FUNDAMENTALS AND SCALING OF PASSIVE SCALAR FIELDS IN ISOTROPIC TURBULENCE

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Abstract

Turbulence is a compelling subject mainly due to its ubiquitous presence in natural and engineering phenomena. In this thesis, the focus is on temperature as a passive scalar in a turbulent velocity field with an imposed cross-stream mean temperature gradient is explored. The fundamentals of passive scalar dynamics mainly through various dimensional arguments, self-preserving solutions and a combination of numerical and experimental data are investigated.

A novel nano-scale sensor that minimizes measurements limitations of current sensors and enables more accurate and reliable data is presented. The conditions for self-similarity of the spectral equation are derived. Experimental and numerical data are used to validate and verify the conditions related the characteristic length scale of the flow, the spectrum of temperature variance and the co-spectrum of temperature and velocity. In particular, it is shown that self-preserving solutions exist for the temperature spectra. In addition, the new temperature sensor allows for data to be acquired in the dissipation range and therefore, self-similarity and scaling of the dissipation range is explored. The analysis reveals that the temperature field can be independently modeled using temperature variables only, as opposed to conventional models where knowledge of the velocity field is required.

In addition, it has been observed that the scalar PDF in the exponential tails as opposed to the velocity PDF. The exponential tails are more pronounced with the new temperature sensor compared to conventional measurement techniques. Therefore the phenomenon is investigated in this study and in particular following the linear eddy model of Kerstein. The analysis reveals more pronounced exponential tails as the low frequency content of the measured signal is excluded.

Lastly, the thesis highlights the exciting dynamics of scalar advection along with the phenomenological differences with the turbulent velocity field. Furthermore, the findings of this study show for the first time that the turbulent scalar field can
be investigated by solely measuring scalar variables with no information about the underlying turbulent velocity field (in this case, temperature). The approach opens a new perspective for analyzing and understanding turbulent flows using scalar measurements that are inherently simpler to conduct.
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# Contents

Abstract ................................................................. iii
Acknowledgements ................................................... v
List of Tables ........................................................... x
List of Figures ........................................................... xi

1 Introduction .......................................................... 1
  1.1 Motivation ......................................................... 1
  1.2 Temperature: A passive scalar with different dynamics .......... 3
  1.3 Similarity laws: Background .................................. 8
  1.4 Measurement limitations: The reason behind a lack of a good understanding of turbulent heat transfer .......... 10
  1.5 Direct Numerical Simulations: A powerful tool ................ 12
  1.6 Outline .......................................................... 13
  1.7 Collaborations ................................................... 14

2 Passive scalar flow field with a mean cross-stream gradient ...... 17
  2.1 Experimental setup: Wind tunnel ............................ 19
  2.2 Direct Numerical Simulations ................................ 21

3 Temperature Measurement: Nano-sensor design, development and characterization ................................................. 24
  3.1 Experimental procedure ....................................... 27
3.2 Data analysis .......................................................... 29
3.3 Model ................................................................. 32
3.4 Results and validation ............................................. 37
3.5 Dynamic correction of temperature signals ................. 44
3.6 Results and validation for convective heat transfer ...... 47
3.7 Development of a new nano-sensor for temperature measurement: T-NSTAP ........................................ 48
  3.7.1 Sensor design and fabrication ............................... 50
  3.7.2 Results .......................................................... 53
3.8 Summary ............................................................. 59

4 Self-similarity analysis of the temperature variance spectrum 61
  4.1 Similarity analysis ................................................. 62
  4.2 Results .............................................................. 65
  4.3 Summary ............................................................. 70

5 Self-similarity analysis of the co-spectrum of temperature and velocity 72
  5.1 Analysis ............................................................. 72
  5.2 Results .............................................................. 74
  5.3 The scalar flux ..................................................... 77
  5.4 Summary ............................................................. 81

6 Scaling and modeling of scalar dissipation spectra .......... 82
  6.1 Results .............................................................. 84
  6.2 Scaling ............................................................. 90
  6.3 Summary ............................................................. 95

7 The integral scale problem ........................................ 97
  7.1 The integral scale: Background ............................... 97
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7.2</td>
<td>Integral scale from the similarity analysis</td>
<td>99</td>
</tr>
<tr>
<td>7.3</td>
<td>Analysis</td>
<td>99</td>
</tr>
<tr>
<td>7.4</td>
<td>Conclusion</td>
<td>101</td>
</tr>
<tr>
<td>8</td>
<td><strong>Investigation of scalar probability distribution function</strong></td>
<td>103</td>
</tr>
<tr>
<td>8.1</td>
<td>Analysis</td>
<td>105</td>
</tr>
<tr>
<td>8.2</td>
<td>Conclusion</td>
<td>114</td>
</tr>
<tr>
<td>9</td>
<td><strong>Concluding remarks</strong></td>
<td>116</td>
</tr>
<tr>
<td></td>
<td><strong>Bibliography</strong></td>
<td>121</td>
</tr>
</tbody>
</table>
List of Tables

2.1 Main parameters of the DNS simulations. ........................................... 22

3.1 Cold wires used to obtain the data presented in this study. The probe
gometry is defined in figure 3.2(a), where the prong distance is $L$, the
prong length is $\ell_3$, the prong diameter is $d_3$, the stubs diameter is $d_2$, the wire length is $\ell_1$, and the wire diameter is $d_1$. .................. 29

5.1 Scalar flux power dependence using the similarity model, equation 5.9 for experimental and DNS data of the present study. Values of previous studies are also shown........................... 80
List of Figures

1.1 Fluorescent dye in a turbulent jet (Shraiman & Siggia, 2000) 4

1.2 PDF downstream a wind tunnel with (blue) and without (red) a mean gradient (Sreenivasan, 1996). 5

1.3 The Kurtosis K of the scalar difference as a function of separation r (Warhaft, 2000). Dotted line represent Gaussian case. 6

1.4 Spectral slope of the inertial range of the temperature spectra as a function of Taylor Reynolds number (Warhaft, 2000). Squares are for shear flows, circles are for grid turbulence. 7

2.1 Schematic of the wind tunnel used for measurements (Yoon & Warhaft, 1990). 22

3.1 (a) Electric circuit of a constant current anemometer (CCA) used in this experiment. (b) Schematic of the experimental setup: the cold wire is heated by a laser source while the beam is cut at a known frequency. 28

3.2 (a) Geometry of a typical probe used in this experiment (Retrieved from the Dantec dynamics website, www.dantecdynamics.com) (b) The placement of the wire in the laser beam, each element of the wire is exposed to a different intensity. 28
3.3 Characteristic cycle extracted by ensemble-average over 100 realizations ........................................ 30

3.4 Bode plot extracted by different methods of data analysis, solid line represents an exponential fit of the combined data covering the full frequency spectrum. Data were obtained for wire 5 of table 3.1. ........ 31

3.5 The lumped capacitance model used to describe the thermal response of a cold wire. The model consists of three RC branches in a parallel arrangement, where each branch corresponds to a different element of the cold wire. ................................................. 33

3.6 Bode plot (amplitude and phase) of a typical frequency response predicted by the model. The response exhibits three poles related to the frequencies characterizing the system. ................................. 37

3.7 The effect of changing the wire filament length: a. Wires 3 and 4 of table 3.1 with \( d_1 = 2.5 \mu m \), b. Wires 1 and 2 of table 3.1 with \( d_1 = 5 \mu m \). An increase in the roll-off frequency corresponds to an increase in amplitude attenuation. ................................................. 39

3.8 The effect of changing the wire filament diameter. Wires 2 and 4 of table 3.1 with \( \ell_1 = 1.25 \text{ mm} \). Similar to figure 3.7, an increase in the roll-off frequency corresponds to an increase in amplitude attenuation. 39

3.9 Effect of changing the wire filament length and diameter on (a) the first amplitude, and (b) the roll-off frequency, illustrating the trade-off between maximizing amplitude and frequency. ................................. 40

3.10 Collapse of the curves shown in figure 3.9 for \( A_1 \) and \( f_1 \). For given desired amplitude, one can obtain the corresponding roll-off frequency \( f_1 \). .............................................................. 41
3.11 (a) Effects of changing the prong length $\ell_3$ and prong diameter $d_3$ on the first amplitude. (b) Collapse of the curves in (a). To maximize the first amplitude, one seeks to minimize $\ell_3/d_3$. ........................................ 42

3.12 The effect of the velocity on the attenuation. The plot is obtained by varying the velocity in the model as applied to wire 6 of table 3.1. For high velocities, the heat transfer coefficient increases while the attenuation observed in the first amplitude decreases. .................. 43

3.13 (a) Comparison between the roll-off frequency as predicted by the model and LaRue et al. (1975) for different wire diameters. (b) Comparison of the $3dB$ cut-off frequency as predicted by LaRue et al. (1975) and the cut-off frequency from the model, legend is the same as in (a). ................................. 44

3.14 The frequency behavior of the transfer function $H(s)$, the inverse $1/H(s)$ and $H_i(s)$ showing the need to use a low pass-filter in $1/H(s)$. Data are for wire 5 of table 3.1. .............................................. 45

3.15 (a) Example of a corrected signal (fluctuating solid line) obtained by applying $H_i(s)$ to the measured signal (low amplitude solid line). The input temperature signal is shown by the dashed line, (b) Correction of the measured signal at each frequency. The expected temperature is shown by the solid line, and the roll-off frequency is shown by the dashed line. Data are for wire 5 of table 3.1. .......................... 46

3.16 Validation of the model in a convective flow, showing a time history of temperature fluctuations $T'$ normalized by the average jet temperature $T_0 = 35^\circ C$. Comparison of the signal measured by the cold wire before and after applying the dynamic correction with a fine-wire used as reference. ................................. 48
3.17 SEM images of the T-NSTAP: (a) Angular view of the probe showing metal side where Pt wire and two Au prongs resting on Si substrate. (b) Free-standing Pt wire and Au prongs. (c) Side view showing the slope by RIE-lag on Si substrate. ............................... 50

3.18 Schematic of an oscillating jet setup used to compare between the frequency response of various designs of the new sensor. ............... 52

3.19 (a) A characteristic cycle of the oscillations for two different design considerations, the first one being the regular NSTAP and the other the optimized T-NSTAP. (b) Ratio between the two sensors, both from experimental data (circles) and the model (solid line). Also shown is the frequency response predicted by the model for both a regular NSTAP and a T-NSTAP. ............................... 52

3.20 One-dimensional temperature spectra measured using a cold-wire and T-NSTAP, for $U = 9m/s$ and $x/M = 160$. Inset shows the same data on semi-logarithmic scale. ............................... 55

3.21 (a) One-dimensional temperature spectra measured using a cold-wire (CW) and T-NSTAP, for $U = 6m/s$ (lower spectra) and $U = 9m/s$ (higher spectra) both for $x/M = 160$. Solid circles represent corrected cold-wire data (CW-corrected). (b) Temperature variance for both a cold-wire and a T-NSTAP for $U = 6m/s$ (lower curves) and $U = 9m/s$ (higher curves) both for different streamwise positions. + represent corrected cold-wire variance. Solid lines represent power law fit to the data. $U = 6m/s$: $x^{0.38}$ (cold-wire), $x^{0.39}$ (T-NSTAP), $U = 9m/s$: $x^{0.42}$ (cold-wire), $x^{0.50}$ (T-NSTAP) ............................... 56
3.22 Bode plot of the ratio between the frequency response of the cold-wire to the frequency response of the T-NSTAP for both $U = 6\, m/s$ and $U = 9\, m/s$. Solid lines represent the behavior obtained from the model while markers represent experimental data.

3.23 One-dimensional temperature dissipation spectra measured using a cold-wire and T-NSTAP, for $U = 9\, m/s$ and $x/M = 160$. Solid circles represent corrected cold-wire data.

4.1 Evolution of velocity and temperature variance with streamwise position $x/M$. Solid lines represent power law fits to the data: $\overline{u^2} \propto x^{-1.3}$ for both initial conditions, $\overline{\theta^2} \propto x^{0.51}$ for Case II and $\overline{\theta^2} \propto x^{0.49}$ for Case I.

4.2 One-dimensional measured temperature spectra for two different initial conditions Case I and Case II for streamwise positions $x/M$ ranging from 40 to 200.

4.3 Coefficient of variation $\sigma$ normalized by the coefficient of variation of the unscaled data $\sigma_0$ versus different exponents, $m$ of the characteristic length scale in the scaling function $E_{s,\theta}$, for two different initial conditions (Case I and Case II). Symbols correspond to different length scales; $\circ$: Taylor microscale, $\triangle$: Batchelor length scale. Vertical dashed line corresponds to the predicted similarity solutions and error bars represent the effect of a virtual origin $x_0/M \pm 3$.

4.4 One-dimensional measured temperature spectra scaled by $E_{s,\theta} = \overline{\theta^2} \ell_\theta$ where $\ell_\theta = (x/M)^m$ with $m = 0.5$, for two different initial conditions (Case I and Case II) for streamwise positions $x/M$ ranging from 40 to 200.

5.1 Spectra of temperature variance for different simulation times.
5.2 Temperature variance spectra scaled with scaling functions $E_{\theta} = \bar{\theta}^2 \ell_\theta$ at different simulations time. .................................................. 75

5.3 Co-spectra of temperature and velocity variance for different simulation times. ................................................................. 76

5.4 Coefficient of variation $\sigma$ normalized by the coefficient of variation of the unscaled data $\sigma_0$ versus different exponents, $m$ of the characteristic length scale ......................................................... 76

5.5 Co-spectra of temperature and velocity variance for different simulation times scaled by $\ell_\theta = (\ell)^{0.5}$. Good collapse is observed at all wavenumbers. ................................................................. 77

5.6 Nusselt number $Nu$ as a function of $Re_\lambda$ (Mydlarski, 2003). .............. 80

6.1 Evolution of temperature and velocity variance with streamwise location $x/M$ (○ temperature data, squares □ velocity data). Solid lines represent power law fit to the data: Temperature $x^{0.51}$ and velocity $x^{-1.35}$. Error bars represent standard error of data acquired in different runs. ......................................................... 85

6.2 (a) One-dimensional temperature spectra measured using a cold wire and T-NSTAP, for Case II and $x/M = 160$. Rectangular selection shows the location of dissipation peak and roll-off (b) Corresponding one-dimensional dissipation spectra .................................................. 86

6.3 (a) One-dimensional temperature dissipation spectra for $x/M$ ranging from 8 to 200 for Case II. (b) Dissipation spectra after noise reduction for Case I and Case II. ......................................................... 88
6.4 (a) $\epsilon_\theta$ for different $x/M$, Case II, calculated with three different methods. Error bars are not plotted for clarity. (b) Logarithmic plot of $\epsilon_\theta$ for Case I and Case II. Solid lines represent power law fit to the data. Case II: $x^{-0.043}$ (region 1), $x^{-0.433}$ (region 2), Case I: $x^{-0.22}$ (region 1), $x^{-0.629}$ (region 2). Error bars represent standard error of data acquired in different runs. 

6.5 One-dimensional temperature dissipation spectra for six $x/M$ locations from 48 to 200 for Case I and Case II (total of twelve curves) scaled with: (a) Batchelor variables: data compared with Pao (1965) with $\gamma = 1.4$, Kraichnan (1968) with $\gamma = 5$ and Pope (2000) with $p_0 = 2, \gamma = 4.3, c_{L_0} = 0.5, c_{\eta_0} = 0.5$ and $C_\theta = 1.1$. (b) Taylor variables: data compared with model 1 $d = 5/3$, model 1 $d = 5/4$ and model 2. Experimental data was diluted for clarity. 

6.6 Scaling of all experimental data for Case I and Case II for $x/M$ from 8 to 40 (total of twenty curves) with: (a) Batchelor/Corrsin variables (b) Taylor variables. 

7.1 Scalar integral scale $L_\theta \propto x^{0.34}$ and scalar Taylor microscale $\lambda_\theta \propto x^{0.48}$ as measured in Case II of the experiments. 

7.2 Measured temperature spectra scaled by the measured scalar integral scale for Case II. 

7.3 Integral scale power law dependence on time for different computational domain size N. Error bars represent power obtained from fitting different points in the simulations, particularly at the beginning of each simulation. Trendline represents a possible fit of the power $m$ as a function of $N/L_\theta$. Dashed represents location of experimental data (wind tunnel size considered as domain size).
8.1 PDF of temperature fluctuations \((x/M = 196)\) using a cold-wire and a T-NSTAP. .......................................................... 106

8.2 PDF of temperature fluctuations at \((x/M = 196)\) high pass filtered at \(f_h = 1000\) Hz. Data acquired using a cold-wire and a T-NSTAP. . . . 106

8.3 PDF of the scalar fluctuation derivative \(\partial \theta/\partial x\) at \(x/M = 196\). Data acquired using both a regular cold-wire and a T-NSTAP. . . . . . . 107

8.4 Kurtosis of the temperature fluctuations and their derivative at different streamwise locations \(x/M\). ................................................. 108

8.5 Effect of high pass filtering on \(a\) in equation 8.3 for the T-NSTAP and cold-wire, both for the pdf and the pdf of the derivative for \(U = 9m/s\), \(\beta = 8K/m\) and \(x/M = 196\). . . . . . . . . . . . . 110

8.6 Effect of high pass filtering on \(a\) in equation 8.3 for the DNS data, both for the pdf and the pdf of the derivative for \(\beta = 8K/m\) and \(Re_{\lambda} = 50\). ......................................................... 111

8.7 The flip concept from the Kerstein model: Eddy of size \(\ell_e\) centered at \(x_0\) flips the scalar around its center producing adjacent parcels of opposite sign and amplitude \(\ell_e\beta\), that can be observed in the fluctuating scalar \(\theta\). ......................................................... 111

8.8 Ratio of the advection timescale to the diffusion timescale, \(K\) as a function of the high-pass filter frequency plotted using DNS data and experimental data. Experimental and numerical flow conditions are the same as figures 8.5 and 8.6, respectively. . . . . . . . . . . . . 114
Chapter 1

Introduction

1.1 Motivation

Turbulence is a compelling subject mainly due to its ubiquitous presence in natural phenomena and engineering applications. There are many opportunities to observe turbulent flows in our everyday surroundings, whether it be smoke, wind or a waterfall. Common to these phenomena is an unsteady, and seemingly random and chaotic behavior. Given that the general solutions of the well-known underlying equations are so far, unknown, scaling and similarity laws and analogies among different flows are often adopted and offer a significant amount of knowledge and insights without the need to solve the otherwise complicated equations.

Among the wide spectrum of turbulence aspects, we focus on turbulent heat transfer and specifically temperature as a passive scalar advected in a turbulent velocity field. Turbulent heat transfer is ubiquitous in nature and one can find it in an overwhelming number of engineering and industrial applications. Although the dynamics of the underlying velocity field is intrinsically nonlinear and much more complex to describe due to its three dimensional nature, it is often assumed that there exist many exact parallels between the statistics of the scalar field and those
of the turbulent velocity. However, as will be explained in subsequent sections, this is far from being true and in fact scalar dynamics exhibits fundamentally different behavior, which makes both exciting and essential to know for an enhanced understanding of turbulent heat transfer.

The basic understanding of turbulent mixing and transport emerged through the work of Taylor, Richardson, Kolmogorov, Obukhov and Corrsin (Monin et al., 1975). One might expect that a passive scalar field, which has no dynamical effect on turbulence, exhibits behavior that is similar to the advecting velocity field; however, the accumulated knowledge over the past years has shown that this assumption fails in several cases as will be further discussed below. Consequently, turbulent advection of a passive scalar is one of the most fascinating aspects of turbulence, primarily due to the phenomenological characteristics that fundamentally differ from that of the advecting velocity field.

In this thesis, the fundamentals of passive scalar dynamics are explored mainly through self-preserving solutions and a combination of numerical and experimental data. We will introduce a novel nano-scale sensor that minimizes measurements limitations of current sensors and enables more accurate and reliable data. The experimental data is used in conjunction with Direct Numerical Simulation, which have become an accessible approach for investigating phenomena in simple turbulent flows. The combination of analytical, experimental and computational work enables a more in depth and wider investigation of essential characteristics of passive scalar dynamics.

There exist three different components that form that core of the present study: First, the focus on temperature as a passive scalar that exhibits intrinsically different dynamics than the velocity field. Second, the main approach that will be adopted in order to shed better light on the scalar dynamics, namely similarity analysis. And lastly, the measurement limitations where conventional measurement techniques
have limited the available data and knowledge related to turbulent heat transfer. In this study, we come one big step closer to understand the dynamics of temperature as passive scalar by introducing a novel fast response temperature sensor in conjunction with self preserving solutions in order to depict novel characteristics that haven’t been observed or analyzed before. In addition, Direct Numerical Simulations are used to investigate quantities that were not acquired experimentally. In what follows is a highlight of the major aspects of this study.

