BANK BALANCE SHEETS, COLLATERAL CONSTRAINTS, AND OPTIMAL FISCAL AND MONETARY POLICY

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Abstract

This thesis studies the revaluation effect of inflation on bank balance sheets and its implications for fiscal and monetary policy.

Chapter 1 offers an empirical assessment of the gains and losses caused by unanticipated higher inflation to the U.S. commercial banks through their exposure to fixed-income instruments. Due to the mismatch of maturity between assets and liabilities, a persistent increase in the inflation rate causes a larger decline in banks' asset value than in liability value. We quantify this effect using the regulatory reports filed by the U.S. commercial banks. Our key finding is that a persistent increase in the inflation rate causes sizable losses to U.S. commercial banks.

Chapter 2 studies the implications of bank balance-sheet costs of inflation for the design of fiscal and monetary policy in response to fiscal shocks. We augment standard models with collateral constraints to account for this cost of inflation. In our model, banks hold nominal government debts, and inflation reduces the real value of government debts and tightens banks' collateral constraints. We study the Ramsey optimal fiscal and monetary policy in this model. Compared to the prescription of perfect tax smoothing in standard models, our model features a much smaller role for inflation in buffering higher government spending. We also extend the model to incorporate price stickiness and long-term government debts. We find that the maturity of government debts crucially impacts the size and persistence of the inflation process in the optimal policy.

Chapter 3 introduces nominal loans into the framework to capture the empirical fact that the majority of banks' fixed-income assets are long-term nominal loans to the business and household sectors. In this model, inflation affects bank balance sheets mainly through loan portfolios, as in the data. In the calibrated model, We find that the bank balance-sheet costs of inflation discourage the use of inflation in the optimal policy. Particularly, the assumption that loans are nominal is important for the quantitative results. In a sticky-price setting, we find that the maturities of firm loans and government debts significantly impact the role of inflation in the optimal policy.

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Contents

Chapter 1

Inflation and Revaluation of Bank Balance Sheets

1.1 Introduction

Since the onset of the 2008 financial crisis, a higher inflation target has been advocated by many economists (e.g., Blanchard et al., 2010; Krugman, 2013; Rogoff, 2008). There are many reasons why a higher inflation target is desirable: to address wage rigidity, reduce household and public debt, and reduce the real interest rate when the nominal interest rate reaches its zero lower bound.

However, higher inflation reduces commercial bank net worth through the revaluation of nominal fixed-income claims. First, commercial banks hold government debt on their balance sheets; monetizing government debt thus directly causes losses for banks.[1] Second, due to the maturity mismatch between bank assets and liabilities, a persistent increase in the inflation rate (e.g., a higher inflation target) causes bank asset values to drop faster than liability values. Commercial banks play an important

[1] For instance, the holdings of Japanese government bonds by Japan's banks equate to 900% of their Tier 1 capital (Jenkins and Nakamoto, 2012). When the Bank of Japan started qualitative and quantitative monetary easing in 2013 with the goal of reaching its 2% inflation target, fears arose that banks would bear large losses if the inflation rate were raised.

role in financial intermediation; because of financial frictions, losses borne by banks hamper credit supply and dampen real economic activities.

In this chapter we quantify the effect of a higher inflation target on U.S. commercial banks' balance sheets. We first document the size and maturity of nominal fixed-income positions on both sides of bank balance sheets. We then perform a simple experiment. Suppose that the inflation target is increased by 1% permanently and unexpectedly and is perfectly priced into yield curves: what would be the effect on the value of banks' nominal fixed-income positions if the only real effect of inflation were to revalue nominal contracts?

To document the size and maturity of bank nominal positions, we use data from the Bank Reports of Condition and Income (usually referred to as call reports) filed quarterly by U.S. commercial banks. The advantage of the call reports is the availability of maturity breakdowns of key nominal instruments on bank balance sheets. We find that during this period, at least 70% of bank assets and liabilities are nominal fixed-income instruments. The gap between the average maturities of assets and liabilities is about five years.

Combining the call reports data with estimated yield curves, we construct streams of future nominal payments generated by bank nominal positions, and use them to conduct the aforementioned experiment and gauge the valuation effect of a higher inflation target.

Our main result is that even a moderate 1% increase in the inflation target causes sizable losses to U.S. commercial banks. The asset-weighted-average capital loss fluctuates between 10–15% in our sample periods from 1997 to 2009. We also estimate the gains and losses contributed by each major class of nominal positions. Two thirds of the overall capital losses are contributed by loans and leases to the household and the corporate sectors, which constitute more than half of bank balance sheets. We also investigate the heterogeneity among banks. Some banks bear larger losses than others. In 2009Q4, 23.3% of banks would bear a capital loss of larger than 20% if inflation were increased by 1% permanently.

Banks may hedge their risks by trading interest rate derivatives. To investigate this possibility, we perform the same analysis on a subgroup of banks with no exposure to interest derivatives. Since these banks do not hedge interest rate risk, the results for this group are cleanest. We find that the size of losses born by this group of banks is also around 10–15% of Tier 1 capital.

We also perform the experiment on large banks with total assets larger than $50 billion, which are more systemically important. The size of losses incurred by these banks is very similar to that of smaller banks.

1.1.1 Related literature

This chapter directly relates to some recent works that document the maturity mismatch of commercial banks and evaluate their exposure to interest rate risk, using bank balance sheets data. Sher and Loiacono (2013) estimate the effect of a 2% parallel shift in the yield curve on loan portfolios held by a sample of large European banks. The Bank of Japan (2013) performs a similar analysis using data on Japanese commercial banks. To our knowledge, our work is the first to perform this analysis on U.S. commercial banks. Our work also features a more rigorous implementation in constructing future nominal payment streams used to evaluate the effect of inflation. For example, we construct payment streams of held-to-maturity claims (e.g., loans) using a recursive method and considering loan refinancing. In comparison, Sher and Loiacono (2013) simply assume that loans are all newly issued.

This chapter also relates to the literature studying the link between banks' interest rate risk exposure and their stock returns and credit supplies. Flannery and James (1984) find that stock prices of publicly traded commercial banks and savings and loan associations react negatively to increases in the general level of interest rates.

They also find that this reaction is stronger for institutions with a larger maturity gap of their assets over their liabilities. Similarly to Flannery and James (1984), English et al. (2014) find that unanticipated increases in both the level and slope of the yield curve associated with the Federal Open Market Committee (FOMC) announcements have large negative effects on bank stock prices. However, the effects are attenuated by a larger maturity gap. Regarding credit supplies, Landier et al. (2013) show that banks' exposure to interest rate risk predicts the sensitivity of bank lending to changes in interest rates.

Banks could use interest rate derivatives, which are off-balance sheet instruments, to offset their on-balance sheet exposure to interest rate risk. However, empirical evidence shows that there has been very limited success, if any, with trading interest rate derivatives. Begenau et al. (2013) replicate both on- and off-balance sheet items of several of the largest U.S. banks with two factors. They find that during the years 1999–2004 and 2007–2011, net derivative positions tended to amplify, not offset, balance sheet exposure to interest rate risk. Landier et al. (2013) also find that holdings of derivatives do not affect the banks' exposure to interest rate risk.

Finally, this chapter is related to recent literature studying the redistribution effect of monetary policy, both empirically and theoretically. Doepke and Schneider (2006) quantify the gains and losses born by various sectors (household, government and foreigners) and age groups under several hypothetical inflation scenarios. Gomes et al. (2014) develop a general equilibrium model with financial frictions to study the effect of inflation on the value of nominal corporate debts and macroeconomic activities.

Roadmap. The rest of the chapter is organized as follows. Section 1.2 discusses the data on the sizes and maturities of banks' nominal positions. Section 1.3 presents the conceptual framework used in our empirical analysis and describes the procedures used to construct streams of future payments. Section 1.4 presents the main results. We conclude in Section 1.5.

1.2 Nominal positions and maturity mismatch of U.S. commercial banks

1.2.1 Data

We use the Bank Reports of Condition and Income, generally referred to as the call reports, filed quarterly by U.S. commercial banks (FFIEC 031 and 041).[2] The call reports contain detailed information of the key items on an institution's income statement and balance sheet. There are two major advantages of the call reports compared to alternative sources of data, such as banks' Securities and Exchange Commission (SEC) filings. First, call reports provide information on the maturity distribution of banks' balance sheet items, such as loans, bonds and mortgage-backed securities (MBSs). Such information is crucial to evaluate banks' risk exposure to changes in the long-run inflation target. Second, the call reports are filed by all banks, including those that are not publicly traded.

We use the reports filed by commercial banks and aggregate bank-level data for all commercial banks owned by the same bank holding company (BHC).[3] We perform this aggregation because common ownership ties could foster risk sharing among bank subsidiaries (Houston et al., 1997). BHCs also file regulatory reports (form FR Y-9C). We do not directly use the reports filed by BHCs because detailed information on maturity distribution is only available in commercial banks' reports.

We build panel data for the sample period 1997Q2-2009Q4, when information on maturity distribution is available. During this period, there were considerable

[2] We acquire the data from Wharton Research Data Services.

[3] We use the variable RSSD9348, which identifies a bank's regulatory high holder.

numbers of mergers and acquisitions among banks and BHCs. We address this issue using data on merger and acquisition activities from the Federal Reserve Bank of Chicago that contain the date of each merger and the identity numbers of the non-surviving and the acquiring bank or BHC.[4] If institution A was acquired by institution B on date t, we add A's balance sheet positions to B's and treat them as one institution prior to date t.

We drop from our sample banks with asset values smaller than $500 million in 2009Q4 and restrict our attention to relatively large banks.[5] We drop banks whose observations are not continuous in the sample. This is because we have adopted a recursive method to construct future payment streams of some asset classes, as we describe in Section 1.3.3. For this purpose, it is important that a bank has continuous observations over the sample period.[6]

Table 1.1 lists the distribution of sample banks in the fourth quarter of each sample year. We segregate banks into three size groups according to their total assets in 2009Q4: large banks have assets greater than $50 billion, medium banks have assets between $10 and $50 billion, and small banks have assets less than $10 billion. There are in total 800–1100 banks in the sample each year, among which more than 90% are small banks. Since larger banks have greater systemic importance in the financial system, we will evaluate the effect of a higher inflation target separately on these three size groups in Section 1.4.1.

1.2.2 Information on maturity breakdowns

Crucial to our analysis are the maturity breakdowns of key items on bank balance sheets. The call reports provide maturity breakdowns for banks' holding of securities,

[4]Data are obtained from `https://www.chicagofed.org/webpages/publications/financial_institution_reports/merger_data.cfm`.

[5]$500 million is the threshold above which a BHC needs to file regulatory report FR Y-9C (after March 2006).

[6]Only 20 banks are dropped due to discontinuous observations, which constitute less than 3% of the total number of banks.

Table 1.1: Number of Bank Holding Companies

Year	Total	Large	Medium	Small
1997	774	22	33	719
1998	808	23	35	750
1999	854	23	39	792
2000	886	23	39	824
2001	913	24	41	848
2002	941	24	41	876
2003	960	24	44	892
2004	982	24	44	914
2005	1,005	24	44	937
2006	1,031	24	44	963
2007	1,054	25	44	985
2008	1,075	28	45	1,002
2009	1,079	29	45	1,005

Note: Large banks have assets larger than $50 billion in 2009Q4, medium banks have assets between $10 and $50 billion in 2009Q4, and small banks have assets less than $10 billion in 2009Q4.

loans, time deposits and other borrowed money.[7] The remaining maturity or the time to next repricing date of each item is categorized in the form of buckets: less than three months, over three months through 12 months, etc. (See Table 1.3 for a complete summary.)

Importantly, the time to the next repricing date, rather than the contractual maturity, is recorded for variable-rate contracts. This information greatly simplifies our analysis because we can treat a variable-rate contract as a fixed-rate contract maturing on the next repricing date.

Maturity breakdowns are available for most items on bank balance sheets as shown in Figure 1.1. For most sample periods, maturity information is known for more than 70% of bank assets and liabilities. Items for which maturity information are not reported include stocks, trading assets and liabilities, and securities purchased (sold)

[7]Other borrowed money includes Federal Home Loan Bank advances and other borrowings (e.g., promissory notes).

under agreements to resell (repurchase). Without this information, we do not consider these items when we evaluate the effect of a higher inflation target.

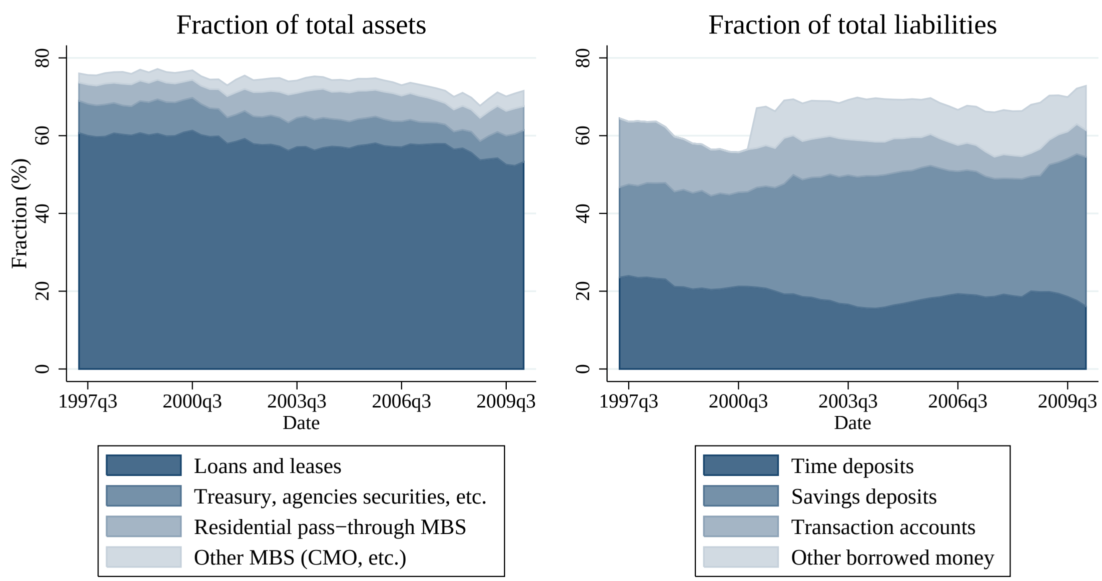

Figure 1.1: Fraction of total assets and liabilities of which maturity breakdowns are reported. We compute fractions for each bank in the sample and report the asset-weighted average statistics in the figure. Note that the maturities of transaction deposits and savings accounts are not reported. In our exercise we assume that they both have maturities of one quarter.

Figure 1.1 also shows the size of key balance-sheet items for which maturity breakdowns are available. On the asset side of bank balance sheets, loans and leases (mortgage, commercial and industrial, etc.) exceed 50% of bank total assets. MBSs, including both pass-through securities[8] and structured products (e.g., CMOs) account for 7% to 10% of total assets. The holdings of securities issued by the U.S. Treasury and government agencies decrease from more than 8% of total assets in 1997Q2 to less than 5% before the 2008 financial crises. They grow back to 8% thereafter, consistent

[8]Pass-through securities are securities of which interest and principal payments from the borrower or homebuyer are directly passed through to holders of the MBSs.

with the flight-to-liquidity and flight-to-quality theories. On the liability side of bank balance sheets, deposits (time deposits, transaction deposits, and savings accounts) cover more than half of bank total liabilities. Since 2001Q1, maturity breakdowns for "other borrowed money" have become available. Other borrowed money accounts for 9–12% of bank total liabilities.

The maturity of transaction deposits and savings accounts deserves special attention. These deposits have zero contractual maturity, and in principal interest rates paid on these deposits can adjust instantly. However, interest rates on these claims are de facto very sticky (Hannan and Berger, 1991). As a result, the effective maturity of these claims can be very long. On the other hand, depositors may withdraw sooner if general interest rates increase but deposit rates remain the same, which shortens the maturity of deposits (e.g., English et al., 2014). Bearing these issues in mind, we follow Doepke and Schneider (2006) and assume that transaction deposits and savings accounts have short maturities (one quarter).

1.2.3 Maturity mismatch between bank assets and liabilities

We now document the degree of maturity mismatch between bank assets and liabilities. The first two panels of Figure 1.2 plot the average maturity/repricing period of the key items on both sides of bank balance sheets. In our calculations, we set the maturity/repricing period within each bucket to the midpoint of that bucket's range.[9]

On the asset side, pass-through MBSs have the longest maturity, increasing from 10 years at the beginning of sample period to 15 years at the end of sample period. Treasury and government agency securities have maturity of around five years. Loans and leases, as well as structured MBSs, have a shorter maturity of around three to four

[9]For example, U.S. Treasury securities with remaining maturity or time to the next repricing date of more than one year but less than or equal to three years are assumed to have a maturity/repricing period of two years, the midpoint of the (1, 3] interval. Claims with remaining maturity or time to the next repricing date of over 15 years are assumed to have a maturity/repricing period of 20 years; claims with remaining maturity or time to the next repricing date of over three years are assumed to have a maturity/repricing period of five years.

years. On the liability side, time deposits and other borrowed money both have very short maturities of one to two years. The maturities of these items remain relatively stable over time.

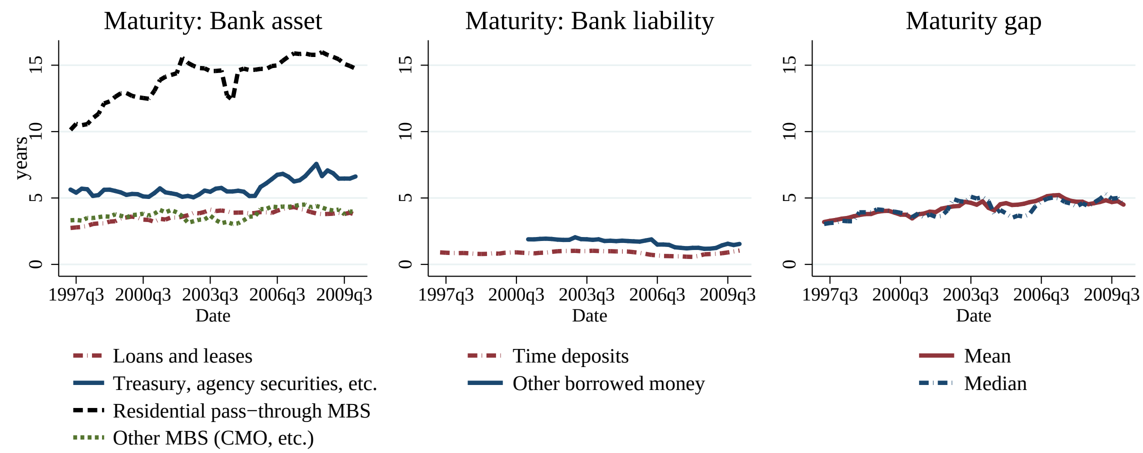

Figure 1.2: Maturities of bank assets and liabilities. We compute statistics for each bank in the sample and report the asset-weighted average statistics in the figure.

To gauge the degree of maturity mismatch, we define maturity gap as the difference between the weighted-average maturity/repricing period of bank assets and liabilities, as in English et al. (2014). We plot the cross-sectional asset-weighted mean and median maturity gap in the third panel of Figure 1.2. Both measures of maturity gaps fluctuate around three to five years over the entire sample period.

Although contractual maturity/repricing periods and maturity gap are useful to describe maturity mismatch as a first pass, they are insufficient and imprecise to characterize the entire distribution of cash payments over future periods. For zero-coupon bonds, it is true that the contractual maturity coincides with the effective duration. But for long-term coupon bonds or amortized mortgages with sizable cash payments before the contractual maturity date, effective durations can be much shorter than the contractual maturities.[10] Therefore, when evaluating bank losses in a higher in-

[10]One way to characterize the effective duration is the Macaulay duration, which is a weighted average of the maturities of the cash payments (see Mishkin and Eakins, 2010, chapter 3).

flation scenario, it is important to know future cash payments of long-term bonds and mortgages. In the next section, we describe our methods for constructing future cash payment streams for various balance-sheet items. We also propose an asset-pricing framework to price the future payments under higher inflation scenarios.

1.3 Methodology

In this section we assess the gains and losses to the U.S. commercial banks induced by an unanticipated arrival of a moderate inflation episode. Suppose that, starting from a benchmark date, the inflation target is increased by one percentage point permanently and unexpectedly. Our goal is to estimate the present-value gains or losses caused by such an inflation episode for the U.S. commercial banks.

To proceed, we first propose an asset-pricing framework to price the future payments under high inflation scenarios. A key assumption we adopt is that the only real effect of higher inflation is revaluating nominal fixed-income claims. More specifically, the real stochastic discount factor and credit risks associated with the financial claims are assumed not to be affected by higher inflation. We then describe our methods to estimate yield curves and construct payment streams generated by bank portfolios.

1.3.1 Conceptual framework

We will first discuss the pricing of zero-coupon bonds. A more general fixed-income claim with coupon payments can be viewed as a portfolio of zero-coupon bonds with different maturities. Therefore, the price of this claim is a linear combination of the prices of zero-coupon bonds.

Pricing nominal zero-coupon bonds. We assume the exogenous fundamentals of the economy are functions of a shock s_t. We let $s^t = (s_0, ..., s_t)$ denote a history of shocks.

We first consider the pricing of a one-period bond issued in date t, that is, a financial claim on date t that pays \$1 in date $t+1$, in all histories $s^{t+1}|s^t$. When considering the credit risk associated with the bond, we assume that it is a pool of many independent borrowers, and therefore the law of large numbers applies. We denote the fraction of borrowers who default in history s^{t+1} by $h(s^{t+1})$. The dollar value of this financial claim in state s^t is

$$w_1(s^t) = \sum_{s^{t+1}|s^t} \Pr(s^{t+1}|s^t) \frac{(1-h(s^{t+1}))\, m_{t,t+1}(s^{t+1})}{\pi_{t,t+1}(s^{t+1})}.$$

Here, $\pi_{t,t+1}(s^{t+1})$ denotes the gross inflation rate between state s^t and s^{t+1}; $m_{t,t+1}(s^{t+1})$ denotes the real stochastic discount factor in state s^t for a payoff in state s^{t+1}; $\Pr(s^{t+1}|s^t)$ is the conditional probability. This pricing equation has the following interpretation. The bond pays $1-h(s^{t+1})$ dollars in state s^{t+1}. For each dollar paid in s^{t+1}, its dollar value in s^t is $\frac{1}{\pi_{t,t+1}(s^{t+1})}$ and should be discounted by the real discount factor $m_{t,t+1}(s^{t+1})$.

We next consider a zero-coupon bond without restricting its maturity to one period. Specifically, a j-period bond that pays \$1 in all states $s^{t+j}|s^t$ for a given $j \geq 1$. It is priced by

$$\begin{aligned} w_j(s^t) &= \sum_{s^{t+j}|s^t} \Pr(s^{t+j}|s^t) \prod_{m=0}^{j-1} \frac{(1-h(s^{t+m+1}))\, m_{t+m,t+m+1}(s^{t+m+1})}{\pi_{t+m,t+m+1}(s^{t+m+1})} \\ &\equiv \frac{1}{(1+i_{t,t+j}(s^t))^j}. \end{aligned} \tag{1.1}$$

The second equality is simply the definition of the j-period zero-coupon yield $i_{t,t+j}(s^t)$. It depends on the expected inflation, the default risks and the real stochastic discount factor.

Now consider an unanticipated one-time announcement by the central bank in state s^t to increase the inflation target by $\Delta\pi$ in all histories after s^t. We assume that this scenario is a surprise to agents in the market, and the expectation of a higher future inflation rate is immediately formed after the announcement. The value of the j-period zero-coupon bond after the announcement becomes

$$\tilde{w}_j(s^t) = \sum_{s^{t+j}|s^t} \Pr(s^{t+j}|s^t) \prod_{m=0}^{j-1} \frac{(1 - h(s^{t+m+1}))\, m_{t+m,t+m+1}(s^{t+m+1})}{\pi_{t+m,t+m+1}(s^{t+m+1}) + \Delta\pi}.$$

The underlying assumption is that the only real effect of higher inflation is to revalue nominal financial claims. More specifically, we assume that the credit risk, characterized by the state-contingent haircut $h(s^{t+m})$, and the real stochastic discount factor $m_{t+m,t+m+1}(s^{t+m+1})$ are both unaffected by the change in the inflation target. In making these assumptions, we essentially restrict our attention to the partial equilibrium effect of inflation. In a general equilibrium model, changes in the inflation target will endogenously affect the default risk, the real stochastic discount factor, and therefore the price of bonds.

When zero coupon yields $i_{t,t+j}(s^t)$ and the change in inflation rate $\Delta\pi$ are small, we can approximate $\tilde{w}_j(s^t)$ by

$$\tilde{w}_j(s^t) \approx \frac{1}{(1 + i_{t,t+j}(s^t) + \Delta\pi)^j}.$$

Intuitively, this equation states that expectations of a higher inflation target are priced into the nominal yield curve, when real interest rates remain unaffected. This is the formula commonly adopted in studies of the redistribution effects of a higher inflation rate (e.g., Doepke and Schneider, 2006).

It follows that the value of the j-period zero-coupon bond drops by

$$\begin{aligned}\Delta w_j(s^t) &= \tilde{w}_j(s^t) - w_j(s^t) \\ &= \frac{1}{(1+i_{t,t+j}(s^t)+\Delta\pi)^j} - \frac{1}{(1+i_{t,t+j}(s^t))^j}.\end{aligned} \tag{1.2}$$

Pricing more general nominal fixed-income claims. Now consider a more general financial claim that pays ν_j dollars in all states $s^{t+j}|s^t$ for $\forall j \geq 1$. By linearity, the decline in its value in a higher inflation scenario of $\Delta\pi$ is

$$\Delta V(s^t) = \sum_j \left[\frac{1}{(1+i_{t,t+j}(s^t)+\Delta\pi)^j} - \frac{1}{(1+i_{t,t+j}(s^t))^j}\right]\nu_j. \tag{1.3}$$

In the rest of the paper, we estimate $\Delta V(s^t)$ for bank portfolios under a scenario of a one percent increase in the inflation target ($\Delta\pi = 0.01$). The estimation involves three steps. First, we estimate the zero-coupon yield curves $\{i_{t,t+j}\}_{j\geq 1}$ for different types of financial claims held by banks. Second, we construct payment streams $\{\nu_j\}_{j\geq 1}$ generated by these financial claims using information on their sizes and maturities. Third, we estimate banks' gains and losses according to Equation (1.3).

1.3.2 Estimating yield curves

To price a given payment stream generated by banks' portfolios at each date, we need to know the zero-coupon yield curve $\{i_{t,t+j}\}_{j\geq 1}$. In principal, asset classes differ in safety, liquidity and other features, and we want to estimate the yield curve for each asset class. Due to limitations on interest rate data, we estimate two yield curves: that of Treasury securities and that of interest rate swap contracts.[11] We use the yield

[11] An interest rate swap contract is an agreement between two parties to exchange fixed and variable interest rate payments on a notional principal amount over a predetermined period. The swap interest rate is the rate of the fixed leg of a swap contract, calculated to make the net present value of the contract equal zero. The swap market is one of the most active segments of the fixed-

curve of Treasury securities to discount banks' holding of safe assets and liabilities (e.g., Treasury and agency securities and consumer deposits). We use the swap yield curve to discount privately issued securities, such as loans and leases.

