor ($\beta = 0.99$) and depreciation ($\delta = 0.025$) are used.

The last set of values is related to how foreign and domestic goods are substituted. We assume an elasticity of substitution of 1.5, which is standard since Backus et al. (1992), and a home good bias of .90\(^{20}\).

In what relates to the shocks, following Corsetti et al. (2008) and Kehoe and Perri (2002), we assume $\rho_A = 0.9$, with 0.004 standard deviation and cross correlation across countries

\(^{18}\)This is in line with the usual assumption of a Cobb-Douglas production function that assumes a unitary elasticity of substitution. Restuccia and Urrutia (2001) could not reject that elasticity is equal to one, while Pessoa et al. (2005) have suggested that the elasticity should be around .7.

\(^{19}\)For the steady state labor supply, we follow Shimer (2010). For the Frisch elasticity, there is a wide range of findings, especially if coming from micro or macro elasticities (Chetty et al. (2011)).

\(^{20}\)By using such number for the elasticity of substitution, we stress that this is not the driving force for getting low consumption risk sharing, since Corsetti et al. (2008) have showed that much lower values are needed.
amounting to .45\textsuperscript{21}.

As we don’t have a strong prior on how should noise shocks behave, we consider them to be normally distributed iid, but we provide three different sizes: small amount of noise ($s.d. = 0.05$), medium amount of noise ($s.d. = 0.1$) and large amount of noise ($s.d. = 0.15$). The small amount of noise shock has unconditional variance less than double of productivity’s, but still relatively large.

The results are obtained from empirically simulating the model 1000 times for 200 periods.

1.5.3 Results

<table>
<thead>
<tr>
<th>All variables logged and HP filtered</th>
<th>Data (no noise)</th>
<th>Benchmark</th>
<th>Small amount of noise</th>
<th>Medium amount of noise</th>
<th>Large amount of noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s.d. (C)$</td>
<td>0.83</td>
<td>0.6208</td>
<td>0.6356</td>
<td>0.6768</td>
<td>0.7376</td>
</tr>
<tr>
<td>$s.d. (I)$</td>
<td>2.78</td>
<td>2.5841</td>
<td>2.6579</td>
<td>2.8729</td>
<td>3.2023</td>
</tr>
<tr>
<td>$corr (e_t, e_{t-1})$</td>
<td>0.89</td>
<td>0.9978</td>
<td>0.9972</td>
<td>0.9958</td>
<td>0.9941</td>
</tr>
<tr>
<td>$corr (I_t, I_{t-1})$</td>
<td>0.91</td>
<td>0.9946</td>
<td>0.9929</td>
<td>0.9889</td>
<td>0.9844</td>
</tr>
<tr>
<td>$corr (Y_t, Y_{t-1})$</td>
<td>0.88</td>
<td>0.9963</td>
<td>0.9963</td>
<td>0.9963</td>
<td>0.9963</td>
</tr>
<tr>
<td>$corr (Y_t, Y^\ast_t)$</td>
<td>0.60</td>
<td>0.3752</td>
<td>0.3710</td>
<td>0.3543</td>
<td>0.3262</td>
</tr>
<tr>
<td>$corr (e_t, c_t^\ast)$</td>
<td>0.38</td>
<td>0.5775</td>
<td>0.5101</td>
<td>0.3405</td>
<td>0.1353</td>
</tr>
<tr>
<td>$corr (I_t, I^\ast_t)$</td>
<td>0.33</td>
<td>0.4115</td>
<td>0.3425</td>
<td>0.1674</td>
<td>-0.0352</td>
</tr>
<tr>
<td>$corr (q, y)$</td>
<td>0.08</td>
<td>0.6840</td>
<td>0.7898</td>
<td>0.7333</td>
<td>0.6053</td>
</tr>
<tr>
<td>$corr (q, c - c^\ast)$</td>
<td>-0.35</td>
<td>0.9386</td>
<td>0.5137</td>
<td>-0.0221</td>
<td>-0.3009</td>
</tr>
<tr>
<td>$s.d. (q_t)$</td>
<td>4.36</td>
<td>0.2808</td>
<td>0.4436</td>
<td>0.0955</td>
<td>0.0732</td>
</tr>
<tr>
<td>$corr (q_t, q_{t-1})$</td>
<td>0.83</td>
<td>0.9957</td>
<td>0.9099</td>
<td>0.9142</td>
<td>0.9314</td>
</tr>
<tr>
<td>$corr (S_t, I_t)$</td>
<td>0.70</td>
<td>0.9887</td>
<td>0.9144</td>
<td>0.7318</td>
<td>0.5300</td>
</tr>
<tr>
<td>$corr \left( i_t, \frac{q_t}{e_t} \right)$</td>
<td>-0.88</td>
<td>0.9340</td>
<td>0.0715</td>
<td>-0.0522</td>
<td>-0.0873</td>
</tr>
</tbody>
</table>

As one can see from Figure 1.5.1, the quantitative implications of the introduction of noise are relevant and help understand some of the puzzles\textsuperscript{22}. The volatility of the real exchange rate increases, even though it is still very low compared to data. Moreover, since

\textsuperscript{21}We also provide a curvature on the unconvered interest parity by allowing the interest rate to be very little sensitive to net foreign assets (0.001), to ensure an equilibrium as in Schmitt-Grohe and Uribe (2003).

\textsuperscript{22}In this section, we perform a first order approximation of the equations shown to obtain the results. Data results come mainly from the following papers: Backus et al. (1992), Coakley et al. (1998) and Froot and Thaler (1990).
we introduce a shock in the risk-sharing equation, the low consumption correlation, even lower than the correlation between outputs for some specifications, can also address this puzzle. Note also that both the Feldstein-Horioka and the uncovered interest parity puzzle are also much improved under the noise shock model\textsuperscript{23}.

More importantly, we obtain a negative coefficient on the relationship between the real exchange rate and the consumption differential, the so-called Backus-Smith puzzle. Such relationship comes entirely from a general equilibrium effect, since the shock is iid. As the exchange rate does not offset entirely the shock, the correlation between relative consumption and the exchange rate is negative\textsuperscript{24}. The shock drives the domestic interest rate compared to the foreign one too high, leading to high savings in the domestic country compared to foreign. Therefore, given the resource constraint, consumption is low in the domestic country explaining the negative correlation between real exchange rate and consumption differentials.

Finally, one should note that, by introducing the noise, the other margins of the model that a business cycle model already matches remain quantitatively unchanged.

\textbf{1.5.4 Empirical Evaluation}

Evaluating how much noise does one observe in the market is a very challenging exercise to undertake. However, the literature has pointed out that, under floating exchange rate regimes, there is much more noise than under fixed exchange rate regimes (see Jeanne and Rose (2002) and the references therein). Therefore, one should observe, if such story is true, puzzles to be more pronounced under floating exchange rate regimes than under pegged

\textsuperscript{23}The relationship between investment and saving arise naturally in such models as first documented by Baxter and Crucini (1993).

\textsuperscript{24}One can rewrite the UIP equation as: $E_t (x_{t+1} - x_t - v_t) = E_t ((c_{t+1} - p_{t+1}) - (c_t - p_t)) - E_t ((c_{t+1}^* - p_{t+1}^*) - (c_t^* - p_t^*))$. But when there is a positive shock in $v_t$ and $E_t (x_{t+1} - x_t)$ does not adjust positively enough, $E_t ((c_{t+1} - p_{t+1}) - (c_t - p_t)) - E_t ((c_{t+1}^* - p_{t+1}^*) - (c_t^* - p_t^*))$ has to reduce and, from the log utility, it should not lead to intertemporal substitution. The same applies for the real exchange rate based on an analogue argument using the Backus-Smith equation.
regimes, *ceteris paribus*.

Devereux and Hnatkovska (2011) have provided evidence that the Backus-Smith puzzle is much stronger in pairs of cities US-Canada than in between cities of US or in between cities of Canada, which could be seen as an evidence in favor of such explanation. We provide further evidence supporting their earlier conjecture that the nominal exchange rate plays a role on the Backus-Smith puzzle. By looking at a panel of countries across time, we find evidence that more pegged regimes are usually associated with a lower violation of the Backus-Smith condition.

This exchange-rate regime dependence is robust to other measures of risk-sharing such as financial integration. By looking at financial integration as suggested by Lane and Milesi-Ferretti (2001) and Lane and Milesi-Ferretti (2007), in which one sums assets and liabilities of a country as a fraction of GDP, we find evidence that floating exchange-rate regimes are associated with lower financial integration. Further investigation suggests that there is lower development of finance, i.e., lower export of currently available goods in return for the promise of future goods (or the reverse). Therefore, it is not a diversification motif, through a wider span of the state space, that distinguish the financial integration between exchange rate regimes.

Finally, we study the relationship between investment and savings as firstly proposed by Feldstein and Horioka (1980). We find that the correlation of investment and savings, which should be zero in a frictionless with perfectly mobile capital world, is closer to one in floating exchange rate regimes.

The results of a lower risk-sharing under floating exchange rate regimes are robust to different measures of pegged regimes. We consider four different measures. A "de jure" index proposed by the IMF and the most popular de facto measures used in the literature: Reinhart et al. (2011), Levy-Yeyati and Sturzenegger (2003) and Shambaugh (2004).25

The frequency of the data studied is annual, since we need to have a long panel with a

\[25\] We describe in further detail each of these indices in the next section.
sizable number of countries. We perform our analysis using Penn World Tables (PWT). As it will become clearer, we consider both the PWT from National Accounts as well as the PPP adjusted.

The second dataset used comes from the work of Lane and Milesi-Ferretti (2007). Such dataset allows us to follow net foreign assets and liabilities of countries across time. The final part on the construction of the datasets is related to how to classify exchange rate regimes. We consider four different classifications: de jure index by the IMF, Reinhart and Rogoff (2004), Shambaugh (2004) and Levy-Yeyati and Sturzenegger (2003) (the details of each of them are in the appendix).

Once we have these results, the question is what do we learn from it. First of all, our empirical results suggest a correlation between exchange rate regimes and risk sharing. However, causality cannot be stated since we do not have an exogenous disturbance that allows a clean identification procedure.

Based on the cumulative evidence that risk-sharing puzzles depend on exchange rate regimes provided here and the difference between intra-national and international evidence in Devereux and Hnatkovska (2011), one has to question what is different across regimes that could help explain these puzzles. Devereux and Hnatkovska (2011) suggest that a standard open economy model with sticky prices cannot explain the regime-dependence of the Backus-Smith puzzle. It is necessary to combine multiple sources of shocks, ex-ante price setting, and incomplete financial markets in order to attain such result.

However, there are two main reasons why the change of an exchange rate regime could matter for risk sharing. The first one, obviously, is through monetary policy. Such approach has been the one followed by Devereux and Hnatkovska (2011), where they introduce price stickiness and evaluate if the change of monetary policy is relevant for understanding risk sharing. However, this explanation falls short on explaining the regime dependence of the puzzles. But one has to question how much the existence of a proper market for bonds and its uncertainty does change the environment. By considering the fact that there is much
more noise trading in floating regimes, as suggested in Jeanne and Rose (2002), these results are in line with what one would expect from a noise model to explain the puzzles.

In the next subsection we discuss the data used in the empirical exercises followed by the results of them.

1.5.5 Methodology

The methodology we propose here is very simple. Suppose that a given relationship that we are interested in lies on the correlation between variables $X$ and $Y$. The method is to evaluate if such relationship changes as one has a pegged or a floating exchange rate regime. Therefore, we consider a panel data to evaluate its relationship enhanced by interactive terms related to a dummy if the regime is pegged or not.

$$X_{it} = \beta_0 + \beta_1 Y_{it} + \beta_2 (Y_{it} \times peg_{it}) + \beta_3 peg_{it} + u_{it}$$

Under floating regimes, the coefficient would be $\beta_1$, while the coefficient under pegged regimes would be $\beta_1 + \beta_2$. Note that we also allow the intercept to change, and we do that just as to have a more flexible functional format.

The first remark one has to make is the reverse causality. In here, we are just evaluating correlations, since exchange rate regimes are endogenous to the macroeconomic policies and outcomes. We provide, however, a broader picture that such pattern arises under different risk-sharing measures and under different classifications of the exchange rate regime.

The second remark is on the definition of the peg. We use the indices discussed above. However, some of them do not have information on the base country with which to peg, we’ll implicitly consider that it is pegged to the dollar\textsuperscript{26}.

On the more econometric side, we will estimate the panel with fixed effects and robust Newey-West standard errors.

\textsuperscript{26}Such assumption is used only in some specification and we will highlight it when using it.
1.5.6 Empirical Results

In this section, we discuss the empirical results for three different dimensions of risk sharing. The first one is the relationship between consumption and real exchange rates. The second one focuses on net foreign assets and the third relates savings and investment.

Consumption and Real Exchange Rate

The first puzzle that we look at is the so-called Backus-Smith puzzle. Under a complete set of state contingent set of securities in a two-country international macro model\(^{27}\),

\[
U_c(C_i^t) RER_i^t = U_c(C_{us}^t) \tag{1.5.1}
\]

We will use all pairs against the U.S., therefore we will have an implicit assumption of pegging against the dollar. From such equation, we assume separability of the utility function between consumption and leisure in order to have an equation that only depends on consumption and on the real exchange rate. By using a CRRA-utility function and subtracting Equation (1.5.1) by its lagged value, we have:

\[
\theta (\Delta c_{it} - \Delta c_{us,t}) = \Delta rer_{i,us}^{i,t}
\]

where \(\theta\) is the degree of relative risk aversion in the utility function \((\theta \geq 1)\).

From this equation, models would suggest that the correlation between the relative consumption and the real exchange rate should be equal to one. However, data suggests that this correlation is close to zero. In order to evaluate the effect of exchange-rate regimes on such correlation, we estimate the following equation

\[
(\Delta c_{it} - \Delta c_{us,t}) = \alpha + \beta rer_{i,us}^{i,t} + peg_{i,t} \left( \alpha_2 + \beta_2 rer_{i,us}^{i,t} \right) + \varepsilon_{it}
\]

\(^{27}\)We provide the derivation of this equation in the appendix, and we normalize the 0-period consumption be equal.
From this equation, there are a couple of comments to make. First of all, we consider real consumption growth both in country \( i \), as well as in U.S.. The real exchange rate is constructed as the nominal exchange rate times the price level in United States over the price level in country \( i \). As we have many specifications, we discuss possible differences on the construction of these variables as we provide the results.

Another remark is that we introduce coefficient \( \alpha \) and \( \alpha_2 \) to have a more flexible format. Figure (1.5.2) provides the results.

**Figure 1.5.2: Backus-Smith condition across regimes**

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>change in RER CPI actual</td>
<td>0.00690**</td>
<td>-0.000614</td>
<td>0.00183</td>
<td>-0.00097</td>
<td>-0.0463</td>
</tr>
<tr>
<td>D.RER CPI actual * Peg(IMF)</td>
<td>0.00192</td>
<td>(0.00578)</td>
<td>0.00021</td>
<td>(0.00413)</td>
<td>(0.0346)</td>
</tr>
<tr>
<td>Dummy Peg(IMF)</td>
<td>-0.0109</td>
<td>(0.00788)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D.RER CPI actual * Peg(IRR Coarse)</td>
<td>0.0128</td>
<td>(0.0130)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy Peg(IRR Coarse)</td>
<td>0.0355***</td>
<td>(0.00853)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D.RER CPI actual * Peg(Shambaugh)</td>
<td></td>
<td></td>
<td></td>
<td>0.260***</td>
<td>(0.0519)</td>
</tr>
<tr>
<td>Dummy Peg(Shambaugh)</td>
<td>0.0172***</td>
<td>(0.00441)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D.RER CPI actual * Peg(Lys)</td>
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<td></td>
<td></td>
<td></td>
<td>0.310***</td>
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<td>0.00781</td>
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<tr>
<td>Constant</td>
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<td>0.0143**</td>
<td>-0.0257***</td>
<td>-0.000716</td>
<td>0.00498</td>
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<td>Observations</td>
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<td>3,712</td>
<td>4,606</td>
<td>3,069</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.0011</td>
<td>0.0022</td>
<td>0.0111</td>
<td>0.0053</td>
<td>0.078</td>
</tr>
<tr>
<td>Number of encodeirr</td>
<td>147</td>
<td>142</td>
<td>140</td>
<td>147</td>
<td>143</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
*** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \)

Consumption in this specification is defined as the sum of household consumption expenditure and final consumption expenditure of Non-profit institutions serving households (at constant prices), plus sum of government collective consumption expenditure and government individual consumption expenditure (at constant prices) less collective consumption of government for public good type activities, like police (at constant prices).

Real exchange rate is the consumer price index from actual consumption, as defined by
the final variable above.

Now, as for the results, we find that, for the Levy-Yeyati and Sturzenegger (2003) and Shambaugh (2004) indices, there are positive and significant effects, while for Reinhart et al. (2011) it is positive, but non-significant. The interpretation of such results is that, under pegged regimes, there is a higher relationship between consumption and real exchange rate, as would be predicted in a model. Therefore, we observe higher risk-sharing under pegged regimes.

Such result is robust to the use of PPP prices and quantities as well as to a fraction of hand-to-mouth agents (results in the appendix). The result is also robust to using consumption as purely the household consumption expenditure plus non-profit institutions. A final noticeable point is that the coefficient is decreasing on the variance of the exchange rate variance\(^{28}\).

Net Foreign Asset Positions

Another dimension in which risk-sharing (or its lack) can be identified is through savings. We benefit from the work of Lane and Milesi-Ferretti (2001) and merge net foreign asset positions to the macroeconomic variables before.

In order to understand if there is higher financial integration in pegged regimes, we construct, following Lane and Milesi-Ferretti (2007), an index of financial integration given by:

\[
IFI = \frac{Total \ Foreign \ Assets + Total \ Foreign \ Liabilities}{GDP}
\]

The first result found is on how financial integration depends on exchange-rate regime as shown in Figure (1.5.3). As one can see, the higher is the exchange rate regime index, i.e. more floating, the lower is the financial integration index. Such result holds both for \textit{de jure}

\(^{28}\)For this last specification, we drop the fixed effect and use random effects.
measures and *de facto* coming from Reinhart and Rogoff (2004)\(^{29}\).

**Figure 1.5.3: Financial Integration and Exchange Rate Classification**

![Diagram showing financial integration and exchange rate classification](image)

In order to investigate in more detail the relationship between international financial integration and exchange rate regimes, we construct a weighted average of international financial integration across time of pegged and non-pegged as can be seen in Figure (1.5.4).

The results suggest that higher international financial integration is observed under pegged regimes. Not only we observe that international financial integration is higher under pegged regimes for all de facto indices as well as the IMF index, but it also has roughly the same patterns. International financial integration has increased under both regimes, even though under pegged regimes it has remained about twice as large as under floating regimes\(^{30}\).

Given that pegged regimes have a higher international financial integration than floating regimes, we try to understand why is this the case. In order to do so, we use Obstfeld (2004) and Obstfeld (2011) to distinguish development finance from diversification trade. He suggests using the Grubel-Lloyd index applied to foreign assets:

\(^{29}\)The other indices for exchange rate regimes are not fine enough to allow an interpretation as this one. For this graph, we just append all country-data values and compute an OLS with confidence intervals.