1.2 Temperature: A passive scalar with different dynamics

A passive scalar is a diffusive contaminant advected in a fluid flow, having no dynamical or negligible effect on the flow (figure 1.1). The concentration of a substance advected by a turbulent velocity field exhibits a complex, chaotically evolving structure over a broad range of space and time scales. This substance could be a pollutant in the atmosphere, dye in a turbulent jet or biological bacteria.
Turbulent advection is important in many natural and engineering settings, ranging from atmospheric phenomena to combustion and biological flows. In many cases, the advected scalar has a strong effect on the fluid flow itself, by generating local forces such as buoyancy however in this thesis we concentrate on the case where the advected substance is passive. The turbulent transport phenomena is by itself a very interesting phenomena where turbulence disperses the scalar in chaotic trajectories, which causes lines of constant scalar concentration to stretch and fold until variations of scalar concentration reach smaller scales (Shraiman & Siggia, 2000). The process eventually leads to an amplification of the local concentration gradient to a point where molecular diffusivity and hence dissipation takes over. This results in rapid acceleration of mixing, which in turns gives rise
to rare large amplitude fluctuations of the scalar field. These fluctuations are revealed in the departure of the probability density function (PDF) from Gaussian behavior and the appearance of exponential tails, in total contrast with the velocity field where no mean gradient (shear flow) exists (figure 1.2). The observed tails are reflected in the Kurtosis deviating from Gaussian value (figure 1.3. The observed tails were predicted in the theory of Pumir et al. (1991) and the linear eddy model of Kerstein (1991); these studies follow mathematical models and assumptions that lead to exponential tails solution. The deviation of the PDF from gaussian behavior, also known as being a manifestation of scalar intermittencies, are one of the major phenomenological differences observed between the scalar field and the advecting velocity field. These differences make the study of scalar advection exciting and extremely appealing.

Figure 1.2: PDF downstream a wind tunnel with (blue) and without (red) a mean gradient (Sreenivasan, 1996).
Figure 1.3: The Kurtosis $K$ of the scalar difference as a function of separation $r$ (Warhaft, 2000). Dotted line represent Gaussian case.

Another important difference between the fields is observed in the spectrum of temperature fluctuations in the well-developed inertial range at low Reynolds number ($Re_λ = \lambda u/ν$) (Jayesh et al., 1994; Warhaft, 2000). For the velocity field, the inertial subrange of the velocity fluctuations spectrum, particularly the one-dimensional temperature spectrum is given by

$$F(k) = C\langle \epsilon \rangle^{2/3} k^{-5/3}$$  \hspace{1cm} (1.1)

where $C$ is regarded as a universal constant, $k$ is the wavenumber and $\langle \epsilon \rangle$ is the mean value of the energy dissipation rate. In parallel, for passive scalar field, Obukhov and Corrsin through a dimensional analysis independently extended the analysis of Kolmogorov and deduced that the inertial-convective subrange follows

$$F_\theta(k) = C_\theta\langle \epsilon \rangle^{-1/3} \langle \epsilon_\theta \rangle k^{-5/3}$$  \hspace{1cm} (1.2)

where $C_\theta$ is known as known as the Obukhov-Corrsin constant. The scaling of the inertial range given in equation 1.2 is referred to as KOC (Kolmogorov-Obukhov-Corrsin) scaling, a classical scaling argument for passive scalars (Obukhov, 1968).
Figure 1.4 shows the variation of the inertial range slope with $Re_\lambda$ and a slow $-5/3$ scaling exponent is observed. The onset of a scaling region in the scalar spectrum is not coincident with that of velocity with the power laws being different for the scalar and velocity fields. Although the power law exponent of the temperature spectrum and its growth with the Reynolds number is consistent with KOC predictions (Obukhov, 1968), the requirement of a high Reynolds number is violated. No such similar scaling is observed in the velocity field, suggesting that a simple explanation of the observed scalar spectrum using KOC arguments is unlikely.

![Figure 1.4: Spectral slope of the inertial range of the temperature spectra as a function of Taylor Reynolds number (Warhaft, 2000). Squares are for shear flows, circles are for grid turbulence.](image)

There has been an increasing development in experimental, numerical and theoretical studies of passive scalar fields in turbulent flows revealing major differences between the turbulent velocity field and scalar dynamics. In this thesis, we aim to further dissect and analyze the fundamentals of turbulent advection for a step
towards a complete understanding. Moreover, the scalar is proving to reveal subtle aspects of turbulence behavior and offer exciting grounds for more research and analysis.

1.3 Similarity laws: Background

Self-similarity has played an important role in shaping our understanding of fluid flows, particularly turbulence theory. Self-similar solutions are invariant under the scaling transformations induced by a change in the system of units. For example, in a time-dependent problem the spatial profile of a solution at one instant of time might be a rescaling of the spatial profile at any other time. These self-similar solutions are often among the few solutions of nonlinear equations that can be obtained analytically, and they can provide valuable insight into the behavior of general solutions. For example, the long-time asymptotics of solutions, or the behavior of solutions at singularities, may be given by suitable self-similar solutions. Several problems in turbulence have been approached by self-preservation or self-similarity hypotheses imposing restrictions on the dynamics of the time dependence of the spectral functions (Monin et al., 1975). The first self-preservation hypothesis of isotropic turbulence was put forward by Von-Karman & Howarth (1938) where the correlations functions or the equivalent spectral function are reduced to functions of a single variable through an appropriate choice of a length scale $L(t)$. In spectral formulation, starting by the spectral form of the Von Kármán-Howarth equation:

$$\frac{\partial E}{\partial t} = -2\nu k^2 E + T$$  \hspace{1cm} (1.3)$$

where $k$ is the wavenumber, $\nu$ is the kinematic viscosity, $E(k,t)$ is the energy spectrum, $T(k,t)$ is the spectral transfer function. Von Kármán hypotheses are written
in the form

\[ E(k, t) = L(t)s(t)^2 f(kL), \quad T(k, t) = s(t)^3 f(kL) \]  

(1.4)

The function \( s(t)^2 = \overline{u(t)^2} \) in the self-preserving part of the spectrum occurs for \( k > k_0 \) with \( k_0 \) corresponding to wavenumbers smaller than the energy-containing disturbances. Arguably, the simplest type of turbulent flow is one in which turbulence is grid-generated. Much effort was focused on studying decaying temperature fluctuations in isotropic turbulence. It is held that the decay of temperature fluctuations is greatly affected by initial conditions such as mean velocity, temperature or grid size (Warhaft & Lumley, 1978). The analysis of self-preservation as conducted by Speziale & Bernard (1992) and George (1992a) offered new approaches into the problem. George (1992a) analyzed the spectral energy equation (1.3) and concluded that it admits self-preservation solutions with the energy \( q^2 \) decaying as \( t^{-m} \) where \( m \) depends on initial conditions. Speziale & Bernard (1992) considered the transport equation for two-point velocity correlations and the viscous rate of dissipation and found two similarity solutions with \( \overline{q^2} \sim t^{-m} \) in the limit of vanishing Reynolds number and \( \overline{q^2} \sim t^{-1} \) at high Reynolds number. George & Gibson (1992) similarly investigated self-preservation of homogeneous shear flow turbulence and identified the velocity scale as obtained from the turbulent kinetic energy \( q^2 \) and Taylor microscale \( \lambda \) as the characteristic length scale. The authors identified two cases of self-preserving solutions; the first case corresponds to constant mean shear, resulting in \( \lambda \) being constant and the second where \( q^2 \) varies exponentially with time and showed that the spectral shape strongly depends on initial conditions. As for the case of passive scalars, George (1992b) considered the spectral equations governing the decay of temperature fluctuations in isotropic turbulence and found that the characteristic length scale is the scalar Taylor microscale, and the scaling function is the scalar variance. The analysis of George (1992b) agrees with the energy spectra of Comte-Bellot & Corrsin (1971) and the
temperature spectra of Warhaft & Lumley (1978) reasonably well, except at low wavenumbers. In the following chapters, self-preservation of the temperature spectral equations is investigated for grid turbulence with the addition of a mean cross-stream temperature gradient. It will be shown that for the case of the temperature variance spectra, similar to equation 1.4, \( E_\theta(k, t) = \ell_\theta(t) s_\theta(t)^2 f_\theta(k \ell_\theta) \) where \( s_\theta(t)^2 \) can be regarded as \( \theta(t)^2 \). For self-preserving solutions to exist, a characteristic length scale \( \ell_\theta(t) \) must be defined. According to Kolmogorov’s hypotheses, the small-scale turbulence is characterized by the kinematic viscosity and the rate of dissipation. Therefore, if expressed in Kolmogorov variables, the probability distributions of small-scale velocity fluctuations for different Reynolds numbers are identical. For temperature fluctuations, the common approach is to consider Corrsin (1964) or Batchelor (1959) scales, each of which applies for different ranges of Prandtl number \( Pr = \nu/\alpha \). For fluids of Prandtl number close to unity, such as air considered in this work, these two scales are equivalent. For consistency, the Batchelor scale \( \eta_\theta \) is adopted to represent these two scales throughout this work.

\[
\eta_\theta = \eta \sqrt{Pr},
\]

where \( \eta \) is the Kolmogorov microscale.

1.4 Measurement limitations: The reason behind a lack of a good understanding of turbulent heat transfer

Some of the most important sources of error in measuring turbulent fluctuations results from the sensors, spatial and temporal. In particular, a shorter sensing element is capable of capturing smaller scales, which occur in high Reynolds number
flows. The increasing need for smaller measurement probes motivated the design and development of nano-scale-sized probes specifically designed for turbulence measurements (NSTAP). Another major factor resides in the frequency response associated with the temporal resolution of the cold wire being attenuated by the heat transfer from the wire filament to its surrounding supports and the probe body itself (LaRue et al., 1975; Petit et al., 1985; Bremhorst & Krebs, 1976). Currently, there are no correction techniques for severely attenuated temperature fluctuations measurements. Modeling the cold wire probe frequency response as a first order system offers a relatively simple way to determine the cut-off frequency and is traditionally used as a benchmark for the roll-off frequency in temperature fluctuation measurements (Mydlarski & Warhaft, 1998). However, other investigations found that the frequency response of a cold wire involves a more complex interaction among the different elements of the probe, giving rise to a higher order system. These studies investigated the dynamic response of a cold wire, partly in an attempt to restore the attenuated signal (Smits et al., 1978; Paranthoen et al., 1982; Petit et al., 1985; Lecordier et al., 1984; Bremhorst & Krebs, 1976; Tagawa et al., 2005; W. & E., 1991). In particular, Petit et al. (1985) provided a theoretical and experimental study of cold wire frequency response and suggested that the system is characterized by three main time constants: the first pertains to the wire filament, and the other two are associated with the prongs, one resulting from conduction of heat from the wire to the prongs, and the other resulting from the thermal boundary layer on the prongs. The result is a transfer function characterized by a plateau that starts at the characteristic frequency describing the heat transfer from the prongs, and ends at the frequency corresponding to the roll-off frequency for the wire itself. An extension of this work was presented by Paranthoen et al. (1982) who studied both effects by considering the ratio of the cold wire length (the part of the wire where its temperature distribution is affected by end conduction) to the thickness
of the thermal boundary layer. They also compared the transfer function of a fully etched wire to a partially etched wire and found that the stubs significantly affect the transfer function. Other investigations of the dynamic characteristics of a cold wire focused on deriving the exact solutions of the heat conduction equations by including rigorous boundary conditions on each element (Tagawa et al., 2005; Tsuji et al., 1992). However, currently and as previously mentioned, there is no effective and practical method to compensate for temporal filtering in a cold wire, and, mostly, there is no fast and accurate alternative sensor.

1.5 Direct Numerical Simulations: A powerful tool

Direct Numerical Simulations aims at solving the Navier-Stokes equations, resolving all scales of motion, with initial and boundary conditions appropriate to the flow considered (Moin et al., 1998). The DNS approach was unfeasible until the 1970s when sufficient computer power became available. Although simulated Reynolds numbers remain lower than those found in most practical applications, continued increases in computer power have made it possible to simulate many laboratory flows, such as grid turbulence or shear flow, at similar Reynolds numbers to those investigated experimentally. This opens up many opportunities for DNS to investigate quantities currently difficult to measure experimentally, such as the scalar flux- a quantity of primary importance in many industrial and engineering applications, especially in atmospheric flows. Where it can be applied, Direct Numerical Simulations (DNS) provides a level of description and accuracy that cannot be matched with other approaches. DNS studies have proved extremely valuable in supplementing our knowledge from experiments of turbulence and turbulent flows. The drawback of DNS is certainly its high computational cost and the fact that the cost increases rapidly with Reynolds number (for example
cost $\propto Re^3$ for homogeneous turbulence) for, meaning that only flows with low or moderate Reynolds numbers can be simulated.

### 1.6 Outline

The work described in this thesis was initiated by the need for a further understanding of turbulent heat transfer. The outline of the chapters presented in this thesis follows:

- **Chapter 2** provides the necessary background and theoretical basis for the flow under investigation, namely isotropic turbulence with a mean cross-stream temperature gradient.

- **Chapter 3** first addresses major problems inherent in conventional high frequency temperature measurements and based on that, a lumped capacitance model that depicts the observed behavior is proposed. Following the model, the development of a new nano-scale temperature sensor (T-NSTAP) designed for temperature measurements at high frequencies is presented along with a comparison between the new sensor and a conventional cold wire in measuring temperature fluctuations in grid turbulence with imposed mean cross-stream gradient. The new nano-sensor offers a significant improvement in temperature data.

- **Chapter 4** presents a similarity analysis of the temperature variance spectra along with validation of the similarity conditions using experimental data acquired by the T-NSTAP.

- **Chapter 5** extends the analysis of Chapter 4 to the co-spectra of velocity and temperature spectra, using Direct Numerical Simulations.
• Chapter 6 investigates scaling of the passive scalar dissipation spectra. It is shown that the dissipation spectra are scaled with the scalar Taylor microscale. Moreover, new models of the that range are proposed and are solely based on temperature variables.

• Chapter 7 addresses the integral scale problem observed in experimental and numerical data and attempts to observe the discrepancy observed between the similarity analysis prediction for the dependence of the integral on time (or space) and the data. A preliminary model is proposed.

• Chapter 8 investigates scalar intermittency and exponential tails observed in the PDF of temperature fluctuations following the linear eddy model of Kerstein. Both experimental and DNS data were used.

1.7 Collaborations

The majority of the work has been described in five publications, and the layout of the remainder of the thesis follows:

• The first part of Chapter 3 consists of a paper that has been published in Measurement Science and Technology (Arwatz et al., 2013). Gilad Arwatz initiated the investigation and the laser experiments. My contribution was in model development and its application to the data acquired. More specifically, I implemented my knowledge in state-space analysis and developed a way to analyze the heat distribution in the wire.

The second part of Chapter 3 consists of a paper that has been published in Measurement Science and Technology (Arwatz et al., 2015). My first contribution is in using the results of the model presented in chapter 3 to design the T-NSTAP. Yuyang Fan fabricated the sensors based on the design guidelines.
The data was taken at Cornell University by Gilad Arwatz and myself. The oscillating jet setup was developed by Gilad Arwatz and data from that setup along with the analysis was performed by Gilad Arwatz and myself. I proposed and guided the necessary theoretical parameters to look at (variance, spectra, dissipation spectra, scalar rate of dissipation, pdf) and worked with Gilad Arwatz on data analysis and validation of the superior performance of the new temperature sensor.

- **Chapter 4** consists of a paper that has been published in the Journal of Fluid Mechanics (Bahri *et al.*, 2015). The analysis was suggested and discussed in length with William George. The theoretical similarity analysis was conducted by me. The data is the same as the data used in chapter 3.7, which was taken by myself and Gilad Arwatz. Data analysis was conducted by me. The coefficient of variation technique was developed by Gilad Arwatz and me.

- **Chapter 6** is the basis of a paper submitted to Physical Review Letters. The data is the same as the data used in chapter 3.7. Noise reduction techniques were proposed by William George and implemented by myself and Gilad Arwatz. The theoretical framework and background was guided by myself and data analysis was conducted by myself and Gilad Arwatz. Gilad Arwatz developed the dissipation spectra models.

- **Chapter 8** is the basis of a paper to be submitted to a peer-reviewed journal. The Direct Numerical Simulations data was conducted by me. The theoretical background related to the exponential tails observed in the PDF was done by me. Gilad Arwatz proposed the use of linear eddy model of Kerstein to investigate the PDF. The Numerical data analysis was done by myself.
Experimental data analysis was done by myself and Gilad Arwatza. The chapter is written in preparation of a future publication.
Chapter 2

Passive scalar flow field with a mean cross-stream gradient

In this thesis, decaying isotropic turbulence with an imposed mean cross-stream linear temperature gradient $\beta$ is investigated. This flow has been investigated in different studies, especially in a wind tunnel at Cornell University by Warhaft (2000). As Corrsin (1952) predicted, in stationary isotropic turbulence with a uniform mean velocity in one direction, an imposed cross-stream mean temperature gradient maintains itself. Several grid turbulence experiments were then reported confirming this prediction, for example the studies of Sirivat & Warhaft (1983). An interesting characteristic of the flow is the fact that the velocity fluctuations are decaying whereas temperature fluctuations $\theta$ are produced due to the mean temperature gradient. The scalar field is governed by the following evolution equation,

$$\frac{1}{2} \frac{d\bar{\theta}^2}{dt} = -\bar{\theta}v\beta - \epsilon_\theta. \quad (2.1)$$
Here, $\overline{\theta^2}$ is the temperature variance, $v$ is the cross-stream velocity fluctuations, the term $-\overline{\theta v}$ is the production of temperature variance and $\epsilon_\theta$ is the scalar rate of dissipation, which is defined as $\epsilon_\theta = \alpha \frac{\partial \theta}{\partial x_i} \frac{\partial \theta}{\partial x_i}$.

The variance of the temperature, $\overline{\theta^2}$, is directly related to the spectrum function, $E_\theta(k)$, as $\frac{1}{2} \overline{\theta^2} = \int_0^\infty E_\theta(k) dk$. The dynamical equation for the scalar spectrum is given by

$$\frac{\partial E_\theta}{\partial t} = -\beta E_{v\theta} + T_\theta - 2\alpha k^2 E_\theta,$$

where $E_{v\theta}$ is the spectrum of scalar flux (co-spectrum), $T_\theta$ is the spectral transfer function and $\alpha$ is the thermal diffusivity.

Note here that passive scalars have one and three-dimensional spectra. These spectra are simpler than velocity spectra because they depend on one variable only, rather than the three components of the velocity vector. The spatial correlation of the temperature fluctuations $\theta(x, t)$ is defined by

$$R_\theta(r) = \overline{\theta(x, t) \theta(x + r, t)}$$

A one-dimensional spectrum $F_\theta(k_1)$ representing the "energy" of waves of wavenumber $k_1$ is defined by:

$$R_\theta([r, 0, 0]) = \int_{-\infty}^{\infty} e^{jk_1r} F_\theta(k_1) dk_1$$

The one-dimensional spectrum is typically measured by deploying a hot or cold wire probe in the direction of the desired measurement, then by acquiring time history data the spectrum is obtained. In addition, a three-dimensional spectrum $E_\theta(k)$ is defined as the spectral density of waves which have the same wavenumber magnitude $k = k.k$ regardless of direction. However, since the three-dimensional spectrum is currently challenging to measure, the assumption of isotropy is used to relate the one-dimensional, measurable, spectrum $F_\theta$ to the spectrum function.
as

\[ E_\theta(k) = -kdF_\theta(k)/dk \]  

(2.5)

Another fundamental aspect is the small scale behavior of the scalar field, and for that, attention is focused on the last term of equation 2.2, which is the dissipation spectrum,

\[ D_\theta(k) = 2\alpha k^2 F_\theta(k). \]  

(2.6)

For homogeneous flows, the dissipation spectrum is related to the scalar dissipation rate \( \epsilon_\theta \) by

\[ \epsilon_\theta = \int_0^\infty D_\theta(k) dk. \]  

(2.7)

The flow is investigated from a theoretical, numerical and experimental point of view. Below, the experimental setup as well as the numerical infrastructure are presented.

### 2.1 Experimental setup: Wind tunnel

The experiments were performed in an open loop wind tunnel at Cornell University (Yoon & Warhaft, 1990), with a 9.1 m long test section, 0.91 m × 0.91 m cross-section, and a passive grid with mesh size \( M = 2.54 \text{ cm} \) (figure 2.1). One of the sidewalls of the tunnel was slightly divergent (pitch angle 0.3°) to ensure a constant centerline mean velocity as the tunnel wall boundary layer develops. The test section was wooden (1.9 cm plywood) and covered with 5.08 cm thick fiber-glass insulating sheets. A diffuser section provides a smooth transition from the square test section to a circular section housing a 48” diameter axial fan. To avoid separation at the diffuser entrance, a fine screen was placed between the end of the test section and the diffuser. For our experiment the mean test section velocity was varied from 6 m/s to 9m/s. Hot air was vented outside the laboratory to avoid heating and unwanted
Feedback. The design principle of the heater, plenum and contraction was to produce a linear temperature profile with minimal background turbulence, so that when the flow reached the turbulence grid there would be a linear temperature profile in almost laminar flow. Thus the temperature fluctuations in the test section would be solely due to the action of the grid-generated turbulence working against the temperature gradient.