We adopt parametric formulations of the yield curves in our estimation. In general, parametric formulations impose smoothness assumptions on the curve, and therefore are more suitable for studying the macroeconomic forces that influence the shape of the curve. In contrast, spline-based methods are better suited for capturing local behaviors of the yield curve.

We follow the standard approach proposed by Svensson (1994) and assume the following parametric form for the instantaneous forward curve at date t:

$$f_t(t+j) = \beta_{0,t} + \beta_{1,t} \exp\left(-\frac{j}{\tau_{1,t}}\right) + \beta_{2,t} \frac{j}{\tau_{1,t}} \exp\left(-\frac{j}{\tau_{1,t}}\right) + \beta_{3,t} \frac{j}{\tau_{2,t}} \exp\left(-\frac{j}{\tau_{2,t}}\right)$$

where $f_t(t+j)$ denotes the instantaneous forward rate j years ahead. Under the expectation hypothesis, the zero-coupon yield curve is given by $i_{t,t+j} = \frac{1}{j}\int_0^j f_t(t+u)du$. At a given point of time t, the zero-coupon yield curve $\{i_{t,t+j}\}_j$ is characterized by six parameters $\{\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \beta_{3,t}, \tau_{1,t}, \tau_{2,t}\}$.

The Svensson yield curve is the most commonly used parametric form by central banks (Reppa, 2008). It is flexible enough to produce curves with two extrema, one maximum and one minimum.

Treasury yield curve. We directly use the result of Gürkaynak et al. (2007), who estimate the Svensson yield curve for the entire maturity range spanned by outstanding Treasury securities from 1961 to present.[12] They show that their estimation is accurate for the entire maturity range, and the prediction error of bond yields lies within one basis point.

income market. Central banks sometimes monitor swap interest rates along with government bonds rates, as the former carries information of credit risks (Bank for International Settlements, 2005).

[12] Available at `http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html`.

Swap yield curve. We use middle rate quotes of the UK-based inter-dealer broker ICAP, accessed through the Reuters database. The maturities of the contracts are 1–10, 12, 15, 20, 25 and 30 years.

In our estimation, we use the fact that a hypothetical bond paying a coupon rate equal to the swap interest rate is priced at par (Lesniewski, 2008). For each quarter of the sample period, we estimate $\{\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \beta_{3,t}, \tau_{1,t}, \tau_{2,t}\}$ by minimizing the weighted sum of squared deviations between actual bond prices and predicted bond prices. The weights are the inverse of the duration of each individual security.[13]

The success at fitting the swap yields is repeated throughout the sample. Table 1.2 shows the time-average absolute yield prediction error in different maturities. As can be seen, all of the errors are quite small over the entire sample, within several basis points.

Table 1.2: Average absolute yield prediction errors by maturity

Maturity	1	2	3	4	5	6	7	8
Error (bps)	1.3	3.3	2.2	2.1	2.0	2.4	5.3	2.6
Maturity	9	10	12	15	20	25	30	
Error (bps)	3.4	3.5	1.5	4.5	2.7	2.5	2.0	

Note: Average absolute yield prediction error in different maturities.

As an example, we report the estimated Treasury and swap yield curves at the beginning of the sample period (1997Q2) and before the crisis (2007Q4) in the appendix (Figure 1.10). Two observations are worth noting. First, the Svensson parametric form is flexible enough to capture two humps in the Treasury yield curve in 2007Q4. Second, our sample period features a large decline in the overall level of interest rate. This pattern of data suggests that when constructing payment streams of long-term

[13] Since a given change in the yield corresponds to a larger change in the price of a bond with a longer duration, fitting prices of each bond given an equal weight irrespective of its duration will lead to over-fitting of the long-term bond prices at the expense of the short-term prices. Therefore we follow the literature by weighting the price error of each bond by a value derived from the inverse of its duration (Bank for International Settlements, 2005). This procedure is approximately equivalent to minimizing the unweighted sum of squared deviations between the actual and predicted yields of securities.

loans, it is important to distinguish loans issued at earlier and later dates since their yields may differ significantly. Therefore, in the spirit of Doepke and Schneider (2006), we adopt a recursive method to construct payment streams for long-term loans and MBSs, as described in the next subsection.

1.3.3 Constructing payment streams

We will now describe the methods used to construct payment streams of major categories of fixed-income instruments on bank balance sheets. In the construction we use size and maturity data on balance-sheet positions as well as yield curves estimated from the previous subsection.

For long-term fixed-income claims, it is important to distinguish between book value and fair value accounting. According to the guidelines of the call reports, most loans are recorded at face value, while most securities (Treasury or agency securities, or privately issued MBSs) are recorded at fair value.

Since maturity data in the call reports are in the form of buckets, we assume that within each bucket the maturity is uniformly distributed and that the maximal maturity is 20 years.

Loans and leases. We assume that all loans and leases are amortized according to the straight-line schedule, which features equal monthly payments until the maturity.

Since most loans and leases are held to maturity, we adopt a recursive method to construct payment streams. In the initial sample period (1997Q2), we assume that all loans were newly issued. For each maturity j, we observe the book value of the loan with maturity of j years. We construct the loans' payment stream $\{\nu_{t,m}\}_m$ according to the fact that the discounted value of payment stream $\{\nu_{t,m}\}_m$ using the swap yield curve must equal its book value. We also determine the remaining face values of the initial vintage of loans in each subsequent sample period. This recursive method distinguishes between loans issued in earlier sample periods when interest rates were high and loans issued in later periods when interest rates were low.

For each subsequent sample period, we compute recursively the face value of new loans issued as well as the expected payments and evolution of face value associated with that period's vintage.

We consider refinancing activities when constructing payment streams. In the late 1990s and early 2000s, many homeowners took advantage of relatively low interest rates to refinance their mortgage loans. As shown in Figure 1.11, 7–13% of outstanding mortgage loans were refinanced each quarter.[14] Therefore, when constructing payment streams after the initial sample period, we take into account that some existing loans are refinanced. We assume that when a mortgage loan is refinanced, the new loan has the same maturity as the remaining maturity of the old loan.

Mortgage-backed securities. We assume that all MBSs are pass-through securities for which principal and interest payments are directly passed on to security holders from mortgage borrowers.[15] To construct payment streams, we adopt a recursive approach similar to that of loans and leases. The only difference is that MBSs are recorded at fair value. Therefore, for each period we compute the fair value of previously issued securities using current interest rates.

Treasuries and agency-bonds. Because these securities are actively traded on the market instead of being held to maturity, the previously mentioned recursive method is not appropriate in constructing payment streams. To proceed, we make two assumptions. First, all securities are newly issued and issued at par; second, a security is a zero-coupon bond if its maturity is less than one year, and a coupon bond otherwise. Then we compute coupon payments using the Treasury yield curve.

[14]To construct the fraction of outstanding mortgage loans being refinanced, we use "mortgage refinance by one- to four-family residences" from Mortgage Bankers Association, and "mortgage debt outstanding by one- to four-family residences" from the Federal Reserve Economic Data (FRED).

[15]As in Figure 1.1, the majority of MBSs are pass-through securities.

Time deposits and other borrowed money. We also adopt the previously mentioned recursive method to construct payment streams for time deposits. The only difference is how payments are distributed across future periods. For time deposits, interests are accrued until maturity; for other borrowed money, we assume that it is in the form of coupon bonds, and their face values are not amortized.

Transaction deposits and savings accounts. As discussed in the previous session, we assume that these deposits have maturity of a quarter and that the interest rates paid on these deposits adjust in a quarter.

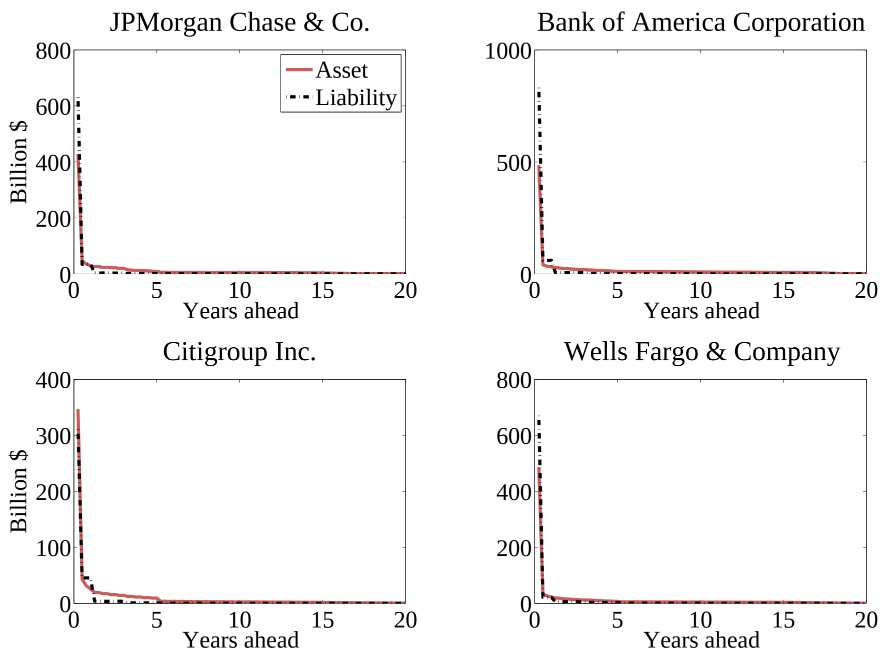

Figure 1.3: Constructed quarterly payment streams for the four largest BHCs

1.3.4 Examples of constructed payment streams

As an example, constructed quarterly payment streams for the four largest BHCs are plotted in Figure 1.3. On both asset and liability sides of bank balance sheets, future payments are very concentrated on short maturities within five years. Consistent with evidence of maturity mismatch discussed in the previous section, payments of bank assets are less concentrated on short maturities compared with payments of bank liabilities.

1.4 Results

In this section, we use the constructed payment streams to evaluate the present value of each category on bank balance sheets. We then assume that the inflation target increases permanently by 1% ($\Delta\pi = 0.01$), and we use Equation (1.3) to gauge the gains or losses of bank balance sheets.

The results are shown in Figure 1.4. For each sample year (fourth quarter), we compute gains and losses as a percentage of Tier 1 capital for each bank in the sample, and report the asset-weighted average statistics in the figure.

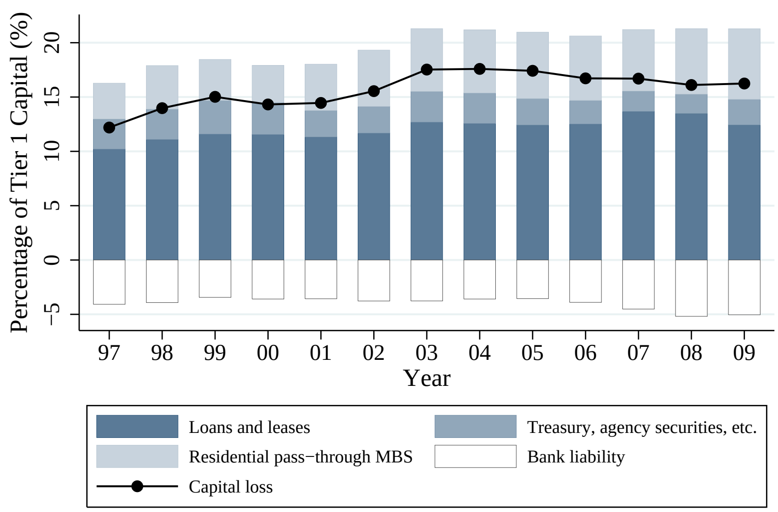

Figure 1.4: Gains and losses caused by a 1% permanent increase in the inflation rate. We compute gains and losses for each bank in the sample and report the asset-weighted average statistics in the figure.

As a result of maturity mismatch, most banks suffer a capital loss after an increase in the inflation rate. Overall losses as a percentage of Tier 1 capital fluctuate at 10–

15% over the sample period. This estimate is comparable with estimates of Japanese banks provided by the Bank of Japan (2013).

As shown in Figure 1.4, most losses are caused by holdings of loans and leases, which are around 10% of Tier 1 capital. This large loss is driven by the large volume of loans and leases, which amounts to more than half of bank total assets (see Figure 1.1). The second largest loss is through holdings of MBSs (3–5% of Tier 1 capital). As seen in the previous section, although MBSs constitute only 10% of bank total assets, they have very long maturities of 10–15 years. Therefore, they contribute a considerable amount of bank loss when inflation rises. Treasury and government agency securities cause a relatively small amount of capital loss, which fluctuates at 1–2%.[16] At the same time, since bank liabilities tend to have very short maturities, they only cause less than 5% of capital gains when the inflation rate rises.

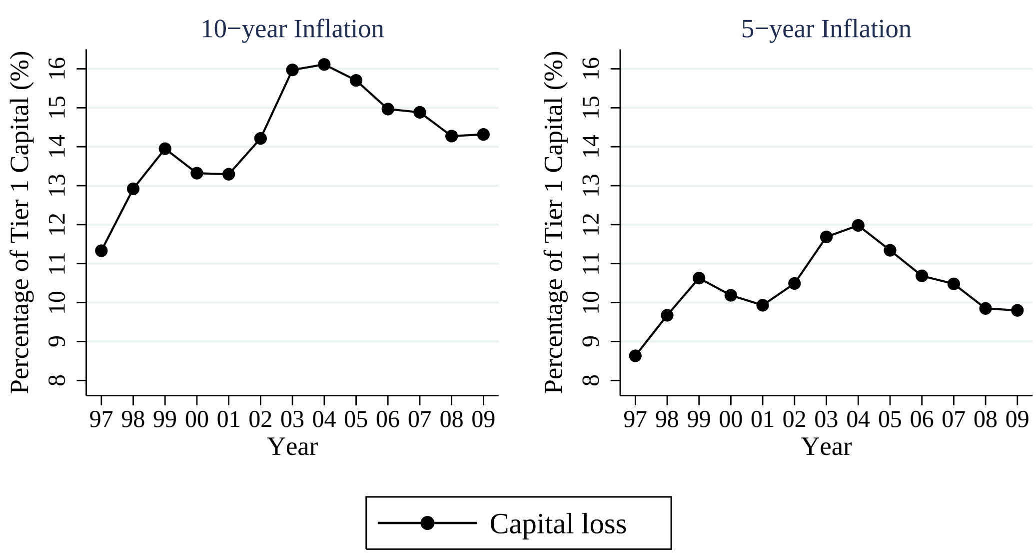

Figure 1.5: Gains and losses caused by a 1% increase in the inflation rate lasting 10 years or 5 years. We compute gains and losses for each bank in the sample and report the asset-weighted average statistics in the figure.

[16] This number is considerably smaller than that of Japanese banks, which is around 10–20%. This is because U.S. commercial banks hold far fewer government bonds on their balance sheets than Japanese banks.

Even if the increase in the inflation rate is not permanent, the bank losses are still sizable if higher inflation is sufficiently long-lasting. Figure 1.5 shows the bank balance-sheet losses assuming that inflation increases by 1% for 10 years (left panel) or 5 years (right panel). We see that the case of 10-year inflation is almost the same as the case of permanent inflation in Figure 1.4. Even if inflation only rises for five years, the amount of Tier 1 capital loss still fluctuates at 10%.

Capital losses born by some banks are much larger than the average. Figure 1.6 presents the cross-sectional distribution of capital losses at the beginning and the end of the sample period (1997Q2 and 2009Q4). In 2009Q4, 23.3% of banks would bear a capital loss larger than 20% if inflation were to increase by 1% permanently. The distribution becomes flatter over time. For example, in 1997Q2 only 8.2% of banks would bear a loss larger than 20%.

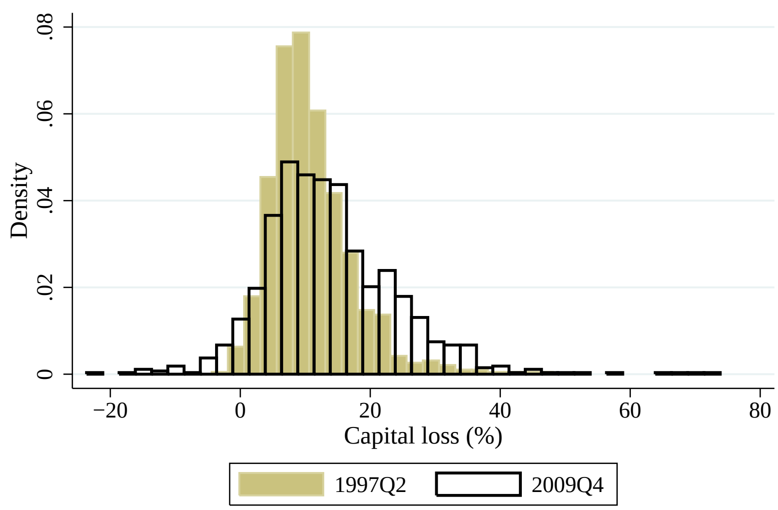

Figure 1.6: Cross-sectional distributions of capital loss. We plot the cross-sectional distributions of capital losses caused by a 1% permanent increase in the inflation rate.

1.4.1 Do losses caused by inflation depend on bank size?

In this subsection we investigate whether bank size affects losses caused by rising inflation. If larger banks have better management of maturity mismatch, we expect that they will suffer smaller capital losses after inflation rises. We perform the same experiment of 1% permanent inflation on three groups of banks categorized according to their total assets in 2009Q4 (see Table 1.1 for sample description). Results of the experiment are plotted in Figure 1.7.

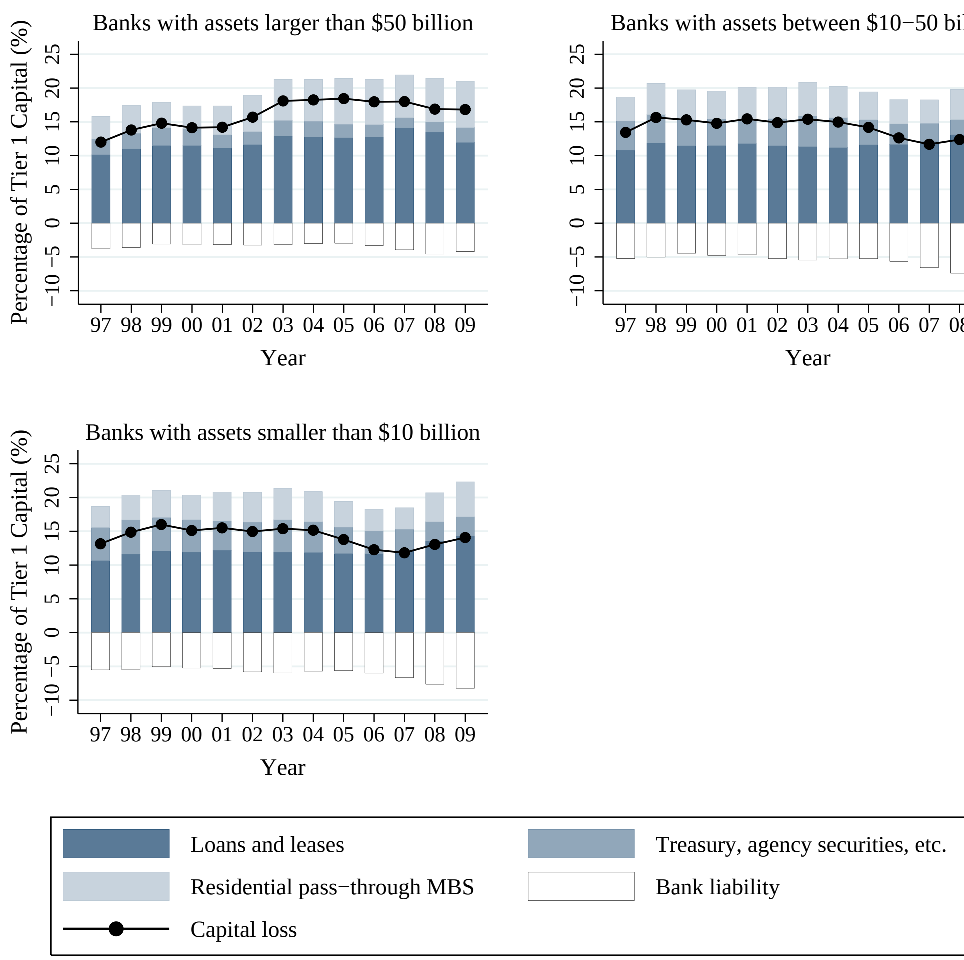

Figure 1.7: Gains and losses by bank size (total assets). We compute gains and losses for each bank in the sample and report the asset-weighted average statistics in the figure.

Overall, the sizes of losses are similar across three groups of banks: around 10–15% percent of Tier 1 capital. If anything, the largest group of banks, with assets larger than $50 billion, bears slightly larger losses than medium- and small-sized banks in the second half of the sample (after 2003). Therefore, inflation causes a substantial loss to big banks that bear more systemic importance.

1.4.2 Do banks hedge risks by holdings of interest rate derivatives?

In this subsection we investigate whether banks hedge interest rate risks through holdings of interest rate derivatives. The call reports record the notational value of interest rate derivatives (swaps, futures, etc.), and they distinguish between derivatives "held for trading" and those "held for purposes other than trading". According to accounting rules, the majority of positions due to market making activity are recorded as "held for trading".[17] We assume that all derivatives held "not for trading" are due to trading on one's own account.

We focus our attention on interest rate derivatives held for purposes other than trading as we are mostly interested in banks' behavior to hedge their own interest rate risks. We plot the fraction of banks holding a positive amount of interest derivatives in Figure 1.8 as well as the size of their holdings as a percentage of total assets, conditional on non-zero holding.

Banks' exposure to interest rate derivatives increases drastically during our sample periods as reflected in the fraction of banks holding interest rate derivatives and the size of their holdings. In 1997, only 5% of banks hold interest rate derivatives, compared to 40% in 2009. Conditional on positive exposure, the average size of

[17] Most interest rate derivatives are traded over the counter, and a few large dealers make the market. In particular, dealers intermediate between two parties by initiating, say, a pay-fixed swap with the first party as well as an offsetting pay-floating swap with the second party. Often one of the parties is another dealer.

exposure also expands quickly, from 5% in 1997 to 40% in 2003, and falls gradually to 20% in 2009. However, it is worth-noting that even in 2009 more than half of the banks do not hold any interest rate derivatives.

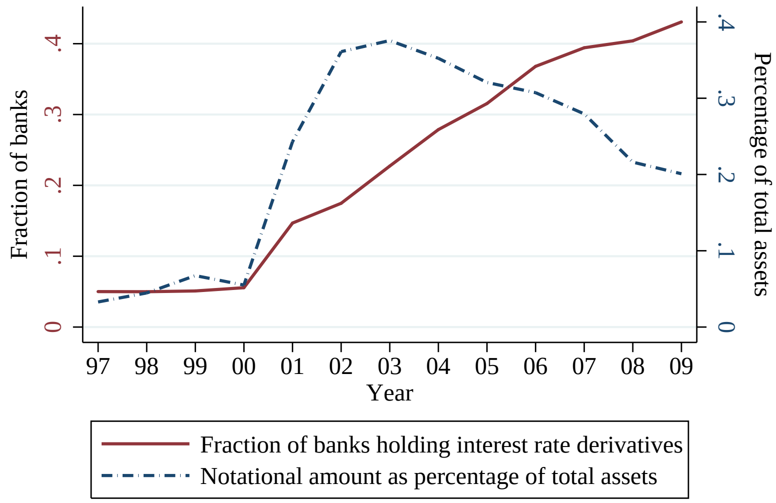

Figure 1.8: Fraction of banks holding interest rate derivatives and the size of their holdings. The dash-dotted line shows the notational amount of interest rate derivatives as a percentage of total assets, conditional on positive holding. We compute the percentage for each bank in the sample and report the asset-weighted average statistics in the figure.

For each sample year, we divide banks into three groups according to their exposure to interest rate derivatives (no exposure, smaller and larger than 20% of total assets). We then repeat our experiment on each group and report the results in Figure 1.9.

The overall size of capital losses caused by higher inflation is similar across the three groups and fluctuates at 10–15%. Without any exposure to interest rate derivatives, the first group of banks does not hedge interest rate risk, and therefore the results are the cleanest for this group.

On the other hand, there is evidence that banks with the largest holdings of interest rate derivatives would bear larger loss through their on-balance-sheet fixed-income portfolios if inflation rate rose. This was particularly the case in the early 2000's before the Great Repression. This result is consistent with the findings of Begenau et al. (2013), that from 2004 to 2007, interest rate swaps and futures were hedging the interest rate risk of on-balance-sheet items.

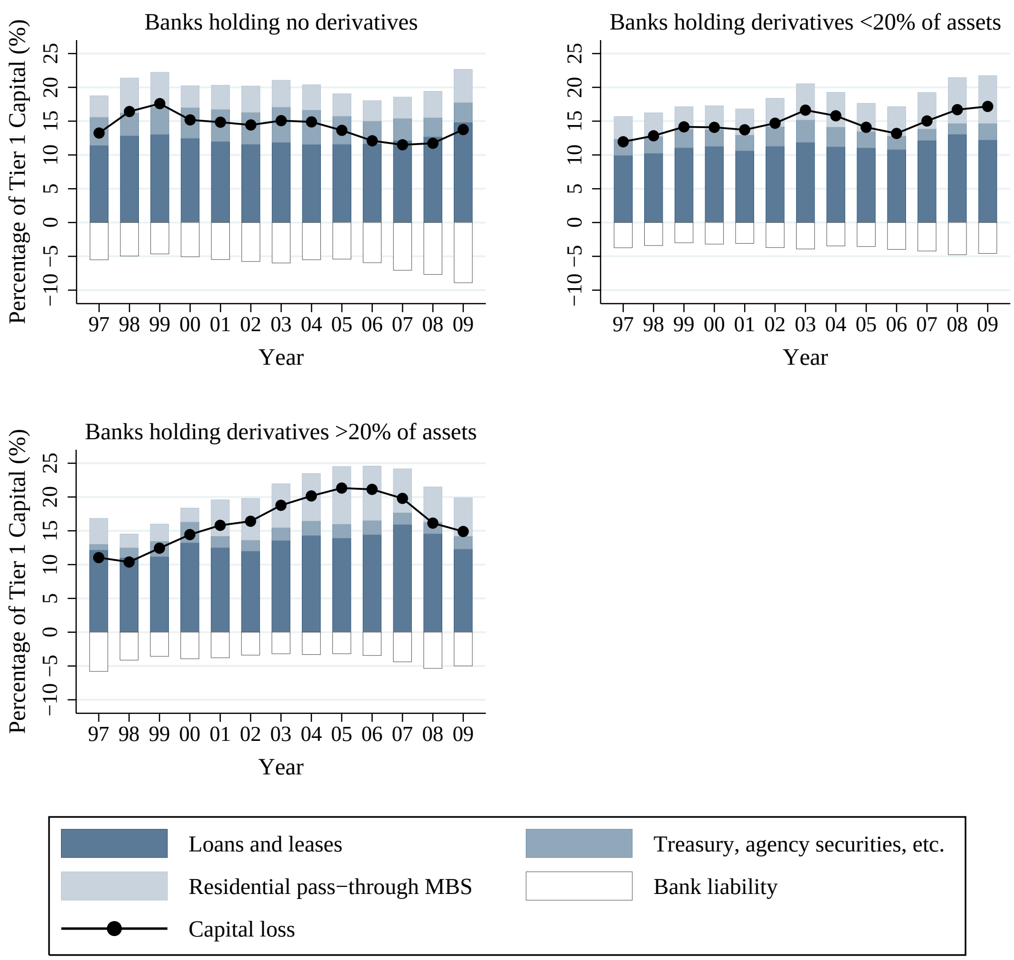

Figure 1.9: Gains and losses by bank derivative holdings. We compute gains and losses for each bank in the sample and report the asset-weighted average statistics in the figure.