\(^{30}\)From the graphs, we also observe that since this result holds since 1970, it is not due to the European Monetary Union or any particular event. Moreover, the results do not arise from the existence of official reserves as shown in the appendix.
If the Grubel-Lloyd index is close to zero, it suggests that net foreign assets and gross foreign assets are comparable. If so, exports are being used for intertemporal trade and would lead to current account imbalances. This would be consistent with the argument of current account imbalances as a way to smooth consumption.

However, if the Grubel-Lloyd index is close to one, in what Obstfeld (2004) calls "diversification trade", there are different claims for future output, and that would be the reason why net and gross positions would not be the same. This would be consistent, for instance, with the existence of assets that pay differently in different states of the world.

As we can see from Figure (1.5.5), there is no clear difference, in the last fifteen years, of the international financial development across regimes. Therefore, even though pegged
regimes provide more financial leverage, there is no evidence that it changes the span of assets being traded.

When looking within the portfolio, we find that floating regimes usually are associated with a smaller fraction of the assets used on reserves and a higher foreign direct investment share (see appendix).

Savings and Investment

The last dimension in which we discuss the lack of risk-sharing in floating regimes if compared to pegged regimes is on the savings vs. investment, as first suggested in Feldstein and Horioka (1980). In a world with perfect capital mobility, savings and investment should be uncorrelated since agents in any given country should look for investment opportunities across countries. However, it is a recurring evidence that investment-saving have a strong
positive correlation\textsuperscript{31}.

Following the same approach discussed before, we compute the relationship between investment and savings in Figure (1.5.6). As one can see, the relationship between investment and saving is stronger under floating exchange rate regimes, suggesting that the lack of risk-sharing, or the Feldstein-Horioka puzzle in this case, is stronger in floating regimes.

Figure 1.5.6: Investment and saving for different regimes (IMF, Reinhart et al. (2011), Shambaugh (2004) and Levy-Yeyati and Sturzenegger (2003), respectively)

Previous literature has investigated the difference between intra-national capital flows and international capital flows. Bayoumi and Rose (1993) have shown that savings and investment are uncorrelated within United Kingdom, while Dekle (1996) and Yamori (1995) have shown that there is no significant correlation using Japanese regional data. Those previous results are consistent with a story that the currency premium plays a role on understanding the Feldstein-Horioka puzzle.

\textsuperscript{31}For a good survey on this matter Apergis and Tsoumas (2009)
Collapsing the annual dataset into a cross-section defined by the pair "country-exchange rate regime" to reduce cyclical endogeneity issues\textsuperscript{32}, we provide a heteroskedastic robust OLS estimate of the relationship between savings and investment shares in Figure (1.5.7). Saving share is obtained by subtracting consumption and government spending from 100\textsuperscript{33}.

![Figure 1.5.7: Feldstein-Horioka puzzle adjusted for Exchange-Rate Regime](image)

<table>
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<td>0.0224</td>
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<td>0.081</td>
<td>0.061</td>
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</table>

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Not only we find that there is a reduction of the correlation between investment and savings in a pegged regime, but that such correlation is not significantly different from zero under pegged regimes. This suggests that the violation of the Feldstein-Horioka perfect capital mobility test occurs only in floating regimes. Moreover, such result is robust to different indices of pegged regimes\textsuperscript{34}.

\textsuperscript{32}Feldstein and Horioka (1980) and Yamori (1995) follow this approach, which has become standard when dealing with the Fedstein-Horioka puzzle.

\textsuperscript{33}We follow the documentation of the Penn World Tables in doing this.

\textsuperscript{34}In the appendix, we provide further robustness checks, discussing other forms of constructing investment and saving shares, weighting the countries, as well as panel data evaluations.
1.6 Conclusion

In this paper, we provide a "map" of the wedge that one should distort to explain some of the puzzles in international macroeconomics. A theoretical and empirical assessment of the wedge suggests that it is a promising venue when constructing models. However, we need a structural interpretation of the wedge to attain the puzzles, which allows us to compare models. Even though capital controls and debt constraints do not seem to be very promising, the simplest noise model already improves significantly the fit to the data, even though the size of the disturbance is uncomfortably large. Further empirical evidence provides backing for this model.

In a more general view, this paper suggests that distortions in the financial markets are key to understanding international open macroeconomy puzzles related to the real economy. Therefore, by considering the regime-dependence of the puzzles and the theoretical results derived, work should be done on microfundamentating exchange rate markets, as in the work of Evans (2010) and Bacchetta and Wincoop (2006) among others, within a quantitative macroeconomic model to evaluate how the real economy is distorted.

1.7 Appendix

1.7.1 Proofs of Section 2

We consider first how to get the real exchange rate in this economy. By taking first order condition with respect to bond holding and consumption in the domestic economy, we have:

\[
\begin{align*}
[c(s^t)] : & \quad u_c(C(s^t), L(s^t)) = \beta \lambda(s^t) P(s^t) \\
[B(s^{t+1})] : & \quad \lambda(s^t) X(s^t) \xi(s^{t+1}|s^t) M^*(s^{t+1}|s^t) = \beta \lambda(s^{t+1}) X(s^{t+1})
\end{align*}
\]
By substituting one into another:

\[ M^* (s^{t+1} | s^t) = \beta \frac{X(s^{t+1})}{X(s^t)} \frac{1}{\xi(s^{t+1} | s^t)} \frac{\lambda(s^{t+1})}{\lambda(s^t)} \]

\[ = \beta \frac{X(s^{t+1})}{X(s^t)} \frac{1}{\xi(s^{t+1} | s^t)} \frac{u_c(C(s^{t+1}), L(s^{t+1}))}{P(s^{t+1})} \frac{P(s^t)}{u_c(C(s^t), L(s^t))} \]

The foreign country gives rise to an analogous equation:

\[ M^* (s^{t+1} | s^t) = \beta \frac{u_c(C^*(s^{t+1}), L^*(s^{t+1}))}{P^*(s^{t+1})} \frac{P^*(s^t)}{u_c(C^*(s^t), L^*(s^t))} \]

By equating the price of Arrow-Debreu state contingent security:

\[ = \frac{u_c(C^*(s^{t+1}), L^*(s^{t+1}))}{P^*(s^{t+1})} \frac{P^*(s^t)}{u_c(C^*(s^t), L^*(s^t))} \frac{X(s^{t+1})}{X(s^t)} \frac{1}{\xi(s^{t+1} | s^t)} \frac{u_c(C(s^{t+1}), L(s^{t+1}))}{P(s^{t+1})} \frac{P(s^t)}{u_c(C^*(s^t), L^*(s^t))} \]

Using the definition of the real exchange rate \( Q(s^{t+1} | s^t) = \frac{X(s^{t+1} | s^t)}{P(s^{t+1} | s^t)} \):

\[ \frac{Q(s^{t+1})}{Q(s^t)} = \xi(s^{t+1} | s^t) \frac{u_c(C^*(s^{t+1}), L^*(s^{t+1}))}{u_c(C^*(s^t), L^*(s^t))} \frac{u_c(C(s^t), L(s^t))}{u_c(C^*(s^{t+1}), L^*(s^{t+1}))} \]

Using the log-format for utilities and assuming separability between consumption and leisure:

\[ \frac{Q(s^{t+1})}{Q(s^t)} = \xi(s^{t+1} | s^t) \frac{C^*(s^t)}{C^*(s^{t+1})} \frac{C(s^{t+1})}{C(s^t)} \]

By computing it recursively, we have:

\[ \frac{Q(s^{t+1})}{Q(s^t)} \frac{Q(s^t)}{Q(s^{t-1})} = \left( \xi(s^{t+1}) \frac{C^*(s^t)}{C^*(s^{t+1})} \frac{C(s^{t+1})}{C(s^t)} \right) \left( \xi(s^t) \frac{C^*(s^{t-1})}{C^*(s^t)} \frac{C(s^t)}{C(s^{t-1})} \right) \]

38
By repeating until \( t = 0 \), we have:

\[
\frac{Q(s^{t+1})}{Q(s^0)} = \frac{C(s^{t+1})}{C^* (s^{t+1})} \prod_{j=0}^{t+1} \xi (s^j)
\]

where \( \Delta \) is the time-0 constant that we normalize to one if consumption of the countries are symmetric and \( Q(s^0) \) is therefore 1.

**Proof.** We will prove Lemma 2.

We have to find the coefficient of the regression:

\[
\beta = \frac{Cov(q_t, c_t - c^*_t)}{V(c_t - c^*_t)} < 0
\]

\[
\frac{Cov\left(c_t - c^*_t + \sum_{j=0}^t \xi, c_t - c^*_t\right)}{V(c_t - c^*_t)} < 0
\]

\[
\frac{Cov\left(\sum_{j=0}^t \xi_j, c_t - c^*_t\right)}{V(c_t - c^*_t)} < -1
\]

\[
\frac{Cov(\kappa, c_t - c^*_t)}{V(c_t - c^*_t)} < -1
\]

\[
\frac{Cov(\kappa, c_t - c^*_t) \sigma_k}{\sigma_{c-c^*}} < -1
\]

\[
\rho_{\kappa,c-c^*} < -\frac{\sigma_{c-c^*}}{\sigma_\kappa}
\]

Now we will consider the proof for the nominal exchange rate, lemma 3:

**Proof.** using the cash-in-advance constraint, we know that \( \mu(s^t) = P(s^t) C(s^t) \) and we can
substitute in the condition stated for the real exchange rate

\[
\frac{Q(s^{t+1})}{Q(s^t)} = \frac{C^*(s^t) C(s^{t+1})}{C^*(s^{t+1}) C(s^t)}
\]

\[
\frac{X(s^{t+1})P^*(s^{t+1})}{X(s^t)P^*(s^t)} = \frac{P^*(s^t) C^*(s^t)}{C(s^t)}
\]

\[
\frac{X(s^t)}{X(s^{t+1})} = \frac{\mu^*(s^t)}{\mu^*(s^{t+1})} \xi(s^t | s^t)
\]

\[
\frac{X(s^{t+1}|s^t)}{X(s^t)} = \frac{\mu(s^{t+1})}{\mu^*(s^{t+1})} \prod_{j=0}^{t+1} \xi(s^j)
\]

Based on this result, we can show the result on Equation 1.2.2

\[\lambda(s^t) X(s^t) \xi(s^{t+1}|s^t) M^*(s^{t+1}|s^t) = \beta \lambda(s^{t+1}) X(s^{t+1})\]
\[\lambda^*(s^t) M^*(s^{t+1}|s^t) = \beta \lambda^*(s^{t+1})\]

\[\beta \frac{\lambda_{t+1}}{\lambda_t} X_{t+1} \frac{1}{X_t} \xi(s^{t+1}|s^t) = \frac{\beta \lambda_{t+1}^*}{\lambda_t^*}\]

Taking expectation under first order approximation:

\[\frac{1}{R_t} \mathbb{E}_t \left( \frac{X_{t+1}}{X_t} \right) \frac{1}{\mathbb{E}_t(\xi_{t+1})} = \frac{1}{R_t^*}\]

\[R_t = \frac{R_t^*}{\mathbb{E}_t(\xi_{t+1})} \mathbb{E}_t \left( \frac{X_{t+1}}{X_t} \right)\]
Given this result, we can show Lemma 4:

**Proof.** We will use two main equations to derive the result:

\[ r_t - r_t^* = E_t (x_{t+1} - x_t - E_t (\xi_{t+1}) \]

\[ x_{t+1} - x_t = \Delta_{\mu\Delta} + \xi_{t+1} \]

By substituting, one into another, we have:

\[ r_t - r_t^* = E_t (\Delta_{\mu\Delta}) \]

But the UIP can be defined as:

\[ \beta = \frac{Cov (r_t - r_t^*, E_t (x_{t+1} - x_t))}{V (E_t x_{t+1} - x_t)} < 0 \]

\[ Cov (r_t - r_t^*, E_t (x_{t+1} - x_t)) < 0 \]

\[ Cov (E_t (\Delta_{\mu\Delta}), E_t (\Delta_{\mu\Delta}) + E_t \xi_{t+1}) < 0 \]

\[ V (E_t (\Delta_{\mu\Delta})) < -Cov (E_t \xi_{t+1}, E_t (\Delta_{\mu\Delta})) \]

\[ \sigma_{E_t(\Delta_{\mu\Delta})}^2 < -\rho_{E_t(\Delta_{\mu\Delta}), E_t(\Delta_{\mu\Delta})} \sigma_{E_t(\Delta_{\mu\Delta})} \sigma_{E_t(\Delta_{\mu\Delta})} \]

\[ \rho_{\Delta_{\mu\Delta}, E_t \xi_{t+1}} < -\frac{\sigma_{\Delta_{\mu\Delta}}}{\sigma_{E_t \xi_{t+1}}} \]

where \( \Delta_{\mu\Delta} = E_t \left( \mu_{t+1} - \mu_t^* \right) - \left( \mu_t - \mu_t^* \right) \)

\[ \square \]

### 1.7.2 Data Used

In order to get wedges, we use annual data from the World Development Indicators, from the World Bank. Consumption data is "Final consumption expenditure, etc. (annual % growth) - NE.CON.TETC.KD.ZG". On constructing the real exchange rate vs. USA,
we use official exchange rates (PA.NUS.FCRF Official exchange rate (LCU per US$, period average)), and inflation (FP.CPI.TOTL.ZG Inflation, consumer prices (annual %)).

Summary Statistics of wedge

See Figure 1.7.1.

1.7.3 Borrowing Constraints

Incomplete Markets

Differently from the complete markets case, in which it is through equating marginal utilities that one gets the results, the rationale for getting the UIP comes from convertibility. If there is an asset traded in euros and another in bonds, by non-arbitrage, UIP should hold if they are subject to the same constraints. We suppose, by simplicity, that the agent is borrow constrained only in the debt not denominated in his own currency.

We consider, therefore, a model in which there are two bonds and the foreign bond is subject to borrowing constraint.

The setup of the consumer side is given by:

\[
\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)
\]

\[
s.t.
\]

\[
P(s^t) C(s^t) + I(s^t) + B(s^{t+1}) + E(s^t) B^f(s^{t+1}) \leq w(s^t) l(s^t) + R(s^t) B(s^t) + r_{k,t} K_t + E(s^t) R^f(s^t) B^f(s^t)
\]

\[
K_{t+1} = (1 - \delta) K_t + I_t
\]

\[
E(s^{t+1}) B^f(s^{t+1}) \geq \circ(s^t)
\]

From an analogous setup for the foreign country, we can find the non-arbitrage condition (UIP)
Lemma 6. Under borrowing constraints, UIP is given by:

\[
R_t = R^*_t \mathbb{E}_t \left( \frac{E_{t+1}}{E_t} \right) \frac{1 - \zeta^*_t c^*_t p^*_t}{1 - \zeta_t c_t p_t}
\]

where \( \zeta_t \) is the Lagrange multiplier on the borrowing constraint and the interest rates are the ones for the non-constrained/partially convertible asset.

Proof. From the first order conditions of the agents, we have different arbitrages depending on which countries point of view:

\[
\lambda_t = \beta R_t \mathbb{E}_t [\lambda_{t+1}]
\]

\[
\frac{1}{R_t} = \beta \mathbb{E}_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right)
\]

The first order condition of the domestic agent for foreign bond is given by

\[
\frac{1 - \frac{\lambda_t}{X_t}}{R^*_t} = \beta \mathbb{E}_t \left( \frac{X_{t+1} \lambda_{t+1}}{\lambda_t X_t} \right)
\]

Therefore, under a first order approximation:

\[
\frac{1}{R_t} = \left( \frac{1 - \frac{\lambda_t}{X_t}}{R^*_t} \right) \frac{X_t}{X_{t+1}}
\]

Therefore:

\[
R_t = R^*_t \mathbb{E}_t \left( \frac{X_{t+1}}{X_t} \right) \left( 1 - \frac{\lambda_t}{X_t} \right)^{-1}
\]

Doing the same steps for the foreign country, we have:
\[ R_t = R_t^* \mathbb{E}_t \left( \frac{X_{t+1}}{X_t} \right) \left( 1 - \frac{\zeta_t^*}{\lambda_t^*} \right) \]

Now as only one of the borrowing constraint binds, the Lagrange multiplier of the other is equal to zero, so we can rewrite in short notation:

\[ R_t = R_t^* \mathbb{E}_t \left( \frac{E_{t+1}}{E_t} \right) \left( \frac{1 - \frac{\zeta_t^*}{\lambda_t^*}}{1 - \frac{\zeta_t}{\lambda_t}} \right) \]

1.7.4 Quantitative Model

The agent problem is standard and it follows from the derivations that we have done in Section 1.2. We should, however, state in more detail how the agent chooses the allocation of the consumption bundle.

Imports and Exports are chosen by each household as usual given the consumption aggregator:

\[ \max C_t \]

s.t.

\[ P_{f,t} C_{f,t} + P_{d,t} C_{d,t} = P_t C_t = Z_t \]

\[ C_t = \left[ \omega_c^{\frac{\rho_c}{1+\rho_c}} C_{f,t}^{\frac{1}{1+\rho_c}} + (1 - \omega_c)^{\frac{\rho_c}{1+\rho_c}} C_{f,t}^{\frac{1}{1+\rho_c}} \right]^{1+\rho_c} \]

Taking first order conditions:

\[ C_t^{\frac{\rho_c}{1+\rho_c}} \omega_c^{\frac{\rho_c}{1+\rho_c}} C_{d,t}^{\frac{\rho_c}{1+\rho_c}} = P_{d,t} \]

\[ C_t^{\frac{\rho_c}{1+\rho_c}} (1 - \omega_c)^{\frac{\rho_c}{1+\rho_c}} C_{f,t}^{\frac{\rho_c}{1+\rho_c}} = P_{f,t} \]
So we substitute back:

\[ C_{d,t} = 1 - \frac{\omega_c}{\omega_c} \frac{P_t}{P_t^{pc}} P_{t,1}^{-\frac{1+\rho_c}{\rho_c}} \]

\[ C_{d,t} = \frac{1 - \omega_c}{\omega_c} \left( \frac{P_{d,t}}{P_t} \right)^{-\frac{1+\rho_c}{\rho_c}} C_t \]

\[ C_{f,t} = \frac{\omega_c}{1 - \omega_c} \left( \frac{P_{f,t}}{P_t} \right)^{-\frac{1+\rho_c}{\rho_c}} C_t \]

We have \( P_d, P_f, P, P^*, C_d, C^*_d, C_f, C^*_f \) and we have seven equations (two first order conditions for each country, the definitions of the aggregator of consumption and the definition of the real exchange rate \( q = E \frac{P_f}{P_t} \)). Therefore, we opt for normalizing \( P_d = 1 \).