The heater, placed at the entrance of the plenum, consisted of 72 horizontal 2.74 m long, equally spaced elements, consisting of nichrome ribbons inserted through 9.53 mm outsider diameter porcelain tubes. The ribbons were 0.127 mm thick and 6.35 mm wide for the top 32 elements (where more heat was required) and 0.1 mm × 4.76 mm for the bottom 40 elements. Their resistances were 1.37 and 2.26 Ω/m respectively. The heater elements were differentially controlled to produce a linear temperature profile.

The heated air produced by the heaters first passed through an air filter which removed all particles. To reduce swirl and lateral mean velocity fluctuations the flow then passed through a low heat conductivity set of honeycombs of 1.905 cm cell diameter and 15.24 cm length. This was followed by eight fine wire screens in order to dampen the turbulence. The solidity of the screens was 0.373, their wire diameter was 0.165 mm and their mesh size was 0.0794 cm. The flow was then accelerated through a 9 : 1 axisymmetric contraction. Both the plenum chamber and the contraction section were insulated with fiberglass sheets to avoid heat loss.

Mean velocity and temperature were measured using standard Pitot tubes and thermocouples. Fluctuating velocity data was acquired using a 2.5 μm hot-wire, at a sampling rate of 300 kHz for 300 s using a 16-bit A/D converter and low-pass filtered with a cut-off frequency of 150 kHz. Two temperature sensors were used: A conventional cold-wire, with 1.27 μm diameter and ℓ/𝑑 = 350, and a new nano-sensor called T-NSTAP (described in details in the following chapter), both mounted on
a linear stage, simultaneously measuring the same flow conditions. More details about both sensors will follow in the next chapter. Fluctuating temperature signals were low-pass filtered (cut-off frequency $f_c = 25 \text{ kHz}$) and digitized at a sampling rate of 50 kHz for 300s to 600s. The cold-wire and T-NSTAP were operated using a constant current circuit with the current being 0.25mA and 0.05mA, respectively. The T-NSTAP and cold wire were statically calibrated against a thermocouple by moving the probes in the cross-stream direction and taking advantage of the existing mean temperature gradient.

Various downstream positions ($x/M$ ranging between 8 and 200) for $M = 2.54cm$ were studied for two different initial conditions: (1) a mean streamwise velocity $U = 6 \text{ m/s}$ and a mean temperature gradient $\beta = 5 \text{ K/m}$, corresponding to $Re_M = MU/\nu = 9090$ (Case I), and (2) a mean velocity of $U = 9\text{ m/s}$, and a mean temperature gradient of $\beta = 8 \text{ K/m}$, corresponding to $Re_M = 13630$ (Case II). By using the same setup, Sirivat & Warhaft (1983) and Jayesh et al. (1994) showed that, for these conditions, temperature behaves as a passive scalar. The Reynolds number $Re_L$, where $L$ is the integral length scale, ranged from 75 to 330 and 130 to 600 for $Re_M = 9090$ and $Re_M = 13630$, respectively. The Taylor Reynolds number $Re_\lambda = u\lambda/\nu$, where $\lambda$ is Taylor microscale, varied from 35 to 100 and from 45 to 120 for $Re_M = 9090$ and $Re_M = 13630$, respectively.

2.2 Direct Numerical Simulations

Direct Numerical Simulations (DNS) of isotropic and homogeneous turbulence with a mean temperature gradient were performed for a Reynolds number, based on the Taylor microscale, of 100 with unity Prandtl number. To fully resolve all the scales in both the velocity and temperature fields, the DNS were performed with 130M grid points ($512 \times 512 \times 512$). In the DNS, a transport equation is solved directly
for the instantaneous temperature fluctuations $\theta$. The simulations presented in this thesis are implemented in NGA, a structured finite difference code for low Mach number turbulent reacting flows. The code utilizes structured, orthogonal grids and is based on the energy-conserving numerical methods of Desjardins et al. (2008). In this code, the momentum and continuity equations are spatially discretized with conservative, centered second-order finite difference schemes. The use of non-dissipative, centered discretization schemes is critical in ensuring that the small scales of the turbulence are not destroyed by numerical dissipation.

The governing equations are integrated in time with a semi-implicit Crank-Nicolson scheme, which allows for a larger time step to be taken while discretely
conserving kinetic energy in constant density flows. The time step in the simulations was taken to be of the order 0.0005 sec. The purpose of the numerical work is to consider a number of important questions left unanswered by experimental data, with a particular motivation of investigating the scalar flux. Different parameters of the DNS are summarized in table 1.1.
Chapter 3

Temperature Measurement: 
Nano-sensor design, development and characterization

As mentioned in chapter 1, current measurements techniques suffer from severe limitations and inherent sources of error. In this chapter, those limitations are modeled and investigated and subsequently used to design and develop and new temperature sensor. Temperature measurements are generally performed using cold wires, which suffer from what is commonly called “end conduction effect”. The end conduction effect is the heat transfer from the sensing element into the supporting structure, which results in a significantly attenuated signal. In this chapter, the dynamical behavior of cold wires and their supporting structure is investigated and a lumped parameter model that accounts for the effects of end conduction and wire response is developed. The model is verified by comparing to experimental data, where the frequency response of the wire is investigated under different heating conditions.
Temperature measurements using cold wires are generally based on static calibration methods that implicitly assume that the sensitivity is independent of frequency, that is, that the frequency response is flat. It is well known, however, that the frequency response of the wire is affected by the heat transfer from the wire to the stubs, to the prong supports, and to the probe body itself, a phenomenon known as end conduction (Smits et al., 1978). Many studies have shown that the dynamic response of the cold wire should include not only the wire thermal-inertia time constant but also time constants associated with the stubs and prongs (Smits et al., 1978; Hojstrup et al., 1976; Antonia et al., 1981; Paranthoen et al., 1982; Petit et al., 1985). In the case of temperature-fluctuation measurements, it was found that because of a disparity in size, the problem can be divided into a fast response system (the wire) and a slow response system (the stubs and the prongs). This analysis then predicts a dip in the frequency response at low frequencies (0.1-1 Hz), which can result in a significant discrepancy between the static and dynamic responses. By a small perturbation analysis of the heat equation together with experiments, Smits et al. (1978) showed that, unless the wire is operated with a length-to-diameter ($\ell/d$) greater than 1000 (generally not suitable for measurements in turbulent flows where spatial resolution is at a premium), end conduction effects are important. Browne & Antonia (1987) also showed that, for their flow, end conduction effects can be a significant source of error in the moments of temperature and its time derivative when $\ell/d < 1500$. More recently, Mydlarski & Warhaft (1998), by investigating temperature fluctuations in grid turbulence, pointed out that minimization of this error by use of a longer wire results in an increase in error from the reduced spatial resolution of the wire. They estimated that, for their experimental conditions, the scalar dissipation was underestimated by 30% when using a wire with 0.63 $\mu m$ diameter and length of $\ell/d = 1500$. Therefore, to obtain accurate measure-
ments of temperature fluctuations, it is necessary to design the probe to minimize end conduction effects, or to employ a dynamic calibration scheme.

Several investigations have aimed at determining the time constant of the wire by considering it to be a first order system (LaRue et al., 1975; Lemay et al., 2003). LaRue et al. (1975) proposed a theoretical form of the wire filament time constant by assuming the sensor to be infinite in length, that is, by neglecting end conduction effects. LaRue compared the result with experiments conducted using electrical excitation and found significant discrepancies. It is important to note that in this method only the wire filament is exposed to heating and therefore the stubs and the prongs play a minor role in the response. More recently, Lemay et al. (2003) used a chopped laser beam focused on the center of the wire and a pulsed-wire technique to determine the wire time constant as a function of a cooling velocity. Here, the prongs and the stubs were also not exposed to the heating. Lemay et al. fitted the response to a first order exponential form where the time constant pertains to the wire filament. They recognized the importance of the Gaussian distribution of the laser beam intensity and the end conduction effects, and introduced two correction factors for these effects to help fit the data better. It is important to note that their correction for end conduction effects assumed the stubs to have same diameter and same material as the wire filament.

Modeling the cold wire probe as a first order system offers a relatively simple way to determine the cut-off frequency and is traditionally used as a benchmark for the roll-off frequency in temperature fluctuation measurements (Mydlarski & Warhaft, 1998). However, other investigations found that the frequency response of a cold wire involves a more complex interaction among the different elements of the probe, giving rise to a higher order system. These studies investigated the dynamic response of a cold wire, partly in an attempt to restore the attenuated
signal (Paranthoen et al., 1982; Petit et al., 1985; Lecordier et al., 1984; Bremhorst & Krebs, 1976; Tagawa et al., 2005; W. & E., 1991).

Here, we describe an experimental method and a related analytical model to more completely quantify the effects of end conduction, and propose a method for dynamically calibrating and correcting the temperature signal. The model takes into account the interaction of the wire filament with the adjacent elements, namely the stubs and the prongs, and the parameters of the model are calculated from the properties and dimensions of the probe. The proposed correction is expressed by a transfer function which provides a simple procedure for predicting the frequency response and is readily applicable to compensating for the attenuation in temperature. First, we investigate the response of the probe to a radiative heat flux, and derive an analytical model for the heat transfer. Second, we modify the model for fluid temperature fluctuations, and demonstrate its accuracy by using the model to correct for end conduction effects experienced in measurements of fluctuating fluid temperatures.

3.1 Experimental procedure

The probe was powered by the constant current circuit shown in figure 3.1(a). To investigate the response of the probe to a radiative heat flux, an Innova-70c Argon laser was used as the heat source with beam diameter of 1.5 mm (at $1/e^2$). Typical values of the laser power and the corresponding wire temperature are 0.4 W and 160°C. Varying the laser power did not change the response and therefore a relatively high power was used to improve signal-to-noise ratio. The wire was mounted on a two-axes stage, which allowed accurate placement in the beam path. An optical chopper MC1000 with a frequency range of 1 Hz to 1 kHz was placed between the laser source and the wire. A schematic diagram of the experimental
setup can be seen in figure 3.1(b). The geometry of a typical probe used in these experiments is illustrated in figure 3.2(a). The illumination of the probe by the laser beam is shown in figure 3.2(b), and we see that the entire probe is exposed to the laser heating. Here, $Y$ is defined as the distance of the wire from the center of the beam and $r$ is the radial coordinate.

Figure 3.1: (a) Electric circuit of a constant current anemometer (CCA) used in this experiment. (b) Schematic of the experimental setup: the cold wire is heated by a laser source while the beam is cut at a known frequency.

Figure 3.2: (a) Geometry of a typical probe used in this experiment (Retrieved from the Dantec dynamics website, www.dantecdynamics.com) (b) The placement of the wire in the laser beam, each element of the wire is exposed to a different intensity.
The unsteady cold wire signals produced by chopping the laser beam were amplified, low-pass filtered (cut-off frequency $f_c = 5$ kHz) and digitized at a sampling rate of 20 kHz using a 16 bit A/D converter.

Table 3.1 lists the different probes used to obtain the results presented in this study. Note that from here on subscript $i = \{1, 2, 3\}$ will indicate the three different elements of the sensor (wire, stub and prongs, respectively). A Wollaston wire was used where the filament is made from platinum rhodium (80%-20%), the stubs are made from silver and the prongs are made from stainless steel passing through a ceramic holder.

<table>
<thead>
<tr>
<th>Wire</th>
<th>$L$ [mm]</th>
<th>$\ell_3$ [mm]</th>
<th>$d_3$ [mm]</th>
<th>$d_2$ [$\mu$m]</th>
<th>$\ell_1$ [mm]</th>
<th>$d_1$ [$\mu$m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire 1</td>
<td>1.6</td>
<td>8</td>
<td>0.3</td>
<td>50</td>
<td>0.4</td>
<td>5</td>
</tr>
<tr>
<td>Wire 2</td>
<td>1.6</td>
<td>8</td>
<td>0.3</td>
<td>50</td>
<td>1.25</td>
<td>5</td>
</tr>
<tr>
<td>Wire 3</td>
<td>1.6</td>
<td>8</td>
<td>0.3</td>
<td>50</td>
<td>0.75</td>
<td>2.5</td>
</tr>
<tr>
<td>Wire 4</td>
<td>1.6</td>
<td>8</td>
<td>0.3</td>
<td>50</td>
<td>1.25</td>
<td>2.5</td>
</tr>
<tr>
<td>Wire 5</td>
<td>1.2</td>
<td>8</td>
<td>0.3</td>
<td>50</td>
<td>0.6</td>
<td>2.5</td>
</tr>
<tr>
<td>Wire 6</td>
<td>3</td>
<td>25</td>
<td>0.3</td>
<td>50</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3.1: Cold wires used to obtain the data presented in this study. The probe geometry is defined in figure 3.2(a), where the prong distance is $L$, the prong length is $\ell_3$, the prong diameter is $d_3$, the stubs diameter is $d_2$, the wire length is $\ell_1$, and the wire diameter is $d_1$.

3.2 Data analysis

The full Bode plot of the probe response was obtained by exposing the probe to the fluctuating heat flux from the laser. To reduce the noise level, the output signal at any given frequency was ensemble-averaged over 100 realizations. Figure 3.3 shows a typical cycle after ensemble-averaging, and in what follows we describe the various methods by which the full bode plot was constructed from these cycles.
The ensemble-averaged cycles were analyzed as a step response. To obtain the low frequency response long measuring times are required, which can introduce noise due to changes in ambient conditions, and it was found that a chopping frequency of 40 Hz was the lowest frequency that could be used for step analysis. To determine the frequency response for frequencies below 40 Hz, the laser beam was chopped once by manually rotating the chopping disc. However, the manual chopping is relatively slow (≤ 5 Hz) and cannot serve as a step response to the system for higher frequencies.

First, the response was found for the entire frequency range by plotting the average amplitude of the measured signal versus the inverse of the elapsed time from the step change. This method is shown in figure 3.4 as “Direct step analysis”. Unfortunately, this approach is very sensitive to the starting point of the step (t = 0), which is difficult to determine accurately. Therefore, this method is shown in conjunction with other methods described below.

Second, the entire frequency range was analyzed as a step response, where the data were transformed from the time domain to the frequency domain. Because a
step response in the frequency domain is only defined at zero, the data were first numerically differentiated to determine the impulse response in the time domain before being transformed to the frequency domain using an FFT. The frequency response extracted through this analysis is presented in figure 3.4 as “Impulse-high FFT” for frequencies above 40 Hz and “Impulse-low FFT” for lower frequencies. The results agree well with those from the direct step analysis.

Lastly, the frequency of the laser heating was varied using the chopper disk from 20 Hz to 1 kHz in increments of 10 Hz. For each frequency, the average amplitude was extracted from the ensemble-averaged cycle and the frequency response was determined by plotting the amplitude versus frequency. The results of this analysis are shown in figure 3.4 as “step amplitude.” and as can be seen they agree well with the previous methods.

![Figure 3.4](image)

Figure 3.4: Bode plot extracted by different methods of data analysis, solid line represents an exponential fit of the combined data covering the full frequency spectrum. Data were obtained for wire 5 of table 3.1.
3.3 Model

By inspecting Bode plots such as figure 3.4 for each wire tested, we noted that the responses can all be characterized by three time constants, and in general the variation with frequency can be described by

\[
\frac{A}{A_i} = 1 - \sum_{i=1}^{3} e^{-2\pi/(f\tau_i)}
\]  

(3.1)

where \(\tau_i\) is a time constant, \(A\) is the output amplitude, and \(A_i\) is the corresponding constant amplitude. Equation 3.1 was fitted to each experiment, and it is shown as a solid line in figure 3.4. The three time constants are associated with the response and interaction of the main elements of a cold wire, namely the wire, the stubs and the prongs.

To model the response of a cold wire, we choose a lumped parameter approach, where the heat transfer rates are modeled with thermal resistors and heat accumulated in each element with thermal capacitors while assuming the temperature to be constant in each of these parts. A key factor in this approach is the Biot number, a dimensionless ratio of convection to conduction resistance to heat transfer given by \(Bi = hL_c/k\), where \(h\) is the heat transfer coefficient, \(L_c\) is a characteristic length and \(k\) is the thermal conductivity. The lumped parameter approximation is valid when Biot number is less than one for each element, which holds for all cases considered here. The lumped parameter approach is expected to provide design guidelines relating basic parameters to system response.

We note that each element of a cold wire (represented by a capacitor) is exposed to a heat flux (represented by a current going through a resistor), and therefore the model should include three series RC circuits in a parallel configuration. Moreover, the elements are connected to each other and therefore heat is conducted between adjacent elements represented by thermal contact resistance. Also, the prongs are
connected to the holder which has a relatively large thermal mass and acts as a heat sink, and so conduction from the prongs to the holder should also be included in the model. Hence, we propose the model presented in figure 3.5, which represents half of the probe under the assumption that the probe behavior is symmetric. The voltage on each node represents the temperature, and the resistances $R_1$, $R_2$ and $R_3$ correspond to the heat transferred (either by radiation or convection) to the wire, the stubs and the prongs, respectively. Each of the resistances is given by $R_i = 1/(h_i S_i)$, $h_i$ is the heat transfer coefficient and $S_i$ is the surface area receiving the applied heat. The capacitors $C_1$, $C_2$ and $C_3$ represent the heat accumulated and are related to the physical properties of each element according to $C_i = \rho_i V_i c_i$, with $\rho_i$ being the density of the element, $V_i$ the volume and $c_i$ the heat capacity. The three elements are coupled to each other through the contact resistances $R_{12}$ and $R_{23}$, given by $1/(k_c S_c)$ where $k_c$ is the thermal contact conductivity and $S_c$ is the contact surface area. Finally, the prongs conduct heat to the holder, an effect modeled by the resistance $R_4$, connecting the prongs to ground and given by $\ell/(kS)$ with $\ell$ being the length of the unheated prongs, $k$ the thermal conductivity and $S$ the cross-sectional area.

![Diagram of the lumped capacitance model](image)

Figure 3.5: The lumped capacitance model used to describe the thermal response of a cold wire. The model consists of three RC branches in a parallel arrangement, where each branch corresponds to a different element of the cold wire.
The heat transfer coefficients $h_i$ depend on the nature of the heat transfer. Here, we will consider radiative heat transfer first, since it is the dominate heat transfer mechanism in our laser heating experiments. We will then consider the case of convective heat transfer, which is relevant for the measurement of fluctuating fluid temperature.

For the heat transfer due to radiation, the heat transfer coefficient is mainly a function of the position of each element with respect to the laser beam and hence the thermal effect of the laser beam is expressed as a space-dependent boundary condition on each element. This effect is due to the fact that the intensity of the laser beam follows a Gaussian distribution with its maximum located at the center. The variation of the heat transfer coefficient with radial distance $r$ is therefore given by

$$h = h_0 e^{-2r^2/w_0^2}$$

(3.2)

where $h_0$ is the maximum heat transfer coefficient at the center of the beam, and $w_0$ is the distance from the center of the beam at which the intensity falls to $1/e^2$ of its maximum value. Figure 3.2(b) showed the placement of the wire in the laser beam. The characteristic heat transfer coefficients for the wire filament, the stubs and the prongs were found by calculating the weighted average through a numerical integration along the element. The maximum heat transfer coefficient $h_0$ is calculated from

$$h_0 = \frac{P}{\pi r_c^2 (T_o - T_\infty)}$$

(3.3)

where $P$ is the laser power, $T_o$ is the temperature of the sensor measured for a corresponding power and $T_\infty$ is the ambient temperature. In addition, $r_c$ is a
characteristic radius of the beam according to

$$r_c = \frac{\int_0^\infty r e^{-r^2/w_0^2} dr}{\int_0^\infty e^{-r^2/w_0^2} dr}$$  (3.4)

For the heat transfer due to convection, the heat transfer coefficient was obtained using Churchill & Bernstein (1977) correlation for the Nusselt number for a cylinder in cross flow, where

$$Nu = \frac{h d_i}{k} = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[ 1 + \left( \frac{Re}{28200} \right)^{5/8} \right]^{4/5}$$  (3.5)

Here, $d_i$ is the diameter of the element (wire filament, stubs or prongs), $k$ is the thermal conductivity of the fluid, $Pr = \nu/\alpha$ is the Prandtl number with $\nu$ the kinematic viscosity and $\alpha$ is the thermal diffusivity, and $Re = d_i U/\nu$ is Reynolds number and $U$ the fluid velocity. Note that other correlations of the heat transfer coefficient may be used for different geometries and flow conditions. A comprehensive review of various correlations can be found in Morgan (1975).