1.5 Conclusion

Our goal in this chapter was to quantitatively assess the effect of inflation on the U.S. commercial banks. We have documented the size and maturity of nominal assets and liabilities on bank balance sheets, and we have used those numbers to compute the capital gains and losses that would be induced by a moderate inflation episode. Our main result is that even moderate inflation leads to sizable capital losses to banks. We also find sizable losses to large banks that are more systemically important, and to banks that hold no interest rate derivatives and thus do not hedge interest rate and inflation risks.

Our findings raise a few questions for the design of fiscal and monetary policy. Will losses born by banks cause a decline in credit supply and the efficiency of resource allocation? If so, how should a country with a fiscal problem trade off between fiscal and monetary policy? Our results suggest that inflation is costly to banks, and this fact should be considered when designing fiscal and monetary policy. We address these questions in the next two chapters.

1.6 Appendix

1.6.1 Maturity breakdowns in the call reports

Table 1.3: Maturity breakdowns in the call reports

Treasury and agencies securities, Residential pass-through MBS, Loans and leases	1. Three months or less. 2. Over three months through 12 months 3. Over one year through three years 4. Over three years through five years 5. Over five years through 15 years 6. Over 15 years
Time deposits, Other borrowed money	1. Three months or less 2. Over three months through 12 months 3. Over one year through three years 4. Over three years
Non pass-through MBS	1. Three months or less 2. Over three months

1.6.2 Estimated yield curves

We report the estimated Treasury and swap yield curves at the beginning of the sample period (1997Q2) and before the crisis (2007Q4) in Figure 1.10. The Svensson parametric form is flexible enough to capture two humps in the Treasury yield curve in 2007Q4. The first hump, which is located at the shorter horizon, probably reflects the expectation of monetary easing prior to crises; the second hump at the longer horizon reflects convexity, which tends to bring down long-term bond yield.

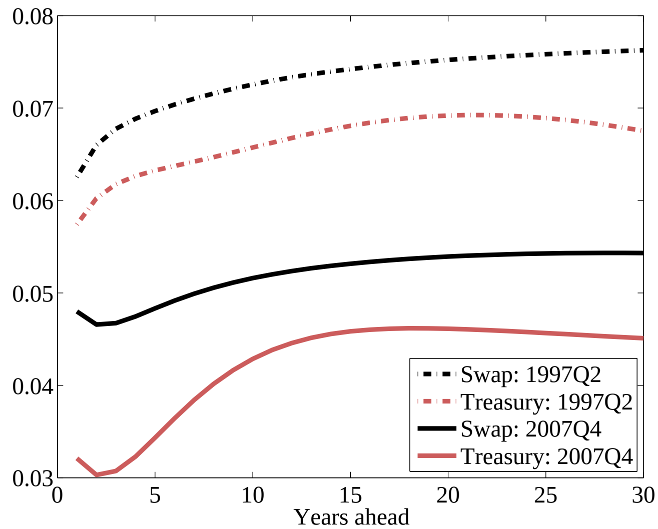

Figure 1.10: Estimated yield curves

1.6.3 Mortgage rate and refinancing activities

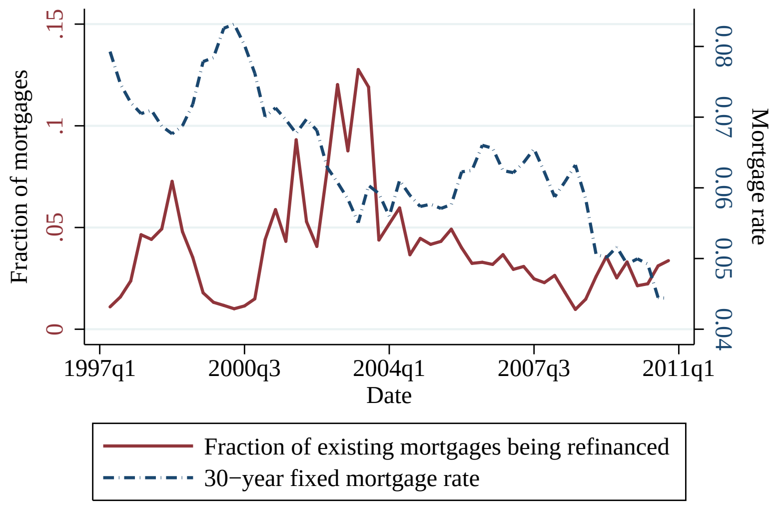

Figure 1.11: Mortgage rate and refinancing activities. The red line plots the (quarterly) percentage of existing mortgage loans being refinanced; the blue dash-dotted line plots the 30-year fixed mortgage rate.

Chapter 2

Optimal Fiscal and Monetary Policy with Collateral Constraints

2.1 Introduction

In this chapter, we investigate how the cost of inflation to commercial banks affects the design of fiscal and monetary policy. Indeed, how a government responds to government spending shocks is a fundamental question in macroeconomics. Without defaulting on the government debts, a country can either increase distortionary taxes or use inflation to reduce the real value of government debts denominated in domestic currency. Since the onset of the 2008 financial crisis, deficits and public debt are approaching historical highs in major economies, and many economists have argued that the inflation target should be raised.

In the optimal policy literature, standard models prescribe the use of state-contingent inflation to smooth tax distortions (Lucas and Stokey, 1983; Chari et al., 1991). In periods of higher fiscal expenditure, generating inflation allows the government to decrease the real value of its outstanding nominal claims. In this way, the government is able to attenuate the increase in taxes required to maintain the present-value budget balance. In these standard models, inflation is a lump-sum tax on government debt holders and is therefore costless from the ex post point of view.[1]

The previous chapter documents a sizable cost of inflation to commercial banks. This cost of inflation has not been considered by the optimal fiscal and monetary policy literature. In this chapter, we fill the gap by adopting the model of Angeletos et al. (2013) and extending it to account for this negative consequence of inflation. We then use the model to study optimal fiscal and monetary policy.

We first consider a simple flexible-price benchmark model to focus exclusively on the cost of inflation to banks. In the benchmark model, the economy is populated with a large number of bankers who provide funds to firms, which are subject to idiosyncratic productivity shocks. For a high-productivity firm to acquire more productive resources, its banker needs to raise external funds through collateralized borrowing. Bankers hold nominal government debt and physical capital, both of which serve as collateral. The government finances fiscal expenditures and interest payments by imposing distortionary labor taxes. When the government generates inflation to reduce the real value of debt, bankers' collateral constraints are tightened, which impedes resource reallocation across heterogeneous firms and distorts investment decisions. In this sense, state-contingent inflation is no longer a lump-sum tax even ex post. When the government optimizes its policies, it should balance the cost of inflation with the cost of distortionary taxes.

We then use the model to study the response of optimal fiscal and monetary policy to fiscal shocks. We first consider a simplified version of the model where agents' preferences are assumed to be quasi-linear. Perfect tax smoothing, which emerges from an otherwise identical model without financial frictions, no longer holds in our

[1] Price stickiness can make state-contingent inflation very costly in models where the government issues short-term debt (Schmitt-Grohé and Uribe, 2004; Siu, 2004). However, when the government issues long-term debt, large changes in the value of the debt can be produced by changes in the nominal interest rate, with much smaller and smoother changes in inflation. Optimal policy therefore still features a sizable contribution of inflation to buffering fiscal shocks (Leeper and Zhou, 2013).

model. Instead, the optimal response to an expenditure shock features a combination of a higher tax rate and a higher inflation rate. For government expenditure processes calibrated to postwar U.S. data, the response of inflation to an expenditure shock is significantly reduced in our model. Following a 10% increase in government spending, the cumulative inflation rate is 7% in our model and 15% in the frictionless model.

We then relax the assumption of quasi-linear preference and further explore the quantitative properties of the model. We perform a decomposition analysis to study the contribution of inflation and taxes to the financing of higher government expenditures. In the frictionless model, inflation finances almost all increases in government expenditure; in our model, inflation only finances 56% of the increase in government expenditures. Our model also reduces the volatility of inflation by half relative to the frictionless model. To the extent that inflation volatility in the frictionless model is extreme and at odds with the data (Chari et al., 1991), our model provides a rationale for small inflation volatility in the optimal policy design.

A natural question for our analysis is whether it is relevant in a more realistic environment with nominal rigidities. For instance, if nominal rigidities already greatly reduce the incentive to engineer state-contingent inflation (Schmitt-Grohé and Uribe, 2004), then the introduction of our collateral channel would have no important implications for inflation.

We then extend the benchmark model to incorporate nominal rigidities in the form of price adjustment costs. We find that price stickiness has limited additional effects as long as government debt has long maturity. When government debt has an average maturity of 10 years, the role played by inflation in fiscal financing in the sticky-price model is very similar to that in the benchmark flexible-price model. Real allocations are also very similar in both economies. This is possible because long-term debt allows large changes in the real return of debt through small but smooth inflation. Naturally, inflation becomes very persistent in this model, and inflation in future periods plays a much larger role than inflation in the current period in the financing of higher fiscal spending.

Related literature. This chapter relates to the literature on optimal fiscal and monetary policy using a Ramsey approach. Our model builds on Angeletos et al. (2013), who consider optimal fiscal policy when real government debt serves as collateral and focus on the determination of long-run debt level. We extend Angeletos et al. (2013) to introduce nominal government debt and argue that collateral constraints are also important in the optimal design of monetary policy. In particular, the behavior of inflation in our model differs substantially from the frictionless benchmark in Lucas and Stokey (1983) and Chari et al. (1991), where government can use inflation to revalue real returns on nominal debt without cost.

By incorporating price stickiness into the model, our work also connects to the literature on optimal fiscal and monetary policy in sticky-price models. Sims (2013), Leeper and Zhou (2013) and Faraglia et al. (2013) argue that the maturity of government debt matters for the contribution of inflation to fiscal financing. We confirm that their findings carry through to models with financial frictions.

This chapter also contributes to the recent literature that examines the redistribution effect of inflation by revaluing nominal contracts in general equilibrium models (Gomes et al., 2014; Garriga et al., 2013; Meh et al., 2010). This literature shares the common notion that nominal contracts create a link between inflation and the real economy and serve as an important source of monetary non-neutrality, even with fully flexible prices. While the previous studies focus on nominal household (mortgage) debt and corporate debt, our work highlights the importance of nominal positions of the banking sector.

At a conceptual level, this chapter also relates to a growing literature on the link between sovereign default and bank fragility (Gennaioli et al., 2014; Sosa-Padilla, 2012; Bolton and Jeanne, 2011). As inflation can be viewed as a partial default on government liabilities, our model shares with this literature the idea that the repudiation of government debt tightens financial constraints on the banking sector. However, our model differs from this literature in two respects. First, the literature suits emerging economies, which borrow in foreign currencies, and members of the eurozone, which do not have control over their own monetary policy. Our study applies to advanced economies such as the U.S. and Japan, which issue debt in their own currencies and have control over their own monetary policy. Second, the literature usually assumes a lack of commitment on the part of the government. We instead study optimal policy under full government commitment and focus exclusively on the frictions in the financial market.

Roadmap. The rest of the chapter is organized as follows. We describe the benchmark model in section 2.2 and describe the Ramsey optimal policy problem in section 2.3. We study the optimal policy problem under the assumption of quasi-linear utility in section 2.4 and then relax this assumption in section 2.5. In section 2.6, we extend the model to incorporate price stickiness and the long maturity of government debt. We conclude in section 2.7.

2.2 Model

2.2.1 Environment

The economy consists of a continuum of identical households. Within each household reside equal masses of bankers $i \in [0, 1]$ and workers $j \in [0, 1]$. Each worker supplies labor in a competitive labor market and earns a wage income. Each banker channels funds to a firm that produces final goods used in consumption and investment. Members in each household share consumption perfectly.

Preference and Technology. Preferences over stochastic processes for the household consumption $\{c_t\}_t$ and labor supply $\{h_{j,t}\}_t$ of each worker j are ordered by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\rho} - 1}{1-\rho} - \chi \frac{\int_0^1 h_{j,t}^{1+\epsilon} dj}{1+\epsilon} \right). \tag{2.1}$$

We ignore financial frictions between a banker and his firm; thus, each banker i effectively owns the firm.[2] We use i to index the firm owned by banker i. Firm i uses $k_{i,t}$ units of physical capital and $n_{i,t}$ units of labor to produce output $y_{i,t}$:

$$y_{i,t} = z_{i,t} F(k_{i,t}, n_{i,t}),$$

where $z_{i,t}$ is an idiosyncratic productivity shock and F is a production function that has decreasing returns to scale, with $F(k,n) = k^\alpha n^\theta$ and $\alpha + \theta < 1$. We assume that $z_{i,t}$ is independent and identically distributed across both bankers i and time t and can take two values:

$$z_{i,t} = \begin{cases} z^H & \text{with probability } \sigma \\ z^L & \text{with probability } 1-\sigma \ . \end{cases}$$

Physical capital depreciates at rate δ. Aggregate capital stock a_t is the sum of the stock of undepreciated capital and current investment i_t:

$$a_t = (1-\delta) a_{t-1} + i_t.$$

Aggregate uncertainty. In this model, the only source of aggregate uncertainty is a stochastic government expenditure g_t. Aggregate history up until time t is denoted

[2] As each banker is the owner of a firm, in the text below we use banker i and firm i interchangeably, with a slight abuse of notation. We abstract from the frictions between bankers and firms by assuming the ownership of a firm by a banker, similar to Gertler and Kiyotaki (2010). This assumption allows us to focus on the bankers' balance sheets and how bankers' borrowing capacity is limited by their net worth.

by $g^t = (g_0, ..., g_t)$, and the time-0 probability of g^t is denoted by $\Pr(g^t)$. To save on notation, we use X_t to denote a random variable that is a function of the history g^t.

Aggregate output y_t is divided between household consumption c_t, investment expenditures, and government expenditures:

$$c_t + a_t + g_t = (1 - \delta)a_{t-1} + y_t. \tag{2.2}$$

Capital market and collateral constraint. The sequence of activities within each time period t is illustrated in Figure 2.1. At the beginning of period t, workers and bankers separate, and they cannot meet each other until the end of the period. We assume that before the separation, each household shares all the assets accumulated during the previous period among all the bankers in the household. Therefore, each banker holds an equal share of the household's assets. A household's assets consist of physical capital a_{t-1} and government-issued one-period nominal bonds B_{t-1}. The bonds issued in period $t-1$ pays a non-contingent gross nominal interest rate R^B_{t-1}.

After the separation of bankers and workers, the idiosyncratic productivity shocks and the aggregate government expenditure shock are realized. High-productivity bankers want to scale up their production and therefore need to finance more labor and capital. As household members are spatially separated at this point, they cannot reshuffle the resources among themselves. To acquire more physical capital, high-productivity bankers can buy it from other bankers in a competitive capital market. A buyer of capital does not pay for the capital until production is finished; therefore, at this stage, he issues private IOUs to the seller.

However, after employment and production take place, buyers could "run away" and repudiate their IOUs. In this case, sellers could confiscate only some fraction of buyers' assets. Ex ante, this lack of commitment limits the amount of IOUs buyers

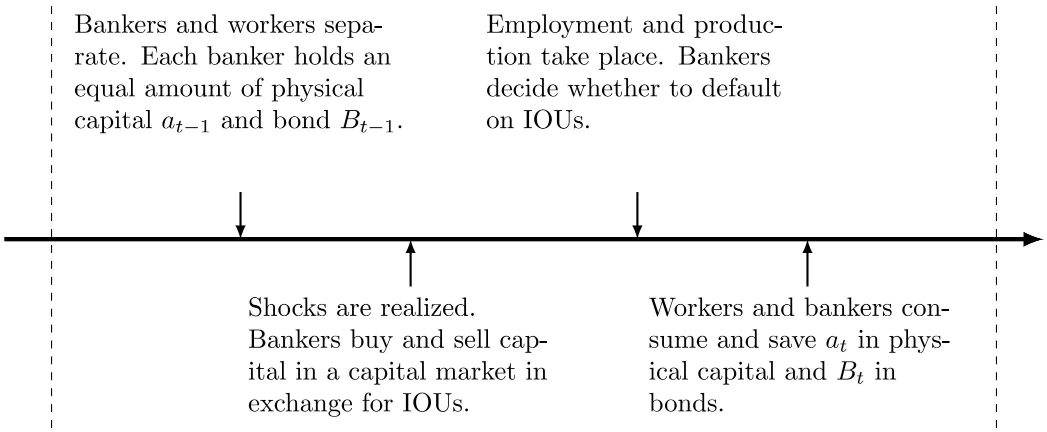

Figure 2.1: Timeline of activities within period t

can issue.[3] At the end of the period, workers and bankers come back to the household and make consumption and saving decisions together.

In terms of notations, let q_t be the price of capital that clears the time-t capital market and $k_{i,t}$ be the amount of capital used in production by banker i.[4] If $k_{i,t} > a_{t-1}$, banker i purchases $k_{i,t} - a_{t-1}$ units of capital and issues IOUs to the seller in order to pay $q_t \left(k_{i,t} - a_{t-1}\right)$ units of consumption goods.

We now describe the limited commitment problem in the capital market in detail. In the case where a buyer of capital i repudiates the IOUs after production, a seller could confiscate ξ fraction of capital installed in buyer i's firm $k_{i,t}$, and the total real payoff from buyer i's government debt holding $R^B_{t-1}B_{t-1}/P_t$.[5] Therefore, a buyer faces the incentive constraint that the total value of IOUs he issues cannot exceed the total

[3] We assume that employment and production happen simultaneously and thus workers can seize bankers' outputs if bankers refuse to pay wages. As a result, there is no friction in the labor market.

[4] As will be clear in the capital demand condition (2.6) in the next subsection, the price of capital q_t equals the gross return of capital for the low-type firm:

$$q_t = 1 - \delta + z^L F_k \left(k^L_t, n^L_t\right).$$

This is because in equilibrium, the bankers who own low-type firms are unconstrained, and are indifferent between selling capital and using it in production.

[5] A banker cannot pledge the wage incomes of workers in the same household as collateral due to the spatial separation of household members during the period. A banker cannot credibly pledge his own future income either. This assumption that human capital is inalienable has been followed in much of the literature on financial frictions since Hart and Moore (1994).

value of the confiscable assets, that is,

$$q_t\left(k_{i,t}-a_{t-1}\right) \leq \xi k_{i,t}+\frac{R_{t-1}^B B_{t-1}}{P_t}.$$

As a buyer's default happens after production when physical capital can be converted to consumption goods one to one, the real price of capital at this point is 1.

Rearranging this inequality constraint yields

$$k_{i,t} \leq \underbrace{\frac{1}{q_t-\xi}}_{\text{leverage}} \times \underbrace{\left(q_t a_{t-1}+\frac{R_{t-1}^B B_{t-1}}{P_t}\right)}_{\text{net worth}}. \tag{2.3}$$

The left-hand side, which is the total amount of capital that can be used by banker i in production, is limited by his total net worth (physical capital and government bonds). $\frac{1}{q_t-\xi}$ is the (within-period) leverage ratio. It has a natural interpretation: for each unit of capital banker i uses in production, he could credibly pledge ξ fraction, and therefore he needs to secure the remaining $q_t-\xi$ fraction using his own net worth.

The bankers' balance sheets in this model are a simplification of the real-world bank balance sheets. In this model, part of the bankers' assets (the government bonds) are in nominal terms. Bankers' liabilities are only within-period, the value of which is not affected by state-contingent inflation. Therefore, they have a shorter maturity than the government bonds that mature in one period. In this sense, this model broadly captures the mismatch of maturity observed in the data.

As a result, this model also captures the negative effect of inflation on bank balance sheets. Other things being equal, when the government engineers inflation and reduces real debt value $R_{t-1}^B B_{t-1}/P_t$, it reduces the net worth of bankers and tightens their collateral constraints. In this sense, state-contingent inflation is no longer a lump-sum tax on bond holders, even ex post.

Remark. We have assumed that bankers in the same household reshuffle assets among themselves at the end of each period. This assumption allows us to study heterogeneity and capital reallocation while maintaining the tractability of the aggregate economy. Absent this assumption, we would need to keep track of the distribution of assets across bankers. This will greatly increase the computational burden, especially because we are interested in the optimal policy response to an aggregate shock.

2.2.2 Households' decision problem

Before idiosyncratic productivity shocks are realized in each period, all bankers and their firms are ex ante the same. Therefore, the production decisions of a firm only depend on its current productivity shock $z_{i,t}$. I denote variables regarding production decisions by superscript s, where $s = L$ if $z_{i,t} = z^L$ and $s = H$ if $z_{i,t} = z^H$.

In each period, a household's income consists of labor income, profits of bankers' firms, and savings income. Each worker in a household earns an after-tax wage income $(1-\tau_t)\, w_t h_t$,[6] where w_t denotes the real wage; each banker earns a profit from his firm:

$$v_t^s = z^s F\left(k_t^s, n_t^s\right) - w_t n_t^s - \left[q_t - (1-\delta)\right] k_t^s. \tag{2.4}$$

A household's end-of-period budget constraint is:

$$c_t + a_t + \frac{B_t}{P_t} = \left[\sigma v_t^H + (1-\sigma) v_t^L\right] + (1-\tau_t)\, w_t h_t + \frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1}. \tag{2.5}$$

A household's decision problem is to choose $\{k_t^s, n_t^s, h_t, c_t, a_t, B_t\}_{t=0}^{\infty}$ to maximize utility (2.1), subject to the end-of period budget constraint (2.5) and the collateral constraint (2.3).

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[6] As workers within each households are identical, they supply the same amount of labor, i.e., $h_{j,t} = h_t$.

Firms' production decision. The labor and capital demand conditions of a type s firm are

$$z^s F_n\left(k_t^s, n_t^s\right) = w_t$$

and

$$z^s F_k\left(k_t^s, n_t^s\right) = q_t - (1-\delta) + \mu_t^s, \tag{2.6}$$

where $\mu_t^s U_{c,t}$ is the multiplier on the collateral constraint. In equilibrium, high- and low-productivity bankers carry the same amount of capital and bonds from previous period a_{t-1} and $\frac{R_{t-1}^B B_{t-1}}{P_t}$. Therefore, the collateral constraint binds at most for the high-productivity bankers, and $\mu_t^L = 0$. The beginning-of-period price capital q_t equals the gross rate of return of the low-productivity bankers.

When the constraint strictly binds for the high type and $\mu_t^H > 0$, the marginal product of capital is greater than the cost of capital $q_t - (1-\delta)$. From the point of view of the aggregate economy, inefficiency occurs because the marginal product of capital does not equalize between the high and low types.[7]

Households' saving decision. The Euler equations for capital and bond holdings are

$$U_{c,t} = \beta \mathbb{E}_t U_{c,t+1} q_{t+1} \left(1 + \frac{\sigma \mu_{t+1}^H}{q_{t+1} - \xi}\right) \tag{2.7}$$

and

$$U_{c,t} = \beta \mathbb{E}_t U_{c,t+1} \frac{R_t^B}{P_{t+1}} \left(1 + \frac{\sigma \mu_{t+1}^H}{q_{t+1} - \xi}\right). \tag{2.8}$$

The Euler equations have natural interpretations. When a household invests in one unit of capital at time t, in $t+1$ the low-type bankers in this household receive a return of q_{t+1}. The σ-fraction of high-type bankers in the household can lever up by $\frac{1}{q_t - \xi}$ to acquire more capital for higher production. For each unit of additional capital,

[7] As there is no friction in the labor market, the marginal product of labor equalizes across the two types of firms.

they receive an additional return μ^H_{t+1}. Similar logic applies to the Euler equation of the government bonds.

The existence of the collateral constraint also introduces a trade-off for the government on the inter-temporal margin. If the collateral constraint strictly binds with positive probability in the following period, the associated Lagrange multiplier introduces a wedge between the rates of return of capital and government bonds and the inter-temporal marginal rate of substitution. This distorts the household's investment decision. On the other hand, government bonds (as well as capital) are priced at a premium relative to an asset that is an equally good form of saving but cannot serve as collateral.[8] A lower interest rate on government debt allows the government to reduce taxes.

2.2.3 Government policy

The government consists of fiscal and monetary authorities. The fiscal authority imposes proportional taxes on labor income with a tax rate τ_t and issues new debt with a nominal amount of B_t. The monetary authority decides upon the nominal interest rate R^B_t. The following consolidated government budget constraint must hold:

$$\tau_t w_t h_t + \frac{B_t}{P_t} = \frac{R^B_{t-1} B_{t-1}}{P_t} + g_t. \tag{2.9}$$

2.2.4 Competitive equilibrium

We now define the competitive equilibrium, taking government policies as given.

Definition 1. *Given initial conditions a_{-1} and $R^B_{-1}B_{-1}$, a* ***competitive equilibrium*** *is a set of allocation $\{k^s_t, n^s_t, h_t, c_t, a_t, B_t\}^\infty_{t=0}$, prices $\{q_t, w_t, P_t\}^\infty_{t=0}$, and fiscal*

[8]This is consistent with the observations that government bonds pay a lower return due to liquidity attributes (Krishnamurthy and Vissing-Jorgensen, 2012) and that the "natural rate of interest" declines as credit gets tighter (Eggertsson and Krugman, 2012).

and monetary policies $\{\tau_t, R_t^B\}_{t=0}^{\infty}$ *satisfying the (consolidated) government budget constraint* (2.9)*, such that*

1. *Given* $\{q_t, w_t\}_{t=0}^{\infty}$*, bankers choose capital and labor demand* $\{k_t^s, n_t^s\}_{t=0}^{\infty}$*.*
2. *Given* $\{w_t, \tau_t\}_{t=0}^{\infty}$*, workers choose labor supply* $\{h_t\}_{t=0}^{\infty}$*.*
3. *Given* $\{q_t, R_t, P_t\}_{t=0}^{\infty}$*, households choose savings* $\{a_t, B_t\}_{t=0}^{\infty}$*.*
4. *Labor, capital, and bond markets clear.*

2.2.5 Aggregation

Next, we characterize the aggregate economy before moving on to the optimal policy problem. To aggregate over the production decisions of the two types of firms, the key is to compute the capital allocation between the two types of firms. Once we know allocations of capital, allocations of labor can be traced down by the equilibrium conditions in the labor market.