**Firms**

\[
\max A_t \left[ \nu (K_t)^{1-\theta} + (L_t)^{1-\theta} \right]^{\frac{1}{1-\theta}} - r_k K_t - w_t L_t
\]

The first order conditions of the firm give:

\[ w_t = \left( \frac{Y_t}{L_t} \right)^{\theta} \]

\[ r_k = \nu \left( \frac{Y_t}{K_t} \right)^{\theta} \]

**Equilibrium**

An equilibrium in this economy is defined by a set of allocations \( c(s^t), c^*(s^t), l(s^t), l^*(s^t), B_d(s^t), B^*_d(s^t), B_f(s^t), B^*_f(s^t) \), with associated consumption baskets with prices \( w(s^t), w^*(s^t), P(s^t), P^*(s^t), P_d(s^t), P_f(s^t) \) and the exogenous variables \( (A^1(s^t), A^2(s^t), v(s^t)) \) such that:

1. Consumers maximize their consumption subject to equilibrium prices and maximize the allocation within a consumption aggregator
2. Firms maximize profits subject to prices.

3. Asset trade market clears:

\[ B_d (s^t) + B_d^* (s^t) = 0 \]

\[ B_f (s^t) + B_f^* (s^t) = 0 \]

4. Resource constraint

**Parametrization**

**UIP**

We consider the uncovered interest parity as:

\[ \frac{1}{R_t} \frac{E_t X_{t+1}}{X_t} \frac{1}{1+\delta t} = \frac{1}{R_t^e - \frac{\sigma_t}{2} X_t \frac{\sigma_t}{C^e}}. \]

**Finding the capital share:**

To simplify, consider the Lagrangian defined by

\[ L = \max \sum \beta^t \left[ \log (c_t) - \frac{\psi_0 l^{1+\psi}}{1+\psi} - \lambda \left[ c_t + k_{t+1} - A \left[ \nu k_t^{1-\theta} + l_t^{1-\theta} \right]^{-\theta} - (1 - \delta) k_t \right] \right] \]

The first order conditions are then defined by:

\[ \psi_0 l^{\psi} = \left( \frac{Y}{l} \right)^\theta \frac{1}{c} \]

\[ c + k = A \left[ \nu k_t^{1-\theta} + l_t^{1-\theta} \right]^{-\theta} + (1 - \delta) k \]

\[ \beta \left[ \nu \left( \frac{Y}{k} \right)^\theta + 1 - \delta \right] = 1 \]

Therefore, \( \nu \) is found as:

\[ \nu = \left[ \frac{1}{\beta} - (1 - \delta) \right] \left( \frac{k}{Y} \right)^\theta \]
Firstly, we find the saving rate:

\[
\frac{c}{y} + \delta \frac{k}{y} = 1
\]

\[
\frac{c}{y} = 1 - \delta \frac{k}{y}
\]

\[
\frac{s}{y} = \delta \frac{k}{y}
\]

\[
\frac{s}{y} = \delta \left( \frac{\nu}{\frac{1}{\beta} - (1 - \delta)} \right)^\frac{1}{\theta}
\]

Finally, we have to find \( \psi_0 \).

\[
\psi_0 (1 - l)^{-\psi} = \left( \frac{Y}{l} \right)^{\theta} \frac{1}{c}
\]

\[
\psi_0 (1 - l)^{-\psi} = \frac{Y}{c l} \left[ \left( \frac{Y}{l} \right)^{\theta - 1} \right]
\]

\[
\psi_0 = (1 - l)^{\psi} \frac{Y}{c l} \left[ \left( \frac{Y}{l} \right)^{\theta - 1} \right]
\]

\[
\psi_0 = (1 - l)^{\psi} \frac{Y}{c l} \left[ \left( \frac{l}{Y} \right)^{1 - \theta} \right]
\]

But one should remember that \( Y = A \left[ \nu k^{1 - \theta} + l^{1 - \theta} \right]^{\frac{1}{1 - \theta}} \), therefore:

\[
\psi_0 = (1 - l)^{\psi} \frac{Y}{c l} \left[ 1 - \nu \left( \frac{k}{Y} \right)^{1 - \theta} \right]
\]

\[
\psi_0 = (1 - l)^{\psi} \frac{Y}{c l} \left[ 1 - \nu \left( \frac{1}{\delta Y} \right) \left( \frac{s}{Y} \right)^{1 - \theta} \right]
\]
For the home good bias, one can see that:

\[
\left( \frac{\partial C_t}{\partial C_{d,t}} \right) = \frac{1}{1 + \rho_c} C_t^{\rho_c} \omega_c^{\rho_c} \left[ C_{d,t}^{\rho_c} - \rho_c \right]
\]

\[
\left( \frac{\partial C_t}{\partial C_{f,t}} \right) = \frac{1}{1 + \rho_c} C_t^{\rho_c} (1 - \omega_c) \left[ C_{f,t}^{\rho_c} - \rho_c \right]
\]

Therefore:

\[
\frac{\omega_c}{1 - \omega_c} = \frac{C_d}{C_f}
\]

1.7.5 Empirical Evaluation of Noise Models

Data

The frequency of the data studied is annual, since we need to have a long panel with a sizable number of countries. We perform our analysis using Penn World Tables (PWT). As it will become clearer, we consider both the PWT from National Accounts, as well as the PPP adjusted.

The second dataset used comes from the work of Lane and Milesi-Ferretti (2007). Such dataset allows us to follow net foreign assets and liabilities of countries across time.

The final part on the construction of the datasets is related to how to classify exchange rate regimes. We consider four different classifications.

The first one is provided by the International Monetary Fund (IMF) and it is a "de jure" index. Such index is based on the regime that the country declares to be running. If the country does not follow what it declares, such index can be very misleading. We decide to use it because it is one of the most used in the literature and it also allows us to understand a bit better if risk-sharing is related to de jure or de facto pegs. A second remark is that
this index is not binary (pegged - non-pegged), so we transform it in the following way: an exchange-rate is pegged if it is classified as a peg, crawling peg or managed floating. If it is freely floating, we consider it as non-pegged.

The second classification comes from the work of Reinhart and Rogoff (2004) and further updated in Reinhart et al. (2011). The key difference on this index is the use of monthly data on market-determined parallel exchange rates going back to 1946 for 153 countries. Furthermore, they show that their index is substantially different from the IMF official classification. A final remark is that we transform it into binary following the same method discussed for the IMF index.

The third classification comes from the work of Shambaugh (2004), updated in Klein and Shambaugh (2006). This index classifies countries into peg and non-peggs based solely on the volatility of the exchange rate (if the exchange rate stayed within +/-2 percent bands against the base currency), being once more a de facto classification, but already in a binary scale.

The final classification considered here comes from Levy-Yeyati and Sturzenegger (2003) and Levy-Yeyati and Sturzenegger (2005). The authors perform a cluster analysis technique that groups countries according to exchange rate volatility, volatility of the exchange rate changes and volatility of reserves. Moreover, we transform into a binary variable by using the minimum and the maximum when using three clusters.

**Robustness checks on the Backus Smith**

**PPP Check**

The first robustness check that we consider is to look at the Backus-Smith condition for PPP adjusted consumption and the respective price levels when computing the real exchange rate. As one can see from Figure (1.7.2), the qualitative results remain.

**Hand-to-mouth agents**

Devereux and Hnatkovska (2011) provide an estimation of the Backus-Smith condition
when there are hand-to-mouth agents. In that case, one has to control for output when computing such correlation. Once more, as Figure (1.7.3) suggests, the qualitative results remain.

**Household consumption**

The last robustness check that we compute is on the use of the consumption variable. In the main text, we have used consumption defined as sum of household consumption expenditure and final consumption expenditure of Non-profit institutions serving households (at constant prices), plus sum of government less collective consumption of government for public good type activities. In here, we use consumption being purely household consumption expenditure plus Non-profit institutions serving household.

**Coefficient and standard deviation of exchange rate**

In this specification, we further explore the relationship between exchange rate and relative consumption as being dependent on the exchange rate. In order to do so, we do not consider a binary variable (pegged vs non-pegged) and we interact with the standard deviation of the real exchange rate

\[
(\Delta c_{it} - \Delta c_{us,t}) = \alpha + \beta \Delta rer_{i,us}^t + \beta_2 \Delta rer_{i,us}^t \cdot \sigma_{rer_i,us} + \varepsilon_{it} 
\]

Differently from before, now we have

\[
\frac{\partial (\Delta c_{it} - \Delta c_{us,t})}{\partial \Delta rer_{i,us}^t} = \beta + \beta_2 \sigma_{rer_i,us} 
\]

Since the partial derivative depends on the standard deviation of the exchange rate in the period, we provide a graph on how it changes as the standard deviation increases in Figure (1.7.4).

---

\(^{35}\)The qualitative results are unaltered if we consider the standard deviation of the nominal exchange rate.
Robustness checks on NFA

On the use of reserves

The first robustness check we do on the international financial integration section is related to the use of official reserves. As we know that in pegged regimes the government has to intervene more often, reserves are higher which could partially explain the difference in the international financial integration. However, in order to address this issue, we construct the following index:

\[
IFI^1 = \frac{Total \ Foreign \ Assets - FX \ Reserves + Total \ Foreign \ Liabilities}{GDP}
\]

FX Reserves also comes from the work of Lane and Milesi-Ferretti (2007) and it does not include gold on it. As we see, the qualitative effects are unaltered and the quantitative effects are not much altered either.

Therefore, as we can see from Figure (1.7.5), pegged regimes are associated with higher financial integration at any point in time and it is not due to higher official reserves.

Trimming 5%

In order to address the criticism that these results could be due to outliers, we trim the dataset excluding the lowest 5% and higher 5% real exchange rate depreciations from the whole sample. Figure (1.7.6) points to the fact that it is not the presence of outliers, or highly volatile exchange-rate countries that drive the result.

Portfolio Shares

Once we have found that there is higher financial integration in pegged regimes, we compute the portfolio shares to evaluate if there is a change within the asset and liabilities allocation from pegged and floating regimes.

Since the definition of the regimes depend on the index we are using, we provide the graphs for each of them.

However, one pattern arises across exchange-rate regime definitions and across time:
floating regimes usually are associated with a smaller fraction of the assets used on reserves and a higher foreign direct investment share. Figures (1.7.7), (1.7.8), (1.7.9) and (1.7.10) provide these results (in each, "0" denotes floating and "1" denotes pegged).

Robustness checks on Feldstein-Horioka puzzles

This section discusses robustness checks for the Feldstein-Horioka puzzles, covering mostly national accounts vs. PPP and the use of a cross-section vs. panel data and the issues on weighting the observations.

PPP measures

Even though the Feldstein-Horioka is usually stated as an investment-saving share in domestic currency, we show in Figure (1.7.11) that the result of a stronger relationship between savings and investment on floating exchange rate regime remains even if one uses PPP.

Weighting and Shares

Weighted cross-section

Following the literature, we have collapsed the panel into a cross-section. However, when collapsing, one has to decide how to take the time-series averages. In the main text, we have used the pure average. In Figure (1.7.12) we provide the results using a weighted sample, where the weights come from the GDP in dollars of the given country.

As can be depicted, the results do not change substantially if we use a weighted or a non-weighted sample.

Different construction for shares

Use of different share on 1.7.13:

Panel data estimation on 1.7.14:
## Figure 1.7.1: Statistics of Wedge

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**Total** 989 -0.4 12.88 -50.1 174.43 650 1.81 10.75 -30.03 80.14
**Figure 1.7.2: Feldstein-Horioka puzzle adjusted for Exchange-Rate Regime**

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Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1
Figure 1.7.3: Feldstein-Horioka puzzle adjusted for Exchange-Rate Regime

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Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Figure 1.7.4: Feldstein-Horioka puzzle adjusted for Exchange-Rate Regime
Figure 1.7.5: International Financial Integration (without official reserves)

Figure 1.7.6: International Financial Integration (trimming 5%)

Figure 1.7.7: Portfolio Shares for IMF index
Figure 1.7.8: Portfolio Shares for Reinhart et al. (2011)

Figure 1.7.9: Portfolio Shares for Shambaugh (2004)

Figure 1.7.10: Portfolio Shares for Levy-Yeyati and Sturzenegger (2003)
Figure 1.7.11: Feldstein-Horioka graphs with PPP

Figure 1.7.12: Feldstein-Horioka puzzle with GDP weighted cross-section averages

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Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1
### Figure 1.7.13: Feldstein-Horioka from current price shares

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Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

### Figure 1.7.14: Panel-data estimation of Feldstein-Horioka

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<tr>
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<td>-0.156***</td>
<td></td>
<td></td>
</tr>
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Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Chapter 2

Optimal Monetary Responses to Asset Price Levels and Fluctuations: The Ramsey Problem and A Primal Approach (with Wei Cui)

2.1 Introduction

The recent financial crisis has shown that asset market liquidity fluctuations can have huge impacts on the real economy. As financial frictions widened, the economy plunged into a recession from 2007Q4 to 2009Q2. However, if one were to consider that one of the driving forces of the crisis was indeed the liquidity aspect transmitted to the real economy, it is surprising that there is no clear understanding on how should a central bank react or even whether it should or should not react, both in the academic literature and in the policy-making arena.
Concerns about the interest rate and the degree of asset market liquidity affect the portfolio allocation of low-return-liquid and high-return-illiquid assets (which by itself affect investment and the real economy). One way of seeing this is by plotting liquid vs. illiquid compositions in portfolio as in Figure 2.1.1\(^1\). During the last recession, the liquid share has increased, even though the equity price, as approximated by S&P 500, was already bouncing back in 2009. This change of liquidity ratio suggests large rebalancing of portfolio in all sectors during the past recession and periods after that. The question we aim to tackle, therefore, is how should optimal monetary policy behave in a liquidity constrained economy.

Our approach emphasizes monetary policy over fiscal policy because, firstly, it has more discretion in the short run and, moreover, we can provide a framework for the recent debate

\(^1\)Liquidity ratio, S&P 500 Index and 3-Months Treasury Bills rate over time. Liquidity ratio is defined as the total liquid assets (check, deposit, tradable receivable and T-Bills while for financial sector with one more source, net over-night interbank lending) over total assets (for financial sectors total assets adjusted by the required reserve in central bank). NNB stands for non-farm and non-corporate business. NCB stands for non-farm and corporate business. FB stands for financial business excluding central bank. Source: 1952Q1-2011Q3 Flow of Funds Table, F102, F103, B102 and B103, Federal Reserve Z1 statistical release. S&P 500 index is obtained from Yahoo Finance and 3-months T-Bills rate is from Federal Reserve Bank at St.Louis. The grey shaded region depicts the NBER recession period and the yellow region depicts the period after 2007-2009 recession until 2011Q3.
on possible types of monetary instruments. The first one uses only liquid assets, usually called the conventional method. Examples of such are steadily increasing or decreasing the money stock, or changing the interest rate of liquid assets. The second one is motivated by the FED’s large purchases of illiquid assets, which we exemplify through open market operations on private equity and label them as unconventional policies\(^2\). The main distinction between them is the introduction of the central bank’s holding of partially liquid assets.

Equity financing and resale constraints (liquidity frictions), as in Kiyotaki and Moore (2011), generate heterogeneity in the economy and create the need for intrinsically valueless liquid assets such as money. The equity financing friction says that an investing entrepreneur can only issue new equity up to a fraction of his investment. At the same time, liquidity frictions set a bound on the fraction of equity that an agent can sell. In financing new investment, the equity issuance constraint limits the outside financing resources while the resale frictions limits the internal financing. Therefore, agents will hold intrinsically valueless fully liquid assets as extra internal resources for future investment. The financial frictions, in a general way, reduce the amount of resources for investment that should be transferred from non-productive agents to productive ones. Not surprisingly, this channel reduces not only the output produced, but also limits consumption smoothing. The most important difference from the standard Kiyotaki and Moore (2011) is that we embed conventional and unconventional policy instruments in a way that we are able to reach optimal policy solution. Monetary policy, therefore, is designed to influence on the return on liquid assets. At the same time, we are able to compare conventional and unconventional policies.

Liquid assets in the economy help lubricate funds transfer in the economy. However, by holding liquid assets, the agent does not internalize its own effect on lowering the return of it. In equilibrium, there is too much holding of liquid assets. Importantly, we ask whether, given the liquidity friction in a competitive economy, a constrained planner (who also respects the liquidity friction) can improve the social welfare. Nevertheless, we are not dealing with policy

\(^2\)We will abuse of notation calling the privately owned equity "private equity", but with a different meaning than the usual jargon.
that can entirely eliminate liquidity friction.

We depart from early monetary policy literature such as Woodford (2003) by focusing on the Ramsey problem of optimal monetary policy and use the primal approach, which we can fully solve analytically\(^3\). We find that the "implementability" condition, that summarizes all the decentralized market conditions, equals the net-worth difference of different types of agents to the total gain if non-resalable capital become resalable. The implementability condition suggests that, as agents switch back and forth from being productive-type (with investment opportunity) to unproductive-type (without investment opportunity), consumption-smoothing will be harder the larger are the financial frictions in the economy. It is worthwhile to consider two extreme cases in which fiat liquid assets disappear:

1. If there is no equity issuance friction, idiosyncratic investment opportunity risk is fully insured and equity resale friction does not matter, leading to zero net-worth difference and perfect consumption smoothing;

2. If there is no equity resale friction (the equity is fully liquid), the value gained by transforming non-resaleable into resaleable is zero since all equity is resaleable. Savings through buying equity will be enough to finance new investment. Again, we have zero net-worth difference and perfect consumption smoothing.

Finally, the implementability constraint also shows that unconventional policies weakly dominate conventional ones theoretically, since the latter can be shown to be a subset of the former.

\(^3\)Negro et al. (2011) and Gertler and Karadi (2011) are the closest to ours as the first one discusses liquidity frictions and the second discusses conventional vs. unconventional policies.
To our best knowledge, we are the first to give an answer to what is the optimal monetary policy in the context of liquidity frictions. Standard New-Keynesian optimal policy uses the second order approximation to the objective function of a representative household, usually finding a balance between output gap and inflation gap\(^4\). Such strategy is not particularly attractive in the context of liquidity friction, where heterogeneous agents and uninsured risks become the central theme. With the help of the implementability condition, heterogeneity can be summarized in one constraint for the policy maker.

We calibrate and structurally estimate the model, using liquid assets data from U.S. flow of funds jointly with aggregate investment from 1991 to 2007. Especially for the shock to resale constraint, we want to estimate the size of the shock such that it will induce private sectors to rebalance the asset portfolio as in the data. Such treatment of data is novel and is directly linked to the question posed.

Our result shows that the monetary authority should “deflate” the economy in steady state, since agents have a propensity to over-save in liquid assets. Intuitively, one way of reducing such problem is by shortening constantly the supply of liquid assets, or, equivalently, an annual 4% real interest rate on liquid assets\(^5\). In the economy in which liquid assets earn higher rate of return due to policy, those who have investment opportunities and liquid assets will have a better internal financing through liquid assets. More wealth is then transferred from agents with funds but no investment opportunity, to those with investment opportunity but not enough funds. The welfare gains compared to no policy amount to almost .4% increase on permanent total consumption. Moreover, the optimal level of interest rate paying is increasing in the liquidity frictions.

---

\(^4\)As we do not follow the strategy of approximating the objective function, we also do not have welfare ranking problems, since in principle one can consider all higher order terms.