After obtaining the characteristic heat transfer coefficient for each element (for either convection or radiation), an energy balance was performed, which translates into a current balance on each node. Because $V_1$, $V_2$ and $V_3$ represent the temperatures of the wire filament, stubs and prongs respectively, the response is described by

$$C_1 \frac{dV_1}{dt} = \frac{V_0 - V_1}{R_1} - \frac{V_1 - V_2}{R_{12}}$$

$$C_2 \frac{dV_2}{dt} = \frac{V_0 - V_2}{R_2} + \frac{V_1 - V_2}{R_{12}} - \frac{V_2 - V_3}{R_{23}}$$

$$C_3 \frac{dV_3}{dt} = \frac{V_0 - V_3}{R_3} + \frac{V_2 - V_3}{R_{23}} - \frac{V_3}{R_4}$$  (3.6)
This system of differential equations may be solved by constructing a state-space representation, solving for each voltage in the frequency domain and transferring the results back to the time domain by performing an inverse Laplace transform.

The solutions are in the form of

\[
\frac{V_i(t)}{V_0} = 1 - \sum_{j=1}^{3} (A_{ji}e^{-2\pi f_jt}) \quad (3.7)
\]

where \( j = \{1, 2, 3\} \) corresponds to the wire filament, stubs and prongs, respectively, \( f_j \) is the frequency and \( A_j \) is the corresponding amplitude. When measuring temperature with a cold wire, one is essentially measuring the temperature of the wire filament and therefore the solution for \( V_1 \) is of interest from which, for \( i = 1 \), equation 3.7 becomes

\[
\frac{T(t)}{T_0} = 1 - \sum_{j=1}^{3} (A_{j}e^{-2\pi f_jt}) \quad (3.8)
\]

where \( T(t) \) is the temperature of the wire filament.

The typical frequency response predicted by our model is presented in an exemplary Bode plot (figure 3.6) with a typical transfer function for a step input given by

\[
H(s) = \frac{(s - z_1)(s - z_2)}{(s - p_1)(s - p_2)(s - p_3)} \quad (3.9)
\]

where \( z_1 \) and \( z_2 \) are the zeros of the system, and \( p_1, p_2 \) and \( p_3 \) are the poles which are related to the frequencies characterizing the system by \( p_i = 2\pi f_i \). Specifically, \( f_1 \) is referred to as the roll-off frequency, which is found by solving for the poles of the system transfer function. In figure 3.6, the plateau observed between the second and third poles is a result of a zero being located in between the poles. The amplitudes \( A_1, A_2 \) and \( A_3 \) are also shown in the same figure. The positions of the poles and zeros can also be seen through the phase plot.
3.4 Results and validation

Here, we present and discuss the frequency response of different cold wires to radiative heat transfer with the physical properties given in table 3.1. Each experimental result is presented in the form of the exponential fit, similar to the fit presented in figure 3.4.

The values of the resistors $R_1, R_2, R_3$ and $R_4$ were calculated using the dimensions of each element and the heat transfer coefficient. The capacitors $C_1, C_2$ and $C_3$ were obtained using $C_i = \rho_i V_i c_i$. The resistors $R_{12}$ and $R_{23}$ are the thermal contact resistances which can be found knowing the thermal contact conductivity $k_c$ and the contact surface area $A_c$. Values of the thermal contact resistance are generally determined empirically and depend on many parameters such as the material properties, the contact pressure, and the surface finish. The value of $R_{12}$ was empirically found to be $270,000 \degree K/W$ which fitted well all the experimental results. Further investigation is required to determine $R_{12}$ and $k_c$ more accurately, but
several trials suggest that this contact resistance depends on the geometry of the tapered region formed at the interface between the wire and the stubs as a result of the etching process. The characteristic length scale of this interface is \((d_2 - d_1)\) and since \(d_1\) is small compared to \(d_2\), the contact resistance is mainly related to \(d_2\) and does not vary significantly with \(d_1\). \(R_{23}\) was found to remain unchanged for the same probe, even for different etched lengths. It was found that the value of \(R_{23}\) is sensitive to the soldering, however it did not vary significantly among different probes and was of the order of 50000°K/W.

To study the interplay among different properties of a cold wire, we first discuss the effect of changing the wire filament length on the frequency response. Figure 3.7(a) shows the response of a cold wire with diameter \(d_1 = 2.5\, \mu\text{m}\) where \(\ell_1/d_1\) was increased from 300 to 500 after etching (wires 3 and 4 in table 3.1). For both cases, the experimental data are compared with the frequency response predicted by the model, where the resistances and capacitances were calculated based on the properties and dimensions of the present wire. Figure 3.7(a) shows that with increasing the wire filament length there is an increase of about 2.5 dB in the first amplitude and a decrease in the roll-off frequency from 260 Hz to 230 Hz. This trend is expected as the frequency mainly depends on the thermal capacity which increases with length. Also, the volume of the wire filament relative to other elements becomes more important which results in decreased attenuation.

Similarly, figure 3.7(b) presents the results of a cold wire with \(d_1 = 5\, \mu\text{m}\) and \(\ell_1/d_1\) varying from 80 to 260 (wires 1 and 2 of table 3.1). The same trend is observed in the frequency response, with a significant increase of around 10 dB in the first amplitude and a decrease in the roll-off frequency of 140 Hz to 80 Hz. In this case, the difference in more pronounced due the more significant change in the wire filament length.
Figure 3.7: The effect of changing the wire filament length: a. Wires 3 and 4 of table 3.1 with $d_1 = 2.5 \, \mu m$, b. Wires 1 and 2 of table 3.1 with $d_1 = 5 \, \mu m$. An increase in the roll-off frequency corresponds to an increase in amplitude attenuation.

The effect of changing the diameter is shown in figure 3.8 where the frequency response of two cold wires with the same length (wires 2 and 4 of table 3.1) is presented. As may be seen, decreasing the diameter attenuates the amplitude while increasing the roll-off frequency.

Figure 3.8: The effect of changing the wire filament diameter. Wires 2 and 4 of table 3.1 with $\ell_1 = 1.25 \, mm$. Similar to figure 3.7, an increase in the roll-off frequency corresponds to an increase in amplitude attenuation.
Figure 3.9: Effect of changing the wire filament length and diameter on (a) the first amplitude, and (b) the roll-off frequency, illustrating the trade-off between maximizing amplitude and frequency.

It is apparent that most of the attenuation of the signal is due to the first amplitude \( A_1 \), while the most important frequency is the corresponding frequency, the roll-off frequency. More insight about the interaction of the wire filament properties and their effect on the frequency response can be obtained by analyzing the first amplitude and the first frequency separately. For that purpose, the model has been used to predict the effect of varying the wire filament length and diameter. In figure 3.9a, the amplitude is plotted against the filament length \( \ell_1 \) for different wire diameters \( d_1 \) ranging from 1 \( \mu \)m to 10 \( \mu \)m, the properties of wire 1 in table 3.1 were used in the model. Note that the wire filament length is normalized by \( L \) which is the distance separating the prongs. We can clearly see that the amplitude \( A_1 \) increases with increasing length and diameter.

Another important parameter is the roll-off frequency of the wire filament \( f_1 \). Figure 3.9(b) shows \( f_1 \) as calculated from the model for different values of \( \ell_1/L \) and \( d_1 \). As expected, increasing the diameter has the undesirable effect of decreasing the roll-off frequency, and hence one faces a trade-off between maximizing amplitude and frequency. This trade-off is illustrated in figure 3.10, where we can see how
the curves for different diameters in figure 3.9(a) collapse to a single curve when plotted against $\ell_1d_1/Ld_2$, where $d_2$ is the prong diameter.

The roll-off frequency is plotted in figure 3.10 where we see that the collapse with $\ell_1d_1/Ld_2$ for the abscissa and $f_1d_1/d_2$ for the ordinate. This figure is useful for design purposes. For example, if one seeks to use the probe at a frequency that attenuates the signal by maximum of $3\,dB$, the corresponding frequency would be $f_1d_1/d_2 = 9$ which means that for $d_1 = 5\,\mu m$ and $d_2 = 50\,\mu m$, the roll-off frequency would be $90\,Hz$.

![Figure 3.10: Collapse of the curves shown in figure 3.9 for $A_1$ and $f_1$.](image)

The maximum amplitude that can be achieved for a specific probe is determined by the remaining elements, namely the prongs, the stubs and the interactions between them. The effect of the stubs is embedded in the results shown above since increasing the filament implies a decrease in the stub dimensions. To illustrate the effect of varying the prong dimensions, figure 3.11(a) shows the first amplitude for wire 1 as a function of prong length and diameter. As can be seen, minimum attenuation is achieved with the largest diameter and the shortest length. A collapse of these curves with $\ell_3d_2/Ld_3$ is shown in figure 3.11(b), illustrating the fact that to
maximize the first amplitude, one seeks to minimize $\ell_3/d_3$, which implies shorter prongs with thicker cross-sections.

![Figure 3.11](image)

Figure 3.11: (a) Effects of changing the prong length $\ell_3$ and prong diameter $d_3$ on the first amplitude. (b) Collapse of the curves in (a). To maximize the first amplitude, one seeks to minimize $\ell_3/d_3$.

The effect of the flow velocity is shown in figure 3.12, using equation 3.5 and the model as applied to wire 6 of table 3.1. For high velocities, the heat transfer coefficient increases while the attenuation observed in the first amplitude decreases. This behavior can be explained by considering that by increasing $h_1$, we effectively decrease $R_1$ in the model (figure 3.3). A similar reasoning applies for the frequency where in figure 3.12, for high velocities, a significant increase in the roll-off frequency can be observed. In general, the performance of the wire improves with increasing velocity.

This trend has been observed in previous studies. Figure 3.13a shows a comparison between the roll-off frequency $f_1$ predicted by our model and the one predicted by the commonly used first-order model of LaRue et al. (1975) for different wire diameters. A similar trend and overall good agreement is observed. The difference is related to the fact that LaRue et al. (1975) neglect end conduction effects while our model takes into account the interaction of the wire filament with adjacent elements. A fundamental difference between the two estimates lies in the fact
Figure 3.12: The effect of the velocity on the attenuation. The plot is obtained by varying the velocity in the model as applied to wire 6 of table 3.1. For high velocities, the heat transfer coefficient increases while the attenuation observed in the first amplitude decreases.

that by considering the cold wire to be a first order system, the roll-off frequency corresponds to a $3\, \text{dB}$ attenuation. In contrast, our model takes into account all the elements in the system which implies three time constants and therefore the roll-off frequency does not necessarily correspond to a $3\, \text{dB}$ attenuation. This is illustrated in figure 3.13b where the theoretical prediction of LaRue et al. (1975) is compared with the frequencies corresponding to $3\, \text{dB}$ attenuation ($f_{3\, \text{dB}}$) as predicted by our model. The figure reveals a significant difference between the two predictions implying significant attenuation in the signal at frequencies lower than those indicated by LaRue. This fact has been observed by previous investigations (Smits et al., 1978; Hojstrup et al., 1976; Antonia et al., 1981; Paranthoen et al., 1982; Petit et al., 1985).
We have shown that our model accurately describes the observed amplitude attenuation through the transfer function $H(s)$ given by equation 3.9, where the zeros and poles of $H(s)$ are dictated by the properties and dimensions of the probe. To correct the signal for this response, the measured signal is processed through the inverse transfer function while ensuring a stable output. From that perspective, it is necessary to write the inverse transfer function according to

$$H_i(s) = \frac{1}{H(s)}H_p(s)$$

(3.10)

where $H_p(s)$ is a low-pass filter necessary to keep the system stable, as well as reduce the noise at high frequencies. We used a second order low-pass filter, although higher order filters can be used. The dynamics of both $H(s)$ and the inverse $H_i(s)$ for wire 5 of table 3.1 are presented in the Bode plot shown in figure 3.14.

Figure 3.15 shows the correction presented above as applied to a measured signal obtained from wire 5 of table 3.1. The figure shows the measured and
corrected temperature in terms of $\theta/\theta_0 = (T - T_\infty)/(T_o - T_\infty)$ where $T_\infty$ is the ambient temperature and $T_o$ is the beam temperature. The input temperature signal applied to the system by chopping the laser beam is the square wave shown using a dashed line, which is also the output of an ideal sensor with a flat frequency response. As can be seen, the output of the measured signal was severely attenuated and showed less than 20% of the actual temperature. To correct for the attenuation, $H(s)$ is first obtained by matching the wire properties with the model, and then the characteristic frequency of the filter $H_{lp}(s)$ is set to be higher than the expected roll-off frequency. Then, by convolution of the measured signal with $H_i(s)$, the corrected signal is obtained. As seen in figure 3.15(a), the square wave is indeed restored and the desired temperature reading can be correctly extracted. This example illustrates the large discrepancy between the measured signal and the true expected reading, and the ability of our model to correct the measured signal and restore the expected reading.
For practical reasons, it should be noted that such large compensation levels come with large uncertainties, and therefore it is always better to use finer wires when conducting temperature fluctuations measurements.

Figure 3.15: (a) Example of a corrected signal (fluctuating solid line) obtained by applying $H_i(s)$ to the measured signal (low amplitude solid line). The input temperature signal is shown by the dashed line, (b) Correction of the measured signal at each frequency. The expected temperature is shown by the solid line, and the roll-off frequency is shown by the dashed line. Data are for wire 5 of table 3.1.

Further validation of the correction process was performed using the temperature readings obtained using wire 5 (table 3.1) at frequencies ranging from 20 Hz to 1 kHz. This specific wire poses a challenge since it shows severe attenuation as a result of its relatively small diameter and short length, as can be observed in figures 3.14 and 3.15a. Figure 3.15b presents the measured and corrected temperature for different frequencies. By looking at the figure, one can see the large discrepancy between the measured signal and the true expected reading. In this example, the correction is applied to frequencies as high as 1000 Hz and a good agreement with the expected temperature can be seen. Note that the roll-off frequency was 340 Hz and the correction still holds for higher frequencies.
3.6 Results and validation for convective heat transfer

Most cold wire temperature measurements are conducted in convective flows. To validate the model for convection, an oscillating jet of air heated by a 400 W coil with an average stream velocity of 2 m/s was used. The experimental setup is similar to the laser heating setup shown in figure 3.1 where the laser source is now replaced by a jet of hot air and the frequency of the jet oscillation was set through the angular velocity of the chopper. Temperature measurements were acquired using wire 6 as listed in table 3.1 and compared with a fine wire thermocouple (Campbell Scientific, with diameter 0.0127 mm type E) capable of accurately measuring fluctuations up to 20 Hz. To apply the proposed correction using the new model, the heat transfer coefficients in $R_1$, $R_2$ and $R_3$ were calculated using equation 3.5. Figure 3.16 shows the measured signal before and after the correction is applied, where a significant improvement is observed. To gain more insight into the signal attenuation, the normalized intensities were obtained through integration of the FFT and normalizing by the intensity of the reference signal (fine wire). While the normalized intensity of the cold wire before correction captured only 0.55 of the intensity acquired by the fine wire, this value becomes 1.08 after compensation using the proposed model. The fact that the model is over-correcting the signal is not surprising, since the fine wire signal can be expected to be slightly attenuated above 20 Hz as well. Consequently, for a convective flow, the proposed correction restores the expected measurement by reversing the effect of the observed attenuation in the cold wire signal.
3.7 Development of a new nano-sensor for temperature measurement: T-NSTAP

Generally, one can simply make the wire length-to-diameter aspect ratio, $\ell/d$, very large to avoid such end-conduction effects. However, since the smallest diameter used, typically, is on the order of 1 $\mu$m this implies that the wires need to be on the order of a millimeter in length. Although a long wire reduces the end-conduction, the spatial resolution is also reduced. Mydlarski & Warhaft (1998) pointed out that minimizing end-conduction effects can result in an increase of the total error due to spatial filtering. Wyngaard (1971), when using a cold-wire with 0.63 $\mu$m diameter and $\ell/d = 1500$, estimated that the scalar dissipation was underestimated by approximately 30%, in their study. Therefore, to obtain accurate measurements...
of temperature fluctuations, it is necessary to design a probe with minimal endconduction effects yet small enough to avoid spatial filtering.

To design an improved sensor for temperature measurements, both the temporal and spatial resolution need to be taken into account. Since the attenuation due to both spatial and temporal filtering is Reynolds number dependent, the optimal design of such a sensor will vary among applications. Spatial filtering is governed by the ratio of the length of the wire and the smallest turbulent length scales in the flow. Thus, a wire that shows severe spatial filtering in one flow can be unaffected by spatial filtering in another. Consequently, minimizing spatial filtering is achieved by reducing the length of the sensing element, $\ell$. A decreased length reduces the thermal mass of the wire filament, which on its own is desired, but it also increases the end conduction effect with low-frequency attenuation as a result. These thermal effects and trade-offs are well known and captured in detail by the lumped parameter model proposed in section 3.3. The model is used to design a new temperature probe based on the model, almost an order of magnitude smaller, with increased roll-off frequency and reduced low frequency attenuation. In addition, the model is used to test the effect of varying different geometric and material properties on the frequency response. The new sensor (T-NSTAP) closely follows the design of the nano-scaled thermal anemometry probe (NSTAP), recently developed at Princeton University. The NSTAP is manufactured using standard semiconductor fabrication techniques and its small size greatly reduces filtering in measurements of velocity fluctuation in high Reynolds number flows (Bailey et al., 2010; Vallikivi et al., 2011; Hultmark et al., 2012).

The new temperature probe (T-NSTAP) was used to measure turbulent temperature fluctuations in grid turbulence with an imposed linear mean temperature gradient. The data acquired with the new temperature sensor is compared to data
acquired simultaneously using a conventional cold-wire. Finally, the differences are compared to what is predicted by the cold-wire model described above.

3.7.1 Sensor design and fabrication

The T-NSTAP (figure 3.17) is designed to have a wire length of 200 µm (considerably smaller than conventional cold-wires, yet more than three times longer than a regular NSTAP). The rectangular cross-section of the wire has a width of approximately 2 µm, and thickness of about 100 nm.

Besides the difference in wire dimensions, a major modification in the design of the new T-NSTAP is the use of two different metals instead of a single layer of Platinum. According to the cold-wire model section 3.3, prongs with higher thermal conductivity are more desirable. Therefore, a two-layer design is adopted with 200 nm layer of gold on the prongs due to high thermal conductivity as compared to Pt. Additionally, as suggested by the model, the prongs were made shorter by 1 mm in order to reduce low-frequency attenuation.

The final design was chosen as a result of an iterative design to optimize the frequency response and minimize attenuation. In this process, the cold-wire model
was used to evaluate the effect of different design parameters and an oscillating jet setup was used to compare the frequency response of various designs (figure 3.18).

In this setup, a rotating chopper wheel was set to alternate between a cold and a hot stream of air, producing a step in temperature. Figure 3.19a shows a characteristic cycle of the oscillations for two different design considerations, the first one being the regular NSTAP and the other the optimized T-NSTAP. In this case, the oscillations were set to 30 Hz, and even at this low frequency a significant difference in the amplitude can be seen between the two sensors. Additionally, a close look at the cycle reveals that the T-NSTAP exhibits a slightly faster response. These effects are better illustrated in the bode plot of figure 3.19b. The oscillating jet setup has a complex dynamic behavior, as the chopper wheel cuts the air jet with a finite angular velocity (limiting the setup to frequencies below 1 kHz), which interferes with the temperature step response. In addition, a mixing process is added to the dynamics making it impossible to decouple the step response from the system. However, this setup can still be used to compare the performance of different sensors, rather than extracting the exact frequency response of any given sensor. This approach, combined with predictions by the cold-wire model was used to evaluate the actual frequency response of the sensors. As shown in section 3.3, we determined the thermal contact resistances in the model empirically by using a chopped laser setup, and found that all tested sensors had similar values. For the remainder of this study those values will be used in the model, for all sensors. Figure 3.19b shows a Bode plot of the predicted response of the regular NSTAP (designed for velocity measurements) and the T-NSTAP, as well as the ratio between the two sensors, both from experimental data and the model. The frequency response predicted by the model for both a regular NSTAP and a T-NSTAP is shown in figure 3.19b. A significant difference between the two sensors is clearly observed where the T-NSTAP performs better over the entire frequency
Figure 3.18: Schematic of an oscillating jet setup used to compare between the frequency response of various designs of the new sensor.

Figure 3.19: (a) A characteristic cycle of the oscillations for two different design considerations, the first one being the regular NSTAP and the other the optimized T-NSTAP. (b) Ratio between the two sensors, both from experimental data (circles) and the model (solid line). Also shown is the frequency response predicted by the model for both a regular NSTAP and a T-NSTAP.

range. In particular, the new temperature sensor exhibits substantially reduced attenuation at low frequencies due to the improvements made to the prongs and the longer wire filament, with a roll-off frequency greater than 10 kHz. Due to the complicated dynamics of the chopped jet setup, only ratios of bode plots can be considered. The experimental ratio agrees convincingly to that predicted by the model, giving further confidence in the model.
3.7.2 Results

The measurements presented in this section directly compare the frequency response of a conventional cold-wire with that of the T-NSTAP in turbulent flow conditions.