Let $x_t \equiv \frac{k_t^H}{a_{t-1}}$ denote the capital used by the high-productivity firms as a fraction of the aggregate capital stock. Then the fraction of capital used by the low-productivity firms is $\frac{k_t^L}{a_{t-1}} = \frac{1-\sigma x_t}{1-\sigma}$. Based on the fact that wage rate equalizes between the two types of firms, the allocation of labor between the two types is

$$\frac{n_t^H}{n_t^L} = \left(\frac{z^H}{z^L}\right)^{\frac{1}{1-\theta}} \left(\frac{x_t - \sigma x_t}{1 - \sigma x_t}\right)^{\frac{\alpha}{1-\theta}}.$$

Let $y_t = \sigma y_t^H + (1-\sigma) y_t^L$ be the aggregate output. In appendix 2.8.1 we show that

$$y_t = \Gamma(x_t) a_{t-1}^{\alpha} h_t^{\theta}, \tag{2.10}$$

where

$$\Gamma(x) = \left[\sigma {z^H}^{\frac{1}{1-\theta}} x^{\frac{\alpha}{1-\theta}} + (1-\sigma) {z^L}^{\frac{1}{1-\theta}} \left(\frac{1-\sigma x}{1-\sigma}\right)^{\frac{\alpha}{1-\theta}}\right]^{1-\theta}.$$

$\Gamma(x)$ is the endogenous aggregate total-factor productivity (TFP) that is affected by the capital allocation between high and low types (measured by x). Since the production technology has decreasing returns to scale, there exists an efficient level x^* in the absence of financial friction, that is, $x^* = \arg\max_x \Gamma(x)$.[9] When the collateral constraint binds, x falls below x^*, and the aggregate TFP falls below the efficient level $\Gamma(x^*)$.

2.3 Ramsey optimal policy

The optimal fiscal and monetary policy is the process $\{\tau_t, R_t^B\}_{t=0}^{\infty}$ associated with the competitive equilibrium that yields the highest social welfare:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\rho}}{1-\rho} - \chi \frac{h_t^{1+\epsilon}}{1+\epsilon} \right).$$

We will follow the primal approach in the Ramsey policy literature, which involves substituting for prices and policy instruments so that the Ramsey planner directly chooses real allocations.

The wage rate and labor tax rate can be backed out using the labor demand and supply conditions. As there is no friction in the labor allocation across firms, the wage rate equals the marginal product of labor using the aggregate production function, that is,

$$w_t = \Gamma(x_t) F_h(a_{t-1}, h_t). \tag{2.11}$$

[9] x^* only depends on exogenous parameters:

$$x^* = \frac{z^{H\,\frac{1}{1-\alpha-\theta}}}{\sigma z^{H\,\frac{1}{1-\alpha-\theta}} + (1-\sigma) z^{L\,\frac{1}{1-\alpha-\theta}}}.$$

The labor tax rate is the wedge between the marginal rate of substitution and the marginal product of labor:

$$\tau_t = 1 + \frac{U_{h,t}}{U_{c,t}} \frac{1}{\Gamma(x_t) F_h(a_{t-1}, h_t)}. \tag{2.12}$$

The price of capital q_t is determined by the marginal product of capital (MPK) of the low-productivity firms, because these firms are not financially constrained. It follows from equation (2.6) that

$$q_t = 1 - \delta + \underbrace{\alpha \Gamma(x_t) a_{t-1}^{\alpha-1} h_t^{\theta}}_{\text{aggregate MPK}} \underbrace{\left[z^L \left(\frac{1 - \sigma x_t}{1 - \sigma} \right)^{\alpha+\theta-1} \Gamma(x_t)^{-1} \right]^{\frac{1}{1-\theta}}}_{\text{deviation from aggregate MPK}} \equiv \boldsymbol{q}(a_{t-1}, h_t, x_t). \tag{2.13}$$

The low-productivity firms' MPK can be decomposed into two terms: the aggregate MPK and the deviation from the aggregate MPK due to capital misallocation. If $x_t = x^*$, then the deviation term is equal to 1, and the low-productivity firms' MPK equals the aggregate MPK. If $x_t < x^*$, capital allocation is suboptimal and too much capital remains in the low-productivity firms. Therefore, these firms' MPK is below the aggregate MPK.[10]

The multiplier for the collateral constraint of the high-productivity firms is also determined by equation (2.6):

$$\mu_t^H = \frac{1}{\sigma} \Gamma'(x_t) \, a_{t-1}^{\alpha-1} h_t^{\theta} \equiv \boldsymbol{\mu}^{\mathbf{H}}(a_{t-1}, h_t, x_t). \tag{2.14}$$

It is strictly positive if and only if capital allocation is suboptimal, that is, $x_t < x^*$.

At last we substitute for the real return on government debt. We denote the real government debt by $b_t \equiv \frac{B_t}{P_t}$, and the real holding-period return on debt by $r_t^b =$

[10]In our numerical analysis, q_t is always greater than 1. However, it is theoretically possible that q_t falls below 1. This happens when the collateral constraint becomes very tight and the low-type firms are sufficiently unproductive. In this situation, the marginal product of capital of the low-type firms becomes very small and drives q_t below 1.

$\frac{R_t^B P_{t-1}}{P_t}$. In Appendix 2.8.2, we show that r_t^b can be substituted for using the household budget constraint (2.5) and the Euler equation of debt (2.8), and we arrive at the flow implementability constraint commonly used in the literature (e.g., Canzoneri et al., 2013).[11]

$$\beta \mathbb{E}_{t-1} \left[U_{c,t} c_t + U_{h,t} h_t - U_{c,t}(1-\alpha-\theta) y_t \right] + \beta \mathbb{E}_{t-1} U_{c,t} \left(a_t + b_t \right) = U_{c,t-1}(a_{t-1} + b_{t-1}). \tag{2.15}$$

Similarly, r_t^b can be substituted for from the collateral constraint by combining it with the government budget constraint:[12]

$$x_t a_{t-1} \left(q_t - \xi \right) \leq q_t a_{t-1} + \left(\theta y_t + \frac{U_{h,t}}{U_{c,t}} h_t + b_t - g_t \right). \tag{2.16}$$

We now establish the equivalence between the primal approach and the original Ramsey problem. The proof is in Appendix 2.8.2.

Lemma 1. *Allocations* $\{a_t, h_t, x_t, c_t, b_t\}_{t=0}^{\infty}$ *satisfying the social resource constraint* (2.2)*, the flow implementability constraint* (2.15)*, the Euler equation* (2.7)*, the collateral constraint* (2.16)*,* $\boldsymbol{\mu}^{\mathbf{H}}(.) \geq 0$*, and the household complementary slackness condition are the same as those in the competitive equilibrium, where price functions* $\boldsymbol{q}(.)$ *and* $\boldsymbol{\mu}^{\mathbf{H}}(.)$ *are defined in equations* (2.13) *and* (2.14)*.*

Remark. In this model, only the real debt b_t and the state-contingent real return on debt r_t^b matter for real allocations.[13] As a result, the Ramsey problem only determines the state-contingent return on debt r_t^b. As the government can only adjust the real return through state-contingent inflation, the Ramsey problem determines the state-

[11] One can obtain the present-value implementability condition by iterating the flow-implementability condition over time.

[12] See Appendix 2.8.2.

[13] In our model, the government issues nominal debt to the household and uses inflation to adjust the ex post real return on debt. It is equivalent to a model where the government issues real debt paying state-contingent returns. It is also equivalent to a model where the government issues Arrow securities to the household.

contingent component of inflation.[14] On the other hand, the expected gross inflation rate $\mathbb{E}_{t-1}\pi_t$ is not determined in the Ramsey problem. Without loss of generality, we assume zero expected inflation, that is, $\mathbb{E}_{t-1}\pi_t = 1$.[15]

2.3.1 Recursive representation

For computational purposes, it is convenient to express the optimal policy problem recursively. As a matter of notation, we use variables with a prime to denote the next-period variables and variables with a minus subscript to denote the last-period variables. For example, g and $g_{\text{-}}$ are the amounts of government spending in the current and previous periods, respectively. We use $\mathbb{E}\left(X|g_{\text{-}}\right)$ to denote the conditional expectation of variable X in the state $g_{\text{-}}$.

With this notation in hand, we can describe the recursive representation for the Ramsey problem. Due to time inconsistency, the time-0 Bellman equation differs from that in the subsequent periods. In the main part of this paper, we focus on the time $t \geq 1$ continuation problem, where the government fully commits to its policy decisions made in the previous period. We discuss the time-0 problem in Appendix 2.8.6. The Ramsey problem with full commitment focuses exclusively on the financial friction and provides a clean benchmark.

The Bellman equation involves four state variables: the value of the capital stock a inherited from the previous period, the real value of government debt issued in the previous period b, the marginal utility of consumption in the previous period $\lambda \equiv U_{c,-}$, and the state of the government expenditure in the previous period $g_{\text{-}}$. The

[14] Denote the gross inflation rate by π_t, then the state-contingent component of inflation is given by

$$\frac{\pi_t}{\mathbb{E}_{t-1}\pi_t} = \frac{\mathbb{E}_{t-1}r_t^b}{r_t^b}.$$

[15] The result of zero inflation can emerge from a sticky price version of our model (see section 2.6). In the literature, expected inflation can be determined either by incorporating price stickiness, which drives the expected inflation rate to 0, or by introducing non-interest-bearing government liability (money stock) that leads to the Friedman rule (e.g., Chari et al., 1991). Both features are absent in our flexible-price model.

Bellman equation is

$$V(a,b,\lambda,g_{\text{-}}) = \max_{b'(g),a'(g),h(g),x(g),c(g),\lambda'(g)} \mathbb{E}\left[\frac{c(g)^{1-\rho}}{1-\rho} - \chi\frac{h(g)^{1+\epsilon}}{1+\epsilon} + \beta V(a'(g),b'(g),\lambda'(g),g)|g_{\text{-}}\right], \tag{2.17}$$

where the maximization is subject to

$$\mathbb{E}\left[\beta U_c(g)b'(g) + U_h(g)h(g) + U_c(g)(1-\alpha-\theta)\Gamma(x(g))a^{\alpha}h(g)^{\theta}|g_{\text{-}}\right] = \lambda(a+b), \tag{2.18}$$

$$\mathbb{E}\left[U_c(g)\boldsymbol{q}(g)\left(1+\frac{\sigma\boldsymbol{\mu}^{\mathbf{H}}(g)}{\boldsymbol{q}(g)-\xi}\right)|g_{\text{-}}\right] = \lambda, \tag{2.19}$$

$$c(g) + g + a'(g) = (1-\delta)a + \Gamma(x(g))a^{\alpha}h(g)^{\theta}, \tag{2.20}$$

$$x(g)a\left[\boldsymbol{q}(g)-\xi\right] \leq \boldsymbol{q}(g)\,a + \left[\theta y(g) + \frac{U_h(g)}{U_c(g)}h(g) + b'(g) - g\right], \tag{2.21}$$

$$\boldsymbol{\mu}^{\mathbf{H}}(g) \geq 0, \quad \text{the household complementary slackness condition}^{16}\text{, and} \tag{2.22}$$

$$\lambda'(g) = U_c(g). \tag{2.23}$$

Equations (2.18) to (3.16) are the implementability condition, the Euler equation of capital, the social resource constraint, and the collateral constraint, respectively. With a slight abuse of notation, we use $X(g)$ to denote variable X in state g. We use $\boldsymbol{q}(g)$ and $\boldsymbol{\mu}^{\mathbf{H}}(g)$ as short notations for $\boldsymbol{q}(a(g),h(g),x(g))$, and $\boldsymbol{\mu}^{\mathbf{H}}(a(g),h(g),x(g))$. Compared to an otherwise identical problem without financial frictions, the Ramsey planner in this problem faces two more constraints: the collateral constraint (3.16) and the household complementary slackness condition (2.22).

In standard Ramsey policy without financial frictions, state-contingent inflation is a lump-sum tax on bond holders from the ex post point of view, and nominal

[16] The complementary slackness condition is

$$\boldsymbol{\mu}^{\mathbf{H}}(g)\left[\boldsymbol{q}(g)\,a + \left(\theta y(g) + \frac{U_h(g)}{U_c(g)}h(g) + b'(g) - g\right) - x(g)a\left[\boldsymbol{q}(g)-\xi\right]\right] = 0.$$

government debt is no more than a shock absorber. When the government receives a high expenditure shock, it engineers inflation to reduce real debt and smooth tax distortions. If the expenditure shock is persistent, the government also issues less debt to save on debt-servicing costs. However, these policies are distortionary in the presence of the collateral constraint. First, inflation reduces bankers' net worth, tightens their constraints, and leads to capital misallocation and TFP loss (equation 2.10). Second, when the government issues less debt, it reduces the amount of collateral in future periods and distorts investment decisions (equation 2.8). The government balances these distortions caused by state-contingent inflation and debt provision with distortions from labor taxes.

2.4 The quasi-linear case

In this section, we assume that the household's utility function is linear in consumption ($\rho = 1$):

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_t - \chi \frac{h_t^{1+\epsilon}}{1+\epsilon} \right).$$

We adopt this utility function for two reasons. First, the computation of the Ramsey problem simplifies drastically when preferences are quasi-linear. The simplification allows us to adopt a global solution method in solving the model. In addition, the quasi-linear preference facilitates the comparison between this model and an otherwise identical model without financial frictions, because optimal policy in the frictionless model features perfect tax smoothing under this quasi-linear specification (see below).[17]

When preferences are quasi-linear, two state variables, real government debt b and government expenditure in the previous period $g_{\text{-}}$, are now sufficient to describe the

[17]I allow for negative consumption. Consumption and investment are determined through interest rates in general equilibrium. For the size of shock we consider in the numerical simulation, negative consumption does not emerge.

state of the economy. Intuitively, the marginal utility λ is now fixed and equal to one; it can therefore be dropped as a state variable. Second, by rearranging terms in the objective function and redefining the Bellman equation, the state variable a (outstanding capital stock in the current period) can be viewed as a control variable at the end of the previous period (Farhi, 2010). To see this, use the social resource constraint to substitute for c_t in the objective function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[y_t + (1-\delta)a_{t-1} - g_t - a_t - \chi \frac{h_t^{1+\epsilon}}{1+\epsilon} \right]$$

$$= \frac{1}{\beta} a_{-1} + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \underbrace{\left[y_t + (1-\delta-\frac{1}{\beta})a_{t-1} - g_t - \chi \frac{h_t^{1+\epsilon}}{1+\epsilon} \right]}_{\text{New period utility}}.$$

We define a new value function $\hat{V}(b, g_{\_})$ where the per-period utility is given by the terms inside the brackets. The value function $\hat{V}(b, g_{\_})$ satisfies the following Bellman equation:

$$\hat{V}(b, g_{\_}) = \max_{b'(g),a,h(g),x(g)} \mathbb{E} \left[y(g) + (1-\delta-\frac{1}{\beta})a - g - \chi \frac{h(g)^{1+\epsilon}}{1+\epsilon} + \beta \hat{V}(b'(g), g) | g_{\_} \right], \tag{2.24}$$

where the maximization is subject to

$$\mathbb{E}\left[\beta b'(g) + (\beta(1-\delta)-1)a + \beta(\alpha+\theta)y(g) - \beta g - \beta\chi h(g)^{1+\epsilon} | g_{\_}\right] = b, \tag{2.25}$$

$$\beta \mathbb{E}\left[\boldsymbol{q}(g) \left(1 + \frac{\sigma \boldsymbol{\mu}^{\mathbf{H}}(g)}{\boldsymbol{q}(g) - \xi} \right) | g_{\_} \right] = 1, \tag{2.26}$$

$$\theta y(g) - \chi x(g) a \left[\boldsymbol{q}(g) - \xi \right] \leq \mathbf{q}(g) a + \left[\theta y(g) + U_h(g) h(g) + b'(g) - g \right], \tag{2.27}$$

$$\boldsymbol{\mu}^{\mathbf{H}}(g) \geq 0 \quad \text{and the household complementary slackness condition.} \tag{2.28}$$

Equations (2.25) to (2.27) are the implementability condition, the Euler equation of capital, and the collateral constraint, respectively. The relationship between the new value function $\hat{V}(b, g_-)$ and the old one $V(a, b, 1, g_-)$ is that $\hat{V}(b, g_-) = \max_a V(a, b, 1, g_-) - \frac{1}{\beta} a$.

The deterministic model. Angeletos et al. (2013) show that the deterministic version of this model features a stable steady state, where the collateral constraint strictly binds.[18] This finding sharply contrasts with standard models in which the long-run debt level equals the initial debt level (Barro, 1979). The existence of the collateral constraint introduces a mean-reverting behavior of government debt. On the one hand, increasing government debt relaxes the collateral constraint and improves resource allocation. On the other hand, increasing government debt reduces the collateral value that bankers assign to the government debt and increases the interest rate and servicing cost of debt. These tradeoffs eventually determine the long-run level of government debt and the tightness of the collateral constraint.

In this paper, we are interested in shocks to government expenditures and the response of state-contingent inflation. We first consider an otherwise identical model without financial frictions. This is a useful benchmark to compare with our model in order to identify how financial frictions and collateral constraints shape the optimal fiscal and monetary policy.

2.4.1 Optimal response to fiscal shocks: without financial frictions

In an otherwise identical model but without financial frictions, we prove the following result (see Appendix 2.8.4).

Proposition 1. *(Ramsey policy without financial frictions.) In the absence of the collateral constraint, the Ramsey problem features a constant tax rate and productions*

[18] See Appendix 2.8.3 for a discussion of key policy functions.

across dates and states,

$$a(g^t) = a^*, \quad \textit{for } t \geq 0 \textit{ and } \forall g^t,$$

$$h(g^t) = h^*, \quad y(g^t) = y^*, \quad \tau(g^t) = \tau^*, \quad \textit{for } t \geq 1 \textit{ and } \forall g^t,$$

where a^*, h^*, y^* *and* τ^* *are constants independent of history* g^t.[19]

Standard Ramsey policy without financial frictions typically finds that the optimal labor tax rate is roughly constant (Chari et al., 1991). Due to the quasi-linear preference, the optimal labor tax rate is exactly constant in our model.[20] As the government spending fluctuates, inflation rate π and the real return on government debt r^b fluctuate to satisfy the government budget constraint. In particular, when a high government expenditure shock is realized, the government optimally generates higher inflation and reduces the real debt $r^b b$ by exactly the sum of expected increases in current and future expenditures. In this way, the government maintains a constant labor tax rate τ^* regardless of the realization of government expenditure shock g. Intuitively, higher inflation and lower real return on debt resemble a lump-sum tax on households' wealth ex post, but proportional labor tax is distortionary and the efficiency loss is convex. Therefore, the Ramsey planner wants to use state-contingent returns to absorb shocks while making the labor tax rate relatively smooth.[21]

[19] The initial period allocations $h(g^0)$ and $y(g^0)$ and the tax rate $\tau(g^0)$ differ from h^*, y^* and τ^* for two reasons. First, the government wants to confiscate the entire stock of outstanding debt by infinite price level. Second, the initial level of capital a_{-1} may differ from a^*.

[20] Our results generalize those of Chari et al. (1991) by incorporating physical capital into the model. On the other hand, it can be shown that in their model, optimal labor tax is constant whenever utility is separable in consumption and leisure, but this is not true in our model.

[21] Proposition 1 also shows that after the initial period, capital $a(g^t)$, labor $h(g^t)$, and output $y(g^t)$ are independent of history and state. This is because government consumption shock is the only aggregate shock in this model. Capital, labor, and output will fluctuate if, for example, an aggregate productivity shock is introduced.

2.4.2 Optimal response to fiscal shocks: with financial frictions

We now return to the case of interest, the Ramsey problem in the presence of financial frictions. We use standard policy function iteration to solve the Ramsey problem described by equations (2.24) to (2.28).

Table 2.1 summarizes the parameters used in the numerical exercise. The model is computed at annual frequency, and the discount factor β is set to be 0.96. We set $\epsilon = 1$, implying a Frisch elasticity of labor supply of 1. This number, in line with the recommendation of Chetty et al. (2011), is appropriate given that our model does not distinguish between intensive and extensive margins of employment.

Regarding the production technology, the overall returns to scale $\alpha + \theta$ are set to 0.85 and the share of labor θ is set to two thirds of 0.85 (Midrigan and Xu, 2014; Basu and Fernald, 1997). We focus on a symmetric productivity shock process by setting $\sigma = 0.5$ and normalize the low realization of productivity z^L to 1. We choose the high realization z^H such that the standard deviation of the logarithm idiosyncratic productivity shock is 0.3. This value is in line with the estimated size of TFP innovations using U.S. manufacturing firms (Asker et al., 2014).[22] The implied value of z^H is 1.822.

The parameter ξ dictates the severity of financial frictions. We calibrate ξ such that the debt-to-GDP ratio after a long series of low government spending shocks g^L converges to 61%, which is the value for the U.S. before the 2008 crises (2007Q3). We choose to target the debt-to-GDP ratio, because the behavior of state-contingent inflation is very sensitive to it (Siu, 2004).[23]

[22] As shown in Asker et al. (2014), the firm-level productivity shock is very persistent, with an autocorrelation coefficient of 0.8. Therefore, the standard deviation of the shock is around $0.3/\sqrt{1-0.8^2} = 0.5$. In our model, idiosyncratic productivity shock is i.i.d.. We perform a conservative calibration by calibrating the size of the shock to the size of productivity innovations rather than the productivity process in the data.

[23] In standard frictionless models, the government wants to use inflation to reduce real debt by exactly the sum of expected increases in the current and future expenditure. As the debt base

Table 2.1: Parameters

	Parameters	Value	Target/Source
Preferences			
Household discount factor	β	0.960	exogenous
Disutility of labor	χ	3.400	Gertler and Karadi (2011)
Inverse Frisch elasticity	ϵ	1.000	in line with Chetty et al. (2011)
Production Technology			
Capital share of output	α	0.283	one third of overall span of control 0.850
Labor share of output	θ	0.566	two thirds of overall span of control 0.850
Depreciation rate of capital	δ	0.100	exogenous
Probability of z^H	σ	0.500	exogenous
High idiosyncratic productivity	z^H	1.822	standard deviation of $\log(z_{i,t})$ is 0.3
Low idiosyncratic productivity	z^L	1.000	normalized
Financial Friction			
Pledgeable share of capital	ξ	0.330	steady-state government debt to GDP ratio is 61%
Government Expenditure			
SS government consumption	$\bar{g}/\bar{y}$	0.151	estimated
High government consumption	g^H	$1.05\bar{g}$	estimated
Low government consumption	g^L	$0.95\bar{g}$	estimated

In the literature on financial frictions, the parameter dictating the tightness of the financial constraint is usually calibrated to match either the yield or the yield spread of assets or statistics such as leverage and share of liquid assets of the constrained agents' balance sheets (Gertler and Karadi, 2011; Gertler and Kiyotaki, 2010; Del Negro et al., 2011). Our choice of ξ implies that the real interest rate on government bonds in the deterministic steady state is 2.9%, very close to the 2.8% estimated by Hall and Sargent (2011) using Treasury securities of all maturity from 1948–2008. Moreover, in our model, government debt is sold at a premium due to its collateral/liquidity value. We define liquidity premium as the difference between interest rates on government debt and an otherwise identical asset with no collateral value, which is $1/\beta - \bar{r}^b$ in the steady state. Under our choice of ξ, the value of the steady state premium is 1.26%. This value is broadly in line with the estimates in the literature, which vary depending on the sample period and the exact type of assets used in the estimation. For instance, Krishnamurthy and Vissing-Jorgensen (2012)

increases, the government is able to generate the same change in real claims with smaller variations in the price level. As a result, the inflation volatility required to achieve cross-state tax smoothing becomes smaller.

estimate that the average liquidity premium from 1926–2008 is 0.46%; Krishnamurthy (2002) documents a liquidity premium of 1.44% in February 2001.

Regarding the bank balance sheets, our parameter choice implies that government debt is 23.3% of total bank assets, and the within-period leverage ratio is 1.45. Compared with U.S. commercial bank data documented in Cao (2014), we overstate the share of government debt as a percentage of bank assets (10%) and understate the leverage ratio of U.S. banks (14). Later in the paper, we will vary the value of ξ and test the robustness of the numerical results.

We adopt an assumption that government expenditure shock follows a two-state Markov process. We use this process to illustrate the transition of the aggregate economy from the low state g^L to the high state g^H. Since g does not generate any utility gain for the agents, we calibrate its process to the government consumption data of the U.S. The U.S. data in the sample period 1949Q1-2007Q4 show that annual government consumption averaged about 15.1% of GDP, with a standard deviation of 1.75% and an autocorrelation of 0.60. The distribution is also very symmetric. Therefore, we set $g^L = 0.194$ and $g^H = 0.213$. We set the transition matrix to

$$\begin{bmatrix} 0.933 & 0.067 \\ 0.067 & 0.933 \end{bmatrix}.$$

The high state g^H is about 10% higher than g^L. The transition probability implies that both states have an average duration of 15 years.

Figure 2.2 shows the dynamics of the model economy as it transitions from the low state g^L to the high state g^H. We compare our economy (the blue line) with an otherwise identical economy in the absence of financial frictions (the black dashed line).

We start the economy with financial frictions at a level of government bond b to which the economy converges after a long sequence of g^L. Then in year 11, government expenditure switches from g^L to g^H and lasts for 10 years. In the frictionless economy, the level of debt in the stochastic steady state (after sufficiently long g^L shocks) is indeterminate and depends on the initial level of debt. We therefore set the debt-to-GDP ratio in the frictionless economy to the value in the economy with frictions (61%). That is, before the government spending shock switches from g^L to g^H, the two economies have the same debt-to-GDP ratio.

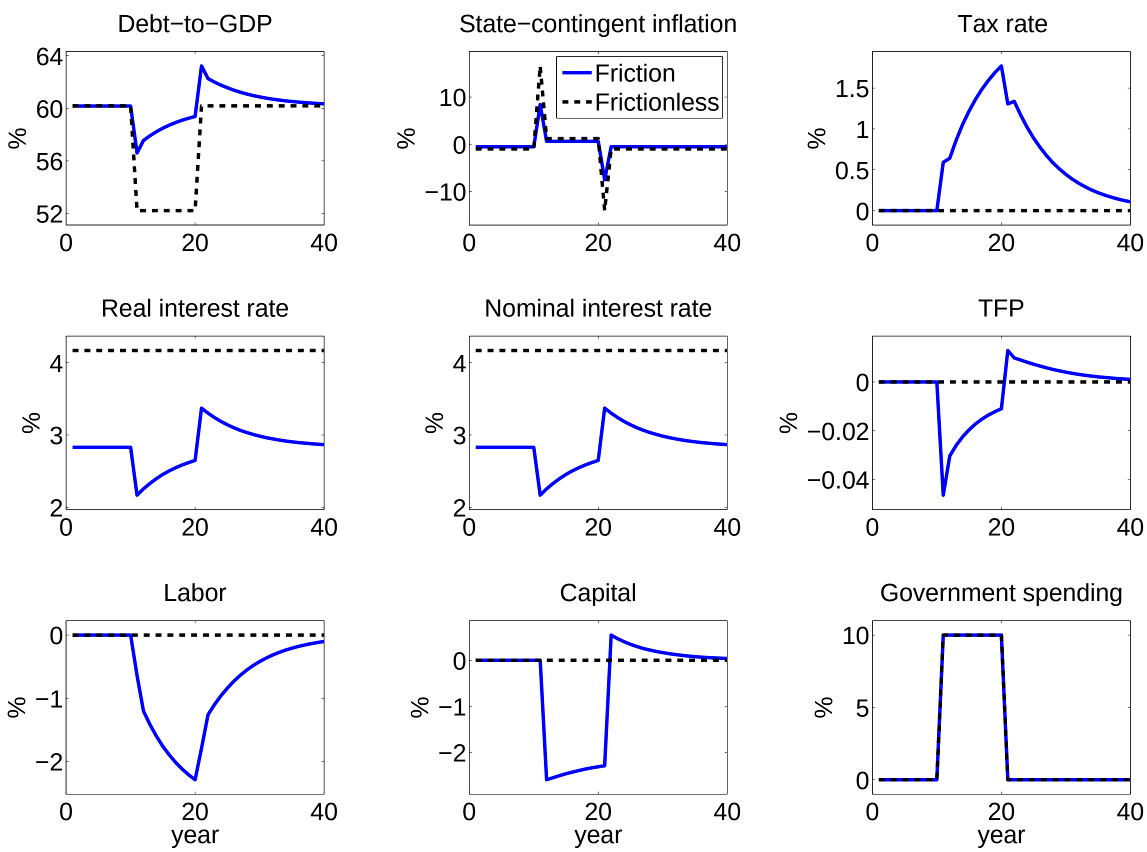

Figure 2.2: Stochastic stimulations. High government expenditure shock g^H occurs in year 11 and lasts for 10 years. The debt-to-GDP ratio, the inflation rate, and the interest rates are measured in percentage points; the tax rate is measured in the deviations from its value in year 10 and other variables are measured in percentage deviations from their values in year 10 (before the g^H shock occurs).