\(^5\)Arguing that optimal policy is deflation abstracts from other margins that we do not consider, such as price stickiness. A more correct interpretation is that liquidity margins suggest an increase on the liquid asset. We do not incorporate sticky price consideration and the calculation of deflation could be thought of average liquid asset return after inflation adjustment. The option for doing this is exactly to highlight the monetary policy under flexible prices, which is rarely discussed.
Even though we have a somewhat similar result to "Friedman rule", the reason behind it is very different. The key reason is the propensity to over-save, instead of the usual opportunity cost of holding money due to transaction needs. Our purpose, however, is not to explain data-observed inflation targets (which the New-Keynesian literature can do using price-stickiness) but how financial frictions alter the optimal level of real interest rate.

Finally, we examine various shocks including productivity shocks and liquidity shocks that lead to a harder resaleability of the illiquid assets. For an unexpected adverse liquidity shock, the policy should aim at help financing investment through increasing the interest rate on liquid assets. Importantly, even though theoretically we prove that the unconventional monetary policy weakly dominates conventional one, quantitatively it is negligibly.

**Related Literature** The literature on financial frictions is vast, spanning mostly borrowing constraints and, more recently, liquidity frictions. Since we are interested in optimal policy with liquidity problems, we build upon the model of Kiyotaki and Moore (2011), as we view it as an otherwise standard business cycle model in which financial frictions are important. The most important difference is that we embed conventional and unconventional policy instruments in a way that we are able to reach optimal policy solution. Monetary policy, therefore, is designed to influence on the return on liquid assets. At the same time, we are able to compare conventional and unconventional policies.

The most related literature to our paper is the one that merges monetary policy and financial frictions. On one hand, some have investigated how "unconventional monetary policies", i.e., that change the central bank's balance sheet, could be used and rationalized

---

6As usual in models with only adverse unexpected liquidity shocks (Kiyotaki and Moore (2011)), flight to liquidity increases the net-worth of agents who hold liquid assets and leads to a bigger change in the supply of illiquid assets. The two effects increase illiquid asset price and increase total consumption, which we do not observe in reality. Thus, we span all the possibilities by examining shocks that provide higher asset prices (productivity shocks), roughly constant asset prices (combination of liquidity and productivity shocks) and lower asset prices (future expected liquidity shocks with current productivity shock).

7On borrowing constraints, the literature is very vast, but the seminal work of Kiyotaki and Moore (1997) and the recent survey of Brunnermeier et al. (2012) are good examples of the broad literature that exists.
Another strand has evaluated financial frictions in a New-Keynesian model with price stickiness, as discussed in Woodford (2003) and, more specifically, in Christiano et al. (2007). Both strands, however, are silent in the policy optimality in an economy with liquidity frictions.

We depart from the literature, first by discussing optimal policy with financial frictions, but also by bringing the Ramsey approach to this literature. Monetary policy in our setup is non-supernatural due to distributive effects, and it is not due to the price stickiness usually assumed.

There is a recent literature that studies pecuniary externality, in which the competitive market is neither efficient, nor constrained efficient. By introducing prices in the budget constraint, the agent does not take into account her own effect on prices and generate an externality. The possibility of a future binding resale constraint lead agents to oversave on the liquid assets. Work by Bianchi (2009), Bianchi (2010), Korinek (2009) and Lorenzoni (2008) looking at how far an economy with financial friction is from the first best are examples on the externality caused by borrowing constraints. We depart from this strand by restricting attention to competitive allocations in a constrained planner’s economy with monetary policy instruments that respect competitive equilibrium outcomes.

2.2 A Canonical Model of Financial Frictions

2.2.1 Set-up

In this section we consider a variant of Kiyotaki and Moore (2011) in which we introduce an inelastic supply of labor and monetary policy. Conventional policies are exemplified by a helicopter drop or drain policy, but the results are entirely equivalent if we were to think

\footnote{Following the jargon, \underline{neutral} means the level of money stock does not matter and \underline{super-neutral} means that growth of money does not matter either.}
about interest rate management on liquid assets if money was thought as a very general liquid asset such as the T-Bill. For unconventional policies, we consider open market operations on purchasing private equity\textsuperscript{9}. We try to stick as much as possible to Kiyotaki and Moore (2011) model in order to evaluate the gains from optimal policy in an otherwise standard model, but some changes are needed to accommodate such policies. Therefore, we’ll be brief in explaining the set-up used.

Time is discrete and infinite. The economy has two types of agents, entrepreneurs with measure 1 and household with measure $L$. Each agent has expected utility of

$$E_t \sum_{s=0}^{\infty} \beta^s \log (c_{t+s})$$

at time $t$. Only entrepreneurs have access to a constant-returns-to-scale technology for producing output from capital and labor. An entrepreneur holding $k_t$ capital at the beginning of period $t$ can employ $l_t$ in a competitive labor market to produce

$$y_t = A_t (k_t)^{\alpha} (l_t)^{1-\alpha}.$$

To produce output, entrepreneurs have to be involved in the production process so that their participation is necessary. Production is completed within period $t$, during which capital depreciates to be $(1-\delta)k_t$, where $0 < \delta < 1$. $A_t = e^{z_t}$ is common to all entrepreneurs and $z_t$ follows

$$z_t = \rho z_{t-1} + \varepsilon_t^z.$$

Entrepreneurs hire each unit of labor at a competitive real wage $w_t$. Due to constant return

\textsuperscript{9}A more detailed explanation of such interpretations is given in Section 4.
to scale technology, profits on capital are linear in individual entrepreneur’s capital\(^{10}\)

\[
y_t - w_t l_t = r_t k_t
\]

where \( r_t \) is the equilibrium profits on capital. The household side is assumed to be supplying \( L \) unit of inelastic labor for simplicity\(^{11}\). After introducing labor, \( r_t \) can now be determined by clearing labor markets. For each entrepreneur with \( k_t \), their decision on hiring labor is

\[
(1 - \alpha) A_t (k_t)^\alpha l_t^{1-\alpha} = w_t, \rightarrow l_t = \left[ \frac{(1 - \alpha) A_t}{w_t} \right]^{\frac{1}{\alpha}} k_t.
\]

Therefore, if aggregate capital stock is \( K_t \), the labor demand is \( \left[ \frac{(1 - \alpha) A_t}{w_t} \right]^{\frac{1}{\alpha}} K_t \). Wage is then \( w_t = (1 - \alpha) A_t (K_t/L)^\alpha \) and the gross profits are \( A_t k_t^{\alpha(1-\alpha)} - w_t l_t = \alpha A_t (K_t/L)^{\alpha-1} k_t \). Thus, profits on capital are

\[
r_t = \alpha A_t \left( \frac{K_t}{L} \right)^{\alpha-1}.
\]  

(2.2.1)

The arrival of an investment opportunity, i.e., the chance to produce new capital from general output, is independently distributed across entrepreneurs (but not across the household) and through time (we assume a constant fraction at every point in time). Investment completed in period \( t \) will be available as capital in period \( t + 1 \):

\[
k_{t+1} = (1 - \delta) k_t + i_t
\]

We assume there is no insurance market against having an investment opportunity, so that the market is incomplete. In order to finance the investment opportunity, an entrepreneur can issue an equity claim to the future output from the investment, but due to the friction only

\(^{10}\)The return on individual capital is linear in their capital stock level, but decreasing in aggregate capital stock level.

\(^{11}\)If we specify them to have the same discount rate \( \beta \) in preference and allow them to buy equity for savings, they will not do so because the equilibrium rate of return will be less than \( \beta \). The use of \( L \) units for workers is a normalization itself, since we keep the entrepreneurs as being 1 unit throughout.
\( \theta \) fraction of the investment can be issued. Such friction can be motivated in a production process in which entrepreneurs have to participate in the production to produce full amount of future output and outsiders may just be able to get \( 1 - \theta \) of the future output. The other friction that we introduce is the equity resale friction; entrepreneurs have difficulties in selling their capital, as they can sell only up to \( \phi \) fraction of their own equity backed by physical capital each period. Resale friction is common especially when information asymmetry is severe.

Table 2.1: Balance Sheet of a Typical Entrepreneur

<table>
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<th>Liquid Asset (Money)</th>
<th>Own Equity Issued</th>
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<tr>
<td>Holding of Others’ Equity</td>
<td>Own Capital Stock</td>
</tr>
<tr>
<td></td>
<td>Net Worth</td>
</tr>
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</table>

An entrepreneur has liquid assets, others’ equity, and un-mortgaged capital stock in his balance sheet as in Table 2.1. For simplicity, both own equity un-mortgaged initially and outside equity can be sold at most \( \phi \) fraction and depreciate at the same rate \( \delta \). Therefore, own equity and outside equity are perfect substitutes. Entrepreneurs can remortgage their previously un-mortgaged capital stock up to \( \phi_t \) fraction of that. There is aggregate uncertainty about resaleability \( \phi_t \) which fluctuates over time to capture the changing of liquidity frictions. Therefore, the exogenous shocks in this economy are summarized by \((z_t, \phi_t)\).

Let \( n_t \) be the equity and let \( m_t \) be money held by an individual entrepreneur at the start of period \( t \). The above discussion can be summarized as

\[
\begin{align*}
n_{t+1} & \geq (1 - \theta) i_t + (1 - \phi_t) (1 - \delta) n_t \quad (2.2.2) \\
m_{t+1} & \geq 0 \quad (2.2.3)
\end{align*}
\]

The first constraint summarizes the amount of equity held in the next period. The minimum equity held would be the sum of un-mortgaged investment and equity that cannot be resold. The second constraint is just a non-negativity constraint on liquid assets (money). Private
agents cannot issue very liquid assets like T-bills. Commercial papers, even some that are very liquid, are still less liquid than assets issued by the government, which are backed up by taxes and government enforcement power. Therefore, any debt that is issued by firms and commercial banks should be thought of as the non-fully resalable assets in the model. The return on private assets in the model thus should be regarded as average return from equity and bonds in reality.

Now, we introduce conventional policy first and leave the unconventional policy in the next subsection. Let $q_t$ be the price of equity and let $p_t$ be the price of liquid assets, in terms of consumption goods\textsuperscript{12}. From now on, we use liquid assets and money interchangeably since money is a case of liquid assets. The flow of funds constraint for an entrepreneur at time $t$ is then given by

$$c_t + i_t + q_t (n_{t+1} - i_t - (1 - \delta) n_t) + p_t (m_{t+1} - m_t - \tilde{M}_t) = r_t n_t$$

where $\tilde{M}_t$ is the new money supply from government at time $t$. We model the conventional monetary policy as a helicopter drop, but, since what matters is the return of money, one could think of changing the return of liquid assets with equivalent results. The individual entrepreneurs take the new increased supply as given and think that they will not affect the equilibrium. If monetary authorities do not react at all, $\tilde{M}_t$ will always be 0. Importantly, we do not restrict the policy to be helicopter drop, i.e., $\tilde{M}_t \geq 0$. The policy can also be a money drain, i.e., $\tilde{M}_t < 0$. In that case, the policy is equivalent to a taxation on all entrepreneurs with $p_t \tilde{M}_t$ and the government use the proceeds to pay interest rate on liquid assets since the policy will change the rate of return on liquid assets.

\subsection*{2.2.2 Recursive Equilibrium}

We want to focus on an economy with valued liquid assets or money. Once money is

\textsuperscript{12}The reason to denote consumption goods as the baseline measure is because it is convenient to think about rate of return on liquid assets.
valued, it is used as an alternative source for savings since equity financing is insufficient due to the resaleability friction. Therefore, productive entrepreneurs will sell up to \( \phi_t \) fraction of equity and their constraints are all binding\(^{13}\). To reach this interesting economy equilibrium, we assume the following, as in Kiyotaki and Moore (2011)\(^{14}\)

\[
\text{Assumption} : \delta \theta + \pi (1 - \delta) \phi < (\beta - 1 + \delta) (1 - \pi).
\]

Entrepreneurs with investment opportunities, under the above assumption, will borrow to the limit so that constraint (2.2.2) will bind and the flow of funds becomes

\[
c^i_t + [1 - \theta q_t] i_t = [r_t + q_t \phi_t (1 - \delta)] n_t + p_t \left( m_t + \bar{M}_t \right).
\]

Using (2.2.2) and (2.2.3), the investing entrepreneur’s consumption is \( 1 - \beta \) fraction of the net-worth, as we have log-utility. Therefore

\[
c^i_t = (1 - \beta) \left\{ r_t n^i_t + [\phi_t q_t + (1 - \phi_t) q^R_t] (1 - \delta) n^i_t + p_t \left( m^i_t + \bar{M}_t \right) \right\}, \tag{2.2.4}
\]

\(^{13}\)We will assume that the optimal policy is also respecting the binding constraint, since we are looking only into policies that can be decentralized into a competitive equilibrium market as such.

\(^{14}\)To understand the assumption, suppose the assumption hold and the steady state capital is \( K \). Note that the following is impossible,

\[
[\delta \theta + \pi (1 - \delta) \phi] K > \delta (1 - \pi) K.
\]

To see this, the right hand side is the saving of non-investing entrepreneurs (with populations \( 1 - \pi \)) in steady state; the left hand side is the sum of new equity issued (\( \delta \theta K \), which is the investment to compensate depreciation) and existing equity sold (\( \pi (1 - \delta) \phi K \)). Then the inequality says that investing entrepreneurs can transfer all the savings from non-investing entrepreneurs, which is not possible by the assumption. Thus, the first best outcome cannot be achieved by individual savings.
where \( q_t^R = \frac{1-\theta_q}{1-\theta} < 1 \) as \( q_t > 1 \). Investment is thus

\[
i_t = \frac{[r_t + q_t \phi_t (1 - \delta)] n_t^i + p_t \left( m_t^i + \hat{M}_t \right) - c_t^i}{1 - \theta q_t}.\]  

For entrepreneurs without investment opportunity

\[
c_t^s + q_t n_{t+1}^s + p_t m_{t+1}^s = r_t n_t^s + q_t (1 - \delta) n_t^s + p_t \left( m_t^s + \hat{M}_t \right),\]

where the consumption can be solved as

\[
c_t^s = (1 - \beta) \left\{ r_t n_t^s + q_t (1 - \delta) n_t^s + p_t \left( m_t^s + \hat{M}_t \right) \right\}.
\]

Meanwhile, these entrepreneurs decide on the portfolio of money and equity. A typical non-investing entrepreneur will be indifferent between money and equity. Therefore, from first order condition, we know that

\[
\begin{align*}
u' \left(c_t^s\right) &= \beta E_t \left\{ \frac{p_{t+1}}{p_t} \left[ (1 - \pi) u' \left(c_{t+1}^{ss}\right) + \pi u' \left(c_{t+1}^{si}\right) \right] \right\} \\
&= \beta (1 - \pi) E_t \left\{ \frac{r_{t+1} + (1 - \delta) q_{t+1}}{q_t} u' \left(c_{t+1}^{ss}\right) \right\} + \beta \pi E_t \left\{ \frac{r_{t+1} + (1 - \delta) \phi_{t+1} q_{t+1} + (1 - \delta) (1 - \phi_{t+1}) q_{t+1}^R}{q_t} u' \left(c_{t+1}^{si}\right) \right\}
\end{align*}
\]

where \( c_{t+1}^{si} \) and \( c_{t+1}^{ss} \) measures the consumption at date \( t + 1 \) without government transfers and subsidies.

We can do aggregation in the economy easily due to the linearity in equity and liquid assets in these equations. But before going into the aggregation, it is appropriate now to introduce another government instrument, the purchasing and selling of private equity.

More recently, the central banks have implemented a new set of policies in which they buy private equity with partial liquidity, such as mortgage backed securities. We consider,
therefore, how open market operations should be used in such context. The coined term "unconventional" for open market operation is due to the fact that the asset that the FED is holding has partial resaleability. Furthermore, it pumps the liquid asset in the economy, which could be thought as money or T-Bills, to inject liquidity in the system. There are possibly indirect instruments for targeted purchases, but we will discuss the direct one for simplicity and also since it had been what the Fed had actually done the most. We, therefore, introduce another instrument, which is $N_t^g$ denoting the equity that can be purchased from private sector, as a "quantity" choice variable of the social planner. When the economy is endowed with $K_t - N_t^g$ and $M_t$ at period $t$, then $\pi (K_t - N_t^g)$ capital and $\pi M_t$ money are in the hands of investing entrepreneurs.

Aggregate investment $I_t$ can be derived from (2.2.5):

\[
(1 - \theta q_t) I_t = [r_t + q_t\phi_t (1 - \delta)] \pi (K_t - N_t^g) + p_t \pi \left(M_t + \tilde{M}_t\right) - C^i_t.
\]

Good's market clearing gives (subtracting labor income and labor consumption on both sides)

\[
r_t K_t = C_t + I_t + G_t,
\]

where $G_t$ is government consumption and $C_t$ is total consumption. Total consumption is defined as

\[
C_t = C^i_t + C^s_t
\]

where consumptions of investing and saving entrepreneurs are

\[
C^i_t = (1 - \beta) \left\{ r_t \pi (K_t - N_t^g) + \left[ \phi_t q_t + (1 - \phi_t) q_t R_t \right] (1 - \delta) \pi (K_t - N_t^g) + p_t \pi \left(M_t + \tilde{M}_t\right) \right\}
\]

\[
C^s_t = (1 - \beta) \left\{ r_t (1 - \pi) (K_t - N_t^g) + q_t (1 - \delta) (1 - \pi) (K_t - N_t^g) + p_t (1 - \pi) \left(M_t + \tilde{M}_t\right) \right\}.
\]
Then we should have an aggregate portfolio choice equation. Define the equity held by entrepreneurs without investment opportunities at the end of period $t$ as $N^s_{t+1}$, where

$$N^s_{t+1} = \theta I_t + [\phi_t \pi (1 - \delta) + (1 - \pi) (1 - \delta)] K_t.$$  

Notice that in (2.2.7), the aggregate version of $c^{ss}_{t+1}$ and $c^{si}_{t+1}$ is

$$C^{ss}_{t+1} = (1 - \pi) \left[ (1 - \beta) (r_{t+1} + (1 - \delta) q_{t+1}) N^s_{t+1} + p_{t+1} \left( M_t + \tilde{M}_t \right) \right]$$

$$C^{ss}_{t+1} = \pi \left[ (r_{t+1} + \phi_{t+1} (1 - \delta) q_{t+1} + (1 - \phi_{t+1}) (1 - \delta) q^{R}_{t+1}) N^s_{t+1} + p_{t+1} \left( M_t + \tilde{M}_t \right) \right]$$

from which one can rewrite (2.2.7) as

$$E_t \left[ \frac{(r_{t+1} + (1 - \delta) q_{t+1}) / q_t - p_{t+1} / p_t}{(r_{t+1} + (1 - \delta) q_{t+1}) N^s_{t+1} + p_{t+1} \left( M_t + \tilde{M}_t \right)} \right] = \pi E_t \left[ \frac{p_{t+1} / p_t - [r_{t+1} + \phi_{t+1} (1 - \delta) q_{t+1} + (1 - \phi_{t+1}) (1 - \delta) q^{R}_{t+1}] / q_t}{[r_{t+1} + \phi_{t+1} (1 - \delta) q_{t+1} + (1 - \phi_{t+1}) (1 - \delta) q^{R}_{t+1}] N^s_{t+1} + p_{t+1} \left( M_t + \tilde{M}_t \right)} \right].$$

(2.2.13)

When we have open market operations, we can think of the government using the money supply and return from previous equity to buy extra holding of private equity $N^g_{t+1} - (1 - \delta) N^g_{t}$, which translates into private sector’s holding of equity of $K_t - N^g_{t}$ at each date $t$. To back out the money spent on open market operations, the government expenditure now has to satisfy:

$$G_t + q_t \left[ N^g_{t+1} - (1 - \delta) N^g_{t} \right] + \psi \left( N^g_{t+1} \right) = r_t N^g_{t+1} + p_t \tilde{M}_t$$

(2.2.14)

In future analysis, we tie our hand by setting $G_t = 0$ to abstract from fiscal part. We view that the marginal cost will be small once government step in to buy equity, while the marginal cost will be very high when the government holds very few or zero private equity, but the specifics of the function is discussed when presenting the parameters. Finally, the
capital evolution is
\[ K_{t+1} - N_{t+1} = (1 - \delta) (K_t - N_t) + I_t \] (2.2.15)

Therefore, we have the following recursive equilibrium definition:

**Definition 1.** A recursive competitive equilibrium is defined as functions \( z_t, \phi_t, C_t, I_t, q_t, p_t, r_t, K_{t+1}, N_{t+1}^g \) that satisfies (2.2.8), (2.2.10), (2.2.11), (2.2.12), (2.2.13), (2.2.14), and (2.2.15), given stochastic processes of \( (z_t, \phi_t) \) and given a sequence of money supply rule \( \{\tilde{M}_t, \bar{M}_t\}_{0}^{\infty} \).