Figure 3.20 shows an example of one-dimensional temperature spectra measured using both a cold-wire and a T-NSTAP in the wind tunnel setup presented in chapter 2, with a constant level of electronic noise subtracted. A clear difference between the two sensors can be seen. At first glance, the difference might seem small and of minor importance. However, a closer look at the inset of figure 3.20 plotted in semi-logarithmic scale reveals a significant difference between the two sensors. This attenuation might be a result of either spatial or temporal filtering. Spatial filtering is first assessed by considering the temperature obtained by averaging over the field in a volume around a point. The one-dimensional spectrum $F_\theta(k_1)$ is obtained by integrating the three-dimensional spectrum $\phi_\theta(\vec{k})$ over $k_2$ and $k_3$ as follows

$$F_\theta(k_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_\theta(\vec{k}) W(\vec{k}) d k_2 d k_3$$  \hspace{1cm} (3.11)

where $W(\vec{k})$ is the averaging volume function around a parallelepiped representing the sensor. $W(\vec{k})$ is the Fourier transform of the product of three delta functions in the limit as the physical dimensions of the sensor go to zero.

For an isotropic flow, $\Phi_\theta(\vec{k})$ is related to $E_\theta(k)$, the three-dimensional spectrum around a sphere of radius $k$ by:

$$\Phi_\theta(\vec{k}) = \frac{E_\theta(k)}{4\pi k^2}$$  \hspace{1cm} (3.12)

where $k^2 = k_1^2 + k_2^2 + k_3^2$
If the probe is effectively a one-dimensional line average in a direction perpendicular to the flow with length $l$, equation 3.11 reduces to:

$$F_{\theta}(k_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{E_{\theta}(k)}{4\pi k^2} \left\{ \frac{\sin(k_2 l/2)}{k_2 l/2} \right\} dk_2 dk_3$$  \hspace{1cm} (3.13)

Applying equation 3.13 to the data with $E_{\theta}(k) = -kdF_{\theta}/dk$ reveals that, by considering the length of the cold wire ($l = 0.4 \text{ mm}$), no visible difference in the spectrum is observed as compared to the spectrum obtained without the effect of spatial filtering. A more quantitative measure of this effect is obtained by looking at the variance, which is related to the one-dimensional spectrum $F_{\theta}(k)$ by

$$\frac{1}{2} \overline{\theta^2} = \int_{0}^{\infty} F_{\theta}(k) dk$$  \hspace{1cm} (3.14)

The measured variance is within 1% of the variance corresponding to a zero length probe ($l = 0$) for all streamwise locations. Therefore, it is concluded that the spatial filtering is not the cause of the difference between the sensors observed in figure 4.4. This result is not surprising as the Batchelor scale is of the order of 0.5 mm compared to 0.4 mm for the cold wire.

Considering temporal filtering and as predicted by the cold-wire model, the cold-wire is significantly attenuated even at low frequencies. Traditionally, the cold-wire is modeled as a first order system offering a relatively simple way to determine the cut-off frequency and is used as a benchmark for the roll-off frequency in temperature fluctuation measurements. The attenuation at low wavenumbers, predicted by the cold-wire model can be clearly observed in figure 3.20. This attenuation, directly, and significantly affects the measured variance, $\overline{\theta^2}$.

Figure 3.21a shows one-dimensional temperature spectra for $x/M = 160$ and two different initial conditions (6 and 9 m/s), measured using both sensors. The model presented in section 3.3 was used to correct the cold-wire data. Specifically,
by considering the properties and dimensions of the cold-wire used to acquire the data, a transfer function is deduced from the model. The inverse transfer function, in conjunction with a low-pass filter necessary to ensure stability of the system, are used to build a correction transfer function. By convolution of the measured signal with the correction transfer function, a corrected signal is obtained (indicated by the solid line in figure 3.21a). The corrected spectrum agrees remarkably well with the T-NSTAP data over all wavenumbers. A demonstration of the significant effect of the low wavenumber attenuation, and the applicability of the model, can be captured by considering the temperature variance shown in figure 3.21b. A remarkable difference is observed between the cold-wire and T-NSTAP data, where the cold-wire data is attenuated by as much as 25% for both initial conditions and over all streamwise positions. Furthermore, it can be seen that the variance of the corrected signal agrees well with the T-NSTAP measurements. Traditionally, at least the low wavenumber data has been assumed to be accurate, which implies...
that any filtering effects on the variances are small. The observed difference in the low wavenumber regime and the resulting variance sheds uncertainty on previous data acquired with cold-wires.

Figure 3.22 shows a Bode plot of the ratio between the frequency response of the cold-wire to the frequency response of the T-NSTAP. Experimental data was obtained at different streamwise locations and the average response is shown. The figure also presents the ratio as predicted by the cold-wire model using the properties and dimensions of the two sensors along with flow conditions. Moreover, the response predicted by the model agrees very well with experimental data for both velocities. An interesting observation is the significant difference between the two cases resulting from the flow conditions. Traditionally, this behavior has not been taken into consideration however it is accurately predicted by the model as can be seen in figure 3.22. In addition, it is clear that the frequency response of the
cold-wire at low frequencies is not flat, contrary to the traditional approach. The attenuation at low frequencies is evident with more than $1dB$ (10%) attenuation, even for frequencies as low as 1-10 Hz. The roll-off observed in the response indicates that the cold-wire has a lower cut-off frequency, which is expected due to the extremely small thermal mass of the T-NSTAP. Further insight into the attenuation at high frequencies can be obtained by inspecting the one-dimensional dissipation spectra given by

$$D_\theta(k) = 2\alpha k^2 F_\theta(k) \quad (3.15)$$

Figure 3.23 shows the dissipation spectra measured using both a cold-wire and a T-NSTAP. In this representation, high frequencies are amplified as seen in equation 3.15. It can be seen that the dissipation peak amplitude measured by the cold-wire is attenuated by 30% compared to the T-NSTAP. This has a significant effect on the estimated scalar rate of dissipation given by

$$\varepsilon_\theta = \int_0^{\infty} D_\theta(k)dk \quad (3.16)$$

which is an important scaling parameter for scalar turbulence. In the case presented in figure 3.23, $\varepsilon_\theta$ obtained from the cold-wire measurements is underestimated by 35% compared to the T-NSTAP measurements. The figure also presents the cold-wire signal corrected by the model, which yields a scalar rate of dissipation within 4% of that measured by the T-NSTAP. As previously indicated, the correction works well at all frequencies and enables almost full recovery of the dissipation peak. Due to the relatively large attenuation at higher wavenumbers, and possible noise contamination, the roll-off of the corrected dissipation spectra is not fully recovered. This fact further emphasizes the need for improved sensors, such as the T-NSTAP.
Figure 3.22: Bode plot of the ratio between the frequency response of the cold-wire to the frequency response of the T-NSTAP for both $U = 6\text{m/s}$ and $U = 9\text{m/s}$. Solid lines represent the behavior obtained from the model while markers represent experimental data.

Figure 3.23: One-dimensional temperature dissipation spectra measured using a cold-wire and T-NSTAP, for $U = 9\text{m/s}$ and $x/M = 160$. Solid circles represent corrected cold-wire data.
3.8 Summary

An experimental investigation was conducted to determine the frequency response of a cold wire for temperature measurement. Although high frequency measurements of temperature are a common practice, the dynamic response is often overlooked and static calibration is conventionally applied. The results of this study emphasize the importance of the frequency response of the probe and therefore the need for dynamic correction.

An experimental method was developed to test different probes under different heating conditions. More importantly, the frequency response of the full spectrum was extracted following different methods that agreed well with each other. The results also agreed well with previous studies and specifically, we observed three different time constants characterizing the response where we believe each time constant pertains to a different element of the wire.

Following these experiments, a lumped-capacitance based model was derived to mimic the dynamic characteristics of cold wire based on its physical properties and dimensions. The model consists of three branches corresponding to each element of the probe, namely the wire filament, the stubs and the prongs. In addition, the model accounts for the interaction and heat transfer among these elements. The model shows good agreement with the experimental data and can serve to predict the response of a cold wire, knowing its properties with minimal experimental effort. The effect of changing wire properties was experimentally investigated and verified by the model. By using the model, we were able to explore large range of parameters affecting the response. By doing so, the model can serve as a sensor design and optimization tool.

Most importantly, we illustrated how our experimental procedure and model can be applied to correct for the attenuation caused by the dynamic response of the sensor for both radiation and convection heat transfer. We developed a
process by which the temperature can be restored from the measured signal. With some modifications, the experimental method and model can be applied to other temperature measurement sensors.

Finally, a new fast-response, sub-miniature, temperature sensor (T-NSTAP) is developed, evaluated and compared to a conventional cold-wire. The design and manufacturing techniques for the T-NSTAP are based on those previously developed for the Nano-Scale Thermal Anemometry Probe (NSTAP), which has proven extremely successful in capturing small scale turbulence at high Reynolds numbers. The T-NSTAP was designed to reduce low frequency attenuation by using a lumped capacitance model previously developed for cold-wire attenuation (section 3.3), and as a result the T-NSTAP has different dimensions as well as a new dual metal-layer construction. It is shown that the T-NSTAP has a dramatically improved frequency response, and that the signal from a conventional cold-wire is severely attenuated even at frequencies as low as 10 Hz. It further shown that temporal filtering and not spatial filtering is responsible for the observed attenuation. The attenuation has widespread effects on most aspects of the measurements, including the measured variances. The attenuation seen in the temperature spectra measured by the cold-wire directly results in the variance being underestimated by at least 25%. High frequency attenuation significantly reduces the dissipation peak and the scalar rate of dissipation by as much as 35%.

With a high confidence in the new temperature sensor T-NSTAP, the next chapters investigate the fundamentals of scalar dynamics using new insights offered by the new data
Chapter 4

Self-similarity analysis of the temperature variance spectrum

Self-similarity has played an important role in shaping our understanding of fluid flows, particularly turbulence theory. With the new data acquired with the T-NSTAP (chapter 3), we hope to highlight self-preserving solutions and dynamics otherwise inaccessible with the conventional measurement techniques. Although isotropic turbulence with a mean temperature gradient has been widely investigated in previous studies, for example in the work of Warhaft (2000), it has not been approached from a self-preservation perspective. In this particular configuration with a mean temperature gradient, the temperature field is passive and governed by equation 2.1 where it can be seen that the rate of change of temperature variance is affected by production and dissipation. In this chapter, we are particularly interested in the spectral equation governing this flow given in equation 2.2. The conditions for which self-preserving solutions exist are derived in section 4.1 and the experimental data are investigated for self-similarity in section 4.2.
4.1 Similarity analysis

In this chapter, the primary objective is to establish self-preserving solutions of equation 2.2, for which all terms remain in relative balance. As can be seen and in contrast to previous investigations, equation 2.2 includes an additional term \((-\beta E_{v\theta})\), resulting from the production of temperature variance.

To seek self-similarity of equation 3.16, a solution of the form

\[
E_\theta = E_{s,\theta}(t)f_1(\gamma) \\
T_\theta = T_{s,\theta}(t)g_1(\gamma) \\
E_{v\theta} = E_{s,v\theta}(t)f_2(\gamma),
\]

is assumed. In the equations, \(\gamma = k\ell_i\) is the similarity variable where \(\ell_i\) is a characteristic length scale of the flow (\(\ell_v\) for the energy spectra equation, \(\ell_\theta\) for the temperature equation and \(\ell_{v\theta}\) for the temperature flux equation), and \(E_{s,\theta}(t), T_{s,\theta}(t), E_{s,v\theta}(t)\) are the respective scaling functions. By assuming the above solution, equation 3.16 reads

\[
\dot{E}_{s,\theta} f_1 + \left[ E_{s,\theta} \frac{\dot{\ell}_\theta}{\ell_\theta} \right] f_1' \gamma = - \left[ \beta E_{s,v\theta} \right] f_2 - \left[ T_{s,\theta} \right] g_1 - \left[ \alpha \frac{E_{s,\theta}}{\ell_{v\theta}^2} \right] 2\gamma^2 f_1,
\]

where prime denotes differentiation with respect to \(\gamma\) and dot is the time rate of change. For self-similarity to hold, the terms in brackets must have same time dependence. Consequently, a set of conditions can be found as

\[
\left[ \dot{E}_{s,\theta} \frac{\ell_{v\theta}^2}{\alpha E_{s,\theta}} \right] = \epsilon_1
\]
\[ [\alpha \ell_{\theta} \dot{\ell}_{\theta}] = \epsilon_2 \quad (4.4) \]
\[ \beta \frac{E_{s,\theta}}{\alpha E_{s,\theta}} \ell_{\theta}^2 = \epsilon_3 \quad (4.5) \]
\[ T_{s,\theta} \frac{\ell_{\theta}^2}{\alpha E_{s,\theta}} = \epsilon_4 \quad (4.6) \]

where \( \epsilon_1, \epsilon_2, \epsilon_3, \) and \( \epsilon_4 \) are constants. For the remainder of the analysis, the primary focus is on the conditions presented in equations 4.3-4.6 but a similar analysis can be performed for the scalar flux, which is governed by

\[ \frac{\partial E_{v\theta}}{\partial t} = -\beta E + (v + \alpha)k^2 E_{v\theta} + T_{v\theta}, \quad (4.7) \]

from which the following set of similarity conditions is reached:

\[ \left[ \dot{E}_{s,v\theta} \frac{\ell_{v\theta}^2}{(\alpha + \nu)E_{s,v\theta}} \right] = \epsilon_{v\theta,1} \quad (4.8) \]
\[ [(\alpha + \nu) \ell_{v\theta} \dot{\ell}_{v\theta}] = \epsilon_{v\theta,2} \quad (4.9) \]
\[ \left[ \beta \frac{E_s}{\alpha E_{s,v\theta}} \ell_{v\theta}^2 \right] = \epsilon_{v\theta,3} \quad (4.10) \]
\[ \left[ \frac{T_{s,v\theta}}{(\alpha + \nu)E_{s,v\theta}} \ell_{v\theta}^2 \right] = \epsilon_{v\theta,4} \quad (4.11) \]

where \( \epsilon_{v\theta,i} \) is a constant.

Similarly, equation 1.3 dictates self-preserving solutions to the spectral equations associated with the velocity field as:

\[ \left[ \dot{E}_s \frac{\ell_v^2}{\nu E_s} \right] = \epsilon_{v,1} \quad (4.12) \]
\[ [\nu \ell_v \dot{\ell}_v] = \epsilon_{v,2} \quad (4.13) \]
\[ \left[ \frac{T_s \ell_v^2}{\nu E_s} \right] = \epsilon_{v,3} \quad (4.14) \]

where \( \epsilon_{v,i} \) is a constant.
Next, the characteristic length scale $\ell_\theta$ is derived from the similarity conditions. Integrating equation 4.4 where the constant is $\epsilon_2 = \alpha A_\theta$ reveals the time dependency of $\ell_\theta$ to be

$$\ell_\theta^2 = \ell_{0\theta}^2 + A_\theta(t - t_0),$$  \hspace{1cm} (4.16)

where $\ell_{0\theta}$ is a constant and $t_0$ is a virtual origin in time. Note here that throughout the thesis, temporal correlations will be approximated by spatial correlations using Taylor’s hypothesis, also known as the frozen turbulence approximation. The approximation can be generally described by assuming that the advection of a turbulence field past a fixed point can be taken to be entirely due to the mean flow and therefore $t = x/U$ is a good approximation. The scaling function $E_{s,\theta}$ can be related to physical quantities, particularly $\ell_\theta$ and the temperature variance $\bar{\theta}^2$ by considering the definition of the spectrum of temperature variance,

$$\bar{\theta}^2 = \int_0^{\infty} E_\theta(k, t)dk = \left(\frac{E_{s,\theta}}{\ell_\theta}\right)\int_0^{\infty} f_1(\gamma)d\gamma,$$  \hspace{1cm} (4.17)

from which we deduce, without loss of generality, that

$$E_{s,\theta} = \bar{\theta}^2 \ell_\theta,$$  \hspace{1cm} (4.18)

Knowing the form of $E_{s,\theta}$ (equation 4.18) and given that equation 4.4 provides a solution for $\ell_\theta$, the similarity conditions dictate the dynamics of the temperature variance $\bar{\theta}^2$ for the existence of a self-preserving solution. In particular, equation 4.3 provides a form of the scaling function $E_{s,\theta}$, given that the characteristic scale follows equation 4.16

$$\ln E_{s,\theta} = C_1 \frac{\alpha}{A_\theta} \ln (\ell_{0\theta}^2 + A_\theta(t - t_0)) + \ln C_2 = C_1 \frac{\alpha}{A_\theta} \ln \ell_\theta^2 + \ln C_2,$$  \hspace{1cm} (4.19)
where \( C_1 \) and \( C_2 \) are constants. Equation 4.19 implies a form for the temperature variance given by

\[
\theta^2 = C_2 \left[ \ell_\theta \right]^{2 \xi_{\theta} \alpha_{A} \theta - 1}.
\] (4.20)

### 4.2 Results

Although thorough studies of this flow have been performed previously (Sirivat & Warhaft, 1983; Warhaft, 2000), this is the first time that self-similarity of the temperature field in grid turbulence with a constant mean temperature gradient has been investigated. In order to relate the similarity analysis described in 4.1 to the one-dimensional measured temperature spectra, the isotropic relation is used as follows

\[
E_\theta = -k \frac{dF_\theta}{dk}
\] (4.21)

where \( F_\theta \) is the measured one-dimensional temperature spectrum. It should be noted that for the present flow, the large-scale thermal field is expected to be anisotropic while the assumption of small scale isotropy is reasonable. Sirivat & Warhaft (1983) observed that the assumption of small scale isotropy is still valid for this flow, by calculating the scalar rate of dissipation using three different methods, two of which assume isotropy. In other flow conditions, such as with the addition of a mean velocity gradient, the temperature derivative shows significant skewness; however, Sirivat & Warhaft (1983) showed that in their experiments, the magnitude of the skewness of the temperature derivative is small (less than 0.15). It has previously been observed that different parameters of the temperature field exhibit homogeneity for \( x/M \gtrsim 40 \). By inspecting the cross-stream profiles of the standard deviation of temperature for grid turbulence with a linear mean temperature profile, Sirivat & Warhaft (1983) observed the establishment of a homogeneous profile.
Figure 4.1: Evolution of velocity and temperature variance with streamwise position $x/M$. Solid lines represent power law fits to the data: $u^2 \propto x^{-1.3}$ for both initial conditions, $\theta^2 \propto x^{0.51}$ for Case II and $\theta^2 \propto x^{0.49}$ for Case I.

by $x/M > 40$. A value of $x/M = 30$ was found by Zhou et al. (2000) for a heated grid setup. As shown by Arwatz et al. (2015), the current data also suggests that a homogeneous flow is established beyond $x/M = 40$. Subsequently, the existence of self-preserving solutions was investigated beyond this limit.

The similarity analysis presented above predicts the scaling function $E_{s,0}$ to be a function of the temperature variance. Figure 4.1 presents the evolution of velocity and temperature variance with streamwise location $x/M$, on a logarithmic scale. As shown, temperature variance increases following a power law due to the imposed mean temperature gradient resulting in a non-zero production term in equation 3.14, while turbulence intensity decays as a power law due to the lack of production of turbulent kinetic energy. As indicated in the figure, velocity variance for both initial conditions decays with the same power. In addition, a power law behavior, consistent with the similarity result presented in equation 5.6, is observed for the temperature variance indicating a similar growth rate for both initial conditions. As the characteristic length scale (4.16) suggests the possibility
Case I

Case II

10

1

10

2

10

3

10

−8

10

−6

10

−4

10

−2

Figure 4.2: One-dimensional measured temperature spectra for two different initial conditions Case I and Case II for streamwise positions $x/M$ ranging from 40 to 200.

of a non-zero virtual origin, the temperature and velocity variance data were used to estimate its location using the method outlined in Comte-Bellot & Corrsin (1966), revealing that $x_0/M < 1$ for both fields. Therefore, the virtual origin is dropped from the equations under consideration; however, the effect of a virtual origin is shown below.

Figure 4.2 shows the unscaled spectra of temperature variance for different streamwise positions and two different initial conditions. In order to investigate the applicability of the similarity conditions to this particular flow and specifically the validity of the scaling function $E_{s,\theta}$ in revealing self-preserving solutions, the spectra are scaled by the scaling function $E_{s,\theta} = \theta^2 x^m$ and the value of $m$ that best scales the data is explored. A new method to determine the quality of the scaling was developed. The coefficient of variation, $\sigma$, of the scaled temperature spectra $E_{\theta}/E_{s,\theta}$ was inspected for different streamwise positions and averaged for all wavenumbers. Figure 4.3 presents $\sigma$ for different exponents, $m$, normalized by the coefficient of variation $\sigma_0$ of the unscaled data. It can be seen that, according
to the data, the coefficient of variation reaches a minimum at $m = 0.495$ for Case I and $m = 0.49$ for Case II.

The symbols shown in the figure relate to the coefficient of variation obtained by scaling the temperature spectra with the exponent, $m$ that corresponds to the Taylor microscale and the Batchelor scale. The similarity analysis presented in 4.1 suggests that if self-preserving solutions exist, the characteristic length scale should follow the form given by equation 4.16. In particular, for a zero virtual origin, equation 4.16 implies a $t^{0.5}$ (or $x^{0.5}$) dependence, marked by the vertical dashed line in figure 4.3. The data suggests that a similarity solution exists, and among the investigated length scales the Taylor microscale scales the spectra better and follows a power that is close to the power found by the similarity analysis.