Consistent with Proposition 1, the frictionless economy features perfect tax smoothing. As the economy switches from g^L to g^H, the government generates 15% state-contingent inflation in one year.[24] The size of state-contingent inflation is sufficient to reduce the real debt $r^b b$ by exactly the sum of expected increases in the current and future government expenditure. Therefore, the labor tax rate and all the real allocations remain constant. As long as the g^H state continues, real debt and the debt-to-GDP ratio remain small, and the government responds to the high expenditure g^H by generating a small amount of inflation. When the economy switches back to the g^L state, the government generates deflation and maintains a constant tax rate and real allocations. Indeed, government debt and state-contingent inflation are purely shock absorbers in this economy.

When financial frictions exist, perfect tax smoothing through the monetizing of outstanding debt is no longer optimal, and the size of the state-contingent inflation is significantly dampened. As the economy switches from g^L to g^H, the optimal policy features a 7% state-contingent inflation, compared with 15% in the frictionless economy. When financial frictions exist, government debt provides collateral value to the economy, and it is no longer purely a fiscal cushion. Consequently, the government faces a tradeoff between the misallocation cost of inflation and the cost of distortionary labor taxes.

The inter-temporal decisions are also affected by government policies. As government expenditure shocks are persistent, after the initial monetization of debt, the government also issues less real debt in the subsequent periods as g^H continues, causing a dearth of collateral in the economy. Bankers are now willing to pay a higher

[24] In this model, we assume that government debt is one-period (year) short-term debt. Therefore, devaluation of debt can only be achieved by inflation in the current period when existing debt matures. If we allow for long-term government debt, then inflation can be spread out into the future. For instance, if the outstanding government debt has a five-year maturity, then roughly speaking, the same debt devaluation could be produced by 3% inflation in the next five years. In this sense, the number we obtain is the cumulative inflation.

price to hold government debt, leading to a lower real interest rate paid on debt.[25] Lower interest rates help the government to reduce the servicing cost of debt and labor taxes. On the other hand, the dearth of collateral makes capital investment less appealing. Consequently, capital investment declines sharply.[26]

Our analysis so far shows the size of inflation starting from a particular level of government bond b (after the economy converges after a long sequence of g^L). In Figure 2.3, we show state-contingent inflation when g^L switches to g^H, $\pi(b, g^L, g^H)$ as a function of outstanding government debt. We plot the policy function on the ergodic distribution of government debt in the economy with financial frictions and transform the horizontal axis to debt-to-GDP ratio.[27]

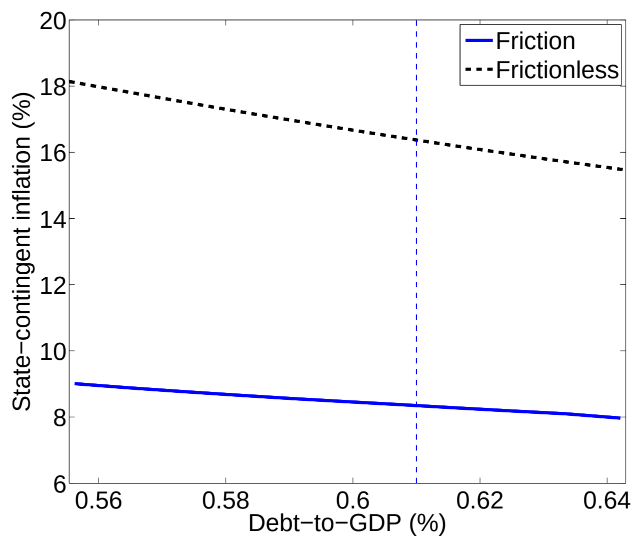

Figure 2.3: Policy function of state-contingent inflation $\pi(b, g^L, g^H)$.

[25]Due to the assumption that the expected inflation rate is zero, the nominal interest rate equals the real interest rate in our model. As shown in the fifth panel, the size of the interest rate decline is not large enough to hit the zero lower bound.

[26]Of course, one reason for the large decline in capital investment is our assumption of quasi-linear preferences. However, we will show that this mechanism is still important after we relax this assumption.

[27]In the frictionless economy, the ergodic distribution of debt depends on the initial debt level. Therefore, to facilitate a comparison, we show the policy function for the same debt-to-GDP ratio as the economy with financial frictions.

Two observations can be made. First, the size of state-contingent inflation in the economy with financial frictions is always significantly smaller compared with that in the friction economy. Second, in both economies, the optimal size of inflation decreases as the debt level increases. In the frictionless economy, a larger debt base allows the government to generate the same change in real claims with smaller variations in the price level. This mechanism also operates in the economy with financial frictions, but it is accompanied by the effect of the collateral constraint. With a larger amount of real government debt, the collateral constraint is more relaxed and the government is more willing to engineer inflation. As a result, although still downward sloping, the policy function becomes flatter relative to the frictionless economy.

2.4.3 Sensitivity analysis by varying ξ

We vary the tightness of the collateral constraint ξ to test the sensitivity of the quantitative results. Higher ξ means that the bankers can credibly pledge a larger fraction of their capital, relaxing the collateral constraint. Therefore, the government issues a smaller amount of debt in the low state of government expenditure (g^L), as in the upper left panel of Figure 2.4. At the same time, government bonds constitute a smaller fraction of total bank assets (lower left panel).

When the economy switches from g^L to g^H, the optimal size of state-contingent inflation is always significantly smaller in the economy with collateral constraint (upper right panel). As the debt base decreases with higher ξ, the size of the optimal state-contingent inflation increases in both economies. As we discussed before, with a smaller debt base, the same value of real debt adjustment can only be achieved through larger inflation.

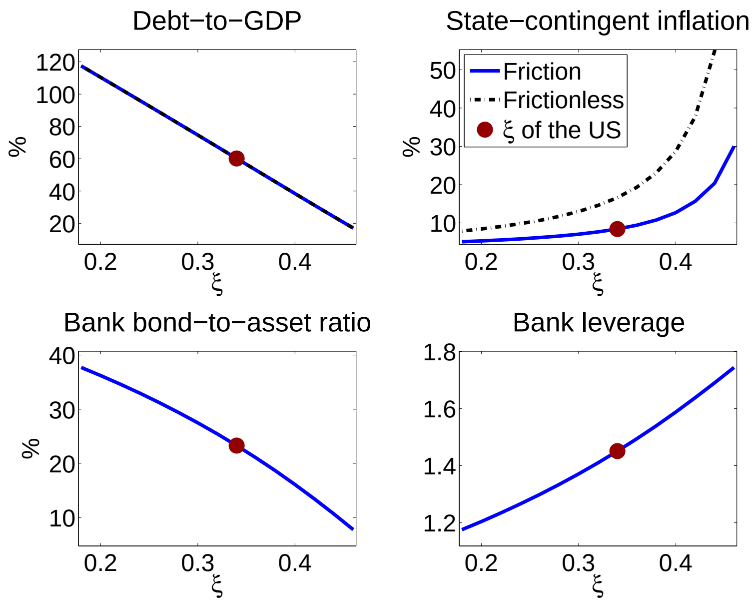

Figure 2.4: Comparative statics of ξ. The debt-to-GDP ratio, bank bond-to-asset ratio, and the bank leverage are values in the stochastic steady state after a long history of g^L. The state-contingent inflation is the size of inflation as the economy switches to g^H after a long history of g^L.

The intra-temporal leverage of bank balance sheets remains small as ξ varies. Leverage equals $1/(q-\xi)$, and capital price q fluctuates around 1. Therefore, the range of leverage is limited.[28]

2.5 General utility functions

We now relax the quasi-linear preference assumption in the previous section and further explore the quantitative property of the model. As the problem now involves two more state variables that add to the computational burden, we now adopt a local solution method. In particular, we approximate the model economy around the non-stochastic steady state where the collateral constraint strictly binds. When solving

[28]In this model, as ξ approaches $\bar{\xi} = \frac{x^*-\beta}{x^*\beta}$, the collateral constraint no longer binds (in the steady state).

the model, we assume that the collateral constraint always binds, and later verify that it is the case for the size of shock we consider.[29]

We calibrate the model to quarterly frequency. For most of the parameters, we take the calibration from the previous session. We set the relative risk aversion ρ to 2. We estimate an AR(1) process for government consumption. The standard deviation is 1.53% and the autocorrelation is 0.89.

Fiscal financing decomposition. To show the contribution of taxes and inflation to the financing of the increases in government spending, we do the following decomposition. Using the first-order-approximation of the inter-temporal government budget constraint, we decompose the increase in government spending g_t into the increase in tax revenue and state-contingent inflation and the decrease in the real interest rate. A higher real interest rate is bad news for fiscal financing, as future primary surpluses are now discounted at a higher rate:

$$\underbrace{\sum_{s=t}^{\infty} \frac{1}{\left(\bar{r}^b\right)^{s-t+1}} \frac{\bar{g}}{\bar{b}} \tilde{g}_s}_{\text{government consumption}} = \underbrace{\sum_{s=t}^{\infty} \frac{1}{\left(\bar{r}^b\right)^{s-t+1}} \frac{\bar{T}}{\bar{b}} \tilde{T}_s}_{\text{tax revenue}} + \underbrace{\frac{1}{\bar{\pi}} \hat{\pi}_t}_{\text{inflation}} - \underbrace{\sum_{s=t+1}^{\infty} \frac{1}{\left(\bar{r}^b\right)^{s-t+1}} \hat{r}^b_s}_{\text{real interest rate}} . \tag{2.29}$$

where we use $\bar{X}$, $\hat{X}$, and $\tilde{X}$ to denote the steady state level, the level deviation from the steady state, and the percentage deviation from the steady state of variable X, respectively. See Appendix 2.8.7 for derivations.

Table 2.2: Decomposition of fiscal financing (as a fraction of total increase in g)

	Frictionless model	Friction model
Tax revenue	-2.25%	51.87%
State-contingent inflation	114.68%	55.65%
Real interest rate	-12.43%	-7.52%

[29] In Appendix 2.8.5, we evaluate the accuracy of linearized solutions by showing that 1) the linearized solution and the global solution for the quasi-linear model are very similar; 2) the linearized and quadratic solutions for the model with the general utility function are very similar.

We feed in a one standard-deviation government expenditure shock. We then compute the fractions of government spending innovation financed by tax, state-contingent inflation, and real interest rate along the path of the shock (Table 2.2). In the frictionless economy, inflation finances more than 100% of the present value of the increase in government expenditure.[30] In the model with financial friction and liquidity value of debt, inflation only finances 56% of higher government expenditure, while tax revenue accounts for 52%. The negative contribution of the real interest rate is smaller because the lower value of real debt causes a decline in the real interest rate.

Volatility of inflation and tax rate. In standard models, optimal policy displays large inflation volatility because state-contingent inflation is costless (Chari et al., 1991). We now show that introducing the collateral constraint significantly reduces the volatility of inflation. In this sense, we provide a new explanation in addition to price stickiness as to why volatile inflation is undesirable.

Quarterly standard deviations of inflation and the labor tax rate are shown in Table 2.3. Without financial friction, volatility of the labor tax rate is near zero, while that of the inflation rate is near 1% per quarter. In contrast, in the model with financial frictions, the standard deviation of inflation is dampened by half, and the labor tax rate becomes much more volatile.

Table 2.3: Standard deviation of tax rate and inflation (quarterly)

	Frictionless model	Friction model
Inflation	0.96%	0.47%
Tax rate	0.02%	0.35%

Robustness as ξ varies. As a robustness check, we vary the tightness of the collateral constraint (Table 2.4). The first three rows show the contributions of tax

[30] This is because of the negative contribution of the real interest rate. After a negative government spending shock hits, consumption drops and grows back to the steady state. Therefore, the real interest rate is higher along this path.

revenues, state-contingent inflation, and the real interest rate to the financing of higher government expenditure. The results remain stable as the collateral constraint is tightened or relaxed. In particular, inflation consistently finances around 50% of the present value of higher government expenditures. The contribution of inflation also remains consistently smaller than in the frictionless model.

Table 2.4: Sensitivity analysis by varying ξ

	$\xi = 0.33$ Baseline		$\xi = 0.23$ Tighter constraint		$\xi = 0.43$ Looser constraint	
	Frictionless	Friction	Frictionless	Friction	Frictionless	Friction
Tax revenue	-2.25%	51.87%	-2.35%	42.65%	-2.13%	55.80%
State-contingent inflation	114.68%	55.65%	122.73%	59.64%	106.56%	49.50%
Real interest rate	-12.43%	-7.52%	-20.39%	-2.29%	-4.43%	-5.30%
SS debt-to-GDP	60.37%	60.37%	99.10%	99.10%	21.49%	21.49%
Volatility of inflation	0.96%	0.47%	0.63%	0.31%	2.51%	1.18%

Note: The first three rows show the results of fiscal financing decomposition.

Changing the values of ξ has a large effect on the debt provision of the government in the long-run steady state. For example, when bankers face tighter collateral constraint (smaller ξ), the government finds it optimal to issue more debt and provide more collateral to the economy. As a result, the steady state debt-to-GDP ratio becomes larger. For the same reason as the quasi-linear case in the previous section, a larger debt base leads to smaller inflation volatility in both economies. The reverse happens when the collateral constraint is relaxed (larger ξ). But importantly, the inflation volatility in our model remains much smaller than in the frictionless model.

To sum up, introducing collateral constraints alone into the standard model significantly reduces the role of inflation in the optimal fiscal and monetary policy, without any sticky-price friction. Inflation now finances only half of higher government expenditures, and it becomes much less volatile compared with the standard model.

2.6 Introducing price stickiness and long-term government debt

We now investigate whether our result in the benchmark flexible-price model is robust in an environment with nominal rigidities, where it is very costly for the government to engineer state-contingent inflation, even in the absence of the collateral channel (Schmitt-Grohé and Uribe, 2004; Siu, 2004).

We extend the benchmark model to incorporate both price stickiness and long-term government debt. In a model calibrated to the degree of price stickiness observed in U.S. data, the quantitative results depend crucially on the average maturity of government debt. When government debt has short maturity, adjustment in the real debt value can only be achieved through large fluctuations in the price level, which has a substantial cost. Consequently, the contribution of inflation to fiscal financing in the optimal policy is significantly smaller than in the benchmark flexible-price model. However, when government debt has long maturity, large changes in the value of the debt can be produced by changes in the nominal interest rate, with much smaller and smoother changes in inflation. As a result, the contribution of inflation to fiscal financing in the optimal policy becomes more similar to that in the benchmark flexible-price model, and the role of the collateral channel remains quantitatively stable.

2.6.1 Model

Price stickiness and retail firms. In order to incorporate price stickiness into the model, we introduce a continuum of retail firms. Retail firms are monopolistic competitors. They buy goods from competitive firms owned by bankers, differentiate these goods costlessly, and resell them to households. The monopoly power of retail firms allows them to set sticky prices above marginal costs; otherwise, they play no role. We assume that profits from retail activity are rebated lump-sum to households.[31]

The final goods used in household consumption and investment are aggregated from the differentiated goods using constant elasticity of substitution (CES) technology. The household optimally chooses their demand or each type of good j.

$$y_{j,t} = y_t \left(\frac{P_{j,t}}{P_t} \right)^{-\nu}, \tag{2.30}$$

where ν measures the elasticity of substitution across goods sold by retail firms, and $\frac{\nu}{\nu-1}$ is the static markup. P_t denotes the aggregate nominal price level, and $P_{j,t}$ denotes the nominal price of type-j good.

We introduce price stickiness through a Rotemberg-style price adjustment costs; to adjust nominal price $P_{j,t}$, retail firm j pays $\frac{\psi}{2}(\frac{P_{j,t}}{P_{j,t-1}} - 1)^2$ units of final goods. Retail firm j sets price $\{P_{j,s}\}_{s \geq t}$ to maximize the expected discounted sum of real profits that it rebates to the household, discounted by the household's real stochastic discount factor $\Lambda_{t,s}$ $(s \geq t)$,

$$\max_{P_{j,s}} \quad \mathbb{E}_t \sum_{s \geq t} \Lambda_{t,s} \left[\frac{P_{j,s}}{P_s} y_{j,s} - m_s y_{j,s} - \frac{\psi}{2} (\frac{P_{j,s}}{P_{j,s-1}} - 1)^2 \right],$$

subject to the demand function for good j in equation (2.30). m_t is the real price (in the units of final goods) to purchase goods from bankers' firms. In other words, m_t is the real marginal cost to produce differentiated goods j.

We focus on a symmetric equilibrium where each retail firm j sets the same price $P_{j,t}$ and $P_{j,t} = P_t$ for all j. The optimality condition of the retail firms takes the form

[31] The separation of competitive and flexible-price firms held by bankers from sticky-price retail firms follows the approach of Bernanke et al. (1999). Directly introducing price stickiness to the firms held by bankers will destroy the tractability of the model because these firms receive idiosyncratic productivity shocks. High-productivity firms would set a lower price and vice versa. Therefore, we would need to keep track of the history and the cross-section distribution of prices.

of the New Keynesian (NK) Phillips curve.[32]

$$[\nu m_t - (\nu - 1)]\, y_t - \psi\left[(\pi_t - 1)\pi_t - \beta \mathbb{E}_t \Lambda_{t,t+1}\left(\pi_{t+1} - 1\right)\pi_{t+1}\right] = 0. \qquad (2.31)$$

If $\psi = 0$, that is, in the case without price stickiness (2.31) reduces to

$$1 = \frac{\nu}{\nu - 1} m_t.$$

Intuitively, the real price of good j, which is 1 because $P_{j,t} = P_t$ in the symmetric equilibrium, equals the product of the static markup $\frac{\nu}{\nu-1}$ and the real marginal cost m_t. The presence of nominal price rigidities alters this optimality condition.

The social resource constraint now takes into account the real adjustment cost from changing prices:

$$c_t + g_t + a_t + \frac{\psi}{2}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 = y_t + (1 - \delta) a_{t-1}.$$

Long-term nominal government debt. We model long-term nominal government debt as a security paying an infinite stream of nominal coupons, which decreases at a constant rate η. In particular, a bond issued in period t promises to pay one dollar in period $t+1$ and $(1-\eta)^{s-1}$ dollars in period $t+s$, with $s \geq 2$. The exogenous parameter η dictates the average maturity of government debts. This way of modeling takes the maturity of government debts as given and abstract away from the maturity composition of the government debt portfolio. It allows us to study long-duration bonds without increasing the dimensionality of the state space, and it is commonly

[32]Log-linearizing equation (2.31), we get the more familiar-looking NK Phillips curve:

$$\tilde{\pi}_t = \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \frac{\nu - 1}{\psi} \bar{y} \tilde{m}_t,$$

$\tilde{\pi}_t$ is the log-linearized inflation rate and $\bar{y}$ is the steady state value of output.

adopted in the literature (e.g., Arellano and Ramanarayanan, 2012; Hatchondo and Padilla, 2013).

As in previous sections, we denote the units of nominal government debt by B_t. We use Q_t^B to denote the nominal price of debt in period t. The household budget constraint in real terms is given by

$$c_t+a_t+\frac{Q_t^B B_t}{P_t} = \left(\sigma v_t^H + (1-\sigma)v_t^L\right)+\nu_t^R+(1-\tau_t)\,w_t h_t+\frac{1+(1-\eta)Q_t^B}{P_t}B_{t-1}+q_t a_{t-1}.$$

This constraint differers from the one in the baseline model (equation 2.5) in terms of the retailers' profits ν_t and the long maturity of government debt η. The household maximizes its utility subject to budget constraint (2.32) and the collateral constraint (2.3).

Government policies need to satisfy the budget constraint (in real terms).

$$\tau_t w_t h_t + \frac{Q_t^B B_t}{P_t} = \frac{1+(1-\eta)Q_t^B}{P_t}B_{t-1} + g_t$$

The role of the maturity of government debt can be seen from this constraint. In the constraint, the amount of real government debt is $\frac{Q_t^B B_t}{P_t}$ and the real holding-period return on government debt is $\frac{1+(1-\eta)Q_t^B}{Q_{t-1}^B \pi_t}$. If $\eta = 1$ and government debt is one-period debt, the only way to adjust real return ex post is through inflation in the current period π_t. Large fluctuations in prices can have a substantial cost in the presence of nominal rigidities. However, if government debt has long maturity, that is, $\eta < 1$, adjustment in the real return ex post can be engineered through changes in bond price Q_t^B (or nominal interest rate), which depends on inflation in future periods. In other words, changes in real debt return can be produced by small and smooth inflation, which is less costly than large fluctuations in inflation. As a result, long-term debt helps the Ramsey government to achieve the desired adjustment in the ex post real return at less cost.

2.6.2 Numerical analysis

To better understand optimal policy response to government expenditure shocks, we perform a simple numerical exercise by calibrating the model to the U.S. data. Parameters that also appear in the baseline model take the same values in Table 2.1. The two parameters new to this model are the elasticity of substitution ν and the degree of price stickiness ψ. We set these parameters to values estimated from U.S. data in Christiano et al. (2005). In particular, we calibrate ν to a 20% markup of retail firms. With respect to nominal rigidities, we set ψ to a value that would replicate, in a linearized setup, the slope of the price Phillips curve derived using Calvo stickiness with an average duration of prices of three quarters.[33]

Similar to the benchmark model, in the steady state of the Ramsey problem, the collateral constraint is strictly binding. We solve the stochastic model by locally approximating the model around the the non-stochastic steady state.[34]

Fiscal financing decomposition. To investigate how the Ramsey optimal policy responds to fiscal shocks, we perform a similar decomposition exercise as in the previous section, using the linear approximation of the inter-temporal government budget constraint. Current inflation, future inflation, taxes, and the real interest rate each finance some fraction of the present value of higher government expenditure g_t, as in

[33]The slope of the Phillips curve in a quarterly Calvo price-setting model is $\frac{(1-\kappa)(1-\beta\kappa)}{\kappa}$, where κ is the probability of not being able to re-optimize price (Galí, 2009). $\kappa = 0.667$ is consistent with the average duration of the wage contract being three quarters. The slope in the Rotemberg model in this paper is $\frac{(\nu-1)\bar{y}}{\psi}$, where $\bar{y}$ is the steady-state value of output. We set $\frac{(\nu-1)\bar{y}}{\psi} = \frac{(1-\kappa)(1-\beta\kappa)}{\kappa}$.

[34]The Ramsey steady state features zero price inflation, that is, $\bar{\pi} = 1$. Intuitively, the only gain from the non-zero inflation rate is to produce state contingency in debt returns in response to fiscal shocks. This gain from inflation does not exist in the non-stochastic steady state in the absence of any fiscal shocks. On the other hand, any deviation from zero inflation leads to a positive adjustment cost in real resources. Thus, we conclude that $\bar{\pi} = 1$.

the following equation:

$$\underbrace{\sum_{s=t}^{\infty}\frac{1}{\left(\bar{r}^{b}\right)^{s-t+1}}\frac{\bar{g}}{\bar{\bar{b}}}\tilde{g}_{s}}_{\text{government consumption}} = \underbrace{\sum_{s=t}^{\infty}\frac{1}{\left(\bar{r}^{b}\right)^{s-t+1}}\frac{\bar{T}}{\bar{\bar{b}}}\tilde{T}_{s}}_{\text{tax revenue}} + \underbrace{\frac{1}{\bar{\pi}}\hat{\pi}_{t}}_{\text{current inflation}} + \underbrace{\sum_{s=t+1}^{\infty}\frac{(1-\eta)^{s-t}}{\left(\bar{r}^{b}\right)^{s-t}\bar{\pi}}\hat{\pi}_{s}}_{\text{future inflation}}$$
$$\underbrace{-\sum_{s=t+1}^{\infty}\frac{1-(1-\eta)^{s-t}}{\left(\bar{r}^{b}\right)^{s-t+1}}\hat{r}_{s}^{b}}_{\text{real interest rate}}. \tag{2.32}$$

See Appendix 2.8.7 for the derivations. Longer maturity of debt (smaller coupon declining rate η) affects the decomposition in two ways. First, other things being equal, the contribution of inflation in future periods becomes larger because inflation in future periods reduces the real value coupon payments in those periods. Second, the contribution of the real interest rate becomes smaller, as the government only needs to roll over a smaller fraction of debt in each period.

Numerical results of the financing decomposition are reported in Table 2.5. The first two columns compare the flexible-price economy ($\psi = 0$) and the economy with price stickiness and one-period (three-month) government debt ($\psi > 0$ and $\eta = 1$). In the second economy, the government can only increase the inflation rate in the current period to reduce the real return on debt, but it is costly to do so due to the presence of the price adjustment cost. Therefore, the contribution of current inflation reduces to 14.71%, in contrast with 65.16% in the flexible-price economy.[35] In addition, the negative contribution of the real interest rate becomes more severe (-45.72%) in the sticky price model. As the government cannot monetize debt adequately, there exists too much collateral in the economy, causing a higher interest rate. The small contributions of inflation and the real interest rate imply a large contribution of taxes.

Inflation plays a much larger role when government debt has long maturity. Column 3 in Table 2.5 shows the fiscal decomposition when the average duration of

[35]The results in column 1 differ from those in the previous section due to the presence of monopolistic competition, that is, $\nu < \infty$.

government debt is five years, which is consistent with the U.S. data.[36] The contribution of current and future inflation sum up to 33.31%, in contrast with 14.71% in the short-term debt economy. Moreover, long-term government debt allows inflation in future periods to devalue coupon payments in those periods. Hence, future inflation plays a much more important role relative to inflation in the current period (27.69% vs 5.62%). In addition, the negative contribution of the real interest rate becomes smaller, as government only rolls over a small fraction of debt compared with the economy with only three-month debt. Column 4 presents the result when government debt has a longer duration of 10 years.[37] The total contribution of inflation is even larger (38.17%).