Notice that the definition of equilibrium already imposes capital market clearing and money market clearing through investment and portfolio balancing equation.

### 2.3 The Optimal Monetary Policy Problem

The approach to reach the optimal policy is in the same spirit of the public finance literature (see Chari and Kehoe (1999) for a survey) on obtaining an “implementability condition”, the so-called primal approach. After constructing the equilibrium conditions of a decentralized market, we solve out prices to depend only on allocations. The problem then becomes of a social planner choosing allocations under two constraints: one that defines a competitive equilibrium and the other that defines resources constraint.

To do so, we first describe how one can obtain an implementability condition with conventional and unconventional policies. Then we show that the conventional policy is actually a particular case of an unconventional setup.
2.3.1 Unconventional and Conventional Policies Together

Implementability Condition

Let \( S_t = K_t - N_t \) be the holding of equity in the private sector. One can solve \( q_t \) from equations (2.2.8) and (2.2.11),

\[
\frac{\beta}{1 - \beta} C^i_t = (1 - \theta q_t) \left[ I_t + \frac{(1 - \delta) (1 - \phi_t)}{1 - \theta} \pi S_t \right] \tag{2.3.1}
\]

Note that \( \frac{1}{1 - \beta} C^i_t \) is the net-worth of the investing agents, so \( \frac{\beta}{1 - \beta} C^i_t \) is the value of their equity holding (including inside and outside equity). On the left-hand side of equation (2.3.1), we have the total equity holding, on the right hand side we sum all the parts that constitute the equity holding: for \( I_t \) investment, \( \theta q_t I_t \) should be subtracted and, out of \( \pi S_t \) initial equity holding, the investing agents have \( (1 - \delta) (1 - \phi_t) \frac{1 - \theta q_t}{1 - \theta} \pi S_t \) after depreciation and equity selling (note that the market value for those that cannot be sold is \( \frac{1 - \theta q_t}{1 - \theta} \)). Therefore,

\[
q_t = \frac{1 - d_t}{\theta} \quad \text{and} \quad q^R_t = \frac{d_t}{1 - \theta},
\]

where

\[
d_t = \frac{\frac{\beta}{1 - \beta} C^i_t}{S_{t+1} - (1 - \delta) S_t + \frac{(1 - \delta)(1 - \phi_t)}{1 - \theta} \pi S_t}.
\]

We can interpret \( d_t \) as the down-payment rate. \( \frac{\beta}{1 - \beta} C^i_t \) is the amount that investors save, while \( S_{t+1} - (1 - \delta) S_t + \frac{(1 - \delta)(1 - \phi_t)}{1 - \theta} \pi S_t \) is the capital that will be used in production. Hence, the price of capital can be interpreted as one minus down-payment rate over the fraction of the investment \( (\theta) \) that can be initially issued. To solve \( p_t \), again from equation (2.2.11), one can express the price of money \( p_t \) as

\[
p_t = \frac{1}{\pi \left( M_t + \bar{M}_t \right)} \left\{ \frac{C^i_t}{1 - \beta} - r_t \pi S_t - \left[ \frac{(1 - d_t)}{\theta} \phi_t + \frac{d_t}{1 - \theta} (1 - \phi_t) \right] (1 - \delta) \pi S_t \right\}
\]
Then plug $p_t$ and $q_t$ into equation (2.2.12) and it yields:

$$
\frac{C^s_t}{(1 - \beta)(1 - \pi)} - \frac{C^i_t}{(1 - \beta)\pi} = (1 - \phi_t) \frac{(1 - \theta)}{\theta(1 - \theta)} (1 - \delta) S_t \tag{2.3.2}
$$

To interpret the implementability condition, recall that $\frac{C^s_t}{1 - \beta}$ is the net worth of the saving agents and $\frac{C^i_t}{1 - \beta}$ is the net worth of the investing agents, so the left hand side is the net worth difference of saving agents and investing agents normalized by the populations. Such difference is determined by the resaleability friction of the equity held after depreciation. This difference occurs since the shadow price for saving agents is $q_t = \frac{1 - d_t}{\theta}$ while for investing agents is $q_t^R = \frac{d_t}{1 - \theta}$. Hence the implementability condition states that the net-worth difference of two types of agents comes exactly from the price difference on non-resalable capital due to financing friction.

In a nutshell, the implementability condition summarizes the frictions. If we relax the financing frication, $q_t = q_t^R$, there will be no net-worth difference. If we relax the resaleability ($\phi = 1$), the net worth difference will also be equal to zero, as one would expect in a usual business cycle model.

**Set Up Ramsey Problem**

**Problem 1.** Now, suppose one wants to assign equal welfare weight to each agent in the economy. The constrained planner’s problem is then given by:

$$
\max \limits_{C^i_t, C^s_t, S_{t+1}, N_{t+1}^g} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \pi \log \left( \frac{C^i_t}{\pi} \right) + (1 - \pi) \log \left( \frac{C^s_t}{1 - \pi} \right) + L \log \left[ (1 - \alpha) A_t \left( \frac{S_{t+1} + N_{t+1}^g}{L} \right) ^{\alpha} / \pi \right] \right\}
$$

subject to:

$$
\frac{C^s_t}{(1 - \beta)(1 - \pi)} - \frac{C^i_t}{(1 - \beta)\pi} = (1 - \phi_t) \frac{(1 - \theta)}{\theta(1 - \theta)} (1 - \delta) S_t
$$

$$
C^i_t + C^s_t + \left( S_{t+1} + N_{t+1}^g \right) + \psi \left( N_{t+1}^g \right) = r_t K_t + (1 - \delta) \left( S_t + N_t^g \right)
$$
where 
\[ d_t = \frac{\beta}{1-\beta} C_i^t \left( K_{t+1} - N_{t+1}^g \right) - (1-\delta)(K_t - N_t^g) + \frac{(1-\delta)(1-\phi_t)}{1-\theta} \pi \left( K_t - N_t^g \right). \]

Consumption can be solved as a function of \( S_t, S_{t+1}, N_g^t \) and \( N_g^{t+1} \), with detailed calculation in the appendix. Not surprisingly we have two instruments, which give rise to two first order conditions and two state variables, one on private equity holding and another on government equity holdings:

\[ \left( \pi \frac{\partial C_i^t}{\partial S_{t+1}} + \beta E_t \pi \frac{\partial C_i^{t+1}}{\partial S_{t+1}} \right) + \left( (1-\pi) \frac{\partial C_i^t}{\partial N_g^t} + \beta E_t (1-\pi) \frac{\partial C_i^{t+1}}{\partial N_g^{t+1}} \right) + \frac{\alpha L}{S_{t+1}} = 0 \]
\[ (2.3.3) \]
\[ \left( \pi \frac{\partial C_i^t}{\partial N_g^{t+1}} + \beta E_t \pi \frac{\partial C_i^{t+1}}{\partial N_g^{t+1}} \right) + \left( (1-\pi) \frac{\partial C_i^t}{\partial N_g^t} + \beta E_t (1-\pi) \frac{\partial C_i^{t+1}}{\partial N_g^{t+1}} \right) + \frac{\alpha L}{N_g^{t+1}} = 0 \]
\[ (2.3.4) \]

We assume the cost function for government holding private equity is a concave function \( (\psi'(.) > 0, \psi''(.) < 0) \) and satisfy that \( \psi(0) = 0, \psi'(0) = 0 \). For small shocks, the deviations from zero open market operation should be small since it is very costly to hold private equity; For large shocks, it becomes necessary for the government to purchase significant amount of private equity, known as unconventional monetary policy to stabilize asset price and enhance liquidity.

A full characterization of each term, as well as some further algebra that simplifies the interpretation of the results, is relegated to the appendix. Finally, the second order condition is checked numerically to ensure a maximum.

2.3.2 Conventional Policies Only

In this section, we restrict attention to the problem when \( N_t^g = 0 \), so that the central bank can only change rates of return on money, whether by a helicopter drop of money or changing interest rates paid on reserves.

Recall that the competitive equilibrium is defined by equations (2.2.8)-(2.2.15) and the additional constraint that \( N_t^g = 0 \). Then we can solve \( p_t \) and \( q_t \) from equations (2.2.8) and (2.2.11) as we did before. By plugging the prices back, with the additional constraint that
the government does not buy illiquid assets, the implementability becomes:

\[
\frac{C_t^s}{(1 - \beta)(1 - \pi)} - \frac{C_t^i}{(1 - \beta)\pi} = (1 - \phi_t) (1 - \theta) - d_t \frac{(1 - \theta)}{\theta} (1 - \delta) K_t
\]  

(2.3.5)

The interpretation is very similar. But now since all asset is privately claimed, we do not need to distinguish between privately and publicly claimed assets. In what regards to the structure of the Ramsey problem, we only have one first order condition, since we have constrained to one instrument.

2.3.3 The Equivalence and Dominance Result

From the implementability conditions, one can see that conventional policy is a subset of all the allocations that can be attained through unconventional policies. Therefore, we have the following equivalence result.

**Proposition 1.** Suppose the government has both conventional and unconventional instruments. The optimal allocation is the same as having only the unconventional instrument.

**Proof.** \(\tilde{M}_t\) does not show up and setting \(\tilde{M}_t = 0\) will not affect the optimal \(S_{t+1}\) if one has unconventional policies to use.

\(\square\)

The immediate dominance result follows:

**Corollary 1.** Unconventional monetary policies dominate conventional ones.

To understand the proposition and corollary, one should recall the central friction in this economy: equity resale friction. Intuitively, the imperfection on the selling equity reduces the rate of return on equity and induces a pecuniary externality, since agents do not take into
account their own effect on holding the liquid asset. Furthermore, agents tend to hold liquid assets which are intrinsically valueless. Both unconventional and conventional policy are intended to correct this externality. However, unconventional policy is more accurate since it targets directly at the illiquid asset and the dominance result becomes straightforward.

2.4 Quantitative Examination of Optimal Policy

In this section, we highlight how important is optimal policy through a series of numerical exercises. Our benchmark is a competitive economy with no policy intervention (constant money supply). We discuss steady-state values as well as impulse response functions under no policy, policy with only conventional instruments and with unconventional instruments. The experiment exercise is to demonstrate how should the optimal policy react both qualitatively and quantitatively (or if it should react at all).

2.4.1 Parameters

Some of the parameters used are standard in the literature such as depreciation rate, capital share in production and discount factor, while more elaboration should be given to $\pi$, $\phi$, $\theta$ and $L$. We assume that 6% of the entrepreneurs are productive every quarter, which is the number to match investment spikes observed from U.S. manufacturing plants in (Doms and Dunne (1998), Cooper et al. (1999) and Negro et al. (2011)). For the financial frictions, previous work by Negro et al. (2011) has assumed the mean values for $\theta$ and $\phi$ to be 19%, matching total treasury bills over outstanding to total assets. We perform another exercise, looking at the ratio of liquid assets to total assets in the economy in the stable period (1991Q1 to 2007Q4) and we confirm these results. Later we will vary $\phi$ to check robustness, which directly measures the resale friction.

Finally, $L$ should show the ratio of workers to entrepreneurs in the economy. The main difference between workers and entrepreneurs is the access to equity markets to fund the
investment opportunity. Therefore, we calibrate this value to be in line with the participation rate of households from SCF in 2009 that we see in the equities market (19%), a number in line with previous studies from Mankiw and Zeldes (1991) and Heaton and Lucas (1999). Such number translates into \( L = 6 \).

When using unconventional monetary policy, the cost for the government of buying private equities is assumed to be

\[
\psi \left( N^g_{t+1} \right) = \mu \left[ \log \left( \frac{1 + N^g_{t+1}}{a} \right) \right]^2
\]

Since we look for a cost on holding assets, and not on the changes of purchase, we consider a function well defined in the positive side (log). Therefore, our task is to find \( \mu \) and \( a \) such that the steady state private holding of equity is the same as in the conventional policy. This requirement leads to the above policy\(^{15}\), together with previous parameters, is summarized in Table 2.2.

Finally, for the evolution of exogenous state variables \( z_t \) and \( \phi_t \), we follow the literature on assuming the productivity an AR(1) process and also take the resaleability as an AR(1).

We estimate the two processes to be:

\[
\begin{align*}
z_t &= 0.9225 z_{t-1} + \epsilon^z_t \\
\phi_t = \phi_{t-1} + \epsilon^\phi_t
\end{align*}
\]

where \( \epsilon^z_t \) are i.i.d zero mean normal random variable with standard deviation 0.0134, \( \epsilon^\phi_t \) are i.i.d zero mean normal random variable with standard deviation 0.0052 and \( corr \left( \epsilon^z_t, \epsilon^\phi_t \right) = 0.495 \).

\(^{15}\)The choice of using this log-function instead of the most common quadratic cost was just to ensure computational tractability to avoid negative values.
The productivity random process is the standard Solow residual process and is taken from estimation of Thomas (2002), in line with previous studies. The AR(1) coefficient of $\phi_t$ process and its residual come from estimating the model with observed rate of return on liquid assets. Namely, we could think of not assuming that the government has already taken the optimal policy, and we use the actual rate of return on liquid assets when estimating it. We estimate the process of $\phi_t$ using observed policy among other direct aggregate variables (investment, liquidity assets value and total asset value) through Bayesian estimates that are detailed in the appendix.

2.4.2 Steady State

We discuss the effects on the steady state in which we provide three scenarios: constant money supply, optimal conventional policies and optimal unconventional policies.

In Table (2.3), we normalize all the variables to be deviations from the no-policy case. The table allows us to not only evaluate the gains from optimal policy, but also to compare how much better is unconventional compared to conventional, since we have already showed that the former dominates the latter.

Firstly, optimal monetary policy plays a role in the steady-state, by changing the rate of return on liquid asset, as one can see in the annualized interest rate, about 3.5% annually. The intuition is that saving entrepreneurs save too much in the liquid asset and use the return to finance investment. However, they create an externality on others because they reduce the return from liquid assets, inducing the others to save even more to finance future investment. A way to overcome this is by reducing the supply of the liquid asset, which increases the rate of return and it leans against the pecuniary externality. By increasing the rate of return on the liquid asset, entrepreneurs will enjoy better return from that for future new investment, as seen by about 35.5% increase in liquid assets value.

The second distinguished feature is the capital stock held in equilibrium. As one would expect, investing agents are constrained due to the financial friction, but due to redistribution policy, the capital stock increases by about 3% and is closer to the first best. Therefore, the
Table 2.3: Steady State Value

<table>
<thead>
<tr>
<th></th>
<th>Conventional Policy</th>
<th>Unconventional Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Interest Rate</td>
<td>+3.527%</td>
<td>+3.444%</td>
</tr>
<tr>
<td>Total Output</td>
<td>+0.917%</td>
<td>0.917%</td>
</tr>
<tr>
<td>$C^i$</td>
<td>+5.416%</td>
<td>+5.318%</td>
</tr>
<tr>
<td>$C^s$</td>
<td>-3.395%</td>
<td>-3.395%</td>
</tr>
<tr>
<td>$C^L$</td>
<td>+0.909%</td>
<td>+0.909%</td>
</tr>
<tr>
<td>$I$</td>
<td>+2.797%</td>
<td>2.803%</td>
</tr>
<tr>
<td>$N_g/S$</td>
<td>0</td>
<td>1%</td>
</tr>
<tr>
<td>$K$</td>
<td>+2.797%</td>
<td>+2.803%</td>
</tr>
<tr>
<td>Asset Price</td>
<td>-9.249%</td>
<td>-9.056%</td>
</tr>
<tr>
<td>Total Money Value</td>
<td>+35.642%</td>
<td>+35.22%</td>
</tr>
<tr>
<td>Liquidity Ratio</td>
<td>+40.719%</td>
<td>+40.146%</td>
</tr>
<tr>
<td>Equivalent Consumption Gains</td>
<td>+0.359%</td>
<td>+0.361%</td>
</tr>
</tbody>
</table>

Normalizing the quantities in the economy with no policy as 1 and comparing the increase or decrease of each variable: $C^i$ is consumption of investing agents, $C^s$ is consumption of saving agents, $C^L$ is consumption of workers, $I$ is investment. $N_g/S$ is the ratio of government held partially-resaleable equity, $K$ is capital, asset price is what we labeled before $q$, total money value we labeled as $M^p$ and liquidity ratio is the ratio of value of liquid assets over total assets. Finally, equivalent consumption gain is how much each agent would increase its consumption permanently by changing to the respective policy.

Asset price $q$, which implicitly measures the degree of financing and resaleability constraint, is closer to 1, the first best outcome in which either financing friction or resaleability friction is eliminated. Thus, even though it is still constrained, the shadow value of relaxing the constraint decreases after policy intervention, leading to a higher capital and therefore a higher welfare.

The welfare gains computed suggest that the benefits from having an optimal monetary policy in such environment are equivalent to increasing the consumption of each agent, permanently, by .36%, a sizable number since it is a permanent change in the economy.

A final comparison is on unconventional and conventional policy outcomes. The quantities and prices from unconventional policies are very similar to what conventional policy can achieve. Interest rate need not be that high since the constrained planner has another instrument to achieve the desired allocation.
2.4.3 Simulations

In this section, we examine how monetary policy responds to shocks, through impulse response functions. We consider four cases: a pure productivity shock, a pure liquidity shock, a combination of a productivity and a liquidity shock and a combination of productivity with expected future liquidity shock. Our focus is mainly on comparing optimal policy (both conventional and unconventional policy) with a constant liquid assets supply, which we label as no-policy. In doing that, we log-linearized the model to solve the rational expectation system\textsuperscript{16}.