This result can also be deduced by inspecting the scalar rate of dissipation and equation 4.18 as follows

$$
\epsilon_\theta = 2\alpha \int_0^\infty k^2 E_\theta(k, t) dk = 2 \left[ \frac{\alpha E_{\theta,\theta}}{\ell_{\theta}^3} \right] \int_0^\infty \gamma^2 f_1(\gamma) d\gamma, \quad (4.22)
$$

from which the following can be deduced:

$$
\epsilon_\theta \propto \frac{\theta^2}{\ell_{\theta}^2}. \quad (4.23)
$$

In addition, considering the following relation of the scalar rate of dissipation to the scalar Taylor microscale $\lambda_\theta$, specifically

$$
\epsilon_\theta = 6\alpha \frac{\theta^2}{\lambda_{\theta}^2}, \quad (4.24)
$$

it can be deduced that $\ell_\theta \propto \lambda_\theta$. Although the data indicates that the virtual origin is close to zero, it is particularly revealing to look at the effect of a virtual origin. The bars shown in figure 4.3 present the effect of a virtual origin of $x_0/M \pm 3$ on the
Figure 4.3: Coefficient of variation $\sigma$ normalized by the coefficient of variation of the unscaled data $\sigma_0$ versus different exponents, $m$ of the characteristic length scale in the scaling function $E_{s,\theta}$, for two different initial conditions (Case I and Case II). Symbols correspond to different length scales; $\circ$-$\bullet$: Taylor microscale, $\vartriangle$-$\bigtriangleup$: Batchelor length scale. Vertical dashed line corresponds to the predicted similarity solutions and error bars represent the effect of a virtual origin $x_0/M \pm 3$.

Figure 4.4a and 4.4b show the one-dimensional spectra of figure 4.2 scaled by $\ell_\theta = (x/M)^{0.5}$, as suggested by the similarity analysis, a value very close to the exponent corresponding to the minimum coefficient of variation and to the Taylor microscale. The uniformity of the collapse for different streamwise locations is remarkable and is good at most wavenumbers, particularly in the inertial and dissipation range. Some deviation from a uniform collapse at low wavenumbers is observed for spectra corresponding to locations closer to the grid, which might be due to initial conditions of the flow or to adjustment of the flow to the mean temperature gradient. However, overall, the collapse is convincing and supports the existence of a similarity solution.
\[
\theta(k)/E_s, \theta
\]

Figure 4.4: One-dimensional measured temperature spectra scaled by \( E_{s,\theta} = \theta^2 \ell_{\theta} \) where \( \ell_{\theta} = (x/M)^m \) with \( m = 0.5 \), for two different initial conditions (Case I and Case II) for streamwise positions \( x/M \) ranging from 40 to 200.

### 4.3 Summary

The conditions for self-preserving solutions for the evolution of a passive scalar in grid turbulence with a mean cross-stream temperature gradient were derived and experimentally investigated. The similarity analysis predicts the scaling function to be related to the temperature variance and a characteristic length scale. The length scale was found to follow \( \ell_{\theta}^2 = \ell_{\theta,0}^2 + A_{\theta}(t - t_0) \), while the analysis predicts the variance to exhibit a power law behavior.

The spectra of temperature variance were investigated for different streamwise positions and two different initial conditions. It was established that the flow reaches cross-stream homogeneity for locations exceeding \( x/M > 40 \), and, therefore, self-similarity is investigated for positions beyond this limit. A new method to objectively assess the quality of the scaling is described. In particular, the minimum of the coefficient of variation for the normalized temperature spectra at different streamwise positions, integrated over all wavenumbers, is investigated. The data for two different initial conditions exhibit a minimum very close to the similarity analysis prediction.
The experimental data confirmed the results of the similarity analysis and further suggests that the virtual origin and initial length are both zero giving rise to the characteristic length scale varying as $x^{0.5}$ (using Taylor’s hypothesis). As predicted by the similarity analysis, the data shows the variance growing as a power law with streamwise position. The collapse of the temperature spectra when scaled with the theoretical result, $x^{0.5}$, is excellent for all wavenumbers with a scatter of 5 to 10% based on the normalized coefficient of variation.
Chapter 5

Self-similarity analysis of the co-spectrum of temperature and velocity

In this chapter, the scalar flux is investigated through the similarity analysis presented in the previous chapter and the implications of a similarity solution on the turbulent heat flux is studied.

5.1 Analysis

Self-preservation of the spectral equations governing the turbulent temperature field was presented in chapter 4. Here, particular attention is given to the co-spectrum of temperature and velocity fluctuations, given its intrinsic connection to heat transfer, where as a result of a mean temperature gradient, a turbulent heat flux exists. Measuring the turbulent heat flux is a challenging task. However, it is of high importance due to its ubiquitous presence in many industrial and engineering applications, especially in environmental and atmospheric flows.
Consequently, our understanding of the underlying physics is very limited, which makes accurate predictions very challenging. Therefore, the common approach is to develop models for the evolution of temperature in a turbulent flow field by drawing heavily on a presumed analogy between the transfer of momentum and heat. However, as discussed in chapter 1, the accumulated knowledge shows that this analogy is incorrect and the dynamics of the temperature fields exhibit major differences from the advecting turbulent field.

The dynamics of the turbulent heat flux is described by the co-spectrum between the transverse velocity and the temperature, $E_{v\theta}(k)$, which, if integrated over all wavenumbers is the flux. Among the set of similarity conditions previously derived, the third similarity condition (equation 4.6) is investigated where, due to the lack of experimental data relating to the heat flux, DNS data as described in chapter 2 is used. The third similarity condition follows

$$E_{s,v\theta} = \beta \frac{E_s}{\alpha \varepsilon_{v\theta}} \ell_{v\theta}^2$$

(5.1)

and involves the scaling function of the co-spectrum, $E_{s,v\theta}$, and thus is closely related to the turbulent heat flux. The scaling function $E_{s,\theta}$ can be related to physical quantities, particularly $\ell_\theta$ and the temperature variance $\bar{\theta}^2$ by considering the definition of the spectrum of temperature variance,

$$\bar{\theta}^2 = \int_0^\infty E_\theta(k, t)dk = \left(\frac{E_{s,\theta}}{\ell_\theta}\right) \int_0^\infty f_1(\gamma)d\gamma,$$

(5.2)

from which we deduce, without loss of generality, that

$$E_{s,\theta} = \bar{\theta}^2 \ell_\theta,$$

(5.3)
Figure 5.1: Spectra of temperature variance for different simulation times.

This, combined with the third similarity condition, yields a form for the scaling function of the co-spectrum as:

$$E_{s,\theta} = \alpha \beta^{2/\ell_\theta}$$

(5.4)

5.2 Results

As shown in equation 5.3, the spectrum of temperature variance exhibits self-preserving solutions with scaling function $E_{s,\theta} = \bar{\theta^2} \ell_\theta$, where $\ell^2_\theta = \ell^2_0 + A_\theta(t - t_0)$, as shown in chapter 4. To validate the numerical data presented in this study, we verify that the spectrum of temperature variance indeed scales with the similarity variables. Figure 5.1 presents the temperature spectra computed from DNS data, while figure 5.2 shows the same data scaled with $E_{s,\theta} = \bar{\theta^2} \ell_\theta$. The spectra collapse convincingly, in agreement with the experimental results presented in chapter 4.

Next, the co-spectrum $E_{\varphi\theta}$ is computed from DNS data and presented in figure 5.3. The co-spectra are noisier than the temperature spectra since there is no
Figure 5.2: Temperature variance spectra scaled with scaling functions $E_{s,\theta} = \theta^2 \ell_\theta$ at different simulations time.

A mathematical limitation to prevent them from changing sign. To plot the co-spectra in logarithmic coordinates, the absolute value of negative excursions has been considered. The method of coefficient of variation presented in chapter 4 to investigate the applicability of the similarity conditions to this particular flow and specifically the validity of the scaling function $E_{s,\theta}$ in revealing self-preserving solutions is followed. The spectra are scaled by the scaling function $E_{s,\theta} = \theta^2 t^{-m}$ where the value of $m$ that best scales the data is explored. In particular, the coefficient of variation is calculated for different powers $m$ of the lengthscale $\ell_\theta$. As shown in figure 5.4 and as required by the similarity analysis, the co-spectrum is best scaled with $m = 0.5$ coinciding with the minimum of the coefficient of variation.

Figure 5.5 shows the co-spectra of figure 5.3 scaled by $\ell_\theta \propto (t)^{-0.5}$, as suggested by the similarity analysis. Note that the data shows that the virtual origin is relatively negligible and therefore, it has been dropped from the similarity equations.
Figure 5.3: Co-spectra of temperature and velocity variance for different simulation times.

Figure 5.4: Coefficient of variation $\sigma$ normalized by the coefficient of variation of the unscaled data $\sigma_0$ versus different exponents, $m$ of the characteristic length scale.
Figure 5.5: Co-spectra of temperature and velocity variance for different simulation times scaled by $\ell_\theta = (t)^{-0.5}$. Good collapse is observed at all wavenumbers.

A remarkable uniformity of the collapse is observed and is convincing over all wavenumbers.

5.3 The scalar flux

In this section, the implication of the similarity analysis on the scalar flux is investigated. A closer look at the third similarity condition reveals an expression for the scalar flux, which is solely based temperature variables. Knowing the form of $E_{s,\theta}$ (equation 5.3), the similarity conditions dictates the dynamics of the temperature variance $\overline{\theta^2}$ for the existence of a self-preserving solution. In particular, equation 4.4 provides a form of the scaling function $E_{s,\theta}$

$$\ln E_{s,\theta} = C_1 \frac{\alpha}{A_\theta} \ln (\ell_{\theta 0}^2 + A_\theta (t - t_0)) + \ln C_2$$

$$= C_1 \frac{\alpha}{A_\theta} \ln \ell_\theta^2 + \ln C_2,$$

(5.5)
where $C_1$ and $C_2$ are constants. Equation 5.5 implies a form for the temperature variance given by

$$\overline{\theta^2} = C_2 \left[ \ell_\theta \right]^{2\gamma \theta - 1}. \quad (5.6)$$

The scalar flux is related to the co-spectrum through:

$$\overline{v\theta} = \int_0^\infty E_{v\theta}(k, t) dk = \left( \frac{E_{s, v\theta}}{\ell_\theta} \right) \int_0^\infty f_2(\gamma) d\gamma. \quad (5.7)$$

from where it can be deduced that:

$$E_{s, v\theta} = \overline{v\theta} \ell_\theta. \quad (5.8)$$

It can be further be shown (using equation 5.4) that the turbulent heat flux can be expressed as:

$$\overline{v\theta} = \frac{\alpha}{\beta} \frac{\overline{\theta^2}}{\ell_\theta^2}. \quad (5.9)$$

Consequently, if the flow exhibits self-similarity, then the scalar flux behaves as a power law as follows:

$$\overline{v\theta} = C_2 \frac{\alpha}{\beta} \left[ \ell_\theta \right]^{2\gamma \theta - 1}. \quad (5.10)$$

It is important to note again that the heat flux is governed only by parameters related to the temperature field. Thus, no information is needed about the velocity field to determine the heat flux in this particular flow configuration. This is of major importance to the turbulence community, and particularly the atmospheric sciences where the scalar flux is of main interest. Measuring the scalar flux is a tedious, often impossible task and therefore one relies on models and dimensional analysis. So far, various models for the scalar flux have used quantities from the velocity field. The form given in equation 5.10 offers for the first time a model of the scalar flux relying only on temperature variables. Practically, it is simpler to measure a scalar,
particularly temperature and especially with the newly developed T-NSTAP and therefore the proposed model offers a more approachable way to obtain the scalar flux, which determines major aspects of turbulent heat transfer.

Moreover, the analytical form of the turbulent heat flux derived here enables us to, for the first time, find an analytical expression for the Nusselt number as:

\[ Nu = \frac{-\bar{v}\theta}{\alpha\beta} = -\frac{\theta^2}{\beta^2 \ell_\theta^2} \]  

(5.11)

The DNS data shows that temperature variance varies as \( \overline{\theta^2} \propto t^{0.54} \) while as previously mentioned, \( \ell_\theta \propto t^{0.5} \). Therefore, according to the similarity analysis, \( \bar{v}\theta \propto t^{-0.46} \). The data for the scalar flux shows that \( \bar{v}\theta \propto t^{-0.43} \), agreeing reasonably close with the predicted value of the similarity analysis.

Generally, the scalar flux is determined by assuming that \( u_{rms} \propto v_{rms} \) and using the correlation coefficient \( \rho_{v\theta} \) where

\[ \bar{v}\theta = \rho_{v\theta} v_{rms} \theta_{rms} \]  

(5.12)

Looking at the experimental data, \( u_{rms} \propto x^{-0.685} \) while \( \theta \propto x^{0.25} \). Assuming \( u_{rms} \propto v_{rms} \) and using equation 5.12 with \( \rho_{v\theta} \propto -0.7 \) (Mydlarski, 2003), the scalar flux follows \( \bar{v}\theta \propto -0.7Cx^{-0.435} \) where C is a constant. With \( \theta \propto x^{0.25} \), the similarity analysis predicts \( \bar{v}\theta \propto x^{-0.48} \), which agrees well with the approximation (within experimental error).

Previous studies (Jayesh & Warhaft, 1992b; Mydlarski & Warhaft, 1998) have experimentally measured the co-spectrum, and thus the turbulent heat flux, using the same flow configuration studied in this thesis. Mydlarski (2003) investigated the heat flux for different Reynolds number \( Re_\lambda \) and established an empirical relation for \( Nu \) as a function of \( Re_\lambda \) (figure 5.6). The authors observed a \( Nu \propto Re_\lambda^{1.1} \) behavior using their own data and \( Nu \propto Re_\lambda^{1.76} \) using the data of Jayesh & Warhaft.
Figure 5.6: Nusselt number $Nu$ as a function of $Re_\lambda$ (Mydlarski, 2003).

Table 5.1: Scalar flux power dependence using the similarity model, equation 5.9 for experimental and DNS data of the present study. Values of previous studies are also shown.

<table>
<thead>
<tr>
<th>Source</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNS</td>
<td>-0.46</td>
</tr>
<tr>
<td>Experiment</td>
<td>-0.48</td>
</tr>
<tr>
<td>Mydlarski (2003)</td>
<td>-0.2</td>
</tr>
<tr>
<td>Jayesh &amp; Warhaft (1992b)</td>
<td>-0.3</td>
</tr>
</tbody>
</table>

The Reynolds number dependence on time follows $Re_\lambda \propto t^{-n/2+0.5}$ where $n = 1.33$ is the velocity variance decay exponent, which translates to $Re_\lambda \propto t^{-0.165}$. Consequently $\overline{\nu \theta} \propto t^{-0.2}$ and $\overline{\nu \theta} \propto t^{-0.3}$ for Mydlarski (2003) and Jayesh & Warhaft (1992b), respectively. Table 5.1 summarizes the findings from the similarity analysis using both DNS and experimental data and data from previous studies.
5.4 Summary

Self-similarity of the co-spectrum of velocity and temperature variance is investigated in decaying grid-turbulence with mean cross-stream stream temperature gradient. A similarity analysis for the evolution equation of the temperature spectrum is performed, and in particular the self-preservation condition for the co-spectrum is studied. Using DNS data, the coefficient of variation is investigated to verify the similarity condition and assess the quality of the collapse of the co-spectra at different simulation times. It is observed that the minimum of the coefficient of variation corresponds to the predictions of the similarity analysis and a good collapse is presented when the co-spectra are scaled with the similarity variables corresponding to the minimum. In addition, the analysis offers an expression for the scalar flux that is solely obtained from temperature variables. To the best of our knowledge, this is the first time such a relation is presented as opposed to empirical relations and allows independent investigation of the scalar flux without a priori knowledge of the velocity field. The approach opens a new perspective for analyzing and understanding turbulent flows using scalar measurements that are inherently simpler to conduct.
Chapter 6

Scaling and modeling of scalar dissipation spectra

Previous chapters presented and verified self-similarity conditions for the flow under consideration, namely passive scalar in isotropic turbulence with mean cross-stream temperature gradient. In this chapter, we consider the dynamics of small scales and in particular the scaling of the passive scalar dissipation spectra. It is shown that the dissipation range exhibits self-similarity, and both the Batchelor/Corrsin scale and the Taylor micro scale offer good collapse of the data measured with two different initial conditions. To the best of our knowledge, the scaling of the temperature dissipation spectra has not been considered previously, mainly due to a lack of high-quality data in this region with limited amount of experimental data available for the scalar dissipation spectra (Warhaft & Lumley, 1978; Mydlarski & Warhaft, 1998). Previous model spectra and their applicability to the measured dissipation range are investigated, which reveal deficiencies inherent in these models. A major reason for the lack of
experimental data is noise and high frequency attenuation in conventional sensing devices, which have prohibited a thorough theoretical validation of this range.

The spectral scalar flux is balanced by the dissipation, and consequently, the dissipation spectra can be expressed in terms of the scalar spectral flux $T_\theta(k)$ as:

$$\frac{-dT_\theta(k)}{dk} = 2\alpha k^2 E_\theta(k)$$  \hspace{1cm} (6.1)

which can be further expressed in terms of the scalar spectrum function $E_\theta(k)$ and the scalar spectral element $\zeta_\theta(k)$ as $T_\theta(k) = \zeta_\theta(k) E_\theta(k)$. To solve equation 6.1, one needs a closure model to relate $T_\theta(k)$ to $E_\theta(k)$. Many models for the scalar spectral flux $T_\theta(k)$ have been developed (Pao, 1965; Corrsin, 1964; Batchelor, 1959; Lin, 1972). Pao (1965) proposed a continuous spectral cascading process with $\zeta_\theta(k)$ depending on $\epsilon_\theta$ and $k$. From dimensional reasoning, $\zeta_\theta(k) = \gamma^{-1}\epsilon^{1/3}k^{5/3}$ where $\gamma$ is a dimensionless constant. By using this form and integrating equation 6.1, Pao (1965) and Corrsin (1964) developed a three-dimensional scalar energy spectrum for isotropic turbulent flow of the form

$$E_\theta(k) = \gamma \epsilon_\theta e^{-1/3} k^{-3/3} \exp\left[-\frac{3}{2}\gamma \alpha e^{-1/3} k^{4/3}\right]$$  \hspace{1cm} (6.2)

Kraichnan (1959) first predicted an exponential decay in the dissipation range and later extended the analysis of Batchelor (1959) of the viscous-convective subrange by taking into account the intermittency of the strain rate and predicted the spectral flux function to be

$$T_\theta(k) = \frac{\xi}{15} k^4 \frac{d}{dk}[k^{-2}E_\theta(k)]$$  \hspace{1cm} (6.3)
where $\xi$ is a function independent of wavenumber. The solution in the steady state was later given by Mjolsness (1975) as

$$E_{\theta}(k) = q\varepsilon_\theta \varepsilon^{-1/2} v^{1/2} (k\eta_\theta)^{-1} (1 + (6q)^{1/2} k\eta_\theta) \exp[-(6q)^{1/2} k\eta_\theta]$$

(6.4)

where $q$ is a constant. It is believed that the exponential decay of this model is the most appropriate representation for large wave numbers dynamics (Meyers & Meneveau, 2008). All of the models presented in this section along with other available models (Batchelor, 1959; Lin, 1972) are intended to represent the scaling exponent of the inertial range and the roll-off of the dissipation range. However, as a result, the roll-off in the inertial range is contaminated by the exponential decay, and so the observed spectral behavior differs from a power law in the inertial range followed by a simple exponential decay.

### 6.1 Results

Figure 6.1 shows the evolution of temperature and velocity variance with streamwise location $x/M$ where $x$ is the position from the grid and $M$ is the grid size. As expected, turbulence intensity decays as a power law due to the lack of production of turbulent kinetic energy, whereas the temperature variance increases as power-law as predicted by chapter 4, due to the imposed mean temperature gradient resulting in a non-zero production term. This fundamental difference between the two fields enhances the dynamical differences and allows a convenient test case for any analogy between them. A detailed investigation of the behavior of the scalar variance is presented in chapter 4, where a theoretical model is introduced and showed to agree well with the experimental data.

Figure 6.2a shows an example of a one-dimensional temperature spectra measured using both a conventional cold wire and a T-NSTAP. The differences might
Figure 6.1: Evolution of temperature and velocity variance with streamwise location $x/M$ (○ temperature data, squares □ velocity data). Solid lines represent power law fit to the data: Temperature $x^{0.51}$ and velocity $x^{-1.35}$. Error bars represent standard error of data acquired in different runs.