Table 2.5: Decomposition of fiscal financing

	Flexible price 3-month debt	Sticky price 3-month debt	Sticky price 5-year debt	Sticky price 10-year debt
Tax revenue	44.90%	131.01%	97.3%	84.21%
Current inflation	65.16%	14.71%	5.62%	3.03%
Future inflation	0.00%	0.00%	27.69%	35.14%
Real interest rate	-10.06%	-45.72%	-30.62%	-22.37%

Note: All four economies are associated with collateral constraints. They only differ in the degree of price stickiness and the maturity of government debt.

Figure 2.5 further illustrates the intuition by showing the optimal policy responses to a one standard-deviation fiscal shock of 1.53% with the first-order autocorrelation coefficient of 0.89. When the government issues short-term debt, the inflation rate rises by 0.12% in the same quarter as the fiscal shock occurs. This amount of inflation is much smaller than the 0.56% inflation that emerges in the flexible-price economy.[38] When the government can issue long-term debt, the optimal response of inflation is

[36] We use the concept of Macaulay duration, which in the steady state is given by

$$D = \frac{1+\bar{r}^b}{\eta+\bar{r}^b}.$$

[37] Average duration of 10 years is consistent with the U.K. data.

[38] Inflation in the flexible-price economy is not plotted for the consideration of presentation, since it has a much larger magnitude than in the other two economies.

much smaller (0.03%) but much more persistent. The cumulative inflation in the 10 years after the occurrence of the fiscal shock is 0.37%. Inflation leads to a large decline in the nominal bond price and facilitates the reduction of real debt return in the period when the shock occurs. As a result, the increase in tax rate is greatly dampened. In terms of real allocations, labor and consumption in the economy with long-term debt becomes much more similar to that in the flexible-price economy than to that in the economy with short-term debt.

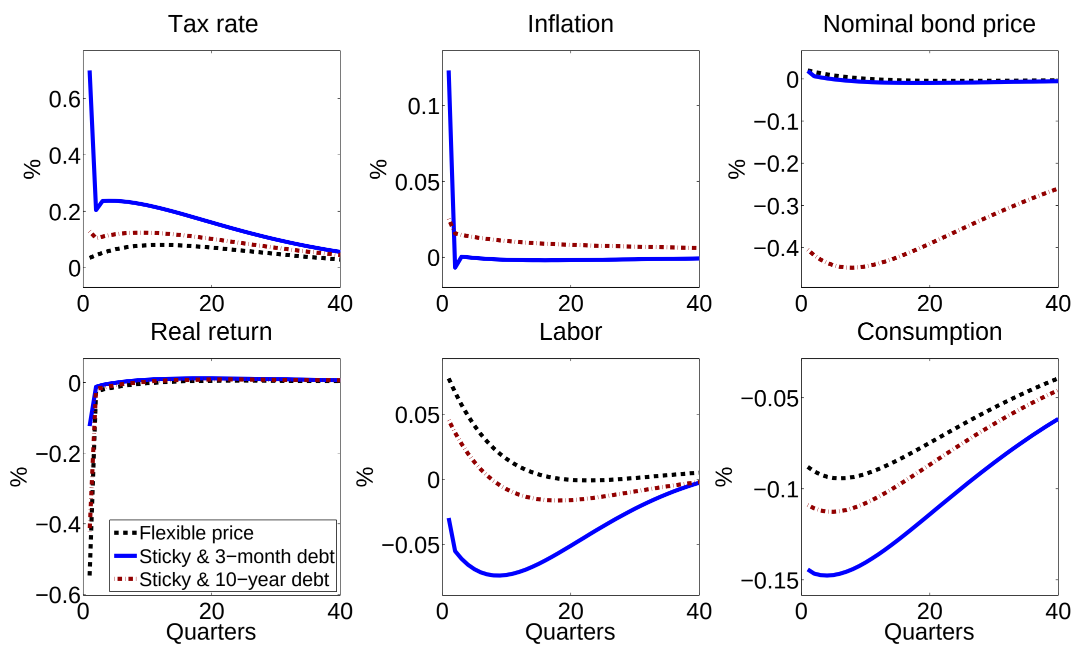

Figure 2.5: Optimal policy responses to one standard-deviation fiscal shock of 1.53% with the first-order autocorrelation coefficient of 0.89.

2.6.3 Application: war financing

As an application, we use the model to study how the U.S. government should optimally finance the wars in Afghanistan and Iraq. The total appropriations for these wars in 2001-2013 amount to $1.54 trillion (Crawford, 2014). As shown in the left panel of Figure 2.6, the budgetary costs of wars amount to 7% of total government consumption at its peak.

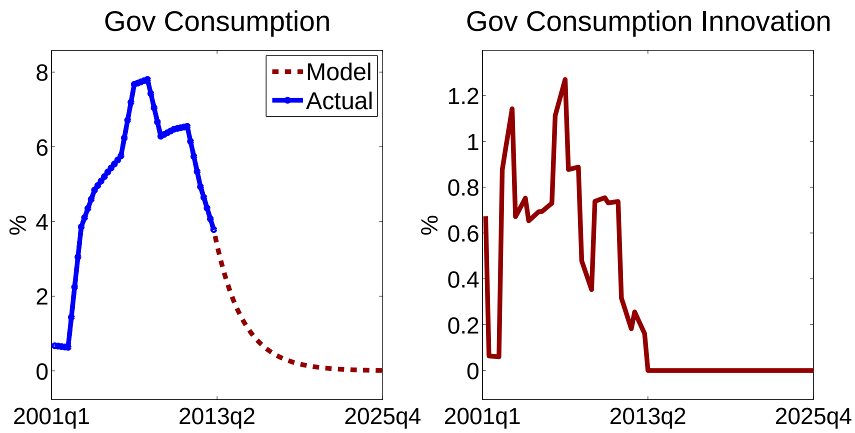

Figure 2.6: Increases in the government consumption caused by wars in Afghanistan and Iraq

We simulate the model with government consumption shocks that match war costs in the data. To do this, we first assume that the government consumption process follows the AR(1) process we estimated in the previous sections. We then calculate the series of shocks that makes the government consumption process match the data in 2001-2013. We assume that after 2013, there are no more shocks, and the government consumption declines at the rate in the AR(1) process. The right panel in Figure 2.6 shows the series of shocks to the government consumption process.

We calibrate the model to the features of the U.S. before the wars. The debt-to-GDP ratio in 2000 is 54.6%, and the average duration of government debt is 5.8 years. We assume that the economy is in the steady state before the series of war shocks arrive.

Figure 2.7 displays the prescriptions of three models for war financing. The black dashed line represents the model without financial friction and price stickiness.[39] The government solely relies on inflation to adjust the real debt value, and the tax rate remains relatively constant. The average annual inflation rate in 25 years is around

[39]Strictly speaking, in this model where prices are perfectly flexible and government debt has long maturity, the optimal path of inflation is indeterminate. This is because the government is indifferent between using current and future inflation to adjust the real value of debt. In the simulation, we determine the optimal path of inflation by setting a tiny degree of price-stickiness.

1.6%. The blue line shows the economy with the existence of the financial friction and collateral constraint. Financial friction dampens the increase in the inflation rate; the average increase in the annual inflation rate is now 1.0%. At the same time, the government raises labor tax rate by an average 0.5 percentage point (from 35 percentage points in the steady state). Finally, the red line represents the economy with both financial friction and prices stickiness, which is the most realistic and preferred calibration. Price stickiness further reduces the use of inflation to 0.4% on average, and the rise in the labor tax rate is now 1 percentage point.

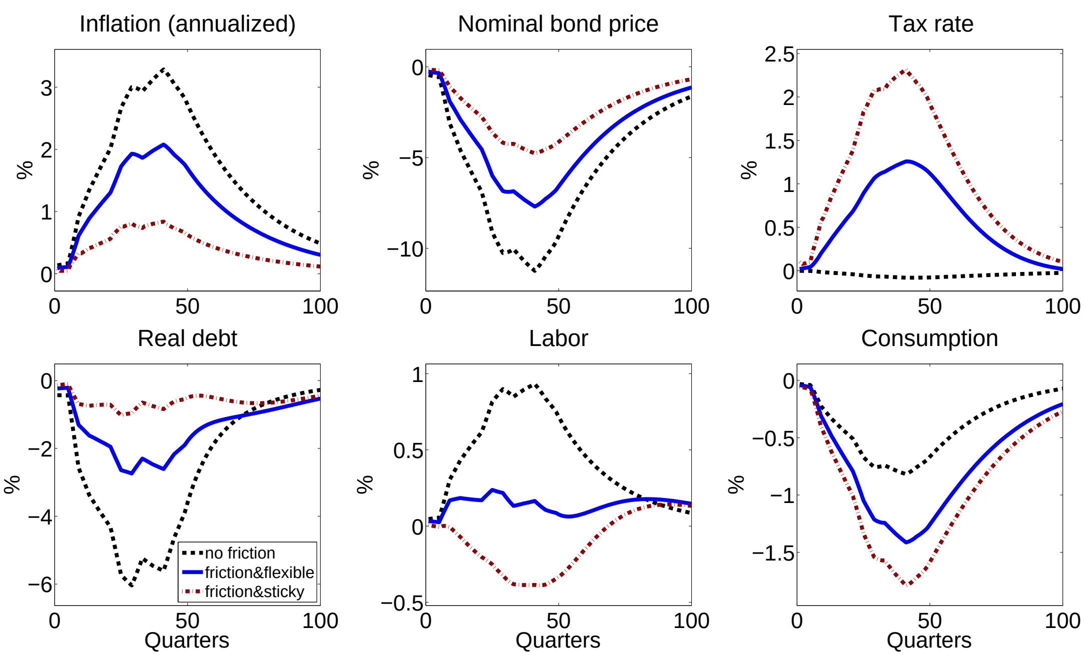

Figure 2.7: War financing

2.7 Conclusion

In this chapter, we have argued that considering the bank net worth effect of inflation substantially changes the optimal fiscal and monetary policy prescriptions. Inflation should play a much smaller role in the financing of higher government spending, compared with standard models where financial frictions and bank balance sheets are not considered. We also extend the model to incorporate price stickiness and long-term government debts. We find that the maturity of government debts crucially impacts the size and persistence of the inflation process in the optimal policy.

2.8 Appendix

2.8.1 Equilibrium characterization and aggregation

We let $x_t = \frac{k_t^H}{a_{t-1}}$ be the amount of capital used by the high-productivity firm as a fraction of the aggregate capital stock. Then

$$\frac{k_t^L}{a_{t-1}} = \frac{1-\sigma x_t}{1-\sigma}.$$

There is no friction in the labor market and therefore marginal product of labor is equalized between high and low-productivity firms, i.e.,

$$\theta z^H (k_t^H)^\alpha (n_t^H)^{\theta-1} = \theta z^L (k_t^L)^\alpha (n_t^L)^{\theta-1}.$$

Therefore the fraction of labor used by the two types of firms are the following:

$$\frac{n_t^H}{h_t} = \frac{(z^H)^{\frac{1}{1-\theta}} (x_t)^{\frac{\alpha}{1-\theta}}}{\sigma (z^H)^{\frac{1}{1-\theta}} (x_t)^{\frac{\alpha}{1-\theta}} + (1-\sigma)(z^L)^{\frac{1}{1-\theta}} \left(\frac{1-\sigma x_t}{1-\sigma}\right)^{\frac{\alpha}{1-\theta}}} = \frac{(z^H)^{\frac{1}{1-\theta}} (x_t)^{\frac{\alpha}{1-\theta}}}{\Gamma(x_t)^{\frac{1}{1-\theta}}},$$

and

$$\frac{n_t^L}{h_t} = \frac{(z^L)^{\frac{1}{1-\theta}} \left(\frac{1-\sigma x_t}{1-\sigma}\right)^{\frac{\alpha}{1-\theta}}}{\sigma (z^H)^{\frac{1}{1-\theta}} (x_t)^{\frac{\alpha}{1-\theta}} + (1-\sigma)(z^L)^{\frac{1}{1-\theta}} \left(\frac{1-\sigma x_t}{1-\sigma}\right)^{\frac{\alpha}{1-\theta}}} = \frac{(z^L)^{\frac{1}{1-\theta}} \left(\frac{1-\sigma x_t}{1-\sigma}\right)^{\frac{\alpha}{1-\theta}}}{\Gamma(x_t)^{\frac{1}{1-\theta}}}.$$

Given the allocations of capital k_t^H/a_{t-1}, k_t^L/a_{t-1} and the allocations of labor n_t^H/h_t and n_t^L/h_t, the aggregate production function can be written as

$$y_t = \sigma z^H (k_t^H)^\alpha (n_t^H)^\theta + (1-\sigma) z^L (k_t^L)^\alpha (n_t^L)^\theta = \Gamma(x_t) a_{t-1}^\alpha h_t^\theta,$$

where $\Gamma(x_t) = \left[\sigma (z^H)^{\frac{1}{1-\theta}} (x_t)^{\frac{\alpha}{1-\theta}} + (1-\sigma)(z^L)^{\frac{1}{1-\theta}} \left(\frac{1-\sigma x_t}{1-\sigma}\right)^{\frac{\alpha}{1-\theta}}\right]^{1-\theta}$.

2.8.2 Proof of Lemma 1

Proof of the "only if"

To prove the "only if" part, we need to show that the set of competitive equilibrium conditions imply the set of constraints in Lemma 1. We proceed by showing that competitive equilibrium conditions imply the implementability constraint (2.15) and the collateral constraint (2.16) in Lemma 1. Other constraints can be derived straightforwardly.

Implementability condition. Plug the expressions for wage rate and tax rate (2.11) and (2.12) into the household budget constraint (2.5), we have

$$c_t + \frac{B_t}{P_t} + a_t = \left(\sigma v_t^H + (1-\sigma) v_t^L\right) - \frac{U_{h,t}}{U_{c,t}} h_t + \frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1}.$$

Using the definition of firm's profit (2.4) we obtain

$$\sigma v_t^H + (1-\sigma) v_t^L = y_t - w_t h_t - [q_t - (1-\delta)] a_{t-1}.$$

The labor and capital demand conditions (2.6) imply that

$$w_t h_t = \theta y_t,$$

and

$$[q_t - (1-\delta)]a_{t-1} = \alpha y_t - \sigma \mu_t^H k_t^H = \alpha y_t - \frac{\sigma \mu_t^H}{q_t - \xi}\left(\frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1}\right),$$

where the last equality holds whether the collateral constraint for the high type binds or not.[40] Because labor market is frictionless, the share of labor income is exactly θ. Due to frictions in capital allocations, the share of capital measured at market price of capital q_t is smaller than α when the collateral constraint strictly binds for the hight type (i.e., $\mu_t^H > 0$).

Plug the expression for profit back into the household budget constraint, we have

$$\begin{aligned}&[U_{c,t}c_t + U_{h,t}h_t - U_{c,t}(1-\alpha-\theta)y_t] + U_{c,t}\left(\frac{B_t}{P_t} + a_t\right)\\ &= U_{c,t}\left(1 + \frac{\sigma\mu_t^H}{q_t - \xi}\right)\left(\frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1}\right). \qquad (2.33)\end{aligned}$$

Taking conditional expectation in date $t-1$ on both sides of the equation, and use the Euler equations for bond and capital (2.8) and (2.7), we arrive at the flow implementability condition in equation (2.15):

$$\beta\mathbb{E}_{t-1}\left[U_{c,t}c_t + U_{h,t}h_t - U_{c,t}(1-\alpha-\theta)y_t\right] + \beta\mathbb{E}_{t-1}U_{c,t}\left(a_t + b_t\right) = U_{c,t-1}(a_{t-1} + b_{t-1}).$$

Collateral constraint. Combining the government budget constraint (2.9) and the expression for tax rate (2.12), we can express the outstanding value of debt at the

[40] To see this, if the collateral constraint binds for the high type, we have

$$k_t^H = \frac{1}{q_t - \xi}\left(\frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1}\right).$$

If the collateral constraint does not bind for the high type, we have

$$\mu_t^H = 0.$$

beginning of period t by

$$r_t^b b_{t-1} = \theta y_t + \frac{U_{h,t}}{U_{c,t}} h_t + b_t - g_t.$$

Substituting for $r_t^b b_{t-1}$ in the collateral constraint, we arrive at the form of collateral constraint in equation (2.16):

$$x_t a_{t-1} \left(q_t - \xi\right) \leq q_t a_{t-1} + \left(\theta y_t + \frac{U_{h,t}}{U_{c,t}} h_t + b_t - g_t\right).$$

Proof of the "if"

To prove the "if" part, we need to show that if allocations $\{a_t, h_t, x_t, c_t, b_t\}_{t=0}^{\infty}$ satisfy the set of constrains in Lemma 1, they also satisfy the set of competitive equilibrium conditions. However, competitive equilibrium conditions also involve prices and policy instruments. We first show here how to construct prices and policy instruments from the set of allocations in the Ramsey problem.

The wage rate w_t, price of capital q_t, multiplier on high type's collateral constraint μ_t^H, tax rate τ_t are implied by equations (2.11), (2.13), (2.14) and (2.12) respectively.

The only remaining price is the the real return on debt r_t^b, can be backed out through the government budget constraint (2.9), i.e.,

$$r_t^b = \frac{\theta y_t + \frac{U_{h,t}}{U_{c,t}} h_t + b_t - g_t}{b_{t-1}}.$$

It is straightforward to check that allocations $\{a_t, h_t, x_t, c_t, b_t\}_{t=0}^{\infty}$, prices w_t, q_t, and policy instruments r_t^b satisfy the competitive equilibrium conditions. □

2.8.3 The deterministic model and the long-run level of debt

In this subsection we discuss the deterministic model and the determination of long-run debt level. Figure 2.8 plot the policy functions of debt issuance, tax rate and real interest rate as a function of outstanding debt.

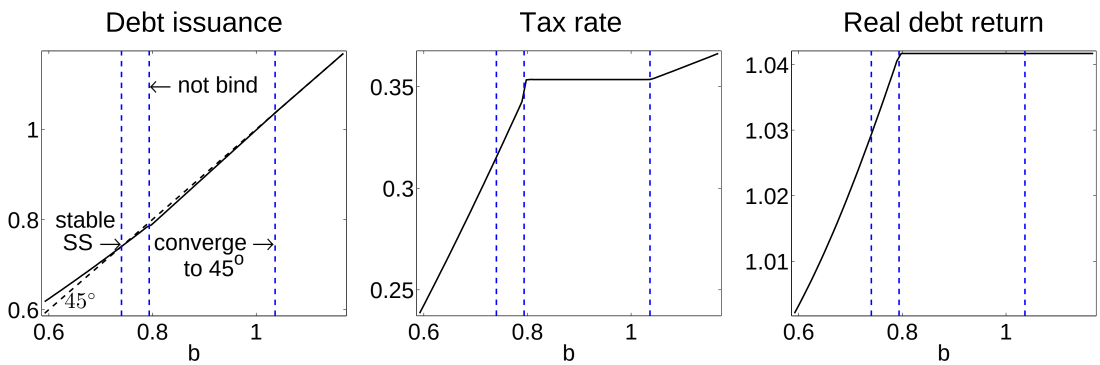

Figure 2.8: Policy function in the deterministic model

The first blue-dashed line indicates a stable deterministic steady state. The amount of debt in this steady state is smaller than the amount that relaxes the collateral constraint of the high-type bankers indicated by the second blue-dashed line. The determination of the long-run debt level contrasts sharply with the random walk behavior of debt in Barro (1979) and Aiyagari et al. (2002). This result reflects the tradeoff between the two key distortions confronted by the Ramsey planner. By reducing the level of debt, the planner tightens the bankers' collateral constraint, which exacerbates the inefficiency in capital reallocation and investment. On the other hand, the tightening of the collateral constraint increases the bankers' willingness to hold government debt, reduces the interest rate on debt and alleviates tax distortions. The steady-state level of debt is determined by balancing of these two effects.[41]

[41] At the debt level where the collateral constraint just binds, the distortion from tightening the collateral constraint is second order, while the tax-saving effect is first order. Therefore it is optimal for the government to reduce debt to the steady state level where the collateral constraint strictly binds.

Note the debt level indicated by the third blue-dashed line, above which the policy function of debt-issuance converges to the 45 degree line, and the optimal policy stops to wind down debt. To reduce debt, the government needs to impose higher taxes in the short run. When the initial debt is too high, this short-run cost of higher taxes together with the long-run cost of tighter constraints outweighs the long-run benefit of lower interest rate and lower taxes.

2.8.4 Proof of Proposition 1 (Ramsey policy without financial friction)

It is convenient to prove Proposition 1 using the sequence-problem formulation. The Ramsey problem without financial frictions is

$$\max_{a_t, h_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_t - \chi \frac{h_t^{1+\epsilon}}{1+\epsilon} \right),$$

subject to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\theta y_t - h_t^{1+\epsilon} - g_t \right) = r_0^b b_{-1}, \tag{2.34}$$

$$\beta \mathbb{E}_t \boldsymbol{q} \left(a_t, h_{t+1}, x^* \right) = 1, \tag{2.35}$$

$$c_t + a_t + g_t = (1-\delta) a_{t-1} + \Gamma(x^*) a_{t-1}^{\alpha} h_t^{\theta}, \tag{2.36}$$

where equations (2.34) to (2.36) are the inter-temporal government budget constraint, the Euler equation of capital, and the social resource constraint, respectively. x^* is the first best value of capital allocation x. In the absence of any financial friction, physical capital is allocated optimally across two types of bankers in each time period. As a result, aggregate TFP is always at the maximum $\Gamma(x^*)$. In the text below we save on notation by denoting $\Gamma^* = \Gamma(x^*)$.

By substituting for consumption c_t in the objection function using the resource constraint and substituting for capital price q_t using the capital demand condition, the Ramsey policy could be written as

$$\max_{a_t,h_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Gamma^* F\left(a_{t-1}, h_t\right) - a_t + (1-\delta)a_{t-1} - g_t - \chi \frac{h_t^{1+\epsilon}}{1+\epsilon} \right],$$

subject to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\theta \Gamma^* F\left(a_{t-1}, h_t\right) - h_t^{1+\epsilon} - g_t \right] = r_0^b b_{-1}, \qquad [\omega]$$

$$\beta \mathbb{E}_t \left[1 - \delta + \Gamma(x^*) F_a\left(a_t, h_{t+1}\right) \right] = 1. \qquad [\gamma_t]$$

In the initial period $t = 0$, the government wants to engineer infinite price level and make $r_0^b = 0$ in order to completely monetize the stock of government debt. Following the literature, we assume that the initial price level and therefore the initial return on government debt r_0^b is given.

The first order condition for capital a_t ($t \geq 0$) is

$$[\partial a_t] \quad (1+\theta\omega)\beta \mathbb{E}_t \Gamma^* F_a\left(a_t, h_{t+1}\right) - 1 + \beta(1-\delta)$$

$$+\beta \mathbb{E}_t \gamma_{t+1} \Gamma^* F_{aa}\left(a_t, h_{t+1}\right) = 0.$$

The first order condition for h_t ($t > 0$) is

$$[\partial h_t] \quad (1+\theta\omega)\Gamma^* F_h\left(a_{t-1}, h_t\right) - \left[1 + (1+\epsilon)\omega\right] h_t^{\epsilon} + \gamma_{t-1} \Gamma^* F_{ah}\left(a_{t-1}, h_t\right) = 0.$$

Government consumption shock g_t does not enter either the first order conditions of the Ramsey planner or the Euler equation of the household, which suggests that h_t, a_t are independent of g_t. More formally, assume $h_t = h^*$, $a_t = a^*$, and $\gamma_t = \gamma^*$, where h^*, s^*, and γ^* are constant and independent of time t and state g^t. Then the first order conditions become

$$(1+\theta\omega)\beta \Gamma^* F_a\left(a^*, h^*\right) - 1 + \beta(1-\delta) + \beta\gamma^* \Gamma^* F_{aa}\left(a^*, h^*\right) = 0,$$

$$(1+\theta\omega)\beta\Gamma^* F_h(a^*,h^*) - [1+(1+\epsilon)\omega]\, h^{*\epsilon} + \gamma^*\Gamma^* F_{ah}(a^*,h^*) = 0.$$

The Euler equation of capital becomes

$$\beta\left[1-\delta+\Gamma^* F_a(a^*,h^*)\right] = 1.$$

Therefore, a^*, h^*, and γ^* are pinned down by the above three equations as functions of parameters and multiplier ω, and ω is determined by making the intertemporal government budget constraint holds. It immediately follows that tax rate τ_t is also constant across states

$$\tau_t = 1 - \frac{h^{*\epsilon}}{F_h(a^*,h^*)}.$$

Government debt b_t equals the discounted sum of expected future primary surplus.

$$b_t = \mathbb{E}_t \sum_{s\geq t+1} \beta^{s-t}\left[\theta F(a^*,h^*) - h^{*1+\epsilon} - g_s\right].$$

If g_t process is i.i.d., then b_t is constant across states; if g_t follows a first-order Markov process, then b_t is a function of g_t and also follows a first-order Markov process.

The real return on government debt r_t^b is

$$r_t^b = \frac{\sum_{s\geq t}\beta^{s-t}\left[\theta F(a^*,h^*) - h^{*1+\epsilon} - g_s\right]}{\sum_{s\geq t}\beta^{s-t+1}\left[\theta F(a^*,h^*) - h^{*1+\epsilon} - g_s\right]}.$$

The same as b_t, when g_t is a first-order Markov process, r_t^b only depends on current state g_t, not on the entire history. □

2.8.5 Accuracy of solution

The quantitative results presented in section 2.5 are based on a log-linear approximation to the first-order conditions of the Ramsey problem. For the simplified model under the assumption of quasi-linear utility, we have computed numerical solutions using global methods. The availability of global solution allows us to evaluate the accuracy of the log-linear solution at least for the case with quasi-linear utility.

Table 2.6: Percentage deviation of the linear solution from the global solution

Real interest rate	**Tax rate**	**Debt issuance**	**Labor**
0.0008	0.0073	0.0008	0.0018
Capital	**x**	**Output**	
0.0072	0.0002	0.0014	

Table 2.6 shows the maximum percentage deviation of the linear solution from the global solution. It shows that the quantitative results obtained using the global solution and the log-linear approximation are very similar. The most noticeable difference concerns the capital and tax rate, with a percentage difference around 0.72%.