Active Conventional Monetary Policy and No Policy

- **Pure Productivity Shock**

The first shock that we consider is a pure productivity shock (Figure 2.4.1). The shock that we investigate is a negative one standard-deviation shock to productivity:

\[ z_t = 0.9225z_{t-1} + \varepsilon_t^z \]

Without policy, such shock drives the price of the equity and money down under a constant money supply policy because there are less resources for agents to save. To emphasize the price level change, the initial triggering of a pure adverse productivity shock leads to inflation (money rate of return decreases). Therefore, with only a negative productivity shock, it will produce inflation pressure in recession.

\textsuperscript{16}Detailed computation can be found in the Matlab code available in the authors’ website
One standard deviation shock to $\phi$ with correlated shock to $\ln A$. Money growth rate path is just its own path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state levels.

With policy intervention, however, when one considers a helicopter drop (drain) type of policy, the equity price drops while the return becomes very high after the shock. To achieve this, we see a positive rate of return on liquid assets that is even higher than the steady state interest rate. The gains from having a conventional policy can be seen in the consumption of savers and investors, even though the total consumption is less affected because workers don’t have their consumption much affected. Overall, aggregate consumption, investment and output do not change significantly under conventional policy and no policy, when only productivity shocks hit. Importantly, the steady state level is still higher under conventional policy. Given that the response in percentage term is similar, the conventional policy still gives a better allocation of resources.

- **Pure Liquidity Shocks**

Now we consider a pure adverse liquidity shock (Figure 2.4.2). We consider, once more, an auto-regressive shock with a one standard deviation shock of 0.0052 obtained from a
Bayesian estimate of the model during the "great moderation", so it can be thought as a small shock during regular periods.

As documented in Kiyotaki and Moore (2011), liquidity shocks usually lead to a flight to liquidity as seen in the figure with a lower rate of return on liquid assets. However, at the same time, the illiquid asset supply drop drastically so that real asset price actually increases even though the nominal price decreases.\textsuperscript{17} Therefore, asset prices \((q_t, p_t)\) increase leading to higher wealth which will lead to initial higher individual consumption, but lower investment (given that output will be initially the same, investment will drop). Overall, pure liquidity shock is at odds with the data too.

Under optimal policy, there is an apparent trade-off between present and future as we can see from the aggregate consumption graph. Aggregate consumption rarely fluctuates. Not surprisingly, the aggregate investment and output are stable as well. This outcome is achieved by changing the liquidity asset return and then changing the consumption gap between saving and investing entrepreneurs. Note that, the larger increase in total investing entrepreneurs’ consumption is a manifestation of more consumption smoothing.

\textsuperscript{17} The nominal asset price is lower after pure liquidity shocks, since money’s real price (in terms of consumption goods) increases more than real asset price initially due to flight to liquidity. We do not plot money’s real price, since we focus on the return from liquid assets instead of the price.
From the last two experiments, we saw that pure productivity or pure liquidity shock misses some important stylized facts observed in recession in the data, no matter whether there is optimal policy or not (mainly consumption path, rate of return on liquid assets and asset resale price). We overcome the odd behavior by considering a liquidity shock accompanied by a productivity shock and estimate its variance-covariance matrix from data (Figure 2.4.3). The shock we investigate is a liquidity shock accompanied by a simultaneous TFP as described before. The correlation between them is also obtained from Bayesian estimates and it comes to be relatively big: 0.49. The economic reason behind such experiment could be, for instance, that financing frictions lead to mis-allocation and reduce TFP, but we do not model it endogenously. Moreover, with such shock we span another possibility which is a liquidity shock that does not translate into an asset price movement in a world without policy.
Without the optimal policy, the adverse $\phi$ shock will reduce the demand on equity because of liquidity run, but not enough to reduce the asset price. Two forces roughly cancel out each other on this exercise: on one hand, the portfolio rebalancing to liquid assets; on the other, as productivity is auto-regressive, the economy becomes more unproductive today and in the future too, overcoming more consumption and less investment today from the liquidity run discussed before. As a result, investment will decrease drastically, while consumption should also fall because it is accompanied by a TFP loss that reduces the available resources. All these features are in line with stylized facts observed in usual recessions, particularly the portfolio rebalancing in recent years.

Figure 2.4.3: No Policy and Conventional Policy: $\phi$ shock and $A$ shock

One standard deviation shock to $\phi$ with correlated shock to $\ln A$. Money growth rate path is just its own path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state levels.

With optimal policy, investment drops, but much less than without an optimal policy to counterbalance such effect. Again, the central bank is redistributing wealth through the payment of liquid assets, which helps on consumption smoothing (from the gap between saving and investing entrepreneurs consumptions).

Not surprisingly, optimal policy achieves a more stable result through redistributing resources. Note, however, that since we are comparing to the steady-state, the optimal policy was already better than the constant money supply case and it becomes even better.
The previous shocks lead to policy response, but without too much impact on stabilizing macroeconomic real variables. We consider expected future liquidity shock, say 4 quarters later, and current productivity jump and examine how much the policy could achieve (Figure 2.4.4). Such experiment is intended to partially capture the fall of Lehman Brothers in 2008Q3. The fall did not immediately stop all the business. In fact, many previous Lehman related business still ran into 2009. However, the fall may have triggered the expectation that in the near future many assets would be very illiquid, which is captured by a 4 quarters later liquidity shock. At the same time, funding froze from the banking sector, limiting efficient production, reducing total factor productivity in the economy, which is seen in the data computed by most policy paper.

The purpose of this exercise is twofold. First, to discuss under which conditions is policy more relevant, and secondly to discuss a liquidity based shock in which the asset price actually goes down. We still take the same structure discussed before, but with shock on current liquidity known 4 quarters before.

Figure 2.4.4: No Policy and Conventional Policy: future $\phi$ shock and $A$ shock

One standard deviation shock to $\phi$ with correlated shock to $\ln A$. Money growth rate path is just its own path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state levels.
Without policy intervention, the macroeconomic variables are very unstable. For example aggregate consumption decreases by 0.7% initially, then increases a lot, decreases back to a level that is lower than the steady state and stay there persistently. The reason for that can been seen from investment, which decreases initially because of low productivity, slightly increases afterward due to less consumption and then incur a big jump because of expected liquidity shock. Persistent low investment thus leads to persistent future low consumption. Not surprisingly, output is persistently lower.

The role of the government policy is remarkable. Interest is kept almost constant until date three, when the rate reduces greatly so that there is a negative interest rate on holding liquid assets at date 3. By increasing money growth in 25% at date 3, it helps on keeping liquid asset accumulation low and a larger room for policy to increase the rate of return from date 4 to date 5. When the real shock hits on date 4, the liquid asset return was maintained high by the central bank which helps on smoothing funds transfer. This experiment demonstrate significantly that monetary policy should move fast in responding to market price and return fluctuation.

Unconventional Policy and Conventional Policy

Now we follow the same structure of the previous version where we have discussed productivity, liquidity and joint productivity-liquidity shocks. The difference, henceforth, is that we want to compare the gains from using unconventional policies vs. conventional ones. We have already established that conventional policies lead to a non-negligible increase in welfare compared to a constant money supply case under steady-state. Moreover, we have theoretically established that unconventional policies are weakly better than conventional ones. A further question that one may ask is: why should we bother understanding conventional instruments if we know that unconventional ones dominate them?

The answer can be depicted from comparing conventional and unconventional policy in
the “Expected Future Liquidity Shocks and Productivity Shocks” experiment (Figure 2.4.5): the optimal path under both policies is roughly the same, being robust to any shock. However, we have assumed that helicopter drain is feasible, which may well not be when truly implementing it. If helicopter drain is not possible, unconventional policies may help attain the desired allocations we have before. Conventional and unconventional policy under other experiments are almost exactly the same (including interest rates) and we will not show them here due to space restriction.

Figure 2.4.5: Unconventional versus Conventional

1 standard deviation shock to future $\phi$ with correlated shock to $\ln A$. Money growth rate path is the level path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state.

Allowing for a new instrument to be used from the FED leads to an increase in equity purchasing of about 3%, a much bigger number than the 0.05% that we had on money growth under conventional policies. However, such difference leads roughly to the same allocation. There is no significant difference on the path of the variables using unconventional or conventional policies.

Even though the path is indistinguishable, the levels under unconventional policy are higher, since we are comparing to a higher steady state. The results for the liquidity shock case only and simultaneous shocks also give indistinguishable paths between conventional and unconventional policies.
Finally, two comments are worth mentioning. First of all, such results do not depend on the cost function used. The intuition is that the unconventional policy, if possible, almost completely reduces the pecuniary externality by changing the rate of return on liquid assets and illiquid equity. The unconventional policy leaves very few room for improvement. For roughly any concave function tested, our results persist. The changes are even smaller if we use convex cost function since marginal cost becomes higher after purchase. Besides that, these results should not be seen as a case against unconventional policies. On the contrary, even though unconventional policies cannot change the paths of variables, they do change the steady state from which we are comparing to. Therefore, the welfare remains higher during all periods after the shock in an unconventional policy. Besides that, if, for instance, helicopter drain policies are not implementable, unconventional policies can substitute them.

**Liquidity Ratio in the Data and the Model**

As can be seen in all the exercises, liquidity ratio fluctuates more under active policies. The key reason is that monetary policies enable the liquid assets to be more valuable and to lubricate funds transfer for investment. Without policy, individuals generate a larger degree of externality and thus make the value of liquid assets very low. Importantly, they have a rigid demand on liquid assets in all experiments, since the other assets give unfavorable return due to illiquidity. Illiquidity shows as lower asset prices in the productivity shocks, or as higher fractions of non-resalable asset in liquidity shocks. Therefore, agents in our model do have large rebalancing to liquid assets as in the data. Given the effective low interest rate policy, optimal policy nevertheless suggests that liquidity ratio probably should be even higher so that rebalancing to liquid assets does not hurt its ability to transfer funds.

**2.4.4 Further Discussion**

**Parametrization Robustness**

We compute the steady-state level of capital for different parametrization to draw com-
parative statics\textsuperscript{18}. Importantly, since one can draw a relationship between capital and the rate of return on liquid assets, one can evaluate the relationship between the financial frictions and the return of liquid assets in the steady-state. The optimal steady-state rate of return on the liquid asset is decreasing in $\phi$.

Relaxing the liquidity constraint from .14 to .25, for instance, the optimal rate of return on the liquid asset in the steady state would jump from around annual 6\% to almost 0\% (Figure (2.4.6)) \textsuperscript{19}. A thorough look at how endogenous variables change as one tighten or loosen liquidity friction is in Table 2.4 \textsuperscript{20}.

Under optimal policy, capital increases less as the friction relaxes. Not surprisingly, the interest rate changes from no-policy to optimal policy is always smaller the higher is $\phi$. Hence, the gains from having an optimal policy are reduced as the importance of financial friction is reduced. Moreover, the allocations under optimal policy and competitive equilibrium become similar as one relaxes the frictions, a result that should be expected, since there would be no role for optimal policy if there was no friction\textsuperscript{21}.

\textsuperscript{18}Such results can be derived from the equations, but we present the results numerically for ease of interpretation.

\textsuperscript{19}For numbers of $\phi$ higher than this, the assumption that ensures that the constraints are binding is not satisfied.

\textsuperscript{20}The title of the column indicates which parameter are we changing compared to our benchmark model presented in Section 4.

\textsuperscript{21}Due to space restriction, we do not show the impulse response functions for different parameters, but the qualitative results discussed previously are the same, and the policy conclusions remain.
### Table 2.4: Robustness of Steady State $\phi$

<table>
<thead>
<tr>
<th>$\phi = 18$</th>
<th>$\phi = 19$</th>
<th>$\phi = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Annual Int. Rate</strong></td>
<td>+4.02%</td>
<td>+3.527%</td>
</tr>
<tr>
<td><strong>Total Output</strong></td>
<td>+1.043%</td>
<td>+0.917%</td>
</tr>
<tr>
<td>$C_i$</td>
<td>+6.148%</td>
<td>+5.416%</td>
</tr>
<tr>
<td>$C_s$</td>
<td>-3.837%</td>
<td>-3.395%</td>
</tr>
<tr>
<td>$C_L$</td>
<td>+1.047%</td>
<td>+0.909%</td>
</tr>
<tr>
<td>$I$</td>
<td>+3.197%</td>
<td>+2.797%</td>
</tr>
<tr>
<td>Asset Price</td>
<td>-10.411%</td>
<td>-9.090%</td>
</tr>
<tr>
<td><strong>Total Money value</strong></td>
<td>+35.070%</td>
<td>+35.642%</td>
</tr>
<tr>
<td>Liquidity Ratio</td>
<td>+40.800%</td>
<td>+40.719%</td>
</tr>
<tr>
<td><strong>Equiv. Perm. Consumption</strong></td>
<td>+0.418%</td>
<td>+0.350%</td>
</tr>
</tbody>
</table>

### 2.5 Conclusion

We study a tractable model of optimal monetary policy instruments dealing with financial frictions, namely equity issuance and resale frictions. We provide an implementability condition that summarizes all the restrictions of a competitive equilibrium allocation in this model. The implementability condition thus allow us to derive the social optimal allocation. By doing so, we avoid the usual ambiguous welfare ranking problem in the optimal monetary policy literature.

Both optimal conventional and unconventional monetary policies should target at paying interest rate on liquid assets. Due to the pecuniary externality arising from the liquidity constraint in a competitive equilibrium, there will always be room for policy to improve welfare in a constrained economy. In the steady-state, permanent aggregate consumption increases by almost .4%, comparing optimal policy to non-policy. Moreover, when hit by an adverse liquidity shock, by using expansionary policy such difference increases even more. Finally, we showed that unconventional policies dominate conventional ones. But in quantitative exercises, the difference that it generates on other macroeconomic variables is very small.
Note that we do not assume sticky price in the economy but the pecuniary externality on holding liquid assets still need policy intervention. Monetary policy, therefore, mainly acts like a redistribution device transferring resources from non-liquid assets holders to liquid assets holders. Whenever the economy runs into problem due to liquidity issues, firms or banks will typically hold more liquid assets, usually more than they should. A usual policy response by lowering interest rate should be reconsidered. Agents in the economy will have a rigid demand on liquid assets if other assets market persistently incur resale (liquidity) problems. In that sense, lowering the interest rate will only hurt the ability for financing future investment, since it exacerbates the incentive to hold even more liquid assets in a world where liquid assets are their favorite choices for savings.

One drawback and potential future work is how the monetary policy will change illiquid asset market resaleability endogenously (how $\phi$ changes endogenously by monetary policies). We viewed it as an exogenous fluctuation but it could certainly depend on market expectation and asset quality. This possibility is left for future work.

2.6 Appendix

In this appendix, we provide the derivations for the Ramsey problem and the details in the estimation.
2.6.1 Appendix to Section 3

Ramsey Problem

Problem 2. From the implementability condition and resources constraint, one can express aggregate investing and saving entrepreneurs’ consumption as

$$C_i^t = \pi \{ r_t K_t - K_{t+1} + (1 - \delta) K_t - G_t - \psi (N_t^g) \}
- (1 - \pi) (1 - \beta) (1 - \phi_k) \left( \frac{(1 - \theta) - d_t}{\theta (1 - \theta)} \right) (1 - \delta) (K_t - N_t^g) \}
$$

$$C_s^t = (1 - \pi) \{ r_t K_t - K_{t+1} + (1 - \delta) K_t - G_t - \psi (N_t^g) \}
+ \pi (1 - \beta) (1 - \phi_k) \left( \frac{(1 - \theta) - d_t}{\theta (1 - \theta)} \right) (1 - \delta) (K_t - N_t^g) \}
$$

Noticing that one can plug in $d_t$ and express $C_i^t$ and $C_s^t$ only in terms of $S_t$, $N_t^g$, $S_{t+1}$ and $N_{t+1}^g$:

$$C_i^t = \pi \frac{1}{1 - B_t} \{ r_t (S_t + N_t^g) - (S_{t+1} + N_{t+1}^g) + (1 - \delta) (S_t + N_t^g) - G_t - \psi (N_{t+1}^g) \}
- (1 - \pi) (1 - \beta) (1 - \phi_k) (1 - \delta) S_t / \theta \} \tag{2.6.1}$$

$$C_s^t = r_t (S_t + N_t^g) - (S_{t+1} + N_{t+1}^g) + (1 - \delta) (S_t + N_t^g) - G_t - \psi (N_{t+1}^g) - C_i^t, \tag{2.6.2}$$

where $B_t = \beta \pi (1 - \pi) S_t / \left\{ \theta \left[ \frac{1 - \theta}{1 - \phi} \left[ \frac{S_{t+1}}{1 - \delta} - S_t \right] + \pi S_t \right] \right\}$. Therefore, one can rewrite the Ramsey problem as

$$\max_{S_t, N_t^g} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \pi \log \left( \frac{C_i^t}{\pi} \right) + (1 - \pi) \log \left( \frac{C_i^t}{1 - \pi} \right) + L \log \left( (1 - \alpha) A_t \left( \frac{S_t + N_t^g}{L} \right) ^{\alpha} / L \right) \right] \right\} \tag{2.6.3}$$

subject to (2.6.1) and (2.6.2).
We now have two instruments, which give rise to two first order necessary conditions (FONCs), one on private equity holding and another on government equity holdings:

\[
\begin{align*}
[S_{t+1}] & : \left( \pi \frac{\partial C_t^i}{C_t^i \partial S_{t+1}} + \beta E_t \frac{\pi}{C_{t+1}^i} \frac{\partial C_{t+1}^i}{\partial S_{t+1}} \right) + \left( \frac{(1 - \pi) \partial C_t^s}{C_t^s \partial S_{t+1}} + \beta E_t \frac{(1 - \pi)}{C_{t+1}^s} \frac{\partial C_{t+1}^s}{\partial S_{t+1}} \right) + \beta \frac{\alpha L}{S_{t+1}} = 0 \\
[N_{t+1}^g] & : \left( \pi \frac{\partial C_t^i}{C_t^i \partial N_{t+1}^g} + \beta E_t \frac{\pi}{C_{t+1}^i} \frac{\partial C_{t+1}^i}{\partial N_{t+1}^g} \right) + \left( \frac{(1 - \pi) \partial C_t^s}{C_t^s \partial N_{t+1}^g} + \beta E_t \frac{(1 - \pi)}{C_{t+1}^s} \frac{\partial C_{t+1}^s}{\partial N_{t+1}^g} \right) + \beta \frac{\alpha L}{N_{t+1}^g} = 0 
\end{align*}
\]

We derive the expression for each term in the FONC for \( S_{t+1} \), while leaving the FONC for \( N_{t+1} \) since it is very similar and even simpler.