It seems negligible, but this is due to the logarithmic scaling of the y-axis. The inset in figure 6.2a is plotted semi-logarithmically, and clearly reveals significant differences between the two sensors over all wavenumbers. It also reveals the difficulty in accurately measuring and analyzing the dissipation spectra. The dashed box shown in the figure corresponds to the dissipation range, including the peak of dissipation and the associated roll-off, illustrating the importance of minimizing sensor attenuation and accurate treatment of noise. All of these factors combined contribute to the limited scalar dissipation data available in previous literature (Warhaft & Lumley, 1978; Mydlarski & Warhaft, 1998).

As observed by Jayesh et al. (1994) and Mydlarski & Warhaft (1998), in contrast to the velocity spectra, the spectra of temperature variance exhibit a power-law region at relatively low Reynolds number, which is clearly seen in figure 6.2a. Our measurements agree with these observations with the average scaling exponent for the cold wire found to be 1.53 and 1.51 for the T-NSTAP. However, even if the
shape of the spectra is similar between the two sensors, there is a clear difference between them; the cold wire measures consistently lower magnitudes than the T-NSTAP. A low frequency attenuation is expected as a result of the heat transfer from the wire to the support elements. Moreover, the attenuation has a significant effect on the peak of dissipation as shown in figure 6.2b, and consequently any conclusions on the scaling of the dissipation spectra. Figure 6.2b presents the corresponding dissipation spectra where the peak height measured by the cold wire is attenuated by 30% compared to the T-NSTAP directly leading to a scalar rate of dissipation underestimated by 35%. This finding is in agreement with Wyngaard (1971) who estimated that the scalar dissipation is underestimated by at least 30% at high wavenumbers. Mydlarski & Warhaft (1998) also indicated reduction of the dissipation for longer cold wires without no significant effect on low wavenumbers. For the remainder of this chapter only the data acquired using the T-NSTAP is considered.

Since high frequency measurements are very sensitive to noise, much attention was given to reduce any effects of noise on the data. Figure 6.3a presents...
one-dimensional dissipation spectra prior to noise reduction. A constant level of noise (white noise) is reflected in this figure as a $k^2$-noise. The decrease of signal-to-noise with wavenumber illustrates the difficulty of measuring and analyzing dissipation spectra and more importantly, accurately estimating the scalar rate of dissipation, an important parameter characterizing the high wavenumber dynamics. Figure 6.3b shows the same one-dimensional dissipation spectra for two different flow conditions (velocities and temperature gradients) after a constant level of noise has been reduced from each spectrum.

Figure 6.4a shows the evolution of the scalar rate of dissipation $\epsilon_\theta$ with streamwise locations. The sensitivity of $\epsilon_\theta$ to noise emphasizes the need to accurately account for the noise in the analysis. To do so, we adopted three approaches to calculate $\epsilon_\theta$. First, the form of $\epsilon_\theta$ for isotropic flow was considered ($\epsilon_\theta = 3\alpha (\partial \theta / \partial x)^2$). Second, the dissipation spectra obtained from the one-dimensional measured temperature spectra was integrated ($\epsilon_\theta = 3\alpha \int_0^\infty k^2 E_\theta dk$). Both these methods require removing a constant level of spectral noise as described above. Lastly, the three-dimensional spectra $E_\theta(k)$ obtained through the isotropic relation is integrated using $\epsilon_\theta = 2\alpha \int_0^\infty k^2 E_\theta(k)dk$. The differential in the isotropic relation (eq. 4.21) removes any sensitivity to a constant noise level, implying that a constant level of noise in the measured one-dimensional spectra translates to a zero level noise and consequently, noise removal is not required for this approach. The convincing agreement between the the three different methods for $x/M > 40$ indicates that the noise is close to constant (white noise). For the remainder of this study, the constant noise level is removed from all analysis.

Another interesting observation can be made in figure 6.4a. The large scatter observed for $x/M < 40$ and the difference between the estimates of $\epsilon_\theta$ obtained by integrating the three-dimensional spectra and the ones obtained by the one-dimensional spectra indicates that the assumption of isotropy does not hold here.
Figure 6.3: (a) One-dimensional temperature dissipation spectra for \( x/M \) ranging from 8 to 200 for Case II. (b) Dissipation spectra after noise reduction for Case I and Case II.

Figure 6.4: (a) \( \epsilon_\theta \) for different \( x/M \), Case II, calculated with three different methods. Error bars are not plotted for clarity. (b) Logarithmic plot of \( \epsilon_\theta \) for Case I and Case II. Solid lines represent power law fit to the data. Case II: \( x^{-0.043} \) (region 1), \( x^{-0.433} \) (region 2), Case I: \( x^{-0.22} \) (region 1), \( x^{-0.629} \) (region 2). Error bars represent standard error of data acquired in different runs.
Figure 6.5: One-dimensional temperature dissipation spectra for six $x/M$ locations from 48 to 200 for Case I and Case II (total of twelve curves) scaled with: (a) Batchelor variables: data compared with Pao (1965) with $\gamma = 1.4$, Kraichnan (1968) with $\gamma = 5$ and Pope (2000) with $p_0 = 2, \gamma = 4.3, c_{L_0} = 0.5, c_{\eta_0} = 0.5$ and $C_\theta = 1.1$. (b) Taylor variables: data compared with model 1 $d = 5/3$, model 1 $d = 5/4$ and model 2. Experimental data was diluted for clarity.

This is in agreement with Sirivat & Warhaft (1983), who observed that a homogeneous profile is established at $x/M = 40$ by inspecting the cross-stream r.m.s temperature profiles. Similarly Zhou et al. (2000) observed a value of $x/M = 30$ for the flow to reach homogeneity for a heated grid setup. However, when the same data is plotted in figure 6.4b it is clear that $\epsilon_\theta$ (obtained by integrating the one-dimensional dissipation spectra) exhibits power-law-like behavior even for $x/M < 40$. The power law behavior is in accordance with Lee et al. (2012) who investigated isotropic turbulence by stretching a grid flow through a secondary contraction and observed that the scalar mean dissipation rate follow a power-law rate of decay that depends on the geometry of the grid. Two distinct regions can be identified (marked by I and II in figure 6.4b), delimited by approximately $x/M = 40$. The exponents are different in the two regions and for the two flow conditions. Inspecting the evolution of temperature variance (equation 6.1), which is being produced and dissipated at a faster rate for the higher velocity case, can explain the difference between the exponents.
6.2 Scaling

Scaling the flow under investigation is non-trivial due to the fact that the turbulence is decaying while temperature variance is generated by interaction with the mean temperature gradient. Therefore, the velocity and temperature fields have opposite trends and the direct analogy between both fields does not hold. We proceed our analysis by considering the scales characterizing this flow, namely Batchelor/Corrsin scale and the temperature Taylor microscale and the integral scale. Traditionally, model spectra are derived by analogy with the velocity field and consequently, scaled with the Kolmogorov microscale or alternatively the Batchelor/Corrsin scale. The Batchelor/Corrsin scale is obtained by solely knowing the parameters of the velocity field, particularly the viscous rate of dissipation in addition to the Prandtl number. For passive scalars, the underlying assumption is that the dynamics of the scalar is governed by the velocity field and that the scalar dissipative scales are proportional to the corresponding scales of the velocity field. Consequently, the Batchelor/Corrsin scale is used in the scaling of the dissipation spectra.

In contrast to Batchelor/Corrsin variables, the temperature Taylor microscale $\lambda_\theta$ depends on the scalar variance and the scalar rate of dissipation and consequently depends only on parameters governing the temperature field. In particular, it relaxes the strict separation requirement among large and fine scales inherent in the universal equilibrium range hypothesis. As recognized by Kraichnan (1968) and later pointed out by Warhaft (2000), the traditional cascade picture as applied to the scalar field is a crude representation and in fact experimental evidence shows that the large and small scales are strongly coupled. The Taylor microscale is often referred to as an intermediate scale that lacks a solid physical interpretation. However, in the scalar field it represents the thermal dissipation timescale ($\overline{\theta^2}/\epsilon_\theta$), which together with the thermal diffusivity forms a length scale.
Figure 6.5a shows the dissipation spectra for six $x/M$ locations ranging from 48 to 200 scaled by Batchelor/Corrsin variables, for two different flow conditions. Comparing this figure with 6.3b reveals an excellent collapse of the data both for different streamwise locations and different flow conditions in velocity and temperature gradient. The peak of the measured one-dimensional dissipation spectra for different $x/M$ scales well with the corresponding Batchelor/Corrsin scale, with the peak observed at $k\eta_\theta = 0.185$, while the peak for the three-dimensional spectra (obtained using the isotropy assumption, equation 2.5) being located at $k\eta_\theta = 0.31$. These values agree well with the observations of Warhaft & Lumley (1978) who measured spectra in heated grid turbulence and found that the three-dimensional spectra exhibited a peak at $k\eta_\theta = 0.3$.

Figure 6.5b presents the same spectra scaled with Taylor variables. Comparing this figure with figure 6.3b also reveals an excellent collapse of the data. The differences observed between the two scales are small and arguments can be made for either scale over different parts of the spectrum. It appears that the Batchelor/Corrsin variables scale the data slightly better around the peak and the beginning of the roll-off, whereas Taylor variables appear to offer a slight improvement over the Batchelor/Corrsin variables for a given initial condition and at the largest wavenumbers. The difference between the two sets of flow conditions was explained by George & Gibson (1992) who studied a turbulent flow with mean velocity gradient through a similarity analysis. They hypothesized and observed that although each flow is self-preserving, no universal spectrum exists and the spectral shape and characteristics vary for each flow according to the given initial conditions. In other words, similarity solutions directly depend on initial conditions.

As previously discussed, the flow exhibits two different regions delimited by $x/M = 40$ above which the flow reaches cross-stream homogeneity and therefore in figure 6.5, we only considered data beyond this limit. However, by considering the
results in figure 6.3b, it seems plausible that the flow might exhibit self-similarity in regions closer to the grid. Figures 6.6a and b show an attempt to scale data for $x/M < 40$ with Batchelor/Corrsin and Taylor variables, respectively. As can be seen, Taylor variables collapse all data curves with different flow conditions quite well, especially when compared with the collapse with Batchelor/Corrsin variables. A closer look at the results reveals that the scatter is not a result of different flow conditions and one may not expect Batchelor/Corrsin scaling to hold close to the grid ($x/M < 40$) since the results may be affected by the conditions close to the grid. This result may indicate that self-preservation of the scalar field holds even before the velocity field has achieved homogeneity. The scatter observed in the collapse of figure 6.6b is a direct result of the scatter shown in figure 6.4 for data taken close to the grid.

Figure 6.5a shows the experimental data together with the models of Pao (1965), Kraichnan (1968) and Pope (2000), scaled with Batchelor/Corrsin variables. All models were integrated to a one-dimensional form using the isotropic relation. An excellent agreement between the data and the prediction of Kraichnan (1968) is observed over all the dissipation range with $\gamma = 4.8$, a value close to $\gamma = 2\sqrt{5}$.
derived by Qian (1990). In particular, the model of Kraichnan (1968) adequately predicts the location of the dissipation peak and represents well the roll-off of the dissipation spectrum at large wave numbers, unlike the model spectrum of Pao (1965) which underpredicts the peak location with the roll-off inadequately cutting back the spectral flux. Note that although the model of Kraichnan (1968) fits the data well, this model is an extension to Batchelor’s uniform strain model representing the viscous-convective subrange, which is only observed for high $Pr$. Analyzing the scaling exponent of the inertial range predicted by the model reveals that the exponential roll-off departs from unity too rapidly. Consequently, the inertial range does not exhibit the expected power law behavior, and the local derivatives vary from $-1.05$ to $-1.45$ at the end of the inertial range, departing significantly from the $k^{-1}$ behavior predicted by Batchelor (1959) uniform strain model. Therefore, the fit of the data is an empirical prediction loosing the fundamental assumptions of the model. Similarly, the models of Pao (1965) and Batchelor (1959) suffer from the same deficiencies and depart significantly from a power law behavior in the inertial range. Pope (2000) recognized these deficiencies and proposed an empirical model for the energy spectrum. To use this model for the temperature field, we adopt KOC scaling for the inertial range as follows

$$E_\theta(k) = C_\theta \epsilon_\theta e^{-1/3} k^{-5/3} f_{L\theta}(kL_\theta) f_{\eta\theta}(k\eta_\theta)$$

$$f_{L\theta}(kL_\theta) = \left( \frac{kL_\theta}{[(kL_\theta)^2 + c_{L\theta}]^{0.5}} \right)^{5/3 + \rho_0}$$

$$f_{\eta\theta}(k\eta_\theta) = \exp[-\gamma((k\eta_\theta)^4 + c_{\eta\theta}^4)^{1/4} - c_{\eta\theta}]]$$

where $L_\theta$ is the integral scale. The function $f_{L\theta}(kL_\theta)$ determines the shape of the energy-containing range and tends to unity for large wavenumbers while $f_{\eta\theta}(k\eta_\theta)$ reflects the dynamics of the fine scale. $C_\theta$ is the Obukhov-Corrsin constant. $c_{L\theta}$ and $c_{\eta\theta}$ are determined by the integral of $E_\theta(k)$ and $D_\theta(k)$ being $\theta^2$ and $\epsilon_\theta$, respectively.
Good agreement between the model derived from Pope (2000) and the data is observed. While both the models of Pope (2000) and Kraichnan (1968) seem to represent the dissipation spectra well, the latter does not account for the energy-containing range and therefore does not represent the entire spectrum.

Based on the observation that the data scales well with Taylor variables (figure 6.5b and 6.6b), a model for the dissipation spectra can be derived using the approach described in equation 6.1 and the closure approximation. This model (model 1) implies that dissipation spectra is solely dependent on the scalar rate of dissipation $\epsilon$, temperature variance $\bar{\theta}^2$ and thermal diffusivity $\alpha$ from which the scalar spectral element $\zeta_\theta(k)$ is

$$\zeta_\theta(k) = \gamma^{-1} \alpha^a \epsilon^b \bar{\theta}^{2c} k^d.$$  (6.6)

This approach implies that the spectral behavior of temperature dissipation can be entirely determined by parameters not directly related to the velocity field. Equation 6.6 together with dimensional considerations leads to a model for the dissipation spectrum with two free parameters $\gamma$ and $d$.

Figure 6.5b presents the data scaled with Taylor variables along with models derived from equation 6.6 for different values of $d$. Best fit for the data is achieved with $d = 5/4$ and $\gamma = 0.14$. As previously discussed, this model suffers from exponential contamination of the inertial range and therefore similar to the model of Kraichnan (1968) discussed above, the local derivative in the corresponding inertial range vary from $-1.25$ to $-1.5$, which explains the good fit to the data (the scaling exponent of the data is presented at the beginning of this section). Therefore, this approach can represent a semi-empirical fit to the dissipation spectrum. Following the dimensional considerations leading to equation 6.6 combined with the model
of Pope (2000), we consider the new model spectrum (model 2) given by

\[ E_\theta(k) = C_\theta \theta^{1/3} \theta^{2/3} k^{-5/3} f_{L\theta}(kL_\theta) f_{\lambda\theta}(k\lambda_\theta) \]  \hspace{1cm} (6.7)

\[ f_{\lambda\theta}(k\lambda_\theta) = \exp\left[-\gamma\left((k\lambda_\theta)^4 + c_{\lambda\theta}^4\right)^{1/4} - c_{\lambda\theta}\right] \]

As expected, a good fit of the data with this model is observed in figure 6.5b with the advantage of appropriate behavior at low wavenumbers. Scaling with Batchelor/Corrsin variables necessitates knowledge of the velocity field parameters, particularly the viscous rate of dissipation \( \epsilon \). In contrast, model spectra that can be derived without any information about the velocity parameters, as in the proposed models, is of great practical use in the study of passive scalars or in flows where velocity measurements are not available.

### 6.3 Summary

The scalar field in decaying grid turbulence with a mean cross-stream temperature gradient was investigated with a focus on the dissipation range. As shown, the new temperature sensor T-NSTAP exhibits major differences when compared with a conventional cold wire, both for the variance and the scalar rate of dissipation. By taking extra care in understanding the effect of noise on the data, \( \epsilon_\theta \) was accurately determined and validated through different methods. The data suggests that \( \epsilon_\theta \) exhibits two clear power law regions with the second corresponding to cross-stream homogeneity.

Scaling of the dissipative spectrum of a passive scalar is generally approached through a direct analogy between the scalar field and the convecting turbulence, and consequently Batchelor/Corrsin scales are normally adopted. Both of these scales rely on parameters that require knowledge about the velocity field, and in
fact, the only parameter connecting the lengthscale to the scalar field is the thermal diffusivity via $Pr$. In contrast, Taylor variables do not necessitate knowledge of the velocity field and therefore allows for investigating the scalar field in an independent manner. By comparing the scaling of the dissipation range with both Batchelor/Corrsin and Taylor variables in the region with cross-stream homogeneity, we observed that the flow exhibits self-similarity and both scales offer excellent collapse of data measured with different flow conditions. Moreover, the flow exhibits self-similarity when scaled with Taylor variables very close to the grid, even before homogeneity is achieved. Given that Batchelor/Corrsin scales are directly related to the Kolmogorov microscale, those scales cannot adequately scale the flow in the region close to the grid where boundaries and flow conditions have significant effects on the flow.

In addition, previous model spectra and their applicability to the dissipation range were investigated, which revealed deficiencies inherent in these models. In particular, the intended power-law behavior in the inertial range is contaminated by the exponential roll-off of the dissipation range resulting in significant departure from the expected power-law behavior in the inertial range, undermining the underlying theoretical foundations. Consequently, while the roll-off of these models might fit the dissipation spectra well, they are not a complete representation of the whole scalar spectra. These deficiencies are remedied by the model of Pope (2000), which was shown to offer good fit with experimental data. Based on our findings a simple alternative model was developed based entirely on temperature related variables and dimensional reasoning. These variables were also combined with the model of Pope (2000) to yield a second model that maintains the spectral behavior in the inertial range. Both models showed a convincing agreement with the experimental data in the dissipation range, and do not require input from the velocity field.
Chapter 7

The integral scale problem

In previous chapters, the existence of self-preserving solutions for isotropic turbulence with mean temperature gradients was investigated. While most of the similarity conditions were verified mainly for the spectra of temperature and the co-spectra of temperature and velocity, it remains to investigate the implication of the analysis on the low wavenumbers range and particularly, the characteristic lengthscale in that domain, namely the integral scale.

7.1 The integral scale: Background

It is important to note that previous studies have observed major subtleties and difficulties in measuring and computing the integral scale and it is claimed that the wind tunnel width or the computational domain size have significant effects on the integral scale measurements and consequently, major errors are introduced. In fact, the integral scale has been the subject of numerous previous investigations both using experiment and more recently DNS (Wang & George, 2002; Comte-Bellot & Corrsin, 1966; Fossen & Ching, 1997; O’Neill et al., 2004; Sirivat & Warhaft, 1983) pointing to the subtlety involved in those measurements. Of particular interest is
how the integral scale varies with time (or space using Taylor’s hypothesis),

\[ L \sim t^m \sim (x/M)^m \]  

(7.1)

where \( L \) is the integral scale. In general, it is observed that the power law exponent \( m \) for the integral scale growth varies between 0.34 and 0.53 (Comte-Bellot & Corrsin, 1966). Sirivat & Warhaft (1983) studied the influence of varying the thermal scale relative to the velocity scale on the evolution of the flow and observed the evolution of the scalar integral scale to be \( m_\sim 0.35 \). Other studies (O’Neill et al., 2004; Wang & George, 2002) assessed and observed a significant effect of the wind tunnel confinement or computational domain size on the large scale fluctuations and therefore on the magnitude and growth of the integral scale and concluded that the integral scale grows as \( \sim x^{0.5} \).

As noted by Wang & George (2002), the problem arises when the peak in the energy spectrum is at wavenumbers not sufficiently greater than the lowest measured or available wavenumber, particularly at later times (or downstream position) as the scales grow. For example, in Direct Numerical Simulations, the simulations provide useful information about averaged quantities only for scales sufficiently smaller than the computational domain where spatial averages can be used to approximate ensemble averages (Wang & George, 2002). Thus, the estimate of integral scales from the spectrum can be severely reduced by the unavailability of the lowest wavenumbers.

Therefore, one the goals of this chapter, in addition to verifying the similarity requirement, is to highlight the subtlety in both measuring and computing the integral scale in a turbulent flow and the discrepancy observed with theoretical predictions. This is done by first comparing the results of the similarity analysis to experimental data. Subsequently and in order to investigate the effect of com-
putational domain or wind tunnel size on the integral scale measurement, direct numerical simulations for various domain sizes are conducted.

7.2 Integral scale from the similarity analysis

As previously mentioned, a major consequence of the similarity solution presented in chapter 4 relates to the integral scale and can be observed by considering equation 4.17, and the definition of the scalar integral scale,

\[ L_\theta = \frac{\pi}{2\theta^2} \int_0^\infty \frac{E_\theta(k)}{k} dk. \]  

(7.2)

If the flow exhibits self-similar solutions, as suggested above, the integral length scale should relate to the characteristic length scale as:

\[ L_\theta \propto \ell_\theta \int_0^\infty f(\gamma) \gamma d\gamma. \]  

(7.3)

Since \( f \) is a function of \( \gamma \) only, \( L_\theta \propto \ell_\theta \), and according to the above observations \( L_\theta \propto \lambda_\theta \). That means that the scalar integral scale and scalar Taylor microscale should remain in constant ratio at any position (or any time). In addition to the scaling of the spectra, this serves as another way to verify if the flow is self-similar.