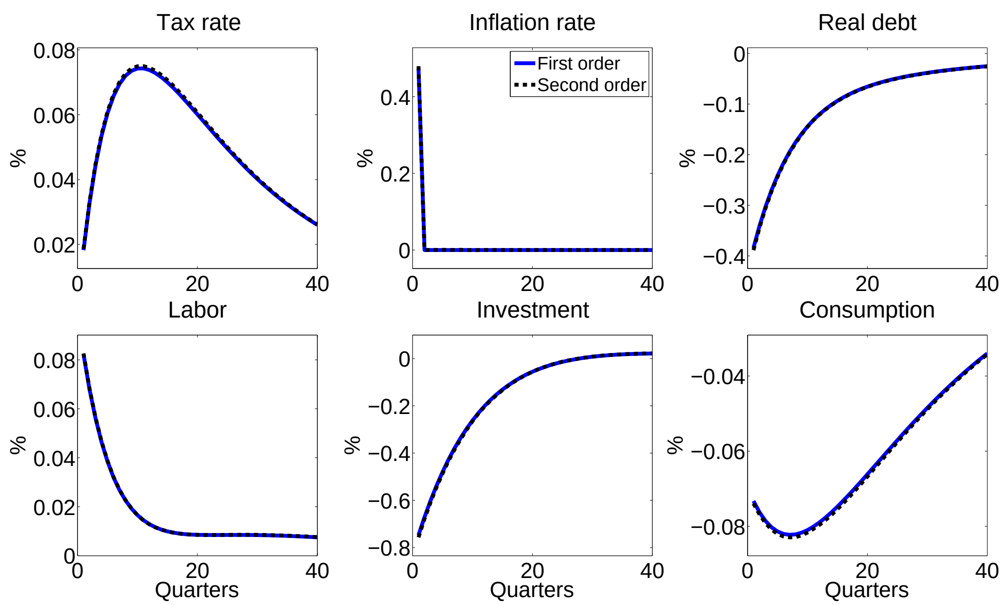

Figure 2.9: Comparing the responses of policies and allocations to a one standard-deviation government spending shock in the first- and second-order approximations

Next, we gauge the accuracy of the first-order approximation to the Ramsey problem with general utility functions by comparing it to the results based on a second-order approximation. Figure 2.9 compares the responses of policies and allocations to a one standard-deviation government spending shock in the first- and second-order approximations. At least for the size of government expenditure shocks experienced by the U.S. economy, the first- and second-order approximations are remarkably similar.

2.8.6 The initial-period Ramsey problem

The initial-period value function involves only two state variables: outstanding capital stock a and government spending shock $g_{\_}$. In the initial period the government is not bound by any precommitment, therefore λ drops out as a state variable. In addition, initial debt level is not a state variable because government can adjusts inflation rate freely to monetize initial debt. The value function $U(a, g_{\_})$ satisfies the following Bellman equation.

$$U(a, g_{\_}) = \max_{b'(g),a'(g),h(g),x(g),c(g),\lambda'(g)} \mathbb{E}\left[\frac{c(g)^{1-\rho}}{1-\rho} - \chi\frac{h(g)^{1+\epsilon}}{1+\epsilon} + \beta V(a'(g), b'(g), \lambda'(g), g)|g_{\_}\right], \tag{2.37}$$

where the maximization is subject to

$$c(g) + g + a'(g) = (1-\delta)a + \Gamma(x(g))a^{\alpha}h(g)^{\theta}, \tag{2.38}$$

$$x(g)a\left[\boldsymbol{q}(g) - \xi\right] \leq \boldsymbol{q}(g)a + \left[\theta y(g) + \frac{U_h(g)}{U_c(g)}h(g) + b'(g) - g\right], \tag{2.39}$$

$$\boldsymbol{\mu}^{\mathbf{H}} \geq 0 \quad \text{and the household complementary slackness condition, and} \tag{2.40}$$

$$\lambda'(g) = U_c(g). \tag{2.41}$$

Compared with the continuation problem after the initial period, in the initial period the government is not bound by the flow implementability constraint and the Euler equation in the previous period.

The initial-period problem in this model contrasts sharply with that in a frictionless model. In a frictionless model, it is well known that a government finds it optimal to confiscate the entire stock of government debt by generating an infinite price level. By doing this the government reduces the future tax distortions. In our model, monetizing debt has the constraint-tightening effect, and monetizing the entire stock of debt is generally not optimal.

An example under assumption of quasi-linear utility. We numerically solve the initial period Ramsey problem under the assumption that the household utility function is quasi-linear. Figure 2.10 plots the time path of the deterministic Ramsey policy starting from the initial period, where initial capital stock equals its steady-state level. Parameters take the same values as in Section 2.4.

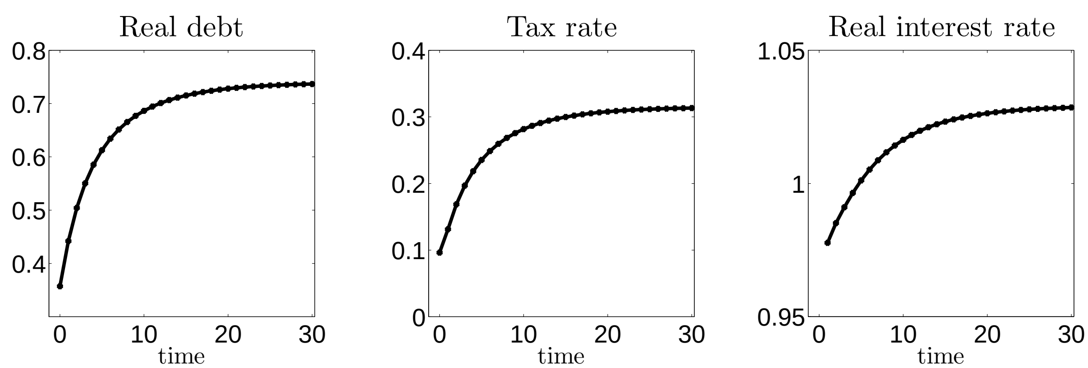

Figure 2.10: The time path of the Ramsey optimal policy starting from the initial period in a deterministic model. We assume that the household utility function is quasi-linear.

Due to lack of commitment in the initial period, the Ramsey government uses inflation to adjust the real value of debt such that real debt in the initial period drops to about half of the long-run steady-state level. It then gradually increases and converges to the steady state. Along this path of growing debt, labor tax rate and real interest rate also increase. Importantly, in our model a positive amount (about half of the steady-state level) of debt can be sustained in the initial period, in contrast with a frictionless model. This result relates to the literature on sovereign debt default that domestic banking sector's exposure to sovereign debt provides a commitment device for the government (Gennaioli et al., 2014; Sosa-Padilla, 2012).

2.8.7 Derivations of the fiscal financing decompositions

One-period nominal government debt

In this subsection, we derive the equation of the fiscal financing decomposition (equation 2.29) under the assumption that the government debt is a one-period nominal debt.

Suppose the government spending shock occurs in date t when the economy was at the steady state. We start from the government budget constraint

$$T_t + b_t = r_b^b b_{t-1} + g_t,$$

where T_t is the labor tax revenue, that is, $T_t = \tau_t w_t h_t$. By linearizing this equation we get

$$\tilde{b}_{t-1} = \frac{1}{\bar{r}^b}\tilde{b}_t - \frac{1}{\bar{r}^b}\hat{r}_t^b - \frac{\bar{g}}{\bar{r}^b\bar{b}}\tilde{g}_t + \frac{\bar{T}}{\bar{r}^b\bar{b}}\tilde{T}_t,$$

where we use $\bar{X}$, $\hat{X}$, and $\tilde{X}$ to denote the steady state level, the level deviation from the steady state, and the percentage deviation from the steady state of variable X, respectively. By iterating this equation forward, we get

$$\tilde{b}_{t-1} = -\sum_{s=t}^{\infty}\frac{1}{(\bar{r}^b)^{s-t+1}}\hat{r}_s^b - \sum_{s=t}^{\infty}\frac{1}{(\bar{r}^b)^{s-t+1}}\frac{\bar{g}}{\bar{b}}\tilde{g}_s + \sum_{s=t}^{\infty}\frac{1}{(\bar{r}^b)^{s-t+1}}\frac{\bar{T}}{\bar{b}}\tilde{T}_s.$$

Using the fact that $\tilde{b}_{t-1} = 0$, we arrive at

$$\sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{g}}{\bar{b}} \tilde{g}_s = \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{T}}{\bar{b}} \tilde{T}_s - \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \hat{r}^b_s. \tag{2.42}$$

In period t when the shock occurs, the real return of debt r^b_t follows the Fisher equation

$$r^b_t = \frac{R^B_{t-1}}{\pi_t}.$$

By linearizing the Fisher equation and using the fact that the nominal interest rate is pre-determined ($\hat{R}^B_{t-1} = 0$), we get

$$\hat{r}^b_t = -\frac{\bar{r}^b}{\bar{\pi}} \hat{\pi}_t.$$

Combining this equation with equation (2.42), we arrive at the fiscal financing decomposition condition (2.29), that is,

$$\sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{g}}{\bar{b}} \tilde{g}_s = \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{T}}{\bar{b}} \tilde{T}_s - \frac{1}{\bar{\pi}} \hat{\pi}_t - \sum_{s=t+1}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \hat{r}^b_s.$$

Long-term nominal government debt

In this subsection, we derive the equation of the fiscal financing decomposition (equation 2.32) under the assumption that the government debt is a long-term nominal debt. In this case, equation (2.42) still holds, but the period-t return on nominal debt becomes

$$r^b_t = \frac{1 + (1-\eta) Q^B_t}{Q^B_{t-1} \pi_t}. \tag{2.43}$$

Linearizing this equation and using the fact that $\tilde{Q}^B_{t-1} = 0$, we have

$$\hat{r}^b_t = \frac{1-\eta}{\bar{\pi}} \tilde{Q}^B_t - \frac{\bar{r}^b}{\bar{\pi}} \hat{\pi}_t.$$

Intuitively, the real return on debt depends on the nominal bond price and the inflation rate in the current period. The nominal bond price Q_t^B is in turn a function of the future real interest rates and inflation rates, which can be shown by iterating $\tilde{Q}_t^B$ forward using a linearized version of equation (2.43):

$$\tilde{Q}_t^B = \sum_{s=t+1}^{\infty} \left(\frac{1-\eta}{\bar{r}^b \bar{\pi}} \right)^{s-t-1} \left(\frac{\hat{\pi}_s}{\bar{\pi}} - \frac{\hat{r}_s^b}{\bar{r}^b} \right).$$

Therefore, we can express the ex-post return r_t^b as a function of current and future inflation rates and future interest rates:

$$\hat{r}_t^b = \sum_{s=t+1}^{\infty} \frac{(1-\eta)^{s-t}}{(\bar{r}^b)^{s-t-1}} \left(\frac{\hat{\pi}_s}{\bar{\pi}} - \frac{\hat{r}_s^b}{\bar{r}^b} \right) - \frac{\bar{r}^b}{\bar{\pi}} \hat{\pi}_t.$$

where we use the fact that $\bar{\pi} = 1$. By combining this equation with equation (2.42), we arrive at the fiscal financing decomposition condition (2.32).

Chapter 3

Long-term Nominal Loans and Optimal Fiscal and Monetary Policy

3.1 Introduction

In this chapter, we extend the stylized model in Chapter 2 to account for an important empirical fact: banks hold sizable long-term nominal loans on their balance sheets. As we documented in Chapter 1, more than 50% of bank balance sheets are loans and leases to households and non-financial businesses. When a government engineers an unexpected and persistent increase in inflation, the majority of bank balance-sheet loss is accounted for by nominal loans and leases. As shown in Figure 1.4, 10% out of the 15% Tier 1 capital loss is caused by banks' holdings of loans and leases, whereas only 1–2% is caused by banks' holdings of treasury securities.

Based on the framework in Chapter 2, we propose a tractable model to account for the long-term nominal loans on the bank balance sheets and then investigate the optimal response of fiscal and monetary policy to an unexpected rise in government expenditures. The model economy is populated with a large number of bankers who hold two types of long-term nominal assets on their balance sheets: long-term loans to a good-producing firm and long-term government debts. Each firm is subject to idiosyncratic productivity shocks. For a high-productivity firm to acquire more productive resources, its partner banker needs to raise external funds by collateralizing its liquid assets, which are government bonds and a fraction of firm loans. When the government generates inflation, the real values of firm loans and government debts decline. Therefore, banks' collateral constraints are tightened, which impedes resource reallocation across firms.

We calibrate the model to match the size and the maturity of bank balance sheets in the data. In particular, the majority of nominal assets are now nominal loans to firms. Using the model, we investigate the response of optimal fiscal and monetary policy to fiscal shocks.

We first study the implications of financial frictions in a flexible-price environment. We show the importance of financial frictions by comparing our model with an otherwise identical model without any financial frictions. To be more specific, we perform a decomposition analysis to study the contributions of inflation and taxes to the financing of higher government expenditures. In the frictionless model, inflation finances almost all of the increase in government expenditures; in our calibrated model, inflation only finances 52% of the increase. This result is quantitatively similar to that in Chapter 2.

We also show that the assumptions of nominal loans are important for the quantitative results of the model. If the loans were real, the government would engineer significantly higher inflation in the optimal policy.

We then consider nominal rigidities and the implications of long-term contracts. When we calibrate the degree of price stickiness and the maturity of nominal contracts to the data, inflation finances about 21% of the increase in government expenditures, and inflation in future periods plays a much larger role than inflation in the initial period. To study the implication of long-term contracts, we consider an alternative model economy where all nominal contracts have only one-quarter maturity. In this economy, inflation only finances 4.9% of the increase in government expenditures. We therefore conclude that price stickiness reduces the use of inflation by a large degree, but its effect is attenuated when nominal contracts have long maturities.

Roadmap. The rest of the chapter is organized as follows. Section 3.2 discusses the model setup and defines the competitive equilibrium and optimal policy problem. Section 3.3 presents the numerical analysis and the key findings. We conclude in Section 3.4.

3.2 Model

3.2.1 Environment

The model economy is similar to the one in Chapter 2. It consists of a continuum of identical households. Within each household reside equal masses of bankers indexed by $i \in [0,1]$ and workers indexed by $j \in [0,1]$. There exists perfect consumption risk sharing within each household.

Preference. Each household's utility function is defined over stochastic processes for the household consumption $\{c_t\}_t$ and the labor supply $\{h_{j,t}\}_t$ of each worker j

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\rho} - 1}{1-\rho} - \chi \frac{\int_0^1 h_{j,t}^{1+\epsilon} dj}{1+\epsilon} \right). \tag{3.1}$$

Technology. There are two types of firms in this economy: wholesale firms and retail firms. Each wholesale firm has a business relationship with a banker. Therefore, we use i to index the firm that has a relationship with banker i. Firm i uses $k_{i,t}$ units of physical capital and $n_{i,t}$ units of labor to produce wholesale goods $y_{i,t}$:

$$y_{i,t} = z_{i,t} F(k_{i,t}, n_{i,t}),$$

where $z_{i,t}$ is an idiosyncratic productivity shock and F is a production function that has decreasing returns to scale, with $F(k,n) = k^{\alpha} n^{\theta}$ and $\alpha + \theta < 1$. We again assume that $z_{i,t}$ is independent and identically distributed across both firm i and time t and can take two values:

$$z_{i,t} = \begin{cases} z^H & \text{with probability } \sigma \\ z^L & \text{with probability } 1 - \sigma \ . \end{cases}$$

There are a measure one of retail firms indexed by s. They buy wholesale goods, differentiate them and resell them to the households. The final goods used in household consumption and investment are aggregated from the differentiated goods using a constant elasticity of substitution (CES) technology

$$y_t = \left[\int_0^1 y_{s,t}^{\frac{\nu-1}{\nu}} ds \right]^{\frac{\nu}{\nu-1}} .$$

We assume that a retail firm s pays a Rotemberg-style price adjustment costs when it adjusts its nominal price $P_{s,t}$. When $P_{s,t}$ differs from $P_{s,t-1}$, the retail firm s pays $\frac{\psi}{2}(\frac{P_{s,t}}{P_{s,t-1}} - 1)^2$ units of final goods. The retail firms rebate any profits or losses to the representative households.

The physical capital used in producing wholesale goods depreciates at rate δ. Aggregate capital stock a_t is the sum of the stock of remaining capital and current investment i_t:

$$a_t = (1 - \delta) a_{t-1} + i_t.$$

Aggregate uncertainty. We consider a stochastic government expenditure g_t in this model. We use X_t to denote a random variable that is a function of the history of g_t shocks.

Aggregate output y_t is divided among household consumption, investment expenditures, government expenditures and the adjustment costs paid by retailers:

$$c_t + a_t + g_t + \int_0^1 \frac{\psi}{2}(\frac{P_{s,t}}{P_{s,t-1}} - 1)^2 ds = (1-\delta)a_{t-1} + y_t. \tag{3.2}$$

Capital markets, timeline and collateral constraints. Figure 3.1 presents the timing and the sequence of activities. Each time period is divided into two subperiods: the beginning and the end. Both aggregate and idiosyncratic shocks arrive at the beginning of a period.

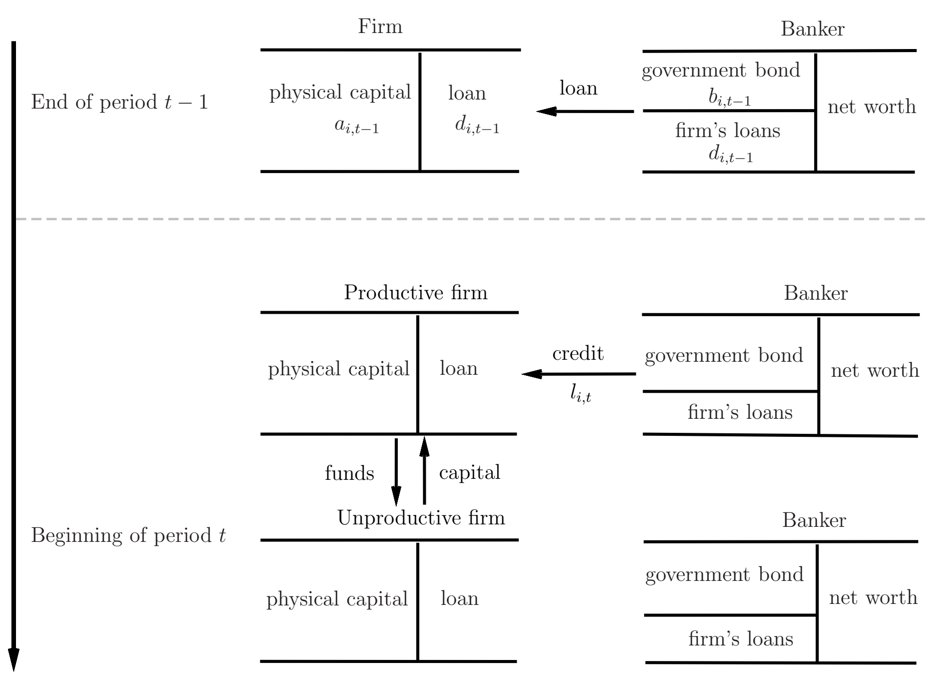

Figure 3.1: Timeline and activities

At the end of period $t-1$, each household shares among its bankers all the resources it accumulated in that period. Each banker then invests in loans to his partner wholesale firm as well as government debts. Each wholesale firm takes loans from its partner banker and use them to repay old loans and finance physical capital investment.

Both loans and government debts are in nominal terms and have long maturities. To keep tractability, we model loans and government debts as securies paying an infinite stream of nominal coupons $\{1, 1-\eta^d, (1-\eta^d)^2, ...\}$ and $\{1, 1-\eta^b, (1-\eta^b)^2, ...\}$, respectively. The exogenous parameters η^d and η^b dictate the average maturities of firm loans and government debts. We denote the real value of the loans and the government debts by $d_{i,t-1}$ and $b_{i,t-1}$.[1]

After bankers make portfolio choices, workers and bankers within the same household separate, and they cannot meet each other until the end of period t. After separation, each banker holds an equal share of the assets he accumulated from the previous period, and the remaining assets are held by the workers in the same household. In particular, a banker carries ω^d fraction of the loans and ω^b fraction of the government debts.

At the beginning of the following period t, the idiosyncratic productivity shocks and the aggregate government expenditure shocks are realized. High-productivity firms want to scale up their production and therefore need to finance more labor and capital. These firms can buy more physical capital from other firms in a competitive capital market. To pay for the purchase, a buyer firm i can get additional credit $l_{i,t}$ from its partner banker, for which the firm pays an interest rate $r^l_{i,t}$ at the end of period t. Therefore, the amount of additional capital that firm i acquires, $k_{i,t} - a_{i,t-1}$,

[1]Denote the units of nominal loans by $D_{i,t}$ and the units of nominal government debts by $B_{i,t}$. Then we have

$$d_{i,t} = \frac{Q^D_t D_{i,t}}{P_t},$$

and

$$b_{i,t} = \frac{Q^B_t B_{i,t}}{P_t},$$

where π_t is the gross inflation rate.

is constrained by the amount of credit $l_{i,t}$, that is,

$$q_t(k_{i,t} - a_{i,t-1}) \leq l_{i,t},$$

where q_t is the price of capital in the intra-firm capital market.

A banker's ability to extend credit is constrained by the amount of liquid assets on his balance sheet. I assume that all the government debts are liquid, and ξ fraction of the loans to firms are liquid. Therefore,

$$l_{i,t} \leq \xi \omega^d r_t^d d_{i,t-1} + \omega^b r_t^b b_{i,t-1}$$

where r_t^d and r_t^b denote real returns to loans and government debts.[2]

As shown in Figure 3.1, the model captures the mismatch of maturity observed in the data. In the model, banks have long-term firm loans and government debts on the asset side of their balance sheets. For tractability reasons, we abstract away from the nominal bank liabilities. This abstraction is not important for quantitative results as long as these liabilities have short maturities and inflation is a persistent process.

This model captures the negative effect of inflation on the economy through the bank maturity mismatch. Other things being equal, higher unexpected inflation in period t (higher π_t) reduces the real returns of loans and government debts (r_t^d and r_t^b). Moreover, if inflation persists into future periods, the nominal prices of long-term loans and debts (Q_t^D and Q_t^B) both fall, which in turn decreases the real returns r_t^d

[2] Denote the nominal price of firm loans in period t by Q_t^D and nominal price of government debts by Q_t^B. Then we have

$$r_t^d = \frac{1 + (1 - \eta^d) Q_t^D}{Q_{t-1}^D \pi_t}, \quad \text{and} \quad r_t^b = \frac{1 + (1 - \eta^b) Q_t^B}{Q_{t-1}^B \pi_t}.$$

and r_t^b. Lower returns on these assets tighten the collateral constraints, reduce the amount of credit extended by a banker and interrupt the reallocation of capital.

In this model, we emphasize the balance sheets of banks instead of the firms. Only the bank balance sheets matter for the real allocations. In fact, unexpected inflation increases firms' net worth by reducing their real debt burden. However, we assume that firms cannot pledge these gains to acquire more productive resources. Instead, they need to rely on their partner bankers. There is a large literature discussing borrowers' net worth and financial constraints (e.g., Bernanke et al., 1999; Gomes et al., 2014). We abstract from the borrowers' balance sheets to focus on the bank balance sheets.

3.2.2 Wholesale firms' problem

We now discuss agents' optimization problems in a competitive equilibrium. To have concrete notations, we write down the models in the form of real loans and real government debts. That is, we let agents choose the real amount of loans $d_{i,t} = D_{i,t}/P_t$, and real government debts $b_{i,t} = B_{i,t}/P_t$, given their real returns

$$r_t^d = \frac{1 + (1-\eta^d)Q_t^D}{Q_{t-1}^D \pi_t},$$

and

$$r_t^b = \frac{1 + (1-\eta^b)Q_t^B}{Q_{t-1}^B \pi_t}.$$

A wholesale firm i maximizes the profits that it transfers to the representative households. It solves the following problem:

$$\max_{k_{i,t}, n_{i,t}, l_{i,t}, a_{i,t}, d_{i,t}} \mathbb{E}_t \sum_{s \geq t} \Lambda_{t,s} \pi_{i,s}, \tag{3.3}$$

$$\text{s.t.} \qquad d_{i,t} \leq a_{i,t}, \tag{3.4}$$

$$q_t\left(k_{i,t} - a_{i,t-1}\right) \leq l_{i,t}, \tag{3.5}$$

where

$$\pi_{i,t} = m_t y_{i,t} - w_t n_{i,t} + (1-\delta)k_{i,t} - q_t\left(k_{i,t} - a_{i,t-1}\right) - r_t^d d_{i,t-1} - \left(r_{i,t}^l - 1\right) l_{i,t} + d_{i,t} - a_{i,t}.$$

Here, m_t is the real price (in the units of final goods) at which a wholesale firm sells its products to retailers. $\Lambda_{t,s}$ is the stochastic discount factor of the household.

At the beginning of the period t, firm i chooses the amount of capital $k_{i,t}$ and labor $n_{i,t}$ used in production and the amount of credit $l_{i,t}$ it takes from the partner bank to finance additional capital acquisition. At the end of period t, firm i invests in capital stock $a_{i,t}$ used in the next period's production. It can take loans $d_{i,t}$ from its partner bank as long as the real value of the loans does not exceed the value of the capital stock.

The firms' first order conditions are

$$[\partial n_{i,t}] \qquad \frac{\theta m_t y_{i,t}}{n_{i,t}} = w_t, \tag{3.6}$$

$$[\partial k_{i,t}] \qquad \frac{\alpha m_t y_{i,t}}{k_{i,t}} + 1 - \delta = q_t\left(1 + \kappa_{i,t}\right), \tag{3.7}$$

$$[\partial l_{i,t}] \qquad r_{i,t}^l - 1 = \kappa_{i,t}, \tag{3.8}$$

$$[\partial d_{i,t}] \qquad \mathbb{E}_t \Lambda_{t,t+1} r_{t+1}^d = 1 - \nu_t, \tag{3.9}$$

$$[\partial a_{i,t}] \qquad \mathbb{E}_t \Lambda_{t,t+1} q_{t+1}\left(1 + \kappa_{i,t+1}\right) = 1 - \nu_t, \tag{3.10}$$

where ν_t and $\kappa_{i,t}$ are the Lagrangian multipliers associated with constraints (3.4) and (3.5).

It is clear from (3.9) and (3.10) that in the equilibrium, all firms choose the same amount of bank loans and capital stock at the end of each period. That is, $d_{i,t} = d_t$

and $a_{i,t} = a_t$ for all $i \in [0,1]$. This is because the idiosyncratic productivity shocks are i.i.d.

As a result, firms only differ in their productivity after the realization of the idiosyncratic shocks. We denote them by superscript $s \in \{H, L\}$. The low-productivity firms' constraint (3.5) does not bind as they are the sellers of capital ($k_t^L < a_{i,t-1}$). High-productivity firms' constraint (3.5) may bind if their partner bankers cannot provide enough credit. In that case, $r_t^{l,H} - 1 > 0$, which means that the firms are willing to pay a positive interest rate on the bank credit.

3.2.3 Bankers' and workers' problem

At time t, bankers and workers solve the following problem:

$$\max_{h_{j,t}, l_{i,t}, d_{i,t}, b_{i,t}, c_t} \quad \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\rho} - 1}{1-\rho} - \chi \frac{\int_0^1 h_{j,t}^{1+\epsilon} dj}{1+\epsilon} \right),$$

$$\text{s.t.} \qquad c_t + \int_0^1 (d_{i,t} + b_{i,t})\, di = \int_0^1 \left[r_t^d d_{i,t-1} + r_t^b b_{i,t-1} + (r_{i,t}^l - 1) l_{i,t} \right] di + \int_0^1 (1-\tau_t)\, w_t h_{j,t} dj + \int_0^1 \pi_{i,t} di + \int_0^1 \pi_{s,t}^r ds \tag{3.11}$$

$$l_{i,t} \leq \xi \omega^d r_t^d d_{i,t-1} + \omega^b r_t^b b_{i,t-1}, \quad \forall i \in [0,1]. \tag{3.12}$$

Be reminded that bankers and workers within a household share consumption risks perfectly. Therefore, there is one single budget constraint (3.11) for each household.