\[
\begin{align*}
\pi \frac{\partial C_t^i}{C_t^i \partial S_{t+1}} &= -\frac{\pi^2}{(1 - B_t) C_t^i} - \frac{\theta (1 - \theta) B_t^2}{(1 - \pi) (1 - \phi) (1 - \delta) (1 - B_t) S_t} \\
(1 - \pi) \frac{\partial C_t^s}{C_t^s \partial S_{t+1}} &= \frac{(1 - \pi)}{(1 - B_t) C_t^s} + \frac{\beta \pi (1 - \phi) (1 - \delta) (1 - B_t) S_t C_t^i}{C_t^s} (1 - \pi).
\end{align*}
\]

\[
\begin{align*}
\beta E_t \frac{\pi}{C_{t+1}^i} \frac{\partial C_{t+1}^i}{\partial S_{t+1}} &= \beta E_t \left( \frac{\pi^2}{\theta^2} \left( \alpha r_{t+1} + (1 - \delta) - (1 - \pi) (1 - \beta) (1 - \phi_t) (1 - \delta) \right) / \theta \right) \\
&\quad \cdot \left( \frac{\partial}{\partial \pi} \frac{C_{t+1}^i}{(1 - B_{t+1})} \right) \left( \frac{B_{t+1}}{S_{t+1}} \right) \left( \frac{\theta \pi - \theta (1 - \theta)}{1 - \phi_t} \right) \left( \frac{1 - \pi}{1 - \phi_t} \right) \left( \frac{1 - B_{t+1}}{1 - B_t} \right) \left( \frac{C_{t+1}^i}{C_{t+1}^s \partial S_{t+1}} \right)
\end{align*}
\]

If the planner cannot purchase equity, the equivalent problem is by setting \( N_{t+1} = 0 \) and \( S_{t+1} = K_{t+1} \) at all \( t \). In computing the optimal policy, simply by replacing \( S_{t+1} = K_{t+1} \) and ignoring the FONC for \( N_{t+1} \)

We do not derive the second order conditions for the Ramsey problems (one and two instruments), since the algebra becomes too tedious. Instead, we check all our calculations numerically by ensure that the FONCs give the welfare maximized solution.
Prices and Policy Instruments

One can simply back out prices including asset price, return on liquid assets and policy instrument from quantity variables. Here, we just explain how prices can be backed out in the steady-state. The steady state version of the portfolio choice equation become

\[
(1 - \pi) \frac{(r + (1 - \delta) q) / q - x}{N^s + M^p} = \pi \frac{x - [r + \phi (1 - \delta) q + (1 - \phi) (1 - \delta) q^R] / q}{(r + \phi (1 - \delta) q + (1 - \phi) (1 - \delta) q^R) N^s + M^p}
\]

where \( x = \frac{p_{t+1}}{p_t} \) measures the return on liquid assets and \( M^p = \frac{C^i}{\pi(1 - \beta)} - rK \)

\[- \left[ \frac{(1-d)\phi+d(1-\phi)}{\theta} \right] (1 - \delta) K \] measures the total value of liquid assets. Rearrange to express \( x \) as

\[
\left[ \frac{1 - \pi}{(r + (1 - \delta) q) N^s + M^p} + \frac{\pi}{(r + \phi (1 - \delta) q + (1 - \phi) (1 - \delta) q^R) N^s + M^p} \right] x = \frac{(1 - \pi)(r + (1 - \delta) q) / q}{(r + (1 - \delta) q) N^s + M^p} + \frac{\pi}{(r + \phi (1 - \delta) q + (1 - \phi) (1 - \delta) q^R) N^s + M^p} \]

where \( N^s = \theta I + \phi \pi (1 - \delta) + (1 - \pi) (1 - \delta) K \), and \( q = (1 - d) / \theta \).

In Kiyotaki and Moore (2011) (constant money supply) economy, the net rate of return on money is always zero in the steady state \( (p_{t+1}/p_t - 1 = 0) \) given that the money supply does not change.

2.6.2 Details on the Estimation

We estimate the model using Bayesian methods. The purpose of the exercise is to obtain the distribution of the shock of \( \phi \) and how it correlates with \( A \) shocks. In order to do so, we estimate the dynamic stochastic general equilibrium model with measurement errors.

As usual, to have identification, we consider the number of shock/measurement errors to be the same as the number of observed variables. We introduce 5 shocks in the estimation: resaleability, productivity shock, resaleability and productivity correlation, as well as measurement errors on the total liquid asset value and the expected interest rate on liquid assets. We have already calibrated the productivity shock from previous literature, leaving
4 shocks to be estimated (with 2 measurement errors).

We consider deviations from the HP trend for aggregate investment. The other variables that we consider are related to portfolio rebalancing. The linkage of portfolio rebalancing and its impact on investment is novel and directly related to our question. For the total liquid asset value, as defined in 2.1.1, we considered check, deposit, tradable receivable and T-Bills. Total assets value also come from the Flow of Funds table and is also defined in 2.1.1. For the rate of return of liquid assets, we considered the 3-Month Treasury bill rate adjusted for expected inflation from the Michigan survey. The sample period used is from 1991 to 2007, in order to consider a stationary and stable period, that we can be sure to be dealing with "normal" times.

Following the literature on setting priors, we consider inverse gamma for standard error of the structural shocks to have a conjugate prior. Table 2.5 summarizes the prior and posterior information. We have tried many different priors and the posteriors are very robust. Interested readers can directly check the code available on the authors’ website.

For the standard deviation of \( \phi \), we consider it small because \( \phi \) itself is already small and we want to have a high probability of staying in the positive domain. The liquid asset expected returns are expected and subject to some errors, and the money value may have some accounting errors too. So we provide a rather flat prior with a small mean on the measurement errors. We leave the mean of the persistence of liquidity shocks \( \rho_\phi \), to be somewhat moderate number 0.9. Finally, we start with a somewhat large productivity-liquidity shocks correlation to since financial impact on TFP maybe large. The posterior

### Table 2.5: Prior and Posterior of the Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Distribution</th>
<th>Mean</th>
<th>Std</th>
<th>Mode</th>
<th>5%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_\phi )</td>
<td>Inverse Gamma</td>
<td>0.01</td>
<td>1</td>
<td>0.0052</td>
<td>0.0051</td>
<td>0.0046</td>
</tr>
<tr>
<td>( \sigma_{ln(x)} )</td>
<td>Inverse Gamma</td>
<td>0.01</td>
<td>1</td>
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<td>0.0314</td>
<td>0.0280</td>
</tr>
<tr>
<td>( \sigma_{ln(PM)} )</td>
<td>Inverse Gamma</td>
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<td>1</td>
<td>0.0086</td>
<td>0.0046</td>
<td>0.0158</td>
</tr>
<tr>
<td>( \sigma_{z,\phi} )</td>
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<td>1</td>
<td>0.4746</td>
<td>0.4950</td>
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</tr>
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<td>( \rho_\phi )</td>
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<td>0.05</td>
<td>0.8929</td>
<td>0.8951</td>
<td>0.8626</td>
</tr>
</tbody>
</table>
Prior distributions are shown in gray color, while posterior distributions are shown in black color. The header of each subplot stands for the parameters estimated. SE\_phi\_shock: $\sigma_{\phi}$. SE\_x\_err: $\sigma_{ln(x)}$. SE\_pm\_err: $\sigma_{ln(Mp)}$. CC\_A\_shock\_phi\_shock: $\sigma_{z,\phi}$. rho\_phi: $\rho_{\phi}$

mode of $\sigma_{\phi}, \sigma_{z,\phi}$ and $\rho_{\phi}$ estimated are used in our numerical analysis. The posterior shape of the 3 parameters are very concentrated, given that we have a relatively flat prior.
Chapter 3

An Information-based Explanation for Price-Setting Patterns

3.1 Introduction

"A pricing analyst described how at one of the early pricing strategy meetings, a representative from the marketing group and one of the members of the sales force "... were shouting back and forth, ..." and the argument became so heated that I thought they were going to throw punches"

Zbaracki et al. (2004)

When one starts reading how a firm works, usually conflicts within the firm are stressed. Managers that do not agree to each other and hierarchies that don’t share the exact same goals are just some examples. Whether it is because of effort, a non-optimal bonus contract or any other reason, it is fair to say that we observe some misalignment of goals within the firm. At the same time, information transmission within the firm is also a major issue; different agents have different information sets and they have to reveal it to their "bosses"
since these are ultimately the decision-makers.

It is actually the interplay of transmission revelation with misalignment of goals that we explore in this paper. Such interplay provides robust results on the price-setting of firms that are supported by microdata.

We explore a framework in which a bottom-layer agent (worker) is privately informed and has to send information to his boss (manager), who decides the price to be set. The private information can be loosely thought as a marginal cost or demand. However, as they have different payment structures not conditioned on the information revelation, the bottom-layer agent may decide not to reveal everything he knows. A particular case of this setup is when the wage / bonus schedule of them is not the same.

Such mechanism sheds light on price patterns that have been extensively discussed in the data recently (Eichenbaum et al. (2008), Nakamura and Steinsson (2008), Bils and Klenow (2004)). We provide an information-based explanation for the nominal rigidity, without having to rely on Menu costs, Calvo lotteries or any other source of cost. We advance on recent work of Nakamura and Steinsson (2009), Matejka (2008) and L’Huillier (2011), as we match micro data moments without loss of tractability, have a clear interpretation for reference and sales prices, discuss how to embed in a general equilibrium structure and match further empirical evidence not directly stressed in the construction of the model.

Even though costs are continuously and independently distributed across time, we find that prices have nominal stickiness. Moreover, in our framework, the existence of sales and reference prices comes from the optimal response to macro and idiosyncratic shocks. We also provide some evidence on the relationship between sales and macroeconomic aggregates and how this is rationalized in the model, as well as we sketch an extension of the model that can also explain multi-product behavior of prices provided in Bhattarai and Schoenle (2010). We discuss two empirical tests of the model. Firstly, in our model, the longer a price stays the same, the larger will be the depth of the sale, which is confirmed, but not robustly, in the data. The second test is related to the fact that firms that have smaller
information misalignment should have higher frequency of price change. We evaluate this by comparing supermarket brands and non-supermarket brand products. In our environment, supermarket brands should have a higher frequency of price changes, under the identifying assumption that the misalignment is smaller within a supermarket, just as we find in the data.

Given that we match microdata moments with such mechanism, we embed it in a simple general equilibrium model, which is straightforward given the tractability of the solution. However, the mechanism does not provide enough inertia on the aggregates, which can be easily done, without loss of tractability, by combining such mechanism with others already suggested in the literature.

The paper is structured as follows: Section 2 provides the microeconomic mechanism, while Section 3 provides the general equilibrium model. In Section 4, we discuss advantages and pitfalls of the mechanism proposed from an empirical and theoretical points of view and, in Section 5, we conclude the paper.

3.2 Microeconomic Mechanism

In this paper, we borrow from the seminal work of Crawford and Sobel (1982) in cheap talk. To make the microeconomic mechanism more concrete, consider the following example: suppose that a firm is comprised of a sales department, that observes privately how is the demand, and the headquarters, that decides the optimal price given the behavior of the market. Moreover, the headquarters gets rewarded by the profit of the firm, while the sales department usually earns bonus if they beat their goals, such as market share or revenue. Furthermore, suppose that the information revealed cannot be contracted upon. This simple description is widely seen in everyday companies. However, in this setup, there is one agent (headquarters) that takes the decision, who is uninformed, and the other (sales) that has
preferences somewhat aligned with the manager, although not complete, but is informed and shall give the information to headquarters.

In such model, there is a finite number of actions possibly chosen in equilibrium since the informed agent mixes within some intervals, i.e., he may only say if demand is high or low, but not the true value of it. The issue, however, is that such discretization is a static result and we are interested in stickiness across periods. However, this translates into dynamic stickiness since some intervals have higher probability to be achieved. Therefore, even under shocks purely independent in a uniform distribution, we obtain nominal stickiness across time\(^1\). Intuitively, the informed agent may decide to partition his information set, for instance, in "really bad" or normal. When a "really bad" shock comes, this could explain a sale, but in normal times, which happen often, the price should be the same, since the headquarters does not actually know more than that it is "normal" times. Given this nutshell explanation of the mechanism, we’ll provide a model to discuss each of the components of the theory.

### 3.2.1 Theoretical Counterpart

As in a usual cheap-talk model, besides the distribution of information, we have to set-up where the private information lies and what is the misalignment of information.

Assuming, as usual, that profits depend on the individual price, aggregate price, aggregate demand and a measure of idiosyncratic demand or marginal cost, we can rewrite the profit function under a second-order approximation to the origin as\(^2\):

\[
\pi(p_{it}, p_t, y_t, z_{it}) = \pi_1 p_{it} + \frac{\pi_{11}}{2} p_{it}^2 + \pi_{12} p_{it} p_t + \pi_{13} p_{it} y_t + \pi_{14} p_{it} z_{it}
\]

where \(p_{it}\) is the price to be set, \(p_t\) is the aggregate price, \(y_t\) is an aggregate shock and \(z_{it}\) is an idiosyncratic shock. \(\pi^i\)'s reflect the respective derivative close to the approximation

\(^1\)We discuss the independence assumption on Section 4.

\(^2\)Mackowiak and Wiederholt (2009) have suggested such approximation for profits.
Given that $\pi_1 = 0$ and in symmetric equilibrium $\pi_{11} = -\pi_{12}$, the optimal price set by the firm is given by:

$$p_{it}^* = p_t + \frac{\pi_{13}}{|\pi_{11}|} y_t + \frac{\pi_{14}}{|\pi_{11}|} z_{it}$$  \hspace{1cm} (3.2.1)$$

Therefore, the loss due to having a suboptimal price would then be given by:

$$L = \left[ p_{it} - \left( p_t + \frac{\pi_{13}}{|\pi_{11}|} y_t + \frac{\pi_{14}}{|\pi_{11}|} z_{it} \right) \right]^2$$

Now, suppose that this same firm has two divisions, one that researches on the macro environment, concerned with the aggregate demand $y_t$, while another sector researches on the idiosyncratic conditions $z_{it}$.

Even though we don’t microfundament it, we assume that, as a result of incentives provision, such sectors don’t have the same objective function as the headquarters; the micro-sales sector should have a higher sensitivity of its own research object on the profits:

$$\pi \left( p_{it}, p_t, y_t, z_{it} \right) = \pi_1 p_{it} + \frac{\pi_{11}}{2} p_{it}^2 + \pi_{12} p_{it} p_t + \pi_{13} p_{it} y_t + \pi_{14}^{\text{micro}} p_{it} z_{it}$$

Analogously, the macro-sales sector has a higher sensitivity on the aggregate demand component:

$$\pi \left( p_{it}, p_t, y_t, z_{it} \right) = \pi_1 p_{it} + \frac{\pi_{11}}{2} p_{it}^2 + \pi_{12} p_{it} p_t + \pi_{13}^{\text{macro}} p_{it} y_t + \pi_{14} p_{it} z_{it}$$

Given such profit functions, we consider that they engage in a game since it is the headquarters that decides the optimal price of the product, even though he’s uninformed. Moreover, we assume that contracts based on the information provided cannot be written. The headquarters talks at the same time with both sectors (macro and micro), which then makes the headquarters misaligned only in one dimension with each sector. The payoffs of
the game can be written as

\[ L = \left( p_{it} - \left( p_t + \frac{\pi_{13}}{|\pi_{11}|} y_t + \frac{\pi_{14}}{|\pi_{11}|} z_{it} \right) \right)^2 \]

\[ L^{\text{micro}} = \left( p_{it} - \left( p_t + \frac{\pi_{13}}{|\pi_{11}|} y_t + \frac{\pi_{14}^{\text{micro}}}{|\pi_{11}|} z_{it} \right) \right)^2 \]

\[ L^{\text{macro}} = \left( p_{it} - \left( p_t + \frac{\pi_{13}^{\text{macro}}}{|\pi_{11}|} y_t + \frac{\pi_{14}}{|\pi_{11}|} z_{it} \right) \right)^2 \]

The equilibrium of such information revelation game is partitioned: the micro sales provides only a finite number of \( z_{it} \), while the macro provides a finite number of \( y_t \). Therefore, even though we have a bi-dimension continuously distributed variable, prices are discrete.

The partitions that the micro and the macro sales sectors provide, respectively, are then given by\(^3\):

\[ z_{n}^{\text{micro}} = C^{\text{micro}} \left[ \left( 1 + 2h - 2\sqrt{h(1+h)} \right)^n - \left( 1 + 2h + 2\sqrt{h(1+h)} \right)^n \right] \]

\[ y_n = C^{\text{macro}} \left[ \left( 1 + 2k - 2\sqrt{k(1+k)} \right)^n - \left( 1 + 2k + 2\sqrt{k(1+k)} \right)^n \right] \] \hspace{1cm} (3.2.2)

\[ p_{it}^{n} = p_t + \frac{|\pi_{11}| y_n + y_{n+1}}{2} + \frac{|\pi_{11}| z_{n}^{\text{micro}} + z_{n+1}^{\text{micro}}}{2} \]

where \( h = \frac{\pi_{14}^{\text{micro}} - \pi_{14}}{\pi_{14}} \) and \( k = \frac{\pi_{13}^{\text{macro}} - \pi_{13}}{\pi_{13}} \), the constants \( C^{\text{micro}} \) and \( C^{\text{macro}} \) are pinned down by introducing more structure on the distribution, but we assume that the lower bound of \( z \) and \( y \) are equal to zero and both follow an iid across time uniform distribution.

There are a couple of comments worth mentioning. First of all, such solution is valid if \( h \) and \( k \) are positive, which means that \( \pi_{14}^{\text{micro}} > \pi_{14} \) and \( \pi_{13}^{\text{macro}} > \pi_{13} \), an intuitive result if one considers that the distortion is such to increase the sensitivity related to the information that the sector has to provide. Another comment to make is that the partitions are monotonic. Therefore, one could easily find the maximum number of possible prices that arise in equilibrium depending on the distribution studied\(^4\). The possible number of prices

\( ^3 \)Details can be found in the appendix.

\( ^4 \) We, however, based on Crawford and Sobel (1982), know that there exists an
achieved in equilibrium is equal to all the possible combinations of intervals coming from the macro and the micro partitions.

3.2.2 Numerical Counterpart

Given the existence of price intervals, we have to evaluate how this leads to temporal stickiness, if so. We therefore focus, as an example, on three partitions in the micro and in the macro sector. The example we highlight is the one in which \( h = 1.8 \) and \( k = 0.9 \) and we have iid shocks following a uniform \([0, 1]\), just to highlight the strength of the argument\(^5\).

Even though the optimal price under perfect information does not have any stickiness, under the informational problem equilibrium, we have stickiness with sales. For clarity, we present the results of our model and of a standard retail price from Dominick’s data in Figure 3.2.1.

Moreover, this simple structure allows us to disentangle what was previously labeled as uninformative babbling equilibrium with only one partition, but we’ll discuss throughout informative equilibria as the literature usually does.