7.3 Analysis

Figure 7.1 presents both the scalar Taylor microscale \( \lambda_\theta \) and the scalar integral scale \( L_\theta \) for case II. As can be seen, the data shows that the scalar Taylor microscale follows \( \lambda_\theta \propto (x/M)^{0.47} \) while the scalar integral scale is \( L_\theta \propto (x/M)^{0.34} \), which do not agree with the similarity requirement.
Figure 7.1: Scalar integral scale $L_0 \propto x^{0.34}$ and scalar Taylor microscale $\lambda_\theta \propto x^{0.48}$ as measured in Case II of the experiments.

Furthermore, following the similarity requirement, the spectra of temperature variance should show a good collapse when scaled with the scalar integral scale, at all wavenumbers. It can be seen in figure 7.2 that while a good collapse is observed at low wavenumbers, the quality of the scaling degrades as the wavenumber increases.

Previous studies suggest that the observed discrepancy between theoretical predictions and experimental data could be related to an experimental bias due to a finite size wind tunnel or large and slow background fluctuations in the temperature field contaminating the integral length scale. However, since building different wind tunnels with different dimensions to investigate the effect of the size on the integral scale is impractical and in order to further investigate the discrepancy and potentially highlight the effect of measurement or numerical setup on the integral scale, direct numerical simulations were conducted with different domain sizes. The integral scale is then calculated for each simulation. Figure 7.3
Figure 7.2: Measured temperature spectra scaled by the measured scalar integral scale for Case II.

presents the power dependence $m$ as a function of the initial number of integral scales in the domain, $N/L_\theta$, where $N$ is the domain size. First, it can be clearly seen that $m$ varies with domain size. Moreover, taking into account the width of the wind tunnel used to obtain the experimental data in this thesis, dashed line in figure 7.3 shows the location of the experimental conditions and shows a dependence of the integral scale to be $m = 0.38$, reasonably close to the measured value of $m = 0.34$. It is evident that further simulations with larger domains is required to investigate any asymptotic behavior at larger domain sizes, however present data demonstrates the effect of the numerical setup on the observed dynamics of the large scales.

### 7.4 Conclusion

The similarity analysis presented in previous chapters dictates the integral scale dependence on time (space). In particular, if self-preserving solutions exist, then the integral scale should follow a power-law behavior $L_\theta \sim t^m \sim t^{0.5}$. The experimental data does not verify this requirement while numerical data shows a dependence of
Figure 7.3: Integral scale power law dependence on time for different computational domain size $N$. Error bars represent power obtained from fitting different points in the simulations, particularly at the beginning of each simulation. Trendline represents a possible fit of the power $m$ as a function of $N/L_\theta$. Dashed represents location of experimental data (wind tunnel size considered as domain size).

the power $m$ on the domain size $N$, which strongly suggests an effect of the domain size on the large scales. Further simulations for larger domain sizes are needed in order to validate the similarity analysis requirement for the integral scale.
Chapter 8

Investigation of scalar probability distribution function

In an effort to seek more insight and understanding of the underlying scalar dynamics, attention is shifted from self-preserving solutions to internal intermittencies, which in recent years became a subject of increasing importance in the scalar turbulence studies. Scalar fields are observed to exhibit small-scale (internal) intermittency characterized by strong variability in mixing rates and scales, and therefore higher order statistics departs from KOC scaling (Monin et al., 1975). In this chapter, scalar intermittency in the presence of a mean cross-stream temperature gradient is investigated. One approach is to look at the probability density function (PDF) where previous investigations have detected the presence of non-Gaussian behavior in the form of exponential tails- in contrast with the velocity fluctuations in homogeneous turbulence, which follow a Gaussian distribution. Although this phenomenon has been widely investigated in recent years, the underlying physics are not well understood, and over the years, a variety of models suggested different interpretations.
The probability density function (PDF) $P(x)$ of a random variable $x$ is defined such as

$$\int_{0}^{\infty} f(x)P(x)dx = \langle f(x) \rangle$$  \hfill (8.1)

for any bounded function $f(x)$, where $\int P(x)dx = 1$.

Unlike the spectrum, the probability density function of a passive scalar has not played a predominant role until recent years. It has been observed (Siggia, 1994) that the PDF of temperature fluctuations goes from a Gaussian distribution to a wider than Gaussian PDF with larger variance and exponential tails. This observation is a manifestation of the phenomenon called passive scalar intermittency.

Pumir et al. (1991) suggested the first analytical model that predicts the existence of the exponential tails in the presence of a mean scalar gradient. The exponential distribution was a direct result of the steady-state solution of a phenomenological model for the scalar undergoing random advection and molecular mixing. Using a linear-eddy model describing the turbulent mixing process in terms of instantaneous local reorganization of the scalar through advection and diffusion, Kerstein (1991) reached a similar result. Warhaft (2000) pointed out that the exponential tails result from a fluid parcel traveling a distance larger than an integral scale without equilibrating. This is known as anomalous mixing and is mainly due to an anomalously long mixing rate (Siggia, 1994). Pumir et al. (1991) considered Kerstein’s model from a numerical approach and observed an exponential distribution of the temperature fluctuations in the presence of a mean scalar gradient. In addition, the authors further studied the turbulent mixing process proposed by Kerstein (1991) and identified different behavior related to the interchange of fluid parcels by advection and diffusion through eddy diffusivity occurring in between consecutive interchanges. This approach for interpreting the phenomena underlying the exponential tails will be further investigated in this chapter.
Jayesh & Warhaft (1992a) investigated the effect of filtering on exponential tails and observed that the use of a high-pass filter to remove large-scale effects helps in revealing the exponential tails. Interestingly, Jayesh & Warhaft (1991) and Jayesh & Warhaft (1992a) observed experimentally that the exponential tails are not evident in the absence of a mean scalar gradient.

8.1 Analysis

In order to assess the deviation of the PDF from a Gaussian distribution, we consider the normalized PDF of temperature fluctuations (figure 8.1) measured using the two sensors, namely a cold-wire and the new temperature sensor T-NSTAP. A hint of exponential tails can be seen in the data measured using the T-NSTAP, while the data measured using the cold wire does not exhibit any deviation from Gaussian. Previous studies (Jayesh & Warhaft, 1991) showed that the PDF exhibits exponential tails when the data is subject to a high-pass filter. As can be seen in figure 8.2 where the data of figure 8.1 is subject to a high pass filter, the PDF measured by the T-NSTAP appears wider and the exponential tails are revealed. Throughout this chapter, the effect of filtering on the exponential tails will be investigated and interpreted through the basic ideas outlined in Kerstein’s model.
Figure 8.1: PDF of temperature fluctuations \((x/M = 196)\) using a cold-wire and a T-NSTAP.

Figure 8.2: PDF of temperature fluctuations at \((x/M = 196)\) high pass filtered at \(f_h = 1000\) Hz. Data acquired using a cold-wire and a T-NSTAP.
Amplification of the exponential tails can be achieved by considering the PDF of the scalar fluctuations derivative, $\partial \theta / \partial t$, clearly shown in figure 8.3. This is consistent with previous investigations (Jayesh & Warhaft, 1992a) that observed broad exponential tails when looking at the derivative PDF.

![Figure 8.3: PDF of the scalar fluctuation derivative $\partial \theta / \partial x$ at $x/M = 196$. Data acquired using both a regular cold-wire and a T-NSTAP.](image)

By looking at the kurtosis (the fourth order moment) given by

$$K_4 = \frac{\bar{\theta^4}}{(\bar{\theta^2})^2}$$  \hspace{1cm} (8.2)

we can further investigate the evolution of the tails dynamics of the tails at different streamwise locations. The Kurtosis for both $\theta$ and $\partial \theta / \partial x$ and different streamwise locations $x/M$ is presented in figure 8.4. A major difference is observed with the kurtosis for $\theta$ tending to the Gaussian value of 3 while the kurtosis of the
fluctuations derivative reaches a value of 8.5, well beyond the Gaussian value. Note that while the kurtosis of $\theta$ gradually decreases with distance from the grid, $K_4$ for $\partial \theta / \partial x$ increases until $x/M = 40$, which could be due to the tails being formed throughout the complex region close to the grid.

Figure 8.4: Kurtosis of the temperature fluctuations and their derivative at different streamwise locations $x/M$.

Here we present a new analysis to study the frequency content of the PDF by changing the weight given to the high frequency content. We note that the exponential tails are considered to follow

$$P(\theta) = e^{-a\theta},$$

(8.3)

where $a$ is the slope of the tails on a semi-logarithmic scale. The results of this analysis is presented in figure 8.5. By using a band-pass filter with variable cut-off frequencies, we were able to investigate the effect of reducing the low wavenumber.
content. It is shown that removing low frequency content dramatically affects the exponential form while small wavenumbers has negligible effect on the tails. While looking at the figure, it is interesting to consider the frequencies corresponding to the main lengthscales of the flow: The dissipative scales are located at $3 \text{kHz}$ and the integral scale corresponds to a frequency of $70 \text{Hz}$, for the flow conditions under consideration.

Figure 8.5 reveals that at cut-off frequencies below $150 \text{Hz}$, both sensors exhibit similar behavior. However, a significant widening of the PDF (decreased value of $a$) is observed as the high-pass cut-off frequency is increased, which reduces the low frequency content of the signal. A closer look at figure 8.5 shows that the tails of the PDF acquired using the cold-wire are not evident at a high-pass cut-off frequency of around $1500 \text{Hz}$ while in contrast, the T-NSTAP data is at its widest point. Further inspection reveals that frequency content contributing the most to intermittencies is located at frequencies of about $1200 \text{Hz}$ and $500 \text{Hz}$, for the T-NSTAP and cold wire data, respectively. It is evident that the T-NSTAP exhibits wider tails than the cold-wire for all cut-off frequencies. A similar behavior can be seen for the PDF of the derivative where the tails being evident at any high-pass frequency.

In order to verify the experimental observation and to ensure that noise in the data is not contaminating the results, DNS data is considered in figure 8.6, which repeats the analysis of figure 8.5. First and in contrast to experimental data, the exponent value reaches a plateau without returning back to the Gaussian value. This fact is expected and is a result of the absence of measurement noise in DNS. In addition, similar to figure 8.5, wider tails are observed with increasing high-pass filter frequency, with the exponential tails evident in the derivative PDF without high-pass filtering.
Figure 8.5: Effect of high pass filtering on $a$ in equation 8.3 for the T-NSTAP and cold-wire, both for the pdf and the pdf of the derivative for $U = 9m/s, \beta = 8K/m$ and $x/M = 196$. 
Figure 8.6: Effect of high pass filtering on $a$ in equation 8.3 for the DNS data, both for the pdf and the pdf of the derivative for $\beta = 8K/m$ and $Re_\lambda = 50$.

Figure 8.7: The flip concept from the Kerstein model: Eddy of size $\ell_e$ centered at $x_0$ flips the scalar around its center producing adjacent parcels of opposite sign and amplitude $\ell_e \beta$, that can be observed in the fluctuating scalar $\theta$. 
Seeking further understanding of the observed exponential tails, the linear eddy model of Kerstein (1991) is considered where the flow is assumed to be a random superposition of eddies. In this particular approach, a scalar $\theta_0$ is flipped around the center $x_0$ under the action of an eddy of size $\ell_e$ and a characteristic timescale $\tau_e$ (figure 8.7). Interchanging scalar concentration around the center of an eddy is referred to as "flip", which creates parcels of amplitude of order $\ell_e \beta$ and opposite sign. Considering $L$ to be the size of the system, we define a space-time density of the flips $\rho$ as follows

$$\rho = (L\tau)^{-1}$$

(8.4)

where $\tau$ is the average time between flips. Given that diffusion and advection are the main players in this approach, a "diffusion" timescale $\tau_e$ can be defined as follows

$$\tau_e = \frac{\ell_e}{u_{rms}},$$

(8.5)

where $u_{rms}$ is the rms of the velocity fluctuations. An advection timescale $\tau_u$ can be determined by the density of flips $\rho$ and the eddy size $\ell_e$ such that

$$\tau_u = \frac{1}{\rho \ell_e},$$

(8.6)

When more eddies are active in a given region, $\tau_u < \tau_e$. In contrast, $\tau_u > \tau_e$ when fewer eddies act on that region. Therefore, in order for the scalar to smooth out the parcel of opposite sign before the following flip occurs, the advection timescale should be of the same order as the diffusion timescale, namely $\tau_u = \tau_e$. In order to quantitatively characterize the different possible states, a non-dimensional time ratio $K$ can be formed using both the diffusion timescale and the advection timescale, as follows

$$K \equiv \frac{\tau_u}{\tau_e} = \frac{u_{rms}}{\ell_e} \left( \frac{1}{\rho \ell_e} \right).$$

(8.7)
Here we consider the limits of the above equation. First, $K \ll 1$ implies weak diffusion, which means that within one diffusion time, several flips occur. In contrast, when $K >> 1$, strong diffusion occurs between flips, therefore giving rise to rare events. The case of $K = 1$ corresponds to the case where the scalar diffuses during a time of the order of $\tau_u$. Finally, in the limit of negligible $K$, the scalar distribution is Gaussian. The non-dimensional time ratio $K$ is presented in figure 8.8 for experimental and DNS data and different high-pass filter frequencies. The dynamics observed in figure 8.8 is consistent with the analysis presented in figure 8.5. First, a closer look at the experimental data shows that $K$ reaches a peak after a continuous increase from $K < 1$. The peak is followed by less evident tails, which is result of the diffusion being dominated by the flip rate. As in figure 8.5, a similar analysis is conducted for DNS data, which exhibits a similar trend with the exception of the decrease in $K$ corresponding to the return to Gaussian form. As previously mentioned, this is due to the absence of measurement noise inherent in experiments data.
Figure 8.8: Ratio of the advection timescale to the diffusion timescale, $\kappa$ as a function of the high-pass filter frequency plotted using DNS data and experimental data. Experimental and numerical flow conditions are the same as figures 8.5 and 8.6, respectively.

### 8.2 Conclusion

Scalar intermittency is investigated using both experimental and numerical data. In particular, this chapter shows that when imposing a high pass filter on the data, the scalar PDF gradually exhibits a major deviation from Gaussian, a phenomena referred to as exponential tails. A new approach to analyze the observed tails was presented, which revealed a significant widening of the PDF as more of the low frequency content is filtered, for both experimental and DNS data. The linear eddy model of Kerstein was investigated and in particular, the exponential tails were interpreted through the interplay of advection and diffusion. Lastly, by defining a
non-dimensional ratio $K$, we were able to reveal the expected behavior with a high pass filter.
Chapter 9

Concluding remarks

In this thesis, the fundamentals of temperature fluctuations in isotropic turbulence were investigated. In particular, statistically homogeneous and isotropic turbulence, with an imposed mean cross-stream linear temperature gradient was studied.

Conventional temperature probes consist of a cold-wire sensor. It was shown that cold wire attenuation has widespread effects on most aspects of the measurements, resulting in the variance being underestimated by at least 25% (for the given flow conditions), while high frequency attenuation reduced the scalar rate of dissipation by as much as 35% for the given flow conditions. A lumped-capacitance based model was derived to mimic the dynamic characteristics of cold wire based on its physical properties and dimensions.

A new fast-response, sub-miniature, temperature sensor (T-NSTAP) is developed based on the proposed model, evaluated and compared to a conventional cold-wire. The design and manufacturing techniques for the T-NSTAP are based on those previously developed for the Nano-Scale Thermal Anemometry Probe (NSTAP), which has proven extremely successful in capturing small scale turbu-
lence at high Reynolds numbers. It is shown that the T-NSTAP has a dramatically improved frequency response.

The conditions for self-preserving solutions for the flow under consideration were derived and investigated using the new nano-sensor and Direct Numerical Simulations. The similarity analysis predicts the scaling function to be related to the temperature variance and a characteristic length scale. The length scale was found to follow \( \ell^2_\theta = \ell^2_{0\theta} + A_{\theta}(t - t_0) \) in time, while the analysis predicts the variance to exhibit a power law behavior. A new method to objectively assess the quality of the scaling is described. In particular, the minimum of the coefficient of variation for the normalized temperature spectra at different streamwise positions, integrated over all wavenumbers, is investigated. The data exhibit a minimum very close to the similarity analysis prediction.

The experimental data confirmed the results of the similarity analysis and as predicted by the similarity analysis, the data shows the variance growing as a power law with streamwise position. In addition, the collapse of the temperature spectra when scaled with the theoretical result, \( x^{0.5} \), is excellent for all wavenumbers with a scatter of 5 to 10% based on the normalized coefficient of variation.

Another consequence of the similarity analysis relates to the integral scale dependence on time (space). In particular, if self-preserving solutions exist, then the integral scale should follow a power-law behavior \( L_\theta \sim t^m \sim t^{0.5} \), proportional to the Taylor microscale. The experimental data does not verify this condition while numerical data shows a dependence of the power \( m \) on the domain size. As previously mentioned, the integral scale measurement has been the subject of numerous investigations and consequently, the current data cannot be used to determine the reason behind the apparent discrepancy observed in this study and in the literature. However, numerical data indeed suggest an effect of the domain size on the large
scales and therefore further investigation of this effect is required in order to assess and model the error resulting from a limited domain (or confined wind tunnel).

Further investigation of self-similarity was conducted and in particular self-similarity of the co-spectrum of velocity and temperature variance. Using DNS data, the coefficient of variation is investigated to verify the similarity condition and assess the quality of the collapse of the co-spectra at different simulation times. It is observed that the minimum of the coefficient of variation corresponds to the predictions of the similarity analysis and a good collapse is presented when the co-spectra are scaled with the similarity variables corresponding to the minimum. In addition, the analysis offers an expression for the scalar flux that is solely obtained from temperature variables. To the best of our knowledge, this is the first time such a relation is presented as opposed to empirical relations. The proposed relation implies that one can infer knowledge about the scalar flux without the need to measure it and by investigating the scalar field alone without any data from the velocity field.

A field of interest in turbulence studies relates to the small scales in the dissipation range. Therefore, scaling of the dissipative spectrum is investigated. By comparing the scaling of the dissipation range with both Batchelor/Corrsin and Taylor variables in the region with cross-stream homogeneity, we observed that the flow exhibits self-similarity and both scales offer excellent collapse of data measured with different flow conditions. Moreover, the flow exhibits self-similarity when scaled with Taylor variables very close to the grid, even before homogeneity is achieved. In addition, previous model spectra and their applicability to the dissipation range were investigate and new models are proposed based entirely on temperature related variables and dimensional reasoning.

Scalar intermittency is investigated by looking at the PDF of temperature fluctuations. When imposing a high pass filter on the data, the PDF gradually exhibits
wider than Gaussian tails of exponential form while the PDF of the scalar derivative is non-Gaussian regardless of the filter. A new technique to analyze the observed tails was presented where the PDF is fitted to a simple exponential form. Further investigations of the nature of the exponential tails was done by considering the linear eddy model of Kerstein where the rare events behind the scalar intermittencies are modeled through the interchange of fluid parcels among different regions by a given eddy. In particular, the interplay between advection and diffusion in the occurrence of the intermittencies is related to the observed exponential tails.

Various aspects of this thesis need future substantial work. First, the scalar flux model needs to be compared against previous models, particularly the widely used gradient diffusion hypothesis. The proposed model shows that the scalar flux is inversely proportional to the mean scalar gradient, while the gradient diffusion hypothesis dictates that the scalar flux is proportional to the mean scalar gradient. The new model could replace previous models but there might be a way to reconcile both approaches as well. Further work needs to be done in this area to validate and answer all these questions. In addition, DNS with larger domains sizes need to be conducted in order to validate first that the integral scale reaches the predicted theoretical value and second that to tune and validate the proposed preliminary model. Finally, the thesis using the linear eddy model does not provide a solid and definite explanation of the exponential tails. Future work in this area might include a dimensional analysis and scaling of the scalar PDF.

The thesis highlights the exciting dynamics of scalar advection along with the phenomenological differences with the turbulent velocity field. It is shows that the flow exhibits self-similar solutions for both the spectra of temperature variance and the co-spectra of velocity and temperature. One particular aspect was that the flow is scaled and characterized by one length scale, which is proportional to the scalar Taylor microscale. Furthermore, the findings of this study show for the
first time that the turbulent scalar field can be investigated by solely measuring scalar variables with no information about the underlying turbulent velocity field (in this case, temperature). Using this fact, the dissipation spectra is model and a new relation for the scalar flux is proposed. The approach opens a new perspective for analyzing and understanding turbulent flows using scalar measurements that are inherently simpler to conduct. It was an exciting journey to take, particularly when discovering that a scalar, which is usually considered as simply carried by turbulence, exhibits new and different dynamics.
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