At the beginning of period t, worker j supplies $h_{j,t}$ units of labor in the competitive labor market (in equilibrium $h_{j,t} = h_t$ for all j); banker i extends credit $l_{i,t}$ to his partner firm and earns interest $r_{i,t}^l$. Banker i's first order condition is

$$r_{i,t}^l - 1 = \mu_{i,t},$$

where $\mu_{i,t}$ is Lagrangian multiplier on constraint (3.12). In equilibrium, banks that have relationship with a high-productivity firm may face a binding constraint ($\mu_t^H > 0$). In that case, the bank earns a positive interest on the credit it extends ($r_t^{l,H} > 0$).

At the end of period t, banker i invests in government debts $b_{i,t}$ and firm loans $d_{i,t}$. The first order conditions are

$$[\partial b_{i,t}] \qquad \mathbb{E}_t \Lambda_{t,t+1} r_{t+1}^b \left(1 + \omega^b \mu_{i,t+1}\right) = 1; \tag{3.13}$$

$$[\partial d_{i,t}] \qquad \mathbb{E}_t \Lambda_{t,t+1} r_{t+1}^d \left(1 + \xi \omega^d \mu_{i,t+1}\right) = 1. \tag{3.14}$$

In the steady state, if the collateral constraint (3.12) strictly binds for high types ($\bar{\mu}^H > 0$), government debts and firm loans both have liquidity values. Therefore, they are priced at a premium compared to an otherwise identical asset without any liquidity value. In other words, $\bar{r}^b < 1/\beta$ and $\bar{r}^d < 1/\beta$.

In this case, from the partner wholesale firms' point of view, bank loans are a cheaper way of financing capital investment than using the households' money. Consequently, they strictly prefer loan financing. That is, $\bar{d} = \bar{a}$.[3]

3.2.4 Retailers' problem

A retail firm s sets price $\{P_{s,\tau}\}_{\tau \geq t}$ of its products to maximize the expected discounted sum of real profits that it rebates to the household. In particular, the firm solves the following problem:

$$\max_{P_{s,\tau}} \quad \mathbb{E}_t \sum_{\tau \geq t} \Lambda_{t,\tau} \left[\frac{P_{s,\tau}}{P_\tau} y_{s,\tau} - m_\tau y_{s,\tau} - \frac{\psi}{2} \left(\frac{P_{s,\tau}}{P_{s,\tau-1}} - 1 \right)^2 \right],$$

[3]Empirically, there is a number of studies showing that equity premium partly represents a illiquidity premium (e.g., Amihud, 2002).

subject to the demand function for good s

$$y_{s,t} = y_t \left(\frac{P_{s,t}}{P_t} \right)^{-\nu},$$

which is a CES demand function with the elasticity of substitution ν. We focus on a symmetric equilibrium where each retail firm s sets the same price $P_{s,t}$ and $P_{s,t} = P_t$ for all s. The optimality condition of the retail firms takes the form of the New Keynesian (NK) Phillips curve.

3.2.5 The government

The government is a union of fiscal and monetary authorities. The fiscal authority sets the tax rate τ_t on labor income and issues new debts with a nominal amount of B_t. The monetary authority decides upon the long-term nominal interest rate $\frac{1+(1-\eta^b)Q_t^B}{Q_{t-1}^B}$.[4] The following consolidated government budget constraint must hold:

$$\tau_t w_t h_t + b_t = r_t^b b_{t-1} + g_t. \tag{3.15}$$

3.2.6 Competitive equilibrium and optimal policy

We now define the competitive equilibrium, taking government policies as given.

Definition 2. *Given initial conditions* a_{-1}, $\left[1 + \left(1 - \eta^d\right) Q_{t-1}^B\right] B_{-1}$, *and* $\left[1 + \left(1 - \eta^d\right) Q_{t-1}^D\right] D_{-1}$, *a* ***competitive equilibrium*** *is a set of allocation* $\{k_t^s, n_t^s, h_t, c_t, a_t, d_t, b_t\}_{t=0}^{\infty}$, *prices* $\{q_t, w_t, m_t, P_t, r_t^{l,s}\}_{t=0}^{\infty}$, *real asset returns* $\{r_t^d, r_t^b\}_{t=0}^{\infty}$ *generated by* $\{Q_t^D, Q_t^B, P_t\}_{t=0}^{\infty}$, *and fiscal and monetary policies* $\{\tau_t, Q_t^B\}_{t=0}^{\infty}$ *satisfying the (consolidated) government budget constraint* (3.15), *such that*

[4] We are essentially assuming that the monetary authority decides the short-term nominal interest rate by supplying zero short-term debt.

1. *Given* $\{q_t, w_t, r_t^{l,s}, m_t\}_{t=0}^{\infty}$ *and real returns* $\{r_t^d\}_{t=0}^{\infty}$*, wholesale firms choose capital and labor demand* $\{k_t^s, n_t^s\}_{t=0}^{\infty}$*, demand for credit* $\{l_t^s\}_{t=0}^{\infty}$*, capital investment* $\{a_t\}_{t=0}^{\infty}$*, and demand for loans* $\{d_t\}_{t=0}^{\infty}$*.*

2. *Given* $\{r_t^{l,s}, r_t^d, r_t^b\}_{t=0}^{\infty}$*, bankers choose supply of credit* l_t^s*, the supply of firm loans* $\{d_t\}_{t=0}^{\infty}$*, and holdings of government debts* $\{b_t\}_{t=0}^{\infty}$*.*

3. *Given* $\{w_t, \tau_t\}_{t=0}^{\infty}$*, workers choose labor supply* $\{h_t\}_{t=0}^{\infty}$*.*

4. *Given* $\{m_t, P_t\}_{t=0}^{\infty}$*, retailers choose their individual prices.*

5. *The markets for labor, capital, firm loans, bank credits, and government debts all clear.*

To characterize the aggregate economy, we only need to know the aggregate state variables as well as the allocation of capital stock between the two types of firms in each period. Let $x_t \equiv \frac{k_t^H}{a_{t-1}}$ denote the capital used by the high-productivity firms as a fraction of the aggregate capital stock. Then following the proof in Section 2.8.1, we have

$$y_t = \Gamma(x_t) a_{t-1}^{\alpha} h_t^{\theta},$$

where

$$\Gamma(x) = \left[\sigma z^{H\frac{1}{1-\theta}} x^{\frac{\alpha}{1-\theta}} + (1-\sigma) z^{L\frac{1}{1-\theta}} \left(\frac{1-\sigma x}{1-\sigma} \right)^{\frac{\alpha}{1-\theta}} \right]^{1-\theta}.$$

The optimal fiscal and monetary policy is the process $\{\tau_t, Q_t^B\}_{t=0}^{\infty}$ associated with the competitive equilibrium that yields the highest social welfare:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\rho}}{1-\rho} - \chi \frac{h_t^{1+\epsilon}}{1+\epsilon} \right).$$

We derive the formal Ramsey problem in Appendix 3.5.1. To solve the problem, we approximate the model economy around the non-stochastic steady state where the collateral constraint strictly binds. When solving the model, we assume that the collateral constraint always binds around the steady state.

3.3 Numerical analysis

3.3.1 Parameters

We calibrate the model to quarterly frequency. For parameters regarding household preferences, production technology, government expenditures, and price stickiness, we take the calibration from the previous chapter.

There are three parameters regarding the financial constraints: the fraction of firms loans held by bankers ω^d, the fraction of government debt held by bankers ω^b and the fraction of pledgeable firm loans ξ. We set $\omega^d = 1$ so that all firm loans are held by bankers. We calibrate ω^b to the fraction of treasury securities held by financial intermediaries that provide liquidity values. We calculate this number from the Flow of Funds Accounts, which is 20.2% in 2007Q3.[5] Finally, we calibrate ξ such that in the steady state of the optimal policy equilibrium, the overall debt-to-GDP ratio matches the data, which is 61% of GDP.[6]

Using these parameter values, we can compute the steady state of the optimal policy equilibrium. In this steady state, bankers' holding of government debts amounts to 5.8% of the overall assets. We compare this number with the data counterpart we document in the U.S. commercial bank call reports. In the call reports, banks hold an average 2.7% of their total assets as treasury securities from 1995–2009. Therefore, we have still overstated banks' holding of government debts, though the number we get is much closer to the data than the previous chapter.

[5]The remaining treasury securities are held by foreigners, households, the non-financial business sector, monetary authorities, etc.

[6]As ξ increases, more firm loans are pledgeable and the collateral constraint is more relaxed, the government issues fewer debts in the optimal policy. The opposite is true as ξ decreases. As a result, we are able to calibrate ξ to match the debt-to-GDP ratio.

Table 3.1: Parameters

	Parameters	Value	Target/Source
Fraction of firm loans held by bankers	ω^d	1.000	
Fraction of government debts held by bankers	ω^b	0.202	Flow of Funds Accounts
Pledgeable fraction of loans	ξ	0.790	Steady-state government debt to GDP ratio 61%

3.3.2 Results in the flexible-price environment

In order to illustrate the role played by the collateral constraints, we first study an flexible-price environment. In this exercise, we assume that firm loans and government debts both have a maturity of one quarter ($\eta^d = 1$ and $\eta^b = 1$). When prices are flexible, whether contracts have short or long maturities is not important.

To show the contribution of taxes and inflation to the financing of the increases in government spending, we again perform the following fiscal financing decomposition using the inter-temporal government budget constraint:

$$\underbrace{\sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{g}}{\bar{b}} \tilde{g}_s}_{\text{government consumption}} = \underbrace{\sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{T}}{\bar{b}} \tilde{T}_s}_{\text{tax revenue}} + \underbrace{\frac{1}{\bar{\pi}} \hat{\pi}_t}_{\text{inflation}} - \underbrace{\sum_{s=t+1}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \hat{r}_s^b}_{\text{real interest rate}}.$$

Table 3.2 compares the results in our model to those in an otherwise identical model without any financial frictions.

Table 3.2: Decomposition of fiscal financing (as a fraction of total increase in g)

	Frictionless model	Friction model
Tax revenue	0.8%	61.3%
State-contingent inflation	111.1%	52.4%
Real interest rate	-11.9%	-13.7%

In the frictionless economy, inflation is solely a lump-sum transfer from the households to the government from an ex post point of view. Therefore, the government should mainly use inflation to respond to fiscal shocks. Indeed, inflation finances more than 100% of the present value of the increase in government expenditure.[7]

In our model, with financial frictions and the liquidity value of government debts and loans, inflation is costly because it reduces the real value of liquid assets (government debts and a fraction of firm loans). This in turn reduces the credits that banks extend to productive firms and firms' ability to acquire additional capital and increase production. In the optimal policy, inflation only finances 52.4% of higher government expenditure, while tax revenues account for 61.3%. These results are similar in magnitude to those in Chapter 2.

Not surprisingly, the inflation process is much less volatile in our model compared to the frictionless economy, as shown in Table 3.3. In the frictionless economy, government responds to fiscal shocks almost entirely through inflation, and the tax rate remains essentially constant. In our model, the standard deviation of inflation is reduced by about half relative to the frictionless model, and the labor tax rate becomes more volatile. Therefore, our model provides a rationale that the inflation rate has small volatility in reality.[8]

Table 3.3: Standard deviation of tax rate and inflation (quarterly)

	Frictionless model	Friction model
Inflation	0.90%	0.43%
Tax rate	0.03%	0.43%

The importance of nominal loans. To illustrate the role played by nominal loans in the quantitative results, we compare our model with an otherwise identical model with inflation-indexed loans. That is, loans are in real terms in this alternative model, and the only nominal asset is the government debt. Table 3.4 presents the results.

[7]This is because of the negative contribution of the real interest rate. After a negative government spending shock hits, consumption drops and grows back to the steady state. Therefore, the real interest rate is higher along this path.

[8]See Figure 3.2 in the Appendix 3.5.2 for the impulse response functions.

When firm loans are indexed to price inflation, inflation now finances 106.1% of the total increase in the fiscal expenditure, which is quite similar to the number in the model without any financial frictions (column 1 in Table 3.2). The intuition is straightforward: when loans are indexed to inflation, unexpected inflation only impacts the real value of the government bonds on the bank balance sheets, which is only 5.8% of total bank assets. Therefore, the cost of inflation to the banks becomes much smaller. As a result, the government optimally uses more inflation to finance higher spending.

Table 3.4: Decomposition of fiscal financing (as a fraction of total increase in g)

	Real-loan model	Friction model
Tax revenue	6.1%	61.3%
State-contingent inflation	106.1%	52.4%
Real interest rate	-12.3%	-13.7%

3.3.3 Sticky prices and long-term contracts

We now investigate a more realistic sticky-price environment and explore the role played by long-term debts. We set parameters η^d and η^b such that the average durations of firm loans and government debts match the durations we have documented from the U.S. call reports. In the 2007Q3 call reports, the average maturity of loans, leases and MBSs on bank balance sheets is 5.4 years, and the average maturity of treasury securities is 6.8 years. The calibrated values of η^d and η^b are 0.049 and 0.031, respectively.[9]

[9] The steady-state duration of an asset that pays a coupon stream $\{1, 1-\eta, (1-\eta)^2, ...\}$ is given by

$$D = \frac{1+\bar{r}}{\eta+\bar{r}},$$

where $\bar{r}$ is the steady-state interest rate of the asset.

We perform the fiscal financing decomposition again. When government debts have long maturity, the decomposition becomes

$$\underbrace{\sum_{s=t}^{\infty}\frac{1}{(\bar{r}^b)^{s-t+1}}\frac{\bar{g}}{\bar{b}}\tilde{g}_s}_{\text{government consumption}} = \underbrace{\sum_{s=t}^{\infty}\frac{1}{(\bar{r}^b)^{s-t+1}}\frac{\bar{T}}{\bar{b}}\tilde{T}_s}_{\text{tax revenue}} + \underbrace{\frac{1}{\bar{\pi}}\hat{\pi}_t}_{\text{current inflation}} + \underbrace{\sum_{s=t+1}^{\infty}\frac{(1-\eta^b)^{s-t}}{(\bar{r}^b)^{s-t}\,\bar{\pi}}\hat{\pi}_s}_{\text{future inflation}}$$
$$\underbrace{-\sum_{s=t+1}^{\infty}\frac{1-(1-\eta^b)^{s-t}}{(\bar{r}^b)^{s-t+1}}\hat{r}^b_s}_{\text{real interest rate}}.$$

Price stickiness is an additional reason why the government finds it undesirable to raise inflation. The first two columns of Table 3.5 compare the current economy with the flexible-price economy. Price stickiness reduces the contribution of inflation from 52.4% to 21%.

Besides, long-term inflation is quantitatively more important than inflation in the initial period. When contracts have long durations, inflation after the initial period also reduces the real value of debts by reducing the real value of repayments in those periods. When prices are sticky, the government wants to smooth out inflation to attenuate the quadratic cost associated with inflation. Therefore, the response of inflation to fiscal shocks is persistent.[10] In the sticky-price economy, the total contribution of current and future inflation amounts to 21%, and future inflation is more than five times as important as current inflation (17.7% vs 3.4%).

Table 3.5: Decomposition of fiscal financing (as a fraction of total increase in g)

	Flexible price	Sticky price long-term contracts	Sticky price 1-quarter contracts
Tax revenue	61.3%	86.8%	109.4%
Current inflation	52.4%	3.4%	4.9%
Future inflation	0.0%	17.6%	0.0%
Real interest rate	-13.7%	-7.7%	-14.3%

[10]See Figure 3.3 in Appendix 3.5.2 for the impulse response functions.

Compared to short-term debts, long-term debts makes it much less costly for the government to inflate away the real value of debt. To illustrate the importance of the long-term contracts, column 3 of Table 3.5 presents the fiscal financing decomposition in a sticky-price economy assuming all nominal contracts have only one-quarter maturity. In this case, the government can only use inflation in the initial period to reduce the real value of the government debts. The contribution of inflation is now only 4.9%. We also show in Figure 3.3 in Appendix 3.5.2 that the dynamics of real allocations in the long-term debt economy are much closer to the flexible-price economy than allocations in the short-term debt economy are.

We can also separate the effects of long-term government debts and long-term firm loans. In column 2 of Table 3.6, we assume that firm loans have an maturity of one quarter ($\eta^d = 1$), whereas government debts still have long maturity. In this economy, the contribution of inflation increases to 28.7%. The intuition is straightforward. In this economy, persistent inflation facilitates the reduction in the government debt value but has little impact on the value of firm loans because of their short maturity. Therefore, persistent inflation has smaller impacts on the bank balance sheets. Consequently, the government resorts to more inflation.

In column 3, we assume that government debts have a maturity of one quarter ($\eta^b = 1$), whereas firm loans still have long maturity. In this case, inflation plays a very small role of 4.8%. This is because inflation now has little impact on the government debt value and fiscal burden, but it would reduce the real value of loans and tighten the bank collateral constraints. As a result, the government is reluctant to raise inflation.[11]

[11]Note that column 3 in Table 3.6 is also very similar to column 3 in Table 3.5, where both government debts and firm loans have short maturities. The logic is that in both economies the government can only use inflation in the initial period to reduce the real government debt value. Inflation in the initial period has the same effect on the real value of firm loans whether they have short or long maturities.

Table 3.6: Decomposition of fiscal financing (as a fraction of total increase in g)

	Long-term debt and loans	Only long-term government debts	Only long-term firm loans
Tax revenue	86.8%	78.8%	109.4%
Current inflation	3.4%	3.5%	4.8%
Future inflation	17.6%	25.2%	0.0%
Real interest rate	-7.7%	-7.5%	-14.2%

3.4 Conclusion

This chapter extends the stylized model in Chapter 2, motivated by two empirical facts documented in Chapter 1: (1) the majority of the assets on bank balance sheets are long-term nominal loans to business and household sectors; (2) an unexpected and persistent increase in the inflation rate causes bank balance-sheet losses mainly through these loans. These facts imply that bank balance-sheet losses constitute a cost of inflation that a government needs to take into consideration when deciding monetary policy.

We propose a model that captures these costs of inflation and study Ramsey optimal fiscal and monetary policy in response to fiscal expenditure shocks. We find that the contribution of inflation in the financing of higher government expenditures in our model is only about a half of that in an economy without financial frictions. We also show that nominal loans play an important quantitative role. If these loans were real, the costs of inflation to banks would be much smaller, and the government would generate much higher inflation in response to fiscal shocks. In a sticky-price setting, we find that the maturities of firm loans and government debts significantly impact the role of inflation in the optimal policy.

3.5 Appendix

3.5.1 The Ramsey problem

In this subsection we derive the Ramsey problem. The Ramsey problem is

$$\max_{c_t,h_t,a_t,x_t,b_t,m_t,Q_t^B,Q_t^D,\pi_t} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\rho}-1}{1-\rho} - \chi \frac{h_t^{1+\epsilon}}{1+\epsilon} \right),$$

$$\text{s.t.} \quad \xi\omega^d r_t^d a_{t-1} + \omega^d r_t^b b_{t-1} = \boldsymbol{q}_t x_t a_{t-1} - \boldsymbol{q}_t a_{t-1}, \tag{3.16}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \left[\boldsymbol{q}_{t+1} \left(1 + \sigma \boldsymbol{\mu}_{t+1}^{\mathbf{H}}\right) - r_{t+1}^d \right] = 0, \tag{3.17}$$

$$\mathbb{E}_t \Lambda_{t,t+1} r_{t+1}^b \left(1 + \omega^b \sigma \boldsymbol{\mu}_{t+1}^{\mathbf{H}}\right) = 1, \tag{3.18}$$

$$\mathbb{E}_t \Lambda_{t,t+1} r_{t+1}^d \left(1 + \xi\omega^d \sigma \boldsymbol{\mu}_{t+1}^{\mathbf{H}}\right) = 1, \tag{3.19}$$

$$\theta m_t y_t + \frac{U_{h,t}}{U_{c,t}} h_t + b_t - r_t^b b_{t-1} - g_t = 0, \tag{3.20}$$

$$c_t + g_t + a_t + \frac{\psi}{2}\left(\pi_t - 1\right)^2 = (1-\delta) a_{t-1} + y_t, \tag{3.21}$$

$$\left(\nu m_t - \nu + 1\right) y_t - \psi \left(\pi_t - 1\right) \pi_t + \beta\psi \mathbb{E}_t \Lambda_{t,t+1} \left(\pi_{t+1} - 1\right) \pi_{t+1} = 0, \tag{3.22}$$

where

$$\boldsymbol{q}_t = 1 - \delta + \alpha m_t \Gamma(x_t) a_{t-1}^{\alpha-1} h_t^{\theta} \left[z^L \left(\frac{1-\sigma x_t}{1-\sigma} \right)^{\alpha+\theta-1} \Gamma(x_t)^{-1} \right]^{\frac{1}{1-\theta}}, \tag{3.23}$$

$$\boldsymbol{\mu}_t^{\mathbf{H}} = \frac{1}{\sigma} m_t \Gamma'\left(x_t\right) a_{t-1}^{\alpha-1} h_t^{\theta}, \tag{3.24}$$

$$r_t^b = \frac{1 + (1-\eta^b) Q_t^B}{Q_{t-1}^B \pi_t}, \tag{3.25}$$

$$r_t^d = \frac{1 + (1-\eta^d) Q_t^D}{Q_{t-1}^D \pi_t}. \tag{3.26}$$

The constraints (3.16)-(3.22) are the collateral constraints, firms' inter-temporal condition, banks' inter-temporal condition of government debts, banks' inter-temporal condition of firm loans, government budget constraints, social resource constraints and the Phillips curve, respectively.

We now derive constraints (3.16)-(3.22). Equation (3.16) is derived from the collateral constraints of firms (3.5) and banks (3.12), with the fact that $d_t = a_t$ and $k_t^H = x_t a_{t-1}$. Constraints (3.17)-(3.18) follow directly the inter-temporal first-order condition of firms (3.9)-(3.10) and banks (3.13)-(3.14). Equation (3.20) is derived from the government budget constraint (3.15) using the fact that total labor income is $w_t h_t = \theta m_t y_t$ and the labor supply condition is

$$(1 - \tau_t) w_t = -\frac{U_{h,t}}{U_{c,t}}.$$

The expressions for $\boldsymbol{q}_t$ and $\boldsymbol{\mu}_t^{\mathbf{H}}$ follow the firm first-order conditions for capital (3.7) and the fact that $k_t^H = x_t a_{t-1}$.

3.5.2 Additional figures

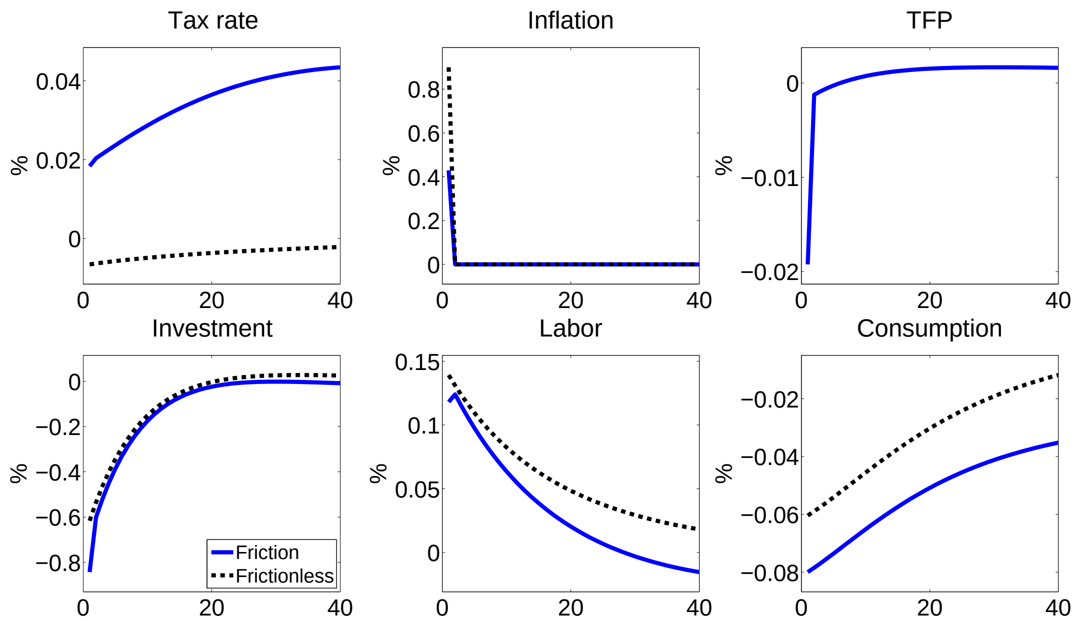

Figure 3.2: Comparing optimal policy responses in our model with those in a frictionless model. The shock is a one standard-deviation fiscal shock of 1.53% with the first-order autocorrelation coefficient of 0.89.

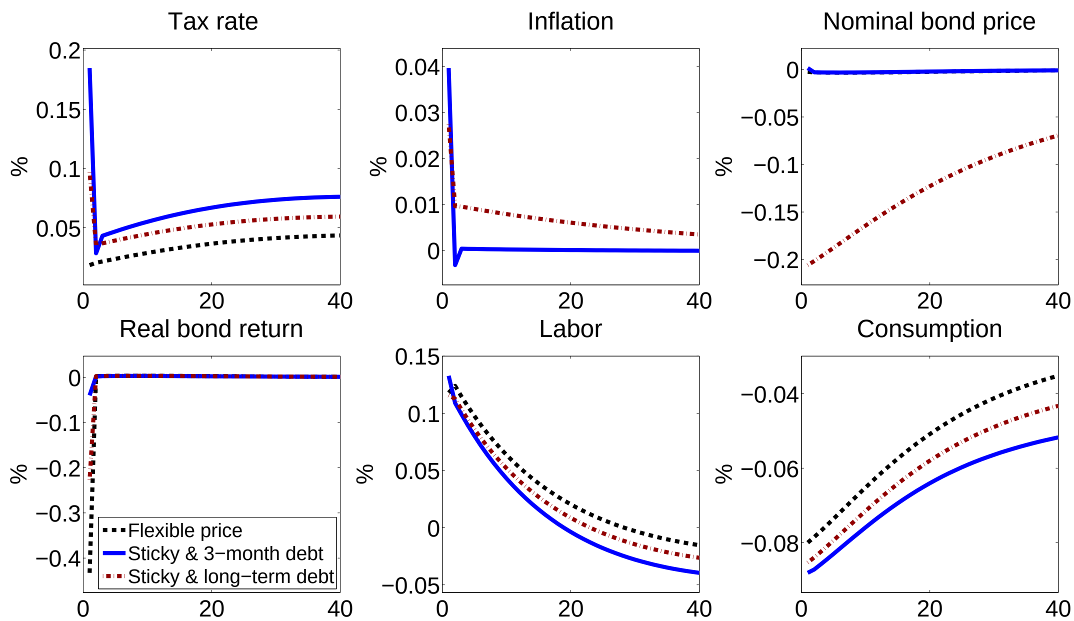

Figure 3.3: Comparing optimal policy responses in models that differ in price stickiness and maturities of contracts. The shock is a one standard-deviation fiscal shock of 1.53% with the first-order autocorrelation coefficient of 0.89.

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