\(^5\) In order to simplify glancing at the graph, just for simplicity, we assume that \( \frac{|\pi_{11}|}{\pi_{14}} = 1 \) and \( \frac{|\pi_{11}|}{\pi_{33}} = 0.1 \) and \( p_t = 1 \), even though the stylized facts are not altered by changing such parameters.
reference prices and sales price\textsuperscript{6}. In our framework, the reference/sale price comes from the optimal response to the shock for which the headquarters has larger misalignment. In that sense, a one-sector model would already explain the reference-sales prices. The introduction of another margin (idiosyncratic) may explain the existence of some changes in prices that would not be a reference price, nor a sale, as discussed in Nakamura and Steinsson (2009). If one thinks that the macroeconomic sector has a larger misalignment, this may help testing the model, since sales would have to respond to macroeconomic conditions, as we further test in the empirical section.

The second point that arises from this model is the fact that the larger is the misalignment of information, the longer will be the nominal stickiness. However, once the shock is big enough, the sale will be deeper. Therefore, we can test another implication of the model: the longer the price remains the same, the deeper should be the sale once it occurs.

3.3 Macroeconomic Model

The mechanism that we have outlined before can be easily put in a general equilibrium model by adding the appropriate equations to close the model. There are three main equations that describe the supply side of the economy. The first one, the optimal price, is given by Equation 3.2.1. Note that such equation is standard in a macroeconomic model, tracing back to Blanchard and Kiyotaki (1987).

However, the information friction that we introduce makes it impossible to set the price equal to the optimal one. The price is staggered in intervals, as shown in Equation 3.2.2. The

\textsuperscript{6}One should further note that in our example the reference prices do not change. This occurs because the distribution of the shock is invariant. If we allow the distribution to change, it certainly changes the partition system, leading to a change in reference prices, but with the same pattern of reference/sales prices. Another possibility that we let for future research is when there is a dynamic interaction between agents or when shocks are auto regressive.
final equation, therefore, that we have to consider is the aggregate price equation. The only difference from the individual price is that idiosyncratic shocks die out and the aggregate price becomes:

\[ p^*_t = p_t + \frac{|\pi_{11}| y_t + y_{t+1}}{\pi_{13}} \]

For the aggregate demand, we consider the simplest specification, the quantitative theory of money:

\[ m_t = p_t + y_t \]

where \( m \) is the nominal GDP. Therefore, the optimal price becomes:

\[ p^*_t = p_t + \frac{|\pi_{11}| (\bar{m}_{t,n} - \bar{p}_{t,n})}{\pi_{13}} \]

where \( \bar{m}_{t,n} = m_{t,n+1} + m_{t,n} \) and \( \bar{p}_{t,n} \) is defined analogously.

From such equation, we see that an equilibrium is when the optimal price and the aggregate price equal the mean-aggregate money conditional on the information obtained. This leads to the conclusion that the price level only attains a finite number of intervals in the simplest case, clearly a simplification at odds with the data.

In order to put the results of a monetary policy shock in perspective, we highlight how it is different from a sticky prices model à la Calvo, that can be summarized by the Phillips curve:

\[ \pi_t = \beta y_t + E_t \pi_{t+1} \]

Besides this, we also consider the backward looking model:

\[ \pi_t = \beta y_t + \pi_{t-1} \]

The calibration used is standard and we follow Mankiw and Reis (2002) for the sticky price and backward looking models and we keep the same parameters of the previous section
for the informational friction model\textsuperscript{7}. The experiment that we consider is a sudden and permanent drop in the aggregate demand of 10%.

Figure 3.3.1: Monetary Policy Effects

![Figure 3.3.1: Monetary Policy Effects](image)

Figure 3.3.1 illustrates a couple of things. First of all, since prices do adjust at every point in time, even though we have nominal stickiness, the mechanism proposed does not have dynamic effects that resemble the ones observed in the data. This should not, though, be confounded with monetary policy having neutral effects on output. Since, in an iid case without learning about private information, the prices do not fully adjust to the right one \( p_t \neq m_t \), there is a permanent impact on activity, even though all effect on inflation occurs immediately.

However, as one can see in the "cheap talk adjusted", by allowing a backward looking mechanism on adjusting prices \( \pi_t = 0.85\pi_{t-1} + 0.15\pi_t^{ij} \), where \( \pi_t^{ij} \) is the inflation obtained under purely the new mechanism proposed), in which just part of the producers can adjust to the optimally information constrained prices, the results become very similar to a usual

\textsuperscript{7}In our case, \( \beta = 0.008333333 \), where \( \beta = \frac{\alpha \lambda^2}{1-\lambda} \), \( \alpha \) is the impact of demand on the optimal price and \( \lambda \) is the probability of changing from one period to another (.25).
sticky prices model, even more than a backward looking model.

The gains, though, of introducing such mechanism even if combined with a sticky price model are numerous. We can have the same usual tractability in the aggregate, but we do attain microdata moments, such as the existence of just a few prices being set, as well as reference and sales prices.

3.4 Discussion

We start with an empirical discussion of how the model behaves and we further discuss some extensions or limitations of the model from a theoretical point of view.

3.4.1 Empirical Evidence

In this section, we investigate some microdata stylized facts previously found, or that we newly bring to highlight how such mechanism is powerful in explaining the microdata.

We start with the evidence from multi-product firms, previously shown by Bhattarai and Schoenle (2010). Our analysis suggests that a simple extension of the model does explain two stylized facts: (1) firms that have higher number of products are less sticky but adjust in a smaller amount and (2) there is synchronization of price adjustment within a firm.

The number of products of a firm and their stickiness would be inversely related in our model if the partitions depend on the cost of many products, i.e., we need cross-elasticity to have mark-ups depending on others’ products. If such is the case, the number of partitions increase with the number of products, leading to less sticky prices but smaller changes. Moreover, since the cost of a given product shows up in the price of more than a product, when the cost of product \( j \) changes, the partition of product \( i \) changes too, leading to high synchronization of price adjustment. Note, however, that we don’t have full adjustment because even though a given cost may change the price of two products, it could be that in one product it is enough to change from one interval to another, while for another product
it could still be in the same interval. In any case, it is undeniable that we have some synchronization of price adjustment within a firm.

We also bring supermarket data to complement the analysis, providing further evidence in favor of the model. The mechanism proposed, in principle, is very hard to test directly. The perfect test would be to see if firms in which the information misalignment is higher, there is a smaller frequency price change. Such test seems impossible, since gathering information-misalignment data is not straightforward.

However, by looking at proxies, we can test it. One possible test that we perform is to look how supermarket brands have different price frequencies compared to other brands using retail data. Such test is important because it is an implication of our model that would not be present in a standard menu cost or Calvo lottery model. In performing this test, we consider the Dominicks’ dataset\(^8\).

Even though the degree of heterogeneity across products in a supermarket is large, as suggested in Figure 3, there is no room to believe that it should be due to supermarket brands and non-supermarket brands. Figure 3 highlights the fact that for most "categories", not only is price change frequency, but also sales frequency different compared to the average one\(^9\).

Using Dominick’s dataset, under a 5% significance level, one can reject the null hypothesis that the mean of the probability of sales frequency is the same for supermarket and non-supermarket brands. More interestingly, the supermarket brand adjusts less (not statistically significant), but discounts are more frequent (statistically significant). Such behavior is entirely consistent with our model if we assume that the misalignment is smaller within the supermarket than in a product that is not part of the supermarket-chain. With a smaller misalignment, more intervals may exist explaining the frequent discounts.

\(^8\)Details on the dataset are left to the appendix.
\(^9\)We provide a more detailed description of each definition and construction of the dataset in the appendix, but we provide the results for price changes, frequency of V-sales, frequency of changes in reference prices as defined by Midrigan and frequency of sales as defined by Midrigan.
One possible source of skepticism is whether it is a composition bias that drives such result, i.e., if supermarkets are biased producing goods that have prices being changed more frequently. In order to address such behavior, we look at the difference between supermarket brand products and non-supermarket brand products for each category\textsuperscript{10}.

\textsuperscript{10}Note that some categories don’t have supermarket brands or enough data, so we show the results for every category for which we have data. The "cleaning" procedures are discussed in the appendix and the categories that we did not use in this table are: bath soap, beer, cigaretes, front-end candies, frozen dinner, frozen entrees, grooming products, laundry detergents, paper towels, oatmeal, shampoos, toothbrushes, toothpastes, bathroom tissues and soaps.
As one can see from the table, especially for the definition of sales following the work of Midrigan, supermarket brands have a higher frequency of sales than non-supermarket brands even within categories, which rules out the composition bias criticism\textsuperscript{11}.

Another implication of our model is that the longer a price stays the same, the deeper

\textsuperscript{11}One could still argue the possibility of composition bias within categories, but this seems implausible.
should be the sale. This comes from the fact that, as an interval is bigger, the probability of remaining in such interval is larger (price the same for more time), but when it changes, it jumps to another interval, leading to a bigger change. In order to address such implication, we evaluate if the depth of a sale is correlated with the number of weeks that a price remained the same\textsuperscript{12}.

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<td>10.30***</td>
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<tr>
<td></td>
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<td>Number of upc</td>
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<td>1,009</td>
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</table>

Standard errors in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

For the Midrigan construction of sales (column 1 in Figure 3.4.3), it behaves as expected; the longer the price remains constant, the larger will be the sales depth. However, it is not robust to the V-shaped sale.

The final question that we aim, which surpasses the mechanism itself, is how sales and reference prices interact with the macroeconomic conditions. In the previous sections, we have considered that sales were an optimal response to aggregate demand shocks. The actual frequency of price changes is very important for monetary policy; if one considers the existence of sales, prices are actually very flexible and monetary policy should not have that big of effect in output. However, when considering reference prices, the opposite happens. In the more theoretical/quantitative side, Kehoe and Midrigan (2010) argue that a sale or a

\textsuperscript{12}A price remains the same if there is no sale or sale is missing. For the case of sales in Midrigan’s approach, we consider the regular price being the same. We consider a UPC-identifier fixed effect panel but we constrain the sample to be only when there is a sale.
reference price change is just a response whether it is a temporary or a permanent change, but not due to responding to different shocks. Guimaraes and Sheedy (2011), however, consider that sales come from the multiplicity of optimality, but they cannot pinpoint the right frequency endogenously. It is therefore in such context that one should understand what is sales responding to.

The literature has not yet investigated the empirical relationship though. In order to evaluate this, we look at Dominick’s dataset and we compute the cross-section change of prices at every week, computing if it was a change on the reference prices or a change in the sales prices.

A visual interpretation may help in such case. We depict, in Figure 3.4.4, the cross-section duration at every point in time for reference prices and sales prices. As one can see, it is hard to find any sizeable relationship between reference prices and the duration during the sample period studied, but there seems to be a relationship between the duration of sales and inflation.

Figure 3.4.4: Duration of prices and Inflation

Such relationship can also be depicted in regressions as in Figure 3.4.5, both for the frequency of sales as dependent variable (first table) and frequency of change of reference prices (second table).\(^{13}\)

\(^{13}\)We provide the results using the approach of Midrigan (2008) to find sales, but it is robust to other definitions such as V-change. We use fixed effects on the UPC identifier the panel regressions. We have used duration instead of frequency in the graph just for clarity.
Figure 3.4.5: Frequency and aggregate variables (dependent variable: sales and reference prices, respectively)

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<tr>
<td></td>
<td>(0.000486)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.163***</td>
<td>0.161***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000411)</td>
<td>(0.000139)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.722***</td>
<td>2.573***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000414)</td>
<td>(0.000441)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>413,424</td>
<td>412,360</td>
<td>317,376</td>
<td>317,376</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.083</td>
<td>0.070</td>
<td>0.057</td>
<td>0.039</td>
</tr>
</tbody>
</table>

Using the frequency of changes on reference prices and sales prices across time, we see that frequency of sales is negatively correlated with inflation, just as shown in the graph (with duration, it is positively correlated). Moreover, using non farm payrolls growth as a proxy for activity to keep it monthly, we see that frequency of sales increases when there is a boom\textsuperscript{14}. In any case, this is supportive evidence that sales and reference prices respond to macroeconomic conditions, as we have outlined in the model.

\textsuperscript{14}As we are using U.S. CPI percentual change, the weight of a given price change in Dominick’s should be negligible, but for the purpose of the model studied, the correlation is already indicative.
3.4.2 Theoretical Remarks

Besides the perennial interest on nominal rigidities, the question of why we would have reference prices is still in discussion in the literature. Matejka (2008) uses rational inattention to explain this, Midrigan (2008) explains through menu costs in a multi-product environment and Nakamura and Steinsson (2009) through forward-looking customer with deep habits.

The mechanism proposed here, more than purely a theoretical construction, is backed by data and is simple to be implemented in general equilibrium.

However, there are a couple of crucial assumptions that we should discuss a bit.

The first set of assumptions lays on the basis of the use of a cheap talk model; the misalignment of information between agents in the economy with non-contingent contracts. This squares nicely with all the principal-manager discussions and empirical evidence as suggested in Zbaracki et al. (2004), but it has the implication that an owner does not know his own profits contemporaneously. However, it is also important that contracts are not written based on the information provided. Even though from the theoretical perspective this is puzzling, such assumption is easily understandable once we look at how contracts are still done, with wages being prevalent and many jobs not being optimally PRP (performance-related pay schemes).

The second set of assumptions is on the specifics of the model proposed, namely the iid across time and independent idiosyncratic-macro shocks. By adding a dynamic interaction between players (Golosov et al. (2011)), or multidimensionality on the uncertainty or the policy outcome (Battaglini (2002)), full revelation, i.e., non-partitioned equilibrium is generically possible. However, a quantitative model that takes such generalizations into account is still to be done to evaluate how the stylized facts would change and we, therefore, leave it for future research.

3.5 Conclusion

In this paper, we have provided a very parsimonious mechanism that explains some of
the stylized facts of price setting. Building upon the idea that there is no contract contingent
on the information (or alternatively that the signal does not show up in the payoff) and that
agents within a firm do not have the same exact payoffs, we obtained the results.

Besides a simple explanation for the price-setting in retail markets, in which sales are
observed, we can also give a structural interpretation for sales. In our case, sales respond
to shocks for which there is more misalignment with respect to the headquarters. If these
are aggregate shocks, the relevant duration of prices from the perspective of a policy-maker
(changes due to macroeconomic environment) is relatively small.

Empirical evidence on price behavior and supermarket brands provides support to our
model, and some implications of the relationship between depth of sale/length of price spell
also provide non-robust support.

Such mechanism is easily introduced in general equilibrium. The mechanism, by itself,
does not provide the dynamics and sluggishness that we observe in aggregate data, but it
can be complemented with other sources of stickiness in the literature to match the observed
stickiness.

Appendix

Obtaining the partitions

We will consider the game between the micro manager and the headquarters, but with
the macro manager is entirely analogous:

We rewrite the objective function for the micro game as

\[ L = \left[ p_{it} - a - \left( \frac{\pi_{14}^{\text{micro}}}{\pi_{14}} \right) \right]^2 \]

\[ L^{\text{micro}} = \left[ p_{it} - a - \left( 1 + \frac{\pi_{14}^{\text{micro}} - \pi_{14}}{\pi_{14}} \right) \frac{\pi_{14}}{\pi_{11}} z_{it} \right]^2 \]
Lemma. If the optimal actions chosen from the micro manager and the headquarters are different, for every realization of $z_{it}$, then $\exists \varepsilon: \forall u, v, |u - v| \geq \varepsilon$, where $u$ and $v$ are actions induced in equilibrium. Further, the set of actions induced in equilibrium is finite.

Proof. The proof is the same as in Crawford and Sobel (1982), where we close the actions set by putting a maximum on the price range that can be offered.

\[ L = \left( p_{it} - a - \frac{\pi_{14}}{|\pi_{11}|} z_{it} \right)^2 \]
\[ L^\text{micro} = \left( p_{it} - a - (1 + h) \frac{\pi_{14}}{|\pi_{11}|} z_{it} \right)^2 \]

The second remark to make is that all equilibria of this game is partition-induced, i.e., the informed agent mixes within some optimally chosen intervals such that:

\[
\left( \frac{\pi_{14}}{|\pi_{11}|} w_q \right)^2 - \left( \frac{\pi_{14}}{|\pi_{11}|} w_{q+1} \right)^2 = 0 \tag{3.5.1}
\]

What this says is that, in the point in which the private marginal cost is equal to $w_q$, the agent is indifferent between reporting one interval or another. In any point inside an interval, he mixes within it.

The headquarters, given such information, would pick the price to maximize its own profits, leading to, under the uniform distribution assumption:

\[ p_{it} = a + \frac{\pi_{14}}{|\pi_{11}|} \frac{w_q + w_{q+1}}{2} \tag{3.5.2} \]
As we substitute 3.5.2 into 3.5.1 and solve out, we have:

\[
\frac{\pi_{14}}{\pi_{11}} w_q - a - (1 + h) \frac{\pi_{14}}{\pi_{11}} w_q = -\left[ \frac{\pi_{14}}{\pi_{11}} \left( w_{q-1} + w_q \right) - a - (1 + h) \frac{\pi_{14}}{\pi_{11}} w_q \right] + \left[ a + \frac{\pi_{14}}{2} \pi_{11} \left( w_{q-1} + w_q \right) - a - (1 + h) \frac{\pi_{14}}{\pi_{11}} w_q \right] - \left[ a + \frac{\pi_{14}}{2} \pi_{11} \left( w_{q+1} + w_q \right) - a - (1 + h) \frac{\pi_{14}}{\pi_{11}} w_q \right] = 0
\]

Once we have such difference equation, this gives us, under the extra requirement that \( w_0 = 0 \):

\[
w_n = \left( 1 + 2h - 2\sqrt{h(1 + h)} \right)^n - \left( 1 + 2h + 2\sqrt{h(1 + h)} \right)^n C[1]
\]

where \( C[1] \) is a constant pinned down by introducing another requirement, such as the maximum value that the private information variable can attain.

**Cleaning data**

The Dominick’s dataset is a supermarket chain microdata on prices and quantities. For this study, we merged all categories (29 files different to be put in one dataset) but used the prices only of the store with largest number of quotes (122), following Midrigan (2008). Moreover, we kept an observation only if the flag "ok" says that such observation can be used. We also considered UPC’s that had at least 80% of weeks having the prices being recorded and with quantities being sold. Finally, we merged the dataset of prices and quantities with one with descriptions of each UPC.

The last point on the construction of the dataset was to merge with macroeconomic
variables. As the variables are monthly (CPI and NonFarm Payrolls), we constructed the monthly percentual change and imputed that, at every week of that month, the percentual change is the same (if CPI of November is 2%, we considered that it is 2% at every week of November). Such step function can be seen whether as an approximate t/t-4 weekly CPI, but under a log approximation it should not affect the results having the inflation equally distributed across the weeks of the month.

Supermarket brand product was found from UPC identifier, given that the last five numbers identify the manufacturer.

Reference and Sales Prices

We consider two broad definitions of regular and sales prices. The change of prices is determined as usual, evaluating when a price changes from one period to another. For the sales, the first possibility is to consider V shaped paths, when the previous and the forward price are higher than the contemporaneous one.

The second definition that we use comes from the work of Midrigan (2008). A sale is a price when it is lower than the regular price, where the regular price is the modal price using a window of 10 periods, provided the modal price is used sufficiently often (at least three times).
Bibliography


